Chapter

Number Sense Performance of Gifted and General Fourth Graders in Taiwan

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Abstract

The study was designed to enable researchers the opportunities to investigate the number sense performance and methods used by both the gifted and general students. A mixed-method design was used, and 48 gifted students and 95 general students in fourth grade from two elementary schools in Southern Taiwan were selected. The sample was chosen using a convenience sampling method. Nine students in each group were randomly selected and interviewed. The results showed that the gifted students performed significantly higher than the general students on the whole test and in each component of number sense. The contributions of this study based on the findings are discussed.

Keywords: fourth graders, general student, gifted student, number sense, elementary school

1. Introduction

Number sense plays an important role in the elementary and middle-grade mathematics curricula [1–5]. Many of number sense-related studies have been conducted worldwide [6–11]. In addition, there are many studies which focus on examining the gifted students' performance on mathematics [12–14]. However, few researchers have investigated the gifted students' number sense performance. Moreover, there are not studies dedicated exclusively to the variations on the topic of number sense between the gifted and general students. Whether the number sense of the gifted students is superior to that of general students should be determined. Results show that number sense is a good predictor for further mathematics achievement [15]. In addition, data also show that the gifted children can apply multiple methods to solve problems flexibly [13, 14]. Children who have good number sense should develop and apply flexible and efficient methods to solve problems [4, 16]. Therefore, the number sense performance and methods used to solve number sense related questions for the gifted students should be examined. The more understanding the thinking and performance of number sense methods used by the gifted students, the more we can design a better approach to help them develop number sense. The kinds of differences should be examined to pursue better mathematics education for the gifted students in the future. Thus, the two research questions are as follows:

1. Is there any significant difference on number sense performance between the gifted and general students?

2. What kinds of differences on number sense-based methods used by the gifted and general students when solving number sense-related problems?

2. Background

2.1 Meaning and components of number sense

Number sense refers to an individual understanding of numbers, operations, the relations between numbers and operations, and the ability to solve real-world problems that involve numbers [1, 17].

Based on previous studies [1, 7, 9, 16, 17], the current study's definition of number sense comprises four components.

C1. Understanding the basic meaning of numbers and operations

This implies an ability to fully understand the meaning of the base-10 number system (e.g., integer, fraction, and decimal), place value, patterns of numbers, multiple methods of representation, and the four basic operations [9]. For example, students should realize that infinite decimals and fractions are found between 0.41 and 0.42 for middle-grade students [18].

C2. Composing and decomposing numbers

Decomposing numbers means to decompose numbers to facilitate the computation, such as 18 = 2 + 16, 18 = 15 + 3; composing numbers means to add numbers to become a number, such 19 + 1 = 20, 37 + 3 = 40 [9]. For example, when encountering a question such as 96 + 76 =? Students can first decompose 76 into 72 and 4, then add 4 to 96, which equals to 100, and finally, add 72. The result is 172.

C3. Ability to judge the reasonableness of a computational result

After obtaining an answer, students can use the information given by a question to determine the reasonableness of a computational result [9]. For example, when students were asked to answer: "How many digits is the sum of 2 three-digit numbers?" They should know that a small three-digit number plus another small three-digit number could be a three-digit number and a large three-digit number plus another large three-digit number could become a four-digit number.

C4. Recognizing relative number size

Students can determine relative number size or determine which number is closer to the target number. For example, "Arrange the following rational numbers in order from the lowest to the highest: $\frac{1}{2}$, 0.65, $\frac{5}{4}$." Students should know that 0.65 is greater than $\frac{1}{2}$, and $\frac{5}{4}$ is greater than 1. Therefore, the order is $\frac{5}{4} > 0.65 > \frac{1}{2}$ [18].

2.2 Gifted students' characteristics and related studies

In this study, gifted students were defined as students who passed two levels test instituted by the Ministry of Education in Taiwan [MEiT] [19]. The first level is the Intelligence Quotient Test, with a passing score of ranking above 93%, and the second level is the Wechsler Intelligence Scale (fourth edition) for Children, with a passing score of ranking above 97%. The IQ test was designed by educators in Taiwan and includes three subtests (e.g., language, mathematics, and graphics) [19]. The IQ test was used to assess students' intelligence. In this test, student's IQ score ranked over 93% is considered to have a high IQ. The Wechsler Intelligence Scale (fourth edition) was designed by Wechsler [20] to measure a child's intellectual ability. The test includes five primary index scores: the Verbal Comprehension Index (**VCI**), Visual Spatial Index (**VSI**), Fluid Reasoning Index (**FRI**), Working Memory Index (**WMI**), and Processing Speed Index (**PSI**). In this study, a student whose score ranked above 93% on the IQ test and ranked above 97% on the Wechsler Intelligence Scale (fourth edition) was defined as a gifted student.

Clark [21] discussed the characteristics of a gifted student from four perspectives, including cognitive, affective, physical, and intuitive perspectives. From a cognitive perspective, a gifted student has excellent memory and comprehension abilities, being capable of fast and flexible thinking, producing different ideas and problem-solving strategies, and so on (Clark). From an affective perspective, a gifted student has a strong motivation to explore new knowledge (Clark). From a physical perspective, a gifted student can absorb a lot of new information at a same time (Clark). From an intuitive perspective, a gifted student has a higher degree of creativity than peers (Clark). Davis and Rimm [22] indicated that gifted students possess many different characteristics, such as excellent analysis, reasoning, and problem-solving ability, ability to use abstract, complex, and high-level logical thinking abilities, and producing effective strategies to solve questions, having good meta-cognitive abilities, and so on. Earlier studies in Taiwan also showed that Taiwanese gifted students are better in abstract thinking, logical reasoning, fast and flexible thinking, using multiple and effective strategies, having good meta-cognitive abilities, and so on [23, 24].

Based on the above studies, the characteristics of gifted students include excellent abilities on concentration, comprehension, and creativity; flexible thinking, good abstract and logical reasoning ability, and insights; strong learning motivation; having the ability to self-reflection and meta-cognition, and so on. In fact, the gifted students in mathematics also have some characteristics that gifted students have [14, 25–29]. Number sense is a foundational content area in mathematics education [5, 9, 16, 30]. However, there is no research that focuses on the examination of the relationship between gifted students and number sense. This motivated the conduct of this study.

2.3 Number sense and gifted students-related studies

Earlier studies showed that there are several common methods used by genera gifted students when solving questions [13, 14, 28, 31]. These methods included (1) composing and decomposing, (2) finding patterns, (3) connecting to prior experience, (4) graphic representation, (5) eliminating possibilities, (6) making and testing conjectures, (7) intuition, and (8) logical reasoning. Some of the methods are similar to the number sense strategies.

Some methods are not specific to number sense. For example, flexibly using pictorial representations, which consists of drawing figures, is typically considered a problem-solving strategy [32]. "Making and testing conjectures" means that students make guesses by observing patterns, test these guesses, and then evaluate the result [33]. Regarding logical reasoning, Greeno [34] asserted that number sense is a set of capabilities for constructing and reasoning with a mental model. Students can estimate the area of a given region by using benchmarks and reasoning, which is an example of logical reasoning. "Intuition" is a type of number sense [28]. According to the previous discussion, the variations in performance and strategies between the gifted and general students in solving number sense questions are not definitive. Therefore, this study is relevant.

Study related to the gifted students on variations in number sense and use of strategies for solving number sense-related questions is lacking. Examining the difference and recognizing the characteristics between the gifted and general students would contribute to the future studies. Hence, this lack of research encouraged us to conduct this study.

3. Methods

A mixed-method approach was used in the current study. For quantitative analysis, the number sense data on the students in this study were collected using a number sense web-based two-tier test system. A statistical analysis was used to evaluate the performance of the gifted students and general students. For qualitative analysis, data were collected through semi-structured interviews.

3.1 Sample

Fourth graders from two public elementary schools (A and B) in Southern Taiwan were selected. Student numbers in each school are over 1,000. School A had 28 gifted students and 320 general students in fourth grade; School B had 20 gifted students and 200 general students in fourth grade. All 48 gifted students from both schools and three classes with 95 general students from School A and B (two classes from school A and one class from school B) were randomly selected to join this study. The families from schools have a wide range of socioeconomic backgrounds. All of the participants were voluntary to join the test and under the agreement of parents and the school administration.

According to the results of the number sense web-based two-tier test, the students in each group were classified into three categories: high-level (top 20%), middle-level (middle 50–60%), and low-level (bottom 20%). Three students at each level of both the gifted and general student groups were randomly selected and interviewed to examine their methods of thinking about number sense problems. Therefore, the sample for the interviews consisted of nine gifted students, coded as low (GL1–3), middle (GM1–3), and high (GH1–3) level students, and nine general students, coded as low (NGL1–3), middle (NGM1–3), and high (NGH1–3).

3.2 Instrument

A Number Sense Web-Based Two-Tier Test System for fourth graders designed by Lin [35, 36] was adopted in this study. The online test system consisted of a two-phase evaluation. The first-tier test (answer-tier) in the two-tier test assesses children's responses to number sense-related questions, and the second-tier test (reason-tier) examines children's reasons for their related choice made in the first-tier test [16, 37]. One example is shown in **Figure 1**.

This test included four components, with eight questions for each component, resulting in 32 total questions. The test was divided into two subtests. Each subtest included 16 items. The items in the web-based two-tier test were written in Chinese and translated into English for writing this manuscript. Each question in the test was

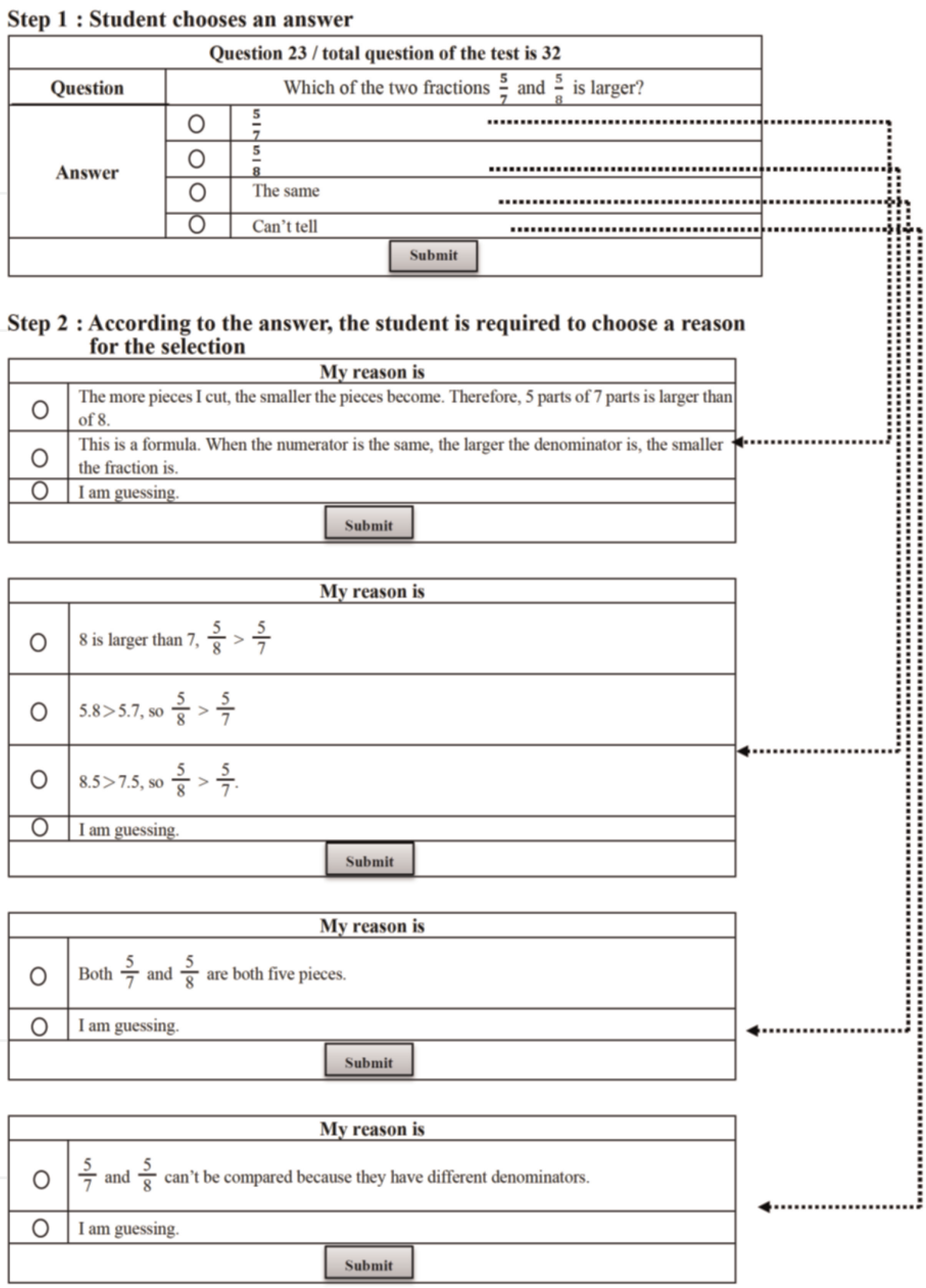

Figure 1.
One example for the two-tier number sense test.

reviewed by mathematics educators and experienced elementary school teachers. They all agreed that the tests are appropriate for the sample students.

To deeply explore students' thinking, a semi-structured interview was used to collect data [38]. Three questions were derived from each of the four components, and a total of 12 questions from the test were selected and used in the interviews to examine the gifted and general students' methods when solving number sense-related problems.

3.3 Procedure

The web-based two-tier test was conducted via an online setting in which students were asked to complete 16 items on computers individually for each subtest. The test included two subtests with 32 items in total. Due to the test included answer-tier and reason-tier, the answer-tier limited 40 seconds and reason-tier limited 60 seconds for students to answer the questions [36, 39]. Hence, participants required about 35 minutes each subtest to complete the test. During the test, students were required to follow the testing rules and procedures: (1) the papers and pencils were not allowed for students to use; (2) log on to the web-based system; (3) key in the individual data; (4) review the rules for the on-line test; (5) practice one item presented on the computer; and (6) begin the formal test.

Each participant was given a booklet during the interview. Each page of the booklet included one item and ample space for allowing students to record their thinking and methods. Each interview took about 40 minutes. Before the interview, the following directions were read aloud to each interviewee: 1. You are encouraged to estimate or mentally compute and do not necessarily to use written computation to find an exact answer on each item; 2. You can write an answer to the question and then briefly explain how you arrived at the answer; and 3. You are welcome to use different approaches to solve questions; the time on each item for you to answer was 3 minutes, so you should not turn to the next page without permission. The interviewer controlled the time to ensure that all interviewees would have an opportunity to answer each question.

3.4 Data collection and analysis

Data were collected through online tests and interviews. Based on the results of the online test, computer software was used to assemble statistical data; in the interview segment, video- and audio-recorded information of the interviews was transcribed into written records.

3.4.1 Quantitative data analysis

The scoring criteria of the two-tier test was calculated based on the students' answer and reason choices. In the first tier, if the students chose the correct answer, 4 points were given. In the second tier, if the students selected the number sense-based method, 4 points were given because the purpose of this study was to examine students' performance on the use of number sense-based method. If the students selected the rule-based method, 2 points were given. Therefore, the highest score was 8 points, and the lowest score was 4 points. If the students chose the wrong answer in the first tier, then 0 points were given in the first and second tiers.

Two independent groups (the gifted and general students) were used in this study; therefore, SPSS statistical software was used to perform the *t* test to determine the variation in number sense performance between the gifted students and the general students. In addition, an ANOVA was used to detect any variation between the gifted students and general students in the use of four components of number sense.

3.4.2 Qualitative data analysis

The students' responses were examined and sorted carefully. In an effort to identify the different methods used by the interviewees, each response (whether correct or incorrect) was sorted according to one of the following categories [17, 38]:

1. Number sense-based method: The students who used meaningful approaches to solve questions were coded as number sense-based method. For example:

   Question 4: "Which answer is equal to $2 \times 42 + 2 \times 58$? (1) 2×100 (2) 4×100 (3) $2 \times 44 \times 58$ (4) 86×58".

   GH1: Because 42 and 58 are all multiplied by 2; therefore, $2 \times 42 + 2 \times 58$ equals $2 \times (42 + 58)$. The answer is 2×100.

   GH1 knew that "$2 \times 42 + 2 \times 58$ equals $2 \times (42 + 58)$." This was coded as "being able to decompose and compose numbers."

2. Rule-based method: The students who used this strategy applied the rules of standard written algorithms to solve problems.

3. Misconception: The students used an incorrect method to solve problems.

4. Other methods: students' responses, except the above methods, were classified.

Two researchers independently reviewed the transcripts and categorized the students' responses for each correct and incorrect answer. These initial reviews produced agreement in over 92% of the categorization of student responses. The remaining responses were reexamined and discussed by the coders until agreement was reached.

3.5 Reliability and validity

The Cronbach's α coefficient of reliability was 0.828, and the construct reliability indices derived from structural equation modeling analysis for the two-tier test was 0.875. In addition, the difficulty indices of the test items were .26–.67, and the discrimination power was .48–.80.

Regarding the content validity, the options (both answer options and response options) in the NS came from earlier number sense studies (e.g., [36]). Especially, the options, including number sense-based method, misconceptions, and so on, used in the reason selections of the test were collected from interviewing over 100 fourth graders from earlier studies (e.g., [39]). Therefore, these options represented students' most frequent responses. In addition, the web-based test was reviewed by several experienced teachers, researchers, and mathematics educators who are experts in number sense to check whether those questions in the test were appropriate and relevant to the fourth graders. They all agreed that all the 32 questions in the test including wording, content, and the reasons for were appropriate for fourth graders.

4. Results

4.1 Variation in number sense performance between the gifted and general students

In **Table 1**, the number sense performance of the gifted and general students is reported. The t test results show statistically significant differences in the number sense performance of the gifted and general students for each number sense component (F1: $t = 9.5$, $p < .000$; F2: $t = 9.51$, $p < .000$; F3: $t = 8.3$, $p < .000$; and F4: $t = 8.96$,

$p < .000$) and total score ($t = 11.65$, $p < .000$). This indicates that the gifted students significantly outperformed the general students in each number sense component and overall number sense performance. Moreover, the results also reached a high effect size ($\eta^2 = .44$). It indicates that the gifted students significantly outperformed the general students in number sense.

4.2 Variations in number sense performance for each component for both groups

To further examine the variations in number sense performance for each component for both groups, a one-factor repeated measures analysis of variance was used. Before proceeding with the statistical analysis, we ensured that these data did not violate the sphericity assumption. The Mauchly values were W = .847 ($\chi^2 = 7.613$, $p > .05$) for the gifted students and W = .969 ($\chi^2 = 2.90$, $p > .05$) for the general students. The results show that the data did not violate the sphericity assumption. Therefore, the one-factor repeated measures analysis of variance could be performed.

In **Tables 2** and **3**, the results of the one-factor repeated measures analysis of variance for both groups are shared. In **Table 2**, the results of ANOVA show that a value of the 48 gifted students did not reach the significance level [$F(3, 141) = 2.444$, $p > .05$]. Therefore, no significant variation was found among the four number sense components for the gifted students. In **Table 3**, the ANOVA value of the general students reached the significance level [$F(3, 282) = 2.962$, $p < .05$], indicating a significant variation among the four number sense components for the general students. The results of post hoc tests showed a significant variation between F4 ($M = 27.95$; recognizing the relative number size) and F2 ($M = 24.08$; ability to decompose and compose numbers). This implies that the general students performed higher on F4 than on F2.

4.3 Similarities and differences in methods used by students of both groups

In **Table 4**, the interview results regarding the methods used by the students of both groups are shown. To explain the three types of methods used by the students, their responses are reported as follows. The interview Question A10 (F4, recognizing the relative number size) asked: "A box had 24 moon cakes. John bought 0.4 of a box, and Mary bought $\frac{1}{2}$ of a box. Who bought more moon cakes?: (a) John; (b) Mary; (c) John bought as many moon cakes as Mary; (d) Cannot be compared."

Variables	Gifted students (n = 48)		General students (n = 95)		t	P	η^2
	M	*SD*	*M*	*SD*			
C1	45.48	10.57	25.29	14.43	9.50	.000**	0.34
C2	48.46	13.67	24.08	14.87	9.51	.000**	0.39
C3	43.77	10.36	26.46	14.15	8.30	.000**	0.29
C4	46.38	10.38	27.95	13.74	8.96	.000**	0.32
Total	184.09	34.01	103.78	47.17	11.65	.000**	0.44

Note. *The total score was 256; each dimension score was 64.*
**p < *.01.*

Table 1.
The statistical analysis of number sense between the gifted students and general students.

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Source of variation	*SS*	*df*	*MS*	*F* value	*P* value
Between (A)	548.292	3	182.764	2.444	.067
Within (error)					
Between (B)	13589.917	47	289.147		
Residual (A*B)	10543.708	141	74.778		
Total	24681.917	191			

Note. *Post hoc: F3 (Recognizing the relative number size) > F2 (Being able to decompose and compose numbers).*
*< .05.

Table 2.
ANOVA analysis of components of number sense for the gifted students.

Source of variation	*SS*	*df*	*MS*	*F* value	*P* value
Between (A)	775.516	3	258.505	2.962	.033*
Within (error)					
Between (B)	52292.447	94	556.303		
Residual (A*B)	24608.984	282	87.266		
Total	77676.95	379	902.074		

Note. *Post hoc: F3 (Recognizing the relative number size) > F2 (Being able to decompose and compose numbers).*
*< .05.

Table 3.
ANOVA analysis of components of number sense for the general students.

	Number sense		Misconception		Written method	
	Gifted	**General**	**Gifted**	**General**	**Gifted**	**General**
C1	25 (93%)	12 (44%)	2 (7%)	15 (56%)	0 (0%)	0 (0%)
C2	23 (85%)	7 (26%)	0 (0%)	16 (61%)	4 (15%)	3 (11%)
C3	25 (93%)	12 (44%)	2 (7%)	10 (37%)	0 (0%)	6 (22%)
C4	18 (67%)	9 (30%)	2 (7%)	9 (33%)	7 (26%)	9 (33%)
Total	91 (84%)	40 (37%)	6 (6%)	50 (46%)	10 (10%)	18 (17%)

Note. *The number of people taken the interview was 108 times (9 people × 12 questions) for each group.*

Table 4.
The frequencies of number sense methods used by students of both groups.

4.3.1 Number sense-based method

The following student response is an example of using a number sense (NS)-based method.

GH3: I think Mary bought more moon cakes than John, because Mary bought half of the box. But what John bought, 0.4 of a box, is less than half. Therefore, Mary bought more moon cakes.

GH3 responded that "0.4 is less than half." This indicated that GH3 could apply $\frac{1}{2}$ as a benchmark and knew that 0.4 is less than $\frac{1}{2}$. Therefore, the response of GH3 was coded as a NS-based method.

4.3.2 Rule-based method

The following student response is an example of using a rule-based method.

GM1: Mary bought more moon cakes, because $\frac{1}{2} = 0.5$, and compared with 0.4, 0.5 is greater. That is why Mary bought more moon cakes.

R: Can you explain it another way?

GM1: It can be solved by comparing $24 \times 0.4 = 9.6$ and $24 \times 0.5 = 12$. Therefore, $12 > 9.6$.

As seen in the previous exchange, GM1 had two ways to solve the problem. One was converting $\frac{1}{2}$ to 0.5 and then comparing it with 0.4. The other was converting $\frac{1}{2}$ to 0.5 and then multiplying it by 24. GM1 also multiplied 0.4 by 24. Both solutions were based on written computation. Therefore, GM1's responses were coded as a rule-based method.

4.3.3 Misconception

The following student response is an example of a misconception.

NGL2: John bought more moon cakes because $24 \times 0.4 = 96$, and Mary bought $\frac{1}{2}$ of them, which is less.

R: Can you do it another way?

NGL2: 0.4 is greater, and $\frac{1}{2}$ is less.

R: How do you determine that 0.4 is greater, and $\frac{1}{2}$ is less?

NGL2: By multiplying the two numbers (meaning 24×0.4). I do not know how to explain it.

The previous explanation shows that NGL2 had a misconception when solving this problem. This was coded as a misconception.

The data showed the gifted students used number sense methods more frequently (84%) than did the general students (37%). By contrast, the general students had more misconceptions (46%) than did the gifted students (6%). The λ^2 test ($\lambda^2_{cri} = 5.991$, $df = 2$, $p = .000$) showed significant variation between the methods used by the gifted and general students. The results of the Marascuilo post hoc test showed significant variation in the use of the NS-based method between the gifted and general students. Moreover, a significant variation was found in the number of misconceptions between the gifted and the general students. However, no significant variation was found in the use of written methods between the gifted and the general students.

5. Discussion and conclusion

The quantitative results show that the gifted students significantly outperformed the general students in number sense and had a high effect size. Previous studies have shown a positive relationship between number sense and mathematics achievement [15, 39]. Earlier studies also agreed that students who have good number sense should be able to develop and apply flexible and efficient strategies (including mental computation and estimation) to handle numerical problems [9, 17, 38, 39]. In addition, several studies reported that the gifted students have the ability to use various methods efficiently and flexibly when solving problems [13, 25, 27, 29]. Therefore, the result that the gifted students significantly outperformed the general students in number sense is reasonable. In addition, the data also show no significant difference between each number sense component for the gifted students. This is probably due to the facts that the gifted students can develop more flexible and efficient methods to solve problems. Therefore, they have more balanced development on each number sense component. However, a significant difference was found between each number sense component for the general students.

The general students' performance on F4 (recognizing the relative number size) was significantly higher than that on F2 (being able to decompose and compose numbers). This is probably because Taiwanese mathematics textbooks typically have several "recognizing the relative number size"-related problems, and Taiwanese students have limited exposure to problems in mathematics that require them to compose and decompose numbers. Therefore, these Taiwanese students performed well on "recognizing the relative number size"-related problems due to they have ample opportunities to solve these kinds of problems. This result is consistent with the findings of a previous study [38, 39]. In addition, decomposing and composing numbers to solve problems need more flexible thinking; therefore, it is reasonable to believe that general students not performed well on this number sense component. Moreover, the teachers of the gifted students also provided several challenging problems to deepen their mathematical learning and thinking. However, these problems are not necessarily related to decomposing and composing numbers.

The interview results showed that the gifted students outperformed the general students in using the number sense-based method. It is reasonable to believe that gifted students can flexibly apply number sense-based methods, including the use of benchmark, estimation, and so on to solve problems. This supports Sands' finding [40] that showed that the gifted students tend to develop multiple methods to solve problems which relate to flexibility in thinking. Therefore, the result that the gifted students outperformed the general students in using the number sense-based method is reasonable. Moreover, the general students obviously had more misconceptions regarding number sense than the gifted students did. This result is probably due to insufficient basic mathematics knowledge exist among the general students. In fact, there is still the lingering question of curricula, learning opportunities, etc. This will lead more studies in the future.

This study was conducted to examine whether the gifted students outperform general students on number sense. Additionally, variations in the use of number sense methods between the gifted and general students when solving number sense-related problems were examined. Although limited by a small sample size, this study provides three major contributions to mathematics education:

1. The findings added a new knowledge about the gifted students' performance on the topic of number sense and the difference on number sense between the gifted and general students. That is the gifted students significantly outperformed the general students in each number sense component and on the whole number sense test.

2. The interview results also added a new knowledge about the methods used by the gifted students when responded to number sense-related questions. The gifted

students are significantly higher in applying number sense-based method to solve problems than the general students.

3. The findings showed that the gifted students performed equally well on each number sense component. This is special and different from the earlier studies that students in Taiwan did not perform equally well on each number sense components [39]. Earlier studies showed that general students performed poor on judging the reasonableness of a computational result (Authors).

We do hope the findings in this study may contribute the future teaching and research relating to number sense and the gifted students.

5.1 Limitations

Due to the sample size and the representativeness of the sample, generalizability of the results is a serious concern. More students with additional grade levels should be invited to participate in this kind of study. These factors should be considered by future researchers. In addition, two important issues not investigated in this study relate to the gifted students and number sense. Do the gifted students naturally have good number sense? How the number sense is developed by the gifted students?

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