Chapter

A Study on Numerical Solution of Fractional Order microRNA in Lung Cancer

Mohammed Baba Abdullahi and Amiru Sule

Abstract

The foremost cause of death resulting from cancer is lung cancer. From the statistics, 2.09 million new cases and 1.7 million deaths from lung cancer were estimated. In this chapter, the analytical solution of the concerned model was studied with help of the Laplace-Adomian Decomposition Method. To obtain the model's numerical scheme of fractional differential equations, the Caputo fractional derivative operator of order $\alpha \in (0, 1]$ is used. To find an approximate solution to a system of nonlinear fractional differential equations, the Laplace-Adomian Decomposition Method is used. Numerical simulations are presented to show the method's reliability and simplicity.

Keywords: fractional order, microRNA, numerical solution, cancer-related deaths, lung cancer

1. Introduction

The largest cause of cancer-related deaths worldwide is lung cancer. In the United States, a projected 236,740 people will receive a lung cancer diagnosis in 2022, making it the 16th most common cancer overall (1 in 15 males and 1 in 17 women). Smoking causes 80% of lung cancer fatalities and is the main risk factor for the disease. Twenty percent of lung cancer deaths occur in people who have never smoked. The second most important risk factor for lung cancer is radon gas exposure [1, 2]. Depending on the average radon level and the incidence of smoking in a nation, radon contributes to anywhere between 3 and 14% of lung cancer cases. Smokers are 25 times more likely than non-smokers to develop lung cancer from radon than non-smokers are likely to develop the cancer [3].

Early detection of high-risk lung cancer cases can reduce the chance of death by up to 20%. If you smoke now or have in the past, ask your doctor if lung cancer screening may be right for you. Approximately 8 million Americans are at high risk for lung cancer and could benefit from a lung cancer screening and yet only 5.7% actually get screened [4]. The dismal statistics associated with lung cancer are a result of both a lack of early detection and a lack of effective target therapy. Therefore, it is likely that developments in both of these areas will end in better results.

MicroRNAs (miRNAs) are a class of short nonprotein-coding RNAs (20–25 nucleotides in length) that predominantly inhibit the expression of target messenger RNAs (mRNAs) by directly interacting with their 30-untranslated regions (30UTRs) [5]. Numerous biological processes, from organismic development to tumor progression, depend on microRNAs in one way or another. These microRNAs have a crucial regulatory function in the pathogenesis of cancer in oncology, which forms the basis for investigating the influences on clinical characteristics using transcriptome data [6]. The seed match architecture between the mRNA seeding and miRNA binding regions determines the fate of the target mRNA. Perfect miRNA complementarity with the seeding sequence induces mRNA degradation, but imperfect or partial complementarities decrease protein translation [5].

Given the fact that a single miRNA may regulate tens to hundreds of genes, understanding the importance of an individual miRNA in cancer biology can be challenging. This is further complicated by observations that the dysregulation of several miRNAs is often required to cause a given phenotype [7]. To date, few models exist to elucidate the mechanisms by which multiple miRNAs contribute both individually and in tandem to promote tumor initiation and progression [8]. However, applying mathematical modeling to miRNA biology provides an opportunity to understand these complex relationships. In the work of Bersimbaev et al. [8], they developed for the first time a mathematical model focusing on miRNAs (miR-9 and let-7) in the context of lung cancer as a mathematical model system.

The organization of this chapter is as follows. In Section 2, some mathematical preliminaries of fractional calculus are needed to demonstrate the main results. The formulation of the Laplace-Adomian Decomposition Method (LADM) and Differential Transform Method (DTM) are given in Section 3. In Section 4, the numerical simulations are presented. In Section 5, the conclusions are given.

For the details of the integer mathematical model see Ref. [8], and the model is given below.

$$
\begin{aligned}
\frac{d}{dt}S(t) &= \mu_S E\frac{S_{tot}-S}{S_{tot}-S+K_{S1}} - \delta_S Ek\frac{S}{S+K_{S2}}\\
\frac{d}{dt}R(t) &= \mu_R S\frac{R_{tot}-R}{R_{tot}-R+K_{R1}}\frac{K_{R2}}{L+K_{R2}} - \delta_R\frac{R}{R+K_{R3}}\\
\frac{d}{dt}Ek(t) &= \mu_{Ek} R\frac{Ek_{tot}-Ek}{Ek_{tot}-Ek+K_{Ek1}} - \delta_{Ek}\frac{Ek}{Ek+K_{Ek2}}\\
\frac{d}{dt}C(t) &= \mu_C Ek - \delta_c C\\
\frac{d}{dt}M(t) &= \mu_M\frac{C^4}{C^4+K_M} - \delta_M M\\
\frac{d}{dt}L(t) &= \mu_L\frac{K_L}{C+K_L} - \delta_L L\\
\frac{d}{dt}H(t) &= \mu_H L\frac{K_H}{M+K_H} - \delta_H H\\
\frac{d}{dt}P(t) &= \mu_P - \delta_P P\frac{H}{H+K_P}
\end{aligned}
\tag{1}
$$

With the given initial condition.

$S(0) = n_1, R(0) = n_2, Ek(0) = n_3, C(0) = n_4, M(0) = n_5, L(0) = n_6, H(0) = n_7, P(0) = n_8$, where tables give the descriptions of the state variables and parameters.

$$
\begin{aligned}
{}^{c}D^{\alpha_1}S(t) &= \mu_S E\frac{S_{tot}-S}{S_{tot}-S+K_{S1}} - \delta_S Ek\frac{S}{S+K_{S2}}\\
{}^{c}D^{\alpha_2}R(t) &= \mu_R S\frac{R_{tot}-R}{R_{tot}-R+K_{R1}}\frac{K_{R2}}{L+K_{R2}} - \delta_R\frac{R}{R+K_{R3}}\\
{}^{c}D^{\alpha_3}Ek(t) &= \mu_{Ek}R\frac{Ek_{tot}-Ek}{Ek_{tot}-Ek+K_{Ek1}} - \delta_{Ek}\frac{Ek}{Ek+K_{Ek2}}\\
{}^{c}D^{\alpha_4}C(t) &= \mu_C Ek - \delta_c C\\
{}^{c}D^{\alpha_5}M(t) &= \mu_M\frac{C^4}{C^4+K_M} - \delta_M M\\
{}^{c}D^{\alpha_6}L(t) &= \mu_L\frac{K_L}{C+K_L} - \delta_L L\\
{}^{c}D^{\alpha_7}H(t) &= \mu_H L\frac{K_H}{M+K_H} - \delta_H H\\
{}^{c}D^{\alpha_8}P(t) &= \mu_P - \delta_P P\frac{H}{H+K_P}
\end{aligned}
\tag{2}
$$

With given initial condition.

$S(0) = n_1, R(0) = n_2, Ek(0) = n_3, C(0) = n_4, M(0) = n_5, L(0) = n_6, H(0) = n_7, P(0) = n_{8,}$ where.

${}^{c}D^{\alpha}\ 0 < x_i \leq 1\ for\ i = 0, 1, 2$ is the Caputo's derivate of fractional order and x shows fractional time derivative.

In model 2, the initial conditions are independent of each other and satisfy the relation.

$N(0) = S(t) + R(t) + Ek(t) + C(t) + M(t) + L(t) + H(t) + P(t)$, where$N(t)$is the total population.

$$S(0) = n_1, R(0) = n_2, Ek(0) = n_3, C(0) = n_4, M(0) = n_5, L(0) = n_6, H(0) = n_7, P(0) = n_8 \tag{3}$$

2. Preliminaries

This section focuses on some basic definitions and outcomes from fractional calculus. For more in-depth, detailed research [9–11].

Definition 2.1. The fractional integral of Riemann-Liouville type of order $\alpha \in (0, 1)$ of a function $f \in L^1([0, T], \Re)$ is defined as:

The Caputo fractional order derivative of a function f on the interval at $[0, T]$ is defined by the following:

$$ {}^{c}D^{\alpha}_{0+}f(t) = \frac{1}{\Gamma(\alpha)}\int_0^1 (t-s)^{n-\alpha-1} f^{(n)}(s)ds, \tag{4}$$

when $n = |x| + 1$ *and* $|x|$ represents the integer part of x. In particularity, for $0 < x < 1$, Caputo derivative becomes

$$ {}^{c}D_{0+}^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{0}^{1} \frac{f(s)}{(1-s)} ds. \tag{5} $$

Lemma 2.1. The next outcome holds for fractional differential equations.

$$ I^{\alpha}({}^{c}D^{\alpha}h)(t) = h(t) + \sum_{i=0}^{n-1} \frac{h^{i}(0)}{i!} t^{i}. \tag{6} $$

for arbitrary $x > 0, i = 0, 1, 2, \ldots, n-1,$ *when* $n = |x| + 1$ *and* $|x|$ *represents* the integer part of x

Definition 2.2. We recall the definition of Laplace transform of Caputo derivative as:

$$ \ell\{{}^{c}D^{\alpha}y(t)\} = s^{\alpha}h(s) - \sum_{k=0}^{n-1} s^{\alpha-i-1} y^{(k)}(0), \;\; n-1 < \alpha < n, \;\; n \in N. \tag{7} $$

for arbitrary $x > 0, i = 0, 1, 2, \ldots, n-1,$ *when* $n = |x| + 1$ *and* $|x|$ *represents* the integer part of x.

2.1 The Laplace-Adomian decomposition method

This section focuses on model (3)'s overall operation under specified initial conditions. When both sides of the model are transformed using the Caputo fractional derivative system (3), the following results are obtained:

$$
\begin{aligned}
L\{{}^{c}D^{\alpha_1}S(t)\} &= \mu_S E \frac{S_{tot} - S}{S_{tot} - S + K_{S1}} - \delta_S Ek \frac{S}{S + K_{S2}} \\
L\{{}^{c}D^{\alpha_2}R(t)\} &= \mu_R S \frac{R_{tot} - R}{R_{tot} - R + K_{R1}} \frac{K_{R2}}{L + K_{R2}} - \delta_R \frac{R}{R + K_{R3}} \\
L\{{}^{c}D^{\alpha_3}Ek(t)\} &= \mu_{Ek} R \frac{Ek_{tot} - Ek}{Ek_{tot} - Ek + K_{Ek1}} - \delta_{Ek} \frac{Ek}{Ek + K_{Ek2}} \\
L\{{}^{c}D^{\alpha_4}C(t)\} &= \mu_C Ek - \delta_c C \\
L\{{}^{c}D^{\alpha_5}M(t)\} &= \mu_M \frac{C^4}{C^4 + K_M} - \delta_M M \\
L\{{}^{c}D^{\alpha_6}L(t)\} &= \mu_L \frac{K_L}{C + K_L} - \delta_L L \\
L\{{}^{c}D^{\alpha_7}H(t)\} &= \mu_H L \frac{K_H}{M + K_H} - \delta_H H \\
L\{{}^{c}D^{\alpha_8}P(t)\} &= \mu_P - \delta_P P \frac{H}{H + K_P}
\end{aligned}
\tag{8}
$$

thus indicates

$$
\begin{aligned}
s^{\alpha_1}L\{{}^cD^{\alpha_1}S(t)\} - s^{\alpha_1-1}S(0) &= L\left\{\mu_S E\frac{S_{tot}-S}{S_{tot}-S+K_{S1}} - \delta_S Ek\frac{S}{S+K_{S2}}\right\}\\
s^{\alpha_2}L\{{}^cD^{\alpha_2}R(t)\} - s^{\alpha_1-1}R(0) &= L\left\{\mu_R S\frac{R_{tot}-R}{R_{tot}-R+K_{R1}}\frac{K_{R2}}{L+K_{R2}} - \delta_R\frac{R}{R+K_{R3}}\right\}\\
s^{\alpha_3}L\{{}^cD^{\alpha_3}Ek(t)\} - s^{\alpha_1-1}Ek(0) &= L\left\{\mu_{Ek} R\frac{Ek_{tot}-Ek}{Ek_{tot}-Ek+K_{Ek1}} - \delta_{Ek}\frac{Ek}{Ek+K_{Ek2}}\right\}\\
s^{\alpha_4}L\{{}^cD^{\alpha_4}C(t)\} - s^{\alpha_1-1}C(0) &= L\{\mu_C Ek - \delta_c C\}\\
s^{\alpha_5}L\{{}^cD^{\alpha_5}M(t)\} - s^{\alpha_1-1}M(0) &= L\left\{\mu_M\frac{C^4}{C^4+K_M} - \delta_M M\right\}\\
s^{\alpha_6}L\{{}^cD^{\alpha_6}L(t)\} - s^{\alpha_1-1}L(0) &= L\left\{\mu_L\frac{K_L}{C+K_L} - \delta_L L\right\}\\
s^{\alpha_7}L\{{}^cD^{\alpha_7}H(t)\} - s^{\alpha_1-1}H(0) &= L\left\{\mu_H L\frac{K_H}{M+K_H} - \delta_H H\right\}\\
s^{\alpha_8}L\{{}^cD^{\alpha_8}P(t)\} - s^{\alpha_1-1}P(0) &= L\left\{\mu_P - \delta_P P\frac{H}{H+K_P}\right\}
\end{aligned}
\tag{9}
$$

Using the initial conditions and taking inverse Laplace transform to system (5), we have:

$$
\begin{aligned}
S(t) &= S_0 + L^{-1}\left\{\mu_S E\frac{S_{tot}-S}{S_{tot}-S+K_{S1}} - \delta_S Ek\frac{S}{S+K_{S2}}\right\}\\
R(t) &= R_0 + L^{-1}\left\{\mu_R S\frac{R_{tot}-R}{R_{tot}-R+K_{R1}}\frac{K_{R2}}{L+K_{R2}} - \delta_R\frac{R}{R+K_{R3}}\right\}\\
Ek(t) &= Ek_0 + L^{-1}\left\{\mu_{Ek} R\frac{Ek_{tot}-Ek}{Ek_{tot}-Ek+K_{Ek1}} - \delta_{Ek}\frac{Ek}{Ek+K_{Ek2}}\right\}\\
C(t) &= C_0 + L^{-1}\{\mu_C Ek - \delta_c C\}\\
M(t) &= M_0 + L^{-1}\left\{\mu_M\frac{C^4}{C^4+K_M} - \delta_M M\right\}\\
L(t) &= L_0 + L^{-1}\left\{\mu_L\frac{K_L}{C+K_L} - \delta_L L\right\}\\
H(t) &= H_0 + L^{-1}\left\{\mu_H L\frac{K_H}{M+K_H} - \delta_H H\right\}\\
P(t) &= P_0 + L^{-1}\left\{\mu_P - \delta_P P\frac{H}{H+K_P}\right\}
\end{aligned}
\tag{10}
$$

Using the values of the initial condition in Eq. (6), we get:

$$
\begin{aligned}
S(t) &= n_1 + L^{-1}\left\{\mu_S E \frac{S_{tot} - S}{S_{tot} - S + K_{S1}} - \delta_S Ek \frac{S}{S + K_{S2}}\right\} \\
R(t) &= n_2 + L^{-1}\left\{\mu_R S \frac{R_{tot} - R}{R_{tot} - R + K_{R1}} \frac{K_{R2}}{L + K_{R2}} - \delta_R \frac{R}{R + K_{R3}}\right\} \\
Ek(t) &= n_3 + L^{-1}\left\{\mu_{Ek} R \frac{Ek_{tot} - Ek}{Ek_{tot} - Ek + K_{Ek1}} - \delta_{Ek} \frac{Ek}{Ek + K_{Ek2}}\right\} \\
C(t) &= n_4 + L^{-1}\{\mu_C Ek - \delta_c C\} \\
M(t) &= n_5 + L^{-1}\left\{\mu_M \frac{C^4}{C^4 + K_M} - \delta_M M\right\} \\
L(t) &= n_6 + L^{-1}\left\{\mu_L \frac{K_L}{C + K_L} - \delta_L L\right\} \\
H(t) &= n_7 + L^{-1}\left\{\mu_H L \frac{K_H}{M + K_H} - \delta_H H\right\} \\
P(t) &= n_8 + L^{-1}\left\{\mu_P - \delta_P P \frac{H}{H + K_P}\right\}
\end{aligned}
\tag{11}
$$

Assume that the solutions, $S(t), R(t), Ek(t), C(t), M(t), L(t), H(t), P(t)$ in the form of infinite series, are given by:

$$
\begin{aligned}
S(t) &= \sum_{n=0}^{\infty} S_n(t), R(t) = \sum_{n=0}^{\infty} R_n(t),\ \ Ek(t) = \sum_{n=0}^{\infty} Ek_n(t).\ C(t) = \sum_{n=0}^{\infty} C_n(t) \\
M(t) &= \sum_{n=0}^{\infty} M_n(t), L(t) = \sum_{n=0}^{\infty} L_n(t),\ \ H(t) = \sum_{n=0}^{\infty} H_n(t).\ P(t) = \sum_{n=0}^{\infty} P_n(t)
\end{aligned}
\tag{12}
$$

While the nonlinear term involved in the model is $S(t)Ek(t), S(t)R(t), F(t)R(t), P(t)H(t)$ and is decomposed as follows, where X_n, Y_n *and* Z_n are the Adomian polynomials defined as are:

$$
\begin{cases}
X_n = \dfrac{1}{\Gamma(n+1)} \dfrac{d^n}{dt^n}\left[\displaystyle\sum_{k=0}^{\infty} \lambda^k S_k \sum_{k=0}^{\infty} \lambda^k Ek_k\right] | \lambda = 0 \\
Y_n = \dfrac{1}{\Gamma(n+1)} \dfrac{d^n}{dt^n}\left[\displaystyle\sum_{k=0}^{\infty} \lambda^k S_k \sum_{k=0}^{\infty} \lambda^k R_k\right] | \lambda = 0 \\
Z_n = \dfrac{1}{\Gamma(n+1)} \dfrac{d^n}{dt^n}\left[\displaystyle\sum_{k=0}^{\infty} \lambda^k Ek_k \sum_{k=0}^{\infty} \lambda^k R_k\right] | \lambda = 0 \\
W_n = \dfrac{1}{\Gamma(n+1)} \dfrac{d^n}{dt^n}\left[\displaystyle\sum_{k=0}^{\infty} \lambda^k P_k \sum_{k=0}^{\infty} \lambda^k H_k\right] | \lambda = 0
\end{cases}
\tag{13}
$$

The first three polynomials are given by:

$$\begin{cases} X_0 = S_0(t)Ek_0(t), \\ X_1 = S_0(t)Ek_1(t) + S_1(t)Ek_0(t) \\ X_2 = 2S_0(t)Ek_2(t) + 2S_1(t)Ek_1(t) + 2S_2(t)Ek_0(t) \\ Y_0 = S_0(t)R_0(t), \\ Y_1 = S_0(t)R_1(t) + S_1(t)R_0(t) \\ Y_2 = 2S_0(t)R_2(t) + 2S_1(t)R_1(t) + 2S_2(t)R_0(t) \\ Z_0 = Ek_0(t)R_0(t), \\ Z_1 = Ek_0(t)R_1(t) + Ek_1(t)R_0(t) \\ Z_2 = 2Ek_0(t)R_2(t) + 2Ek_1(t)R_1(t) + 2Ek_2(t)R_0(t) \\ W_0 = P_0(t)H_0(t), \\ W_1 = P_0(t)H_1(t) + P_1(t)H_0(t) \\ W_2 = 2P_0(t)H_2(t) + 2P_1(t)H_1(t) + 2P_2(t)H_0 \end{cases} \tag{14}$$

Using Eqs. (8) and (10) in model (6), yields

$$\begin{aligned} L\left\{\sum_{n=0}^{\infty} S_k(t)\right\} &= \frac{S_0}{s} + \left[\frac{1}{s^\alpha} L\left\{\mu_S E \frac{S_{tot} - S}{S_{tot} - S + K_{S1}} - \delta_S Ek \frac{S}{S + K_{S2}}\right\}\right] \\ L\left\{\sum_{n=0}^{\infty} R_k(t)\right\} &= \frac{R_0}{s} + \left[\frac{1}{s^\alpha} L\left\{\mu_R S \frac{R_{tot} - R}{R_{tot} - R + K_{R1}} \frac{K_{R2}}{L + K_{R2}} - \delta_R \frac{R}{R + K_{R3}}\right\}\right] \\ L\left\{\sum_{n=0}^{\infty} Ek_k(t)\right\} &= \frac{Ek_0}{s} + \left[\frac{1}{s^\alpha} L\left\{\mu_{Ek} R \frac{Ek_{tot} - Ek}{Ek_{tot} - Ek + K_{Ek1}} - \delta_{Ek} \frac{Ek}{Ek + K_{Ek2}}\right\}\right] \\ L\left\{\sum_{n=0}^{\infty} C_k(t)\right\} &= \frac{C_0}{s} + \left[\frac{1}{s^\alpha} L\{\mu_C Ek - \delta_c C\}\right] \\ L\left\{\sum_{n=0}^{\infty} M_k(t)\right\} &= \frac{M_0}{s} + \left[\frac{1}{s^\alpha} L\left\{\mu_M \frac{C^4}{C^4 + K_M} - \delta_M M\right\}\right] \\ L\left\{\sum_{n=0}^{\infty} L_k(t)\right\} &= \frac{L_0}{s} + \left[\frac{1}{s^\alpha} L\left\{\mu_L \frac{K_L}{C + K_L} - \delta_L L\right\}\right] \\ L\left\{\sum_{n=0}^{\infty} H_k(t)\right\} &= \frac{H_0}{s} + \left[\frac{1}{s^\alpha} L\left\{\mu_H L \frac{K_H}{M + K_H} - \delta_H H\right\}\right] \\ L\left\{\sum_{n=0}^{\infty} P_k(t)\right\} &= \frac{P_0}{s} + \left[\frac{1}{s^\alpha} L\left\{\mu_P - \delta_P P \frac{H}{H + K_P}\right\}\right] \end{aligned} \tag{15}$$

Now, comparing like terms on both sides, yields

$$
\begin{cases}
L[S_0(t)] = \dfrac{n_1}{s}, L[R_0(t)] = \dfrac{n_2}{s}, L[Ek_0(t)] = \dfrac{n_3}{s}, L[C_0(t)] = \dfrac{n_4}{s}, \\
L[M_0(t)] = \dfrac{n_5}{s}, L[L_0(t)] = \dfrac{n_6}{s}, L[H_0(t)] = \dfrac{n_7}{s}, L[P_0(t)] = \dfrac{n_8}{s}, \\
L(S_1) = \left(\dfrac{\mu_S E}{s^\alpha}\dfrac{S_{tot}-S_0}{S_{tot}-S_0+K_{S1}} - \dfrac{\delta_S}{s^\alpha}\dfrac{X_0}{S_0+K_{S2}}\right)\dfrac{1}{s^{\alpha_1+1}}, \\
L(R_1) = \left(\mu_R S_0 \dfrac{R_{tot}}{R_{tot}-R_0+K_{R1}}\dfrac{K_{R2}}{L_0+K_{R2}} - \mu_R \dfrac{Y_0}{R_{tot}-R_0+K_{R1}}\dfrac{K_{R2}}{L_0+K_{R2}} - \delta_R \dfrac{R_0}{R_0+K_{R3}}\right)\dfrac{1}{s^{\alpha_2+1}}, \\
L(Ek_1) = \left(\mu_{Ek} R_0 \dfrac{Ek_{tot}}{Ek_{tot}-Ek_0+K_{Ek1}} - \mu_{Ek}\dfrac{Z_0}{Ek_{tot}-Ek_0+K_{Ek1}} - \delta_{Ek}\dfrac{Ek_0}{Ek_0+K_{Ek2}}\right)\dfrac{1}{s^{\alpha_3+1}}, \\
L(C_1) = (\mu_C Ek_0 - \delta_c C_0)\dfrac{1}{s^{\alpha_4+1}}, \\
L(M_1) = \left(\mu_M \dfrac{{C_0}^4}{{C_0}^4+K_M} - \delta_M M_0\right)\dfrac{1}{s^{\alpha_5+1}}, \\
L(L_1) = \left(\mu_L \dfrac{K_L}{C_0+K_L} - \delta_L L_0\right)\dfrac{1}{s^{\alpha_6+1}}, \\
L(H_1) = \left(\mu_H L_0 \dfrac{K_H}{M_0+K_H} - \delta_H H_0\right)\dfrac{1}{s^{\alpha_7+1}}, \\
L(P_1) = \left(\mu_P - \delta_P \dfrac{W_0}{H_0+K_P}\right)\dfrac{1}{s^{\alpha_8+1}}, \\
\ldots \\
L(S_{n+1}) = \left(\mu_S E \dfrac{S_{tot}-S_n}{S_{tot}-S_n+K_{S1}} - \delta_S \dfrac{X_n}{S_n+K_{S2}}\right)\dfrac{1}{s^{\alpha_1+1}}, \\
L(R_{n+1}) = \left(\mu_R S_n \dfrac{R_{tot}}{R_{tot}-R_n+K_{R1}}\dfrac{K_{R2}}{L_n+K_{R2}} - \mu_R \dfrac{Y_n}{R_{tot}-R_n+K_{R1}}\dfrac{K_{R2}}{L_n+K_{R2}} - \delta_R \dfrac{R_n}{R_n+K_{R3}}\right)\dfrac{1}{s^{\alpha_2+1}}, \\
L(Ek_{n+1}) = \left(\mu_{Ek} R_n \dfrac{Ek_{tot}}{Ek_{tot}-Ek_n+K_{Ek1}} - \mu_{Ek}\dfrac{Z_n}{Ek_{tot}-Ek_n+K_{Ek1}} - \delta_{Ek}\dfrac{Ek_n}{Ek_n+K_{Ek2}}\right)\dfrac{1}{s^{\alpha_3+1}}, \\
L(C_{n+1}) = (\mu_C Ek_n - \delta_C C_n)\dfrac{1}{s^{\alpha_4+1}}, \\
L(M_{n+1}) = \left(\mu_M \dfrac{{C_n}^4}{{C_n}^4+K_M} - \delta_M M_n\right)\dfrac{1}{s^{\alpha_5+1}}, \\
L(L_{n+1}) = \left(\mu_L \dfrac{K_L}{C_n+K_L} - \delta_L L_n\right)\dfrac{1}{s^{\alpha_6+1}}, \\
L(H_{n+1}) = \left(\mu_H L_n \dfrac{K_H}{M_n+K_H} - \delta_H H_n\right)\dfrac{1}{s^{\alpha_7+1}}, \\
L(P_{n+1}) = \left(\mu_P - \delta_P \dfrac{W_n}{H_n+K_P}\right)\dfrac{1}{s^{\alpha_8+1}}.
\end{cases}
\tag{16}
$$

Taking Laplace inverse of (11) and considering the first two terms at different values of$\alpha = 1, 0.95, 0.85$ and 0.75: and using the following values in **Tables 1** and **2**.

Variables	Description	Values	References
$S(t)$	Active SOS concentration	0.0298 μm	[7]
$R(t)$	Active Ras concentration	0.0053 μm	[7]
$Ek(t)$	Active ERK concentration	0.2488 μm	[7]
$C(t)$	MYC protein concentration	0.2189 μm	[7]
$M(t)$	miR-9 concentration	1.8x10^{-5} μm	[7]

Variables	Description	Values	References
$L(t)$	let-7 concentration	0.0023 μm	[7]
$H(t)$	E-Cadherin concentration	0.1 μm	[7]
$P(t)$	MMP mRNA concentration	1.157x10^{-13} μm	[7]

Table 1.
The state variables of the model.

Variables	Description	Values	References
E_0	Concentration of EGF-EGFR complex (Constant)	0.2488 $\mu M.\ \mu m$	[7]
S_{tot}	Total concentration of SOS	0.2120 μm	[7]
R_{tot}	Total concentration of Ras	0.2120 μm	[7]
Ek_{tot}	Total concentration of ERK	1.0599 μm	[7]
K_{S1}	Saturation of inactive SOS on active SOS	10.7515 μm	[7]
K_{S2}	Saturation of active SOS on inactive SOS	0.0023 μm	[7]
$H(t)$	Saturation of inactive Ras on active Ras	0.0635 μm	[7]
K_{R2}	Control of let-7 on Ras	0.0230 μm	[7]
K_{R3}	Saturation of active Ras on inactive Ras	2.5305 μm	[7]
K_{EK1}	Saturation of inactive ERK on active ERK	1.7795 μm	[7]
K_{EK2}	Saturation of active ERK on inactive ERK	6.1768 μm	[7]
K_M	Saturation of MYC on miR-9	22.9606 μm	[7]
K_L	Control of MYC on let-7	0.2189 μm	[7]
K_H	Control of MYC on E-Cadherin	1.8×10^{-5} μm	[7]
K_P	Control of E-Cadherin on MMP mRNA	0.1 μm	[7]
μ_S	Catalytic production rate of active SOS	394.5868/μm min	[7]
μ_{R0}	Catalytic production rate of active Ras	32.344/ min	[7]
μ_{Ek}	Catalytic production rate of active ERK	49.2683/ min	[7]
μ_C	Catalytic production rate of MYC	0.0184/ min	[7]
μ_M	Catalytic production rate of miR-9	0.0026/μm min	[7]
μ_L	Catalytic production rate of let-7	$1.3340 \times 10^{-5} \mu m$/ min	[7]
μ_H	Catalytic production rate of E-Cadherin	0.2087/ min	[7]
μ_P	Catalytic production rate of MMP	$9.8379 \times 10^{-17} \mu m$/ min	[7]
δ_{S0}	Degradation rate of active SOS	322.3940/ min	[7]
δ_R	Degradation rate of active Ras	319.9672 μm/ min	[7]
δ_{Ek}	Degradation rate of active ERK	1.8848 μm/ min	[7]
δ_C	Degradation rate of MYC protein	0.0231/ min	[7]
δ_M	Degradation rate of miR-9	0.0144/ min	[7]
δ_L	Degradation rate of let-7	0.0029/ min	[7]
δ_H	Degradation rate of E-Cadherin	0.0024/ min	[7]
δ_P	Degradation rate of MMP mRNA	0.0017/ min	[7]

Table 2.
The parameters of the model.

From $\alpha = 1$, (12) *obtained*

$$\begin{cases} S(t) = 394.5868 + 21.03377825t + 39.51795700t^2 \\ R(t) = 0.0053 + 8874.951815t + 6221.509715t^2 \\ Ek(t) = 0.2488 - 0.00877591439t + 62646.35055t^2 \\ C(t) = 0.2189 - 0.00047867t + 0.0000862605090t^2 \\ M(t) = 0.000018 + 0.00002672931626t - 1.924510749x10^{-7}t^2 \\ L(t) = 0.0023 + 0.000006684617295t^2 \\ H(t) = 0.1 - 0.00023333t + 0.0000029641441673t^2 \\ P(t) = 1.157x10^{-13} + 3.4x10^{-20}t + 4.941947609x10^{-17}t^2 \end{cases} \tag{17}$$

From $\alpha = 0.95$, (12) *obtained*

$$\begin{cases} S(t) = 394.5868 + 21.46565321t^{0.95} + 41.36344349t^{1.90} \\ R(t) = 0.0053 + 9057.176303t^{0.95} + 6512.053888t^{1.90} \\ Ek(t) = 0.2488 - 0.0047867t^{0.95} + 65571.93179t^{1.90} \\ C(t) = 0.2189 - 0.0004884982670t^{0.95} + 0.00009029571759t^{1.90} \\ M(t) = 0.000018 + 0.00002727813456t^{0.95} - 2.014385299x10^{-7}t^{1.90} \\ L(t) = 0.0023 + 0.000006996788568t^{1.90} \\ H(t) = 0.1 - 0.000238120836t^{0.95} + 0.000003102566932t^{1.90} \\ P(t) = 1.157x10^{-13} + 3.469810324x10^{-20}t^{0.95} + 5.17227363x10^{-17}t^{1.90} \end{cases} \tag{18}$$

From $\alpha = 0.85$, $(10 - 13)$ *obtained*

$$\begin{cases} S(t) = 394.5868 + 22.2435804t^{0.85} + 45.17936838t^{1.70} \\ R(t) = 0.0053 + 935.413409t^{0.85} + 7112.814038t^{1.70} \\ Ek(t) = 0.2488 - 0.009280679638t^{0.85} + 71621.71589t^{1.70} \\ C(t) = 0.2189 - 0.0005062017158t^{0.85} + 0.0000986258194t^{1.70} \\ M(t) = 0.000018 + 0.00002826670933t^{0.85} - 2.200219512x10^{-7}t^{1.70} \\ L(t) = 0.0023 + 0.000007642267217t^{1.70} \\ H(t) = 0.1 - 0.0002467504677t^{0.85} + 0.000003388789774t^{1.70} \\ P(t) = 1.157x10^{-13} + 3.5955818010^{-20}t^{0.85} + 5.649939635x10^{-17}t^{1.70} \end{cases} \tag{19}$$

From $\alpha = 0.75$, (12) *obtained*

$$\begin{cases} S(t) = 394.5868 + 22.88612324t^{0.75} + 49.1470382t^{1.50} \\ R(t) = 0.0053 + 9656.526685t^{0.75} + 7736.466899t^{1.50} \\ Ek(t) = 0.2488 - 0.009548767504t^{0.75} + 77900.93395t^{1.50} \\ C(t) = 0.2189 - 0.0005208241943t^{0.75} + 0.000107273391t^{1.50} \\ M(t) = 0.000018 + 0.00002908324024t^{0.75} - 2.393135170x10^{-7}t^{1.50} \\ L(t) = 0.0023 + 0.000008312342631t^{1.50} \\ H(t) = 0.1 - 0.0002538782653t^{0.75} + 0.000003685919493t^{1.50} \\ P(t) = 1.157x10^{-13} + 3.699421857x10^{-20}t^{0.75} + 6.145327396x10^{-17}t^{1.50} \end{cases} \tag{20}$$

2.2 Differential transform method

The following recurrence relation to the system (2) with respect to time (t) is obtained.

$$S(k+1)=\frac{1}{k+1}\left[\mu_S E\frac{S_{tot}-S(k)}{S_{tot}-S(k)+K_{S1}}\partial(k)-\delta_S\frac{\sum_{i=0}^{n}S(l)Ek(k-l)}{S(k)+K_{S2}}\right],$$

$$R(k+1)=\frac{1}{k+1}\left[\mu_R S(k)\frac{R_{tot}}{R_{tot}-R(k)+K_{R1}}\frac{K_{R2}}{L(k)+K_{R2}}-\mu_R\frac{\sum_{i=0}^{n}S(l)R(k-l)}{R_{tot}-R(k)+K_{R1}}\frac{K_{R2}}{L(k)+K_{R2}}-\delta_R\frac{R(k)}{R(k)+K_{R3}}\right],$$

$$Ek(k+1)=\frac{1}{k+1}\left[\mu_{Ek}R(k)\frac{Ek_{tot}}{Ek_{tot}-Ek(k)+K_{Ek1}}-\mu_{Ek}\frac{\sum_{i=0}^{n}Ek(l)R(k-l)}{Ek_{tot}-Ek(k)+K_{Ek1}}-\delta_{Ek}\frac{Ek(k)}{Ek(k)+K_{Ek2}}\right],$$

$$C(k+1)=\frac{1}{k+1}[\mu_C Ek(k)-\delta_c C(k)],$$

$$M(k+1)=\frac{1}{k+1}\left[\mu_M\frac{C^4(k)}{C^4(k)+K_M}-\delta_M M(k)\right],$$

$$L(k+1)=\frac{1}{k+1}\left[\mu_L\frac{K_L}{C_0+K_L}-\delta_L L(k)\right],$$

$$H(k+1)=\frac{1}{k+1}\left[\mu_H L(k)\frac{K_H}{M(k)+K_H}-\delta_H H(k)\right],$$

$$P(k+1)=\frac{1}{k+1}\left[\mu_P-\delta_P\frac{\sum_{i=0}^{n}P(l)H(k-l)}{H(k)+K_P}\right],\tag{21}$$

The inverse differential transform of $S(k)$is defined as: When $t_0$is taken as zero, the given function $y(x)$ is declared by a finite series and the above equation can be written in the form $S(t)=\sum_{i=0}^{2}S(k)i^k$.

By solving the above equation for

$$S(k+1),R(k+1),Ek(k+1)\,C(k+1),M(k+1),L(k+1),H(k+1)and\ P(k+1)\tag{22}$$

up to order 2, we get the function.

$S(k),R(k),Ek(k)\,C(k),M(k),L(k),H(k)and\ P(k)S(k),E(k),I(k)\ and\ R(k)$ of respectively

$$S(t)=\sum_{i=0}^{2}S(k)i^k,R(t)=\sum_{i=0}^{2}R(k)i^k,\ \ Ek(t)=\sum_{n=0}^{\infty}Ek(k)i^k,C(t)=\sum_{n=0}^{\infty}C(k)i^k$$
$$M(t)=\sum_{n=0}^{\infty}M(k)i^k,L(t)=\sum_{n=0}^{\infty}L(k)i^k,\ \ H(t)=\sum_{n=0}^{\infty}H(k)i^k,\ \ P(t)=\sum_{n=0}^{\infty}P(k)i^k\tag{23}$$

$$\begin{cases} S(t) = 394.5868 + 21.03377825t + 40.55133050t^2 \\ R(t) = 0.0053 + 8874.951815t + 6221.509905t^2 \\ Ek(t) = 0.2488 - 0.00877591439t + 62646.35060t^2 \\ C(t) = 0.2189 - 0.00047867t + 0.0000862605090t^2 \\ M(t) = 0.000018 + 0.00002672931626t - 1.924510768x10^{-7}t^2 \\ L(t) = 0.0023 + 0.000006684617295t^2 \\ H(t) = 0.1 - 0.00023333t + 0.0000029641441658t^2 \\ P(t) = 1.157x10^{-13} + 3.4x10^{-20}t + 4.941947609x10^{-17}t^2 \end{cases} \tag{24}$$

3. Numerical results

The plots below show the population of each compartment for different values of $\alpha_i (i = 1, 2, 3, 4)$ (**Figures 1–8**).

3.1 The comparison plots of the LADM and DTM of different compartments

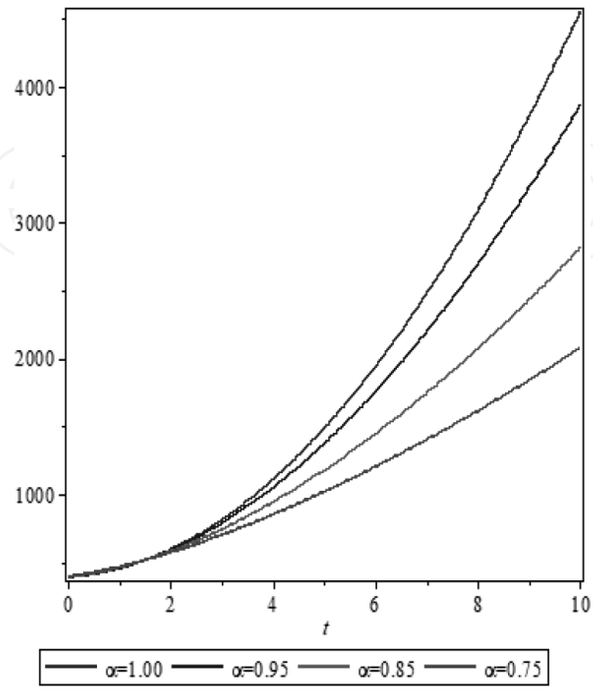

Figure 1.
The plot shows the population of active SOS concentration for $\alpha_i, (i = 1, 2, 3)$.

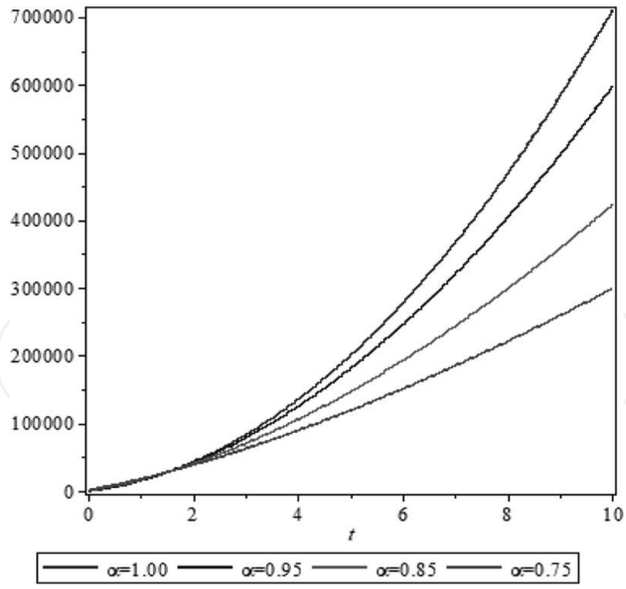

Figure 2.
The plot shows the population of active Ras concentration for α_i, $(i = 1, 2, 3)$.

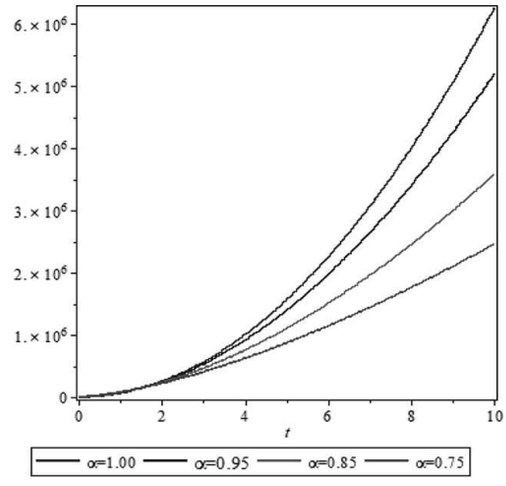

Figure 3.
The plot shows the population of active ERK concentration for α_i, $(i = 1, 2, 3)$.

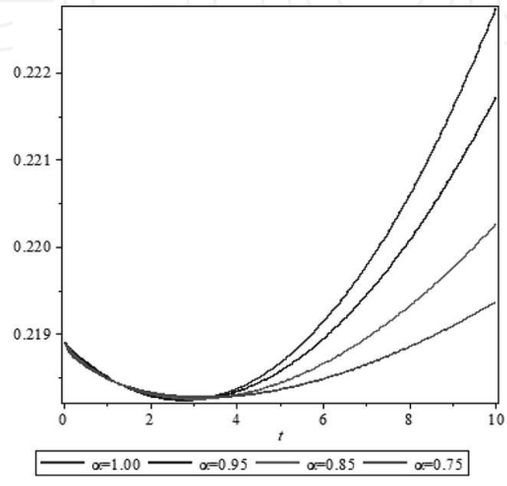

Figure 4.
The plot shows the population of active MYC protein concentration for α_i, $(i = 1, 2, 3)$.

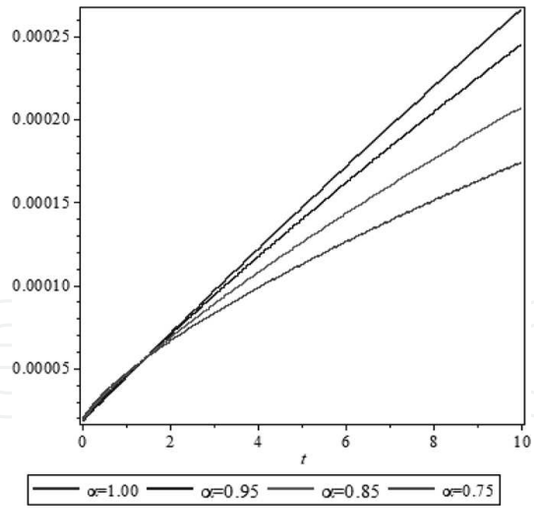

Figure 5.
The plot shows the population of miR-9 concentration for α_i, $(i = 1, 2, 3)$.

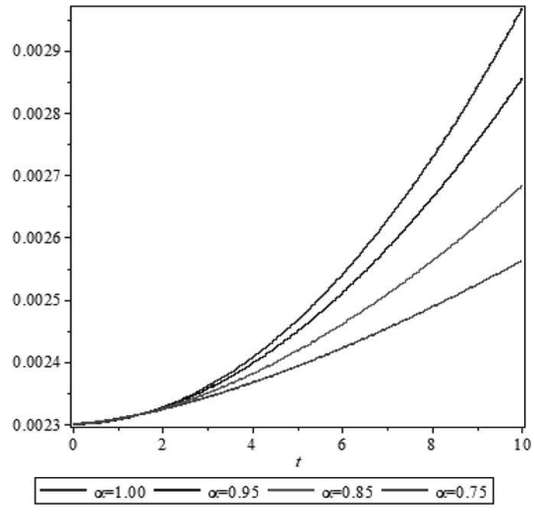

Figure 6.
The plot shows the population of let − 7 concentration for α_i, $(i = 1, 2, 3)$.

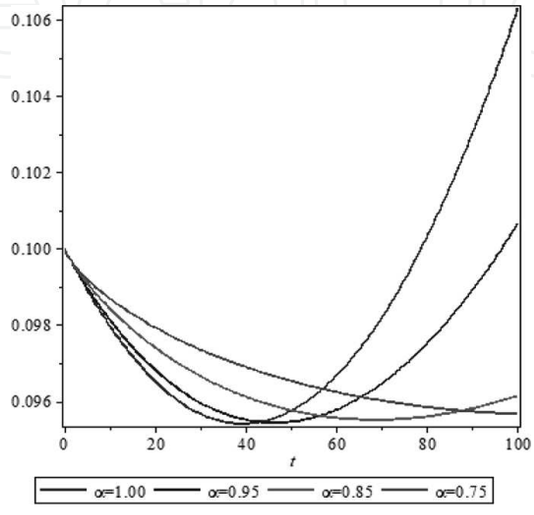

Figure 7.
The plot shows the population of E-cadherin concentration for α_i, $(i = 1, 2, 3)$.

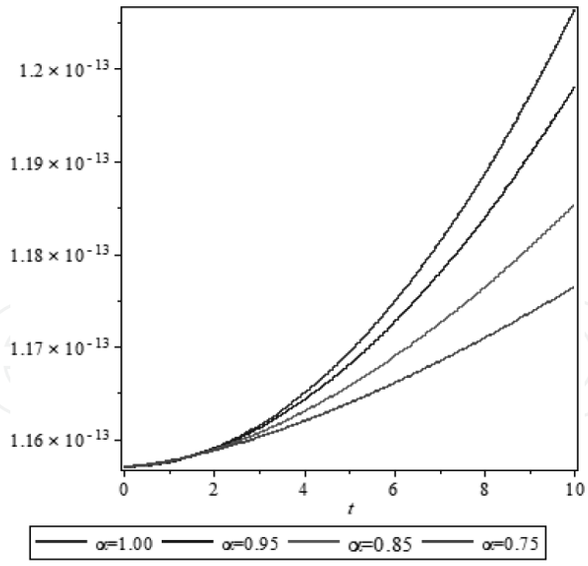

Figure 8.
The plot shows the population of MMP in RNA concentration for α_i, $(i = 1, 2, 3)$.

4. Conclusion

In this chapter, a fractional order differential equation model is considered. The model was investigated and a scheme for the numerical solution for the fractional order differential equation microRNA in lung cancer using LADM (**Figures 9–16**). The LADM is an effective technique to solve nonlinear mathematical models and

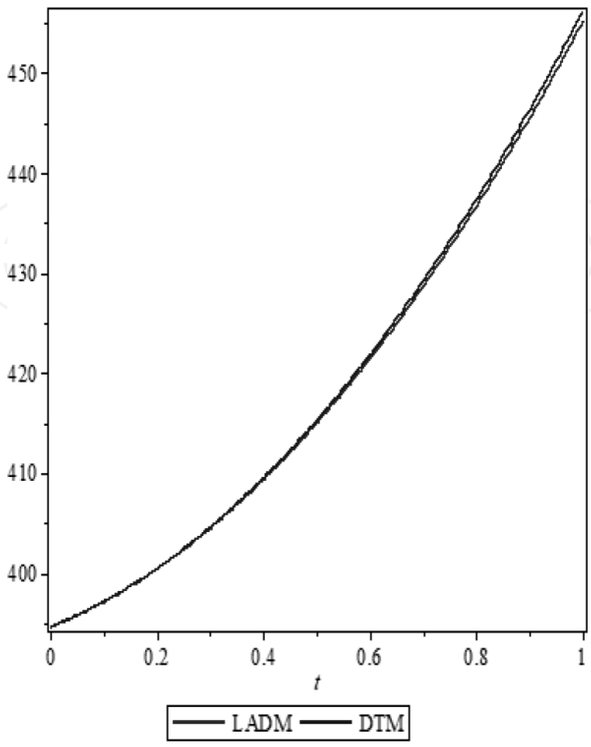

Figure 9.
The comparison plots of the dynamics of active SOS concentration using LADM and DTM.

is extensively applied in engineering and applied mathematics. Applying Laplace-Adomian Decomposition Method to obtain the series solution of fractional the model and comparing the results of the model at $\alpha \in (0,1]$ with the classical Differential

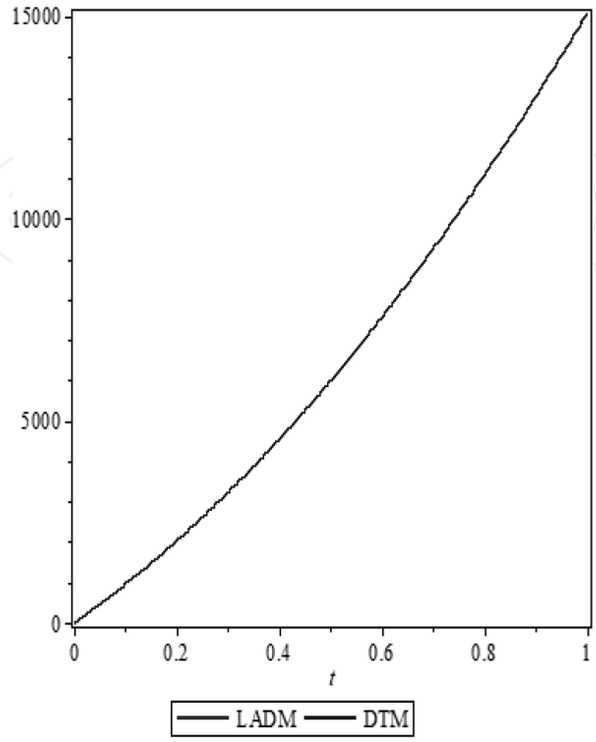

Figure 10.
The comparison plots of the dynamics of active Ras concentration using LADM and DTM.

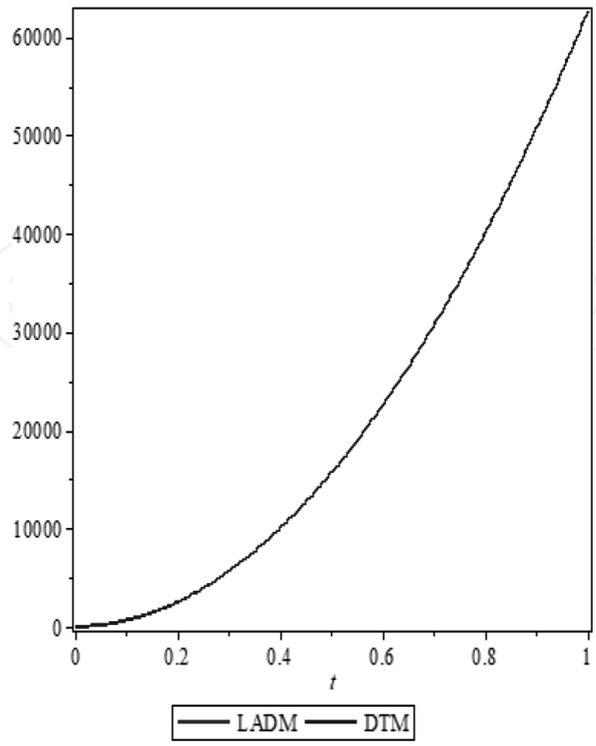

Figure 11.
The comparison plots of the dynamics of active ERK concentration using LADM and DTM.

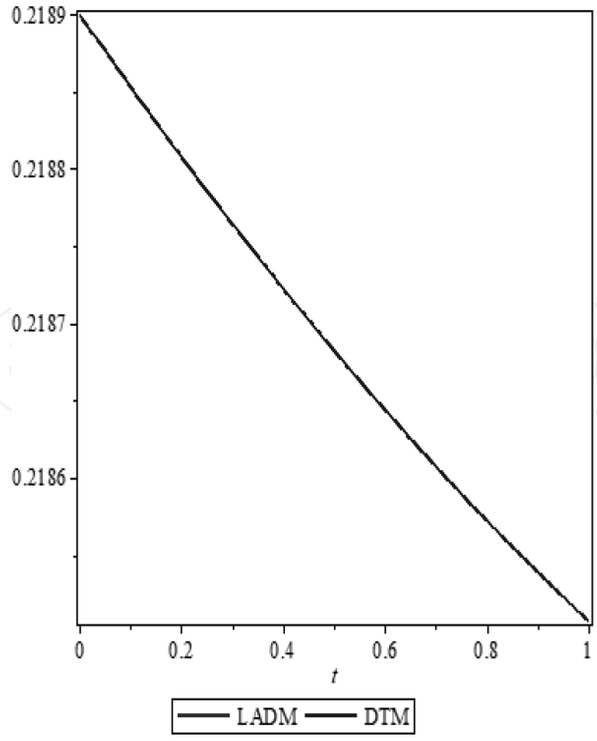

Figure 12.
The comparison plots of the dynamics of MYC protein concentration using LADM and DTM.

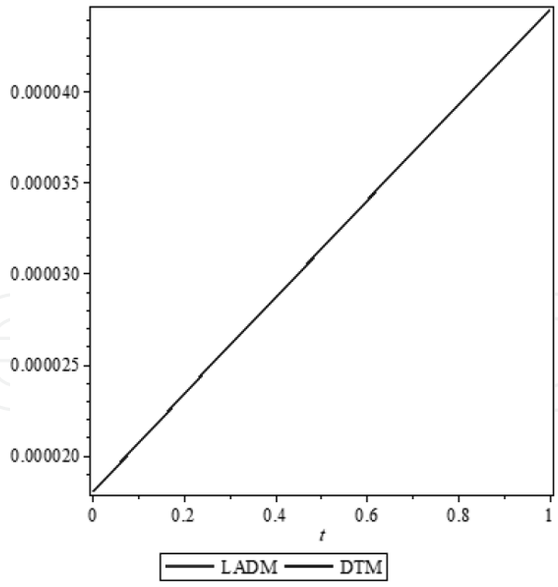

Figure 13.
The comparison plots of the dynamics of miR-9 concentration using LADM and DTM.

Transform Method is the main contribution of the work. The solution obtained through this method strongly agrees with DTM as shown in **Figures 1–16**. The effect of fractional parameters on our obtained solutions is presented through graphs.

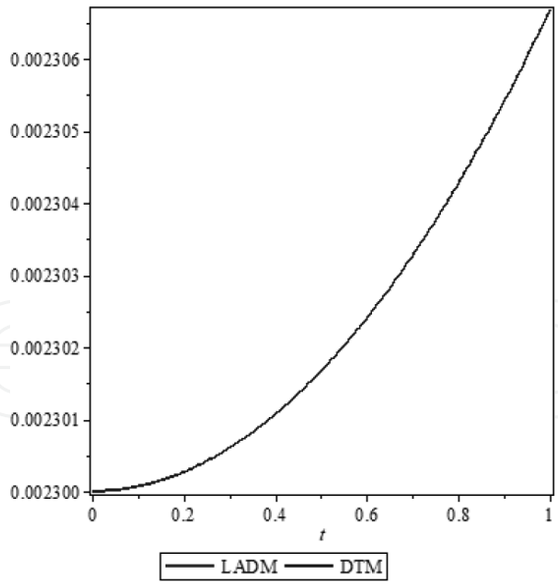

Figure 14.
The comparison plots of the dynamics of let − 7 concentration using LADM and DTM.

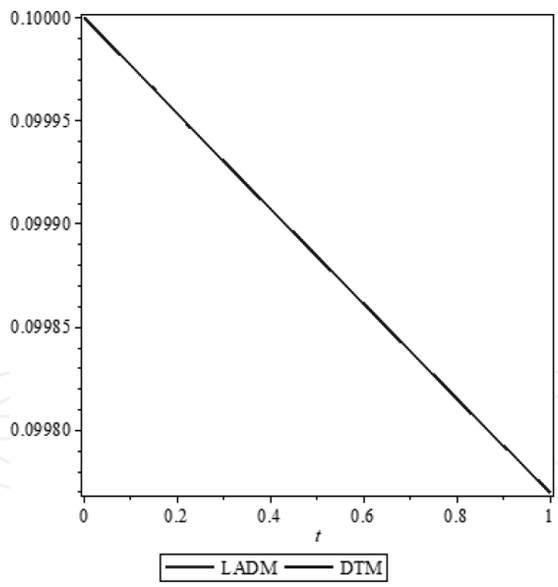

Figure 15.
The comparison plots of the dynamics of E-cadherin concentration using LADM and DTM.

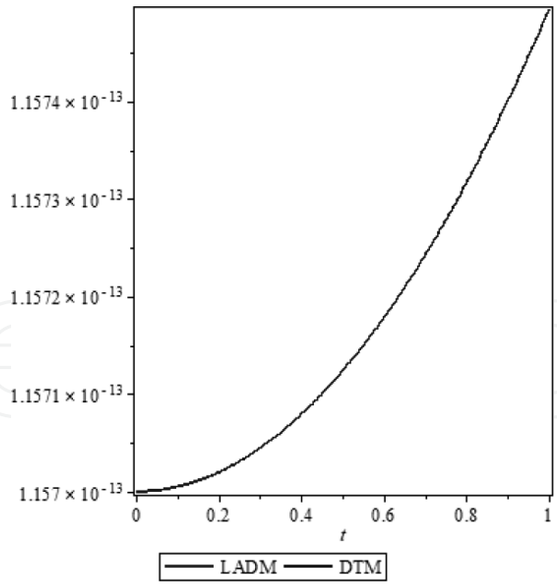

Figure 16.
The comparison plots of the dynamics of MMP mRNA concentration using LADM and DTM.

Author details

Mohammed Baba Abdullahi[1*] and Amiru Sule[2]

1 Department of Mathematics/Statistics, School of Mathematics and Computing, Kampala International University, Kampala, Uganda

2 Faculty of Science, Department of Mathematical Science, Federal University Gusau, Nigeria

*Address all correspondence to: kutigibaba@gmail.com

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