Chapter

The Paradigm of Complex Probability and the Theory of Metarelativity: A Simplified Model of MCPP

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"Subtle is the Lord. Malicious, He is not."

Albert Einstein.

"Mathematics, rightly viewed, possesses not only truth but supreme beauty ... "

Bertrand Russell.

"Logic will get you from A to Z; imagination will get you everywhere."

Albert Einstein.

"There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy."

Hamlet (1601), William Shakespeare.

"Ex nihilo nihil fit: Nothing comes from nothing."

Parmenides.

"All is One. From One all things."

Heraclitus.

Abstract

All our work in classical probability theory is to compute probabilities. The original idea in this research work is to add new dimensions to our random experiment, which will make the work deterministic. In fact, probability theory is a nondeterministic theory by nature; which means that the outcome of the events is due to chance and luck. By adding new dimensions to the event in the real set of probabilities $\mathcal{R}$, we make the work deterministic, and hence a random experiment will have a certain outcome in the complex set of probabilities and total universe $\mathbf{G} = \mathcal{C}$. It is of great importance that the stochastic system, like in real-world problems, becomes totally predictable since we will be totally knowledgeable to foretell the outcome of chaotic and random events that occur in nature, for example, in statistical mechanics or in all stochastic processes. Therefore, the work that should be done is to add to the real set of probabilities $\mathcal{R}$ the contributions of $\mathcal{M}$, which is the imaginary set of probabilities that will make the event in $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M}$ deterministic. If this is found to be fruitful, then a new theory in statistical sciences and in science, in general, is elaborated and this is to understand absolutely deterministically those phenomena that used to be random phenomena in $\mathcal{R}$. This paradigm was initiated and developed in my previous 21 publications. Moreover, this model will be related to my theory of Metarelativity, which takes into account faster-than-light matter and energy. This is what I called "The Metarelativistic Complex Probability Paradigm (*MCPP*)," which will be elaborated on in the present two chapters 1 and 2.

Keywords: chaotic factor, degree of our knowledge, complex random vector, probability norm, complex probability set $\mathcal{C}$, metarelativistic transformations, imaginary number, imaginary dimensions, superluminal velocities, metaparticles, dark matter, metamatter, dark energy, metaenergy, metaentropy, universe $\mathbf{G}_1$, metauniverse $\mathbf{G}_2$, luminal universe $\mathbf{G}_3$, the total universe $\mathbf{G}$

1. Introduction

The development of a new theory in physics, which I called the theory of Metarelativity creates a new continuum or space-time in which a new matter interacts [1–3]. This newly discovered matter is surely not the ordinary matter but a new kind of matter that can be easily identified to be the dark matter that astronomers, astrophysicists, and cosmologists seek to find. In fact, my novel theory shows that this new matter is superluminal by nature and is related to the new meta-space-time that lies in the metauniverse $\mathbf{G_2}$ in the same fashion that ordinary matter is related to the ordinary space-time and that lies in the universe $\mathbf{G_1}$ that we know. From what has been proved in Metarelativity, it was shown that the theory does not destroy Albert Einstein's theory of relativity that we know at all but on the contrary, it proves its veracity and expands it to the superluminal velocities' realm. The new space-time is "imaginary" since it exists in the domain of imaginary numbers and is now called meta-space-time or metauniverse because it lays beyond the ordinary "real" space-time that exists in the domain of real numbers as well as the matter and the energy interacting within them. Now the relation between both matter and metamatter is shown in the theory of Metarelativity. The first space-time is called the universe and the second space-time is called the metauniverse, which is another universe if we can say as material and as real as the first one but at a different level of experience because it is superluminal relative to the first one. It is similar to the atomic world that exists and is real but at a different level of physical experience, in the sense that we have discovered its laws in the theory of quantum mechanics where we deal with atoms and particles like when we deal in astronomy and astrophysics with planets and galaxies. In fact, astronomy is also real in the sense that we have discovered the laws governing the stars and planets but it lays at a different level of reality from our everyday world and experience. Metarelativity comes now to enlarge once more the scope of our understanding to encompass a new level of physical reality.

Furthermore, my Metarelativity will be bonded to my Complex Probability Paradigm (*CPP*), which was developed in my 21 previous research works. In fact, the system of axioms for probability theory laid in 1933 by Andrey Nikolaevich Kolmogorov can be extended to encompass the imaginary set of numbers, and this by adding to his original five axioms an additional three axioms. Therefore, we create the complex probability set $\mathcal{C}$, which is the sum of the real set $\mathcal{R}$ with its corresponding real probability and the imaginary set $\mathcal{M}$ with its corresponding imaginary probability. Hence, all stochastic and random experiments are performed now in the complex set $\mathcal{C}$ instead of the real set $\mathcal{R}$. The objective is then to evaluate the complex probabilities by considering supplementary new imaginary dimensions to the event occurring in the "real" laboratory. Consequently, the corresponding probability in the whole set $\mathcal{C}$ is always equal to one and the outcome of all random experiments that follow any probability distribution in $\mathcal{R}$ is now predicted totally and absolutely in $\mathcal{C}$. Subsequently, it follows that chance and luck in $\mathcal{R}$ are replaced by total determinism in $\mathcal{C}$. Consequently, by subtracting the chaotic factor from the degree of our knowledge of the stochastic system, we evaluate the probability of any random phenomenon in $\mathcal{C}$. My innovative Metarelativistic Complex Probability Paradigm (*MCPP*) will be developed in this work in order to express all probabilistic phenomena completely deterministically in the total universe $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M} = \mathbf{G}_1 + \mathbf{G}_2 + \mathbf{G}_3$.

Finally, and to conclude, this research work is organized as follows: After the introduction in Section 1, the purpose and the advantages of the present work are presented in Section 2. Afterward, in Section 3, we will review and recapitulate the complex probability paradigm (*CPP*) with its original parameters and interpretation. In Section 4, a concise review of Metarelativity will be explained and summarized. Also, in Section 5, I will show the road and explain the steps that will lead us to the final *MCPP* theory, which will be developed in the subsequent sections. Therefore, in Section 6, and after extending Albert Einstein's relativity to the imaginary and complex sets, I will link my original theory of Metarelativity to my novel complex probability paradigm; hence, the first simplified model of *MCPP* will be developed. Finally, in Section 7, we will present the conclusion of the first chapter and then mention the list of references cited in the current research work. Moreover, in the second following chapter and in Section 1, a more general second model will be established. Furthermore, in Section 2, a wider third model will be presented. And in Section 3, the final and the most general model of *MCPP*, which takes into account the case of electromagnetic waves will be elaborated. Additionally, in Section 4, we will present some very important consequences of the *MCPP* paradigm. Finally, I conclude the work by doing a comprehensive summary in Section 5 and then present the list of references cited in the second research chapter.

2. The purpose and the advantages of the current publication

To summarize, the advantages and the purposes of this current work are to [4–24]:

1. Extend the theory of classical probability to encompass the complex numbers set; hence, to bond the theory of probability to the field of complex variables and analysis in mathematics. This mission was elaborated on and initiated in my earlier 21 papers.

2. Apply the novel probability axioms and *CPP* paradigm to Metarelativity and hence to bond my Metarelativity theory to my Complex Probability Paradigm and thus show that:

$$\begin{aligned} \mathcal{C} &= \mathcal{R} + \mathcal{M} \\ &= (R_1 + R_2 + R_3) + (M_1 + M_2 + M_3) \\ &= (R_1 + M_1) + (R_2 + M_2) + (R_3 + M_3) \\ &= \mathcal{C}_1 + \mathcal{C}_2 + \mathcal{C}_3 = \mathbf{G}_1 + \mathbf{G}_2 + \mathbf{G}_3 = \mathbf{G}. \end{aligned}$$

3. Show that all nondeterministic phenomena like in the problems considered here can be expressed deterministically in the complex probabilities set and total universe $\mathbf{G} = \mathcal{C}$.

4. Compute and quantify both the degree of our knowledge and the chaotic factor of the probability distributions and *MCPP* in the sets $\mathcal{R}$, $\mathcal{M}$, and $\mathcal{C}$.

5. Represent and show the graphs of the functions and parameters of the innovative paradigm related to Metarelativity.

6. Demonstrate that the classical concept of probability is permanently equal to one in the set of complex probabilities; hence, no randomness, no chaos, no ignorance, no uncertainty, no nondeterminism, no unpredictability, and no information loss or gain exist in:

$$\begin{aligned} &\mathbf{G}\ (\text{complex matter and energy set}) = \mathcal{C}\ (\text{complex probabilities set}) \\ &\quad = \mathcal{R}\ (\text{real probabilities set}) + \mathcal{M}\ (\text{imaginary probabilities set}). \end{aligned}$$

7. Explain the existence of dark matter and dark energy that exist in $\mathbf{G}_2 \subset \mathbf{G}$.

8. Prepare to implement this creative model to other topics and problems in physics. These will be the job to be accomplished in my future research publications.

Concerning some applications of the novel-founded paradigm and as a future work, it can be applied to any nondeterministic phenomenon in science. And compared with existing literature, the major contribution of the current research work is to apply the innovative paradigm of *CPP* to Metarelativity and to express it completely deterministically as well as to determine the corresponding mass and energy of dark matter and dark energy.

The next figure displays the major purposes of the Metarelativistic Complex Probability Paradigm (*MCPP*) (**Figure 1**).

3. The complex probability paradigm

3.1 The original Andrey Nikolaevich Kolmogorov system of axioms

The simplicity of Kolmogorov's system of axioms may be surprising [4–24]. Let E be a collection of elements $\{E_1, E_2, \ldots\}$ called elementary events and let F be a set of subsets of E called random events [25–29]. The five axioms for a finite set E are:

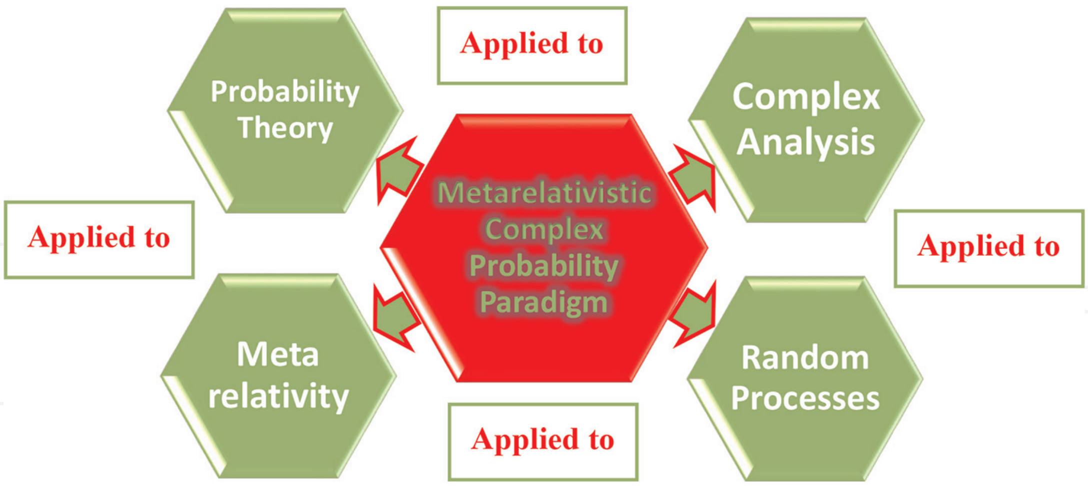

Figure 1.
The diagram of the Metarelativistic Complex Probability Paradigm's major purposes and goals.

Axiom 1: F is a field of sets.
Axiom 2: F contains the set E.
Axiom 3: A nonnegative real number $P_{rob}(A)$, called the probability of A, is assigned to each set A in F. We have always $0 \le P_{rob}(A) \le 1$.
Axiom 4: $P_{rob}(E)$ equals 1.
Axiom 5: If A and B have no elements in common, the number assigned to their union is:

$$P_{rob}(A \cup B) = P_{rob}(A) + P_{rob}(B)$$

Hence, we say that A and B are disjoint; otherwise, we have:

$$P_{rob}(A \cup B) = P_{rob}(A) + P_{rob}(B) - P_{rob}(A \cap B)$$

And we say also that: $P_{rob}(A \cap B) = P_{rob}(A) \times P_{rob}(B/A) = P_{rob}(B) \times P_{rob}(A/B)$ which is the conditional probability. If both A and B are independent then:

$$P_{rob}(A \cap B) = P_{rob}(A) \times P_{rob}(B).$$

Moreover, we can generalize and say that for N disjoint (mutually exclusive) events $A_1, A_2, \dots, A_j, \dots, A_N$ (for $1 \le j \le N$), we have the following additivity rule:

$$P_{rob}\left(\bigcup_{j=1}^{N} A_j\right) = \sum_{j=1}^{N} P_{rob}(A_j)$$

And we say also that for N independent events $A_1, A_2, \dots, A_j, \dots, A_N$ (for $1 \le j \le N$), we have the following product rule:

$$P_{rob}\left(\bigcap_{j=1}^{N} A_j\right) = \prod_{j=1}^{N} P_{rob}(A_j)$$

3.2 Adding the imaginary part $\mathcal{M}$

Now, we can add to this system of axioms an imaginary part such that:

Axiom 6: Let $P_m = i \times (1 - P_r)$ be the probability of an associated complementary event in $\mathcal{M}$ (the imaginary part or probability universe) to the event A in $\mathcal{R}$ (the real part or probability universe). It follows that $P_r + P_m/i = 1$, where i is the imaginary number with $i = \sqrt{-1}$ or $i^2 = -1$.

Axiom 7: We construct the complex number or vector $Z = P_r + P_m = P_r + i(1 - P_r)$ having a norm $|Z|$ such that:

$$|Z|^2 = P_r^2 + (P_m/i)^2.$$

Axiom 8: Let Pc denote the probability of an event in the complex probability set and universe $\mathcal{C}$, where $\mathcal{C} = \mathcal{R} + \mathcal{M}$. We say that Pc is the probability of an event A in $\mathcal{R}$ with its associated and complementary event in $\mathcal{M}$ such that:

$$Pc^2 = (P_r + P_m/i)^2 = |Z|^2 - 2iP_rP_m \text{ and is always equal to 1.}$$

We can see that by taking into consideration the set of imaginary probabilities we added three new and original axioms and consequently the system of axioms defined by Kolmogorov was hence expanded to encompass the set of imaginary numbers and realm.

3.3 A concise interpretation of the original *CPP* paradigm

To summarize the novel *CPP* paradigm, we state that in the real probability universe $\mathcal{R}$ the degree of our certain knowledge is undesirably imperfect and hence unsatisfactory, thus we extend our analysis to the set of complex numbers $\mathcal{C}$, which incorporates the contributions of both the set of real probabilities, which is $\mathcal{R}$ and the complementary set of imaginary probabilities, which is $\mathcal{M}$. Afterward, this will yield an absolute and perfect degree of our knowledge in the probability universe $\mathcal{C} = \mathcal{R} + \mathcal{M}$ because $Pc = 1$ constantly and permanently. As a matter of fact, the work in the universe $\mathcal{C}$ of complex probabilities gives way to a sure forecast of any stochastic experiment, since in $\mathcal{C}$ we remove and subtract from the computed degree of our knowledge the measured chaotic factor. This will generate in the universe $\mathcal{C}$ a probability equal to 1 as it is shown and proved in the following equation: $Pc^2 = DOK - Chf = DOK + MChf = 1 = Pc$. Many applications which take into consideration numerous continuous and discrete probability distributions in my 21 previous research papers confirm this hypothesis and innovative paradigm [4–24]. The Extended Kolmogorov Axioms (*EKA* for short) or the Complex Probability Paradigm (*CPP* for short) can be shown and summarized in the next illustration (**Figure 2**):

4. A concise review of Metarelativity

4.1 The new Metarelativistic transformations

Consider the following two inertial systems of referential (**Figure 3**) [1]:

Assume that v becomes greater than c, which is the velocity of light, the system of equations called metarelativistic transformations, which are the extension of relativistic Lorentz transformations to the imaginary space-time are:

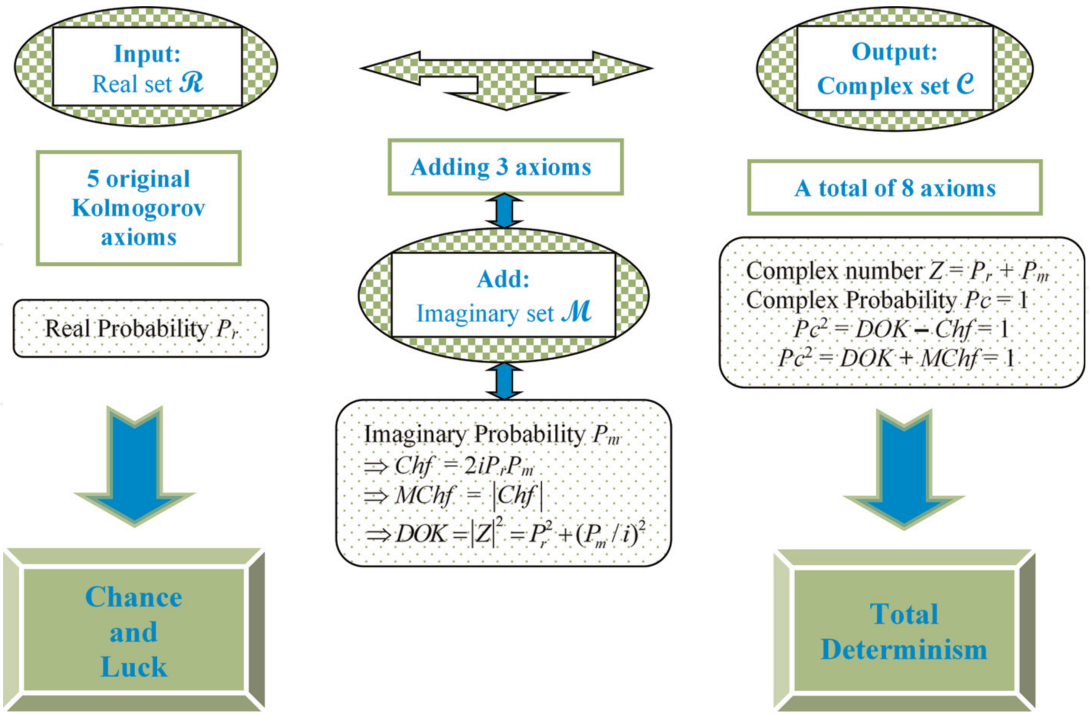

Figure 2.
The EKA *or the* CPP *diagram.*

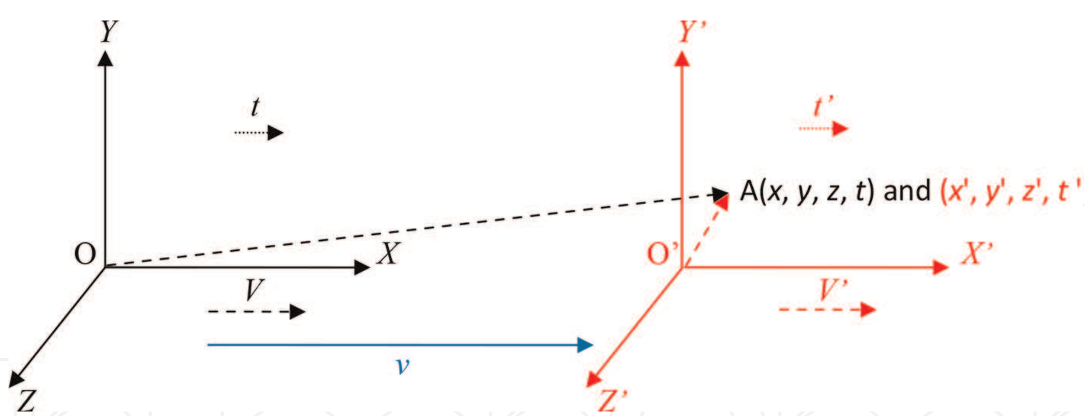

Figure 3.
Two inertial systems of referential.

$$\begin{cases} x' = \dfrac{(x-vt)}{\sqrt{(-1)\times\left(\dfrac{v^2}{c^2}-1\right)}} = \dfrac{(x-vt)}{\sqrt{-1}\times\sqrt{\dfrac{v^2}{c^2}-1}} = \dfrac{(x-vt)}{\sqrt{i^2}\times\sqrt{\dfrac{v^2}{c^2}-1}} = \dfrac{(x-vt)}{\pm i\times\sqrt{\dfrac{v^2}{c^2}-1}} = \dfrac{\pm i\times(x-vt)}{\sqrt{\dfrac{v^2}{c^2}-1}} \\ y' = y \\ z' = z \\ t' = \dfrac{\left(t-\dfrac{vx}{c^2}\right)}{\sqrt{(-1)\times\left(\dfrac{v^2}{c^2}-1\right)}} = \dfrac{\left(t-\dfrac{vx}{c^2}\right)}{\sqrt{-1}\times\sqrt{\dfrac{v^2}{c^2}-1}} = \dfrac{\left(t-\dfrac{vx}{c^2}\right)}{\sqrt{i^2}\times\sqrt{\dfrac{v^2}{c^2}-1}} = \dfrac{\left(t-\dfrac{vx}{c^2}\right)}{\pm i\times\sqrt{\dfrac{v^2}{c^2}-1}} = \dfrac{\pm i\times\left(t-\dfrac{vx}{c^2}\right)}{\sqrt{\dfrac{v^2}{c^2}-1}} \end{cases}$$

where i is the imaginary number such that: $i^2 = -1$ and $\sqrt{-1} = \pm i$ and $\frac{1}{\pm i} = \mp i$.

4.2 The mass of matter and metamatter

We have in special relativity: $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = m_{G1}$.

If v becomes greater than $c = 300,000$ km/s, then m becomes equal to:

$$m = \frac{m_0}{\sqrt{(-1) \times \left(\frac{v^2}{c^2} - 1\right)}} = \frac{m_0}{\sqrt{-1} \times \sqrt{\frac{v^2}{c^2} - 1}} = \frac{m_0}{\sqrt{i^2} \times \sqrt{\frac{v^2}{c^2} - 1}} = \frac{m_0}{\pm i \times \sqrt{\frac{v^2}{c^2} - 1}}$$
$$= \frac{\pm i \times m_0}{\sqrt{\frac{v^2}{c^2} - 1}} = m_{G2}$$

yielding hence two imaginary and superluminal particles $+i|m_{G2}|$ and $-i|m_{G2}|$ where $|m_{G2}|$ is the module or the norm of the imaginary mass m_{G2}. They are called imaginary in the sense that they lay in the four-dimensional superluminal universe of imaginary numbers that we call $\mathbf{G_2}$ or the metauniverse. Matter which has increased throughout the whole process of special relativity will become equal to infinity when velocity reaches c as it is apparent in the equations, and in the new dimensions matter is imaginary due to the imaginary dimensions that we defined in the theory of Metarelativity. We say that beneath c we are working in the subluminal universe $\mathbf{G_1}$ or in the universe, and beyond c that we are working in $\mathbf{G_2}$ or in the metauniverse. Additionally, if the velocity is equal to c, we say that we are working in the luminal universe of electromagnetic waves and that is denoted by $\mathbf{G_3}$.

Firstly, in the first following equation:
$m = \frac{+im_0}{\sqrt{\frac{v^2}{c^2}-1}} = m_{G2}(\text{MetaParticle})$.

That means that matter now will decrease till it vanishes whenever the velocity reaches infinity, which means that mass is equal to zero at the velocity infinity.

Secondly, in the second following equation:
$m = \frac{-im_0}{\sqrt{\frac{v^2}{c^2}-1}} = m_{G2}(\text{MetaAntiParticle})$.

we say mathematically, that if v tends to infinity, m tends to zero. Matter now will continue increasing as in the equation till it vanishes whenever the velocity reaches infinity, which means that mass is equal to zero at the velocity infinity.

Thirdly, electromagnetic waves (EW), which travel at the velocity of light c exist in the universe $\mathbf{G_3}$ and have a mass:

$$m_{G3} = m_{EW} = \frac{hf}{c^2}$$

where h is Planck's constant and f is the frequency of the EW.

In the total universe $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2} + \mathbf{G_3}$ we have:

$$m_G = m_{G1} + m_{G2} + m_{G3} = m_{G1} \pm i|m_{G2}| + \frac{hf}{c^2}$$

And we can notice that m_G belongs to the set of complex numbers denoted in mathematics by $\mathbb{C}$. The following graphs illustrate these facts (**Figures 4** and **5**).

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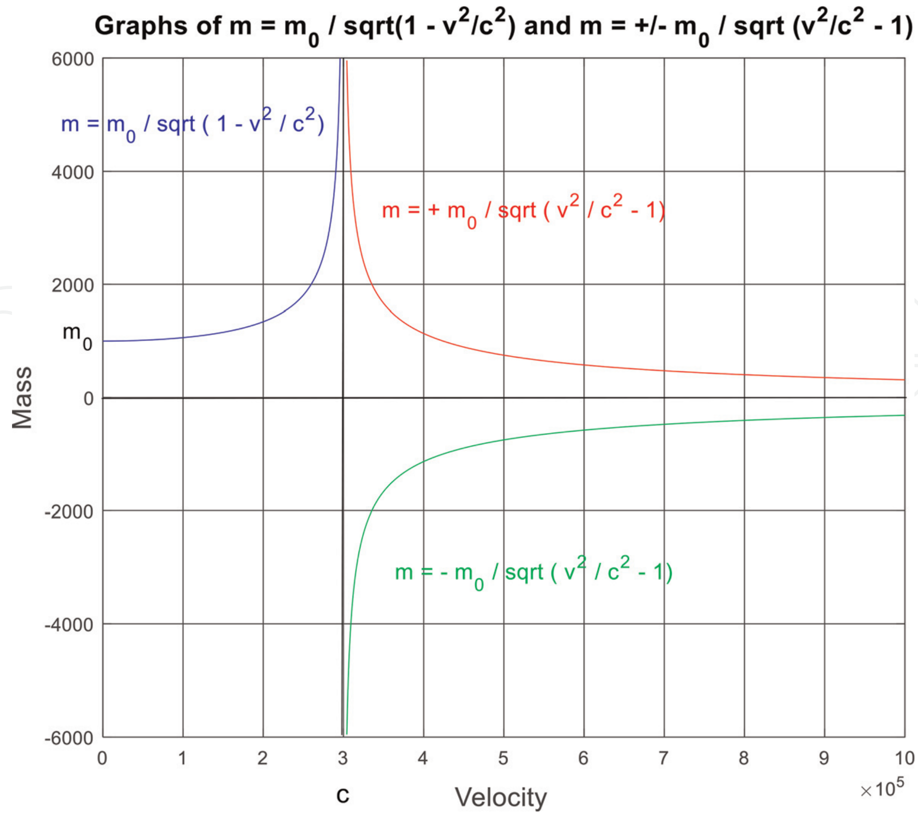

Figure 4.
The graphs of $m = m_0/\sqrt{1-\frac{v^2}{c^2}}$ *in blue for* $0 \leq v < c$ *and of* $m = +m_0/\sqrt{\frac{v^2}{c^2}-1}$ *in red and of* $m = -m_0/\sqrt{\frac{v^2}{c^2}-1}$ *in green for* $v > c$.

Graphically, we can represent the two complementary metaparticles $+i|m_{G2}|$ and $-i|m_{G2}|$ and their annihilation in **Figure 6** as in the following reactions:

$+i\,|m_{G2}| - i\,|m_{G2}| \rightarrow 2m_{G1}$ and $+i\,|m_{G2}| - i\,|m_{G2}| \rightarrow 2$ photons.

4.3 The energy and the metaenergy

We know from special relativity that energy is given by:

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1-\frac{v^2}{c^2}}} = E_{G1}$$

In Metarelativity, we have accordingly the imaginary energy or metaenergy. It is clear from the equation above that this metaenergy can be positive as:

$E = \frac{+im_0 \times c^2}{\sqrt{\frac{v^2}{c^2}-1}} = E_{G2}$ (MetaParticle Energy or MetaEnergy).

or it can be negative as:

$E = \frac{-im_0 \times c^2}{\sqrt{\frac{v^2}{c^2}-1}} = E_{G2}$(MetaAntiParticle Energy or MetaAntiEnergy).

Additionally, the luminal electromagnetic waves (EW) in the universe $\mathbf{G_3}$ have energy:

$$E_{G3} = E_{EW} = hf$$

where h is Planck's constant and f is the frequency of the EW.

Therefore, in the total universe $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2} + \mathbf{G_3}$ we have:

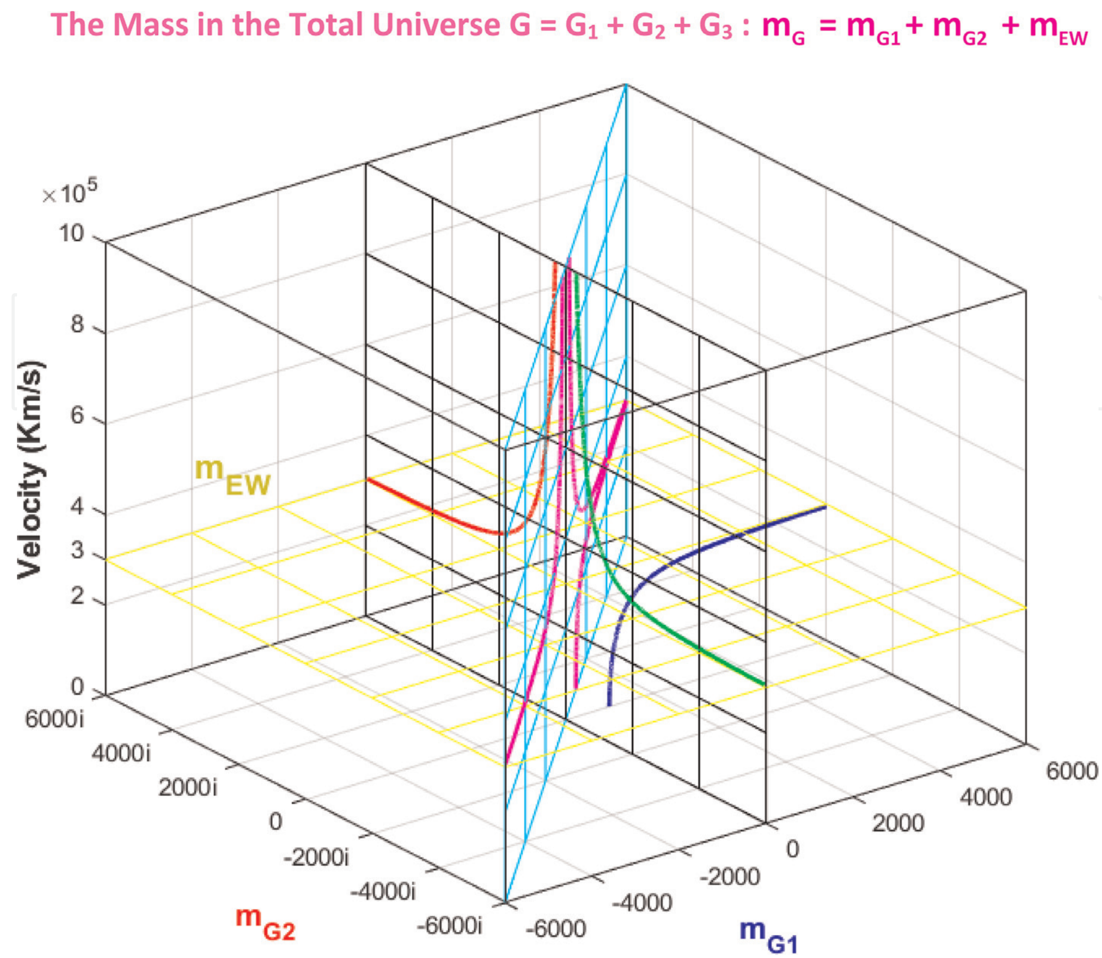

Figure 5.
The graphs of $m_{G1}(v)$ in blue, of $+i|m_{G2}(v)|$ in red, of $-i|m_{G2}(v)|$ in green, where m_{EW} lies in the yellow plane of equation v = c, and of $m_G = m_{G1} + m_{G2} + m_{EW} = m_{G1} \pm i|m_{G2}| + \frac{hf}{c^2}$ in magenta for $0 \leq v < +\infty$ in the complex plane in cyan.

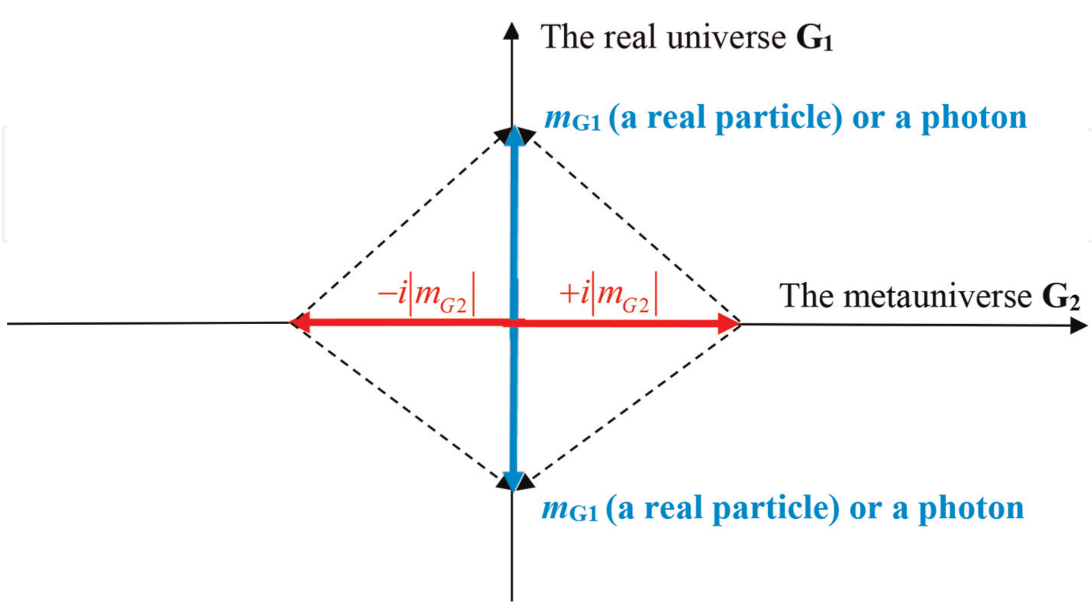

Figure 6.
The two complementary particles of metamatter $+i|m_{G2}|$ and $-i|m_{G2}|$ where $|m_{G2}|$ is the module or the norm of the imaginary mass m_{G2} in the metauniverse G_2 and their annihilation into the real matter in the universe G_1 or into photons in the universe G_3.

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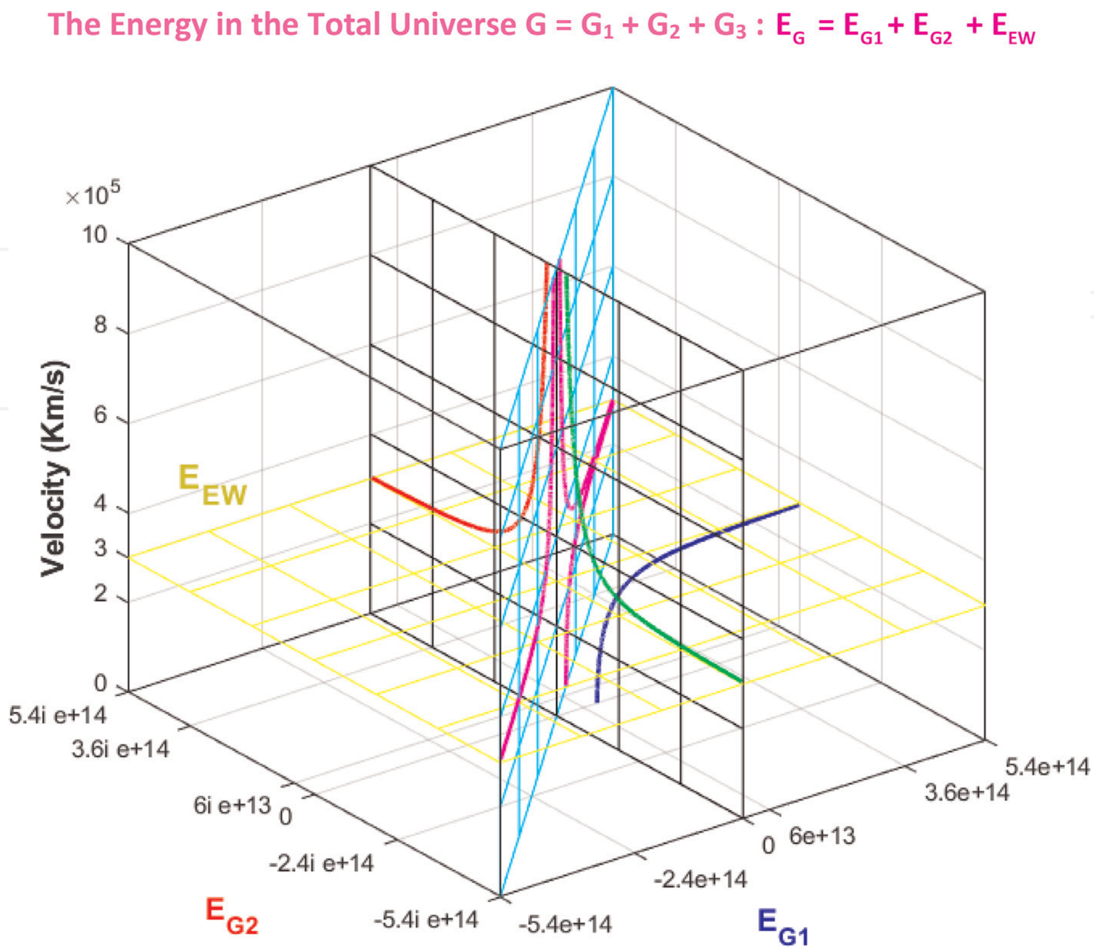

Figure 7.
The graphs of $E_{G1}(v)$ in blue, of $+i|E_{G2}(v)|$ in red, of $-i|E_{G2}(v)|$ in green, where E_{EW} lies in the yellow plane of equation v = c, and of $E_G = E_{G1} + E_{G2} + E_{EW} = E_{G1} \pm i|E_{G2}| + hf$ in magenta for $0 \le v < +\infty$ in the complex plane in cyan.

$$E_G = E_{G1} + E_{G2} + E_{G3} = E_{G1} \pm i|E_{G2}| + hf.$$

And we can notice that E_G belongs to the set of complex numbers denoted in mathematics by $\mathbb{C}$. Additionally, $|E_{G2}|$ is the module or the norm of the imaginary energy E_{G2} in the metauniverse $\mathbf{G_2}$. The following graph illustrates these facts (**Figure 7**).

4.4 Time intervals and imaginary time

When $v > c$, we get:

$$T' = \frac{\pm i \times T}{\sqrt{\frac{v^2}{c^2} - 1}}$$

If $T' = \frac{+i\times T}{\sqrt{\frac{v^2}{c^2}-1}}$ then this means that when v increases, T' decreases (time contraction).

And if $T' = \frac{-i\times T}{\sqrt{\frac{v^2}{c^2}-1}}$ then this means that when v increases, T' increases (time dilation).

Figure 8.
The flow of time in both the universe $\mathbf{G_1}$ *and in a part of the metauniverse* $\mathbf{G_2}$.

Concerning the explanation of this is that firstly time goes clockwise in the new four-dimensional continuum $\mathbf{G_2}$ relative to the universe $\mathbf{G_1}$ since it is positive, and secondly it goes counterclockwise relative to the universe $\mathbf{G_1}$ since it is negative. It is to say once more that the imaginary number "i" identifies the new four dimensions that define $\mathbf{G_2}$ (**Figure 8**).

4.5 The real and imaginary lengths

When $v > c$, we will have:

$$L' = \pm i \times L\sqrt{\frac{v^2}{c^2} - 1}$$

If $L' = +i \times L\sqrt{\frac{v^2}{c^2} - 1}$ this means that when v increases so L' increases (Length dilation).

And if $L' = -i \times L\sqrt{\frac{v^2}{c^2} - 1}$ this means that when v increases so L' decreases (Length contraction).

In fact, the minus sign confirms the fact that a length contraction can occur in $\mathbf{G_2}$ when $v > c$ similar to the length contraction in the region where $v < c$ that means in the universe $\mathbf{G_1}$.

4.6 The entropy and the metaentropy

To understand the meaning of negative time in $\mathbf{G_2}$ relative to $\mathbf{G_1}$, then entropy is the best tool. We know that entropy is defined as $dS \geq 0$ in the second principle of thermodynamics. We say that when time grows, then entropy increases. Due to the fact that time is negative as one possible solution in $\mathbf{G_2}$, this implies that we can have $dS \leq 0$. Consequently, and for this case, we say that when time flows, then entropy (or metaentropy) decreases. This means directly the following: The direction of evolution in a part of $\mathbf{G_2}$ is the opposite to that in $\mathbf{G_1}$.

4.7 The transformation of velocities

We have from special relativity: $v = \frac{V' - V}{\left(\frac{VV'}{c^2} - 1\right)} \Leftrightarrow v = \frac{c^2(V' - V)}{(VV' - c^2)}$.

First Case:

This is the case of two bodies in $\mathbf{G_1}$, where their velocities are smaller than c.

$$\overrightarrow{\mathrm{G}_1} \qquad\qquad \overrightarrow{\mathrm{G}_1}$$

We note that: $V = fc$, where $0 \leq f < 1$ and $V' = f'c$, where $0 \leq f' < 1$.

This implies that:

$$v = \frac{c^2(f'c - fc)}{(ff'c^2 - c^2)} = \frac{c^2 \times c \times (f' - f)}{c^2 \times (ff' - 1)} = \frac{c(f' - f)}{ff' - 1}$$

This relation is the one we use in relativistic computations. So, it is not new to us and just as predicted by special relativity.

Second Case:

This is the case of a body in $\mathbf{G_1}$ (where the velocity is $< c$) and a beam of light (where the velocity is c).

$$\overrightarrow{\text{Light}} \qquad\qquad \overrightarrow{\text{G}_1}$$

We have now: $V = c$ and $V' = f'c$, where $0 \leq f' < 1$. Then:

$$v = \frac{c^2(f'c - c)}{((c \times f'c) - c^2))} = \frac{c^3 \times (f' - 1)}{c^2 \times (f' - 1)} = c$$

This means that light is the limit velocity in $\mathbf{G_1}$ and is constant in it whatever the velocity of the body in $\mathbf{G_1}$ relative to the beam of light. So just like Albert Einstein's special relativity has predicted.

Third Case:

This is the case of a beam of light relative to another beam of light.

$$\overrightarrow{\text{Light}} \qquad\qquad \overrightarrow{\text{Light}}$$

$$\lim_{V' \to c} v = \lim_{V' \to c}\left[\frac{c^2(V' - V)}{(VV' - c^2)}\right] = \lim_{f' \to 1}\left[\frac{c \times (f' - 1)}{(f' - 1)}\right] = c$$

We know that we have here $V = c$ and $V' = c$. In this case, $v = c$, this is not new to us also. It is the consequence of the relativistic transformation also.

Fourth Case:

This is the case of a beam of light relative to a moving body in $\mathbf{G_2}$, where the velocity is greater than c.

$$\overrightarrow{\text{Light}} \qquad\qquad \overrightarrow{\text{G}_2}$$

We have now: $V = c$ and $V' = f'c$, where $1 < f' < +\infty$. Then:

$$v = \frac{c^2(f'c - c)}{((c \times f'c) - c^2))} = \frac{c^3 \times (f' - 1)}{c^2 \times (f' - 1)} = c$$

This means that relative to $\mathbf{G_2}$, light is still the limit velocity and is still constant. In other words, $\mathbf{G_2}$ relative to Light is similar to $\mathbf{G_1}$ relative to Light. Light is the limit velocity in both $\mathbf{G_1}$ and $\mathbf{G_2}$. This fact will be more clarified and more understood in the fifth case.

Fifth Case:

This is the case of a moving body in $\mathbf{G_2}$ relative to another moving body in $\mathbf{G_2}$.

$$\overrightarrow{G_2} \qquad\qquad \overrightarrow{G_2}$$

We have from Metarelativity: $m = \frac{\pm i \times m_0}{\sqrt{\frac{v^2}{c^2}-1}} \Leftrightarrow m_{G2} = \frac{m_{0,G2}}{\sqrt{\frac{v^2}{c^2}-1}}$.where $m_{0,G2}$ is the starting or the smallest mass in $\mathbf{G_2}$.

$$\text{So, if } m_{G2} = m_{0,G2} \Leftrightarrow m_{0,G2} = \frac{m_{0,G2}}{\sqrt{\frac{V^2}{c^2}-1}} \Leftrightarrow 1 = \frac{1}{\sqrt{\frac{V^2}{c^2}-1}}$$

$$\Leftrightarrow \sqrt{\frac{V^2}{c^2}-1} = 1 \Leftrightarrow \frac{V^2}{c^2} - 1 = 1^2 = 1 \Leftrightarrow \frac{V^2}{c^2} = 2 \Leftrightarrow V^2 = 2c^2$$
$$\Leftrightarrow V = c\sqrt{2}$$

That means that the starting mass which is $m_{0,G2}$ in the metauniverse $\mathbf{G_2}$ occurs when $V = c\sqrt{2}$.

Assume that $V = c\sqrt{2}$ (the smallest velocity in $\mathbf{G_2}$) and that $V' = \beta c\sqrt{2}$ that is any velocity greater or equal to the starting velocity, in other words: $\beta \geq 1$. This implies that:

$$v = \frac{c^2\left(\beta c\sqrt{2} - c\sqrt{2}\right)}{(2\beta c^2 - c^2)} = \frac{c\sqrt{2}(\beta - 1)}{(2\beta - 1)}$$

If $\beta = 1$ then $v = 0$.

If $\beta \to +\infty$ then $v \to \frac{c\sqrt{2}}{2} = 0.7071c < c$.

This is similar to the relativistic transformations since we have:
$0 \leq v \leq \frac{c\sqrt{2}}{2} = 0.7071c < c$. As if we are working in $\mathbf{G_1}$ exactly. This means that the universe $\mathbf{G_2}$ relatively to itself behaves like the universe $\mathbf{G_1}$ relative to itself since the velocity of $\mathbf{G_2}$ relative to $\mathbf{G_2}$ is smaller than c just like the velocity of $\mathbf{G_1}$ relative to $\mathbf{G_1}$. This fact can be also explained as follows: $\mathbf{G_2}$ is as real as $\mathbf{G_1}$ relative to itself but at a different level of experience and in higher dimensions. Accordingly, we can say that $\mathbf{G_2}$ relative to itself is a "real" universe but relative to $\mathbf{G_1}$ is an "imaginary" universe as it will be shown in the sixth and seventh cases.

Sixth Case:

This is the case of $\mathbf{G_2}$ relative to $\mathbf{G_1}$.

$$\overrightarrow{G_1} \qquad\qquad \overrightarrow{G_2}$$

We note that: $V = fc$, where $0 \leq f < 1$ and $V' = f'c$, where $1 < f' < +\infty$.

This implies that:

$$v = \frac{c^2\left(f'c - fc\right)}{\left(ff'c^2 - c^2\right)} = \frac{c^2 \times c \times \left(f' - f\right)}{c^2 \times \left(ff' - 1\right)} = \frac{c\left(f' - f\right)}{ff' - 1}$$

So, if $f = 0 \Rightarrow v = \frac{c\left(f'-0\right)}{\left(0 \times f'\right)-1} = \frac{f'c}{-1} = -f'c \Rightarrow |v| = f'c > c$ since $1 < f' < +\infty$.

And if $f \to 1 \Rightarrow v \to \frac{c\left(f'-1\right)}{\left(1 \times f'\right)-1} = \frac{c\left(f'-1\right)}{f'-1} = c$.

We have here $|v| > c$. This implies that the metarelativistic transformations are needed here and for the first time.

Seventh Case:

This is the case of $\mathbf{G_1}$ relative to $\mathbf{G_2}$.

$$\overrightarrow{G_2} \qquad\qquad \overrightarrow{G_1}$$

We note that: $V = fc$, where $1 < f < +\infty$ and $V' = f'c$, where $0 \le f' < 1$.

This implies that:

$$v = \frac{c^2(f'c - fc)}{(ff'c^2 - c^2)} = \frac{c^2 \times c \times (f' - f)}{c^2 \times (ff' - 1)} = \frac{c(f' - f)}{ff' - 1}$$

So, if $f' = 0 \Rightarrow v = \frac{c(0-f)}{(f\times 0)-1} = \frac{-fc}{-1} = fc > c$ since $1 < f < +\infty$.

And if $f' \to 1 \Rightarrow v \to \frac{c(1-f)}{(f\times 1)-1} = \frac{(1-f)c}{f-1} = -c \Rightarrow |v| \to c$.

We have here $|v| > c$. This implies that the metarelativistic transformations are needed here also.

4.8 The new principle of Metarelativity

Now, if we want to elaborate on the new principle of Metarelativity, it will be:

"Inertial observers must correlate their observations by means of relativistic Lorentz transformations if the velocity is smaller than c and by means of the metarelativistic transformations if the velocity is greater than c, and all physical quantities must transform from one inertial system to another in such a way that the expression of the physical laws is the same for all inertial observers. The subluminal universe is denoted by $\mathbf{G_1}$, the superluminal universe is denoted by $\mathbf{G_2}$, and the luminal universe of frequencies is denoted by $\mathbf{G_3}$. The sum of $\mathbf{G_1}$, of the electromagnetic waves **EW**, and of $\mathbf{G_2}$ is denoted by:

$$\mathbf{G} = \mathbf{G_1} + \mathbf{G_3} + \mathbf{G_2} = \mathbf{G_1} + \boldsymbol{Light} + \mathbf{G_2}$$

and all electromagnetic waves (that include light) are at constant velocity in both $\mathbf{G_1}$ and $\mathbf{G_2}$."

As it was shown that the new theory does not destroy Einstein's theory of relativity that we know at all but on the contrary, it proves its veracity and then expands it to the set of complex masses, time, lengths, and energies, which is the eight-dimensional complex hyperspace $\mathbb{C}$ or equivalently in the total universe **G**.

5. The four models of *MCPP*: The road to the final and most general model of *MCPP*

In this work and in the following sections in chapters 1 and 2 we will consider three simplified models of *MCPP* then present at the end the final and most general model [1–24, 30–43]. Each model is an enhanced and wider model than the previous one. In all four models, we will consider the velocities of the bodies moving in **G** to be random variables that follow certain probability distributions (*PDFs*) and certain corresponding cumulative probability distribution functions (*CDFs*) in both the subluminal universe $\mathbf{G_1}$ and the superluminal metauniverse $\mathbf{G_2}$. I have followed this methodology in order to develop gradually and systematically the *MCPP* paradigm and in order to reach the final and most general model of *MCPP* that can be adopted in any possible and imaginable situation. Even the deterministic case, which is a special case of the general random *CPP,* was also presented and considered in order to show that *MCPP* is valid everywhere either in the deterministic or in the random case. Consequently, and in each model, we evaluate the associated real, imaginary, and complex probabilities as well as all the related *MCPP* parameters in the probabilities sets $\mathcal{R}$, $\mathcal{M}$, and $\mathcal{C}$; hence, in the universes $\mathbf{G_1}$, $\mathbf{G_2}$, $\mathbf{G_3}$, and $\mathbf{G}$. Thus, we connect successfully *CPP* with Metarelativity to unify both theories in a general and a unified paradigm that we called *MCPP*.

In the first simplified model, the body velocities PDF_1 in $\mathbf{G_1}$ and PDF_2 in $\mathbf{G_2}$ are taken to be both Gaussian and normal, in addition, the velocity in the metauniverse $\mathbf{G_2}$ varies here between c (light velocity) and $2c$. In this reduced model, we restricted our study to $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2}$.

In the second simplified and more general model, the body velocities PDF_1 in $\mathbf{G_1}$ and PDF_2 in $\mathbf{G_2}$ are taken also to be both Gaussian and normal, in addition, the velocity in $\mathbf{G_2}$ varies between c and nc, where n is an arbitrary and predetermined number having: $\forall n, n \in \mathbb{R}^+ : n > 1 \Leftrightarrow n \in (1, +\infty)$. This was done since the velocities in $\mathbf{G_2}$ according to Metarelativity vary between c and infinity. Hence, the second model is an improved version of the first one. In this model, we restricted our study also to $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2}$.

In the third simplified and wider model, the body velocities PDF_1 in $\mathbf{G_1}$ and PDF_2 in $\mathbf{G_2}$ are taken to follow any possible probability distribution whether discrete or continuous and they do not have to be similar at all like in the previous two models, this in order to be realistic. In addition, the velocity in $\mathbf{G_2}$ varies between c and nc also. Thus, the third model is an enhanced model. In this model, we have considered also only the universe $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2}$.

The final and most general model is the sought model of *MCPP*. It is the goal of all the calculations made and of the methodology adopted. Here, the body velocities PDF_1 in $\mathbf{G_1}$ and PDF_2 in $\mathbf{G_2}$ are taken to follow any possible probability distribution whether discrete or continuous and they do not have to be similar at all in order to be totally realistic. The velocity in $\mathbf{G_2}$ varies between c and nc also. Additionally, we have included the contributions of the luminal universe of electromagnetic waves, which is $\mathbf{G_3}$, where the velocity of the EW is c and the corresponding frequency follows any possible probability distribution PDF_3 and CDF_3, respectively. Therefore, we have considered here the most general case which is the total universe $\mathbf{G} = \mathbf{G_1} + \mathbf{G_2} + \mathbf{G_3}$. This final model links definitively and in the most general way *CPP* with Metarelativity into the unified paradigm of *MCPP*.

Furthermore, in all four models in the two chapters, we have defined, calculated, simulated, illustrated, and drawn all the probabilities and all the *MCPP* parameters in $\mathcal{R}$, $\mathcal{M}$, and $\mathcal{C} = \mathbf{G}$.

6. The Metarelativistic complex probability paradigm (*MCPP*): A first model

6.1 The real and imaginary probabilities

Let v_1 be the velocity of a body in R_1 with $0 \le v_1 < c$ and let it be a random variable that follows the normal distribution: $N(\overline{v}_1 = c/2, \sigma_{v1} = c/6)$ where $\overline{v}_1$ is the mean or the expectation of this symmetric normal probability distribution of v_1 or $PDF_1(v_1)$ and σ_{v1} is its corresponding standard deviation.

And let v_2 be the velocity of a body in R_2 with $c < v_2 \le 2c$ and let it be a random variable that follows the normal distribution: $N(\overline{v}_2 = 3c/2, \sigma_{v2} = c/6)$ where $\overline{v}_2$ is the mean or the expectation of this symmetric normal probability distribution of v_2 or $PDF_2(v_2)$ and σ_{v2} is its corresponding standard deviation.

Then, $P_{R1} = P_{rob}(0 \le V \le v_1) = CDF_1(0 \le V \le v_1) = \int_0^{v1} PDF_1(v)dv = \int_0^{v1} N(\overline{v} = c/2, \sigma_v = c/6)dv.$

If $v_1 < 0 \Rightarrow P_{R1} = P_{rob}(V < 0) = CDF_1(V < 0) = 0.$

If $v_1 = 0 \Rightarrow P_{R1} = P_{rob}(V \le 0) = CDF_1(V \le 0) = \int_0^0 PDF_1(v)dv = 0.$

If $v_1 = \overline{v}_1 = c/2 \Rightarrow P_{R1} = P_{rob}(0 \le V \le c/2) = CDF_1(0 \le V \le c/2) = \int_0^{c/2} PDF_1(v)dv = 0.5.$

If $v_1 \to c^- \Rightarrow P_{R1} \to P_{rob}(0 \le V < c) = CDF_1(0 \le V < c) = \int_0^c PDF_1(v)dv = \int_0^c N(\overline{v} = c/2, \sigma_v = c/6)dv = 1.$

If $v_1 > c \Rightarrow P_{R1} = P_{rob}(V > c) = CDF_1(V > c) = \int_0^{v1} PDF_1(v)dv = \left\{\int_0^c PDF_1(v)dv + \int_c^{v1} PDF_1(v)dv\right\} = (1 + 0) = 1.$

And $P_{R2} = P_{rob}(c < V \le v_2) = CDF_2(c < V \le v_2) = \int_c^{v2} PDF_2(v)dv = \int_c^{v2} N(\overline{v} = 3c/2, \sigma_v = c/6)dv.$

If $v_2 < c \Rightarrow P_{R2} = P_{rob}(V < c) = CDF_2(V < c) = 0.$

If $v_2 \to c^+ \Rightarrow P_{R2} \to \int_c^c PDF_2(v)dv = \int_c^c N(\overline{v} = 3c/2, \sigma_v = c/6)dv = 0.$

If $v_2 = \overline{v}_2 = 3c/2 \Rightarrow P_{R2} = P_{rob}(c < V \le 3c/2) = CDF_2(c < V \le 3c/2) = \int_c^{3c/2} PDF_2(v)dv = 0.5.$

If $v_2 = 2c \Rightarrow P_{R2} = P_{rob}(c < V \le 2c) = CDF_2(c < V \le 2c) = \int_c^{2c} PDF_2(v)dv = \int_c^{2c} N(\overline{v} = 3c/2, \sigma_v = c/6)dv = 1.$

If $v_2 > 2c$

$$\Rightarrow P_{R2} = P_{rob}(V > 2c) = CDF_2(V > 2c) = \int_c^{v2} PDF_2(v)dv = \left\{\int_c^{2c} PDF_2(v)dv + \int_{2c}^{v2} PDF_2(v)dv\right\} = (1 + 0) = 1$$

Moreover,

$$P_{M1} = i(1 - P_{R1}) = i[1 - P_{rob}(0 \le V \le v_1)] = i[1 - CDF_1(0 \le V \le v_1)] = iCDF_1(v_1 < V < c) = i\left[1 - \int_0^{v1} PDF_1(v)dv\right] = i\int_{v1}^c PDF_1(v)dv = i\int_{v1}^c N(\overline{v} = c/2, \sigma_v = c/6)dv$$

If $v_1 < 0 \Rightarrow$

$P_{M1} = i\int_{v1}^c PDF_1(v)dv = i\left\{\int_{v1}^0 PDF_1(v)dv + \int_0^c PDF_1(v)dv\right\} = i(0 + 1) = i \Rightarrow P_{M1}/i = 1.$

If $v_1 = 0$

$\Rightarrow P_{M1} = i[1 - P_{rob}(V \le 0)] = i[1 - CDF_1(V \le 0)] = i(1 - 0) = i \Rightarrow P_{M1}/i = 1.$

If $v_1 = \overline{v}_1 = c/2$

$$\Rightarrow P_{M1} = i[1 - P_{rob}(0 \le V \le c/2)] = i[1 - CDF_1(0 \le V \le c/2)]$$
$$= i\left[1 - \int_0^{c/2} PDF_1(v)dv\right] = i\int_{c/2}^{c} PDF_1(v)dv = i(1-0.5) = 0.5i \Rightarrow P_{M1}/i = 0.5$$

If $v_1 \to c^-$
$\Rightarrow P_{M1} \to i[1 - P_{rob}(0 \le V < c)] = i[1 - CDF_1(0 \le V < c)] = i\left[1 - \int_0^c PDF_1(v)dv\right] = i(1-1) = 0$
$\Rightarrow P_{M1}/i \to 0$
If $v_1 > c \Rightarrow P_{M1} = 0 \Rightarrow P_{M1}/i = 0$.
And

$$P_{M2} = i(1 - P_{R2}) = i[1 - P_{rob}(c < V \le v_2)] = i[1 - CDF_2(c < V \le v_2)] = iCDF_2(v_2 < V \le 2c)$$
$$= i\left[1 - \int_c^{v2} PDF_2(v)dv\right] = i\int_{v2}^{2c} PDF_2(v)dv = i\int_{v2}^{2c} N(\overline{v} = 3c/2, \sigma_v = c/6)dv$$

If $v_2 < c \Rightarrow P_{M2} = i\int_{v2}^{2c} PDF_2(v)dv = i\left\{\int_{v2}^{c} PDF_2(v)dv + \int_c^{2c} PDF_2(v)dv\right\} = i(0+1) = i$
$\Rightarrow P_{M2}/i = 1$.
If
$v_2 \to c^+ \Rightarrow P_{M2} \to i[1 - P_{rob}(V \le v_2)] = iP_{rob}(c < V \le 2c) = i \times 1 = i \Rightarrow P_{M2}/i \to 1$.
If $v_2 = \overline{v}_2 = 3c/2$

$$\Rightarrow P_{M2} = i[1 - P_{rob}(c < V \le 3c/2)] = i[1 - CDF_2(c < V \le 3c/2)]$$
$$= i\left[1 - \int_c^{3c/2} PDF_2(v)dv\right] = i\int_{3c/2}^{2c} PDF_2(v)dv = i(1-0.5) = 0.5i \Rightarrow P_{M2}/i = 0.5$$

If $v_2 = 2c$
$\Rightarrow P_{M2} = i[1 - P_{rob}(c < V \le 2c)] = i[1 - CDF_2(c < V \le 2c)] = i\left[1 - \int_c^{2c} PDF_2(v)dv\right] = i(1-1) = 0$
$\Rightarrow P_{M2}/i = 0$
If $v_2 > 2c \Rightarrow P_{M2} = 0 \Rightarrow P_{M2}/i = 0$.
We have $\mathcal{R} = R_1(0 \le v < c) + R_2(c < v \le 2c)$.

Now, let $P_R = \frac{P_{R1}+P_{R2}}{2}$ and it is equal to half of the sum of the cumulative probability that $0 \le V \le v_1$ in R_1 and the cumulative probability that $c < V \le v_2$ in R_2.

$$\Rightarrow P_R = \frac{CDF_1(0 \le V \le v_1) + CDF_2(c < V \le v_2)}{2}$$
$$= \frac{1}{2}\left\{\int_0^{v1} PDF_1(v)dv + \int_c^{v2} PDF_2(v)dv\right\}$$
$$= \frac{1}{2}\left\{\int_0^{v1} N(\overline{v} = c/2, \sigma_v = c/6)dv + \int_c^{v2} N(\overline{v} = 3c/2, \sigma_v = c/6)dv\right\}$$

We have in $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M} = \mathbf{G_1} + \mathbf{G_2}$: $0 \le v \le 2c$ with $v \ne c$.
So, if $0 \le v < c \Rightarrow P_{R1} = P_{rob}(0 \le V \le v) = CDF_1(0 \le V \le v)$.
And $P_{R2} = P_{rob}(V < c) = CDF_2(V < c) = 0$

$$\Rightarrow P_R = \frac{CDF_1(0 \le V \le v) + 0}{2} = \frac{CDF_1(0 \le V \le v)}{2} = \frac{P_{R1}}{2}$$

Therefore, we say here that we are working in the real probability universe $\mathcal{R} = R_1$ alone.
And if $c < v \le 2c \Rightarrow P_{R1} = P_{rob}(V > c) = CDF_1(V > c) = 1$.
And $P_{R2} = P_{rob}(c < V \le v) = CDF_2(c < V \le v)$

$$\Rightarrow P_R = \frac{1 + CDF_2(c < V \le v)}{2} = \frac{1 + P_{R2}}{2}$$

Therefore, we say here that we are working in the real probability universe $\mathcal{R} = R_2$ alone.
And, if $0 \le v \le 2c$ with $v \ne c \Rightarrow P_{R1} = P_{rob}(0 \le V \le v) = CDF_1(0 \le V \le v)$.
And $P_{R2} = P_{rob}(c < V \le v) = CDF_2(c < V \le v)$

$$\Rightarrow P_R = \frac{CDF_1(0 \le V \le v) + CDF_2(c < V \le v)}{2} = \frac{P_{R1} + P_{R2}}{2}$$

Therefore, we say here that we are working in the real probability universe $\mathcal{R} = R_1 + R_2$.
And consequently,
if $v < 0 \Rightarrow P_R = \frac{CDF_1(V<0)}{2} = \frac{0}{2} = 0$.
if $v = c/2 \Rightarrow P_R = \frac{CDF_1(0 \le V \le c/2) + CDF_2(V<c)}{2} = \frac{0.5+0}{2} = 0.25$.
if $v \to c^- \Rightarrow P_R \to \frac{CDF_1(0 \le V < c) + CDF_2(V<c)}{2} = \frac{1+0}{2} = 0.5$.
if $v = 3c/2 \Rightarrow P_R = \frac{CDF_1(0 \le V < c) + CDF_2(c < V \le 3c/2)}{2} = \frac{1+0.5}{2} = 0.75$.
if $v = 2c \Rightarrow P_R = \frac{CDF_1(0 \le V < c) + CDF_2(c < V \le 2c)}{2} = \frac{1+1}{2} = 1$.
We have $\mathcal{M} = M_1(0 \le v < c) + M_2(c < v \le 2c)$.
Now, let $P_M = \frac{P_{M1}+P_{M2}}{2}$ and it is equal to half of the sum of the complement of the cumulative probability that $0 \le V \le v_1$ in M_1 and the complement of the cumulative probability that $c < V \le v_2$ in M_2.

$$\Rightarrow P_M = \frac{i(1 - P_{R1}) + i(1 - P_{R2})}{2}$$
$$= \frac{2i - i(P_{R1} + P_{R2})}{2} = i - \frac{i(P_{R1} + P_{R2})}{2} = i\left[1 - \frac{(P_{R1} + P_{R2})}{2}\right] = i(1 - P_R)$$
$$\Rightarrow P_M = \frac{i[1 - CDF_1(0 \le V \le v_1)] + i[1 - CDF_2(c < V \le v_2)]}{2}$$
$$= \frac{i}{2}\left\{\int_0^{v1}[1 - PDF_1(v)]dv + \int_c^{v2}[1 - PDF_2(v)]dv\right\}$$
$$= \frac{i}{2}\left\{\int_0^{v1}[1 - N(\overline{v} = c/2, \sigma_v = c/6)]dv + \int_c^{v2}[1 - N(\overline{v} = 3c/2, \sigma_v = c/6)]dv\right\}$$
$$= \frac{i}{2}\left\{\int_{v1}^{c} N(\overline{v} = c/2, \sigma_v = c/6)dv + \int_{v2}^{2c} N(\overline{v} = 3c/2, \sigma_v = c/6)dv\right\}$$

We have in $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M} = \mathbf{G_1} + \mathbf{G_2}$: $0 \le v \le 2c$ with $v \neq c$.
So, if $0 \le v < c \Rightarrow P_{M1} = i[1 - P_{rob}(0 \le V \le v)] = i[1 - CDF_1(0 \le V \le v)]$.
And $P_{M2} = i[1 - P_{rob}(V < c)] = i[1 - CDF_2(V < c)] = i(1 - 0) = i$

$$\Rightarrow P_M = \frac{i[1 - CDF_1(0 \le V \le v)] + i}{2} = \frac{i + P_{M1}}{2} = i\left[1 - \frac{P_{R1}}{2}\right]$$

Therefore, we say here that we are working in the imaginary probability universe $\mathcal{M} = M_1$ alone.
And if $c < v \le 2c \Rightarrow P_{M1} = i[1 - P_{rob}(V > c)] = i[1 - CDF_1(V > c)] = i(1 - 1) = 0$.
And $P_{M2} = i[1 - P_{rob}(c < V \le v)] = i[1 - CDF_2(c < V \le v)]$

$$\Rightarrow P_M = \frac{0 + i[1 - CDF_2(c < V \le v)]}{2} = \frac{P_{M2}}{2} = i\left[\frac{1 - P_{R2}}{2}\right]$$

Therefore, we say here that we are working in the imaginary probability universe $\mathcal{M} = M_2$ alone.
And, if
$0 \le v \le 2c$ with $v \neq c \Rightarrow P_{M1} = i[1 - P_{rob}(0 \le V \le v)] = i[1 - CDF_1(0 \le V \le v)]$.
And $P_{M2} = i[1 - P_{rob}(c < V \le v)] = i[1 - CDF_2(c < V \le v)]$
$\Rightarrow P_M = \frac{i[1 - CDF_1(0 \le V < c)] + i[1 - CDF_2(c < V \le v)]}{2} = \frac{P_{M1} + P_{M2}}{2} = i\left[1 - \frac{P_{R1} + P_{R2}}{2}\right] = i[1 - P_R]$.
Therefore, we say here that we are working in the imaginary probability universe $\mathcal{M} = M_1 + M_2$.
And consequently, if $v < 0 \Rightarrow P_M = i\left[1 - \frac{CDF_1(V < 0)}{2}\right] = i\left[1 - \frac{0}{2}\right] = i \Rightarrow P_M/i = 1$.
if $v = c/2 \Rightarrow P_M = i\left[1 - \frac{CDF_1(0 \le V \le c/2) + CDF_2(V < c)}{2}\right] = i\left[1 - \frac{0.5 + 0}{2}\right] = 0.75i$

$$\Rightarrow P_M/i = 0.75$$

if $v \to c^- \Rightarrow P_M \to i\left[1 - \frac{CDF_1(0 \le V < c) + CDF_2(V < c)}{2}\right] = i\left[1 - \frac{1 + 0}{2}\right] = 0.5i \Rightarrow P_M/i \to 0.5$.
if $v = 3c/2 \Rightarrow P_M = i\left[1 - \frac{CDF_1(0 \le V < c) + CDF_2(c < V \le 3c/2)}{2}\right] = i\left[1 - \frac{1 + 0.5}{2}\right] = 0.25i$

$$\Rightarrow P_M/i = 0.25$$

if $v = 2c \Rightarrow P_M = i\left[1 - \frac{CDF_1(0 \le V < c) + CDF_2(c < V \le 2c)}{2}\right] = i\left[1 - \frac{1 + 1}{2}\right] = i(1 - 1) = 0$

$$\Rightarrow P_M/i = 0$$

In addition, since $v = c$ is an axis of symmetry then we can deduce from calculus that:

$$P_{R1}[0 \le v_1 < c] = P_{M2}[c < (v_2 = 2c - v_1) \le 2c]/i = 1 - P_{R2}[c < (v_2 = 2c - v_1) \le 2c]$$

Check that: $P_{R1}[0 \le (v_1 = 0) < c] = P_{M2}[c < (v_2 = 2c - v_1 = 2c - 0 = 2c) \le 2c]/i = 0$

$$P_{R1}[0 \le (v_1 = c/2) < c] = P_{M2}[c < (v_2 = 2c - v_1 = 2c - c/2 = 3c/2) \le 2c]/i = 0.5$$
$$P_{R1}[0 \le (v_1 \to c) < c] = P_{M2}[c < (v_2 = 2c - v_1 \to 2c - c = c) \le 2c]/i = 1$$

And,

$$P_{R2}[c < v_2 \le 2c] = P_{M1}[0 \le (v_1 = 2c - v_2) < c]/i = 1 - P_{R1}[0 \le (v_1 = 2c - v_2) < c]$$

Check that: $P_{R2}[c < (v_2 = 2c) \le 2c] = P_{M1}[0 \le (v_1 = 2c - v_2 = 2c - 2c = 0) < c]/i = 1$

$$P_{R2}[c < (v_2 = 3c/2) \le 2c] = P_{M1}[0 \le (v_1 = 2c - v_2 = 2c - 3c/2 = c/2) < c]/i = 0.5$$
$$P_{R2}[c < (v_2 \to c) \le 2c] = P_{M1}[0 \le (v_1 = 2c - v_2 \to 2c - c = c) < c]/i = 0$$

Therefore, for any value of $0 \le v \le 2c$ with $v \ne c$, we can write without any confusion that:

$$P_{M1} = i(1 - P_{R1}) \text{ and } P_{R1} = 1 - P_{M1}/i$$

hence, M_1 is the imaginary complementary probability universe to the real probability universe R_1.

$$\text{And } P_{M2} = i(1 - P_{R2}) \text{ and } P_{R2} = 1 - P_{M2}/i$$

hence, M_2 is the imaginary complementary probability universe to the real probability universe R_2.

Additionally, in condition and in the case where $v_2 = 2c - v_1$, we have:

$$P_{R1}[0 \le v_1 < c] + P_{R2}[c < (v_2 = 2c - v_1) \le 2c] = 1$$

hence, R_2 is the real complementary probability universe to the real probability universe R_1.

$$\text{And, } P_{M1}[0 \le v_1 < c] + P_{M2}[c < (v_2 = 2c - v_1) \le 2c] = i$$

hence, M_2 is the imaginary complementary probability universe to the imaginary probability universe M_1.

Moreover, in all cases and for any value of $v : 0 \le v \le 2c$ with $v \ne c$, we have:

$P_R = \frac{P_{R1} + P_{R2}}{2}$ where $\mathcal{R} = R_1 + R_2$.

And $P_M = \frac{P_{M1} + P_{M2}}{2}$ where $\mathcal{M} = M_1 + M_2$.

Check that:

$$P_M = \frac{i(1 - P_{R1}) + i(1 - P_{R2})}{2}$$
$$= \frac{2i - i(P_{R1} + P_{R2})}{2} = i - \frac{i(P_{R1} + P_{R2})}{2} = i\left[1 - \frac{(P_{R1} + P_{R2})}{2}\right] = i(1 - P_R)$$

Hence, $\mathcal{M}$ is the imaginary complementary probability universe to the real probability universe $\mathcal{R}$.

Moreover, we have in $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M}$, where $0 \le v \le 2c$ with $v \ne c$:

$\mathcal{C} = (R_1 + R_2) + (M_1 + M_2) = (R_1 + M_1) + (R_2 + M_2) = \mathcal{C}_1 + \mathcal{C}_2$.

In fact, in $\mathcal{C}_1$ we have: $Pc_1 = P_{R1} + P_{M1}/i = P_{R1} + (1 - P_{R1}) = 1$.

And, in $\mathcal{C}_2$ we have: $Pc_2 = P_{R2} + P_{M2}/i = P_{R2} + (1 - P_{R2}) = 1$.

And, in $\mathcal{C}$ we have:

$$Pc = P_R + P_M/i = \frac{P_{R1} + P_{R2}}{2} + \left[\frac{P_{M1} + P_{M2}}{2}\right]/i$$
$$= \frac{P_{R1} + P_{R2}}{2} + \left[\frac{i(1 - P_{R1}) + i(1 - P_{R2})}{2}\right]/i$$
$$= \frac{P_{R1} + P_{R2}}{2} + \frac{(1 - P_{R1}) + (1 - P_{R2})}{2} = \frac{P_{R1} + P_{R2}}{2} + 1 - \frac{P_{R1} + P_{R2}}{2}$$
$$= 1$$

We can write also:

$$\begin{aligned} Pc &= P_R + P_M/i \\ &= \frac{P_{R1}+P_{R2}}{2} + \left[\frac{P_{M1}+P_{M2}}{2}\right]/i = \frac{P_{R1}+P_{M1}/i}{2} + \frac{P_{R2}+P_{M2}/i}{2} = \frac{Pc_1}{2} + \frac{Pc_2}{2} \\ &= \frac{Pc_1+Pc_2}{2} = \frac{1+1}{2} = 1 \end{aligned}$$

Consequently: $Pc = Pc_1 = Pc_2 = 1$, in accordance with *CPP* axioms.

Furthermore, we have:

$\mathbf{G} = \mathbf{G_1}\ (0 \le v < c) + \mathbf{G_2}\ (c < v \le 2c)$, which means that the total universe **G** is the sum of the real subluminal universe $\mathbf{G_1}$ and the imaginary superluminal universe or metauniverse $\mathbf{G_2}$.

Additionally, the real subluminal universe $\mathbf{G_1}$ corresponds to the complex probability universe $\mathcal{C}_1$, which is also subluminal, hence: $\mathbf{G_1} = \mathcal{C}_1 = R_1 + M_1$ with $(0 \le v < c)$. And the imaginary superluminal universe $\mathbf{G_2}$ or metauniverse corresponds to the complex probability universe $\mathcal{C}_2$, which is also superluminal; hence, $\mathbf{G_2} = \mathcal{C}_2 = R_2 + M_2$ with $(c < v \le 2c)$.

Therefore,

$$P_{G1} = Pc_1 = P_{R1} + P_{M1}/i = P_{R1} + (1 - P_{R1}) = 1 \text{ and}$$
$$P_{G2} = Pc_2 = P_{R2} + P_{M2}/i = P_{R2} + (1 - P_{R2}) = 1.$$

Consequently, the complex total universe $\mathbf{G} = \mathbf{G_1}\ (0 \le v < c) + \mathbf{G_2}\ (c < v \le 2c)$, which is the sum of the universe and the metauniverse corresponds to the complex probability universe $\mathcal{C}$ having:

$\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M} = (R_1 + R_2) + (M_1 + M_2) = (R_1 + M_1) + (R_2 + M_2)$
$= \mathcal{C}_1\ (0 \le v < c) + \mathcal{C}_2\ (c < v \le 2c) = \mathbf{G_1}\ (0 \le v < c) + \mathbf{G_2}\ (c < v \le 2c)$ with $0 \le v \le 2c$ and $v \ne c$,

Hence,

$$P_G = Pc = \frac{P_{G1}+P_{G2}}{2} = \frac{Pc_1+Pc_2}{2} = \frac{1+1}{2} = 1$$

Hence, $Pc = 1$, in accordance with *CPP* axioms.

Thus, we can conclude that, by adding the complementary imaginary probabilities universes M_1, M_2 and $\mathcal{M}$ to the real probabilities universes R_1, R_2 and $\mathcal{R}$ then all random phenomena in the complex probabilities' universes $\mathcal{C}_1$, $\mathcal{C}_2$, and $\mathcal{C}$, and hence in the subluminal universe $\mathbf{G_1}$, in the superluminal universe $\mathbf{G_2}$, and in the total and complex universe **G**, become absolutely and perfectly deterministic with probabilities:

$$Pc = Pc_1 = Pc_2 = 1 \text{ and } P_G = P_{G1} = P_{G2} = 1.$$

6.2 The *MCPP* parameters of the first model

The real probabilities in $\mathcal{R} = R_1 + R_2$:

$$P_R = \frac{P_{R1}+P_{R2}}{2}$$

The imaginary complementary probabilities in $\mathcal{M} = M_1 + M_2$:

$$P_M = \frac{P_{M1} + P_{M2}}{2} = i(1 - P_R) = i\left[1 - \frac{(P_{R1} + P_{R2})}{2}\right]$$

The real complementary probabilities in $\mathcal{R} = R_1 + R_2$:

$$P_M/i = \left[\frac{P_{M1} + P_{M2}}{2}\right]/i = \frac{P_{M1}/i + P_{M2}/i}{2}$$
$$= i(1 - P_R)/i = 1 - \frac{(P_{R1} + P_{R2})}{2}$$

The Complex Random Vectors in $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M} = \mathbf{G_1} + \mathbf{G_2}$.
We have: In $\mathbf{G_1}$: $Z_1 = P_{R1} + P_{M1}$ and in $\mathbf{G_2}$: $Z_2 = P_{R2} + P_{M2}$.
Then, in $\mathbf{G}$: $Z = P_R + P_M = \frac{P_{R1}+P_{R2}}{2} + \frac{P_{M1}+P_{M2}}{2} = \frac{P_{R1}+P_{M1}}{2} + \frac{P_{R2}+P_{M2}}{2} = \frac{Z_1+Z_2}{2}$.
The degrees of our knowledge:
We have: In $\mathbf{G_1}$: $DOK_1 = |Z_1|^2 = |P_{R1} + P_{M1}|^2 = P_{R1}^2 + [P_{M1}/i]^2 = P_{R1}^2 + [1 - P_{R1}]^2$.
In $\mathbf{G_2}$: $DOK_2 = |Z_2|^2 = |P_{R2} + P_{M2}|^2 = P_{R2}^2 + [P_{M2}/i]^2 = P_{R2}^2 + [1 - P_{R2}]^2$.
Then, in $\mathbf{G}$:

$$DOK = |Z|^2 = |P_R + P_M|^2 = P_R^2 + [P_M/i]^2 = P_R^2 + [1 - P_R]^2$$
$$= \left[\frac{P_{R1} + P_{R2}}{2}\right]^2 + \left[1 - \left(\frac{P_{R1} + P_{R2}}{2}\right)\right]^2$$

The chaotic factors:
We have: In $\mathbf{G_1}$:
$Chf_1 = 2iP_{R1}P_{M1} = 2iP_{R1}i(1 - P_{R1}) = 2i^2P_{R1}(1 - P_{R1}) = -2P_{R1}(1 - P_{R1})$.
In $\mathbf{G_2}$: $Chf_2 = 2iP_{R2}P_{M2} = 2iP_{R2}i(1 - P_{R2}) = 2i^2P_{R2}(1 - P_{R2}) = -2P_{R2}(1 - P_{R2})$.
Then, in $\mathbf{G}$:

$$Chf = 2iP_RP_M = 2iP_Ri(1 - P_R) = 2i^2P_R(1 - P_R) = -2P_R(1 - P_R)$$
$$= -2\left[\frac{P_{R1} + P_{R2}}{2}\right]\left[1 - \left(\frac{P_{R1} + P_{R2}}{2}\right)\right]$$

The magnitudes of the chaotic factors:
We have: In $\mathbf{G_1}$:
$MChf_1 = |Chf_1| = -2iP_{R1}P_{M1} = -2iP_{R1}i(1 - P_{R1}) = -2i^2P_{R1}(1 - P_{R1}) = 2P_{R1}(1 - P_{R1})$.
In $\mathbf{G_2}$: $MChf_2 = |Chf_2| = -2iP_{R2}P_{M2} = -2iP_{R2}i(1 - P_{R2}) = -2i^2P_{R2}(1 - P_{R2}) = 2P_{R2}(1 - P_{R2})$.
Then, in $\mathbf{G}$:

$$MChf = |Chf| = -2iP_RP_M = -2iP_Ri(1 - P_R) = -2i^2P_R(1 - P_R) = 2P_R(1 - P_R)$$
$$= 2\left[\frac{P_{R1} + P_{R2}}{2}\right]\left[1 - \left(\frac{P_{R1} + P_{R2}}{2}\right)\right]$$

The deterministic probabilities in $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M}$.
We have: In $\mathbf{G_1}$:

$$Pc_1^2 = [P_{R1} + P_{M1}/i]^2 = [P_{R1} + (1 - P_{R1})]^2 = 1^2 = 1$$
$$= DOK_1 - Chf_1 = 1$$
$$= DOK_1 + MChf_1 = 1$$
$$= Pc_1$$

In $\mathbf{G_2}$:

$$\begin{aligned} Pc_2^2 &= [P_{R2} + P_{M2}/i]^2 = [P_{R2} + (1 - P_{R2})]^2 = 1^2 = 1 \\ &= DOK_2 - Chf_2 = 1 \\ &= DOK_2 + MChf_2 = 1 \\ &= Pc_2 \end{aligned}$$

Then, in $\mathbf{G}$:

$$\begin{aligned} Pc^2 &= [P_R + P_M/i]^2 = [P_R + (1 - P_R)]^2 = 1^2 = 1 \\ &= DOK - Chf = 1 \\ &= DOK + MChf = 1 \\ &= Pc \end{aligned}$$

6.3 The deterministic cases and the *MCPP* parameters of the first model

Additionally, it is crucial and very important to mention here that if the real probabilities P_{R1} and P_{R2} are equal to one or zero, then we will return directly to the deterministic theory, which is a special nonrandom case of the probabilistic complex probability paradigm (*MCPP*) general case. Hence, this certainly proves that *MCPP* is always valid in the deterministic case or in the probabilistic and random case.

Consequently, in the deterministic situation, we will have these possible four cases:

Case 1:

$P_{R1} = 1$ and $P_{R2} = 1$, that means we are working in $\mathbf{G} = \mathbf{G_1}$ $(0 \le v < c)$ + $\mathbf{G_2}$ $(c < v \le 2c)$.

$P_{M1}=i(1-P_{R1})= i(1-1)= 0$ and $P_{M2}=i(1-P_{R2})= i(1-1)= 0$.

$P_{M1}/i= i(1-P_{R1})/i = 1 - P_{R1}= 1-1 =0$ and $P_{M2}/i= i(1-P_{R2})/i = 1-P_{R2}= 1-1 =0$.

$P_{R1} + P_{M1}/i = P_{R1} + i(1-P_{R1})/i = 1 + 0 = 1 = P_{G1} = Pc_1$ and $P_{R2} + P_{M2}/i = P_{R2} + i(1-P_{R2})/i = 1 + 0 = 1 = P_{G2} = Pc_2$.

Hence, $P_R = \frac{P_{R1}+P_{R2}}{2} = \frac{1+1}{2} = 1$ and $P_M = \frac{P_{M1}+P_{M2}}{2} = \frac{0+0}{2} = 0$.

So, $P_R + P_M/i = P_R + i(1-P_R)/i = 1 + 0 = 1 = Pc$.

And $P_G = \frac{P_{G1}+P_{G2}}{2} = \frac{1+1}{2} = 1 = Pc$

$$Z_1 = P_{R1} + P_{M1} = 1 + 0 = 1$$

$$Z_2 = P_{R2} + P_{M2} = 1 + 0 = 1$$

$$Z = P_R + P_M = 1 + 0 = 1 = \frac{Z_1 + Z_2}{2} = \frac{1+1}{2} = 1$$

$$DOK_1 = |Z_1|^2 = P_{R1}^2 + (P_{M1}/i)^2 = 1^2 + 0^2 = 1$$

$$DOK_2 = |Z_2|^2 = P_{R2}^2 + (P_{M2}/i)^2 = 1^2 + 0^2 = 1$$

$$DOK = |Z|^2 = P_R^2 + (P_M/i)^2 = 1^2 + 0^2 = 1$$

$$Chf_1 = 2iP_{R1}P_{M1} = 2i \times 1 \times 0 = 0$$

$$Chf_2 = 2iP_{R2}P_{M2} = 2i \times 1 \times 0 = 0$$

$$Chf = 2iP_RP_M = 2i \times 1 \times 0 = 0$$

$$MChf_1 = |Chf_1| = -2iP_{R1}P_{M1} = -2i \times 1 \times 0 = 0$$

$$MChf_2 = |Chf_2| = -2iP_{R2}P_{M2} = -2i \times 1 \times 0 = 0$$

$$MChf = |Chf| = -2iP_RP_M = -2i \times 1 \times 0 = 0$$

$$\begin{aligned} Pc_1^2 &= (P_{R1} + P_{M1}/i)^2 = (1+0)^2 = 1^2 = 1 \\ &= DOK_1 - Chf_1 = 1 - 0 = 1 \\ &= DOK_1 + MChf_1 = 1 + 0 = 1 \\ &= Pc_1 \end{aligned}$$

$$\begin{aligned} Pc_2^2 &= (P_{R2} + P_{M2}/i)^2 = (1+0)^2 = 1^2 = 1 \\ &= DOK_2 - Chf_2 = 1 - 0 = 1 \\ &= DOK_2 + MChf_2 = 1 + 0 = 1 \\ &= Pc_2 \end{aligned}$$

$$\begin{aligned} Pc^2 &= (P_R + P_M/i)^2 = (1+0)^2 = 1^2 = 1 \\ &= DOK - Chf = 1 - 0 = 1 \\ &= DOK + MChf = 1 + 0 = 1 \\ &= Pc \end{aligned}$$

Case 2:

$P_{R1} = 1$ and $P_{R2} = 0$, that means we are working in $\mathbf{G} = \mathbf{G_1}$ $(0 \le v < c)$ alone.

$P_{M1} = i(1 - P_{R1}) = i(1-1) = 0$ and $P_{M2} = i(1 - P_{R2}) = i(1-0) = i$.

$P_{M1}/i = i(1 - P_{R1})/i = 1 - P_{R1} = 1 - 1 = 0$ and $P_{M2}/i = i(1 - P_{R2})/i = 1 - P_{R2} = 1 - 0 = 1$.

$P_{R1} + P_{M1}/i = P_{R1} + i(1 - P_{R1})/i = 1 + 0 = 1 = P_{G1} = Pc_1$ and $P_{R2} + P_{M2}/i = P_{R2} + i(1 - P_{R2})/i = 0 + 1 = 1 = P_{G2} = Pc_2$.

Hence, $P_R = \frac{P_{R1}+P_{R2}}{2} = \frac{1+0}{2} = 0.5$ and $P_M = \frac{P_{M1}+P_{M2}}{2} = \frac{0+i}{2} = 0.5i$.

So, $P_R + P_M/i = P_R + i(1 - P_R)/i = 0.5 + 0.5 = 1 = Pc$.

And $P_G = \frac{P_{G1}+P_{G2}}{2} = \frac{1+1}{2} = 1 = Pc$

$$Z_1 = P_{R1} + P_{M1} = 1 + 0 = 1$$

$$Z_2 = P_{R2} + P_{M2} = 0 + i = i$$

$$Z = P_R + P_M = 0.5 + 0.5i = \frac{Z_1 + Z_2}{2} = \frac{1+i}{2} = \frac{1}{2} + \frac{i}{2} = 0.5 + 0.5i$$

$$DOK_1 = |Z_1|^2 = P_{R1}^2 + (P_{M1}/i)^2 = 1^2 + 0^2 = 1$$

$$DOK_2 = |Z_2|^2 = P_{R2}^2 + (P_{M2}/i)^2 = 0^2 + 1^2 = 1$$

$$DOK = |Z|^2 = P_R^2 + (P_M/i)^2 = 0.5^2 + 0.5^2 = 0.5$$

$$Chf_1 = 2iP_{R1}P_{M1} = 2i \times 1 \times 0 = 0$$

$$Chf_2 = 2iP_{R2}P_{M2} = 2i \times 0 \times i = 0$$

$$Chf = 2iP_RP_M = 2i \times 0.5 \times 0.5i = -0.5$$

$$MChf_1 = |Chf_1| = -2iP_{R1}P_{M1} = -2i \times 1 \times 0 = 0$$

$$MChf_2 = |Chf_2| = -2iP_{R2}P_{M2} = -2i \times 0 \times i = 0$$

$$MChf = |Chf| = -2iP_RP_M = -2i \times 0.5 \times 0.5i = 0.5$$

$$\begin{aligned} Pc_1^2 &= (P_{R1} + P_{M1}/i)^2 = (1+0)^2 = 1^2 = 1 \\ &= DOK_1 - Chf_1 = 1 - 0 = 1 \\ &= DOK_1 + MChf_1 = 1 + 0 = 1 \\ &= Pc_1 \end{aligned}$$

$$\begin{aligned} Pc_2^2 &= (P_{R2} + P_{M2}/i)^2 = (0+1)^2 = 1^2 = 1 \\ &= DOK_2 - Chf_2 = 1 - 0 = 1 \\ &= DOK_2 + MChf_2 = 1 + 0 = 1 \\ &= Pc_2 \end{aligned}$$

$$\begin{aligned} Pc^2 &= (P_R + P_M/i)^2 = (0.5+0.5)^2 = 1^2 = 1 \\ &= DOK - Chf = 0.5 - (-0.5) = 0.5 + 0.5 = 1 \\ &= DOK + MChf = 0.5 + 0.5 = 1 \\ &= Pc \end{aligned}$$

These results can be understood and explained since we are considering in this case only the subluminal universe $\mathbf{G}_1$ and discarding totally the superluminal universe $\mathbf{G}_2$; hence, we are looking at one part or half of the whole picture, which is the total complex universe $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$. Additionally, both $\mathbf{G}_1$ and $\mathbf{G}_2$ have equal probabilities to be considered in $\mathbf{G}$ and which are $P_R = P_M/i = \frac{1}{2} = 0.5$. Moreover, and for the same reason, *DOK* which is the degree of our knowledge in the whole universe $\mathbf{G}$ is minimum and is equal to 0.5; hence, accordingly *MChf,* which measures the magnitude of chaos and ignorance in $\mathbf{G}$ is maximum and is equal to 0.5. Knowing that *Pc* in $\mathbf{G}$, which is computed by subtracting and eliminating chaos materialized by *Chf* from the experiment and after adding the contributions of $\mathcal{M}$ to $\mathbf{G}$, is always maintained as equal to 1 = 100%.

Case 3:

$P_{R1} = 0$ and $P_{R2} = 1$, that means we are working in $\mathbf{G} = \mathbf{G_2}$ $(c < v \le 2c)$ alone.

$P_{M1} = i(1 - P_{R1}) = i(1-0) = i$ and $P_{M2} = i(1 - P_{R2}) = i(1-1) = 0$.

$P_{M1}/i = i(1 - P_{R1})/i = 1 - P_{R1} = 1 - 0 = 1$ and $P_{M2}/i = i(1 - P_{R2})/i = 1 - P_{R2} = 1 - 1 = 0$.

$P_{R1} + P_{M1}/i = P_{R1} + i(1 - P_{R1})/i = 0 + 1 = 1 = P_{G1} = Pc_1$ and $P_{R2} + P_{M2}/i = P_{R2} + i(1 - P_{R2})/i = 1 + 0 = 1 = P_{G2} = Pc_2$.

Hence, $P_R = \frac{P_{R1}+P_{R2}}{2} = \frac{0+1}{2} = 0.5$ and $P_M = \frac{P_{M1}+P_{M2}}{2} = \frac{i+0}{2} = 0.5i$.

So, $P_R + P_M/i = P_R + i(1 - P_R)/i = 0.5 + 0.5 = 1 = Pc$.

And $P_G = \frac{P_{G1}+P_{G2}}{2} = \frac{1+1}{2} = 1 = Pc$

$$Z_1 = P_{R1} + P_{M1} = 0 + i = i$$

$$Z_2 = P_{R2} + P_{M2} = 1 + 0 = 1$$

$$Z = P_R + P_M = 0.5 + 0.5i = \frac{Z_1 + Z_2}{2} = \frac{i+1}{2} = \frac{i}{2} + \frac{1}{2} = 0.5 + 0.5i$$

$$DOK_1 = |Z_1|^2 = P_{R1}^2 + (P_{M1}/i)^2 = 0^2 + 1^2 = 1$$

$$DOK_2 = |Z_2|^2 = P_{R2}^2 + (P_{M2}/i)^2 = 1^2 + 0^2 = 1$$

$$DOK = |Z|^2 = P_R^2 + (P_M/i)^2 = 0.5^2 + 0.5^2 = 0.5$$

$$Chf_1 = 2iP_{R1}P_{M1} = 2i \times 0 \times i = 0$$

$$Chf_2 = 2iP_{R2}P_{M2} = 2i \times 1 \times 0 = 0$$

$$Chf = 2iP_RP_M = 2i \times 0.5 \times 0.5i = -0.5$$

$$MChf_1 = |Chf_1| = -2iP_{R1}P_{M1} = -2i \times 0 \times i = 0$$

$$MChf_2 = |Chf_2| = -2iP_{R2}P_{M2} = -2i \times 1 \times 0 = 0$$

$$MChf = |Chf| = -2iP_RP_M = -2i \times 0.5 \times 0.5i = 0.5$$

$$\begin{aligned} Pc_1^2 &= (P_{R1} + P_{M1}/i)^2 = (0+1)^2 = 1^2 = 1 \\ &= DOK_1 - Chf_1 = 1 - 0 = 1 \\ &= DOK_1 + MChf_1 = 1 + 0 = 1 \\ &= Pc_1 \end{aligned}$$

$$\begin{aligned} Pc_2^2 &= (P_{R2} + P_{M2}/i)^2 = (1+0)^2 = 1^2 = 1 \\ &= DOK_2 - Chf_2 = 1 - 0 = 1 \\ &= DOK_2 + MChf_2 = 1 + 0 = 1 \\ &= Pc_2 \end{aligned}$$

$$\begin{aligned} Pc^2 &= (P_R + P_M/i)^2 = (0.5 + 0.5)^2 = 1^2 = 1 \\ &= DOK - Chf = 0.5 - (-0.5) = 0.5 + 0.5 = 1 \\ &= DOK + MChf = 0.5 + 0.5 = 1 \\ &= Pc \end{aligned}$$

These results can be understood and explained since we are considering in this case only the superluminal universe $\mathbf{G}_2$ and discarding totally the subluminal universe $\mathbf{G}_1$; hence, we are looking at one part or half of the whole picture, which is the total complex universe $\mathbf{G} = \mathbf{G}_1 + \mathbf{G}_2$. Additionally, both $\mathbf{G}_1$ and $\mathbf{G}_2$ have equal probabilities to be considered in $\mathbf{G}$ and which are $P_R = P_M/i = \frac{1}{2} = 0.5$. Moreover, and for the same reason, *DOK* which is the degree of our knowledge in the whole universe $\mathbf{G}$ is minimum and is equal to 0.5; hence, accordingly *MChf* which measures the magnitude of chaos and ignorance in $\mathbf{G}$ is maximum and is equal to 0.5. Knowing that *Pc* in $\mathbf{G}$, which is computed by subtracting and eliminating chaos materialized by *Chf* from the experiment and after adding the contributions of $\mathcal{M}$ to $\mathbf{G}$, is always maintained as equal to 1 = 100%.

Case 4:

$P_{R1} = 0$ and $P_{R2} = 0$, that means that we have impossible events and experiments in the whole $\mathbf{G}$.

$P_{M1} = i(1 - P_{R1}) = i(1 - 0) = i$ and $P_{M2} = i(1 - P_{R2}) = i(1 - 0) = i$.

$P_{M1}/i = i(1-P_{R1})/i = 1-P_{R1} = 1-0 = 1$ and $P_{M2}/i = i(1-P_{R2})/i = 1-P_{R2} = 1-0 = 1$.

$P_{R1}+P_{M1}/i = P_{R1}+i(1-P_{R1})/i = 0+1 = 1 = P_{G1} = Pc_1$ and $P_{R2}+P_{M2}/i = P_{R2}+i(1-P_{R2})/i = 0+1 = 1 = P_{G2} = Pc_2$.

Hence, $P_R = \frac{P_{R1}+P_{R2}}{2} = \frac{0+0}{2} = 0$ and $P_M = \frac{P_{M1}+P_{M2}}{2} = \frac{i+i}{2} = i$.

So, $P_R + P_M/i = P_R + i(1-P_R)/i = 0+1 = 1 = Pc$.

And $P_G = \frac{P_{G1}+P_{G2}}{2} = \frac{1+1}{2} = 1 = Pc$

$$Z_1 = P_{R1} + P_{M1} = 0 + i = i$$

$$Z_2 = P_{R2} + P_{M2} = 0 + i = i$$

$$Z = P_R + P_M = 0 + i = i = \frac{Z_1+Z_2}{2} = \frac{i+i}{2} = \frac{2i}{2} = i$$

$$DOK_1 = |Z_1|^2 = P_{R1}^2 + (P_{M1}/i)^2 = 0^2 + 1^2 = 1$$

$$DOK_2 = |Z_2|^2 = P_{R2}^2 + (P_{M2}/i)^2 = 0^2 + 1^2 = 1$$

$$DOK = |Z|^2 = P_R^2 + (P_M/i)^2 = 0^2 + 1^2 = 1$$

$$Chf_1 = 2iP_{R1}P_{M1} = 2i \times 0 \times i = 0$$

$$Chf_2 = 2iP_{R2}P_{M2} = 2i \times 0 \times i = 0$$

$$Chf = 2iP_RP_M = 2i \times 0 \times i = 0$$

$$MChf_1 = |Chf_1| = -2iP_{R1}P_{M1} = -2i \times 0 \times i = 0$$

$$MChf_2 = |Chf_2| = -2iP_{R2}P_{M2} = -2i \times 0 \times i = 0$$

$$MChf = |Chf| = -2iP_RP_M = -2i \times 0 \times i = 0$$

$$\begin{aligned} Pc_1^2 &= (P_{R1} + P_{M1}/i)^2 = (0+1)^2 = 1^2 = 1 \\ &= DOK_1 - Chf_1 = 1 - 0 = 1 \\ &= DOK_1 + MChf_1 = 1 + 0 = 1 \\ &= Pc_1 \end{aligned}$$

$$\begin{aligned} Pc_2^2 &= (P_{R2} + P_{M2}/i)^2 = (0+1)^2 = 1^2 = 1 \\ &= DOK_2 - Chf_2 = 1 - 0 = 1 \\ &= DOK_2 + MChf_2 = 1 + 0 = 1 \\ &= Pc_2 \end{aligned}$$

$$\begin{aligned} Pc^2 &= (P_R + P_M/i)^2 = (0+1)^2 = 1^2 = 1 \\ &= DOK - Chf = 1 - 0 = 1 \\ &= DOK + MChf = 1 + 0 = 1 \\ &= Pc \end{aligned}$$

6.4 The first model simulations

We note that in the following simulations, P_{R3} is the real probability in the luminal universe $\mathbf{G_3}$ for $(v = c)$ in yellow in the simulations, where we have $\forall P_{R3} : 0 \le P_{R3} \le 1$

and that it will be included in the final most general model of *MCPP*. Thus, the current model is a simplified first model. The simulations from **Figures 9–12** illustrate the first model.

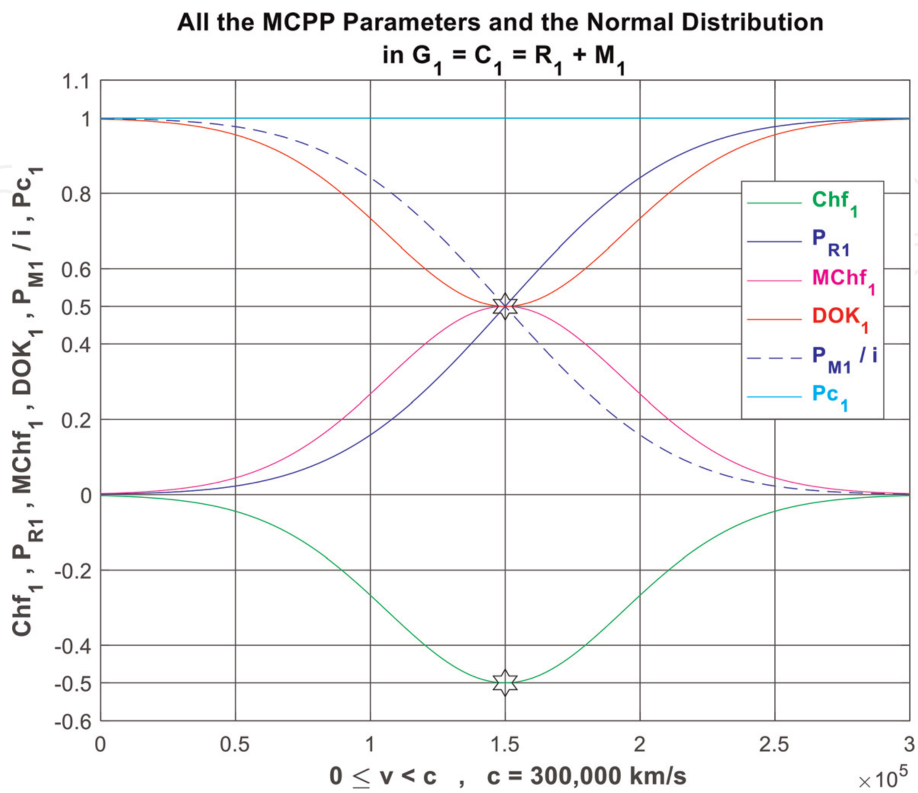

Figure 9.
The MCPP *first model parameters and the normal distribution in* G_1.

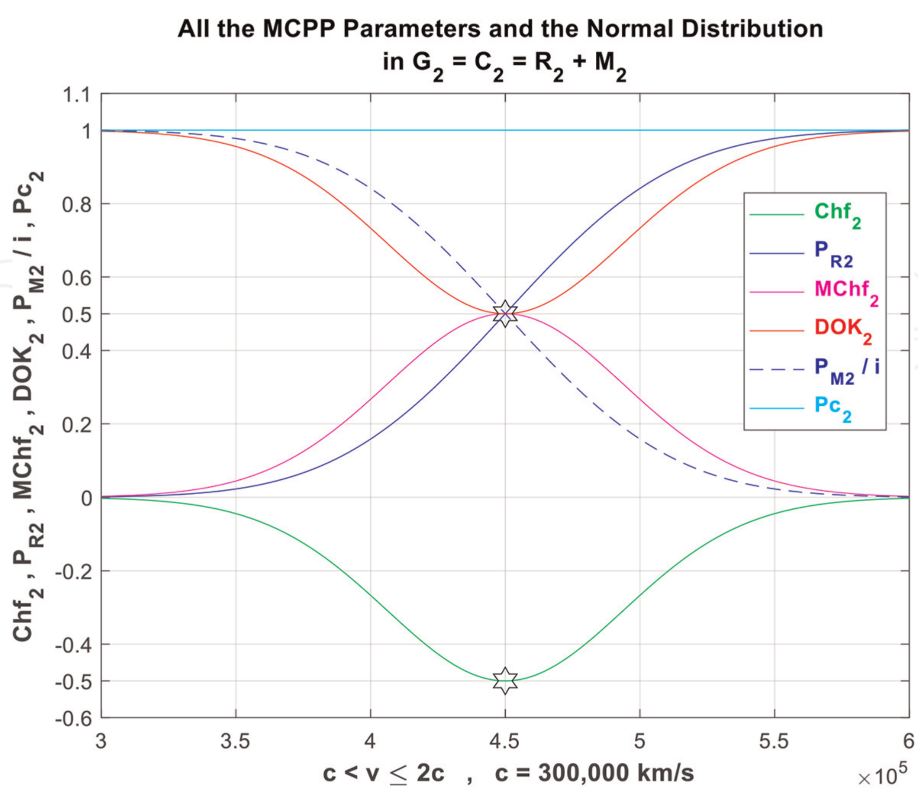

Figure 10.
The MCPP *first model parameters and the normal distribution in* G_2.

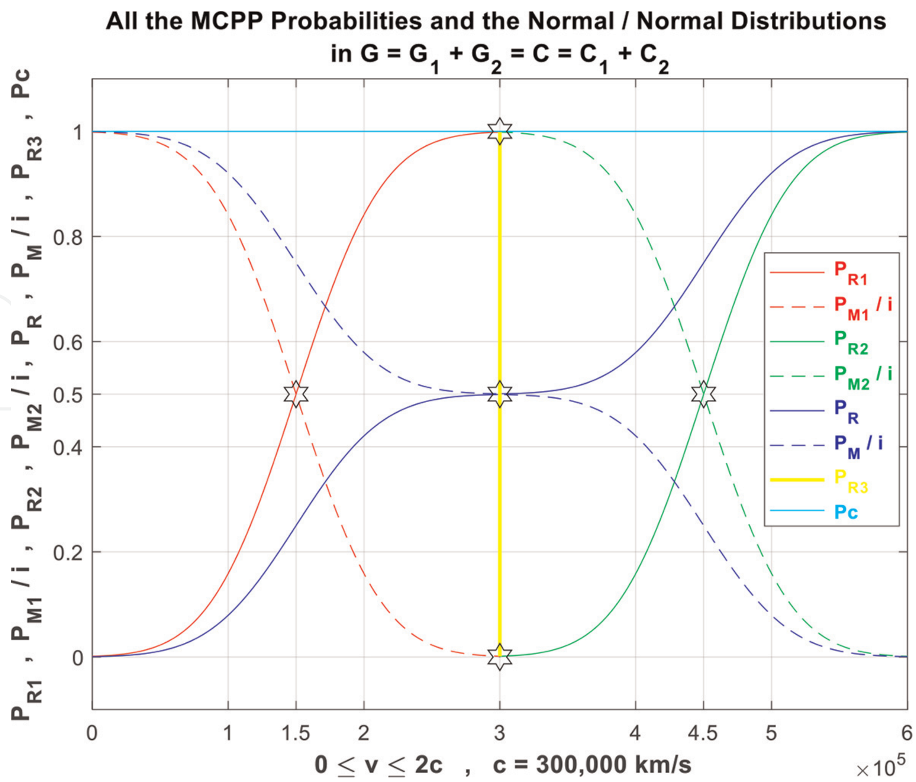

Figure 11.
The MCPP *first model probabilities and the normal/normal distributions in* **G***.*

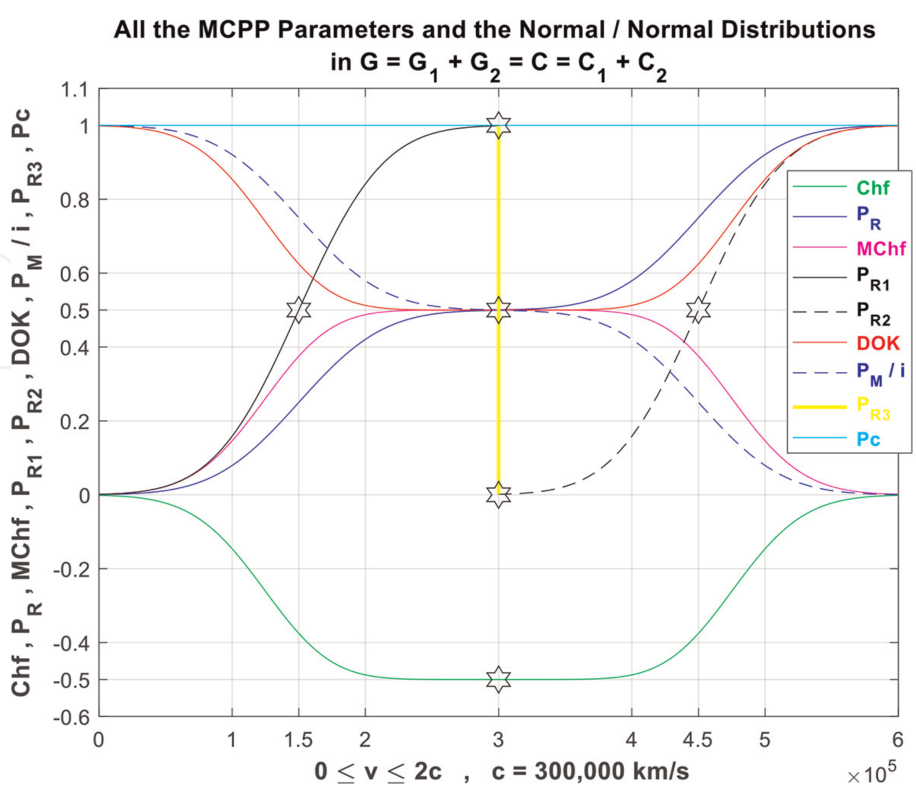

Figure 12.
The MCPP *first model parameters and the normal/normal distributions in* **G***.*

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7. Conclusion

In the current research work, the original extended model of eight axioms (*EKA*) of A. N. Kolmogorov was connected and applied to Metarelativity theory. Thus, a tight link between Metarelativity and the novel paradigm (*CPP*) was achieved. Consequently, the model of "Complex Probability" was more developed beyond the scope of my 21 previous research works on this topic.

Additionally, as it was proved and verified in the novel model, before the beginning of the random phenomenon simulation and at its end we have the chaotic factor (*Chf* and *MChf*) is zero and the degree of our knowledge (*DOK*) is one since the stochastic fluctuations and effects have either not started yet or they have terminated and finished their task on the probabilistic phenomenon. During the execution of the nondeterministic phenomenon and experiment, we also have $0.5 \leq DOK < 1$, $-0.5 \leq Chf < 0$, and $0 < MChf \leq 0.5$. We can see that during this entire process, we have incessantly and continually $Pc^2 = DOK - Chf = DOK + MChf = 1 = Pc$, which means that the simulation that behaved randomly and stochastically in the real probability set $\mathcal{R}$ is now certain and deterministic in the complex probability set and total universe $\mathbf{G} = \mathcal{C} = \mathcal{R} + \mathcal{M}$, and this after adding to the random experiment executed in the real probability set $\mathcal{R}$ the contributions of the imaginary probability set $\mathcal{M}$ and hence after eliminating and subtracting the chaotic factor from the degree of our knowledge as it is shown in the equation above. Furthermore, the real, imaginary, complex, and deterministic probabilities and that correspond to each value of the velocity random variable have been determined in the three probabilities sets, which are $\mathcal{R}$, $\mathcal{M}$, and $\mathbf{G} = \mathcal{C}$ by P_r, P_m, Z, and Pc, respectively. Consequently, at each value of v the novel *MCPP* parameters P_r, P_m, P_m/i, DOK, Chf, $MChf$, Pc, and Z are surely and perfectly predicted in the complex probabilities set and total universe $\mathbf{G} = \mathcal{C} = \mathbf{G_1} + \mathbf{G_2} + \mathbf{G_3}$ with Pc maintained equal to one permanently and repeatedly.

Nomenclature

EKA	Extended Kolmogorov's Axioms
CPP	Complex Probability Paradigm
MCPP	Metarelativistic Complex Probability Paradigm
i	the imaginary number where $i = \sqrt{-1}$ or $i^2 = -1$
$\mathbf{G_1}$	real universe of matter and energy = subluminal universe
$\mathbf{G_2}$	imaginary universe of matter and energy = superluminal universe or metauniverse
$\mathbf{G_3}$	luminal universe of electromagnetic waves
$\mathbf{G}$	total universe of matter and energy $= \mathbf{G_1}\ (v < c) + \mathbf{G_2}\ (v > c) + \mathbf{G_3}\ (v = c)$ = complex universe
R_1	real probabilities set in $\mathbf{G_1} (v < c)$
M_1	imaginary complementary probabilities set to R_1 in $\mathbf{G_1}\ (v < c)$
R_2	real probabilities set in $\mathbf{G_2}\ (v > c)$
M_2	imaginary complementary probabilities set to R_2 in $\mathbf{G_2}\ (v > c)$
R_3	real probabilities set in $\mathbf{G_3}\ (v = c)$
M_3	imaginary complementary probabilities set to R_3 in $\mathbf{G_3}\ (v = c)$
$\mathcal{C}_1$	$R_1 + M_1$ = complex set of probabilities in $\mathbf{G_1}\ (v < c)$
$\mathcal{C}_2$	$R_2 + M_2$ = complex set of probabilities in $\mathbf{G_2}\ (v > c)$

$\mathcal{C}_3$	$R_3 + M_3$ = complex set of probabilities in $\mathbf{G_3}$ $(v = c)$
$\mathcal{R}$	$R_1 + R_2 + R_3$ = real set of events and probabilities in **G**
$\mathcal{M}$	$M_1 + M_2 + M_3$ = imaginary set of events and probabilities in **G**
$\mathcal{C}$	complex set of events and probabilities in **G**, $\mathcal{C} = \mathcal{R} + \mathcal{M}$
P_{rob}	probability of any event
P_{R1}	probability in the real set $\mathcal{R}$ in $\mathbf{G_1}$
P_{R2}	probability in the real set $\mathcal{R}$ in $\mathbf{G_2}$
P_{R3}	probability in the real set $\mathcal{R}$ in $\mathbf{G_3}$
P_{M1}	probability in the imaginary set $\mathcal{M}$ in $\mathbf{G_1}$
P_{M2}	probability in the imaginary set $\mathcal{M}$ in $\mathbf{G_2}$
P_{M3}	probability in the imaginary set $\mathcal{M}$ in $\mathbf{G_3}$
P_R	probability in the real set $\mathcal{R}$ in **G**
P_M	probability in the imaginary set $\mathcal{M}$ in **G**
Pc_1	probability in the complex set $\mathcal{C}_1$ in $\mathbf{G_1}$
Pc_2	probability in the complex set $\mathcal{C}_2$ in $\mathbf{G_2}$
Pc_3	probability in the complex set $\mathcal{C}_3$ in $\mathbf{G_3}$
Pc	probability of a real event in $\mathcal{R}$ with its associated complementary imaginary event in $\mathcal{M}$ = probability in the complex probability set $\mathcal{C}$ in the total universe **G**
Z	complex probability number = sum of P_R and P_M = complex random vector
DOK	$=\lvert Z\rvert^2$= the degree of our knowledge of the random system or experiment, it is the square of the norm of Z
Chf	the chaotic factor of Z
$MChf$	the magnitude of the chaotic factor of Z
c	light velocity $\cong$ 300,000 Km/s = 3×10^8 m/s $\cong$ 186,000 miles/s in vacuum
m_{G1}	mass in the real subluminal universe $\mathbf{G_1}$ of matter
m_{G2}	mass in the imaginary superluminal universe $\mathbf{G_2}$ of matter or metamatter
m_{G3}	mass in the luminal universe $\mathbf{G_3}$
m_G	mass in the complex total universe of matter **G**
E_{G1}	energy in the real subluminal universe $\mathbf{G_1}$ of energy
E_{G2}	energy in the imaginary superluminal universe $\mathbf{G_2}$ of energy
E_{G3}	energy of the electromagnetic waves in the luminal universe $\mathbf{G_3}$ of energy
E_G	energy in the complex total universe **G** of energy
EW	electromagnetic waves in $\mathbf{G_3}$
f	frequency of the electromagnetic waves in $\mathbf{G_3}$
PDF	probability density function
CDF	cumulative probability distribution function

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References

[1] Abou Jaoude A. The theory of Metarelativity: Beyond Albert Einstein's relativity. Physics International, Science Publications. 2013;**4**(2):97-109

[2] Wikipedia, the Free Encyclopedia, Special Relativity. https://en.wikipedia.org/

[3] Wikipedia, the Free Encyclopedia, General Relativity. https://en.wikipedia.org/

[4] Abou Jaoude A, El-Tawil K, Kadry S. Prediction in complex dimension using Kolmogorov's set of axioms. Journal of Mathematics and Statistics, Science Publications. 2010;**6**(2):116-124

[5] Abou Jaoude A. The Complex statistics paradigm and the law of large numbers. Journal of Mathematics and Statistics, Science Publications. 2013;**9**(4):289-304

[6] Abou Jaoude A. The theory of complex probability and the first order reliability method. Journal of Mathematics and Statistics, Science Publications. 2013;**9**(4):310-324

[7] Abou Jaoude A. Complex probability theory and prognostic. Journal of Mathematics and Statistics, Science Publications. 2014;**10**(1):1-24

[8] Abou Jaoude A. The complex probability paradigm and analytic linear prognostic for vehicle suspension systems. American Journal of Engineering and Applied Sciences, Science Publications. 2015;**8**(1):147-175

[9] Abou Jaoude A. The paradigm of complex probability and the Brownian motion. Systems Science and Control Engineering, Taylor and Francis Publishers. 2015;**3**(1):478-503

[10] Abou Jaoude A. The paradigm of complex probability and Chebyshev's inequality. Systems Science and Control Engineering, Taylor and Francis Publishers. 2016;**4**(1):99-137

[11] Abou Jaoude A. The paradigm of complex probability and analytic nonlinear prognostic for vehicle suspension systems. Systems Science and Control Engineering. Taylor and Francis Publishers. 2016;**4**(1):99-137

[12] Abou Jaoude A. The paradigm of complex probability and analytic linear prognostic for unburied petrochemical pipelines. Systems Science and Control Engineering. Taylor and Francis Publishers. 2017;**5**(1):178-214

[13] Abou Jaoude A. The paradigm of complex probability and Claude Shannon's information theory. Systems Science and Control Engineering. Taylor and Francis Publishers. 2017;**5**(1):380-425

[14] Abou Jaoude A. The paradigm of complex probability and analytic nonlinear prognostic for unburied petrochemical pipelines. Systems Science and Control Engineering. 2017;**5**(1):495-534

[15] Abou Jaoude A. The paradigm of complex probability and Ludwig Boltzmann's entropy. Systems Science and Control Engineering. Taylor and Francis Publishers. 2018;**6**(1):108-149

[16] Abou Jaoude A. The paradigm of complex probability and Monte Carlo methods. Systems Science and Control Engineering, Taylor and Francis Publishers. 2019;**7**(1):407-451

[17] Abou Jaoude A. Analytic prognostic in the linear damage case applied to

buried petrochemical pipelines and the complex probability paradigm. Fault Detection, Diagnosis and Prognosis, IntechOpen. 2020;**1**(5):65-103. DOI: 10.5772/intechopen.90157

[18] Abou Jaoude A. The Monte Carlo techniques and the complex probability paradigm. Forecasting in Mathematics - Recent Advances, New Perspectives and Applications. IntechOpen. 2020;**1**(1):1-29. DOI: 10.5772/intechopen.93048

[19] Abou Jaoude A. The paradigm of complex probability and prognostic using FORM. London Journal of Research in Science: Natural and Formal (LJRS). 2020;**20**(4):1-65. London Journals Press, Print ISSN: 2631-8490, Online ISSN: 2631-8504. DOI: 10.17472/LJRS

[20] Abou Jaoude A. The paradigm of complex probability and the central limit theorem. London Journal of Research in Science: Natural and Formal (LJRS). 2020;**20**(5):1-57. London Journals Press, Print ISSN: 2631-8490, Online ISSN: 2631-8504. DOI: 10.17472/LJRS

[21] Abou Jaoude A. The paradigm of complex probability and Thomas Bayes' theorem. The Monte Carlo Methods - Recent Advances, New Perspectives and Applications. IntechOpen. 2021;**1**(1):1-44. ISBN: 978-1-83968-760-0. DOI: 10.5772/intechopen.98340

[22] Abou Jaoude A. The paradigm of complex probability and Isaac Newton's classical mechanics: On the Foundation of Statistical Physics. The Monte Carlo Methods - Recent Advances, New Perspectives and Applications. IntechOpen. 2021;**1**(2):45-116. ISBN: 978-1-83968-760-0. DOI: 10.5772/intechopen.98341

[23] Abou Jaoude A. The Paradigm of Complex Probability and Quantum Mechanics: The Infinite Potential Well Problem - the Position Wave Function. Applied Probability Theory - New Perspectives, Recent Advances and Trends. London, UK: IntechOpen; 2022;**1**(1):1-44. DOI: 10.5772/intechopen.107300

[24] Abou Jaoude A. The paradigm of complex probability and quantum mechanics: The infinite potential well problem – The momentum Wavefunction and the Wavefunction entropies. Applied Probability Theory – New Perspectives, Recent Advances and Trends. IntechOpen. 2022;**1**(2):45-88. DOI: 10.5772/intechopen.107665

[25] Benton W. Probability, Encyclopedia Britannica. Vol. 18. Chicago: Encyclopedia Britannica Inc.; 1966. pp. 570-574

[26] Benton W. Mathematical Probability, Encyclopedia Britannica. Vol. 18. Chicago: Encyclopedia Britannica Inc.; 1966. pp. 574-579

[27] Feller W. An Introduction to Probability Theory and its Applications. 3rd ed. New York: Wiley; 1968

[28] Walpole R, Myers R, Myers S, Ye K. Probability and Statistics for Engineers and Scientists. 7th ed. New Jersey: Prentice Hall; 2002

[29] Freund JE. Introduction to Probability. New York: Dover Publications; 1973

[30] Abou Jaoude A. The Computer Simulation of Monté Carlo Methods and Random Phenomena. United Kingdom: Cambridge Scholars Publishing; 2019

[31] Abou Jaoude A. The Analysis of Selected Algorithms for the Stochastic

Paradigm. United Kingdom: Cambridge Scholars Publishing; 2019

[32] Abou Jaoude A. The Analysis of Selected Algorithms for the Statistical Paradigm. Vol. 1. The Republic of Moldova: Generis Publishing; 2021

[33] Abou Jaoude A. The Analysis of Selected Algorithms for the Statistical Paradigm. Vol. 2. The Republic of Moldova: Generis Publishing; 2021

[34] Abou Jaoude A. Forecasting in Mathematics - Recent Advances, New Perspectives and Applications. London, UK, London, United Kingdom: IntechOpen; 2021. ISBN: 978-1-83880-825-9

[35] Abou Jaoude A. The Monte Carlo Methods - Recent Advances, New Perspectives and Applications. London, UK, London, United Kingdom: IntechOpen; 2022. ISBN: 978-1-83968-760-0

[36] Abou Jaoude A. Applied Probability Theory – New Perspectives, Recent Advances and Trends. London, UK, London, United Kingdom: IntechOpen; 2023. ISBN: 978-1-83768-296-6

[37] Abou Jaoude A. Ph. D. Thesis in Applied Mathematics: Numerical Methods and Algorithms for Applied Mathematicians. Spain: Bircham International University; 2004. http://www.bircham.edu

[38] Abou Jaoude A. Ph. D. Thesis in Computer Science: Computer Simulation of Monté Carlo Methods and Random Phenomena. Spain: Bircham International University; 2005. http://www.bircham.edu

[39] Abou Jaoude A. Ph. D. Thesis in Applied Statistics and Probability: Analysis and Algorithms for the Statistical and Stochastic Paradigm. Spain: Bircham International University; 2007. http://www.bircham.edu

[40] Wikipedia, the Free Encyclopedia, Dark Matter. https://en.wikipedia.org/

[41] Wikipedia, the Free Encyclopedia, Dark Energy. https://en.wikipedia.org/

[42] Wikipedia, the Free Encyclopedia, Lambda-CDM Model. https://en.wikipedia.org/

[43] Wikipedia, the Free Encyclopedia, Relativity: The Special and the General Theory. https://en.wikipedia.org/