Distributed coordinate tracking control of multiple wheeled mobile robots

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DISTRIBUTED COORDINATE TRACKING CONTROL OF MULTIPLE WHEELED MOBILE ROBOTS

A Thesis

by

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ABSTRACT

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In this thesis, distributed coordinate tracking control of multiple wheeled-mobile robots is studied. Control algorithms are proposed for both kinematic and dynamic models. All vehicle agents share the same mechanical structure. The communication topology is leader-follower topology and the reference signal is generated by the virtual leader.

We will introduce two common kinematic models of WMR and control algorithms are proposed for both kinematic models with the aid of graph theory. Since it is more realistic that the control inputs are torques so dynamic extension is studied following by the kinematics. Torque controllers are designed with the aid of backstepping method so that the velocities of the mobile robots converge to the desired velocities. Because of the fact that in practice, the inertial parameter of WMR maybe not exactly known or even unknown, so both dynamics with and without inertial uncertainties are considered in this thesis.

DEDICATION

I would like to dedicate this thesis work to my family and friends for their endless love, support and encouragement. A special feeling of gratitude to my loving parents, Daqi Xing and Hongmei Zhang, who have always loved me unconditionally and whose good examples have taught me to work hard for the things that I aspire to achieve, thank you both for supporting me all the way since the beginning of my studies.

I would like to dedicate this thesis work to my girlfriend, Cheng Dong, who has been a constant source of support and encouragement throughout the challenges of graduate school and life, you are my best cheerleader.

I also would like to dedicate this thesis work to my advisor, Dr. Dong, who has always been extremely helpful throughout this study, especially thanks for your confidence in me, the completion of this thesis would be impossible without your guidance.

Finally, to all my friends, thanks for your understanding and encouragement in many moments of crisis, your friendships make my life a wonderful experience. I cannot list all the names here but you are always on my mind.

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I dedicate my heartfelt thanks to Dr. Wenjie Dong, not only my committee chair but also my advisor and mentor, whose guidance, support, countless hours of reflecting, and patience are invaluable. You have shown me, by example, how a good scientist should be and I believe I learned from the best. Also thank you for offering me research assistantship and finally supported me.

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CHAPTER I

INTRODUCTION

A wheeled mobile robot (WMR) is defined as a wheeled vehicle that can move autonomously without assistance from external human operator. Every WMR is equipped with a set of motorized actuators and an array of sensors, which help it to carry out useful work. In order to govern its motion, usually, there will be an on-board computer to command the motors to drive, based on reference inputs and the signals gathered by the sensors.

Unlike the majority of industrial robots that can only move with a fixed frame in a specific workspace, the wheeled mobile robot has a distinct future of moving around freely within its predefined workspace to fulfill a desired task. The mobility of wheeled mobile robot makes it suitable for a variety of applications in structured as well as in unstructured environments. For example, NASA has already developed a WMR that successfully demonstrated its ability to achieve the mission goals in exploring and running experiments on other plants such as moon. In military and high-risk hazardous environments, a miniature remotely operated vehicle that has been in use in many military and law enforcement organizations worldwide. It provides distinct advantages over human operators to complete critical missions in a safe manner. The wheeled mobile robots can also be found in other field of applications such as mining, transportation, entertainment and so on.

We should expect WMR to have stronger autonomous capabilities and higher agility, be able to self-learn and reliable for continuous operation regardless of time and environment in the future. The ever increasing demand and applications of wheeled mobile robots justify the research needs and potentials of this very fascinating topic.

In general, the research on WMR can be divided into several components namely the modeling of the WMR, the planning and the navigation strategies, the localization techniques, the communication system and the mobility (i.e., control task). The research in mobility of the WMR is related to understand the physical mechanics of the Mobile robot platform, the model of the interaction between the robot and its environment as well as the overall effect of control algorithm on the WMR. In localization, the research objective is to estimate the location, attitude, velocity and acceleration of the WMR. Navigation is concerned with the acquisition of and response to external sensed information to execute the mission. Meanwhile, research in planning is related to behaviors, trajectories or waypoints generation for the robot mission. Lastly, the goal of communication research is to provide the link between WMR and any remaining elements in the whole system, including system operators or other wheeled mobile robots.

For the theoretical part of this thesis, we need some basic knowledge of kinematics, dynamics, control theory, and mathematics and for the application part we require some idea on robotic systems. A good introduction on kinematics, dynamics and control of robotic system can be found in [1]. Majority of the car-like mobile robots platforms we studied in this research use standard wheels over omni-directional wheels due to the inherent mechanical simplicity. These car-like mobile robots are called nonholonomic mobile robots because of the velocity constrains imposed due to the structure of the wheels. A car is an example of a four-wheel vehicle system that shares many similarities with a WMR system due to the same wheel structure. The nonholonomic nature of the car-like robot is related to the assumption that the robot wheels roll without slipping, WMR is the simplest nonholonomic vehicle that displays the general characteristics. A review from WMR literatures indicates that conventional modeling of a WMR assumes nonholonomic, no-slip constraints at the contact point between the wheel and the ground surface [2]-[5]. The dynamics of general nonholonomic systems was thoroughly analyzed in [6], a controllability study for kinematic models of car-like robots with trailers was presented in [7].

The subject of this thesis is the control problem for nonholonomic wheeled mobile robots moving on the plane, and in particular the use of feedback techniques for achieving a given motion task. A number of works have dealt with the problem of controlling via feedback. In automatic control, feedback improves system performance by allowing the successful completion of a task even in the presence of external disturbances. The trajectory tracking problem was solved in [8] by means of a local feedback action. Use of dynamic feedback linearization was proposed in [9]. For car-like robots, the trajectory tracking problem was also addressed in [10] through the use of dynamic feedback linearization.

In the past two decades, multiple vehicle cooperative control has received significant attention in the system and control society, multiple agents system are composed of multiple interacting agents and can solve complex problems which cannot be achieved by monolithic system. Multiple agents system have been useful in many applications, different applications of multiple agents system are discussed in [11]-[24], including distributed sensor networks [11] [12] and cooperative control of unmanned air vehicles [13] [14], flocking [15]-[17], formation control [18]-[21] and rendezvous [22]-[24].

A distributed approach used in tracking control of multiple car-like mobile robots is called consensus, it means that a group of vehicles reach an agreement on a common value by interacting with local neighbors. With the aid of new techniques such as graph theory [25]-[26] and distributed computing [31]-[33], consensus problem for multiple WMRs system has been studied intensively in recent years. Consensus for multiple WMRs system means all the agents can reach an agreement in respect with a certain quantity of interests. In real-life operations, it is common that multiple agents are expected to be operated synchronically and preserve common quantity of states to accomplish tasks cooperatively. Consensus with dynamics leader is called consensus tracking has been studied from different perspective. The authors in [27] proposed and analyzed a consensus tracking algorithm under a variable undirected network topology. In [28], the authors proposed a proportional-and-derivative-like consensus tracking algorithm under a directed network topology in both continuous-time and discrete-time settings. In [29], the authors studied a leader-follower consensus tracking algorithm with time-varying delays.

The authors in [34]-[40] focus on the different common forms of dynamic agents, distributed control of multiple single-integrator systems are discussed in [34]-[36], and in [37]-[40] multiple systems with higher order are addressed. Due to the fact that different dynamic systems can be modeled by the combination of first-order, second order and even multiply order systems, so it is necessary to study control algorithms for those simplified systems. [34] is concerned with sampled-data consensus of first-order delayed multiply agent systems with delayed-state-derivative feedback. In [35], the sampled-data based quantized consensus problem has been investigated for multi-agent system with first order dynamics. In [36], the author proposed two distributed adaptive consensus tracking controllers for multiple nonlinear first-order systems with unknown parameters and external disturbances, it is shown that perfect tracking performance can be accomplished. In [37], cooperative tracking control of higher-order nonlinear systems are studied, a practical design method is developed for cooperative tracking control of higher-order nonlinear systems with a dynamic leader. In [38], the authors have considered the consensus problem for the first-order linear network and the second-order linear network with time-delays, and the relationship between network mathematical relationships and consensus has proposed. In [39], the author addressed second-order consensus algorithm and performs a convergence analysis under a fixed directed information exchange topology, a second order protocol for information consensus among multiple vehicles are proposed. In [40], the author extended the consensus algorithm for double integrator dynamics in [39] to the case that the information exchange topologies switch randomly with time and to the case that the final consensus value evolves according to a given nonlinear reference model.

Most works on consensus focus on algorithms taking the form of first-order dynamics and second-order dynamics, however, in this thesis we extend the algorithms to third-order dynamics.

Besides the different forms of dynamics agents, different assumptions of the communication topologies such as time-delays of communication links, communication switching and node link failure and different models of agent systems also are very important in distributed control consensus problem.

The theoretical framework for modeling and solving consensus problems for networked dynamics systems are introduced in [41] and [42]. In [41], the average consensus problems were solved in a distributed way, both linear consensus protocol and nonlinear consensus protocol are proposed for networks of dynamic agents under undirected communication graph, also the analysis for the case of networks with non-ideal links such as communication time-delays is addressed. In [42], based on several different theories such as matrix theory, graph theory and control theory, the authors discussed both fixed and switching communication topologies for the consensus problems for networks of dynamic agents. Both fixed and switching topologies are studied under directed and undirected graph. For networks with and without time-delays, two consensus protocols are presented for each case. In [43], the authors discussed the consensus problem with the aid of directed information flow, also the disturbances such as time-delays and link/node failures are considered. Consensus algorithms control law for multiple WMR networked systems are proposed under the assumption that the communication graph is directed communication graph. In [44], the authors focus on linear discrete-time multiple WMR systems under the undirected and fixed graph, communication delay among the mobile agents are considered. All the WMRs' inputs are assumed to be a constant, albeit possibly unknown time delay. It is shown that communication delay between WMRs will affect the restrictions on the communication topologies and will generally influence the consensus. In [45], the consensus problem for discrete-time multiple systems under directed and fixed graph is studied, a distributed dynamic output feedback control law is proposed.

The motivation behind multiple wheeled mobile robots cooperative control is that a group of vehicles working together can achieve great benefits including increasing reliability and efficiency, low cost, high adaptively and easy maintenance compared to single robot operation, especially in military. One of the most important and interesting problem of multiple mobile vehicle system is coordination of each single agent. The problem can be solved either by controlling each mobile agent respectively or by applying distributed control law with the aid of communication between all the mobile robots. The communication between different vehicles can be described by communication graph [25] [26] [27], each mobile system is considered as one node in the graph and there are communication links between different systems if one can receive others' information or can send out the information of itself. The links can be both directed and undirected based on different types of communication. In reality case, all WMRs are equipped with different sensors such as sonar that will detect and provide neighbors' information to them. It is known that if the communication graph satisfies some specific requirements, all the mobile systems will converge to a consensus states by using distributed control laws.

One common problem of multiple WMRs system coordination is the navigation problem. The navigation problem can be divided into three basic problems: coordinated tracking a reference trajectory, following a path and point stabilization. Some nonlinear feedback controllers have been proposed to solve these problems, the main idea behind all of these algorithms is to define velocity control inputs which stabilize the closed-loop system. Here in this thesis, our focus is the first problem which is coordinated tracking problem of a reference trajectory. In [46], a new control law for determining vehicles' linear and rotational velocities are given, the authors proposed a stable tracking control law for nonholonomic vehicles. In [30], the authors considered distributed tracking control problem for both first-order kinematics and second-order dynamics when there is a virtual leader and the leader's information is only known to a subset of a group of followers. For the first-order kinematics, the authors proposed a distributed consensus tracking algorithm without knowing the velocity and showed it can be achieved in finite time. For the second-order dynamics, the authors proposed two distributed tracking algorithm without knowing the acceleration. In [47], the authors proposed and analyzed a neighbor-based observer consensus tracking algorithm for a group of mobile agents with a virtual leader moving with an unknown velocity under a variable undirected communication graph. In [48], the authors studied a leader-follower consensus tracking problem for a multiple-agent system with a vary-velocity dynamic leader with time-varying delays. In [16], the author studied a flocking algorithm under the assumption that the leader's velocity is constant and is available to all followers. In [49], the flocking algorithm proposed in [16] is extended from two directions: the leader's accurate position and velocity are needed if the leader has a constant velocity and the leader's position is required when the leader has a varying velocity. In [50], the authors studied flocking problems of a group of autonomous vehicles with dynamic leader and a set of switching law relies on the state information of its neighbors and external signals were proposed to solve the problem.

All these controllers consider only the kinematic model, in other words, the steering system of the mobile robot and the "perfect velocity" tracking is assumed to generate the actual vehicle control inputs. However, it is more realistic that the control inputs are torques, so the dynamic model of mobile robots should be considered.

In [51], a dynamic extension and a method for dynamic model were proposed. The authors first designed a kinematic controller that makes the tracking error between the follower robots and the leader robot converges to zero. Second, a torque controller (i.e. dynamic controller) was designed by the aid of backstepping method to guarantee the velocities of the follower robots converge to the desired velocities. In [53], the authors considered three common problems of motion stabilization of a nonholonomic vehicle to a predefined trajectory, and a control laws for dynamic model of mobile robot was proposed. In [55], the authors presented a method to design an adaptive tracking controller for the dynamic model of a nonholonomic mobile robot with unknown parameters with the aid of backstepping method. In [58], the authors proposed a novel controller for wheeled mobile robots that can make all states of the closed-loop dynamic model of WMRs with unknown dynamics globally track a given trajectory. In [59], the authors studied the dynamic tracking problem of a class of the nonholonomic systems with unknown constant inertial parameters. The proposed tracking controller not only ensures all the follower robots asymptotically track the leader's trajectory, but also is characterized by low dimension and the absence of singular points. In [60], the authors considered the tracking control problems of a nonholonomic mobile robot with both parameter uncertainty and non-parameter uncertainty, a robust adaptive controller was proposed with the aid of backstepping method and robust control techniques.

In this thesis, distributed coordinate tracking control of multiple wheeled mobile robots is studied and both kinematic model and dynamic model are considered. Control algorithms are proposed for both kinematic model and dynamic model. All the vehicle agents share the same control and structural properties and are assumed to be able to receive specific neighbors' information or send out the information of itself to specific neighbors. The communication topology is leader-follower topology and the reference signal is generated by the virtual leader, in other words, the reference trajectory is the trajectory of the leader WMR, we define the trajectory of the virtual leader is a unit circle throughout this thesis.

In the text of kinematics, we introduced two common kinematic models of mobile system, one is two-wheel model vehicle with three generalized states including two Cartesian coordinates with respect to x-axis and y-axis, and the heading angle with respect to x-axis while the other one is four-wheel model vehicle with four generalized states including two Cartesian coordinates with respect to x-axis and y-axis, the orientation of the car body with respect to x-axis and the steering angle. We first transform both of the nonlinear car-like mobile robot models to linear models. In control theory, it is well known that if the number of generalized coordinates equals the number of input commands, a nonlinear static state feedback law can be used in order to transform the nonlinear robot kinematics and dynamics into linear system. It is easier and straightforward to design controllers for tracking control problems in linear model. There are two types of linearization algorithms can be considered, one is full-state linearization and another one is input-out linearization. Full-state linearization transforms the whole set of differential equations into linear system via feedback while only the input-output differential map is made linear in input-out linearization, it is obviously that the input-output linearization gives a weaker result. In the text of linearization we used full-state linearization algorithms for four-wheel model vehicles and input-out linearization algorithms for two-wheel model vehicles. After linearization, new distributed control methods were proposed for both kinematic systems with the aid of cascaded system theory and graph theory. For the four generalized states, the results of first-order kinematics and second-order dynamics controllers in [30] are extended to the third-order dynamics in this thesis. We assume that only a few mobile robots in the system can receive trajectory information from the leader robot directly and all the rest followers can only access the information of their neighbors from the communication topology. Our results showed that follower robots that are not neighbor of the leader can still be able to track the trajectory of the leader by using the state information of neighbor vehicles with the aid of communication graph. The control inputs in kinematic model are translational and rotational velocities.

However, dynamic extension was studied following by the kinematics because that it is more realistic that the control inputs are torques. In the text of dynamics, the control velocities that designed in kinematics are used as intermediate variables. Torque controllers are designed with the aid of backstepping method so that the velocities of the mobile robots converge to the desired velocities, which are obtained by the kinematic controller that we designed at the first step. Because of the fact that in practice, the inertial parameter of WMR maybe not exactly known or unknown, so both dynamics with and without inertial uncertainties are considered in this thesis, distributed tracking control laws are proposed for both scenarios. In order to verify the effectiveness of the proposed control laws, all simulations were conducted under the environment of Simulink in Matlab and all the simulation results showed that the proposed control laws are effective.

CHAPTER II

BACKGROUND AND PRELIMINARIES

In this chapter, some useful theories such as graph theory and Laplacian matrix throughout this thesis are introduced, all the theories are important analytical tools for distributed coordinate tracking control of multiple wheeled mobile robots. The reminder of this chapter is organized as follow. First, graph theory is introduced to model the information exchange between different agents in a multiple robots system, then Laplacian matrix is studied to analyze the communication graph mathematically, last stability of nonlinear system will be addressed.

2.1 Local Interaction

Suppose that there is a team consists of multiple WMRs, it is natural to model information exchange between different WMRs by using local interaction known as directed or undirected graphs. Conceptually, a graph $\mathcal{G}$ is formed by a group of vertices and edges that connecting those vertices. Formally, a graph is a pair of sets $\mathcal{G} = (V, E)$, where $V = \{v_1, v_2, \dots, v_n\}$ is defined as the set of vertices and E is defined as the set of edges and formed by pairs of vertices, $E \subseteq V \times V$, the pairs of vertices is ordered if the graph is directed otherwise it is unordered. E is a multiset which means the elements in E can occur more than once so that every element has a multiplicity.

Based on different characteristics of the elements in E, a communication graph $\mathcal{G}$ can be either directed or undirected, if the links between different nodes are directional, then the communication graph is directed graph, in other words, if the links between different nodes are bidirectional, then the communication graph is undirected graph. If there is a directed edge from node v_i to node v_j, then v_i is defined as the parent node and v_j is defined as the child node, $e_{ij} = (v_i, v_j)$. Here we show an example of a directed communication topology in Figure 2.1 to better demonstrate the above conceptions.

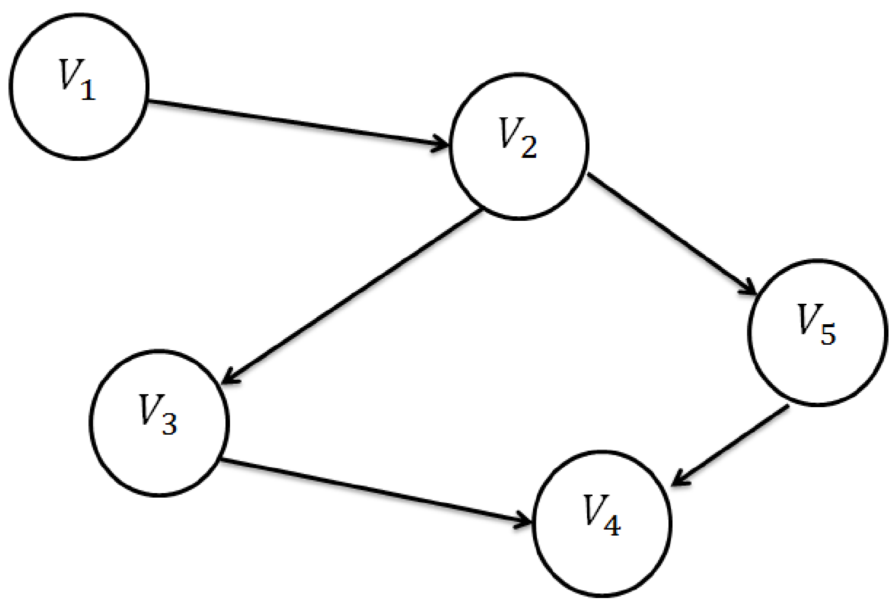

Figure 2.1 Communication graph I

In figure 2.1, we can see that there are five vertices in the graph and the all the communication links are directional, so $\mathcal{G} = (V, E)$ where $V = \{v_1, v_2, v_3, v_4, v_5\}$ and $E = \{e_{12}, e_{23}, e_{34}, e_{25}, e_{54}\}$.

2.2 Properties in Communication Graph

From subsection 2.1 we discussed the basic idea and expression of communication graph and it is learned that the communication system of multiple car-like systems can be either directed or undirected. In order to have a deeper understanding of communication graph, in this section, we will introduce some definitions such as path and subpath as well as some fundamental properties such as connectivity and tree structure in communication graph, which are very crucial to the end of controller design.

Some definitions and properties in communication graph:

1) In a directed graph or digraph, a directed path is a sequence of ordered edges of the form $(v_1, v_2), (v_2, v_3), \ldots$, where $v_i \in V$.
2) If there exists a subpath $\mathcal{G}_2$ of graph $\mathcal{G}_1$, then the subpath $\mathcal{G}_2$ is defined as $V(\mathcal{G}_2) \subseteq V(\mathcal{G}_1)$ and $E(\mathcal{G}_2) \subseteq E(\mathcal{G}_1)$.
3) If there exists a path starts and ends at the same node, then the path is defined as a cycle.
4) A directed graph is called strongly connected if there is a directed path form every node to every other node.
5) An undirected graph is called connected if there is a path between any distinct pair nodes.
6) For a node v_i in a communication graph $\mathcal{G} = (V, E)$, the in-degree and out-degree of node v_i are, respectively, defined as $deg_{in}(v_i) = \sum_{j=1}^{n} a_{ji}$ and $deg_{out}(v_i) = \sum_{j=1}^{n} a_{ij}$.

7) Given a graph $\mathcal{G} = (V, E)$, two vertices v_i and v_j are said to be neighbors or adjacent nodes if $(v_i, v_j) \in E$, also if $\mathcal{G}$ is directed, we distinguish between incoming neighbors of v_i (those vertex $v_j \in V$ such that $(v_j, v_i) \in E$) and outgoing neighbors of v_i (those vertex $v_j \in V$ such that $(v_i, v_j) \in E$).
8) In the case of undirected graph, a tree is defined as a graph that every pair of vertices is connected by only one path.
9) In the case of directed graph, a directed tree is defined that every node has exactly one parent except for one node which is known as root, root has no parent and has directed path to every other node. Note that there is no circle in directed tree and every edge has a natural orientation away from the root.
10) A directed spanning tree of a graph is defined as a directed tree that containing graph edges that connect all the nodes of the graph, in other words, a graph has or contains a directed spanning tree if there is a directed spanning tree being a subset of the graph. The condition that there exists a node having a directed path to all the other nodes is equivalent to the case that the digraph has a directed spanning tree.

Remark:

In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected, however, it does not apply for directed graphs. In the case of directed graphs, having a directed spanning tree is a weaker condition than being strongly connected.

2.3 Laplacian Matrix

It is shown in sections 2.1 and 2.2 that information exchange between different mobile agents in multiple car-like robots system can be modeled by a communication graph, in a communication graph each vehicle is considered as a vertex in the graph. In order to have a better understanding of graph theory and to design distributed tracking controller later, Laplacian matrix will be studied in this section. The Laplacian matrix can be used to find many other properties of the graph.

In the mathematical field of graph theory, the Laplacian matrix is a matrix representation of a graph, with the aid of Laplacian matrix, one can analyzes communication graph mathematically.

For a communication graph $\mathcal{G}_n$ that contains n vehicles $\mathcal{G}_n = (V_n, E_n)$, an adjacency matrix $\mathcal{A}$ associated with $\mathcal{G}_n$ is defined as $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$, where $a_{ij} = 1$ if v_j is able to receive information from v_i, if v_j cannot receive information from v_i then $a_{ij} = 0$. Because of the fact that a mobile robot is always able to acquire its own information without communicating with any other vehicles, so a_{ii} are assumed to be zero for all i, unless stated otherwise. An additional weight factor $\mathcal{A}_w = [w_{ij}a_{ij}] \in \mathbb{R}^{n\times n}$ is needed to describe the communication more accurately due to the fact that in real life the communication intensity between different links is usually uneven. It is proved that the additional weights have no effects on the overall system consensus performance but only affect the convergence time.

Given a simple graph $\mathcal{G}_n$ with n vehicles $\mathcal{G}_n = (V_n, E_n)$, its Laplacian matrix $L \coloneqq (l_{ij})_{n\times n}$ is defined as

$$L = D - \mathcal{A} \tag{2.1}$$

where $D = diag(\sum_{j=1}^{n} a_{1j}, \sum_{j=1}^{n} a_{2j}, \ldots, \sum_{j=1}^{n} a_{nj})$ is the degree matrix and $\mathcal{A}$ is the adjacency matrix of the graph. Note that the degree matrix D of a digraph is a diagonal matrix $D = [D_{ij}]$ where $D_{ij} = 0$ for all $i \neq j$ and $D_{ii} = deg_{out}(v_i)$.

Take the undirected communication topology shown in Figure 2.2 as an example.

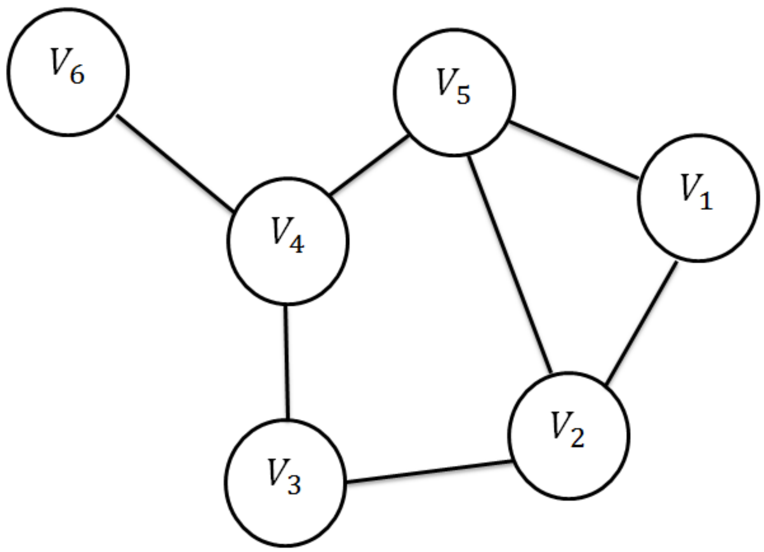

Figure 2.2 Communication graph II

Figure 2.2 shows a graph contains six vertexes and all the links between different vertexes are bidirectional, so the communication graph is undirected. According to (2.1), the Laplacian matrix that associated with the graph shown in Figure 2.2 is

$$L = D - \mathcal{A}$$

$$= \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

$$
= \begin{bmatrix} 2 & -1 & 0 & 0 & -1 & 0 \\ -1 & 3 & -1 & 0 & -1 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & -1 & 3 & -1 & -1 \\ -1 & -1 & 0 & -1 & 3 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}
$$

Note that for an undirected graph, there exists a property that the Laplacian matrix L is a symmetric positive semi-definite matrix, however, the Laplacian matrix L for a directed graph does not have this property.

2.4 Properties of Laplacian Matrix

From section 2.3, some basic concepts and notations in Laplacian matrix are introduced. Consider that all the distributed tracking controllers we designed in this thesis are under a simple directed communication topology, so in this subsection some fundamental properties of Laplaican matrix of digraphs will be studied.

Theorem 2.1: Consider a weighted digraph $\mathcal{G} = (V, E, \mathcal{A})$ with Laplacian matrix L, then $rank(L) = n - 1$ if $\mathcal{G}$ is strongly connected.

A direct relation between the strongly connected property of a directed graph and the rank of its Laplacian is established by the above theorem, according to theorem 2.1, the Laplacian of a strongly connected directed graph always has an isolated eigenvalue at zero. For an undirected graph $\mathcal{G}$, $\mathcal{G}$ is connected if and only of $rank(L) = n - 1$. Note that the opposite side of theorem does not hold.

Proof: The proof of theorem 2.1 is given in [42] ■

Definition 2.1: Gershgorin disc $D(k_{ii}, R_i)$ is defined as a closed disc centered at k_{ii} with radius R_i, where K is a complex matrix with entries k_{ij}. For $i \in \{1, \dots n\}$ let $R_i = \sum_{j \neq i} |k_{ij}|$ be the sum of the absolute values of the non-diagonal entries in the i^{th} row.

Theorem 2.2: Every eigenvalue of K lies within at least one of the Gershgorin discs $D(k_{ii}, R_i)$.

Theorem 2.3: Let $\mathcal{G} = (V, E, \mathcal{A})$ be a weighted digraph with Laplacian matrix L, all the eigenvalues of L are located in the following disk:

$$D(G) = \{|z - d_{max}(\mathcal{G})| \leq d_{max}(\mathcal{G})\}$$

The disc is centered at $z = d_{max}(\mathcal{G}) + 0j$ in the complex plane (Figure 2.3). Where $d_{max}(\mathcal{G}) = max_i deg_{out}(v_i)$ denotes the maximum node out-degree of the digraph $\mathcal{G}$.

Proof: The proof of theorem 2.3 is given in [42] ■

By definition, the sum of every row of the Laplacian matrix is zero, therefore, the laplacain matrix always has a zero eigenvalue.

Theorem 2.4: Let $\mathcal{G} = (V, E)$ be a graph with Laplacian matrix L, all eigenvalues of L are located in the right half of the complex plane but one is zero if $\mathcal{G}$ has a spanning tree.

Theorem 2.5: Let $\mathcal{G} = (V, E, \mathcal{A})$ be a weighted digraph with Laplacian matrix L, all eigenvalues of L are located in the right half of the complex plane but one is zero if $\mathcal{G}$ is strongly connected.

Theorem 2.6: Assume $\mathcal{G}$ is a strongly connected digraph with Laplacian matrix L satisfying $L_{wr} = 0$, $w_l^T L = 0$ and $w_l^T w_r = 1$, then

$$R = \lim_{t \to \infty} exp(-Lt) = w_r w_l^T \epsilon M_n$$

where w_r and w_l are the right and left eigenvectors of the Laplacian matrix L associated with $\lambda_1 = 0$, respectively. M_n are the set of square $n \times n$ matrices.

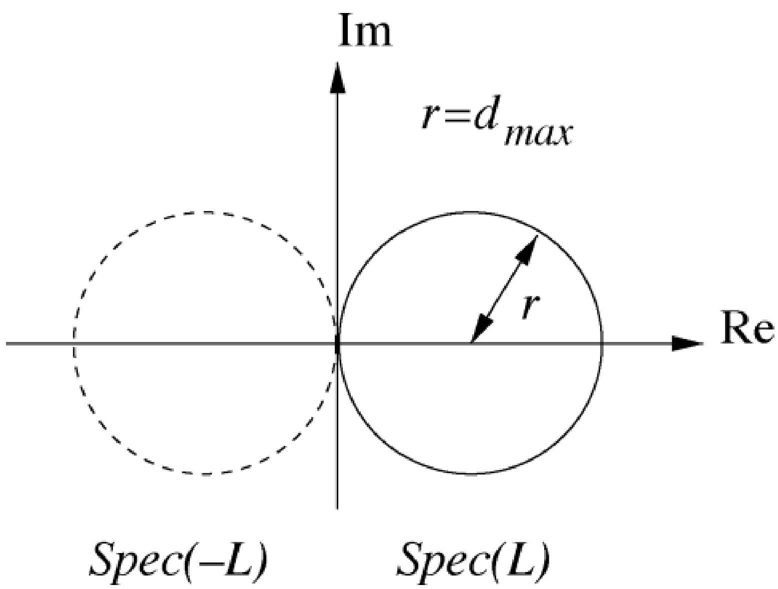

Figure 2.3 Gershgorin disc

2.5 Stability of Nonlinear System

A nonlinear system refers to a set of nonlinear equations such as algebraic, differential, integral or abstract equations used to describe a physical device which is mobile robot in our case. Stability theory plays a central role in system engineering, especially in the field of control systems and automation. The basic concept of stability emerged from the study of an equilibrium state of a mechanical system. In general, stability means the outputs of the system and its internal signals are bounded within admissible limits (bounded-input/bounded output stability) or more strictly, the system outputs tend to an equilibrium state of interest (asymptotic stable).

A continuous-time nonlinear control system in generally described by a differential equation following the form

$$\dot{x} = f(x, t; u), \quad t \in [t_0, \infty) \tag{2.2}$$

where $x = x(t)$ is the state of the system, u is the control input and f is a Lipschitz or continuously differentiable nonlinear function. System (2.2) has a unique solution for each control input and suitable initial condition $x(t_0) = x_0$, the initial time used here is $t_0 \geq 0$ unless otherwise indicated. Since function f depends explicitly on time variable t, so the system is called non-autonomous, a special case of the nonlinear system (2.2) is that the time variable t is independent from the state vector in the system function f, in this case the nonlinear system (2.2) is called autonomous. For example,

$$\dot{x} = f(x), \quad x(t_0) = x_0 \tag{2.3}$$

with a state feedback control $u(t) = h(x(t))$.

An equilibrium or fixed point x^* of system (2.3), if exists, must be a constant state and is defined as

$$f(x^*) = 0$$

An equilibrium is stable if some nearby trajectories of the system states approach it, it is unstable if some nearby trajectories of the system states move away from it.

In an autonomous system, suppose that $x^* \neq 0$ and consider the a variable $y = x - x^*$. The derivative of y is given by

$$\dot{y} = \dot{x} = f(x) = f(y + x^*) \stackrel{\text{def}}{=} g(y)$$

where $g(0) = 0$, so system has an equilibrium at 0 with the new variable y. Therefore, without loss generality, we can study the stability of the origin $x = 0$ with the assumption that $f(x)$ satisfies $f(0) = 0$.

Definition 2.2: The equilibrium point $x = 0$ of autonomous system (2.3) is stable if for each $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$ such that

$$\|x(0)\| < \delta \Rightarrow \|x(t)\| < \varepsilon, \quad \forall\, t \geq 0$$

Definition 2.3: The equilibrium point $x = 0$ of autonomous system (2.3) is asymptotically stable if it is stable and δ can be chosen such that

$$\|x(t_0)\| < \delta \Rightarrow \lim_{t\to\infty} x(t) = 0$$

Having defined stability and asymptotic stability of equilibrium points in definition 2.2 and 2.3, now we show the ways to determine stability. Let us define the energy $E(x)$ as the sum of the system's potential and kinematic energies, here we choose the reference of the potential energy such that $E(0) = 0$. When friction is neglected, E is a constant, in other words, $dE/dt = 0$ while the system is in motion, so we can again say that $x = 0$ is a stable equilibrium. When friction is not neglected, $dE/dt \leq 0$ while the system is in motion, and the trajectory tends to $x = 0$ as $t \to \infty$ sicne E cannot remain as a constant and keeps decreasing until it eventually reaches zero. So one can determine the stability of the equilibrium point by examining the derivative of E. However, Lyapunov showed that particular functions can be used instead of energy to determine stability. Let $V: D \to R$ be a continuously differentiable function defined in a domain $D \in R^n$ that contains the origin, the derivative of V of system (2.3) is given by

$$\dot{V}(x) = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} \dot{x}_i = \sum_{i=1}^{n} \frac{\partial V}{\partial x_i} f_i(x)$$

$$= \left[\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \dots, \frac{\partial V}{\partial x_n}\right] \begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_n(x) \end{bmatrix}$$

$$= \frac{\partial V}{\partial x} f(x)$$

The derivative of V is dependent on the system's equation, so $\dot{V}(x)$ will be different for different systems. Assume $\phi(t, x)$ is the solution of (2.3) that starts at initial state x at $t = 0$, then

$$\dot{V}(x) = \frac{d}{dt} V\big(\phi(t, x)\big) \text{ at } t = 0$$

therefore, if $\dot{V}(x)$ is negative, V will decrease along the solution of (2.3). Now Lyapunov's stability theorem is ready to be used.

Theorem 2.7: Let $x = 0$ is an equilibrium point for system (2.3) and $D \in R^n$ be a domain that contains $x = 0$. Let $V: D \to R$ be a continuously differentiable function such that

$$V(0) = 0 \quad \text{and} \quad V(x) > 0 \text{ in } D - \{0\} \tag{2.4}$$

$$\dot{V}(x) \leq 0 \text{ in } D \tag{2.5}$$

then $x = 0$ is stable. Moreover, if

$$\dot{V}(x) < 0 \text{ in } D$$

then $x = 0$ is asymptotically stable.

Proof: The proof of theorem 2.7 is given in [61] ■

A continuously differentiable function $V(x)$ satisfying (2.4) and (2.5) is called a Lyapunov function. If a function $V(x)$ satisfying (2.4), then it is positive definite. If it satisfies a weaker condition $V(x) \geq 0$ for $x \neq 0$, then it is positive semi-definite. A function $V(x)$ is called negative definite if $-V(x)$ is positive definite and negative semi-definite if $-V(x)$ is positive semi-definite.

Theorem 2.8: Let $x = 0$ is an equilibrium point for system (2.3). Let $V: R^n \to R$ be a continuously differentiable function such that

$$V(0) = 0 \quad \text{and} \quad V(x) > 0 \;\; \forall\, x \neq 0 \tag{2.6}$$

$$\|x\| \to \infty \;\Rightarrow\; V(x) \to \infty \tag{2.7}$$

$$\dot{V}(x) < 0, \;\; \forall\, x \neq 0 \tag{2.8}$$

then x is globally asymptotically stable.

Proof: The proof of theorem 2.8 is given in [61] ■

So far in this section, we discussed the stability of equilibrium in autonomous systems which is a special case of the nonlinear system (2.2). Now we consider the non-autonomous system (2.2), for simplicity discussion, if we combine the control input $u(t) = h(x(t), t)$ in (2.2) into the system function f, then we have a general non-autonomous system

$$\dot{x} = f(x, t), \quad x(t_0) = x_0 \tag{2.9}$$

Definition 2.4: System (2.9) is stable in the sense of Lyapunov (Lyapunov stable) with respect to the equilibrium $x^* = 0$, for any $\varepsilon > 0$ and any initial time $t_0 > 0$, if there exists a constant $\delta = \delta(\varepsilon, t_0)$ such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| < \varepsilon, \quad \forall\, t \geq t_0$$

Note that the constant δ is always depend on both ε and t_0. Conceptually, Lyapunov stability of an equilibrium means that solutions starting with a distance δ from the equilibrium which is close enough, and remain that distance forever. This must be true for any ε that one may choose.

Definition 2.5: System (2.9) is said to be asymptotically stable with respect to the equilibrium $x^* = 0$, if it is Lyapunov stable, moreover, if there exists a constant $\delta = \delta(t_0) > 0$ such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| \to 0, \quad \text{as } t \to \infty$$

Conceptually, asymptotic stability means that solutions not only remain close enough to the equilibrium, but also converge to the equilibrium eventually. Note that the nonlinear system (2.9) is called uniform asymptotically stable if the constant δ is independent on $t_0 \in [0, \infty)$, and is called global asymptotically stable if $\|x(t)\| \to 0$ is not dependent on the initial state $x(t_0)$.

Definition 2.6: System (2.9) is exponential stable with respect to the equilibrium $x^* = 0$, if it is asymptotically stable, furthermore, if for two positive constants c and σ such that

$$\|x(t_0)\| < \delta \Rightarrow \|x(t)\| \leq ce^{-\sigma t}$$

Exponential stability means that solutions not only converge to the equilibrium, but also converge faster than or at least as fast as a particular known rate.

It is shown in definition 2.4 to definition 2.6 that exponential stability implies asymptotic stability and asymptotic stability implies the Lyapunov stability.

CHAPTER III

DISTRIBUTED TRACKING CONTROL OF MULTIPLE KINEMATIC WMRS

In this chapter, we shall derive two common types of kinematic equations for wheeled mobile robots, distributed tracking control laws are proposed for both kinematic models with the aid of the analytical tools that are introduced in previous chapter II. The main feature of a kinematic model of wheeled mobile robot is the presence of nonholonomic constrains due to the rolling without slipping condition between the wheels and ground.

First, we introduce two common kinematic models of car-like robots, two-wheel model and four-wheel model. Two-wheel model vehicle has three generalized states including two Cartesian coordinates with respect to x-axis and y-axis, and the heading angle with respect to x-axis while four-wheel model vehicle contains four generalized states including two Cartesian coordinates with respect to x-axis and y-axis, the orientation of the car body with respect to x-axis and the steering angle. Second, linearization algorithms are used to transform the nonlinear robot kinematics and dynamics into linear system since it is easier and straightforward to design the controller for tracking control problems in linear model. After linearization, new distributed control methods will be proposed for both kinematic systems with the aid of cascaded system theory and graph theory, for the four-wheel model kinematics, the results of first-order kinematics and second-order dynamics controllers that introduced in [30] are extended to the third-order dynamics. The control inputs in kinematic model are translational and rotational velocities.

3.1 Kinematic Model of Four-Wheel Mobile Robots

Now we consider multiple four-wheel Mobile Robots with the kinematics as shown in Figure 3.1. For simplicity discussion, we use two fixed points located at the midpoint of front axle and rear axle as references for each vehicle and the front wheels are steering wheels. The generalized coordinates of four-wheel vehicles are defined as $q_i = (x_i, y_i, \theta_i, \phi_i)$, where $i = 0, 1,2, \dots \dots, n$, x_i and y_i are the Cartesian coordinates of the rear reference points with respect to x-axis and y-axis, θ_i measures the orientation of the car body with respect to x-axis and ϕ_i is the steering angle.

Because of the nonholonomic constrains in the presence of rolling without slipping condition between wheels and ground, so the system shown in Figure 3.1 subject to two nonholonomic constraints:

$$\dot{x}_{if}\, sin(\theta_i + \phi_i) - \dot{y}_{if} \cos(\theta_i + \phi_i) = 0$$

$$\dot{x}_i sin\theta_i - \dot{y}_i cos\theta_i = 0$$

where x_{if} and y_{if} are the Cartesian coordinates of the front wheel and are defined as

$$x_{if} = x_i + l_i cos\theta_i$$

$$y_{if} = y_i + l_i sin\theta_i$$

where l_i is the distance between the fixed midpoint of the front axle and rear axle and all the vehicle's l_i are equal because we assume that all vehicles have the same mechanical structure.

Then the first constraint can be rewritten as

$$\dot{x}_i sin(\theta_i + \phi_i) - \dot{y}_i\, cos(\theta_i + \phi_i) - \dot{\theta}_i l_i cos\phi_i = 0$$

The Pfaffian constraint matrix is

$$C(q_i) = \begin{bmatrix} sin(\theta_i + \phi_i) & -cos(\theta_i + \phi_i) & -l_i cos\phi_i & 0 \\ sin\, \theta_i & -\cos\theta_i & 0 & 0 \end{bmatrix}$$

Note that the rank of the Pfaffian constraint matrix is 2.

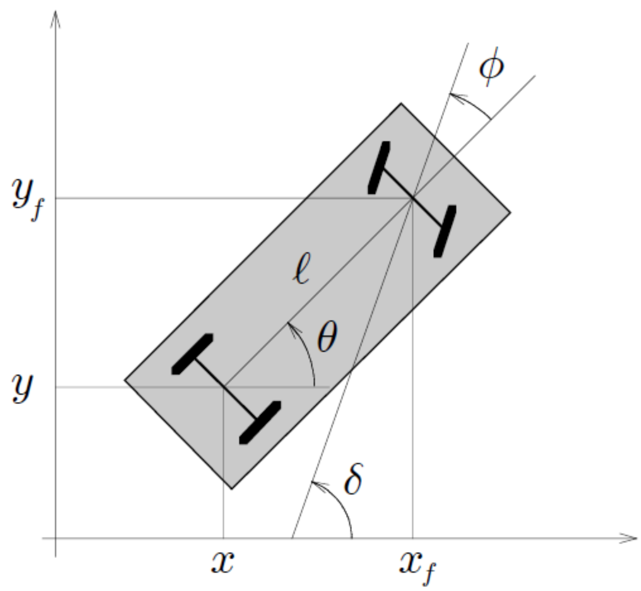

Figure 3.1 Generalized coordinates of four-wheel vehicles

The four-wheel vehicle systems can be either driven by front wheels or rear wheels, and kinematic models are different based on different systems model, but throughout this thesis we only focus on the model of rear-wheel driving. The kinematic model of rear-wheel driving model is derived as

$$\begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \\ \dot{\phi}_i \end{bmatrix} = \begin{bmatrix} cos\,\theta_i \\ sin\,\theta_i \\ \frac{tan\,\phi_i}{l_i} \\ 0 \end{bmatrix} v_{i1} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_{i2} \tag{3.1}$$

where v_{i1} and v_{i2} are the linear velocity and the angular velocity input, respectively, for each vehicle. Note that the steering angle ϕ_i cannot reach or over 90° which makes perfect sense in reality. System (3.1) can be rewritten as

$$\dot{q}_i = g_{i1}(q_i)v_{i1} + g_{i2}(q_i)v_{i2} \tag{3.2}$$

where

$$g_{i1} = \begin{bmatrix} cos\,\theta_i \\ sin\,\theta_i \\ \frac{tan\,\phi_i}{l_i} \\ 0 \end{bmatrix} \quad \text{and} \quad g_{i2} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The above system (3.2) is nonlinear and driftless which means in order to make any motion of the system, inputs must be implemented and cannot be zero. Note that there are less control inputs than generalized coordinates in (3.2).

3.2 Full-State Linearization via Dynamic Feedback

It is well known in control theory that if the number of generalized coordinates equals the number of input commands, then a nonlinear static state feedback law can be used to transform exactly the nonlinear robot kinematics and dynamics into a linear system. For a linear system, it is easier and straightforward to design the distributed tracking controllers for multiple WMRs system, so in this subsection a linearization algorithm called full-state linearization is used to transform the nonlinear robot systems into linear systems and another linearization algorithm called input-output linearization will be introduced later.

Suppose that a team consists of n vehicles, in accordance with the task definition, we choose the two system outputs as

$$z_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

Namely the x_i and y_i coordiantes of each robot. Differentiating z_i with respect to time, we have

$$\dot{z}_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \end{bmatrix} = \begin{bmatrix} v_{i1}\, cos\, \theta_i \\ v_{i1}\, sin\, \theta_i \end{bmatrix} = \begin{bmatrix} cos\, \theta_i & 0 \\ sin\, \theta_i & 0 \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \end{bmatrix} \tag{3.3}$$

where both v_{i1} and v_{i2} are the control inputs. Note that the decoupling matrix is singular because the input v_{i2} does not appear. In order to proceed with differentiation, an integrator with state denoted by ξ_{i1} is added on the first input.

$$v_{i1} = \xi_{i1} \qquad\qquad \dot{\xi_{i1}} = v_{i1}{}' \tag{3.4}$$

where $v_{i1}{}'$ is a new auxiliary input. Using equation (3.4) we can rewrite the first derivative of the output (3.3) as

$$\dot{z}_i = \begin{bmatrix} \xi_{i1}\, cos\, \theta_i \\ \xi_{i1}\, sin\, \theta_i \end{bmatrix}$$

which is independent from the inputs v_{i1}' and v_{i2} of the extended system. In this way, differentiation of the original input signal at the next step of the procedure is avoided, then we have

$$\ddot{z}_i = \begin{bmatrix} v_{i1}' \cos\theta_i - \xi_{i1}^{\,2} \sin\theta_i \dfrac{\tan\theta_i}{l_i} \\ v_{i1}' \sin\theta_i - \xi_{i1}^{\,2} \cos\theta_i \dfrac{\tan\theta_i}{l_i} \end{bmatrix} = \begin{bmatrix} -\xi_{i1}^{\,2} \sin\theta_i \dfrac{\tan\theta_i}{l_i} \\ \xi_{i1}^{\,2} \cos\theta_i \dfrac{\tan\theta_i}{l_i} \end{bmatrix} + \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \end{bmatrix} \begin{bmatrix} v_{i1}' \\ v_{i2} \end{bmatrix}$$

As v_{i2} still does not appear yet, another integrator is needed on the input v_{i1}', the state of integrator is denoted by ξ_{i2}

$$v_{i1}' = \xi_{i2} \qquad \dot{\xi}_{i2} = v_{i1}'' \tag{3.5}$$

then we have

$$\ddot{z}_i = \begin{bmatrix} \dfrac{-\xi_{i1}^{\,2} \tan\phi_i \sin\theta_i}{l_i} + \xi_{i2} \cos\theta_i \\ \dfrac{\xi_{i1}^{\,2} \tan\phi_i \cos\theta_i}{l_i} + \xi_{i2} \sin\theta_i \end{bmatrix}$$

finally, the last differentiation gives us

$$\dddot{z}_i = \begin{bmatrix} v_{i1}'' \cos\theta_i - \xi_{i2} \sin\theta_i \dfrac{\tan\theta_i}{l_i} \xi_{i1} - \dfrac{\sec^2\phi_i v_{i2}}{l_i} \xi_{i1}^{\,2} \sin\theta_i + \dfrac{\tan\phi_i}{l_i} 2\xi_{i1}\xi_{i2} + \dfrac{\tan^2\phi_i}{l_i} v_{i1}\xi_{i1}^{\,2} \cos\theta_i \\ v_{i1}'' \sin\theta_i + \xi_{i2} \cos\theta_i \dfrac{\tan\phi_i}{l_i} \xi_{i1} + \dfrac{\sec^2\phi_i v_{i2}}{l_i} \xi_{i1}^{\,2} \cos\theta_i + \dfrac{\tan\phi_i}{l_i} 2\xi_{i1}\xi_{i2} - \dfrac{\tan^2\phi_i}{l_i} v_{i1}\xi_{i1}^{\,2} \sin\theta_i \end{bmatrix} \tag{3.6}$$

We set $\dddot{z}_{i1} = r_{i1}$ and $\dddot{z}_{i2} = r_{i2}$ which are both auxiliary inputs values to solve equation (3.6) for

$$\begin{bmatrix} v_{i1}'' \\ v_{i2} \end{bmatrix} = \begin{bmatrix} {\xi_{i1}}^3 \frac{tan^2\phi_i}{{l_i}^2} + r_{i1} \, cos\,\theta_i + r_{i2} \, sin\,\theta_i \\ \frac{-3\xi_{i2} cos^2\phi_i \, tan\,\phi_i}{\xi_{i1}} - \frac{l_i r_{i1} cos^2\phi_i \, sin\,\theta_i}{{\xi_{i1}}^2} + \frac{l_i r_{i2} cos^2\phi_i \, sin\,\theta_i}{{\xi_{i1}}^2} \end{bmatrix} \tag{3.7}$$

Putting together the extension equation (3.4) and (3.5) with equation (3.7), the resulting nonlinear dynamic feedback controllers

$$v_{i1} = \xi_{i1}$$

$$v_{i2} = \frac{-3\xi_{i2} cos^2\phi_i \tan\phi_i}{\xi_{i1}} - \frac{l_i r_{i1} cos^2\phi_i \sin\theta_i}{{\xi_{i1}}^2} + \frac{l_i r_{i2} cos^2\phi_i \cos\theta_i}{{\xi_{i1}}^2}$$

$$\dot{\xi}_{i1} = \xi_{i2}$$

$$\dot{\xi}_{i2} = {\xi_{i1}}^3 \frac{tan^2\phi_i}{{l_i}^2} + r_{i1} \cos\theta_i + r_{i2} \sin\theta_i$$

transforms the original nonlinear system into a two linear decoupled system with three integrators

$$\dddot{z}_{i1} = r_{i1}$$
$$\dddot{z}_{i2} = r_{i2} \tag{3.8}$$

The original system has four states and the dynamic controllers have two additional states (states of two integrators) and all these six states can be found in the above input-output description (3.8), so there are no internal dynamics left. Now we can say that the full-state linearization has been achieved. On the linear and decoupled system, it is easier to complete the control design with a globally stabilizing feedback for the desired trajectory. Note that if $i = 0$, then it means it is the controllers of the virtual leader of the system which are assumed to be already known, so here in this thesis we only focus on designing the controller of the follower WMRs.

3.3 Distributed Control of Kinematic Four-Wheel Mobile Robots

In section 3.2, all the nonlinear robotic systems are transferred into linear systems, on the linear and decoupled system, it is easier to complete the controller design with a globally stabilizing feedback for the desired trajectory which is, in our case, the trajectory of the virtual leader. Suppose that in a multiple-vehicle system, there are n vehicles, labeled 1 to n, called follower robots and there exists a virtual leader which is labeled as vehicle 0 with a positon $z_0 = [x_0, y_0]$ and velocity $\dot{z}_0 = [\dot{x}_0, \dot{y}_0]$. We assume that throughout this thesis, $\dot{x}_0 \leq \gamma_\ell$ and $\dot{y}_0 \leq \gamma_\ell$, where γ_ℓ is a positive constant.

Problem statement: In this section, our control objective is to design distributed controllers for all follower robots in the multiple-vehicle system by using its own state information as well as its neighbor's state information such that

$$\lim_{t\to\infty}(x_i - x_0) = 0$$

$$\lim_{t\to\infty}(y_i - y_0) = 0$$

where $(i = 1, 2, \dots \dots, n)$.

We assume that not all vehicles in the system are neighbors of the leader robot, which means only a few subsets of group of followers have the leader robot's state information.

Now consider followers with third-order dynamics given by

$$\dot{z}_{i1} = \varepsilon_{i1}\,, \quad \dot{\varepsilon}_{i1} = \delta_{i1}\,, \quad \dot{\delta}_{i1} = r_{i1}\,, \quad i = 1, 2, \dots\dots, n \tag{3.9}$$

$$\dot{z}_{i2} = \varepsilon_{i2}\,, \quad \dot{\varepsilon}_{i2} = \delta_{i2}\,, \quad \dot{\delta}_{i2} = r_{i2}\,, \quad i = 1, 2, \dots\dots, n \tag{3.10}$$

where $z_{i1} \epsilon \mathbb{R}$, $\varepsilon_{i1} \epsilon \mathbb{R}$ and $\delta_{i1} \epsilon \mathbb{R}$ are, respectively, the x-axis position, velocity and acceleration of x-axis direction of follower i, $z_{i2} \epsilon \mathbb{R}$, $\varepsilon_{i2} \epsilon \mathbb{R}$ and $\delta_{i2} \epsilon \mathbb{R}$ are, respectively, the y-axis position, velocity and acceleration of y-axis direction of follower i. Both $r_{i1} \epsilon \mathbb{R}$ and $r_{i2} \epsilon \mathbb{R}$ are control inputs.

In this subsection, we only consider the case that when all mobile robots are in a one-dimensional space, but all the results hereafter are still valid for the n-dimensional with the aid of Kronecker product. The objective here is to design r_{i1} in (3.9) and r_{i2} in (3.10) for third-order dynamics such that all followers are able to track the trajectory of the virtual leader with local interaction.

Now consider system (3.9), with the aid of new variables defined below (3.11), the system (3.9) can be rewritten as

$$\dot{z}_{i1} = -k_1 z_{i1} + k_1 z_{i1} + \varepsilon_{i1} = -k_1 z_{i1} + x_{i2}$$

$$\dot{x}_{i2} = k_1 \dot{z}_{i1} + \delta_{i1} = -k_2 x_{i2} + k_2 x_{i2} + k_1 \dot{z}_{i1} + \delta_{i1} = -k_2 x_{i2} + x_{i3}$$

$$\dot{x}_{i3} = k_2 \dot{x}_{i2} + k_1 \ddot{z}_{i1} + r_{i1} = u_{i1}$$

where $k_1 > 0$ and $k_2 > 0$. We have

$$x_{i2} = k_1 z_{i1} + \varepsilon_{i1}$$

$$x_{i3} = k_2 x_{i2} + k_1 \dot{z}_{i1} + \delta_{i1} \tag{3.11}$$

$$u_{i1} = k_2 \dot{y}_{i2} + k_1 \ddot{z}_{i1} + r_{i1}$$

now system (3.9) becomes to

$$\dot{z}_{i1} = -k_1 z_{i1} + x_{i2}$$

$$\dot{x}_{i2} = -k_2 \varepsilon_{i1} + x_{i3} \tag{3.12}$$

$$\dot{x}_{i3} = u_{i1}$$

Lemma 3.1: For the variables that defined in (3.11), if we have

$$\lim_{t\to\infty}(z_{i1} - z_{01}) = 0$$

$$\lim_{t\to\infty}(x_{i2} - x_{02}) = 0$$

$$\lim_{t\to\infty}(x_{i3} - x_{03}) = 0$$

then

$$\lim_{t\to\infty}(z_{i1} - z_{01}) = 0$$

$$\lim_{t\to\infty}(\varepsilon_{i1} - \varepsilon_{01}) = 0$$

$$\lim_{t\to\infty}(\delta_{i1} - \delta_{01}) = 0$$

are guaranteed.

Proof: First we have

$$\lim_{t\to\infty}(x_{i2} - x_{02}) = 0$$

so by definition, we get

$$\lim_{t\to\infty}[(k_1 z_{i1} + \varepsilon_{i1}) - (k_1 z_{01} + \varepsilon_{01})] = 0$$

since k_1 is a positive constant and $\lim_{t\to\infty}(z_{i1} - z_{01}) = 0$, so that

$$\lim_{t\to\infty}(\varepsilon_{i1} - \varepsilon_{01}) = 0$$

is achieved.

Also according to (3.11)

$$\lim_{t\to\infty}(x_{i3} - x_{03}) = \lim_{t\to\infty}[(k_2 x_{i2} + k_1 \dot{z}_{i1} + \delta_{i1}) - (k_2 x_{02} + k_1 \dot{z}_{01} + \delta_{01})]$$

$$= \lim_{t\to\infty}[(k_2 x_{i2} + k_1 \varepsilon_{i1} + \delta_{i1}) - (k_2 x_{02} + k_1 \varepsilon_{01} + \delta_{01})]$$

$$= 0$$

since

$$\lim_{t\to\infty}(x_{i2} - x_{02}) = 0 \quad \text{and} \quad \lim_{t\to\infty}(\varepsilon_{i1} - \varepsilon_{01}) = 0$$

so

$$\lim_{t\to\infty}(\delta_{i1} - \delta_{01}) = 0$$

is achieved. ■

Lemma 3.2: If we have $\lim_{t\to\infty}(x_{i3} - x_{03}) = 0$, then

$$\lim_{t\to\infty}(x_{i2} - x_{02}) = 0$$

$$\lim_{t\to\infty}(z_{i1} - z_{01}) = 0$$

are guaranteed.

Proof: According to (3.11)

$$\lim_{t\to\infty}(x_{i3} - x_{03}) = \lim_{t\to\infty}[(k_2 x_{i2} + k_1 \dot{z}_{i1} + \delta_{i1}) - (k_2 x_{02} + k_1 \dot{z}_{01} + \delta_{01})]$$

$$= \lim_{t\to\infty}[(k_2 x_{i2} + \dot{x}_{i2}) - (k_2 x_{02} + \dot{x}_{02})]$$

$$= 0$$

since k_1 is a positive constant, so we have

$$\lim_{t\to\infty}(x_{i2} - x_{02}) = 0$$

and we know that

$$\lim_{t\to\infty}[(k_2 x_{i2} + \dot{x}_{i2}) - (k_2 x_{02} + \dot{x}_{02})] = \lim_{t\to\infty}[(k_2 x_{i2} - k_2 \varepsilon_{i1} + x_{i3}) - (k_2 x_{02} - k_2 \varepsilon_{01} + x_{03})]$$

since

$$\lim_{t\to\infty}(\,x_{i3}-x_{03})=0$$

so we have

$$\lim_{t\to\infty}[(k_2y_{i2}-k_2\varepsilon_{i1})-(k_2y_{02}-k_2\varepsilon_{01})]$$

$$=\lim_{t\to\infty}\{[k_2(k_1z_{i1}+\varepsilon_{i1})-k_2\varepsilon_{i1}]-[k_2(k_1z_{01}+\varepsilon_{01})-k_2\varepsilon_{01}]\}$$

$$=\lim_{t\to\infty}\{[k_2(k_1z_{i1}+\varepsilon_{i1})-k_2\varepsilon_{i1}]-[k_2(k_1z_{01}+\varepsilon_{01})-k_2\varepsilon_{01}]\}$$

$$=\lim_{t\to\infty}[(k_2k_1z_{i1})-(k_2k_1z_{01})]$$

$$=0$$

Finally we get

$$\lim_{t\to\infty}(z_{i1}-z_{01})=0$$ ∎

Based on lemma 3.1, lemma 3.2 and the definition of new variables (3.11), we showed that a first-order kinematic controller $\dot{x}_{i3}=u_{i1}$ can be used to solve the third-order dynamics tracking control problem. A first-order kinematic controller for distributed tracking control problems is proposed in [30] as follows

$$u_{i1}=-\alpha\sum_{j=0}^{n}a_{ij}\left(x_{i3}-x_{j3}\right)-\beta sgn\sum_{j=0}^{n}a_{ij}\left[\left(x_{i3}-x_{j3}\right)\right] \tag{3.13}$$

where a_{ij} $(i=1,\dots,n,\ j=0,\dots,n)$ is the $(i,j)^{th}$ entry of the adjacency matrix $\mathcal{A}$, a_{i0} is a positive constant if the leader robot's state information is available to follower robot i and $a_{i0}=0$ otherwise. α and β are two positive constants and $sgn(\cdot)$ is the signum function.

Theorem 3.1: Suppose that a fixed undirected graph $\mathcal{G}$ and the leader robot is the root of the spanning. Using (3.13) for the new system (3.12), it guarantees that

$$\lim_{t\to\infty}(x_{i3} - x_{03}) = 0$$

Proof: The proof of theorem 3.1 is given in [30] ■

Similar to system (3.9), we can make the same changes to system (3.10), the new variables are defined as

$$\dot{z}_{i2} = -k_3 z_{i2} + k_3 z_{i2} + \varepsilon_{i2} = -k_3 z_{i2} + y_{i2}$$

$$\dot{y}_{i2} = k_3 \dot{z}_{i2} + \delta_{i2} = -k_4 y_{i2} + k_4 y_{i2} + k_3 \dot{z}_{i2} + \delta_{i2} = -k_4 y_{i2} + y_{i3}$$

$$\dot{y}_{i3} = k_4 \dot{y}_{i2} + k_3 \ddot{z}_{i2} + r_{i2} = u_{i2}$$

where $k_3 > 0$ and $k_4 > 0$.

We now define

$$\begin{aligned} y_{i2} &= k_3 z_{i2} + \varepsilon_{i2} \\ y_{i3} &= k_4 y_{i2} + k_3 \dot{z}_{i2} + \delta_{i2} \\ u_{i2} &= k_4 \dot{y}_{i2} + k_3 \ddot{z}_{i2} + r_{i2} \end{aligned} \tag{3.14}$$

now system (3.10) becomes to

$$\begin{aligned} \dot{z}_{i2} &= -k_3 z_{i2} + y_{i2} \\ \dot{y}_{i2} &= -k_4 \varepsilon_{i2} + y_{i3} \\ \dot{y}_{i3} &= u_{i2} \end{aligned} \tag{3.15}$$

Lemma 3.3: For the variables that defined in (3.14), if we have

$$\lim_{t\to\infty}(z_{i2} - z_{02}) = 0$$

$$\lim_{t\to\infty}(y_{i2} - y_{02}) = 0$$

$$\lim_{t\to\infty}(y_{i3} - y_{03}) = 0$$

then

$$\lim_{t\to\infty}(z_{i2} - z_{02}) = 0$$

$$\lim_{t\to\infty}(\varepsilon_{i2} - \varepsilon_{02}) = 0$$

$$\lim_{t\to\infty}(\delta_{i2} - \delta_{02}) = 0$$

are guaranteed.

Proof: The proof of lemma 3.3 is similar to lemma 3.1 ■

Lemma 3.4: If $\lim_{t\to\infty}(y_{i3} - y_{03}) = 0$, then

$$\lim_{t\to\infty}(z_{i2} - z_{02}) = 0$$

$$\lim_{t\to\infty}(y_{i2} - y_{02}) = 0$$

are guaranteed. The proof of lemma 3.4 is similar to the proof of lemma 3.2.

Theorem 3.2: Suppose that a fixed undirected graph $\mathcal{G}$ and the leader robot is the root of the spanning tree. Using the following tracking controller (3.16) for system (3.15), then it guarantees that

$$\lim_{t\to\infty}(y_{i3}-y_{03})=0$$

The controller is given as

$$u_{i2}=-\alpha\sum_{j=0}^{n}a_{ij}\left(y_{i3}-y_{j3}\right)-\beta sgn\sum_{j=0}^{n}a_{ij}\left[\left(y_{i3}-y_{j3}\right)\right] \tag{3.16}$$

where a_{ij} $(i=1,\dots,n,\ j=0,\dots,n)$ is the $(i,j)^{th}$ entry of the adjacency matrix $\mathcal{A}$, a_{i0} is a positive constant if the leader robot's state information is available to follower robot i and $a_{i0}=0$ otherwise. α and β are two positive constants and $sgn(\cdot)$ is the signum function.

Theorem 3.3: Suppose that a fixed undirected graph $\mathcal{G}$ and the leader robot is the root of the spanning tree. Using controller (3.13) for system (3.12) as x_i in kinematic description (3.1) and controller (3.16) for system (3.15) as y_i in kinematic description (3.1), then both

$$\lim_{t\to\infty}(x_{i3}-x_{03})=0$$

$$\lim_{t\to\infty}(y_{i3}-y_{03})=0$$

are guaranteed. The control inputs v_{i1} and v_{i2} are

$$v_{i1}=\xi_{i1}$$

$$v_{i2}=\frac{-3\xi_{i2}cos^2\phi_i\tan\phi_i}{\xi_{i1}}-\frac{l_iu_{i1}cos^2\phi_i\sin\theta_i}{\xi_{i1}{}^2}+\frac{l_iu_{i2}cos^2\phi_i\cos\theta_i}{\xi_{i1}{}^2}$$

where ξ_{i1} and ξ_{i2} are states of integrators that are defined in (3.4) and (3.5). u_{i1} and u_{i2} are defined in (3.13) and (3.16)

Proof: The proof is given in lemma 3.1 to lemma 3.4. Note that v_{i1} and v_{i2} can be obtained by putting together the dynamic extension (3.4) and (3.5) with equation (3.7). ■

3.4 Simulation

In this section, in order to show the effectiveness of the proposed controllers in section 3.3, we will present several simulation examples of a group of three wheeled mobile robots with a virtual leader, all the vehicles are assumed to have the same mechanical structure as shown in Figure 3.1. The communication graph of the multiple-vehicle system is shown in Figure 3.2, vertex v_0 represents the leader robot.

Figure 3.2 is a simple directed communication graph contains three follower robots and one leader robot. It is shown that the virtual leader is a neighbor to follower 1 only which means with local communication, the state information of the leader robot is only available to follower 1, the state information of follower 1 is only available to follower 2, the state information of follower 2 is only available to follower 3 and all followers are aware of its own state information.

We choose $\alpha = 1$ and $\beta = 1$ and define the trajectory of the leader vehicle (the desired trajectory) is a unit circle such that $x_0 = sin(t)$ and $y_0 = cos(t)$. The tracking errors of $x_i - x_0$ and $y_i - y_0$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 3.3, Figure 3.4 and Figure 3.5. The state information $q_i = [x_i, y_i, \theta_i, \phi_i]$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 3.6, Figure 3.7 and Figure 3.8.

It can be seen that the tracking errors of each vehicle ultimately converge to zero and state parameters of each vehicle satisfy the condition of being a unit circle, so the proposed controllers are effective.

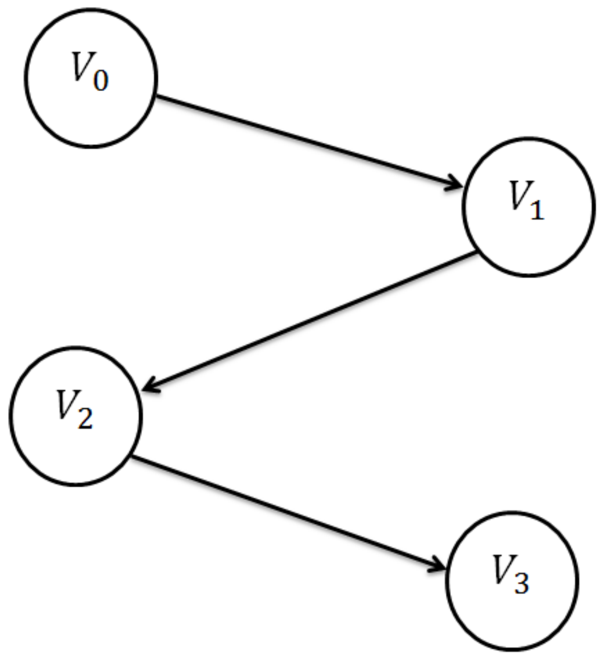

Figure 3.2 Communication graph III

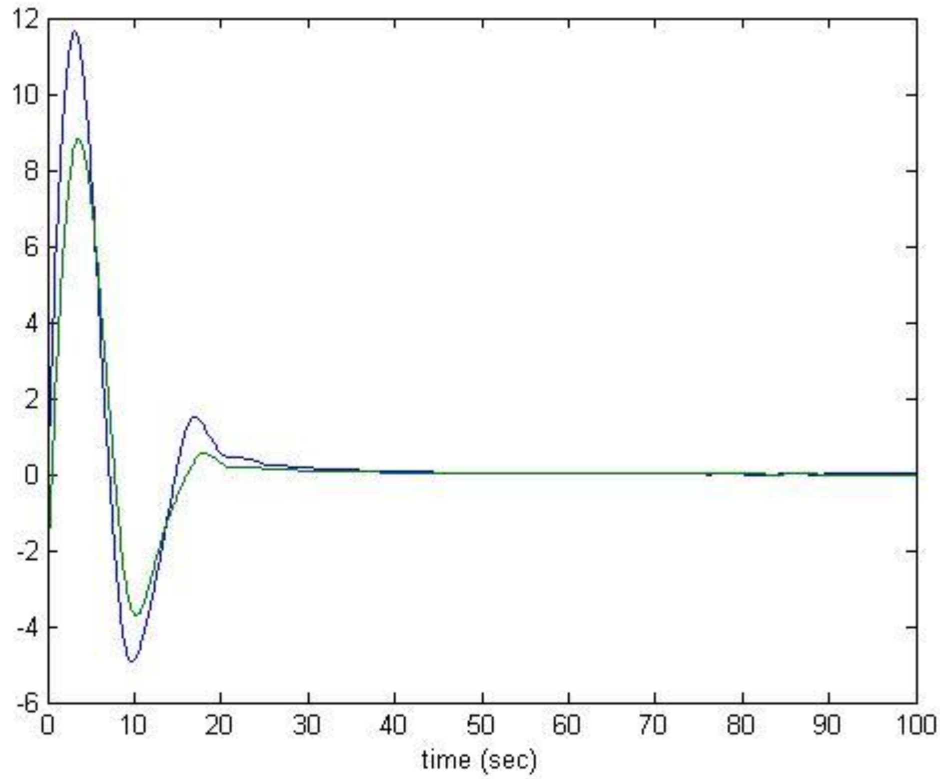

Figure 3.3 Responses of $(x_1 - x_0)$ and $(y_1 - y_0)$

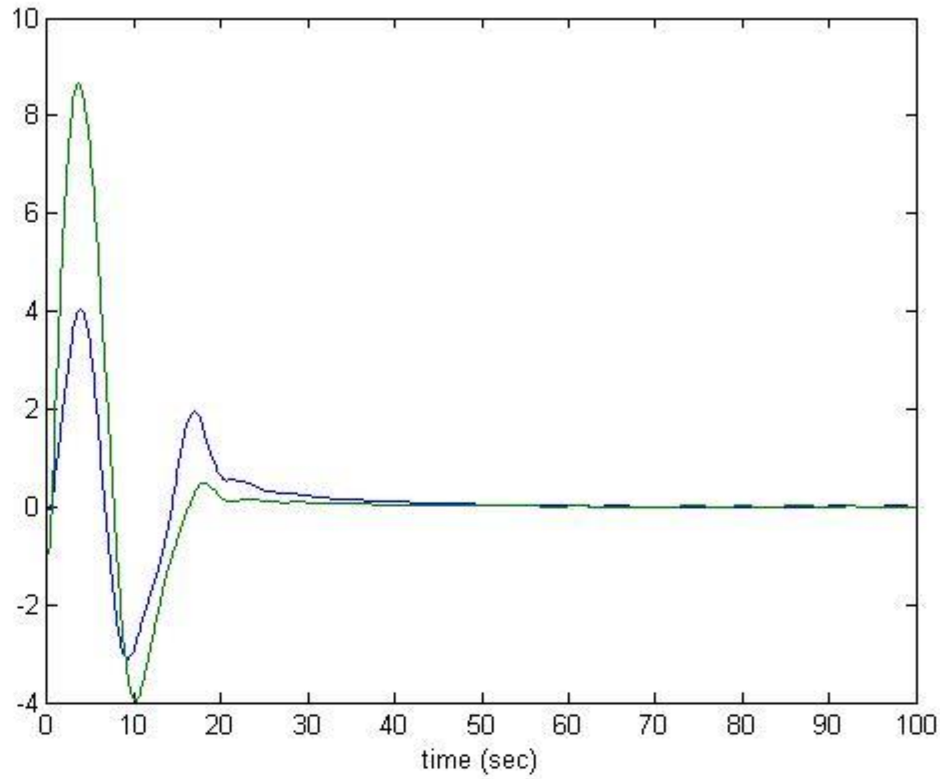

Figure 3.4 Responses of $(x_2 - x_0)$ and $(y_2 - y_0)$

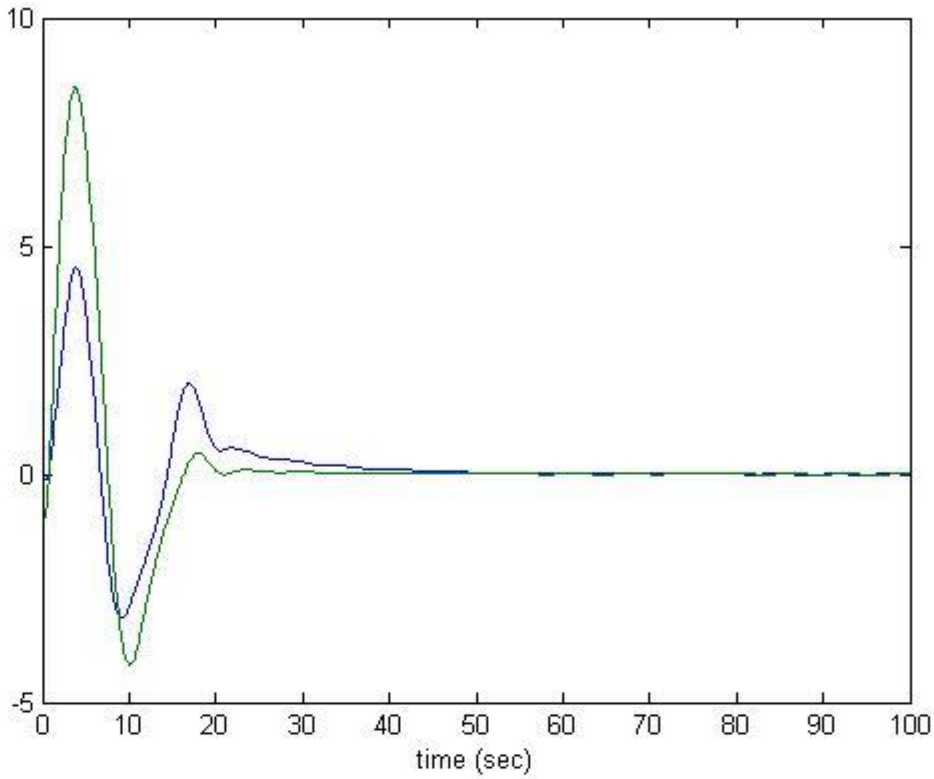

Figure 3.5 Responses of $(x_3 - x_0)$ and $(y_3 - y_0)$

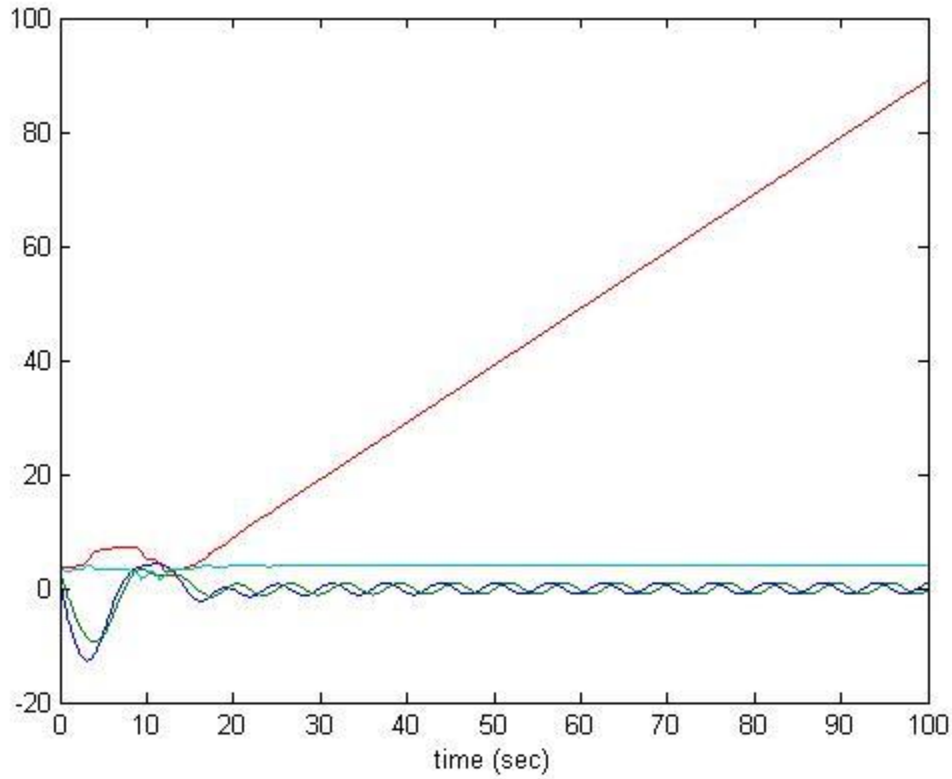

Figure 3.6 Responses of $q_1 = [x_1, y_1, \theta_1, \phi_1]$

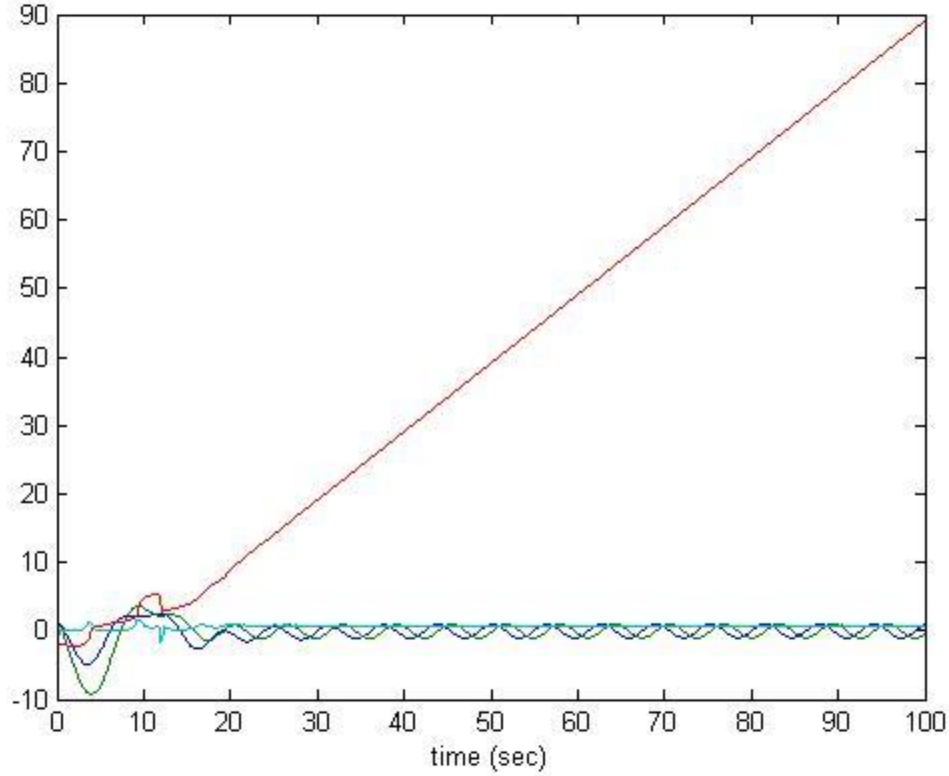

Figure 3.7 Responses of $q_2 = [x_2, y_2, \theta_2, \phi_2]$

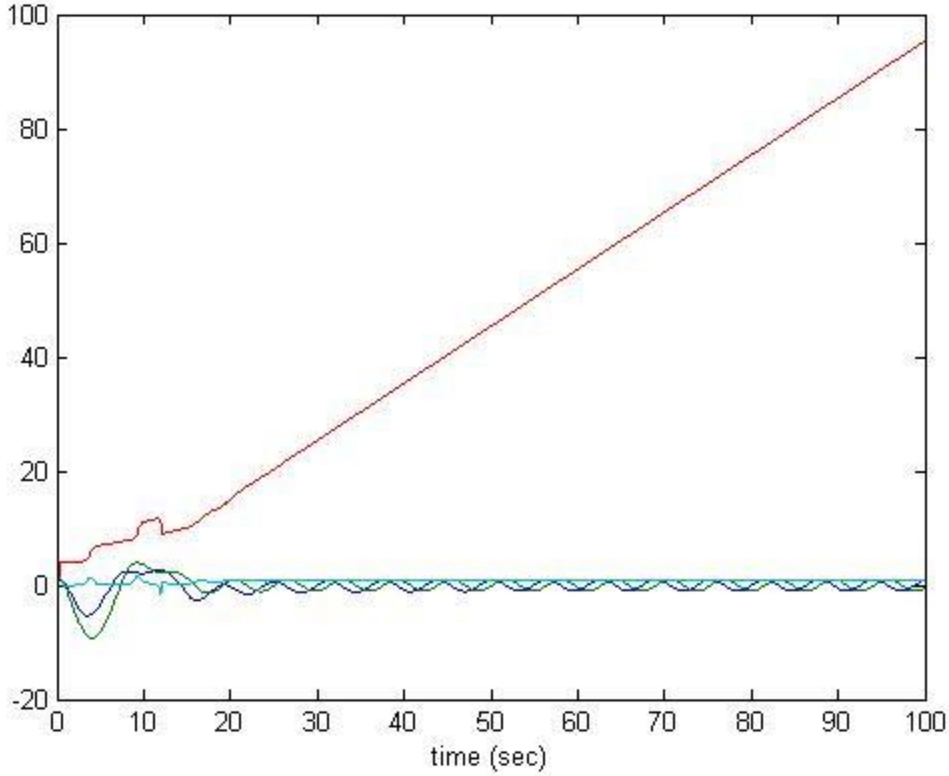

Figure 3.8 Responses of $q_3 = [x_3, y_3, \theta_3, \phi_3]$

3.5 Kinematic Model of Two-Wheel Mobile Robots

From section 3.1 to section 3.4, kinematic model of four-wheel Mobile Robots are discussed and tracking controllers are proposed accordingly. In this section, we will introduce the kinematic model of another mobile robot with two actual wheels on each side and the controllers will be proposed later.

The Two-Wheel Mobile Robots model shown in Figure 3.9 is another typical example of a nonoholonomic mechanical system. It consists of a vehicle with two driving wheels mounted on the same axis. The motion and orientation of the vehicle are achieved by independent actuator, which means DC motors provide the necessary power to the driving wheels. We can describe the configuration of the two-wheel Mobile Robots model system with a vector $q_i = (x_i, y_i, \theta_i)$ $i = 1, 2, \ldots, n$. Because of the nonholonomic constrains in the presence of pure rolling and non-slipping condition, $\dot{q}_i$ must satisfy

$$[sin\theta_i \quad -cos\theta_i \quad 0] \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = 0$$

where (x_i, y_i) are the coordinates of intersection $\boldsymbol{P}$ of the left and right wheel and θ_i are the heading angles of the mobile robots, so the kinematic model is derived as

$$\dot{q}_i = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = \begin{bmatrix} cos\,\theta_i \\ sin\,\theta_i \\ 0 \end{bmatrix} v_{i1} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_{i2} \tag{3.17}$$

where one input v_{i1} is the linear velocity and another input v_{i2} is the angular velocity.

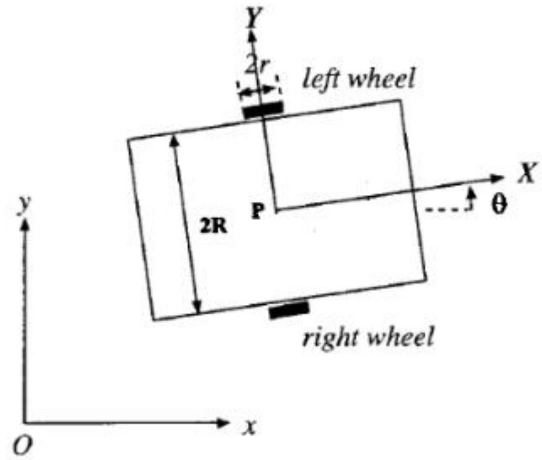

Figure 3.9 Generalized coordinates of two-wheel vehicles

3.6 Input-Output Linearization via Static Feedback

For a car-like robot kinematic model (3.8), a natural output choice for the trajectory tracking is

$$z_i = \begin{bmatrix} x_i \\ y_i \end{bmatrix}$$

The linearization algorithm begins by computing

$$\dot{z}_i = \begin{bmatrix} cos\theta_i & 0 \\ sin\theta_i & 0 \end{bmatrix} \begin{bmatrix} v_{i1} \\ v_{i2} \end{bmatrix} = S(\theta_i) v_i$$

At least one input appears in both components of $\dot{z}_i$, so that $S(\theta_i)$ is the actual decoupling matrix of the system and the system is singular. This well-known result states that linearization control techniques fail at point $\boldsymbol{P}$. A common solution to this problem is to define a new reference point located at a certain distance d from point $\boldsymbol{P}$ as shown in Figure 3.10. $2\boldsymbol{R}$ is the width of the mobile robot and r is the radius of the wheel. $O - xy$ is the world coordinate system and $P - XY$ is the coordinate system fixed to the mobile robot. In this thesis we choose a fixed point $\boldsymbol{C}$ and use this point as a reference point hereafter.

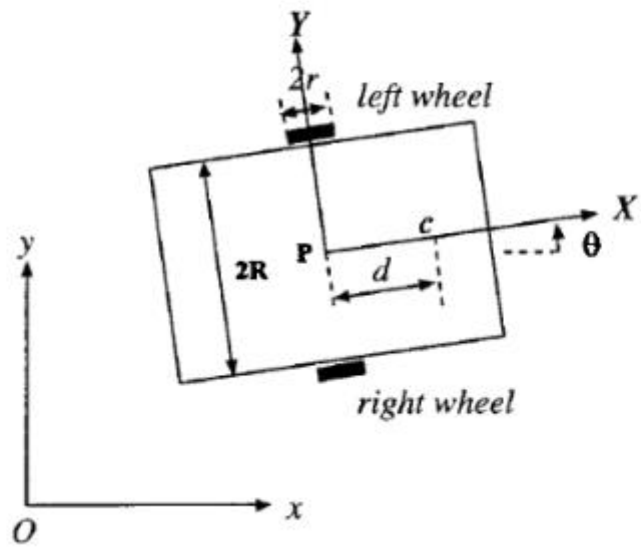

Figure 3.10 Generalized coordinates of two-wheel vehicles with new reference point

By choosing the new reference point $\boldsymbol{C}$, the nonholonomic constraints of pure rolling and non-slipping can be stated by

$$\dot{y}_{ic}\, cos\theta_{ic} - \dot{x}_{ic}\, sin\theta_{ic} - d\dot{\theta}_{ic} = 0$$

It is easy to derive the kinematic equations of point $\boldsymbol{C}$ in terms of its linear velocity and angular velocities are

$$\begin{bmatrix} \dot{x}_{ic} \\ \dot{y}_{ic} \\ \dot{\theta}_{ic} \end{bmatrix} = \begin{bmatrix} cos\theta_{ic} & -dsin\theta_{ic} \\ sin\theta_{ic} & dcos\theta_{ic} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_{ic1} \\ v_{ic2} \end{bmatrix} \tag{3.18}$$

where we assume $v_{ic1} < \gamma_\ell$ and $v_{ic2} < \gamma_\ell$, γ_ℓ is a positive constant. System (3.18) is called the steering system.

Now the output of the system can be redefined as

$$z_{ic} = \begin{bmatrix} x_{ic} \\ y_{ic} \end{bmatrix} = \begin{bmatrix} x_i + dcos\theta_{ic} \\ y_i + dsin\theta_{ic} \end{bmatrix}$$

with $d \neq 0$.

Differentiation of this new output gives

$$\dot{z}_{ic} = \begin{bmatrix} cos\theta_{ic} & -dsin\theta_{ic} \\ sin\theta_{ic} & dcos\theta_{ic} \end{bmatrix} \begin{bmatrix} v_{ic1} \\ v_{ic2} \end{bmatrix} = S(\theta_{ic})v_{ic}$$

Now we can set $\dot{z}_{ic} = r_{ic}$ as auxiliary input values and solve for in inputs v_{ic} as

$$v_{ic} = S^{-1}(\theta_{ic})v_{ic}$$

In globally defined transformed coordinates $(x_{ic}, y_{ic}, \theta_{ic})$ the closed-loop system becomes

$$\dot{x}_{ic} = \dot{z}_{ic1} = r_{ic1}$$

$$\dot{y}_{ic} = \dot{z}_{ic1} = r_{ic2}$$

$$\dot{\theta}_{ic} = v_{ic2} = -\frac{sin\theta_{ic}}{d} r_{ic1} + \frac{cos\theta_{ic}}{d} r_{ic2}$$

which is input-output linear and decoupled.

After using the input-output linearization algorithm, the nonlinear systems are transformed into linear systems, now we are ready to design the distributed tracking controllers for two-wheel mobile robots.

3.7 Distributed Control of Kinematic Two-Wheel Mobile Robots

In section 3.6, all the nonlinear robotic systems are transformed into linear systems, for a linear and decoupled system, it is easier to complete the controller design with a globally stabilizing feedback for the desired trajectory which is, in our case, the trajectory of the virtual leader. Suppose that in a multiple-vehicle system, there are n follower vehicles, labeled 1 to n, and there exists a virtual leader which is labeled as vehicle 0 with a positon $z_0 = [x_0, y_0]$ and velocity $\dot{z}_0 = [\dot{x}_0, \dot{y}_0]$. We assume that throughout this thesis, $\dot{x}_0 \le \gamma_\ell$ and $\dot{y}_0 \le \gamma_\ell$, where γ_ℓ is a positive constant.

Problem statement: In this section, our control objective is to design distributed controllers for all follower robots in the multiple-vehicle system by using its own state information as well as its neighbor's state information such that

$$\lim_{t\to\infty}(x_{ic} - x_0) = 0$$

$$\lim_{t\to\infty}(y_{ic} - y_0) = 0$$

where $(i = 1, 2, \dots\dots, \mathrm{n})$. We assume that not all vehicles in the system are neighbors of the leader robot which means only a subset of groups of followers have the leader robot's state information.

We have the kinematics of the leader robot in terms of the reference point $\boldsymbol{C}$ is given by

$$\dot{q}_{0c} = \begin{bmatrix} \dot{x}_{0c} \\ \dot{y}_{0c} \\ \dot{\theta}_{0c} \end{bmatrix} = \begin{bmatrix} cos\,\theta_{0c} \\ sin\,\theta_{0c} \\ 0 \end{bmatrix} v_{0c1} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} v_{0c2}$$

where x_{0c}, y_{0c} and θ_{0c} are the configure of the leader robot and v_{0c1} and v_{0c2} are its reference inputs and are already known. Similar to follower vehicles, we assume $v_{0c1} < \gamma_\ell$ and $v_{0c2} < \gamma_\ell$, where γ_ℓ is a positive constant.

Now consider the following system:

$$\dot{z}_{ic1} = r_{ic1} \tag{3.19}$$

$$\dot{z}_{ic2} = r_{ic2} \tag{3.20}$$

where $z_{ic1} \epsilon \mathbb{R}$ and $z_{ic2} \epsilon \mathbb{R}$ are the position of the reference point $\boldsymbol{c}$ of each follower mobile robot, and $r_{ic1} \epsilon \mathbb{R}$ and $r_{ic2} \epsilon \mathbb{R}$ are the control inputs for each vehicle.

The two control inputs r_{ic1} and r_{ic2} that make the tracking error converge to zero are given by the following:

$$r_{ic1} = -\alpha \sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right) - \beta \frac{\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)}{\sqrt{\left[\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)\right]^2 + \varepsilon}} \tag{3.21}$$

$$r_{ic2} = -\alpha \sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right) - \beta \frac{\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)}{\sqrt{\left[\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)\right]^2 + \varepsilon}} \tag{3.22}$$

where $\varepsilon = e^{-2qt}$, q is a positive constant, $a_{ij}(i = 0, 1, \dots, n, j = 1,2, \dots, n)$ is the $(i,j)^{th}$ entry of the adjacency matrix $\mathcal{A}$ associated with the system, a_{i0} is a positive constant if the virtual leader's position is available to follower i, otherwise $a_{i0} = 0$. α is a nonnegative constant and β is a positive constant.

Theorem 3.3: Suppose that a fixed undirected graph $\mathcal{G}$ and the leader robot is the root of spanning tree, then by using the control inputs (3.21) and (3.22) for kinematic system (3.18), if we choose $\beta > \gamma_\ell$, then

$$\lim_{t\to\infty}(x_{ic} - x_0) = 0$$

$$\lim_{t\to\infty}(y_{ic} - y_0) = 0$$

are guaranteed.

Proof: For simplicity discussion, here we only provide the proof of controller (3.21), but the same procedure also applies to the proof of controller (3.22). We define that $\tilde{x}_{ic} = x_{ic} - x_{0c}$, so the closed loop system (3.18) can be rewritten as

$$\dot{\tilde{q}}_{ic1} = \dot{\tilde{x}}_{ic} = -\alpha \sum_{j=0}^{n} a_{ij}\left(\tilde{x}_{ic} - \tilde{x}_{jc}\right) - \beta \frac{\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)}{\sqrt{\left[\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_j\right)\right]^2 + \varepsilon}} - \dot{x}_0 \qquad (3.23)$$

Equation (3.23) can be rewritten in matrix form as

$$\dot{\tilde{x}} = \begin{bmatrix} \dot{\tilde{x}}_{1c} \\ \vdots \\ \dot{\tilde{x}}_{nc} \end{bmatrix} = -\alpha M \begin{bmatrix} \tilde{x}_{1c} \\ \vdots \\ \tilde{x}_{nc} \end{bmatrix} - \beta \begin{bmatrix} \dfrac{a_{10}(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})}{\sqrt{[a_{10}(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})]^2 + \varepsilon}} \\ \vdots \\ \dfrac{a_{n0}(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})}{\sqrt{[a_{n0}(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})]^2 + \varepsilon}} \end{bmatrix} - \mathbf{1}\dot{x}_0$$

where $M = L + diag(a_{10}, \dots, a_{n0})$ with L being the Lapacian matrix associated with the system. Because the fixed undirected graph $\mathcal{G}$ is connected and at least one a_{i0} is nonzero (and hence it is positive), so M is symmetric positive definite.

Now consider the following Lyapunov function candidate

$$V = \frac{1}{2}\tilde{x}^T M \tilde{x}$$

where $(i = 0, 1, 2, \dots \dots, n)$.

Take derivative of V gives

$$\dot{V} = \tilde{x}^T M \left[-\alpha M \tilde{x} - \beta \begin{pmatrix} \frac{a_{10}(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})}{\sqrt{[a_{10} b(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})]^2 + \varepsilon}} \\ \vdots \\ \frac{a_{n0}(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})}{\sqrt{[a_{n0} b(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})]^2 + \varepsilon}} \end{pmatrix} - \mathbf{1}\dot{x}_0 \right]$$

$$= -\alpha \tilde{x}^T M^2 \tilde{x} - \beta \tilde{x}^T M \begin{pmatrix} \frac{a_{10}(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})}{\sqrt{[a_{10}(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})]^2 + \varepsilon}} \\ \vdots \\ \frac{a_{n0}(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})}{\sqrt{[a_{n0}(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})]^2 + \varepsilon}} \end{pmatrix} - \tilde{x}^T M \mathbf{1} \dot{x}_0$$

Define $Y = [Y_1 \quad \cdots \quad Y_n] = \tilde{x}^T M$, then

$$\dot{V} = -\alpha \tilde{x}^T M^2 \tilde{x} - \beta Y \begin{pmatrix} \frac{a_{10}(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})}{\sqrt{[a_{10}(x_{1c} - x_{0c}) \cdots + a_{1n}(x_{1c} - x_{nc})]^2 + \varepsilon}} \\ \vdots \\ \frac{a_{n0}(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})}{\sqrt{[a_{n0}(x_{nc} - x_{0c}) \cdots + a_{nn-1}(x_{nc} - x_{n-1c})]^2 + \varepsilon}} \end{pmatrix} - \tilde{x}^T M \mathbf{1} \dot{x}_0$$

$$\le -\alpha \tilde{x}^T M^2 \tilde{x} + |\dot{x}_0| \sum_{i=1}^{n} |Y_i| - \beta \sum_{i=1}^{n} Y_i \frac{a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{cn})}{\sqrt{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon}}$$

$$\leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} |Y_i| - \beta \sum_{i=1}^{n} Y_i \frac{a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})}{\sqrt{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon}}$$

If we choose $\beta = \gamma_\ell$, then

$$\dot{V} \leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} (|Y_i| - Y_i \frac{a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})}{\sqrt{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon}})$$

$$\leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} (|Y_i| - \frac{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2}{\sqrt{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon}})$$

$$\leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} (|Y_i| - \frac{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon - \varepsilon}{\sqrt{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon}})$$

$$\leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} (|Y_i| - \sqrt{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon}$$
$$+ \frac{\varepsilon}{\sqrt{[a_{i0}(x_{ic} - x_{0c}) \cdots + a_{in}(x_{ic} - x_{nc})]^2 + \varepsilon}})$$

$$\leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} (|Y_i| - \sqrt{|Y_i|^2 + \varepsilon} + \frac{\varepsilon}{\sqrt{|Y_i|^2 + \varepsilon}})$$

Since $\sqrt{|Y_i|^2 + \varepsilon} > 0$, then

$$\dot{V} \leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} \frac{\varepsilon}{\sqrt{|Y_i|^2 + \varepsilon}}$$

$$\leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell \sum_{i=1}^{n} \sqrt{\varepsilon}$$

$$\leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell n\sqrt{\varepsilon}$$

$$\leq \gamma_\ell n\sqrt{\varepsilon}$$

Since $|v_{01}| \leq \gamma_\ell$, we have $|\dot{x}_0| \leq \gamma_\ell$, so the second inequality is derived. Note that M^2 is symmetric positive definite, α is nonnegative, ε is a sufficient small positive constant and $\beta > \gamma_\ell$. Integrate $\dot{V}$ we have

$$\int_0^t \dot{V}\,dt \leq \int_0^t \gamma_\ell n\sqrt{\varepsilon}\,dt$$

$$\leq \gamma_\ell n \int_0^t e^{-qt}\,dt$$

$$\leq \gamma_\ell n \frac{1}{q}(1 - e^{-qt})$$

where $t \in [0, \infty)$. Now $V(t)$ is shown to be bounded.

Since

$$\dot{V} \leq -\alpha\tilde{x}^T M^2 \tilde{x} + \gamma_\ell n\sqrt{\varepsilon}$$

Now we have

$$\int_0^t \dot{V}\,dt \leq -\alpha \int_0^t \tilde{x}^T M^2 \tilde{x}\,dt + \gamma_\ell n \int_0^t \sqrt{\varepsilon}\,dt$$

so $\tilde{x}$ is bounded.

$$\alpha \int_0^t \tilde{x}^T M^2 \tilde{x}\, dt \leq \gamma_\ell n \int_0^t \sqrt{\varepsilon}\, dt - \int_0^t \dot{V}\, dt$$

The boundedness of the right side of the above inequality is already shown, so we can see that the left side of the inequality is also bounded. Note that M^2 is symmetric positive definite, so $\int_0^t \tilde{x}^2$ is bounded.

Now we show the boundedness of both $\tilde{x}$ and $\int_0^t \tilde{x}^2$, it then follows lemma 3.2.5 with $p = 2$ in [62] that $\lim_{t\to\infty} \tilde{x} = 0$. ■

3.8 Simulation

In this section, we will present several simulation examples of a group of three wheeled mobile robots with a virtual leader to show the effectiveness of the proposed controllers in section 3.7. All the vehicles are assumed to have the same mechanical structure as shown in Figure 3.10 and the communication graph is shown in Figure 3.11, vertex v_0 represents the leader robot.

We choose $\alpha = 1$, $q = 1$ and $\beta = 1$ for the proposed controllers, also define the desired trajectory is a unit circle with $x_0 = sin(t)$ and $y_0 = cos(t)$. The tracking errors $x_i - x_0$ and $y_i - y_0$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 3.12, Figure 3.13 and Figure 3.14. The state information $q_i = [x_i, y_i, \theta_i]$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 3.15, Figure 3.16 and Figure 3.17.

It can be seen that the tracking errors of each vehicle ultimately converge to zero and state parameters of each vehicle satisfy the condition of being a unit circle, so the proposed controllers are effective.

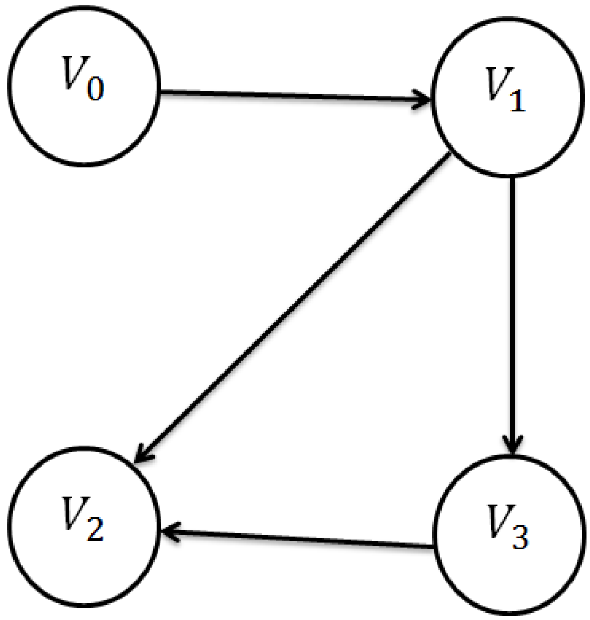

Figure 3.11 Communication graph IV

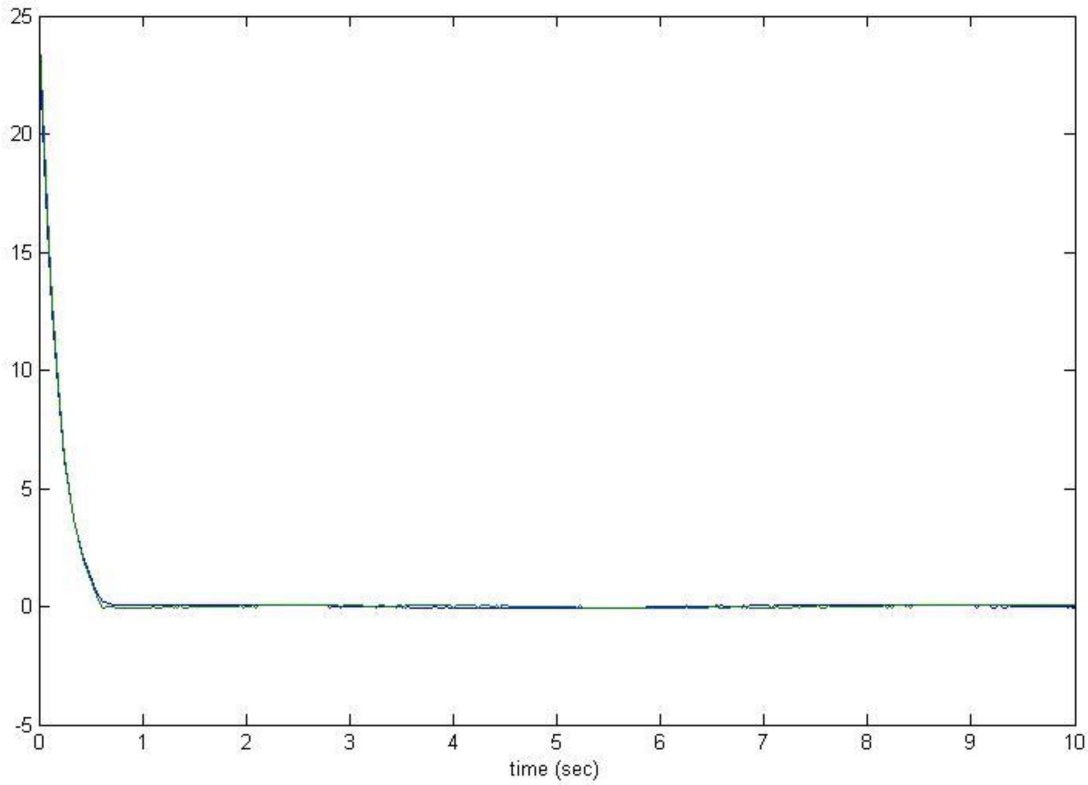

Figure 3.12 Responses of $(x_1 - x_0)$ and $(y_1 - y_0)$

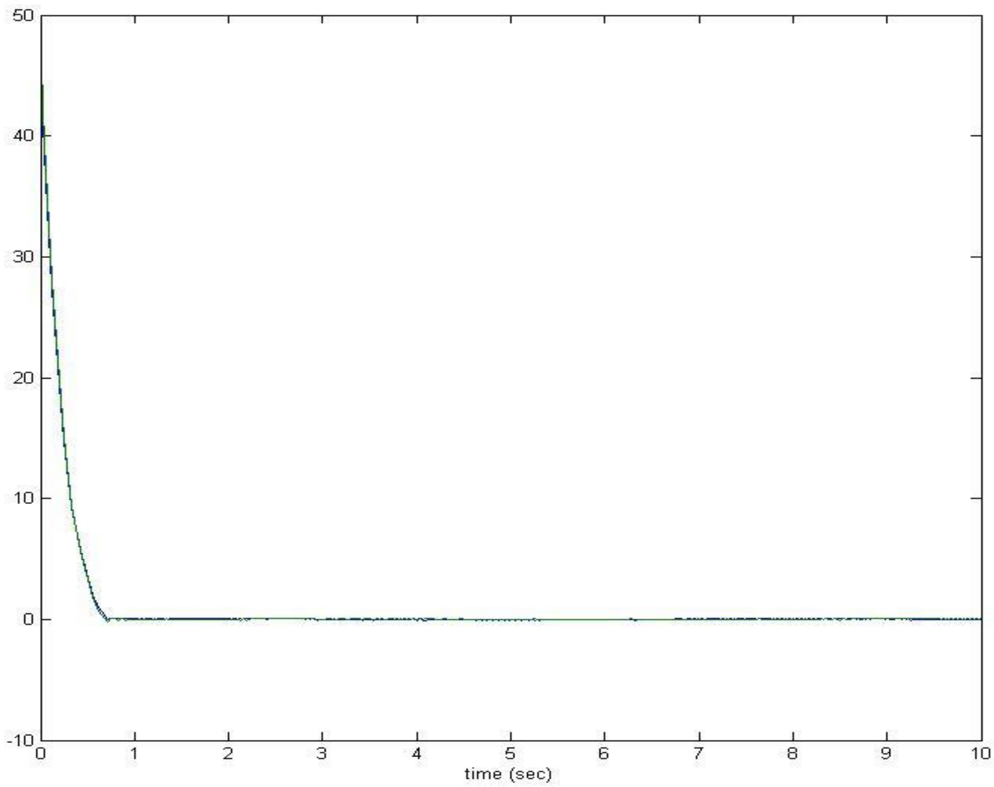

Figure 3.13 Responses of $(x_2 - x_0)$ and $(y_2 - y_0)$

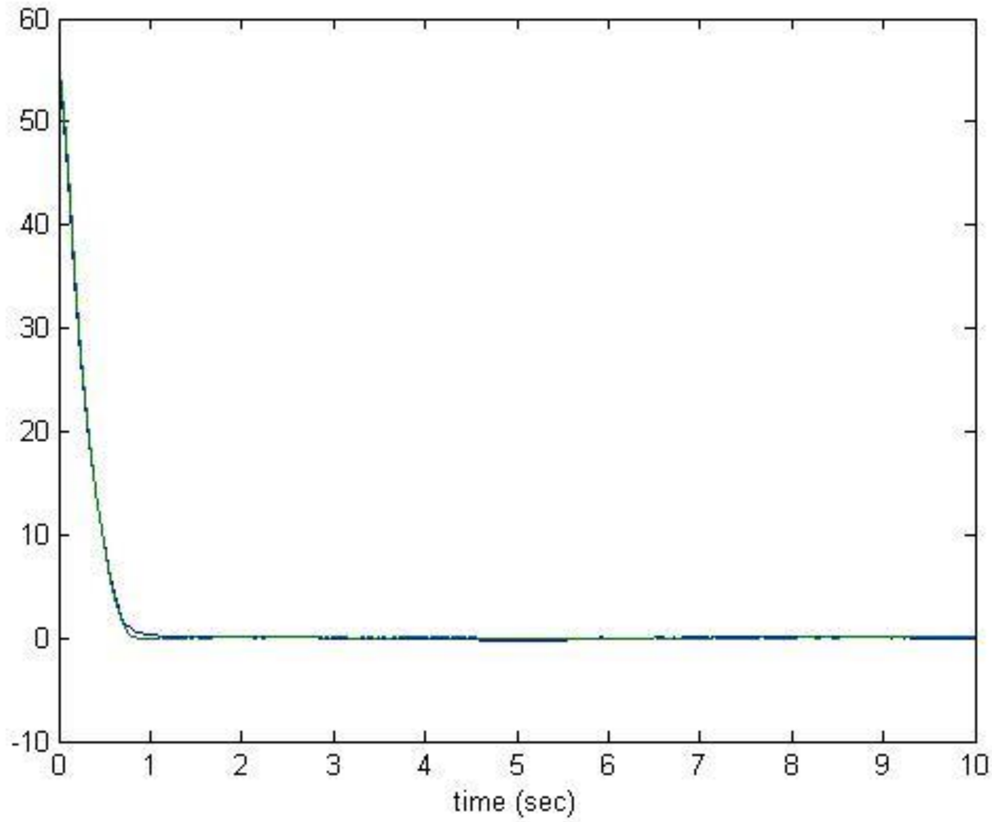

Figure 3.14 Responses of $(x_3 - x_0)$ and $(y_3 - y_0)$

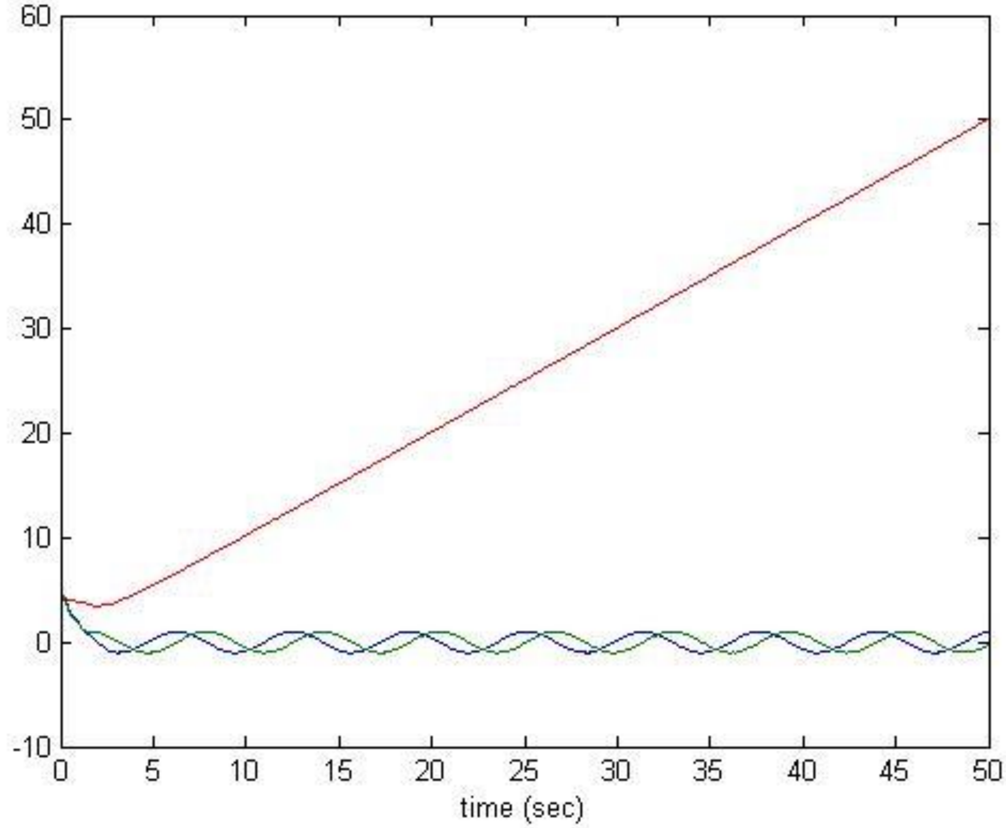

Figure 3.15 Responses of $q_1 = [x_1, y_1, \theta_1]$

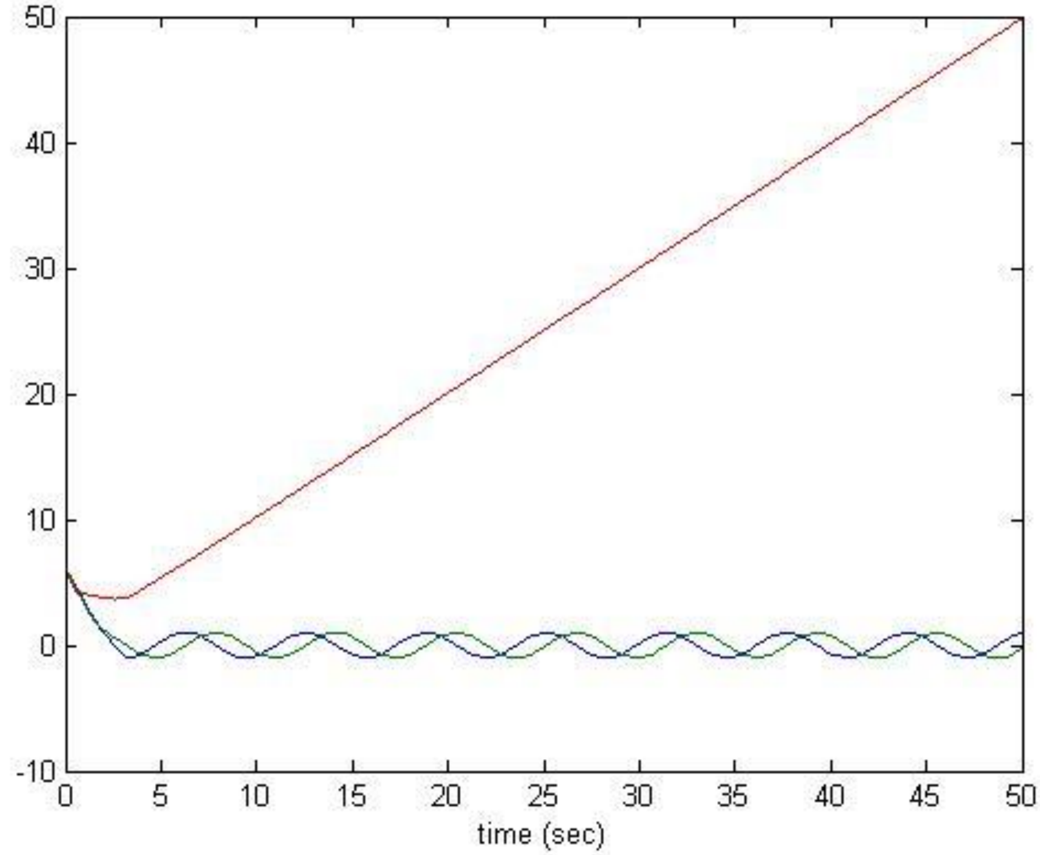

Figure 3.16 Responses of $q_2 = [x_2, y_2, \theta_2]$

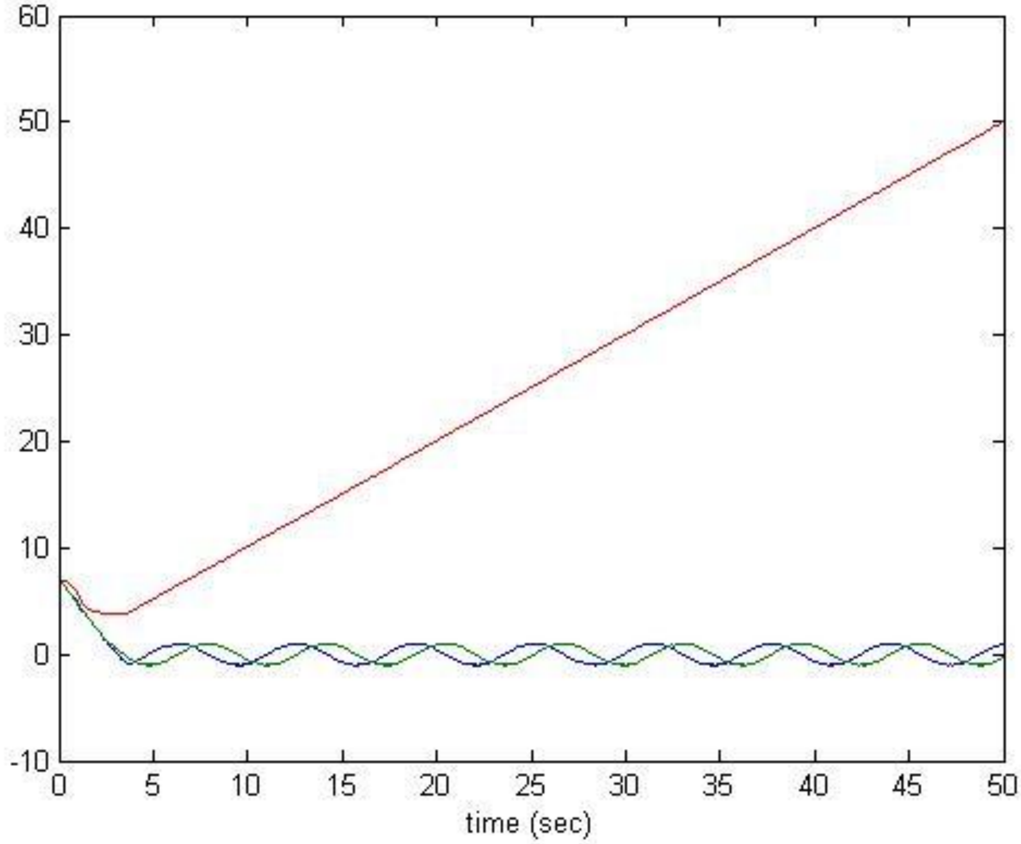

Figure 3.17 Responses of $q_3 = [x_3, y_3, \theta_3]$

3.9 Summary

In this chapter, distributed coordinate tracking control of multiple wheeled mobile robots is addressed for two different kinematic models. Cooperative coordinate tracking control is achieved with the aid of exact feedback control, variable transformations are utilized to transfer the original nonlinear kinematics into linear system, and also graph theory is utilized in order to design the tracking controllers.

Control via exact feedback is used in this chapter, in section 3.2 and 3.6 we transform both the third-order dynamic four-wheel mobile robots nonlinear system and two-wheel mobile robots system to linear systems by using full-state linearization algorithm and input-output linearization algorithm respectively. In section 3.3, we extended the results of first-order kinematics and second-order dynamics controller that introduced in [30] to the third-order dynamics. In section 3.7, distributed tacking controllers for two-wheel kinematic vehicles are proposed. The effectiveness of all control laws proposed in this chapter are shown by the simulation results and a follower robot is able to track leader robots trajectory without knowing leader's state information.

CHAPTER IV

DISTRIBUTED TRACKING CONTROL OF MULTIPLE DYNAMIC WMRS

In chapter III, kinematic models of wheeled mobile robots are discussed and control laws for kinematic systems are proposed accordingly. The control inputs of the kinematic controllers are generally velocities, however, it is more realistic that the control inputs are torques generated by engine or motor of the vehicle. So in this chapter, torque controllers will be designed in dynamic model of the vehicle by using backstepping method to make sure the velocities of the vehicle converge to the desired velocities. The desired velocities are given by the kinematic controllers that designed in chapter III and asymptotic stability is guaranteed by Lyapunov theory.

The dynamic extension proposed in this chapter using the method of taking account the specific vehicle dynamics to convert a steering system command into control inputs for the actual vehicle. The method is considered as backstepping control approach. First, feedback velocity control inputs are designed for kinematic system to make the position error asymptotically stable, which is already done in chapter III. Second, feedback velocities following control laws are designed such that the vehicle's velocities converge asymptotically to the given velocity inputs. Finally, the torque feedback controller uses the second control signal to compute the required torque for the actual mobile robots.

In real life, it is hard to calculate some inertial parameters of WMRs such as the inertial tensors, also the payload for missions may be different, so in this chapter, we will consider dynamics that with and without inertial uncertainties.

4.1 Dynamic Modeling of Wheeled Mobile Robots

Consider the nonholonomic mobile robot shown in Figure 3.10, the motion and orientation are achieved by independent actuators, which means DC motors providing the necessary torques to the rear wheels. We consider all the mobile robots are subject to m constraints

$$D(q_i)\ddot{q}_i + C(q_i, \dot{q}_i)\dot{q}_i + G(q_i) = B(q_i)\tau_i - A^T(q_i)\lambda_i \tag{4.1}$$

where q_i are defined in (3.19), $\tau_i \in \mathbb{R}^r$ are input vectors, $\lambda_i \in \mathbb{R}^m$ are the vectors of constraint forces, $D(q_i) \in \mathbb{R}^{n\times n}$ are symmetric and positive-definite inertial matrix, $C(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ are the centripetal and Coriolis matrix, $G(q_i) \in \mathbb{R}^n$ are the gravitational vectors, $B(q_i) \in \mathbb{R}^{n\times r}$ are the input transformation matrixes and $A(q_i) \in \mathbb{R}^{m\times n}$ are the matrixes associated with the constraints. In the following of this chapter, we assume that $r = n - m$.

Consider that all kinematic equality constraints are independent of time, and can be expressed as

$$A(q_i)\dot{q}_i = 0 \tag{4.2}$$

Let $S(q_i)$ be a full rank matrix formed by a set of smooth and linearly independent vector fields spanning the null space of $A(q_i)$

$$S^T(q_i)A^T(q_i) = 0 \tag{4.3}$$

Based on equation (4.2) and (4.3), it is possible to find an auxiliary vector time function $v_i(t) \in \mathbb{R}^{n-m}$, such that for all t

$$\dot{q}_i = S(q_i)v_i(t) \tag{4.4}$$

As we already discussed that for a car-like agent shown in Figure 3.10, linearization control algorithm fail at point $\boldsymbol{P}$ which is the intersection of the left and right wheel, so a fixed point $\boldsymbol{C}$ located at a distance d from point $\boldsymbol{P}$ is redefined as reference point and use this reference point to develop the mathematical model.

The nonholonomic constraints state that wheels of all the mobile robots roll and do not slip

$$\dot{y}_{ic}cos\theta_{ic} - \dot{x}_{ic}sin\theta_{ic} - d\dot{\theta}_{ic} = 0$$

It is easy to verify that $S(q_i)$ is given by

$$S(q_i) = \begin{bmatrix} cos\theta_{ic} & -dsin\theta_{ic} \\ sin\theta_{ic} & dcos\theta_{ic} \\ 0 & 1 \end{bmatrix} \tag{4.5}$$

It matches the kinematics model of point $\boldsymbol{C}$ we derived in (3.18). System (4.5) is called the steering system of a vehicle.

The Lagrange formalism is used to derive the dynamic equations of mobile robots, in the case that $G(q_i) = 0$, the dynamic equation of the mobile robots shown in Figure 3.10 can be expressed as

$$D(q_i) = \begin{bmatrix} m & 0 & mdsin\theta_{ic} \\ 0 & m & -mdcos\theta_{ic} \\ mdsin\theta_{ic} & -mdcos\theta_{ic} & I \end{bmatrix}$$

$$C(q_i, \dot{q}_i)\dot{q}_i = \begin{bmatrix} md\dot{\theta}_{ic}{}^2 cos\theta_{ic} \\ md\dot{\theta}_{ic}{}^2 sin\theta_{ic} \\ 0 \end{bmatrix}$$

$$B(q_i) = \frac{1}{r}\begin{bmatrix} cos\theta_{ic} & cos\theta_{ic} \\ sin\theta_i c & sin\theta_{ic} \\ R & -R \end{bmatrix}$$

$$A^T(q_i) = \begin{bmatrix} -sin\theta_{ic} \\ cos\theta_{ic} \\ -d \end{bmatrix}$$

$$\lambda_i = -m(\dot{x}_{ic}cos\theta_{ic} + \dot{y}_{ic}sin\theta_{ic})\dot{\theta}_{ic}$$

For the nonholonomic mobile robots in (4.1), the following properties hold:

Property 1: $D(q_i)$ and $C(q_i, \dot{q}_i)$ are bounded.

Property 2: The matrix $\dot{D} - 2C$ is skew-symmetric, that is, $x^T(\dot{D} - 2C)x = 0$.

Differentiating equation (4.4),

$$\ddot{q}_i = \dot{S}v_i + S\dot{v}_i$$

Substituting the result in (4.1) gives

$$D(\dot{S}v_i + S\dot{v}_i) + CSv_i = B(q_i)\tau_i - A^T\lambda_i$$

Then multiply by S^{T}

$$S^T D(\dot{S}v_i + S\dot{v}_i) + S^T CSv_i = S^T B(q_i)\tau_i - S^T A^T \lambda_i$$

Since $S^T A^T = 0$, so we have

$$S^T DS\ \dot{v}_i + S^T(D\dot{S} + CSv_i S)v_i = S^T B\ \tau_i$$

Then the complete equation of motion of nonholonomic mobile robots are transformed and divided into the following two equations which is a more appropriate representation for control purposes.

$$\dot{q}_i = S(q_i)v_i(t) \tag{4.6}$$

$$\overline{D}(q_i)\dot{v}_i + \overline{C}(q_i, \dot{q}_i)v_i = \overline{B}(q_i)\tau_i \tag{4.7}$$

where $v_i \in \mathbb{R}^{n-m}$ is a velocity vector, $\overline{D} = S^T DS$ is a symmetric and positive-definite inertial matrix, $\overline{C} = S^T(D\dot{S} + CSv_i S)$ is the centripetal and $\overline{B} = S^T B$ is the input transformation matrix and $\tau_i \in \mathbb{R}^{r\times r}$ is the input vector. Equation (4.7) describes the behavior of the nonholonomic system in a new local coordinates. If $r = n - m$, then $\overline{B}(q_i)$ is nonsigular.

For system (4.7), similar properties hold:

Property 3: $\overline{D}(q_i)$ and $\overline{C}(q_i, \dot{q}_i)$ are bounded.

Property 4: The matrix $\dot{\overline{D}} - 2\overline{C}$ is skew-symmetric.

4.2 Distributed Control of Dynamic Systems

Without Inertial Uncertainties

The complete dynamics system (4.6) and (4.7) consist of the kinematic steering system (4.6) plus some extra dynamics (4.7), standard approaches to design nonholonomic controllers only deal with (4.6), just like chapter III, ignoring the actual vehicle dynamics. So in this chapter, we will correct this omission.

Let u_i be an auxiliary input, then by applying the nonlinear feedback

$$\tau_i = f_{\tau_i}(q_i, \dot{q}_i, v_i, u_i) = \overline{B}^{-1}(q_i)\left[\overline{D}(q_i)u_i + \overline{C}(q_i, \dot{q}_i)v_i\right] \tag{4.8}$$

If we assume all the dynamical quantities $\overline{D}(q_i)$ and $\overline{C}(q_i, \dot{q}_i)$ are exactly known, then one can convert the dynamic control problem into kinematic problem.

$$\dot{q}_i = S(q_i)v_i \tag{4.9}$$

$$\dot{v}_i = u_i \tag{4.10}$$

For system (4.9), as we discussed in section (3.6) that the natural output choice for trajectory tracking task is

$$q_{ic} = \begin{bmatrix} x_i + dcos\theta_i \\ y_i + dsin\theta_i \end{bmatrix}$$

After linearization, in global defined transformed coordinates $q_{ic} = (x_{ic}, y_{ic}, \theta_{ic})$ the closed-loop system become

$$\dot{x}_{ic} = r_{ic1}$$

$$y_{ic} = r_{ic2}$$

$$\dot{\theta}_{ic} = -\frac{sin\theta_{ic}}{d} r_{ic1} + \frac{cos\theta_{ic}}{d} r_{ic2}$$

which are input-output linear and decoupled. So we can rewrite system (4.9) and (4.10) to

$$\dot{q}_{ic} = r_{ic} \tag{4.11}$$

$$\dot{r}_{ic} = u_i \tag{4.12}$$

In this section, we will convert the prescribed control $v_i(t)$ in chapter III into a torque control τ_i for the actual vehicles. So our objective is select τ_i in (4.7) such that (4.6) and (4.7) exhibit the desired behavior.

The auxiliary velocity control input that achieves distributed tracking control for system (4.11) is given by

$$r_{ic1} = -\alpha \sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right) - \beta \frac{\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)}{\sqrt{[\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)]^2 + \varepsilon}}$$

$$r_{ic2} = -\alpha \sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right) - \beta \frac{\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)}{\sqrt{[\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)]^2 + \varepsilon}}$$

where α, β, a_{ij} and ε are all defined in (3.21) and (3.22), then the derivation of r_{ic1} and r_{ic1} becomes

$$\dot{r}_{ic1} = -\alpha \sum_{j=0}^{n} a_{ij}\left(\dot{x}_{ic} - \dot{x}_{jc}\right) - 2a_{ij}\left(x_{ic} - yx_{jc}\right) * (\dot{x}_{ic} - \dot{x}_{jc})\beta \frac{\sqrt{[\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)]^2 + \varepsilon} - \frac{\left(x_{ic} - x_{jc}\right)}{2\sqrt{[\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)]^2 + \varepsilon}}}{[\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)]^2 + \varepsilon})$$

$$\dot{r}_{ic2} = -\alpha \sum_{j=0}^{n} a_{ij}\left(\dot{y}_{ic} - \dot{y}_{jc}\right) - 2a_{ij}\left(y_{ic} - y_{jc}\right) * (\dot{y}_{ic} - \dot{y}_{jc})\beta \frac{\sqrt{[\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)]^2 + \varepsilon} - \frac{\left(y_{ic} - y_{jc}\right)}{2\sqrt{[\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)]^2 + \varepsilon}}}{[\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)]^2 + \varepsilon})$$

We propose the nonlinear feedback control input is

$$u_i = \dot{r}_{ic} + K_5(r_{ic} - v_i) \tag{4.13}$$

where K_5 is a positive definite, diagonal matrix given by

$$K_5 = k_5\boldsymbol{I}$$

where k_5 is a positive constant.

Theorem 4.1: Given nonholonomic systems (4.6) and (4.7) with n generalized coordinates q, m independents and r actuators, let the conditions satisfied:

1) The number of actuators is equal to the number of degree of freedom (*i.e.* $r = n - m$).

2) Smooth auxiliary velocities control input r_{ic} are given by (3.21) and (3.22).

3) k_5 is a positive constant.

If the leader robot is the root of spanning tree and let the nonlinear feedback control u_i given by (4.13) and the mobile robot input commands be given by (4.8), then the velocities of mobile robots satisfy

$$\lim_{t\to\infty}(v_i - r_{ic}) = 0$$

Proof: Define an auxiliary velocity error

$$e_{vi} = v_i - r_{ic} = \begin{bmatrix} e_{vi1} \\ e_{vi2} \end{bmatrix} = \begin{bmatrix} v_{i1} + \alpha \sum_{j=0}^{n} a_{ij}(x_{ic} - x_{jc}) + \beta \frac{\sum_{j=0}^{n} a_{ij}(x_{ic} - x_{jc})}{\sqrt{[\sum_{j=0}^{n} a_{ij}(x_{ic} - x_{jc})]^2 + \varepsilon}} \\ v_{i2} + \alpha \sum_{j=0}^{n} a_{ij}(y_{ic} - y_{jc}) + \beta \frac{\sum_{j=0}^{n} a_{ij}(y_{ic} - y_{jc})}{\sqrt{[\sum_{j=0}^{n} a_{ij}(y_{ic} - y_{jc})]^2 + \varepsilon}} \end{bmatrix} \quad (4.14)$$

by using (4.13), we have

$$\dot{r}_{ic} = u_i - K_5(r_{ic} - v_i) \quad (4.15)$$

substituting (4.15) gives

$$\dot{e}_{vi} = \dot{v}_i - \dot{r}_{ic} = \dot{v}_i - u_i + K_5(r_{ic} - v_i)$$

by using (4.13) we obtain

$$\dot{e}_{vi} = K_5(r_{ic} - v_i) = -K_5 e_{vi}$$

Under the assumption that k_5 is a positive constant the auxiliary velocity vector e converges exponentially to zero, so the velocity vectors of vehicles satisfy

$$\lim_{t\to\infty}(v_i - r_{ic}) = 0 \qquad \blacksquare$$

4.3 Simulation

In this section, we will present several simulation examples of a group of three wheeled mobile robots with a virtual leader to show the effectiveness of the proposed controllers in section 4.2. All the vehicles are assumed to have the same mechanical structure as shown in Figure 3.10. The communication graph is shown in Figure 4.1, vertex v_0 represents the leader robot.

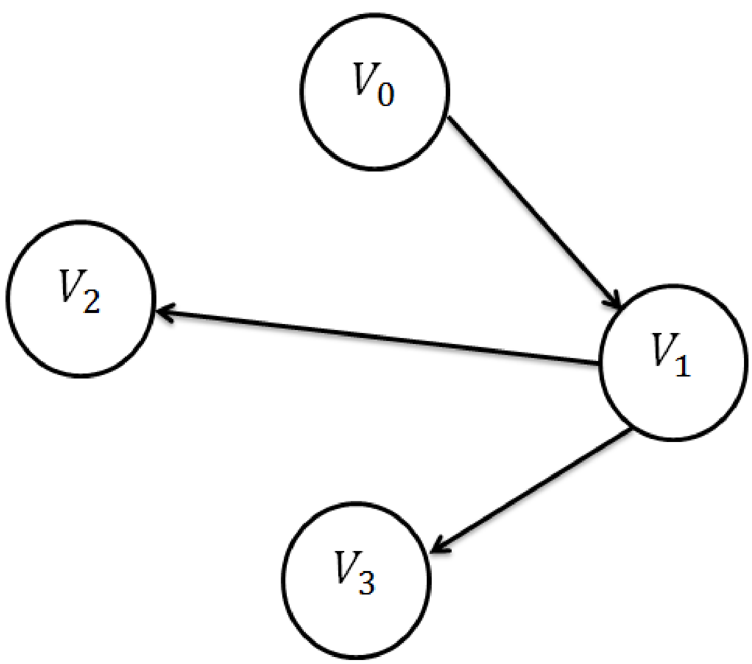

Figure 4.1 Communication graph V

We choose $\alpha = 1$ and $\beta = 1$, also define the desired trajectory is a unit circle with $x_0 = sin(t)$ and $y_0 = cos(t)$. The tracking errors of $x_i - x_0$ and $y_i - y_0$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 4.2, Figure 4.3 and Figure 4.4. The state information $q_i = [x_i, y_i, \theta_i]$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 4.5, Figure 4.6 and Figure 4.7.

It can be seen that the tracking errors of each vehicle ultimately converge to zero and state parameters of each vehicle satisfy the condition of being a unit circle, so the proposed controllers are effective.

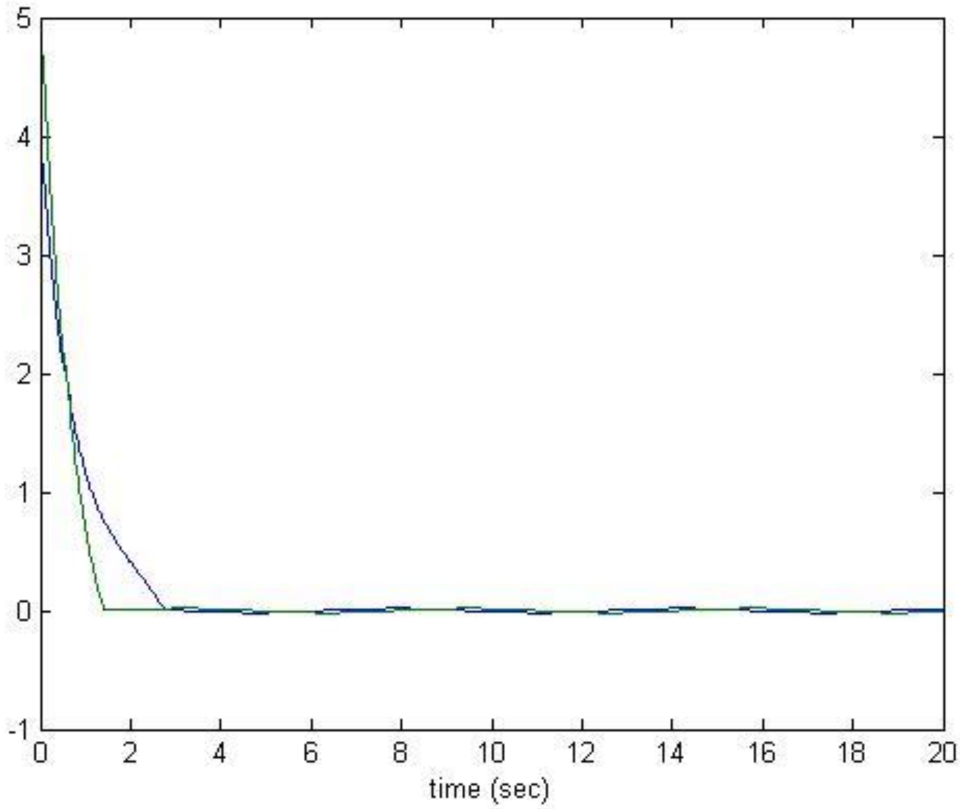

Figure 4.2 Responses of $(x_1 - x_0)$ and $(y_1 - y_0)$

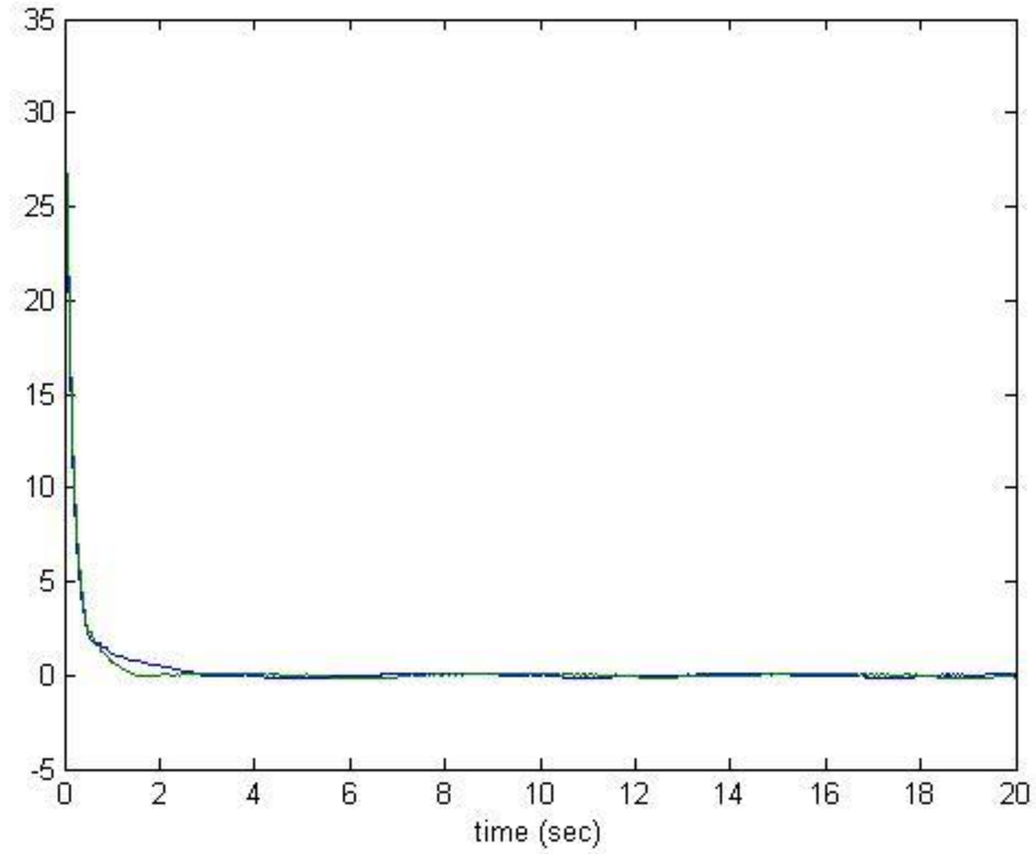

Figure 4.3 Responses of $(x_2 - x_0)$ and $(y_2 - y_0)$

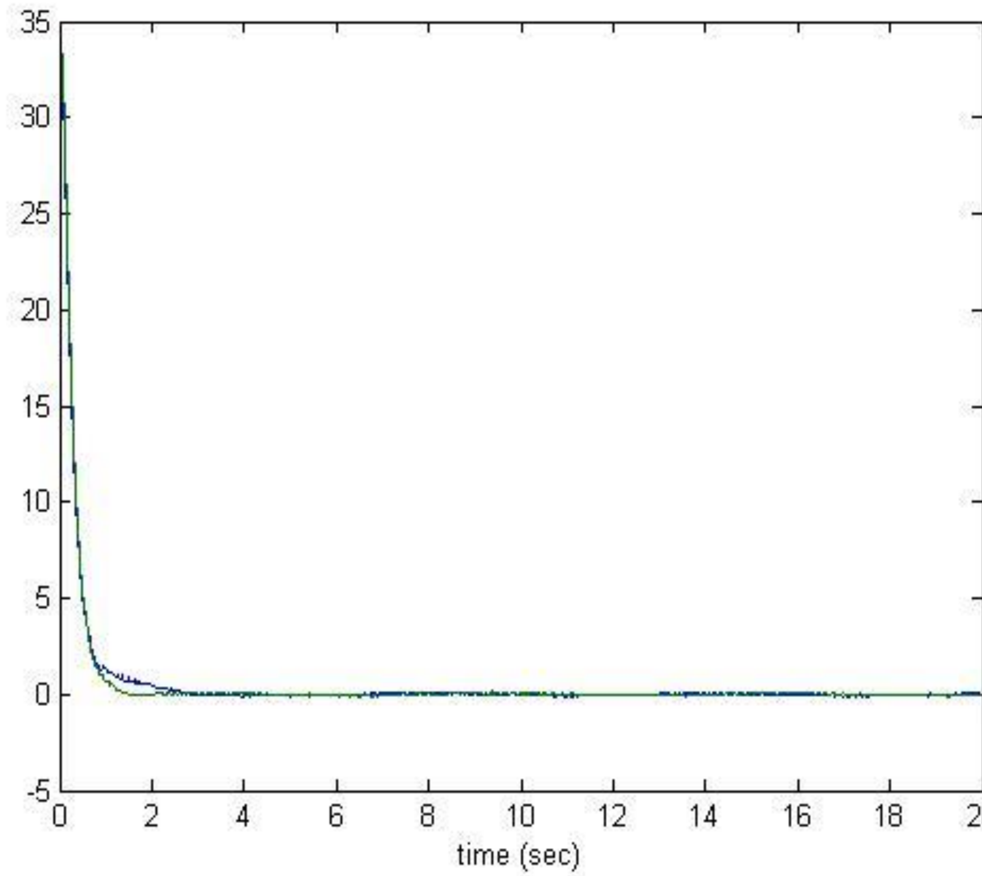

Figure 4.4 Responses of $(x_3 - x_0)$ and $(y_3 - y_0)$

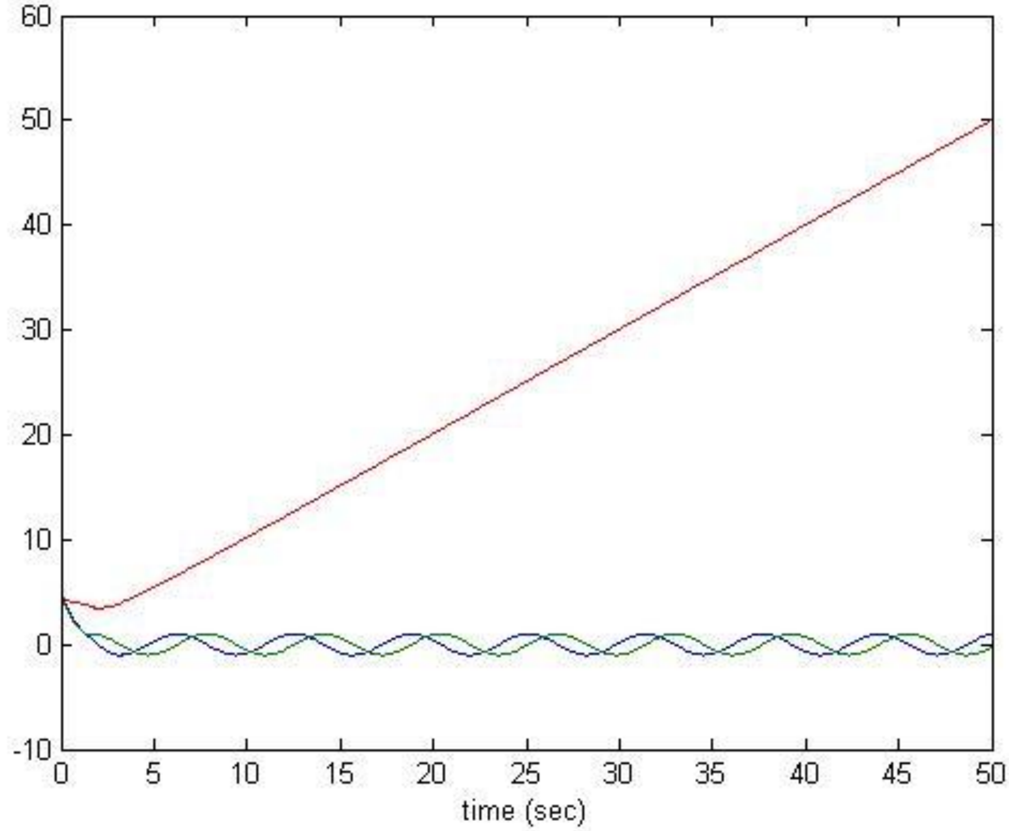

Figure 4.5 Responses of $q_1 = [x_1, y_1, \theta_1]$

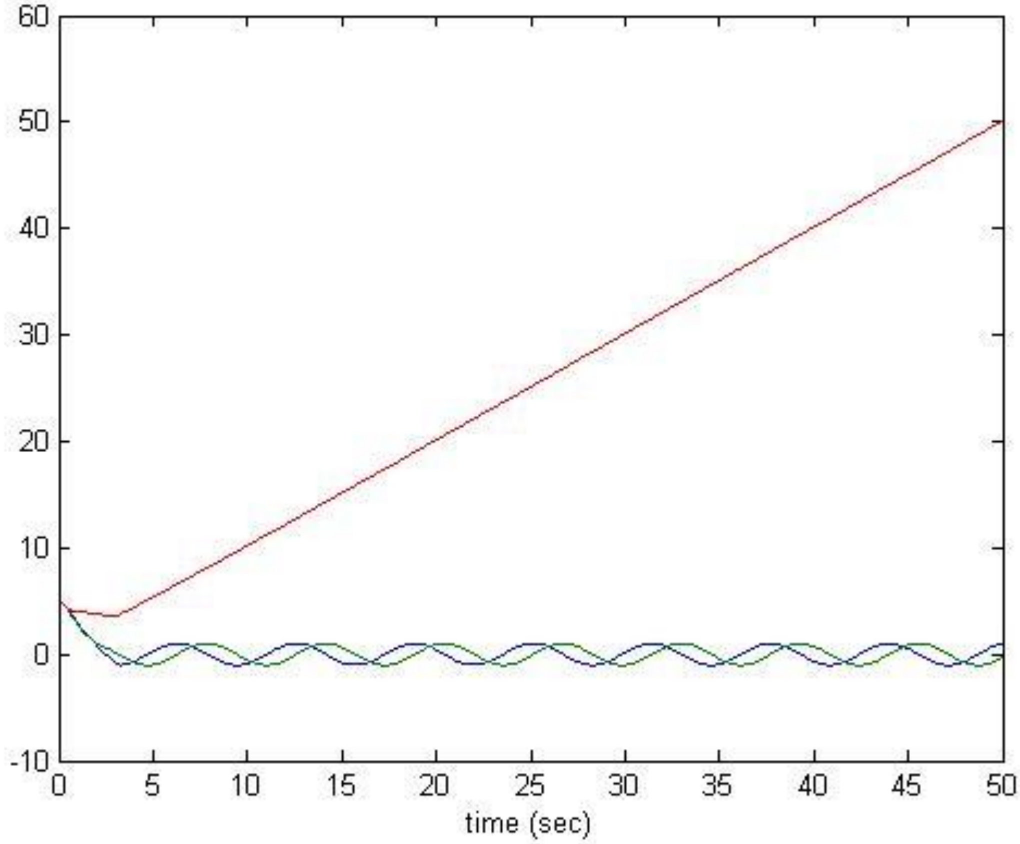

Figure 4.6 Responses of $q_2 = [x_2, y_2, \theta_2]$

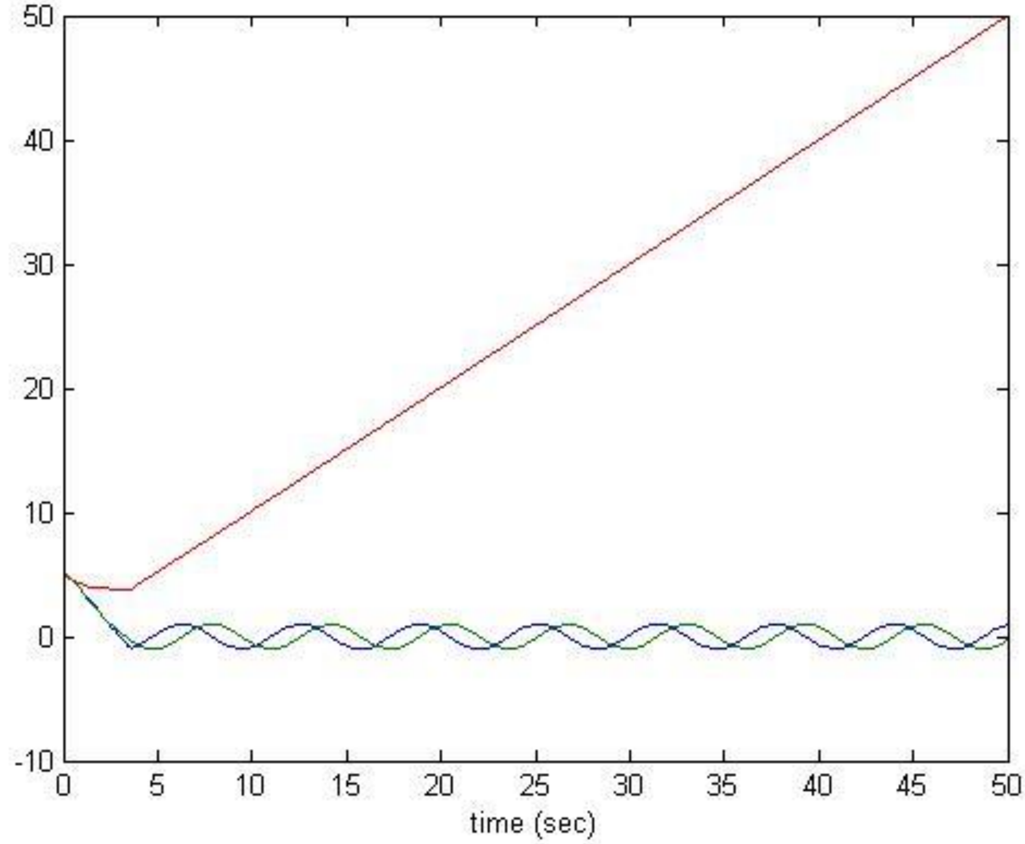

Figure 4.7 Responses of $q_3 = [x_3, y_3, \theta_3]$

4.4 Distributed Control of Dynamic Systems

With Inertial Uncertainties

In section 4.2, distributed tracking controllers are proposed for dynamic systems without inertial uncertainties, we assume that $\overline{D}(q_i)$ and $\overline{C}(q_i, \dot{q}_i)$ are exactly known in system (4.7), however, it is more realistic that the inertial parameters are not exactly known or even not known.

Consider the general dynamics of mobile robots (4.1), there exists a differentiable vector a_i with components depending on mechanical parameters (masses, moments of inertia, etc.), such that

$$D(q_i)\dot{\eta}_i + C(q_i, \dot{q}_i)\eta_i + G(q_i) = Y(q_i, \dot{q}_i, \eta_i, \dot{\eta}_i)a_i$$

where a_i is an inertia parameter vector which is related to the mass and moment of inertia and the regressor matrix $Y(q_i, \dot{q}_i, \eta_i, \dot{\eta}_i)$ is known as function of $q_i, \dot{q}_i, \eta_i$ and $\dot{\eta}_i$. In practice, the inertial parameter vector a_i is not exactly known or unknown due to the following reasons:

1) It is hard to calculate some inertial parameters, such as the inertial tensors.
2) The payload for missions may be different.

So in this section, we assume the inertial parameter vector a_i is a constant vector and remains unknown.

Problem statement: Our control objective in this section is, under the assumption that the inertial parameter vector a_i are unknown, to design distributed controllers τ_i for all follower robots described in (4.1) such that

$$\lim_{t\to\infty}(x_{ic} - x_0) = 0$$

$$\lim_{t\to\infty}(y_{ic} - y_0) = 0$$

As we already discussed in section 3.6, a fixed point **C** can be used to redefine the output of the system as

$$q_{ic} = \begin{bmatrix} x_{ic} \\ y_{ic} \end{bmatrix} = \begin{bmatrix} x_i + dcos\theta_i \\ y_i + dsin\theta_i \end{bmatrix}$$

with $d \neq 0$.

Differentiating this new output gives

$$\dot{q}_{ic} = \begin{bmatrix} cos\theta_i & -dsin\theta_i \\ sin\theta_i & dcos\theta_i \end{bmatrix} \begin{bmatrix} v_{ic1} \\ v_{ic2} \end{bmatrix} = S(\theta_{ic})v_{ic}$$

Now we can set $\dot{q}_{ic} = r_{ic}$ (an auxiliary input value) and solve for the inputs v_{ic} as

$$v_{ic} = S^{-1}(\theta_{ic})\dot{q}_{ic}$$

In globally defined transformed coordinates $(x_{ic}, y_{ic}, \theta_i)$ the closed-loop system becomes

$$\dot{q}_{ic1} = r_{ic1}$$

$$\dot{q}_{ic2} = r_{ic2}$$

$$\dot{\theta}_i = v_{ic2}$$

which is input-output linear and decoupled.

Now we have

$$v_{ic} = S^{-1}(\theta_{ic})r_{ic}$$

Differentiating v_{ic} gives

$$\dot{v}_{ic} = \dot{S^{-1}} r_{ic} + S^{-1} \dot{r}_{ic}$$

Substituting in (4.7)

$$\overline{D}(\dot{S^{-1}} r_{ic} + S^{-1}\dot{r}_{ic}) + \overline{C} S^{-1}(\theta_{ic}) r_{ic} = \overline{B}(q_{ic})\tau_i$$

We have

$$S^{-1^T} \overline{D} S^{-1} \dot{r}_{ic} + S^{-1^T}(\overline{D}\dot{S^{-1}} + \bar{C} S^{-1}) r_{ic} = S^{-1^T} \bar{B} \tau_i$$

Now the complete equations of motion of nonholonomic mobile robots are transformed and divided into the following two equations which are more appreciate representations for control purposes. We can rewrite systems (4.6) and (4.7) to

$$\dot{q}_{ic} = r_{ic}(t) \tag{4.16}$$

$$\overline{\overline{D}}(q_i)\dot{r}_{ic} + \bar{\bar{C}}(q_i, \dot{q}_i) r_{ic} = \bar{\bar{B}}(q_i)\tau_i \tag{4.17}$$

where

$$\overline{\overline{D}} = S^{-1^T} \overline{D} S^{-1}$$

$$\bar{\bar{C}} = S^{-1^T}(\overline{D}\dot{S^{-1}} + \bar{C} S^{-1}) r_{ic}$$

$$\bar{\bar{B}} = S^{-1^T} \bar{B}$$

System (4.16) and (4.17) have cascade structure. $\bar{\bar{B}}$ is known and assumed to be a full-rank matrix. A controller can be designed with the aid of backstepping technique.

We define

$$\bar{\bar{D}}(q_i)\dot{\eta}_{ic} + \bar{\bar{C}}(q_i, \dot{q}_i)\eta_{ic} = \bar{\bar{Y}}(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic})a_i$$

where the regressor matrix $\bar{\bar{Y}}(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic})$ is a known function of $q_i, \dot{q}_i, \eta_i$ and $\dot{\eta}_i$. a_i is an inertial parameter vector which is related to the mass and moment of inertia and is not exactly known or unknown.

Step 1: For system (4.16), we assume r_{ic} is control input, so controller will be designed for r_{ic} such that

$$\lim_{t\to\infty}(q_{ic} - q_0) = 0$$

To this end, we define $\tilde{q}_{ic} = q_{ic} - q_0$, then we have

$$\dot{\tilde{q}}_{ic} = r_{ic} - \dot{q}_0$$

Define

$$\tilde{x} = \begin{bmatrix} \tilde{x}_{1c} \\ \vdots \\ \tilde{x}_{nc} \end{bmatrix} \qquad \tilde{y} = \begin{bmatrix} \tilde{y}_{1c} \\ \vdots \\ \tilde{y}_{nc} \end{bmatrix}$$

We choose a Lyapunov function candidate

$$V_1 = \frac{1}{2}\tilde{x}^T M \tilde{x} + \frac{1}{2}\tilde{y}^T M \tilde{y}$$

where $M = L + diag(a_{10}, \dots, a_{n0})$ with L being the Laplacian matrix, V_1 is positive definite for $\tilde{x}$ and $\tilde{y}$, then

$$\dot{V}_1 = \tilde{x}^T M \dot{\tilde{x}} + \tilde{y}^T M \dot{\tilde{y}}$$

We choose the controller (3.21) and (3.22) we designed in chapter III,

$$\eta_{ic1} = -\alpha \sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right) - \beta \frac{\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)}{\sqrt{[\sum_{j=0}^{n} a_{ij}\left(x_{ic} - x_{jc}\right)]^2 + \varepsilon}}$$

$$\eta_{ic2} = -\alpha \sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right) - \beta \frac{\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)}{\sqrt{[\sum_{j=0}^{n} a_{ij}\left(y_{ic} - y_{jc}\right)]^2 + \varepsilon}}$$

where $\varepsilon = e^{-2qt}$, $a_{ij}(i = 0, 1, \dots, n, j = 1,2, \dots, n)$ is the $(i,j)^{th}$ entry of the adjacency matrix $\mathcal{A}$ associated with the system, a_{i0} is a positive constant if the virtual leader's position is available to follower i, otherwise $a_{i0} = 0$. α is a nonnegative constant and β is a positive constant.

If $r_{ic} = \eta_{ic}$, then we already shown in chapter III that

$$\dot{V}_1 \leq -\alpha \tilde{x}^T M^2 \tilde{x} - \alpha \tilde{y}^T M^2 \tilde{y} + 2\gamma_\ell n\sqrt{\varepsilon}$$

Step 2: Since r_{ic} is not the actual control input, so we define $\tilde{r}_{ic} = r_{ic} - \eta_{ic}$, then system (4.16) becomes to

$$\dot{q}_{ic} = \begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{y}} \end{bmatrix} = \begin{bmatrix} \tilde{r}_{ic1} + \eta_{ic1} \\ \tilde{r}_{ic2} + \eta_{ic2} \end{bmatrix} \tag{4.18}$$

Define

$$\tilde{r}_{c1} = \begin{bmatrix} \tilde{r}_{1c1} \\ \vdots \\ \tilde{r}_{nc1} \end{bmatrix} \qquad \tilde{r}_{c2} = \begin{bmatrix} \tilde{r}_{1c2} \\ \vdots \\ \tilde{r}_{nc2} \end{bmatrix} \tag{4.19}$$

and

$$\eta_{c1} = \begin{bmatrix} \eta_{1c1} \\ \vdots \\ \eta_{nc1} \end{bmatrix} \qquad \eta_{c2} = \begin{bmatrix} \eta_{1c2} \\ \vdots \\ \eta_{nc2} \end{bmatrix} \tag{4.20}$$

then we have

$$\bar{\bar{D}}(q_i)\dot{\tilde{r}}_{ic} + \bar{\bar{C}}(q_i,\dot{q}_i)\tilde{r}_{ic} = \bar{\bar{B}}(q_i)\tau_i - [\bar{\bar{D}}(q_i)\dot{\eta}_{ic} + \bar{\bar{C}}(q_i,\dot{q}_i)\eta_{ic}]$$

$$= \bar{\bar{B}}(q_i)\tau_i - \bar{\bar{Y}}(q_i,\dot{q}_i,\eta_{ic},\dot{\eta}_{ic})a_i$$

Theorem 4.2: If the leader robot is the root of spanning tree and given nonholonomic systems (4.17) with unknown inertial parameter vectors a_i, the adaptive controllers

$$\tau_i = \overline{B}(q_i)^{-1}\left[Y(q_i,\dot{q}_i,\eta_{ic},\dot{\eta}_{ic})\hat{a}_i - \begin{bmatrix} a_{i0}(\tilde{x}_{ic} - \tilde{x}_{0c}) + \cdots + a_{in}(\tilde{x}_{ic} - \tilde{x}_{nc}) \\ a_{i0}(\tilde{y}_{ic} - \tilde{y}_{0c}) + \cdots + a_{in}(\tilde{y}_{ic} - \tilde{y}_{nc}) \end{bmatrix} - K_6\tilde{r}_{ic}\right] \tag{4.21}$$

$$\dot{\hat{a}}_i = -\Gamma Y^T(q_i,\dot{q}_i,\eta_{ic},\dot{\eta}_{ic})\tilde{r}_{ic} \tag{4.22}$$

ensures that

$$\lim_{t\to\infty}(x_{ic} - x_0) = 0$$

$$\lim_{t\to\infty}(y_{ic} - y_0) = 0$$

where K_6 and Γ are positive definite matrices, $\dot{\hat{a}}_i$ is bounded.

Proof: Consider the following Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}\sum_{i=1}^{n} \tilde{r}_{ic}{}^T \overline{D}_i \tilde{r}_{ic} + \frac{1}{2}\sum_{i=1}^{n} (\hat{a}_i - a_i)^T \Gamma^{-1} (\hat{a}_i - a_i)$$

Take derivative of V_2 with respect to time and substitute the result in (4.18), (4.19) and (4.20), then we have

$$\dot{V}_2 = \tilde{x}^T M \dot{\tilde{x}} + \tilde{y}^T M \dot{\tilde{y}} + \sum_{i=1}^{n} \tilde{r}_{ic}{}^T \overline{D}_i \dot{\tilde{r}}_{ic} + \frac{1}{2}\sum_{i=1}^{n} \tilde{r}_{ic}{}^T \dot{\overline{D}}_i \tilde{r}_{ic} + \sum_{i=1}^{n} (\hat{a}_i - a_i)^T \Gamma^{-1} \dot{\hat{a}}_i$$

$$= \tilde{x}^T M (\tilde{r}_{c1} + \eta_{c1}) + \tilde{y}^T M (\tilde{r}_{c2} + \eta_{c2}) + \sum_{i=1}^{n} \tilde{r}_{ic}{}^T \overline{D}_i \dot{\tilde{r}}_{ic} + \frac{1}{2}\sum_{i=1}^{n} \tilde{r}_{ic}{}^T \dot{\overline{D}}_i \tilde{r}_{ic}$$

$$+ \sum_{i=1}^{n} (\hat{a}_i - a_i)^T \Gamma^{-1} \dot{\hat{a}}_i$$

$$\leq \tilde{x}^T M \tilde{r}_{c1} + \tilde{y}^T M \tilde{r}_{c2} - \alpha \tilde{x}^T M^2 \tilde{x} - \alpha \tilde{y}^T M^2 \tilde{y} + 2n\gamma_\ell \sqrt{\varepsilon}$$

$$+ \sum_{i=1}^{n} \tilde{r}_{ic}{}^T \left[\overline{B}_i \tau_i - \overline{C}_i \tilde{r}_{ic} - Y(q_i, \dot{q}_i, \eta_i, \dot{\eta}_i) a_i\right] + \sum_{i=1}^{n} \frac{1}{2} \tilde{r}_{ic}{}^T \dot{\overline{D}}_{ic} \tilde{r}_{ic}$$

$$+ \sum_{i=1}^{n} (\hat{a}_i - a_i)^T \Gamma^{-1} \dot{\hat{a}}_i$$

$$\leq \tilde{x}^T M \tilde{r}_{c1} + \tilde{y}^T M \tilde{r}_{c2} - \alpha \tilde{x}^T M^2 \tilde{x} - \alpha \tilde{y}^T M^2 \tilde{y} + 2n\gamma_\ell \sqrt{\varepsilon}$$

$$+ \sum_{i=1}^{n} \tilde{r}_{ic}{}^T \left[\overline{B}_i \tau_i - Y(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic}) a_i\right] + \frac{1}{2}\sum_{i=1}^{n} \tilde{r}_{ic}{}^T \left(\dot{\overline{D}}_i - 2\overline{C}_i\right) \tilde{r}_{ic}$$

$$+ \sum_{i=1}^{n} (\hat{a}_i - a_i)^T \Gamma^{-1} \dot{\hat{a}}_i$$

$$\leq \tilde{x}^T M \tilde{r}_{c1} + \tilde{y}^T M \tilde{r}_{c2} - \alpha \tilde{x}^T M^2 \tilde{x} - \alpha \tilde{y}^T M^2 \tilde{y} + 2n\gamma_\ell \sqrt{\varepsilon}$$

$$+ \sum_{i=1}^{n} \tilde{r}_{ic}{}^T \left[\overline{B}_i \tau_i - Y(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic}) a_i\right] + \sum_{i=1}^{n} (\hat{a}_i - a_i)^T \Gamma^{-1} \dot{\hat{a}}_i$$

$$\leq \tilde{r}_{c1}{}^T M\tilde{x} + \tilde{r}_{c2}{}^T M\tilde{y} - \alpha\tilde{x}^T M^2\tilde{x} - \alpha\tilde{y}^T M^2\tilde{y} + 2n\gamma_\ell\sqrt{\varepsilon}$$

$$+ \sum_{i=1}^{n} \tilde{r}_{ic}{}^T\left[\overline{B}_i\tau_i - Y(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic})a_i\right] + \sum_{i=1}^{n} (\hat{a}_i - a_i)^T\Gamma^{-1}\dot{\hat{a}}_i$$

$$\leq \tilde{r}_{c1}\sum_{j=0}^{n} a_{ij}\left(\tilde{x}_{ic} - \tilde{x}_{jc}\right) + \tilde{r}_{c2}\sum_{j=0}^{n} a_{ij}\left(\tilde{y}_{ic} - \tilde{y}_{jc}\right) - \alpha\tilde{x}^T M^2\tilde{x} - \alpha\tilde{y}^T M^2\tilde{y} + 2n\gamma_\ell\sqrt{\varepsilon}$$

$$+ \sum_{i=1}^{n} \tilde{r}_{ic}{}^T\left[\overline{B}_i\tau_i - Y(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic})a_i\right] + \sum_{i=1}^{n} (\hat{a}_i - a_i)^T\Gamma^{-1}\dot{\hat{a}}_i$$

$$\leq \sum_{i=1}^{n} \tilde{r}_{ic1}[a_{i0}(\tilde{x}_{ic} - \tilde{x}_{0c}) + \cdots + a_{in}(\tilde{x}_{ic} - \tilde{x}_{nc})]$$

$$+ \sum_{i=1}^{n} \tilde{r}_{ic2}[a_{i0}(\tilde{y}_{ic} - \tilde{y}_{0c}) + \cdots + a_{in}(\tilde{y}_{ic} - \tilde{y}_{nc})]$$

$$-\alpha\tilde{x}^T M^2\tilde{x} - \alpha\tilde{y}^T M^2\tilde{y} + 2n\gamma_\ell\sqrt{\varepsilon}$$

$$+ \sum_{i=1}^{n} \tilde{r}_{ic}{}^T\left[\overline{B}_i\tau_i - Y(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic})a_i\right] + \sum_{i=1}^{n} (\hat{a}_i - a_i)^T\Gamma^{-1}\dot{\hat{a}}_i$$

We choose the control input (4.21)

$$\tau_i = \overline{B}(q_i)^{-1}[Y(q_i, \dot{q}_i, \eta_{ic}, \dot{\eta}_{ic})\hat{a}_i - \begin{bmatrix} a_{i0}(\tilde{x}_{ic} - \tilde{x}_{0c}) + \cdots + a_{in}(\tilde{x}_{ic} - \tilde{x}_{nc}) \\ a_{i0}(\tilde{y}_{ic} - \tilde{y}_{0c}) + \cdots + a_{in}(\tilde{y}_{ic} - \tilde{y}_{nc}) \end{bmatrix} - K_6\tilde{r}_{ic}]$$

where K_6 is a positive definite matrix, with the control input we have

$$\dot{V}_2 \leq \sum_{i=1}^{n} \tilde{r}_{ic1}[a_{i0}(\tilde{x}_{ic} - \tilde{x}_{0c}) + \cdots + a_{in}(\tilde{x}_{ic} - \tilde{x}_{nc})]$$

$$- \sum_{i=1}^{n} \tilde{r}_{ic1}[a_{i0}(\tilde{x}_{ic} - \tilde{x}_{0c}) + \cdots + a_{in}(\tilde{x}_{ic} - \tilde{x}_{nc})]$$

$$+\sum_{i=1}^{n}\tilde{r}_{ic2}[a_{i0}(\tilde{y}_{ic}-\tilde{y}_{0c})+\cdots+a_{in}(\tilde{y}_{ic}-\tilde{y}_{nc})]$$

$$-\sum_{i=1}^{n}\tilde{r}_{ic2}[a_{i0}(\tilde{y}_{ic}-\tilde{y}_{0c})+\cdots+a_{in}(\tilde{y}_{ic}-\tilde{y}_{nc})]$$

$$-\alpha\tilde{x}^T M^2\tilde{x}-\alpha\tilde{y}^T M^2\tilde{y}+2n\gamma_\ell\sqrt{\varepsilon}-\sum_{i=1}^{n}\tilde{r}_{ic}{}^T K_6\tilde{r}_{ic}$$

$$-\sum_{i=1}^{n}\tilde{r}_{ic}{}^T Y(q_i,\dot{q}_i,\eta_{ic},\dot{\eta}_{ic})(\hat{a}_i-a_i)+\sum_{i=1}^{n}(\hat{a}_i-a_i)^T\Gamma^{-1}\dot{\hat{a}}_i$$

$$\leq-\alpha\tilde{x}^T M^2\tilde{x}-\alpha\tilde{y}^T M^2\tilde{y}+2n\gamma_\ell\sqrt{\varepsilon}-\sum_{i=1}^{n}\tilde{r}_{ic}{}^T K_6\tilde{r}_{ic}$$

$$+\sum_{i=1}^{n}(\hat{a}_i-a_i)^T\Gamma^{-1}\left[\dot{\hat{a}}_i+\Gamma Y^T(q_i,\dot{q}_i,\eta_{ic},\dot{\eta}_{ic})\tilde{r}_{ic}\right]$$

Now we use the update law of $\hat{a}_i$ as (4.22)

$$\dot{\hat{a}}_i=-\Gamma Y^T(q_i,\dot{q}_i,\eta_{ic},\dot{\eta}_{ic})\tilde{r}_{ic}$$

where Γ is a positive definite matrix, then

$$\dot{V}_2\leq-\alpha\tilde{x}^T M^2\tilde{x}-\alpha\tilde{y}^T M^2\tilde{y}+2n\gamma_\ell\sqrt{\varepsilon}-\sum_{i=1}^{n}\tilde{r}_{ic}{}^T K_6\tilde{r}_{ic}$$

$$\leq 2n\gamma_\ell\sqrt{\varepsilon}$$

Note that M^2 is symmetric positive definite and K_6 is positive definite matrix.

Integrating $\dot{V}_2$ gives

$$\int_0^t\dot{V}_2\,dt\leq\int_0^t 2n\gamma_\ell\sqrt{\varepsilon}\,dt$$

$$\leq 2n\gamma_{\ell} \int_0^t e^{-qt}\, dt$$

$$\leq 2n\gamma_{\ell} \frac{1}{q}(1 - e^{-qt})$$

where $t \in [0, \infty)$. Now we can see that $V_2(t)$ is bounded.

Since

$$\dot{V}_2 \leq -\alpha \tilde{x}^T M^2 \tilde{x} - \alpha \tilde{y}^T M^2 \tilde{y} + 2n\gamma_{\ell}\sqrt{\varepsilon} - \tilde{r}_c{}^T K_6 \tilde{r}_c$$

We get

$$\int_0^t \dot{V}_2\, dt \leq -\alpha \int_0^t \tilde{x}^T M^2 \tilde{x}\, dt - \alpha \int_0^t \tilde{y}^T M^2 \tilde{y}\, dt - \int_0^t \tilde{r}_c{}^T K_1 \tilde{r}_c + 2\gamma_{\ell} n \int_0^t \sqrt{\varepsilon}\, dt$$

So $\tilde{x}$, $\tilde{y}$ and $\tilde{r}_c$ are bounded.

$$\alpha \int_0^t \tilde{x}^T M^2 \tilde{x}\, dt\alpha + \int_0^t \tilde{y}^T M^2 \tilde{y}\, dt + \int_0^t \tilde{r}_c{}^T K_6 \tilde{r}_c \leq 2\gamma_{\ell} n \int_0^t \sqrt{\varepsilon}\, dt - \int_0^t \dot{V}_2\, dt$$

The boundedness of the right side of the inequality is already shown, so we can see the left side of the inequality is also bounded. Note that M^2 and K_6 are both symmetric positive definite, so $\int_0^t \tilde{y}^2$, $\int_0^t \tilde{x}^2$ and $\int_0^t \tilde{r}_c{}^2$ are bounded.

Now we have the boundedness of both $\tilde{q}_i$ and $\int_0^t \tilde{q}_{ic}{}^2 dt$, $\tilde{r}_{ic}$ and $\int_0^t \tilde{r}_{ic}{}^2 dt$, then with the aid of Barbalat's lemma 3.2.5 with $p = 2$ in [62], it can be shown that

$$\lim_{t\to\infty} \tilde{x} = 0$$

$$\lim_{t\to\infty} \tilde{y} = 0$$

$$\lim_{t \to \infty} \tilde{r}_c = 0 \qquad \blacksquare$$

Remark:

$K_6 \tilde{r}_{ic}$ is used to make $\tilde{r}_{ic}$ converge to zero and $Y\hat{a}_i$ is used to cancel the term Ya_i. The advantage of this controllers is the tracking error converge to zero even if the inertial parameter vectors are unknown, however, the inertial parameter vector must be constant.

4.5 Simulation

In this section, we will present several simulation examples of a group of three wheeled mobile robots with a virtual leader to show the effectiveness of the proposed controllers in section 4.4. All the vehicles are assumed to have the same mechanical and system structure as shown in Figure 3.10. The communication graph is shown in Figure 4.8, vertex v_0 represents the leader robot.

We choose $\alpha = 1$ and $\beta = 1$, also define the desired trajectory is a unit circle with $x_0 = sin(t)$ and $y_0 = cos(t)$. With the proposed controllers, the control parameters are chosen as: $k_6 = 1, \Gamma = diag[1,1,1]$.

The tracking errors of $x_i - x_0$ and $y_i - y_0$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 4.9, Figure 4.10 and Figure 4.11. The state information $q_i = [x_i, y_i, \theta_i]$ of follower 1, follower 2 and follower 3 are shown in, respectively, Figure 4.12, Figure 4.13 and Figure 4.14.

It can be seen that the tracking errors of each vehicle ultimately converge to zero and state parameters of each vehicle satisfy the condition of being a unit circle, so the proposed controllers are effective.

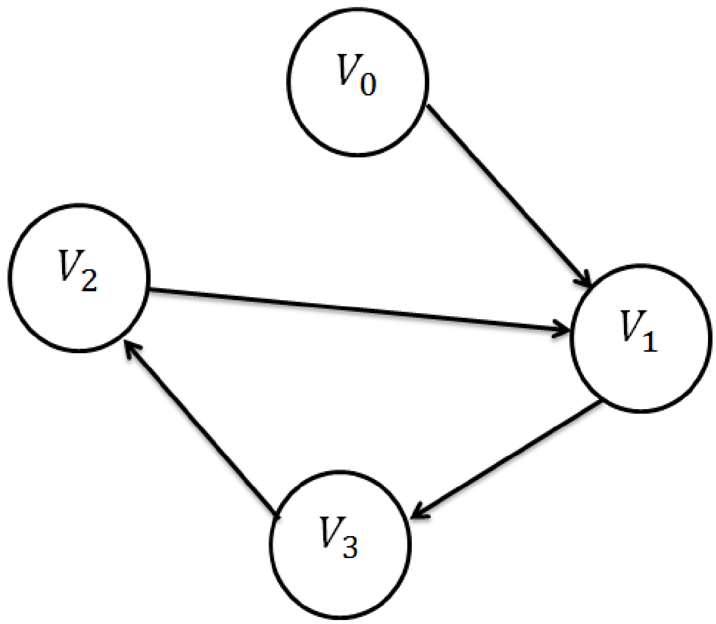

Figure 4.8 Communication graph VI

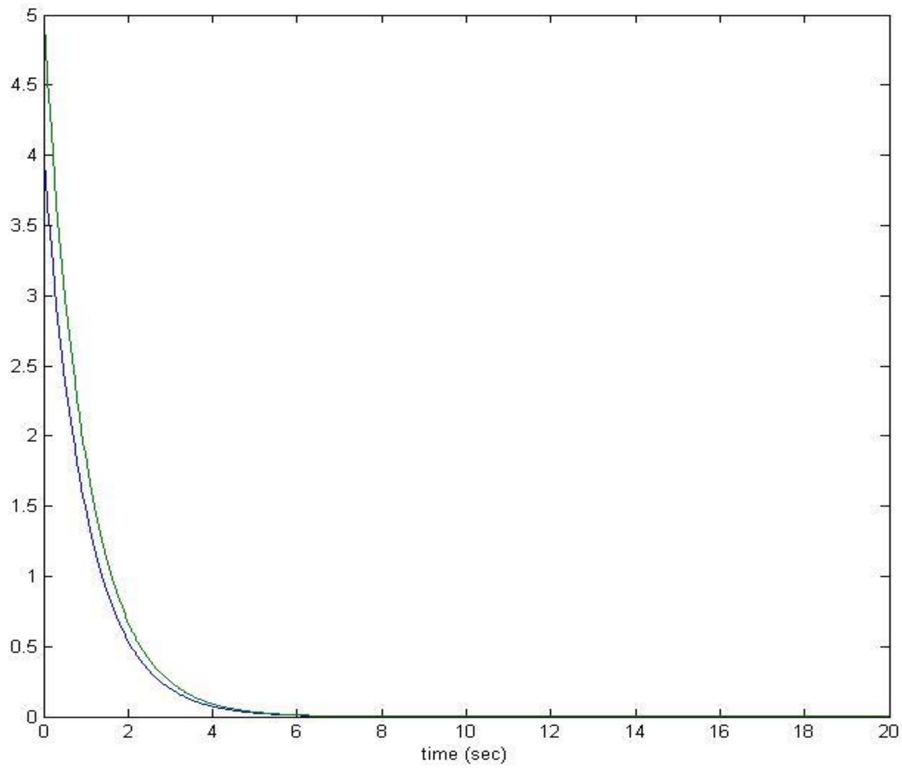

Figure 4.9 Responses of $(x_1 - x_0)$ and $(y_1 - y_0)$

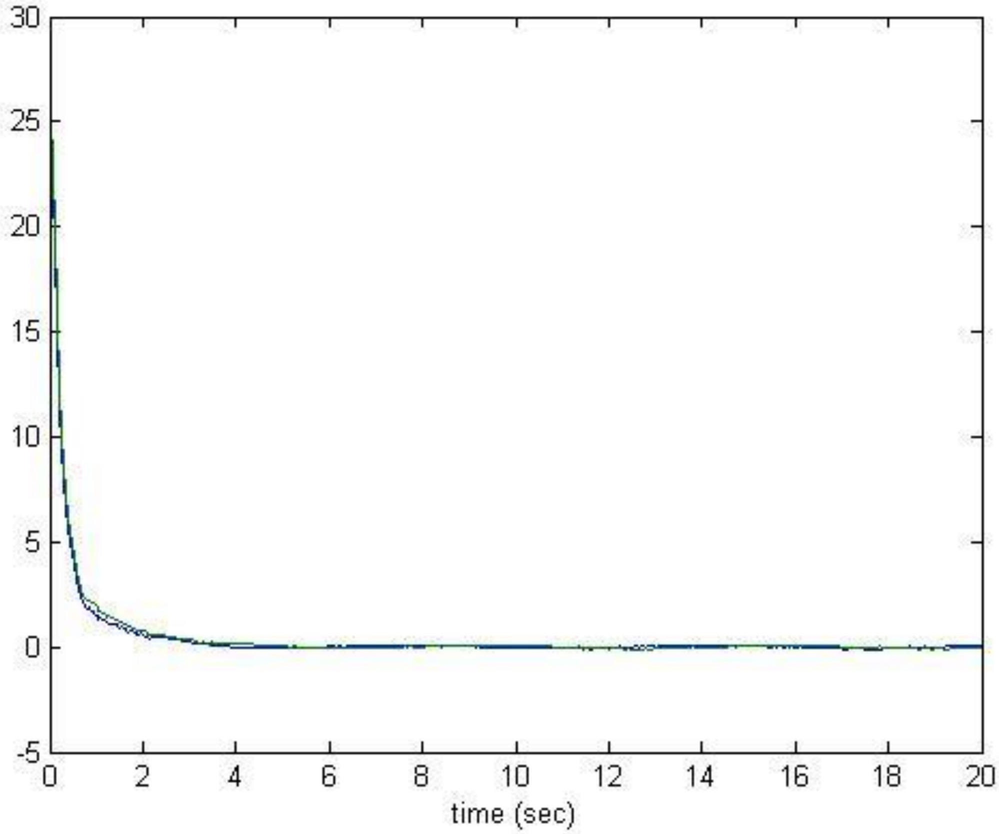

Figure 4.10 Responses of $(x_2 - x_0)$ and $(y_2 - y_0)$

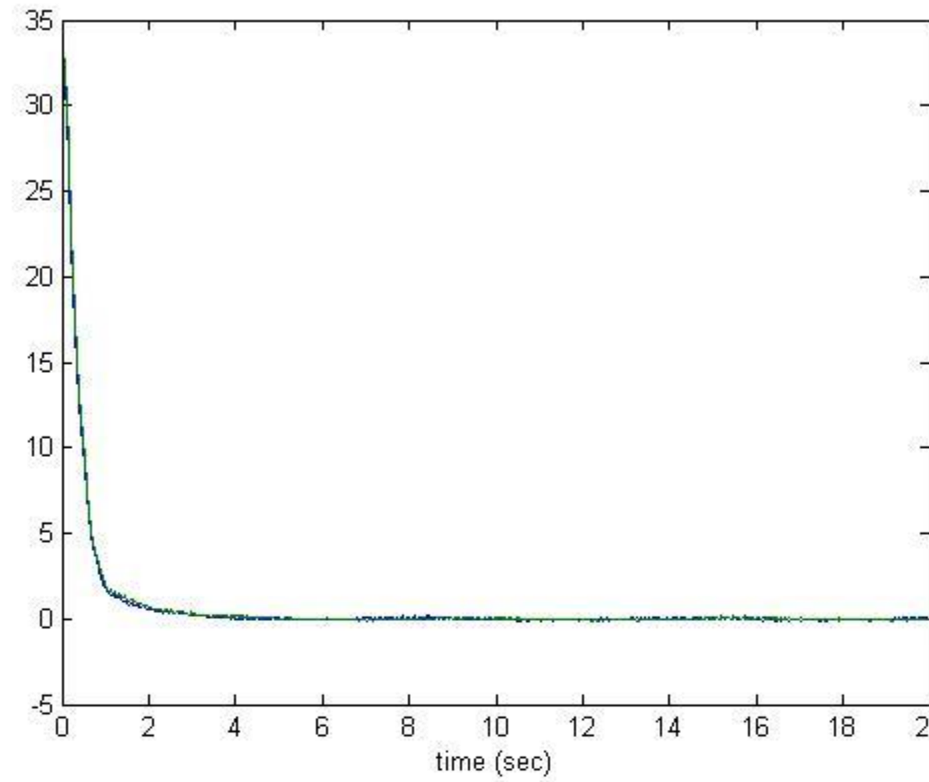

Figure 4.11 Responses of $(x_3 - x_0)$ and $(y_3 - y_0)$

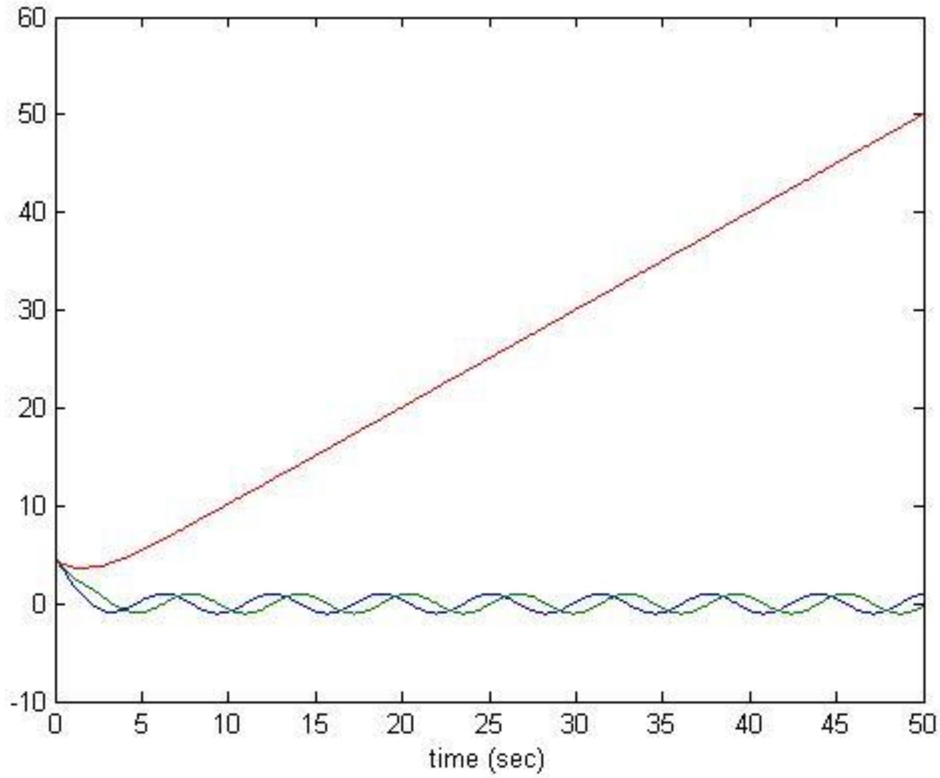

Figure 4.12 Responses of $q_1 = [x_1, y_1, \theta_1]$

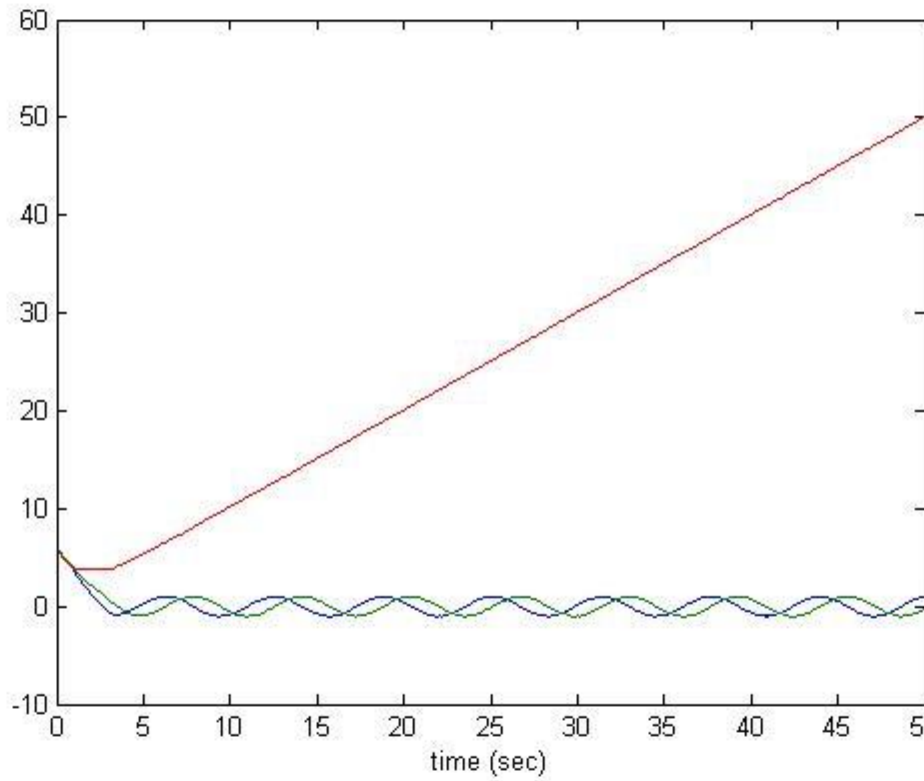

Figure 4.13 Responses of $q_2 = [x_2, y_2, \theta_2]$

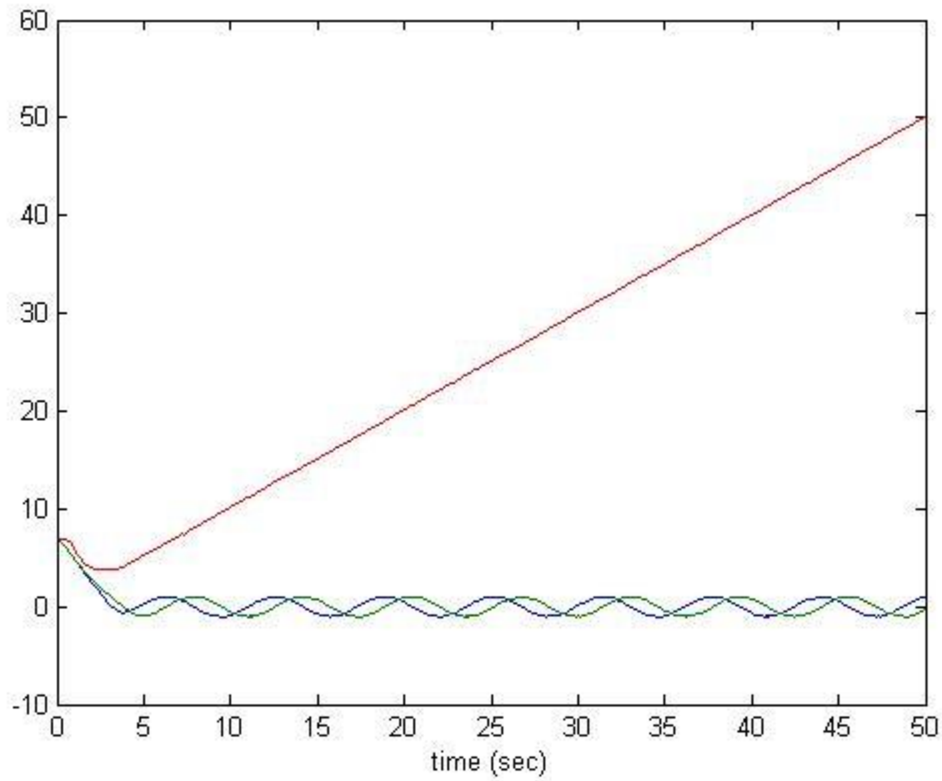

Figure 4.13 Responses of $q_3 = [x_3, y_3, \theta_3]$

4.6 Summary

Distributed coordinate tracking control of multiple WMRs with dynamic extension is studied, standard approaches to design nonholonomic controllers only deal with the kinematics of the vehicle system and ignore the actual vehicle dynamics, so we correct this omission in this chapter.

Instead of designing velocities inputs for kinematic model, we proposed torque inputs for dynamics model which is more realistic in real life. The controllers proposed in this chapter are based on the kinematic controllers proposed in chapter III and backstepping technique. Both dynamics with and without inertial uncertainties are considered and control laws are proposed accordingly, all the proposed control laws are proved to be effective.

CHAPTER V

CONCLUSION

In this thesis, distributed coordinate tracking control of multiple wheeled-mobile robots is studied. All WMRs are assumed to have same mechanical structure and are able to send out information or receive others state information. The desired trajectory is predefined as the trajectory of the virtual leader of the multiple WMRs system. Note that not all the follower robots are able to receive the state information of the virtual leader, we assume only a subset of groups of the followers can communicate to the virtual leader directly. The control objective is to design control laws for each WMR in the system, such that all follower vehicles are able to track the leader vehicle's trajectory eventually with the aid of graph theory and Laplacian matrix. Graph theory is utilized to model the information exchange between vehicles and Laplacian matrix is used to analysis the system mathematically. Dynamics of WMRs are considered following by kinematics of WMRs since it is more realistic that the control inputs are torques not the velocities, the velocity inputs in kinematic model are considered as intermediate variables as well as backstepping technique to design the torque inputs.

In the text of kinematics, we introduced two common kinematic models of mobile system, two-wheel model car-like robot with three generalized states and four-wheel model car-like robot with four generalized states. First the nonlinear car-like mobile robot models are transformed to linear models with the aid of full-state linearization technique and input-output linearization technique, then new distributed control methods are proposed for both kinematic systems with the aid of cascaded system theory and graph theory. The results showed that follower robots that are not neighbors of the leader can still be able to follow the leader robot by using the proposed controllers. The control inputs in kinematic model are translational and rotational velocities.

In the text of dynamics, the control velocities designed in kinematic section are used as intermediate variables. Torque controllers are designed with the aid of backstepping method so that the velocities of the mobile robots converge to the desired velocities, which are obtained by the kinematic controller designed at the first step. Because of the fact that in practice, the inertial parameter of WMR maybe not exactly known or unknown, so both dynamics with and without inertial uncertainties are considered, distributed tracking control laws are proposed for both scenarios.

Contributions of this thesis are: first, simpler and higher order distributed tracking algorithms are proposed based on the feedback control and stability of cascaded system. The results of first-order kinematics and second-order dynamics controller in [30] are extended to the third-order dynamics in this thesis and the variable transformation used in chapter III makes the control algorithm much more concise. Second, the dynamic extensions are considered. Standard approaches to design nonholonomic controllers only deal with the kinematics of the vehicle system and ignore the actual vehicle dynamics, so we correct this omission. Both vehicles dynamics with and without inertial uncertainties are addressed based on the velocity control inputs and backsteeping method.

Some future works and improvements including collision avoidance algorithm between WMRs during the tracking process need to be established.

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