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HYBRID KALMAN FILTERING ALGORITHM WITH WAVELET PACKET DATA PROCESSING FOR LINEAR DYNAMICAL SYSTEMS

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Abstract

The paper develops a hybrid algorithm for predicting a linear dynamic system based on a combination of an adaptive Kalman filter with preprocessing using a wavelet packet analysis of the initial data of the background of the system under study.

Being based on Fourier analysis, wavelet analysis and wavelet packet analysis are quite acceptable for time-frequency analysis of a signal, but they cannot be performed recursively and in real time and, therefore, cannot be used for dynamic analysis of random processes. In combination with the Kalman filter, a combination of the characteristics of the multiple-resolution wavelet transform and the recurrent formulas of the Kalman filter in real time is obtained.

Since the original signal is usually given in the form of discrete measurements, to implement their convolution used in the Kalman filter, it is necessary to use cyclic convolutions with periodic continuation of the signal for any time interval. In the case of different values of the original signal at the ends of the considered time interval $[0,T]$, the periodized signal can have large values and sharp different amplitude at the ends of the periodization interval.

To smooth out the values of the periodized signal at the ends of the periodization interval, a cascade decomposition and recovery algorithm was used using Dobshy boundary wavelets with a finite number of moments. Signal recovery is performed in a series of operations comparable to the duration of the time interval under consideration.

The smoothed signal obtained in this way is used as a Kalman filter platform for predicting the dynamic system under study.

Taking into account that the correlation functions of the noise in the observation equation and the phase state of the system are usually unknown, the adaptation of the Kalman filter to these noises (interference) is carried out on the basis of a zeroing sequence. The manuscript does not contain related data.

Keywords: hybrid algorithm, wavelet packet analysis, Kalman filter, dynamic system.

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1. Introduction

With the usual orthogonal multiple-scale (multi-resolution) wavelet decomposition (wavelet analysis) of the signal [1], the approximating coefficients are decomposed into approximating and detailing coefficients of a lower level, and then this procedure is applied at a subsequent level to the obtained approximating coefficients at the previous level. At the same time, the detailing coefficients are not considered further [2].

In contrast to the wavelet analysis, in the wavelet packet analysis [3], the same decomposition method also decomposes the detailing coefficients. This approach provides a richer analysis based on a binary coefficient tree starting from the original signal [4, 5].

With the help of wavelet packet analysis, elementary measurements can be divided into two components: the «trend» of measurements, which characterizes the main (correct) course of the process, and the «fluctuation» of measurements generated by high-frequency noise (interference) of measurements.

Wavelet packet analysis (WPA), as a well-known powerful method, is intensively used in the study of propagation processes of various signals [6–10].

Being based on Fourier analysis, wavelet and wavelet packet analysis are well acceptable for time frequency analysis of a signal. However, a multiscale analysis based on the decomposition of wavelet coefficients is carried out over a certain period of time and cannot be performed recursively and in real time (real-time). Therefore, by itself it cannot be used for dynamic analysis of random processes [11]. But in combination with the Kalman filter [12, 13], a combination of the multi-resolving characteristics of the wavelet transform and the recursive characteristics of the Kalman filter in real time is obtained. This combination is successfully used for the analysis and evaluation of a random process and dynamic systems in various fields of science and technology [14, 15].

The aim of this study is to develop an algorithm for the numerical implementation of a combination of wavelet-packet analysis of discrete signals given by time series of observations over a finite period of time with a Kalman filter for linear dynamical systems.

2. Materials and methods

Problem statement. Following [11], let's consider a linear dynamical system:

$$\left.\begin{aligned} v_{k+1} &= A_k v_k + B_k u_k + \Gamma_k \xi_k \\ w_k &= C_k v_k + D_k u_k + \eta_k \end{aligned}\right\}, \tag{1}$$

where A_k, B_k, Γ_k, C_k, D_k, u_k are the given functions of the time parameter k.

Let's assume that the phase state of the system is one-dimensional $\{\xi_k\}$ and $\{\eta_k\}$ are sequences of white Gaussian noises (i.e. noises with independent normally distributed values) and the following characteristics are given:

$$E[x_0] = \overline{x_0}, D[x_0] = P_0, E[\xi_k] = \xi_k, \operatorname{cov}[\xi_k \xi_j] = Q_k \delta_{kj},$$

$$\operatorname{cov}[\xi_k, x(0)] = 0, E[\eta_k] = \overline{\eta_k}, \operatorname{cov}[\eta_k, \eta_j] = R_k \delta_{kj}, \operatorname{cov}[\eta_k, x(0)] = 0,$$

where E and D are the signs of mathematical expectation and variance, respectively; δ_{kj} is the Kronecker symbol.

Because of the presence in the system as deterministic quantities stochastic variables $\{\xi_k\}$ and $\{\eta_k\}$ (1) is called a linear deterministic stochastic system. Let's decompose this system into the sum of a linear deterministic system:

$$\left.\begin{aligned} z_{k+1} &= A_k z_k + B_k u_k, \\ s_k &= C_k z_k + D_k u_k, \end{aligned}\right\}, \tag{2}$$

and linear (purely) stochastic system:

$$\left.\begin{aligned} x_{k+1} &= A_k x_k + \Gamma_k \xi_k, \\ y_k &= C_k x_k + \eta_k, \end{aligned}\right\}, \tag{3}$$

where $w_k = s_k + y_k$ and $u_k = z_k + x_k$. The advantage of this decomposition is that the solution of the linear deterministic system (2) is well known and is represented by the so-called *transfer equation*:

$$z_k = (A_{k-1} - A_0) z_0 + \sum_{i=1}^{k} (A_{k-1} - A_{i-1}) \cdot B_{i-1} u_{i-1}. \tag{4}$$

Therefore, it is sufficient to describe the optimal estimate of $\hat{x}_k$ for x_k from the stochastic system (5). Then the estimate of $\hat{u}_k$ for u_k will be written as:

$$\hat{u}_k = z_k + \hat{x}_k. \qquad (5)$$

According to [11], the optimal estimate $\hat{x}_{k/j}$ of the value x_k, taking into account the one-dimensionality of the phase state of the system, is determined by the least squares method and is written as:

$$\hat{x}_k = \left(C_k R_k\right)^{-3}\left(y_k - D_k u_k\right). \qquad (6)$$

The estimate (6) is denoted by $\hat{x}_{k/j}$ for the given values $y_0, y_1, \ldots, y_j$. By definition [11]:

1) when $j = k\hat{x}_k = x_{k/k}$ and the evaluation process is called the digital filtering process;

2) when $j < k\hat{x}_{k/j}$ is called the forecast for x_k, and the evaluation process is called the digital prediction process;

3) when $j > k\hat{x}_{k/j}$ is called smoothing estimation, and the process of obtaining it is called the process of digital smoothing.

Since the values of R_k are usually unknown in practice, let's propose (section 2) to construct an estimate of $\hat{x}_k$ using a wavelet packet analysis that smooths out the possible «outliers» of the signal, $x[n]$, given at $n\in 0, 1, \ldots, N$–1.

The Kalman algorithm for system (3) is described by recurrent relations:

$$\begin{cases} \hat{x}_k(+) = \hat{x}_k(-) + K_k\left[y_k - C_k x_k(-)\right],\ \hat{x}_0(+) = \bar{x}_0, \\ \hat{x}_k(-) = A_{k-1}\hat{x}_{k-1}(+) + B_{k-1}u_{k-1}, \end{cases} \qquad (7)$$

and the gain function K_k is calculated from the relations:

$$\begin{cases} P_k(-) = A^2_{(k-1)}P_{(k-1)}(+) + Q_{(k-1)}, \\ K_k = P_k(-)C_k[C_k^2 \cdot P_k(-) + R(k)]^{-1}, \\ P_k(+) = P_k(-) - K_k C_k P_k(-),\ P_0(-) = P_0, \end{cases} \qquad (8)$$

(7), (8) with substitutions $\hat{x}_k(-) = x^*(k), \hat{x}_k(+) = \hat{x}_k, \hat{x}_{k-1}(+) = \hat{x}_k(-)$ and $P_k(-) = P^*(k)$, $P_k(+) = \tilde{P}(k)$ coincide with the corresponding formulas from [12].

Covariance functions Q_k, R_k are usually unknown in practice. In this regard, in [16] a method of estimating Q_k, R_k by their a priori estimates was proposed (in our calculations, as in [17]) $Q_k \equiv R_k \equiv 10^{-5}$ was taken as a priori estimates based on the updating sequence:

$$v_k = y_k - \hat{y}_k(-), \qquad (9)$$

where $\hat{y}_k(-) = C_k\hat{x}_k(-)$ In this case, the estimates for R_k and Q_{k-1}, in (10) are calculated using the formulas:

$$\hat{R}_k = \hat{C}_{v_k} - C_k^2 \cdot P_k(-), Q_{k-1} = K_k^2 \cdot C_{v_k}, \qquad (10)$$

$$\hat{C}_{v_k} = \frac{1}{N}\sum_{j=j_0}^{k} v_j^2\ \left(j_0 = k - N_0 + 1\right).$$

Estimates $\hat{C}_{v_k}$ are calculated by averaging over a sliding time window of size N_0, $N_0 \ll N$ (N is the total number of observations y_k).

The result is an adaptive Kalman filter resulting in optimal estimates of x_k states that can be used to construct a one-step predictor:

$$x^{np}_{k+p} = A_k\hat{x}_k(+), \qquad (11)$$

and p-step predictor:

$$x_{k+p}^{np}=\left(\prod_{j=k}^{k+p-1}A_j\hat{x}_k(+)\right),\tag{12}$$

$$E\left[e_{k+1}\right]=A_k\left(y_k-x_k(+)\right),\ \sigma_{\varepsilon k+1}^2=P_k(-);$$

$$E\left[e_{k+p}\right]=\left(\prod_{j=k}^{k+p-1}A_j\right)\cdot E\left[k+1\right],\tag{13}$$

$$\sigma_{\varepsilon k+1}^2=\left(\prod_{j=k}^{k+p-1}A_j^2\right)P_k(-)+\hat{Q}_{k+p-1}+\sum_{j=k}^{k+p-2}\left(\prod_{i=j+1}^{k+p-1}A_j^2\right)Q_j.$$

The forecast based on these formulas is obtained under the assumption that the process model remains unchanged at the lead interval. The mean value and variance of the forecast error are calculated using the formulas [13].

2. Materials and methods

In wavelet packet analysis (WPA), the improvement of wavelet analysis (WA) is carried out by additional processing of the high-frequency component of the analyzed signal. As a result, a «complete» balanced tree is obtained, the branches of which correspond to a set of subspaces with bases constructed as for a one-sided tree according to WA. The functions and filters that generate these bases are called wavelet packets and packet filters, respectively.

Orthonormal bases of wavelet packets use so-called conjugate mirror filters, with the help of which the frequency axis is divided into separate intervals of various sizes. Let's consider a wavelet packet analysis based on a discrete wavelet transform (discrete wavelet trans form – DWT) for discrete signals $f[n]$, $n\in Z$ (Z is a set of integers, except 0) belonging to a real linear Hilbert space $I^2(Z)$ with a scalar product:

$$<f[n],g[n]>=\sum_{n=-\infty}^{+\infty}f[n]g[n]\tag{14}$$

with a finite norm:

$$<f,f>=\sum_{n=-\infty}^{+\infty}f^2[n]<\infty\tag{15}$$

determining the energy of the signal.

By definition [2], a discrete filter $h[n]$, whose Fourier transform, $\hat{h}[k]$ satisfies the conditions:

$$\left|\hat{h}(k)\right|^2+\left|\hat{h}[k+\pi]\right|^2=2,\ k\in Z,\tag{16}$$

$$\left|\hat{h}[0]\right|=2,\tag{17}$$

it is called a conjugate mirror filter. It is known [18] that if $\varphi[n]€I_2(Z)$ periodized with period N, where N is the length of the carrier $h[n]$, the Fourier transform $\hat{h}[k]$ satisfies conditions (16), (17) and, therefore, such a function $h[n]$ is a conjugate mirror filter.

In WPA, the space V_j of a multiple-scale approximation is decomposed into the sum of the space V_{j+1} of lower resolution $2^{-(j+1)}$ and the space of details W_{j+1} (complement V_{j+1} to V_j) by splitting the orthogonal basis $\{\varphi_j(t-2^j\cdot n)\}_{n\in z}$ of the spaces V_j on two new orthogonal bases $\{\varphi_{j+1}(t-2^j\cdot n)\}_{n\in z}$ of the space V_{j+1} and $\{\Psi_{j+1}(t-2^{j+1}\cdot n)\}_{n\in Z}$ of the spaces W_{j+1}. The expansion of functions $\varphi_{j+1,p}\in V_{j+1}$ and $\Psi_{j+1,p}\in W_{j+1}$ on the basis) $\varphi_{j,n}=\{\varphi_j(t-2^j\cdot n)\}_{n\in Z}$ space V_j:

$$\varphi_{j+1,p}=\sum_{n=-\infty}^{+\infty}h[n-2p]\cdot\varphi_{j,n},\ \psi_{j+1,p}=\sum_{n=-\infty}^{+\infty}g[n-2p]\cdot\varphi_{j,n}\tag{19}$$

defined by a pair of conjugate mirror filters $h[n]$ and $g[n]$, where $g[n]$ is related to $h[n]$ by equality:

$$g[n]=(-1)^{1-n}h[1-n]. \tag{20}$$

According to [3], for any space U_j with an orthonormal basis $\{\theta_j(t-2^j\cdot n)\}_{n\in Z}$, using a pair of conjugate mirror filters $h[n]$ and $g[n]$ connected by condition (20), it is possible to construct families:

$$\theta^0_{j+1}(t)=\sum_{n=-\infty}^{+\infty}h[n]\theta_j(t-2^j\cdot n),$$

$$\theta^1_{j+1}(t)=\sum_{n=-\infty}^{+\infty}g[n]\,\theta_j(t-2^j\cdot n), \tag{21}$$

combining which into a family:

$$\left\{\theta^0_{j+1}(t-2^{j+1}\cdot n),\ \theta^1_{j+1}(t-2^{j+1}\cdot n)\right\}_{n\in Z}, \tag{22}$$

there is an orthonormal basis of the space U_j.

In WPA, instead of splitting approximation spaces V_j to construct spaces W_{j+1} (as is done in WA), it is possible to put $U_j=W_j$ and split these spaces to obtain new bases (21). Continuing recursively splitting (21), let's obtain a binary tree whose root is the approximation space V_j, if the signal in question is approximated with a scale of 2^L. For a finite signal of length N, let's assume. $N^{-1}=2^L$ ($L<0$).

The V_L space admits an orthogonal basis of scaling functions $\{\varphi_L(t-2^j\cdot n)\}_{n\in Z}$ with:

$$\varphi_L(t)=2^{-L/2}\cdot\varphi(2^{-L}t).$$

Let's mark with indexes (j, p) any node of a binary tree, where $j-L\geq 0$ is the depth of the node on the tree and p is the number of nodes on the left at the same depth $j-L$. Each node (j, p) corresponds to a space, W_j^p which admits an orthonormal basis $\{\psi_j(t-2^j\cdot n)\}_{n\in Z}$ for all nodes lying on the path down the tree with a starting point from node (j, p). On at the root of the tree there is $W_L^0=V_L$.

Let $B=\{\psi_j^p(t-2^j\cdot n)\}_{n\in Z}$ $\{\psi_j(t-2^j\cdot n)\}_{n\in Z}$ be an orthonormal basis for, W_j^p constructed in node (j, p). The two wavelet packets of orthogonal bases generated by this node are determined by the relations (21):

$$\psi^{2p}_{j+1}(t)=\sum_{n=-\infty}^{+\infty}h[n]\psi^p_j(t-2^j\cdot n),$$

$$\psi^{2p+1}_{j+1}(t)=\sum_{n=-\infty}^{+\infty}g[n]\psi^p_j(t-2^j\cdot n). \tag{23}$$

Since the family $\left\{\psi_j(t-2^j\cdot n)\right\}_{n\in Z}$ is orthonormal, then:

$$h(n)=<\psi^{2p}_{j+1}(u)\psi^p_j(u-2n)>,\ g(n)=<\psi^{2p+1}_{j+1}(u),\psi^p_j(u-2n)>. \tag{24}$$

In accordance with (20) of the family:

$$B^{2p}_{j+1}=\left\{\psi^{2p}_{j+1}(t-2^{j+1}\cdot n)\right\}_{n\in Z},\ B^{2p+1}_{j+1}=\left\{\psi^{2p+1}_{j+1}(t-2^{j+1}\cdot n)\right\}_{n\in Z} \tag{25}$$

are orthonormal bases of the spaces W^{2p}_{j+1} and W^{2p+1}_{j+1} such that:

$$W^p_j=W^{2p}_{j+1}\oplus W^{2p+1}_{j+1}, \tag{26}$$

where $\oplus$ is the sign of the direct sum of spaces in the sense that $W^{2p}_{j+1}\cap W^{2p+1}_{j+1}=\{0\}$ and any function $f^p_j(t)\in W^p_j$ can only be represented as $f^p_j(t)=f^{2p}_{j+1}(t)+f^{2p+1}_{j+1}(t)$ with functions $f^{2p}_{j+1}(t)\in W^{2p}_{j+1}$ and $f^{2p+1}_{j+1}(t)\in W^{2p+1}_{j+1}$ {0} is the sign of an empty set.

Recursive splitting (26) defines a binary tree of wavelet packet spaces. Each parent node of this tree has two branches (two descendants) representing two orthogonal subspaces.

The most common input discrete signal $b[n]=f(N^{-1}\cdot n)$ is obtained using a finite resolution device that averages the input analog signal. If the sampling step is equal to N^{-1}, then to calculate the wavelet coefficients, it is necessary to associate with $b[n]$ the function V_L, approximated with a scale of $2^L=N^{-1}$ in order to calculate $a_L[n]=<f, \varphi_{L,n}>$, where $\varphi_{j,n}=\frac{1}{\sqrt{2j}}\varphi\left(\frac{t-n}{2j}\right)$ and the family $\{\varphi_{j,n}\}_{n\in Z}$ is an orthonormal basis of the space V_j for all $j\in Z$.

For this purpose, the relation is used in [2]:

$$b[n]=N^{-1/2}a_L[n]\approx f(N^{-1}\cdot n), \tag{27}$$

from where is determined:

$$a_L[n]=b[n]\cdot N^{-1/2}\approx f\left(N^{-1}\cdot n\right)/N^{1/2}, \tag{28}$$

where $N^{1/2}$ is the normalizing factor.

With a discrete signal at the input, $b[n]$, s1elected with a step $N^{-1}=2^L$ is associated, the $f\in V_L$ of which expansion coefficients $a_L[n]=<f, \varphi_{L,n}>$ satisfy the equality (28). For any node (j, p) of the wavelet packet tree, the coefficients of the wavelet packet are denoted as:

$$d_j^p[n]=<f(t),\psi_j^p\left(t-2^j\cdot n\right)>. \tag{29}$$

For the root of the tree, $d_L^0[n]=a_l[n]$ are calculated according to (28).

Denote by $\hat{x}$ the signal obtained by substituting zeros between each two consecutive counts of x (decimation ↑2), i.e. an incomplete sample is formed, determined by the function:

$$\hat{x}[n]\begin{cases}x[n], & n=2p,\\ 0, & n=2p+1.\end{cases} \tag{30}$$

Such an incomplete sample is used in decomposition, and in reconstruction, an incomplete sample obtained using decimation ↓2 is used, according to which all odd samples are removed from the signal $x[n]$. Then, when decomposing, the coefficients are calculated as:

$$\begin{gathered}d_{j+1}^{2p}[k]=d_j^p*\bar{h}[2k],\\ d_{j+1}^{2p+1}[k]=d_j^p*\bar{g}[2k]\end{gathered} \tag{31}$$

and when restoring (reconstruction) as:

$$d_j^p[k]=\hat{d}_{j+1}^{2p+1}*h[k]+\hat{d}_{j+1}^{2p+1}*g[k]. \tag{32}$$

Here * is the sign of a discrete convolution sample of two signals $f_1[n]$ and $f_2[n]$, defined by the formula:

$$f_3[n]=\sum_{p=-\infty}^{+\infty}f_1[n]\cdot f_2[n-p]=f_1*f_2[p]=f_2*f_1[p]=\sum_{p=-\infty}^{+\infty}f_2[n]\cdot f_1[n-p]. \tag{33}$$

Denoting by $\bar{x}[n]=x[-n]$ formulas (31) and (32) in expanded form, taking into account the definition of discrete convolution (32) can be written as:

$$d_{j+1}^{2p}[k]=\sum_{n=-\infty}^{+\infty}h[n-2k]d_j^p[n], d_{j+1}^{2p+1}[k]=\sum_{n=-\infty}^{+\infty}g[n-2k]d_j^p[n], \tag{34}$$

$$d_j^2[k]=\sum_{n=-\infty}^{+\infty} h[k-n]\hat{d}_j^{2p}[n]+\sum_{n=-\infty}^{+\infty} g[k-n]\hat{d}_j^{2p+1}[n]. \quad (35)$$

Iteration of equations (31) (or, which is still (34) along the branches of the wavelet packet tree calculates all the coefficients of the wavelet packet (**Fig. 1**, ***a*** [2]). And by the coefficients of the wavelet packet on the leaves of the internal nodes (j, p) of the tree, D_L^0 is restored at the top of the tree (**Fig. 1, *b*** [2]).

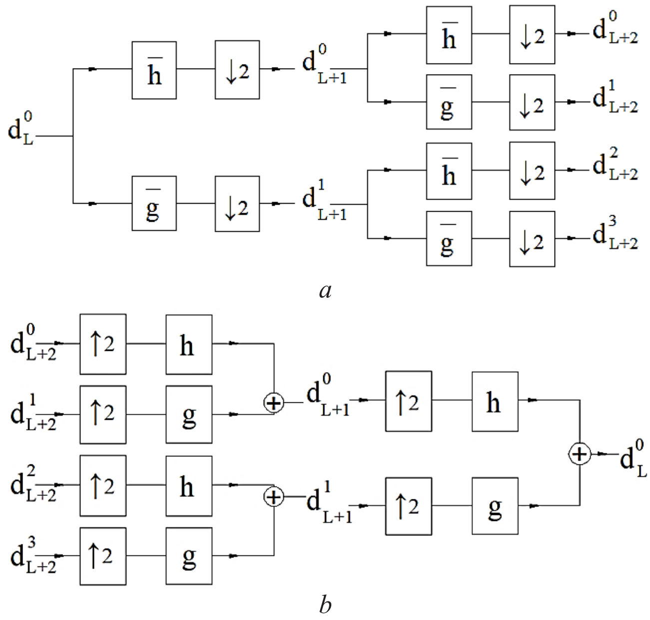

Fig. 1. A set of filters for the decomposition of a wavelet packet:
a – with subsequent filtering; *b* – recovery with substitution and filtering at the output.

In this case, the convolutions in (31)–(35) are replaced by cyclic convolutions if a_L is a finite signal of length $2^L = N$. Previously, the signals $f[n]$ $h[n]$ and $g[n]$ continue periodically with a period of N, according to the definition $\hat{f}[n]=f[n \bmod N]$, $\tilde{h}[n]=h[n \bmod N]$, $\tilde{g}[n]=g[n \bmod N]$, where $n \bmod N$ is the remainder when dividing n by N (for $n<N$, $n \bmod N=n$ is assumed).

The cyclic convolution of two signals $f^{nep}[n]$ and $f^{nep}[n]$ with period N (denoted by ⊙) is defined as the sum of their period:

$$f^{nep}\odot h^{nep}[n]=\sum_{p=0}^{N-1} f^{nep}[p]h^{nep}[n-p]=\sum_{p=0}^{N-1} f^{nep}[n-p]h^{nep}[p], \quad (36)$$

the signal received in this case, depending on n, also has a period of N.

As a filter $h[n]$, a Daubechies filter with a compact carrier of length M is usually used, then the values of the filter $g[n]$ for $n=k$ are determined by the formula:

$$g_k=(-1)^k h_{2M-1-k}. \quad (37)$$

In calculations using formulas (31), (32), it is easiest to use the Haar filter (a special case of the Daubechies filter for $M=1$):

$$h[n]=\begin{cases}1/\sqrt{2}, & \text{when } n=0.1,\\ 0, & \text{with others } n,\end{cases} \quad (38)$$

since the Haar filter h_H is the only conjugate mirror filter with a compact carrier, the transfer function (i.e. the Fourier transform $\hat{h}_H(u)$ of which has a linear complex phase [19].

3. The results

It should be borne in mind that the periodized signals in the case of $f(0)\neq f(1)$ may have large values and sharp different amplitude at the ends of $t=0$ and $t=1$ of the interval [0, 1]. For discrete signals given by counts at $n=0,1,\ldots,N–1$ these sharp transitions will take place at the boundaries $n=0$ and $n=N–1$ of the interval $[0, N–1]$. To avoid this, the boundary wavelets described in [2] are used.

Since for the combination of wavelet packet analysis with the Kalman filter, we are mainly interested in the behavior of the wavelet packet coefficients at point $n=N–1$, then as a platform for applying the Kalman filter, the reconstructed signal $a_L[n]$ obtained as a result of the wavelet packet analysis will be replaced at points $n=0$ and $N=1$ by the values the source signal $b[n]$. Based on the corrected signal $a[n]$ obtained in this way, let's calculate, for $n=N–1$, a boundary wavelet constructed on the basis of a Daubechies wavelet with a carrier of length $2p$. Let's shift the Daubechies scaling function so that its carrier is $[–p+1, p]$ and the Daubechies wavelet has p zero moments.

For $p=2$, let's use the boundary filters, $H_{k,e}^{np}$, $G_{k,e}^{np}$, $h_{k,m}^{np}$, $g_{k,m}^{np}$ the values of which are given in [2] and are given below in the form of **Table 1**.

Table 1

Left and right boundary coefficients for a Daubechies wavelet with $p=2$ zero moments

k	**l**	H_{k1}^{le}	G_{k1}^{le}	**k**	**m**	h_{k1}^{le}	g_{k1}^{le}
0	0	0.603332511	–0,796543616	0	2	–0.398312997	–0.258792248
0	1	0.690895531	0.546392714	1	2	0.850088102	0.227428117
1	0	0.037517460	0.010037224	1	3	0.223820357	–0.836602821
1	1	0.457327659	0.122351043	1	4	–0.129222743	0.483012921
k	*l*	H_{k1}^{np}	G_{k1}^{np}	*k*	*m*	h_{k1}^{np}	g_{k1}^{np}
–2	–2	0.190151418	–0.363906959	–2	–5	0.443149049	0.235575950
–2	–1	–0.194233407	0.371718966	–2	–4	0.767556669	0.401069519
–1	–2	0.434896998	0.801422962	–2	–3	0.374955331	–0.717579999
–2	–1	0.870508753	–0.257512919	–1	–3	0.230389043	–0.539822500
–	–	*h*[–1]	*h*[0]	–	–	*h*[1]	*h*[2]
–	–	0.482962913	0.8365303	–	–	0.224143868	–0.129409522

When $N^{-1}=2^L$, $L=-4$, it is necessary to restore the value of the signal a_4 [15]. In this case, it is possible to apply it in a truncated form (only for the right filters) for $N>p$, formulas for the wavelet coefficients $a_j[k]$ and $d_j[k]$, proposed by Cohen A., Daubechies I., Vial P. [20].

If $p\leq k<2^{-j}-p$,

$$a_j[k]=\sum_{l=-\infty}^{+\infty} h[l-2k]\,a_{j-1}[l],$$

$$d_j[k]=\sum_{l=-\infty}^{+\infty} g[l-2k]\,a_{j-1}[l]. \tag{39}$$

If $-p\leq k<0$,

$$a_j\left[2^{-j}+k\right]=\sum_{l=-p}^{-1} H_{k,l}^{np}\cdot a_{j-1}\left[2^{-j+1}+l\right]+\sum_{m=-p+2k+1}^{-p-1} h_{k,m}^{np}\cdot j,$$

$$d_j\left[2^{-j}+k\right]=\sum_{m=-p+2k+1}^{-p-1} G_{k,l}^{np}\cdot a_{j-1}\left[2^{-j+1}+l\right]+\sum_{m=-p+2k+1}^{-p-1} g_{k,m}^{np}\cdot a_{j-1}\left[2^{-j+1}+m\right]. \tag{40}$$

The cascade algorithm (39)–(40) decomposes $a_L[n]$ into a discrete wavelet transform $a_J[n]$, $\{d_j\}$ in $O(N)$ operations. The largest scale 2^J must satisfy the inequality $2^J\leq(2p)^{-1}$ since at all scales the number of boundary coefficients remains equal to $2p$.

Recovery formulas.

If $3p-1\leq l\leq 2^{-j+1}-3p$,

$$a_{j-1[l]}=\sum_{k=-\infty}^{+\infty} h[l-2k]a_j[k]+\sum_{k=-\infty}^{+\infty} g[l-2k]d_j[k].$$

If $-p-1 \geq l \geq -3p+1$,

$$a_{j-1}\left[2^{-j+1}+l\right]=\sum_{k=-p}^{(l+p-1)/2} h_{k,l}^{np} a_j\left[2^{-j}+k\right]+\sum_{k=-\infty}^{+\infty} h[l-2k]\, a_j\left[2^{-j}+k\right]+$$
$$+\sum_{k=-p}^{(l+p-1)} g_{k,l}^{np} d_j[2^{-j}+k]+\sum_{k=-\infty}^{+\infty} g[l-2k] d_j[2^{-js}+k]. \quad (42)$$

If $-1 \geq l \geq -p$,

$$a_{j-1}\left[2^{-j+1}+l\right]=\sum_{k=-p}^{-1} H_{k,l}^{np} a_j\left[2^{-j}+k\right]+\sum_{k=-p}^{-1} G_{k,l}^{np} d_j\left[2^{-j}+k\right]. \quad (43)$$

The original signal $a_L[n]$ is reconstructed from the orthogonal wavelet representation $a_j[n],\{d_j[n]\}_{[L\leq j\leq J]}$ by sequential recurrent use of equations (38)–(43). Recovery is performed in $O(N)$ operations.

In formulas (39)–(43) $h[n]$ and $g[n]$ are low-frequency and high-frequency Dob filters with the number of moments p. When $p=2$,

$$h_0=\frac{1}{4\sqrt{2}}\left(3+\sqrt{3}\right),\ h_2=\frac{1}{4\sqrt{2}}\left(3-\sqrt{3}\right),\ h_3=\frac{1}{4\sqrt{2}}\left(1-\sqrt{3}\right) \text{ for } n\neq 0,1,2,3,\ h_n=0.$$

The coefficients g_k are calculated by the formula (32) with $M=2$.

The order of using formulas (39), (43) will be shown by an example.

Example. Let the signal $a[n]$ be given by counts at points $n=0,1,\ldots,N-1$, where $N^{-1}=2^L$, $L=-4$. It is required to restore $a_L[n]$ at $n=15$, i.e. at the end of the interval [0, 15] using boundary wavelets and Daubechies wavelets with $p=2$.

Solution: According to the discrete convolution formula (33), using a low-frequency filter $h[n]$ Daubechies with $p=2$ and the corresponding high-frequency filter $g[n]$, determined by the ratio (43), after the periodization of these filters with period N, let's calculate:

$$a_{-3}[6]=\sum_{l=-\infty}^{+\infty} h[l-12]a_{-4}[l],$$
$$d_{-3}[6]=\sum_{l=-\infty}^{+\infty} g[l-12]a_{-4}[l],$$
$$a_{-3}[7]=\sum_{l=-\infty}^{+\infty} h[l-14]a_{-4}[l],\ d_{-3}[7]=\sum_{L=-\infty}^{+\infty} g[l-14]a_{-4}[l]. \quad (44)$$

Assuming now $j=-3$, $l=-1$, $k=-2.1$, using the filters $H_{k,l}^{np}$ and $G_{k,l}^{np}$ from **Table 1**, using the formula (43) let's find:

$$a_{-4}[15]=H_{-1,1}^{np}\cdot a_{-3}[7]+H_{-2,-1}^{np}\cdot a_{-3}[6]+G_{-1,1}^{np}d_{-3}[7]+G_{-2,-1}^{np}d_{-3}[6].$$

Let's now assume that in the signal $d_{-4}^{0}[k]=a_{-4}[k]$ reconstructed using (42), the wavelet packet coefficient $a_{-4}[15]$ is replaced by the value of the boundary wavelet and let's consider the received signal $a_{-4}[k]$ with counts $k=1,2,\ldots,N-1$ as a platform for the adaptive Kalman filter. The value $a_{-4}[k]$ at $k=0$ is excluded from further consideration, due to the possible appearance of a large value $a_{-4}[0]$ in the wavelet–packet analysis of the periodized (for the use of discrete convolution) signal $a[n]$, $n=0,1,\ldots,N-1$.

4. Discussion

The obtained results of the study are interpreted as follows.

The wavelet packet signal processing includes the identification of both the low-frequency component and a more detailed recovery of the high-frequency component, characterizing the noise (errors) of observations and external interference. Thus, it is possible to estimate the correlation functions of the equation of state and observations. The main difference between the wavelet packet analysis and the classical wavelet analysis is the most refined interference estimation used by us when adapting the Kalman filter. Multiple-scale time-frequency analysis of a signal in the case of wavelet-packet decomposition is carried out over a certain period of time and therefore cannot be performed separately recursively and in real time, which limits their use for dynamic analysis of random processes, in particular, for dynamic systems with a random fluctuation component. The combination of a wavelet analysis package with a Kalman filter allows to analyze and obtain predictive estimates of a random process and dynamic systems. In the absence of information about correlation functions, optimal signal estimation using the least squares method becomes inapplicable for state noise and observations. Such an assessment was carried out by us on the basis of preliminary wavelet packet signal processing. To predict a discrete signal given by a finite number of observations N, a cyclic discrete convolution is used, continued outside the observation interval $[0,T]$ with a period N. Taking into account the possible appearance of discontinuities of the first kind at the ends of time intervals extreme T, let's propose to replace the wavelet packet value of the signal at $t=T$ with the value of the boundary wavelet. To avoid the appearance of wavelet coefficients with large amplitudes at the boundaries of the periodization segments, it is necessary to construct boundary wavelets having as many zero moments as the original signals. For this purpose, Daubechies wavelet with the number of zero moments of the signal, and a fast cascade discrete algorithm Cohen, Daubechies, Vial [18] decomposing the studied signal into a discrete wavelet transform in O(N) operations were used. When implementing the wavelet-packet decomposition into $[0,T]$, the most convenient Haar filter in numerical calculations was used, which, in comparison with other Daubechies filters, is the only conjugate mirror filter with a compact carrier that has a linear complex phase [2].

To construct an adaptive Kalman filter, it is proposed [20] a priori estimates with averaging over a time window of size $N_0 \ll N$ for calculating the correlation functions of noise. The hybrid Kalman filtering algorithm described in the paper with preliminary wavelet packet processing is designed for the analysis and prediction of linear dynamical systems. For further development of this approach, its application is envisaged for the extended Kalman filter in the case of nonlinear dynamical systems based on their linearization using the approximating Taylor decomposition.

5. Conclusions

Due to the lack in the general case of information about the covariance functions Q_k and R_k for the state and observations noises, it is impossible to use an optimal estimate $\hat{x}_k$ of the signal x_k depending on R_k. Therefore, before applying the Kalman filter, it is proposed to perform pre-wavelet batch processing of the signal.

Wavelet packet decomposition and signal recovery is associated with the use of cyclic discrete convolution of a periodized signal with period N. However, the periodization of the signal can lead to sharp changes in its amplitude at the ends of the considered time interval. In this regard, it is necessary to replace the wavelet packet value of the signal at the right end of the interval with the value of the boundary wavelet.

The smoothed signal value obtained in this way (with the exception of the wavelet packet value at the left end of the interval) is used as a platform for the adaptive Kalman filter used in the work for one-step and multi-step prediction of the dynamic system under study.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

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Data availability

Manuscript has no associated data.

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