Chapter

PID Cascade Controller Design for an Unstable System

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Abstract

In control engineering, the control of an unstable system is very concerning. An example of an unstable system that uses control principles is a ball and beam balancer system. This system consists of a long beam attached to a motor at its midpoint. A steel ball moves on the top of the beam with an acceleration proportional to the beam angle. If the system is uncontrolled well, the steel ball may fall from the beam. This paper presents an approach to modeling and controlling the ball position for the ball and beam balancer system. Starting with a mathematical equation for the nonlinear system, the model of the system is produced. Then, it demonstrates the design of the PID cascade controller system to stabilize the system and regulate the ball to its reference position. The performance of the system was evaluated and tested for set-point tracking signal and disturbance rejection test. The simulation studies were done using Matlab Simulink and the results indicated that the proposed approach yields robust closed-loop performance.

Keywords: unstable system, ball and beam balancer, modeling, controller design, cascade, PID, tuning, disturbance rejection

1. Introduction

Most industrial processes can be considered nonlinear and uncertain processes. The control of these processes is considered dangerous if they were unstable. An unstable system such as a ball and beam balancer exists in some control laboratories. It is used by control engineering students to design and apply some of the controller techniques to control the ball position [1]. It is easy to understand that many classical and modern controller design techniques still can be applied to this system. Besides that, due to its simple construction, it can be classified as a feedback control system and it is used to learn how to stabilize the unstable system. It is normally related to real control problems such as horizontally stabilizing an airplane during landing and in turbulent airflow [2, 3].

The ball and beam system is seen as a standard control engineering system whose fundamental theory can be used to stabilize problems for various systems. For example, the balancing problem in moving robots that use to carry and spacecraft position control systems in aerospace engineering [4].

The ball and beam system is considered a nonlinear system [5]. Its nonlinearity is due to the dead zone and saturation characteristic, DC motor, and pulley drive nonlinearity and discontinuity of position measurement. The ball and beam system can show a general nonlinear control object. The model of the ball and beam balancer system can be used as a control item such as balancing mobile robots in goods carrying, controlling spacecraft, control of nonlinear actuators, and position control of space vehicles in aerospace engineering can all use the model, the ball, and the beam system as the control object [6].

A steel ball is positioned on the beam as shown in **Figure 1**. When the current control signal is applied to the motor, the beam is titled about its center axis. This causes the rolling of the ball on the beam. The control objective is to regulate the ball position on the beam automatically by changing the manipulated beam angle. This is a difficult mission due to the rolling of the ball on the length of the beam. This movement has an acceleration proportional to the angle of the beam [7]. The system is bounded input unbounded output because bounded beam angle gives unbounded ball position. Hence, the system must be controlled to position the ball at the desired position.

Most real systems are nonlinear and closed-loop controllers that can be a valuable approach to ensure sufficient performance. Many nonlinear control systems have been documented for academic education to learn about feedback control in control engineering courses. As a standard, the ball and beam is a classic example of such a system [8].

The ball and beam system has been studied and controlled using several control techniques by many researchers. In the previous, a good variety of methods is applied to this system [9–13].

2. System mathematical modeling

The ball and beam balancer system is a multi-loop system that contains two parts. The modeling of DC motor and the model derivation ball and beam. The system free-body of the ball and beam balancer system as shown in **Figure 2** illustrates two DOF systems. One is moving the ball up and down on the beam and the other one is the beam rotating around its center. The demonstration of the whole system dynamics is

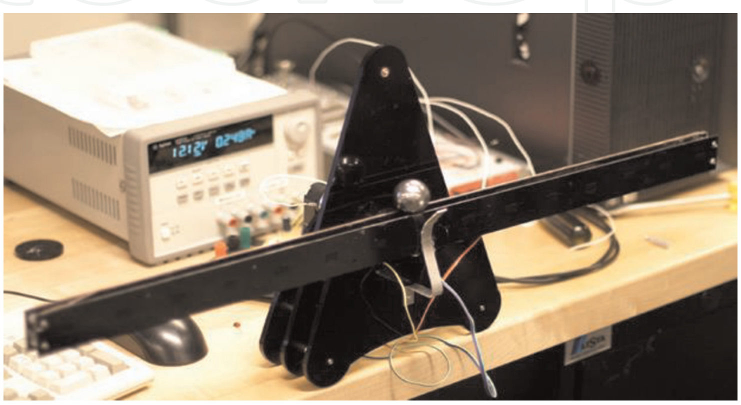

Figure 1.
Ball beam balancer system.

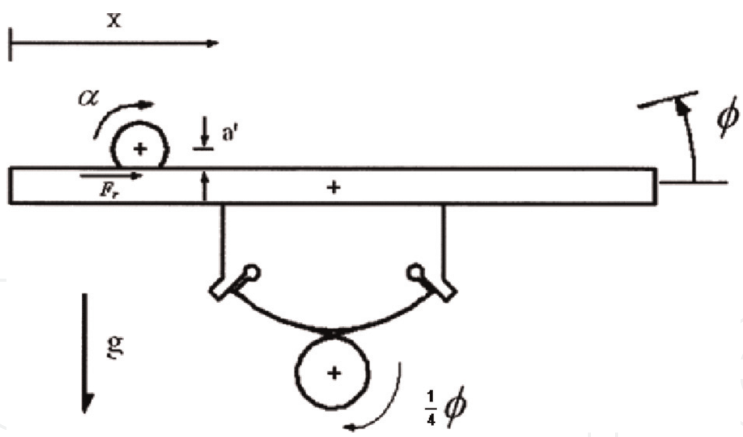

Figure 2.
System free body [14].

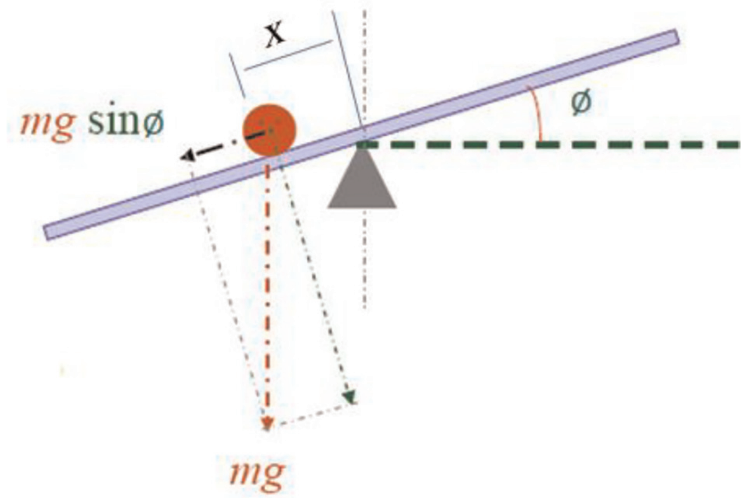

Figure 3.
Ball and beam balancer motion.

complex therefore a simple mathematical model derivation is done to design the system controller.

The ball and beam balancer motion is shown in **Figure 3**. It can be seen that three forces acting on the ball are the rolling constraint force, the sliding component force due to gravity, which depends on the beam angle ∅, and the reacting force.

The balancing forces are;

$$\sum F_b = mg\,sin\,\varnothing - F_r = m\ddot{x} \tag{1}$$

F_r is the ball rolling force and x is the ball position on the beam. By geometry, the ball position is rewritten as

$$x = \alpha \bullet \acute{a} \tag{2}$$

α = angular displacement of the ball.$\acute{a}$ = distance between the axis of the ball and the point of contact of the ball with the beam

The ball balancing torque τ_b is:

$$\sum \tau_b = F_r \acute{a} = J_b \ddot{\alpha} \tag{3}$$

$$J_b = \frac{2}{5} m a^2 \tag{4}$$

whereJ_b = ball moment inertia.a = ball radius.

The beam and motor torque can be derived. Since the beam bears the load of the ball and the input motor torque. The torque balance equation is:

$$\sum \tau_{bm} = \tau_{in} = J_{bm} \ddot{\varnothing} \tag{5}$$

where bm indicates the beam and motor and τ_{in} represents the torque produced by the motor.

$$\tau_{in} = k_t I_{in} \tag{6}$$

Then, the obtained equations after simplification and substitution are;

$$\left[1 + \frac{2}{5}\left(\frac{a}{\acute{\alpha}}\right)^2\right]\ddot{x} = \mathrm{g}\sin\varnothing \tag{7}$$

and

$$J_{bm}\ddot{\varnothing} = k_t I_{in} \tag{8}$$

For linearization, it is assumed that the system operates at about 0° beam angle, and for small angles approximation $\sin\varnothing = \varnothing$. Then Eq. (7) can be rewritten as

$$\left[1 + \frac{2}{5}\left(\frac{a}{\acute{\alpha}}\right)^2\right]\ddot{x} = \mathrm{g}\varnothing \tag{9}$$

$$\left(\frac{a}{\acute{\alpha}}\right) \cong 1$$

$$\left[1 + \frac{2}{5}\right]\ddot{x} = \mathrm{g}\varnothing \tag{10}$$

Two transfer functions can be obtained, the first transfer function for the motor system relates to beam angles $\varnothing$ and the current inputI_{in}. This transfer function is the inner loop transfer function. By taking Laplace transform of Eq. (8)

$$\frac{\varnothing(s)}{I_{in}(s)} = \frac{k_t}{J_{bm}s^2} \tag{11}$$

The second transfer function is for the ball and beam system that relates to the ball position x and beam angle $\varnothing$is obtained by taking Laplace to transform of Eq. (10). The parameters of the system that have been used in this paper are given in **Table 1**.

$$\frac{x(s)}{\varnothing(s)} = \frac{5\mathrm{g}}{7} \bullet \frac{1}{s^2} \tag{12}$$

Parameter	Symbol	Value	Unit
Beam and motor moment of inertia	J_{bm}	0.062	$kg.m^2$
Gravitational constant	g	9.81	m/s^2
Motor torque constant	k_t	5.27	Nm/A
Mass of the ball	m	0.01	kg
Ball radius	α	0.015	m
Beam length	l	0.40	m
Beam width	w	0.004	m

Table 1.
Parameters of the system.

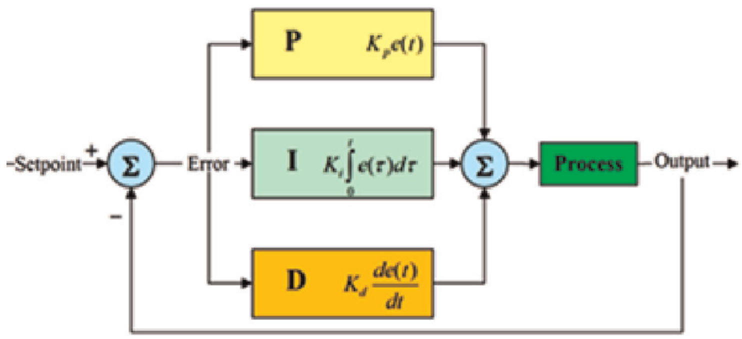

Figure 4.
Feedback control system with PID controller.

The overall open-loop system transfer function related to ball position to the current input can be written as.

$$\frac{x(s)}{I_{in}(s)} = \frac{5gk_t}{7J_{bm}} \bullet \frac{1}{s^4} \tag{13}$$

3. PID controller

A proportional–integral–derivative controller (PID controller) is a generic feedback control mechanism as shown in **Figure 4**. PID controller is the most commonly used feedback controller. The controller is used to minimize the difference between the reference input and the controlled variable by manipulating the manipulated variable. PID controllers are the top first choice in the unknown process. However, the PID controller gains should be tuned carefully to obtain the greatest performance.

The PID controller design includes three parameters: proportional, integral, and derivative gains, denoted as P, I, and D, respectively [15, 16]. The proportional gain determines the reaction based on the present error, the integral gain calculates the reaction based on the sum of recent errors, and the derivative gain determines the reaction based on the rate of change of error. The PID controller can operate in different control mods since some processes need to use one mode or two mods to obtain the desired control. This is done by eliminating undesired control action and putting its gain to zero. PI, PD, P, or I controller mods is derived from standard PID controller in the nonappearance of the relevant control actions. PI controllers are the most commonly used in industrial applications since derivative action is sensitive to noise. On the other hand, the presence of an integral action makes the system output reaches its setpoint.

The PID controller operates on the error signal $e(t)$, which is the difference between the desired setpoint $r(t)$ and the measured output $y(t)$ to yield a control signal $u(t)$. PID controller has the general form.

$$u(t) = K_p e(t) + K_i \int_0^t e(t)dt + K_d \frac{d}{dt} e(t) \tag{14}$$

The desired closed-loop system characteristics are achieved by correcting the controller parameters K_p, K_i, and K_d, often by "tuning." The proportional term may only achieve stability. The integral term guarantees the disturbance rejection. The derivative term is responsible for response shaping. Even though, the PID controllers are the most widely used class of control systems. It cannot be used in MIMO systems due to system complicity.

Applying Laplace transformation of Eq. (14) results in the transformed PID controller equation

$$\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s \tag{15}$$

where the controller parameters are:

Proportional gain, K_p.

A large value gives a faster response and a very large value may cause an oscillatory and unstable system.

Integral gain, K_i.

A large value eliminates the steady-state error more quickly and increases overshoot. Very large values invite instability, integrator windup, or actuator saturation.

Derivative gain, K_d with a.

With a larger value, the response reaches the desired response faster, decreases overshoot, slows the transient response, and may cause instability.

The desired characteristic specification involves a good tuning control using adjusting the controller parameters to their optimum values. Sometimes PID controllers often offer satisfactory control; even their parameters did not tune. However, carefully tuning can improve the system performance, and poor tuning gives an unacceptable performance.

PID tuning is a challenging problem, the complex performance criteria should be achieved even with the limitations of PID control [17]. Different methods for loop tuning and techniques are more sophisticated and subjected to patents. In manual methods for loop tuning, if the system is online, the tuning method requires setting integral and derivative gains to zero and increasing the proportional gain until the output response becomes oscillatory. According to the "quarter amplitude decay" response, this gain is set to half of that value. Next, increase the integral gain until zero error is obtained for the process. Finally, increase the derivative gain, if necessary, until the output response reaches its steady state in presence of disturbance. However, too much high derivative gain will cause the output response overshoots the desired input and some systems cannot accept overshoot. **Table 2** describes the effectiveness

Controller parameter	Rise time	Overshoot	Settling time	Steady state error
K_P	Decrease	Increase	Small change	Decrease
K_i	Decrease	Increase	Increase	Eliminate
K_d	Small change	Decrease	Decrease	None

Table 2.
Effect of increasing K_p, K_i, and K_d parameters in the closed-loop system.

Method	Advantages	Disadvantages
Manual Tuning	The online method does not require math	Time-consuming, expert persons are required
Ziegler-Nichols	Established Method, online method	Some trial and error is required, aggressive tuning
Software Tools	Regular tuning, online or offline method.	Costs and training involved
Cohen-Coon	Good process method	Offline method, Math required, deal with first-order processes only

Table 3.
PID controller tuning methods.

of increasing controller gains on the system response. The table is used as a reference and only help in calculating the values of controller parameters. Because these parameters are dependent on each other, changing one of these parameters can change the effect of the other two.

There are different control techniques for tuning a PID controller. Many of these techniques require the transfer function of the system, then selecting controller gains K_p, K_i, and K_d depending on the system dynamics. The PID controller tuning method should be carefully chosen, for example, in practice, some loops take minutes or longer to reach steady state. Therefore, manual tuning could be an inefficient choice. In addition, tuning method selection depends on the type of tuning online or offline method. In the case of offline systems, some tuning methods require applying a step change to the system and measuring the output response. Then, use this reopens to calculate the controller gains. **Table 3** illustrates the comparison between some tuning methods.

4. PID cascade controller design for ball beam balancer system

The suggested block diagram of the ball and beam balancer system consists of two-loop as shown in **Figure 5**. The inner loop is the motor control loop, and the outer loop, which is the ball beam control loop. In this paper, the design approach has to stabilize the inner loop first, and then the outer loop is controlled. The motor loop control is in series with ball and beam loop control.

In this research, the design of the PID cascade controller [18–21] includes two control loops, an inner loop with a primary PID controller to control the beam angle, and an outer loop with a secondary PID controller to control the ball position.

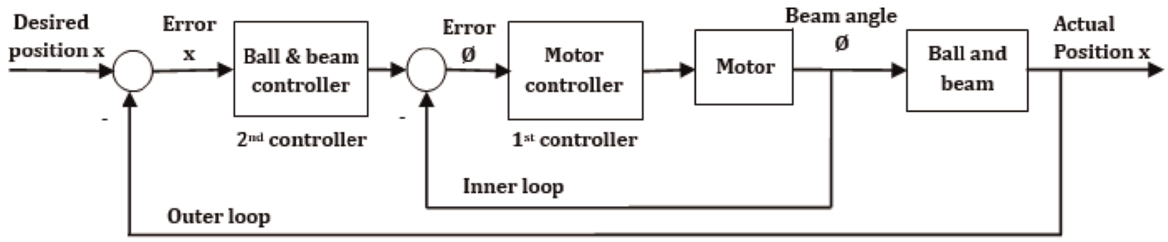

Figure 5.
Block diagram of ball and beam balancer system.

The secondary controller is designed to offset the effect of disturbances before it significantly affects the output of the controlled system. While the output of the first controller provides the reference for the second loop. This reduces any unexpected changes from the inner loop. The secondary controller adjusts the motor. There is a relation between motor angle and beam angle, any change in the motor angle causes a change in the beam angle. Therefore, the ball position is controlled by changing the beam angle.

The PID controller parameters are hard to tune when system parameters besides control action change while the system operates. So, in this research, the controller gains are tuned using a tuning tool in Simulink control design based on the performance and robustness of the system. The obtained PID controller parameters to control the system are illustrated in **Table 4**. The simulation of the PID cascade diagram of the ball and beam balancer system is shown in **Figure 6**.

In this design, the ITAE criterion is used to compute the best PID controller gains. The ITAE index is the best selectivity of the performance indices because it measures the integration of error for a specific time as the system parameters are varied [7]. The ITAE index formula is:

Controller	K_p	K_i	K_d
PID (inner loop)	0.1087	0.0029	1.1465
PID (outer loop)	26.2173	11.8563	12.8809

Table 4.
The best tuned PID controller parameters.

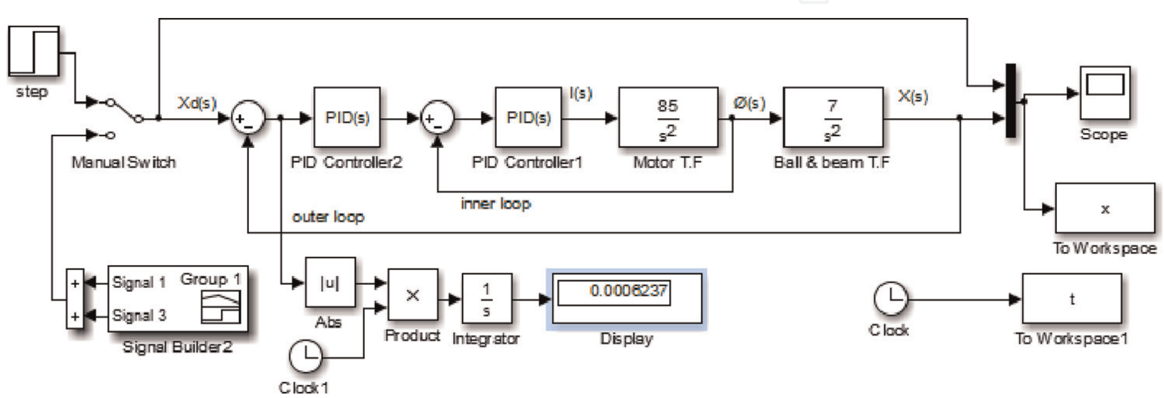

Figure 6.
Simulation diagram of ball and beam balancer system.

$$ITAE = \int_0^t t|e(t)|dt$$

5. Simulation results and discussion

The system is an unstable system with poles at the origin. So a controller is needed to control the system by looking at the response of the system that specifies some criteria. One of the first things that must be done is to decide upon a criterion for measuring how good the response is. For example, it does not matter how the response reaches a steady state but the steady state itself either has a large error or not. However, transient behavior is also important in determining the best response. For that, some criterion is introduced such as settling time, rise time, overshoot, and steady-state error.

PID cascade controller was designed and built using Matlab Simulink as shown in the previous section. To verify its performance in regulating the ball position of the ball and beam balancer system for different positions, such as step signal, step-change tracking signal, and sinusoidal signals were used as input test types for ball position in the simulation model. To test the robustness of the controlled system, the input disturbance step signal was used.

Firstly, the inner loop was controlled by tuning the controller parameters of the first PID until getting the best performance of the inner loop. After that, the second PID in the outer loop was tuned to get a stable response from the system. **Figure 7** shows the step response of ball position using PID controllers, where the best parameters that give this response were tableted in **Table 4**.

The designed controller stabilized the system with zero steady-state error. The system has a fast response and reached the steady-state value in short time. However, the system has an overshoot; but it is still an acceptable overshoot because the ball was still on the specified beam length. The performance specifications of the system using the PID cascade controller are summarized in **Table 5**.

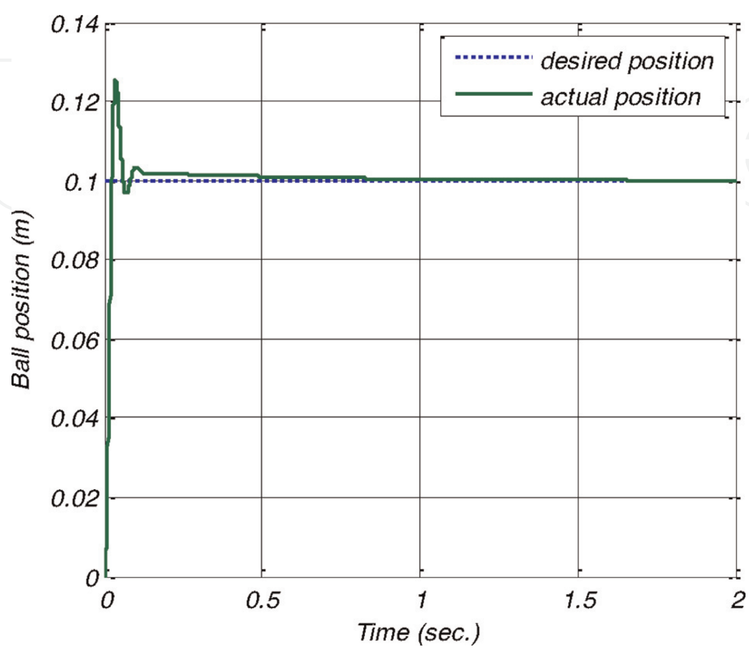

Figure 7.
Step response of ball position using PID controller.

Performance specification	
Overshoot (OS %)	25.4%
Peak Time	0.036 sec.
Settling Time	0.122 sec.
Steady State Error	0
ITAE	0.0006237

Table 5.
performance specifications of the system using PID controller.

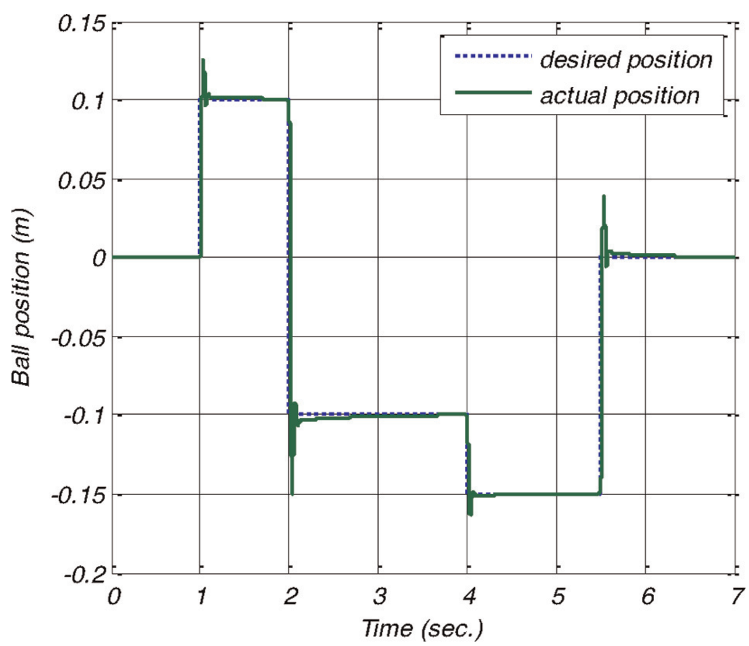

Figure 8.
Step tracking response of the system.

The setpoint change was performed, starting with the ball at the middle of the beam (origin). At 1 second, the ball position changed to 0.1-meter position of the beam center, at 2 seconds the desired ball position changed to the left side of the beam at position 0.1 meter, the ball position was changed again to 0.15 meter at 4 seconds on the left side, and return to the middle of the beam at 5.5 seconds.

The setpoint-tracking signal contained changing the set point during the operation as shown in **Figure 8**. Using the PID controller, the system output perfectly tracked the setpoint changes of ball position.

In this research, the ball and beam balancer system was subjected to input disturbance with a force acting on a ball with 0.05 step at a time of 2 sec. The output response under effective of this disturbance is shown in **Figure 9**.

The result showed that the disturbance affected the system at a time between 2 and 2.5 seconds. The designed controller can overcome this disturbance and reposition the ball to its setpoint in approximately 0.5 seconds.

The controlled system is tested for sinusoidal input and the result of the ball position is shown in **Figure 10**. As shown, the output response reached the desired input in a fast time, without overshoot and almost zero steady-state error.

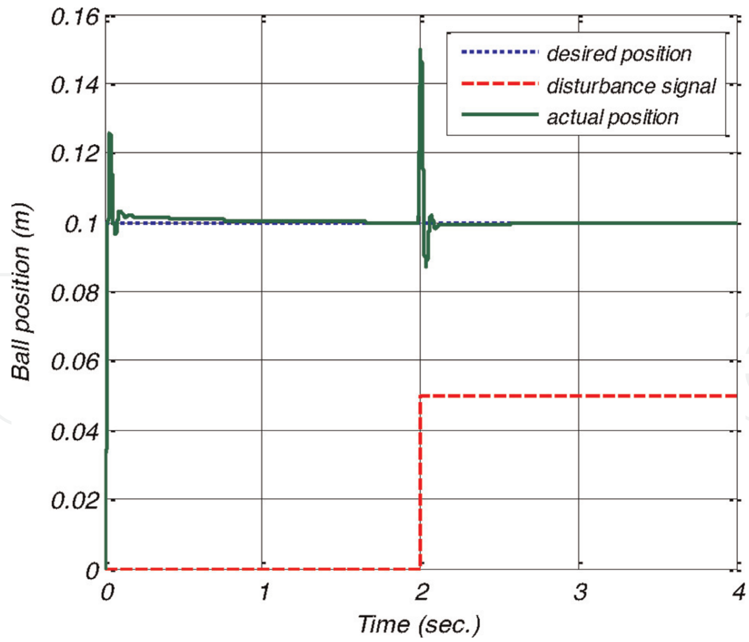

Figure 9.
Output response of the system with input disturbance.

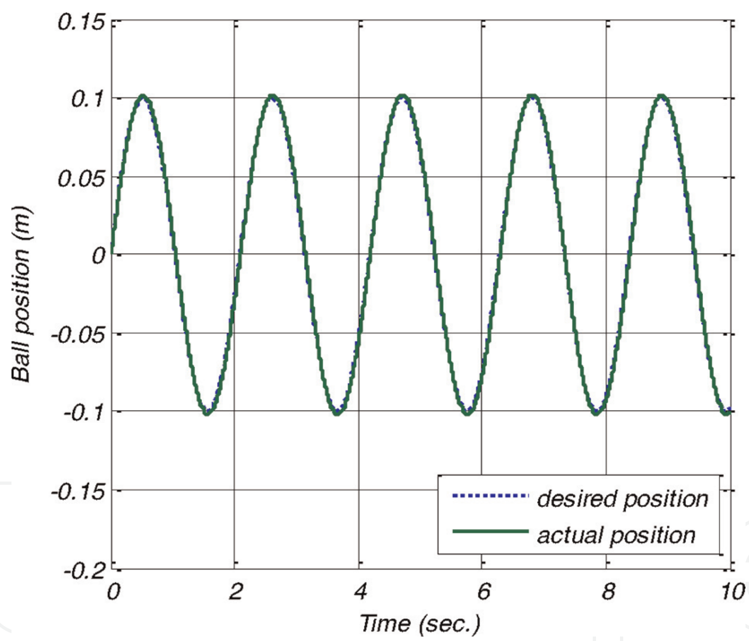

Figure 10.
Output response for sinusoidal input.

6. Conclusion

This research was successfully elaborated and the PID cascade controllers were magnificently been designed. A model for a ball and beam system is well derived such that the free-rolling ball of the ball and beam system can be positioned at any desired location on the beam without falling. For a ball and beam balancer system, the most required criterion is that the system has a small or no overshoot and zero steady-state error. From the results and discussion in the previous section. The simulation results have shown that the suggested approach can stabilize the system efficiently. Furthermore, the performance during the transient period of the system is good where a small overshoot was obtained. In addition, the PID cascade controller provided a very small settling time and zero steady-state error. Moreover, the proposed controller has successfully tracked the step tracking signal and sinusoidal signal. For the disturbance, the designed controller approved its ability to reject the effect of disturbance. Therefore, it can be concluded that the robustness of PID cascade controllers was achieved.

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