# A stochastic production cost model for remanufacturing systems 

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# A STOCHASTIC PRODUCTION COST MODEL FOR REMANUFACTURING SYSTEMS 

A Thesis

By

## GAURANG.S.PATEL

# Submitted to the Graduate School of the University of Texas-Pan American In partial fulfillment of the requirements for the degree of 

 MASTER OF SCIENCEJuly 2006

Major Subject: Manufacturing Engineering

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## by

Gaurang. S. Patel

2006

# A STOCHASTIC PRODUCTION COST MODEL FOR REMANUFACTURING SYSTEMS 

## A Thesis

By

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Approved as to style and content by:


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July 2006


#### Abstract

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Growing environmental concern throughout this decade coupled with enforced legislation, customer expectations and economic incentives have forced increasing number of manufacturers to take back their products from end user. Managing this reverse flow of products is called reverse logistics. The stochastic nature of returned products complicates the production planning for remanufacturing systems in reverse logistics. In this thesis a stochastic production cost model, considering various costs associated under different situations for remanufacturing systems of returned products is developed. A search algorithm method is recommended to determine optimal production run size, production rate and space for remanufacturing systems in reverse logistics. This production cost model will help the businesses or "third party" logistics service provider to effectively manage remanufacturing systems of returned products.


Key words: Reverse logistics, Production planning, Total cost

## DEDICATION

This work is dedicated to my parents and family members, who have supported me all along this special journey.

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## CHAPTER 1

## INTRODUCTION

## 1 Reverse Logistics

The traditional view of a supply chain, reflected in textbooks and articles, is a linear structure, conveying goods from suppliers to manufacturers, wholesalers, retailers and finally to the consumer. But with growing concern for environment, coupled with the economic incentives and legislative compulsions has enhanced producer's responsibilities to take back end of life products. This upstream flow of product or materials in the opposite direction to traditional flow for the purpose of creating or recapturing value, or for proper disposal is called "Reverse logistics". In short reverse logistics may be defined as the management of returned materials from customers, including their restoration, reengineering, recycling, liquidating or disposal of waste in an environmentally friendly manner. The objective is to minimize the handling cost while maximize the value from the goods, or proper disposal.

Though the conception of reverse logistics dates back from long time ago, the denomination of the term is difficult to trace. Stock [29] published the first known definition of reverse logistics. Later the council of logistics research defined the exact definition of reverse logistics as "The process of planning, implementing, and controlling
the efficient, cost effective flow of raw materials, in process inventory, finished goods and related information from point of consumption to the point of origin for the purpose of recapturing value or proper disposal". Reverse logistics includes all activities required to move products from point of use to point of disposition. This includes return; recycling; reuse; product substitution; disposal; product return for warranties; source reduction; recall and refurbishing.

### 1.1 Reverse Logistics Activities:

Reverse logistics includes following activities

- Local Screening: Local screening is done at the point of collection of the returned products. Often products enter the supply chain that should not enter in the first place and cause unnecessary transportation, administration and handling costs. In an ideal reverse supply chain, products are screened at the point of collection according to specifications of the manufacturer.
- Collection: It refers to bringing the products from customer to a point of recovery. There are many different ways to collect the products that are destined to enter the reverse supply chain. Retailers often have to send their return products back to their suppliers' different warehouses throughout the country.
- Sorting: products are sorted according to the planned recovery option and within each option; products are sorted according to their quality state and recovery route. Sorting process will help firms make better and quicker disposition decisions, and cycle times will improve, resulting in better asset recovery and higher customer satisfaction.

Figure 1.1 provides an overview of reverse logistics network


Fig: 1.1 Reverse logistics network
As discussed, there are multiple tasks involved in reverse logistics activities like Collection and sorting; storage; Transportation and distribution; Compaction, Shredding and densification; Processing and filtration; remanufacturing; full disposal. Multiple tasks involve in reverse logistics activities make the process more complicated. A summary of general tasks flows involve in reverse logistics is illustrated below.

- Reprocessing or direct recovery:

Reprocessing: It includes
Repair: The purpose of repair is to return used product to working order. For example warranty returns need repair.

Refurbishing: The purpose of refurbishing is to bring used product up to specified quality. Generally expensive products or civil objects are refurbished.

Remanufacturing: Products are disassembled and their parts are used in manufacturing of the same products. For example car parts can be remanufactured and reused.

Recycling: In recycling the identity and functionality of the product is lost. For example plastic products, soda cans are recyclable.

Incineration: Products are burned and released energy is captured.

- Direct re- use or re- sale:

Re-use: return products contains valuable components that can be directly reuse.

Re-sale: Return products are redirected and sold in secondary market.

- Dispose:

Scrap: Scrap the product to obtain scrap value
Donate: Donate products to charity. Act as social responsibility and helps customer image.

Dispose in secured manner: Disposal in secured manner to avoid potential misuse in future.

A task flow chart of activities involved in reverse logistics is explained.


Figure: 1.2. Task flow chart for reverse logistics [23]

The final function of all task involved in reverse logistics is to achieve the following objectives of reverse logistics.

### 1.1.2 Objectives of Reverse Logistics are

- Customer satisfaction: After sales service, warranties etc.
- Asset recovery: Recover as much residual value as possible.
- Responsible to environment: Waste recycling, hazardous waste management
- Cost reduction: Recycling.


### 1.1.3. Comparison of Direct Logistics, Reverse Logistics and Green Logistics

Table 1.1: Comparison of forward logistics and reverse logistics

| Direct Logistics | Reverse Logistics |
| :---: | :---: |
| Quality of product is consistent | Quality of products is uncertain |
| Product flow is certain | Product flow is uncertain |
| Inventory management consistent | Inventory management not consistent |
| Market demand and product pricing is <br> unambiguous | Market demand and product pricing is <br> ambiguous |
| Process is more transparent | Process is less Transparent |
| Process is manageable | Process is less manageable |

Since the mid-nineties, in Europe it was accompanied with legal enforcement for manufacturers for product and material recovery or proper disposal. In U. S. landfill tolls became a lot more expensive and restrictions on cross-State transportation of waste rose substantially. In addition to these, competition, marketing and strategic arguments have pushed businesses into generous take back policies. This has compel businesses to redesign their business model to include more environmentally friendly products, returned flow of products, and recycling / remanufacturing and disposal strategies so that it can service forward, reverse and green logistics activities efficiently.


Figure 1.3: Forward and Reverse logistics coordination

### 1.1.4. Reasons for Reverse Logistics

There are many reasons that drive the current practices in reverse logistics. These driving forces are broadly classified in three categories.
(a) Economics (direct and indirect): A reverse logistics program can bring direct gains to companies by abating cost, dwindling use of materials, act as source of valuable spare parts or reducing cost of disposal. The indirect gains of reverse logistics include green image, improved customer relations, preparation for future legislation etc.
(b) Legislation: It relates to any jurisdiction indicating that a company should recover its products or accept them back. Many countries in world are also actively seeking legislation for product take back like E.U. packaging (2003), E.U. battery directive (2006), China WEEE (2007), CA ROHS (2006) etc.
(c) Extended Responsibilities: It concerns set of values or principles that impel businesses to become responsibly engaged in reverse logistics. Customer friendly product takes back programs helps in attracting more customers and potential revenue. For example Cole shoes, Hanna Anderson clothes offer 20 percent discount to customers returning old clothes, also Nike encourages consumer to bring used shoes to stores, and these shoes are shipped back to Nike where they are shredded to make basketball courts or, running tacks for communities.

Blumberg in his book [4] lists out the number of forces driving reverse logistics: Heightened consumer awareness and green laws imposed by government requiring safe return of products; Shortened life cycle of the products obsolescence; Increasing
customer demand for customer service; Desire cost reduction by the manufacturers to lower working capital; Shift in consumer buying behavior from in store to e commerce purchase; Existence of many types of return option for the buyer, like warranty returns, leasing of products, product recall; Increased utilization of reusable container; Increased in demand for service and support by purchasing organization, including repairs, upgrades, recalibration etc. All these factors have contributed to increase in reverse logistics activities.

### 1.1.5. Key Elements of Reverse Logistics:

For evaluation of reverse logistics system, the key elements of reverse logistics can be determined as follows.


Fig 1.4: Key elements of reverse logistics [23]

- Gate control: it serves as gatekeeper to rest of reverse logistic pipeline.
- Transportation: ways to control transportation use regional consolidation depots, negotiation for rates with several transporters, automated system for shipping etc
- Facility and Equipment: principles of facility and equipment set up for ideal reverse logistic operations are that for every major project should have dedicated processing area; major project should have assigned loading dock, traffic and processing flow consideration etc.
- Work flow: Work flow in reverse logistics has pivotal impact on the bottom line of reverse logistics.
- Communication: It should be clear and well defined
- Information system: It should be flexible and easy to integrate, comprehensive and a have real time capability. It can be classified as Product related information; Location related information; Utilization related information; Legislative information; Market information; Process information.


### 1.2 Characteristics of Reverse Logistics

The activities involved in reverse logistics contribute to typical characteristics of reverse logistics system compared to forward logistics. These include:

- Uncertain flow of material: Usually businesses do not know when an item will be returned or disposed.
- Uncertain quality and wide variety of return products: Businesses have to deal with wide and random variety of product with uncertain quality.
- Customer dependent: The return flow is highly diverse and dependent on end user or customer.
- Timing: Routing of material is stochastic and processing time is highly uncertain.
- Uncertain potential residual value: Uncertain value of asset returned.
- Uncertain market demand: Price and demand of secondary market of product is uncertain.


### 1.2.1. Reverse Logistics System Requirements

The highly stochastic characteristics of reverse logistics process have resulted in number of system requirements for developing an efficient reverse logistics system.

- Special collection centers: Efficient collection centers for wide range of products types are required.
- Classification system: A well defined classification system is required for wide range and quality of products.
- Inventory policy: A flexible inventory policy is required for handling uncertainty associated with reverse flow.
- Scheduling policy: Priority base scheduling policies is required for quick asset recovery and avoiding any environmental damage.
- Information flow: A highly efficient information flow system is required to manage the returns process, tracking costs etc.
- Flexibility: Need to design flexible system in terms of capacity, processing, transportation etc.
- Multi-parties coordination: Coordination of various actors involved in the system is essential.


### 1.2.2. Barriers in Reverse Logistics

According to research, relative unimportance of the issue of reverse logistics is the largest (39.2\%) barrier in developing a good reverse logistics management. But with
increasing competition has force many businesses to change their business practices. Lack of reliable system is also major hurdle for implementing reverse logistics practices. Other issues like competition, financial resources, personnel resources and legal issues has also restrict the application of reverse logistics in many industries.

### 1.3 Reverse Logistics Practices

Logistics cost are estimated to account for approximately 10.7 percent of the U.S. economy. The study shows that reverse logistics cost accounted for approximately four percent of total logistics costs that results in a half percent of total U.S. G.D.P. The study also estimated the market value of reverse logistics in 1997 at $\$ 35$ billion dollars and would be around $\$ 80$ billion for year 2005. The magnitude and impact of reverse logistics varies with industry. It is clear that the overall amount of reverse logistics activities in the economy is large and still growing.

According to Carnegie Mellon University research, approximately 15 million used computers could end up in U.S. landfills by 2005 costing 1 billion dollars As a result, several U.S. computer manufacturers have started to recycle processor boards and power supplies along with other reusable components such as batteries and printer cartridges etc. Dell computers through its asset recovery program, "Dell Exchange" is offering product take back program of used computers to its customers. Also, HP through its "Planet Partners Program" takes back used computers. Apple Inc. also has I-pod and computer recycling programs.

Besides this many companies like Kodak, remanufactures circuit board of disposable cameras and Xerox, remanufactures office equipments, makes substantial profits by incorporating reverse logistics practices in their business. Stringent electronic
products recycling legislation of European Union has led to development of companies like Zerlegezentrum Grevnbroich, which has different recycling lines to treat different products such as refrigerators, electronic consumer goods etc. Mierc Bv, a subsidiary of Phillips, treats electronic consumer products from Phillips and other companies. In the U.S. there are many third party logistics service provider like Jabil, Image Microsystems, and ATC Logistics \& Electronics etc. that provides product recovery services to electronic industry. Also there are many specialize reverse logistics service providers like Yellow Logistics services, FedEx Logistics, UPS Worldwide, Genco, Burnham that provides specialty solution for reverse logistics to several industries.

### 1.3.1. Comparative Study between European and U.S. Reverse Logistics Practices

It has been found that US businesses differ from their European counterparts in terms of green activism and awareness. For example, research by the Global Logistics Research Team at Michigan State University (1995) found that European firms demonstrate high levels of environmental sensitivity compared to U.S. businesses. Governments in Europe play a role of regulators, facilitators, and buyers in preserving the environment. This active participation of government and industry has made the reverse logistics industry in Europe more advance both in terms of technological development and applications compared to their U.S. counterparts.

It is found that most of the reverse logistics activities in U.S. are outsourced to third party logistics provider while in Europe the reverse logistics practices are carried in house. There are number of advantages and disadvantages associated with in house processing as well as outsourcing of reverse logistics activities. In house processing recovers the returned products at much faster rate; Companies are in direct touch with
their customers so their opinion about products can act as future product design input; and companies have complete control over revenue from the recovered products. But there are also some negatives associated with in house processing like return products disrupt the material planning of forward manufacturing process leading to complexities in inventory control, production planning etc. The advantages associated with outsourcing reverse logistics practices are: third party logistics provider are more resourceful and efficient in collection and recovery of returned products; Third party logistics provider have more expertise in product recovery process resulting in waste minimization and full potential recovery of returned products; third party logistics provider being separate companies their operations has no interference with original manufacturer production line simplifying the operations. There are also some negatives associated with outsourcing like: the product and client secrecy is compromise and the original manufacturer is unable to take full advantage of the revenues generated by product recovery.

### 1.4. Production Planning and Control for Remanufacturing System in Reverse Logistics:

Axsater [2] stated that the objective of production planning is to balance conflicting goals of keeping inventory level down to make resources available for other purposes as well as balance the production line. With this regard, the production planning for remanufacturing / recovery of returned products is a complicated issue due uncertainties associated with returned products.

Guide [13] lists out complicating characteristics associated with production planning and control for product recovery/remanufacturing and points out the research issues that needs to be studied to deal with the modeling of complicating characteristics
of production planning for product recovery/remanufacturing. Uncertainty in timing and quantity of returns can be studied by forecasting models, reliability models and inventory control models. Disassembly of return products can be studied by models for the design and models support in planning what parts and components to recover in disassembly. Materials recovery uncertainty can be studied by models that support materials recovery planning and models that can predict amount of materials that can be recovered based on age, usage rate etc. Reverse logistics network can be studied by models and system for products acquisition, models for optimizing channel choice for remnufacturers etc. Material handling can be studied by models for shop floor control and coordination and models for information flow for material tracking. Designing of appropriate production control model for product recovery / remanufacturing that ensured that both business and social objective are achieved is a complex issue.

The production planning and control for product recovery/remanufacturing has to deal with complicated tasks like demand management, capacity planning, materials planning and production scheduling. Demand management has to tackle the problem of balancing demand for remanufactured products with return products. Since in reverse logistics the remanufacturing capabilities are restricted by inflow of return products, the demand planning for product recovery/remanufacturing system depends upon degree of knowledge of inflow process. The resource and capacity planning faces uncertain processing operations and uncertain resource requirements due to variation in quality, type and quantity of return products this leads to competition for resources between returned products. To balance this competition between return products for resources one has to simultaneously deal with three important aspects of production planning viz.

- Productions run size. (How many units should be produced)
- Scheduling (When should each item be produced)
- Priority (In what order should item be produced).

The challenges involved with coordination of all the tasks makes the production planning in remanufacturing more complicated then traditional production planning.

It can be concluded that good reverse logistics program requires sufficient resources for planning, implementing and controlling of remanufacturing systems. Companies must recognize that reverse logistics is not solely a supply chain issue. It requires direct involvement at every level of organization. Many production systems are ill equipped to initiate high levels of interaction with the reverse flow of products. This has created a need for development of sophisticated production execution systems that offer dedicated supply chain visibility and interaction applications with returned flow of products. These allow companies to share data and management control with diverse parts of their organization, empowering designated managers to streamline the administration of returns and make a tangible contribution to customer service and competitive differentiation.

## CHAPTER 2

## LITERATURE REVIEW

Though the conception of reverse logistics dates back from long time ago, the denomination of the term is difficult to trace with precision. According to council of Logistics management (CLM) the first known definition of reverse logistics was published by Stock [29] in year 1992 was quite general. In the end of nineties, Rogers and Tibben-Lembke [28] defined reverse logistics and discusses current practices, barriers and trends in reverse logistics. The research was based on survey concerning current reverse logistics practices in U.S. and develops information system surrounding trends in reverse logistics.

Brito and Dekker [5] studied the framework of reverse logistics in context to driving forces, decision framework, and types of return products. The issues in reverse logistics like why products are returned; the driving reasons like legislative, commercial and public image, how products are returned; the actors and processor involve in reverse logistics and what products are returned; product composition, conditions were studied. A recovery option pyramid was developed with recovery options at the top of the pyramid were of high value, while option close to bottom recovers less value these pyramid to Facilitate decision process for recovery of returned products. At the end, a decision framework based on long, medium and short term perspective was discussed.

Blumberg [4] in his book illustrated both qualitative and quantitative measurement of reverse logistics and repair service market opportunities and segmentation in U.S. It was concluded that under current business practices there was a need to integrate reverse logistics activities with forward logistics activities to serve the market needs and achieve greater economic benefits.

Fleischmann, M et al., [11] identified general characteristic of product recovery networks and compared them with traditional forward logistics structures. It was found that supply uncertainty in a wide sense appears to be a major distinguishing factor between product recovery and traditional production distribution network. A review of case studies supported the development of the classification of reverse logistics network on basis of degree of centralization, number of levels, and links with other network. It was concluded that mathematical models that capture uncertainty and structural consideration for product recovery network can act as a vital tool for quantitative analysis of product recovery network

Richey, R et al., [27] investigated the issues for developing effective reverse logistics programs on basis of survey in automobile aftermarket industry. It was found that the influence of program design characteristics like returns policy restrictiveness has the most direct influence on performance of reverse logistics system, while characteristics like innovation and formalization had decreasing order influence on performance of reverse logistics system.

Lee, J. [23] identified the critical issues that OEMs face in managing the reverse flow of products. The study identified the key elements like gate control for product returns, transportation control, facility / equipment configuration, work flow control,
information system and communication channel for reverse logistics system. It was found that these key elements could serve as important decision parameters in establishing an effective reverse logistics system.

Beullens, P. [3] identified the obstacles that may arise when introducing product recovery system in context to economic landscape. It was found that the factors like economic incentives associated with product recovery, the problems associated with operational planning of remanufacturing and recycling facilities due to uncertainty in variety, quality, quantity and timing of returned products, the design of reverse logistics network complicated by collection and vehicle routing needs to be evaluated in economic context before implementing a product recovery system.

Inderfurth, K. [14] studied the issue of uncertainty in return and demand in context to stationary demand and return process for stochastic remanufacturing model. The study investigated a closed loop system to determine optimal product recovery and production policy. A numerical analysis illustrated that cost-efficient decision making affects the product recovery behavior positively. The sensitivity analysis evaluated various problems that influence the preferences for product recovery. Specifically, the impacts of different sources of uncertainty were investigated. It was concluded that the impact of uncertainties in the underlying reverse logistics context might be less serious when newly produced items and remanufactured items were sold in different markets so that respective OEM reverse logistics activities are less tightly coupled.

Brito, M et al., [6] classified an overview of scientific literature that describes reverse logistics activities in practice. Numerous case studies with different classification criteria for reverse logistics models were studied. For example they classified the reverse
logistics models in different categories on basis of network structure: public and private reverse logistics network; Type of returns: commercial returns, service returns, and end of life returns. Also number of case studies describing the actual planning and control of product recovery activities involving uncertainty in arrival and recovery of the return products were studied.

Kirkke, H et al., [18] studied a case study at Roteb, the municipal waste company of the city of Rotterdam, Netherlands. The study helps in determination of optimal recovery strategy for recycling of discarded computer monitors. The decision rules were formulated on handling of return products in terms of disassembly, recovery and disposal. Two strategy were discussed, one at product level and other at group level, for recovery and disposal. A stochastic dynamic programming model was developed for one product type with maximal net profit, taking into account relevant technical, and ecological and economical feasibility criteria at product level. It was observed that PRD (product recovery and disposal) strategy was primarily meant for a remanufacturing situation with quality dependent reuse option and high return rate products. In GRD (Group recovery and disposal) strategy, it was found that fixed cost proved to be very important for the economic viability for product recovery. It was concluded that factors like type, quality and timing of returned products need to considered before applying the either of discussed strategies to achieve greater economic advantages.

Lu and Stuart [24] investigated a case study of end of life electronic products recycling for industrial and residential returns where different reprocessing options were studied for industrial versus residential returns. A short term bulk recycling planning (SBRP) model was developed based on mixed integer programming model that
maximizes a recyclers profit by accounting for incoming product net revenues, high value output material sales, equipment processing cost, inventory cost and disposal costs. The model helped recyclers to make decision regarding economic production run size, products acceptance policies and scheduling for processing. It was found that SBRP model was attractive for industrial as well as domestic return. It was also illustrated by case study that the SBRP model can help recyclers to identify the decisions that were sensitive to market conditions and return source.

Fleischmann, M et al., [9] reviewed the quantative models for reverse logistics on basis of three fields, namely distribution planning, inventory control, and production planning Reverse logistics models were studied and it was concluded that characteristics like uncertainty in arrival, quality, quantity, and variety of products makes the reverse logistics process more complicated. It was found that planning of inventory becomes more difficult as processor has little control on return flow in terms of quantity, quality and timing. It was concluded that traditional material resource planning (MRP) system fail for remanufacturing system and extended approaches were required for planning production activities related with product and material reuse.

Fleischmann, M et al., [10] studied a basic single item stochastic inventory model encompassing random item returns. It was found that the model can be transformed into equivalent standard inventory model without returns. An optimal control policy ( $\mathrm{s}, \mathrm{S}$ ) was derived using cost function of long run expected average cost per time for parameters $s$ (reorder level) and S (replenishment level). It was illustrated by example that ( $\mathrm{s}, \mathrm{S}$ ) order policy was the average cost optimal in the return flow model. From numerical calculations it was concluded that total cost of the system increase steeply as return ratio
approached one. Holding cost showed the similar behavior as total cost as return ration close to one. It was found that for high return ratio replenishments orders are rare and inventory behaves as M/M/1 queue.

Kleber, R et al., [19] studied a single stage recoverable inventory with dynamic demand and return under linear cost regime. The recovery system was characterized with serviceable and recoverable inventory. The dynamic optimal control problem was solved by applying pontryagin`s maximum principal for linear cost model with constraints that allows determining returns, collection and recovery time intervals for optimal control. It was concluded that same framework was suitable in determining economic value of the returned products by not only accounting remanufacturing cost advantage but also considered holding cost for return to reuse.

Kiesmuller, G. [15] studied the problem of a stochastic recovery system with different lead-times for production and remanufacturing. The production and remanufacturing decisions were based on two inventory positions, recoverable inventory and serviceable inventory. The demand and return were modeled as independent and identically distributed random variable. Two decision variables, (S,M) policy where S denoted order up to level and $\mathbf{M}$ denotes Remanufacture up to level was used, to determine average minimum cost. A numerical comparison of the cost performance for two different inventory position and single inventory position concluded that for larger remanufacturing lead time in general it was first decided how much to produce and afterwards remanufacturing order was determined resulting in no stock in recoverable inventory (push policy) was optimal. Further, for push policy with two inventory positions resulted in less safety stock in serviceable inventory for larger lead time
differences resulting in reduced cost. For the case with larger production lead time outstanding orders and the net stock of serviceable inventory resulting in returned items in recoverable inventory (pull policy) was found optimal. It was found that for policy with two inventory positions under pull policy, the stock on hand in recoverable inventory increases while serviceable inventory decreases. Since holding cost for recoverable inventory was less, the cost can be reduced by keeping the returned items in recoverable inventory for longer period instead of pushing them into the system.

Laan and Salomon [21] studied a stochastic inventory system with production, remanufacturing, and disposal operations. A simple two point, serviceable and remanufacturable inventory control system was considered. A numerical study indicated that, the plan disposal was effective way to reduce system cost as it reduces variably in the systems` inventories. It was concluded that the most important difference between push and pull control was timing of remanufacturing and disposal operations. In push policy, control of start of the remanufacturing operation was solely based on number of products in remanufacturable inventory and disposal decision was based on inventory position. For pull strategy the start depends on both the inventory position and number of products in remanufcturable inventory. The disposal decision depends, on hand remanufacturable inventory.

Laan, E. [22] investigated a single product, single echelon production and inventory system with product returns, remanufacturing and disposal. Three different procurement and inventory control strategies i.e. $\left(s_{p}, Q_{p}, s_{d}, N\right)$ strategy, $\left(s_{p}, Q_{p}, s_{d}\right.$, strategy $\left(s_{p}, Q_{p}, N\right)$ strategy were discussed. For each strategy exact expression was derived of the total expected costs as function of the control parameters. A numerical
study indicated that that ( $s_{p}, Q_{p}, s_{d}$ ) strategy outperform ( $s_{p}, Q_{p}, N$ ). But in overall case combined strategy ( $\mathrm{s}_{\mathrm{p}}, \mathrm{Q}_{\mathrm{p}}, \mathrm{s}_{\mathrm{d}}, \mathrm{N}$ ) resulted in most cost reduction.

Teunter and Vlachos [30] carried out a simulation study in a single item hybrid production system with manufacturing and remanufacturing under assumption that remanufacturing was profitable then manufacturing and there were more demands than returns. It was found that it was only profitable to consider disposal of slow moving items (less demand) when remanufacturing is as expensive as manufacturing.

Teunter, R et al., [31] studied a hybrid manufacturing / remanufacturing system with very short lead time for remanufacturing. A new class of push/pull strategy was proposed called separate push/pull strategy where manufacturing decision and remanufacturing decision were separated as much as possible. Underlying logic was that long term manufacturing decision should control stock in system while short term remanufacturing decisions should control serviceable stock on hand. A numerical comparison was carried out by comparing relevant costs under standard push/ pulls strategy, separate push/ pull and adjusted push/pull strategy. It was concluded that for fast remanufacturing separate pull strategy that was proposed in the article perform much better than standard and adjusted strategy.

Fleischmann, M et al., [10] studied a special form of (S-1, S) base stock model, where if the inventory position (stock on hand plus stock on order minus back orders) drops below certain target level $S$ at the review instant, the firm places an order to return the inventory level to $S$. To satisfy practical situation of possible other supply source. A simulation study was carried out where six alternative policies were compared based on two alternative channel design and three alternative coordination mechanism. The three
coordination approaches were analogous to those in the analytic model namely, optimal coordination, reactive approach and netting. A situation without dismantling supply forms the sixth policy. In all the cases it was analyzed that the procurement cost largely outweighed inventory related cost. It was also found that time spent in dismantling part in inventory almost never reached the critical level beyond which disposal would have been preferable. Only when supply rose as high as 90 percent of demand did the inventory cost exceeded procurement cost saving. The use of either push strategy or pull strategy for dismantling had limited impact on the total cost. In general, postponing the testing of dismantled parts yielded a slight cost advantage in the case of reactive coordination and it did so some times in case of netting.

Aras, N et al., [1] assessed the impact of quality based categorization of returned products in designing control policies for remanufacturing system. A continuous time markov chain model of make to stock production system was developed. It was assumed that returned products could be categorized as high quality and low quality returns. This categorization enable to analyze and compare two alternative strategies, each giving priority to one quality type when there was demand for remanufacturing. The numerical analysis on hybrid manufacturing-remanufacturing system, found that incorporation of returned products quality in the remanufacturing and disposal decision could lead to significant cost savings when the quality difference between return types is high and the quality of both return types is low (i.e. cost of remanufacturing is superior to manufacturing), the return rate is high relative to demand rate and the demand rate is low (slow moving products). Finally it was concluded that quality base categorization could lead to cost saving of approximate 10 percent.

Korugan and Gupta [20] studied a two echelon inventory systems with return flow, where mutually independent demands and returns were considered. The problem was modeled as an open queuing network with finite buffers to determine total expected inventory cost. An expansion methodology was used to analyze the queuing network model. In order to observe the effect of the system parameters on cost function, a computer code was developed that calculates the total expected cost for all points in predetermined subspace like high cost lost sales, manufacturing, remanufacturing, transportation cost change, high disposal cost etc. It was found that moderate increase in holding cost did not affect decision parameter even though they caused little fluctuation in total cost. A significant increase in manufacturing cost forced a slight increase in serviceable inventory, while low disposal cost encouraged the system to hold the minimum possible inventory. It was also observed that higher return rate resulted in lower holding cost indicating remanufacturing of used products may have positive effect on overall cost reduction for production system.

Kim, E. [16] used a continuous time markov decision process to develop, an optimal control policy expression that minimizes the expected discounted costs over infinite horizon for finite capacity and non instantaneous replenishment inventory control system. A procedure that jointly finds optimal buffer size and order quantity was developed with a necessary condition that guarantees the existence of the optimal policy for a finite capacity system. The model was extended to multiple outstanding orders and was found to be cost effective.

Kim and Oyen [17] studied finite capacity constrain for queuing system of single product with multi product criterion with finite capacity. For model including rejection
penalties, a heuristic policy called capacitated modified index rule (CMIR) for capacitated scheduling with customer loss penalties was developed. Comparison of the simulation results of CMIR method with that of optimal result concluded that CMIR model works well with small buffer sizes as well as for asymmetric and high cost rejection systems.

Guide, V et al., [12] examined the priority dispatching rules and disassembly release mechanism using simulation model in remanufacturing environment. Four disassembly release mechanism viz. first off, first to shop (FCFS), Last off, first to shop (LCFS), Flush and Time phased delay were studied with fifteen different priority dispatching rules. Statistical analysis using ANOVA concluded that disassembly release mechanism did not have much effect on system and was recommended to use the simplest form (preferably first off, first to shops). It was also found that due date based priority scheduling dispatching procedure performed generally well under all conditions. The same conclusion was derived in (Guide, V. 1997). It was recommended not to use reassembly accelerator rules to pro actively expedite parts as they made no significant difference in any performance measure.

McGovern and Gupta [25] studied a greedy algorithm to expedite scheduling of environmentally hazardous and high demand products. The algorithm was based on the First Fit Decreasing (FFD) algorithm effectively used in computer processors scheduling and enhanced to preserve precedence relationship within the product being disassembled. The FFD was further modified to a multi objective algorithm that seeks to minimize the number of workstations while attempting to remove hazardous and high demand product component as early as possible.

## CHAPTER 3

## METHODOLOGY

### 3.1 Problem Statement

Accomplishment of the goal of this project will demonstrate a production cost model for remanufacturing system of returned products in reverse Logistics. This production cost model will help the businesses or "third party" logistics service provider to effectively manage remanufacturing of returned products.

The purpose of the thesis is to develop a production cost model considering various costs associated with remanufacturing system in reverse logistics. The major concern with production planning of remanufacturing system of in reverse logistics is the stochastic or uncertain arrival of the returned products. The remanufacturers or recyclers have no advance knowledge of the quantity and quality of the products they are going to receive for processing. This uncertainty associated with reverse logistics process makes the resource planning for remanufacturing system more complicated giving rise higher cost and frequent system disruption. To avoid these adverse conditions the remanufacturers are keen on development of a production cost model for remanufacturing system that would provide them with accurate information to determine optimal value for decision variables under different situations of returned products in reverse logistics.

### 3.2 Stages in Model Development

### 3.2.1 The Model Structure

Based on actual business practices, a basic remanufacturing system model for returned products involves following stages. The products returned by consumer were received in receiving area. The received products were then inspected and sorted for priority. The sorted products were stored in remanufacturable inventory before being processed. The products from remanufacturable inventory were processed according the control policies and final finished products were shipped to secondary market. A detail schematic of the process is shown in figure 3.1.


Fig 3.1 Schematic diagram of Production System in Reverse logistics

### 3.2.2 The Model Dimensions

It is observed in the literature review that researchers have used time, cost and decisions as dimensions to model the remanufacturing system in reverse logistics. Time

In the literature there was a clear distinction between discrete modeling approach and continuous modeling approach of time. Continuous time approach seems to be more flexible with respect to cost structure. Therefore in this research continuous time approach was used to model the system.

## Decisions

The decision modeling for production system of remanufacturing system in reverse logistics follow largely traditional decision modeling. Most models in the literature incorporate production run size or replenishment orders as control parameters to control traditional manufacturing system. Additional decision of re-distribution or disposal was also considered for remanufacturing system of returned products models. In this research decision dimensions like production run size, production rate and space were used as control parameters in system modeling.

## Costs

As far as cost modeling was concerned, the cost associated with returned products was more complex than in traditional manufacturing system. A detailed analysis of the costs associated with remanufacturing system of returned products was analyzed. In this research the cost dimensions like total cost, holding cost, setup cost were used to model the system output that can be utilized for optimization of the remanufacturing system of returned products.

### 3.2.3 Cost Influence diagram

An influence diagram of the costs involved with their interdependent relation was developed with fundamental objective function of reducing the overall total cost of remanufacturing system of returned products. Various costs involved with returned products were analyzed and their interdependent relations were evaluated. A detailed cost structure of various costs associated with remanufacturing production system was obtained by categorizing the total cost into four major costs viz. Opportunity cost, Setup cost, holding cost and Remanufacturing cost. The foremost cost contributing to the total cost was the holding cost. The holding cost can be further subdivided into three cost categories viz. depreciation cost, material handling cost and opportunity cost. Depreciation cost involves, loss incurred due to new products arriving in the market; returned products may become obsolete because their functionality becomes outdated. Depreciation cost depends on type of product as well as waiting time required before products can be processed. Material handling cost contributing to holding cost involves the utilities associated with storage and moving of products. Opportunity cost was the cost incurred due to loss of opportunity when the investment made in acquiring returned products to the time product were resold in the secondary market could have been invested in other profitable venture. The opportunity cost depends on the MARR (maximum acceptable rate of return) value and recoverable value of the returned product, which depend upon product type. The other cost contributing to total cost is the re distribution cost, it was the cost incurred when the remanufacturers were unable to process the products due to variety of reasons and have to redistribute the received products to other remanufacturers or dispose the products in the landfills. The costs
contributing to re distribution cost were transportation cost involved in transporting goods to other remanufacturers or landfills and opportunity loss cost that counts the loss of financial advantage that would have been resulted from resale value of the remanufactured products instead of re distributing the products to other remanufacturers. The other cost contributing to the total cost for remanufacturing of returned products was the set up cost. Setup cost was the cost involved in switching production from one product to another. The costs contributing to set up cost were labor cost involved in switching and the utilities involved during the switch. The frequency of switch would depend upon several factors like type of products received by remanufacturer with certain products having higher priority or depreciation rate, resources available for processing of the products etc. Remanufacturing cost associated with processing of the returned products was also key cost in determining the total cost of the system. The costs contributing to remanufacturing cost mainly consist of material handling cost and labor cost. The material handling cost would involve, fixed costs like utilities cost involved in processing of the products, other utilities like water, power etc. The labor cost was directly associated with number of labors involved. The discussion of cost analysis concluded that various costs involved and their interdependent relations in the remanufacturing system of returned products resulted in a complex cost structure. A detailed influence diagram of the cost associated with remanufacturing system is illustrated in fig 3.2.


Figure 3.2 Influence Diagram of Cost for Production System in Reverse logistics

### 3.2.4 The Arrival Process

The stochastic nature of the returned product makes the production planning of remanufacturing system complex. In the past many researchers have assumed the product return process as poisson process. This assumption was base on memory less property of the poisson process that the time elapsed since last arrival gives no information about how long to wait until next arrival. This lack of memory of poisson process explains the mathematical tractability of the process. In poisson arrival process the product arrive singly while in compound poisson process the products arrive in batches. The batching characteristic of compound poisson process makes it suitable for modeling the bulk arrival of return products. For example, consider a remanufacturer who receives different type of returned products according to poisson process and different type of products have batch sizes that were independent and identically distributed. Then the cumulative
product of particular type of product up to time $t$ is a compound poisson variable. Tijms [32] has derived a mathematical equation for determining the probability distribution of compound poisson variable at any time (t) during the process.

$$
\begin{equation*}
r_{j}(t)=\frac{\lambda t}{j} \sum_{k=0}^{j-1}(j-k) a_{j-k} r_{k}(t) d t \tag{1}
\end{equation*}
$$

The equation developed by the author was suitably modified for modeling a stochastic batch arrival of returned products from consumer to remanufacturer. The quantity of receiving $j$ products was represented as economic production run size of $Q_{A}$ and $Q_{B}$. The term $\mathrm{a}_{\mathrm{j}-\mathrm{k}}$ was represented as distribution of batch size. The distribution for the batch size of the received products was assumed as uniform distribution to simplify the model. For products that were being processed, the probability of having certain number of products was determined by using netting approach (i.e. Adding arriving products to the products being processed to obtain net number of products ) to determine net number of products accumulated during processing run.

### 3.2.5 Network Diagram

The uncertainty in arrival process and varying processing rate with different type of products returned products makes the remanufacturing system highly unstable. These variations in arrival rate and processing rate would result in different situations for inventory with different probabilities associated with situation for different arrival and processing rate. To analyze these different situations and different probabilities associated with these situations, a network diagram was developed for all possible situations with probabilities associated with it. A schematic of network diagram is shown in figure 3.3.


Figure 3.3 Network Diagram for Different Situations and Conditions
3.3 The Model Assumptions:

A control model for remanufacturing system of returned products in reverse logistics was developed under following assumptions

Assumptions:

- Only two products types were considered.
- One product had higher priority then other.
- No back orders were allowed.
- Demand of remanufactured products was constant.
- Labor cost per worker is constant i.e. there was one flat rate for worker compensation (no overtime).
- One location for remanufacturing facility with fix number of servers
- Demand and supply were independent.
- No variation in production rate for one type of product.
- Only low priority products were re distributed.

A detailed study of the network diagram and arrival process of the returned products from the consumer to remanufacturers under above assumptions resulted in four possible situations for different arrival and processing rate. A detailed analysis of the all four situations was carried out in the next section.

### 3.4 Situation 1

### 3.4.1 Description

In this situation the time required to process the entire economic production run size of product $B$ was less then time required by product $A$ to reach economic production run size. Also the time required to process entire production run size of product A was less
then time required by product $B$ to reach economic production run size. This situation leads to duration of time when Product B (during first part of cycle) and product A(during second part of cycle) were continuously processed as they arrived, since the other arriving product due to slower arrival rate has not reach the economic production run size.


Figure 3.4: Schematic diagram of Inventory Profile for situation 1

### 3.4.2 Inventory Profile for Situation 1

A brief procedure for developing an inventory profile of the above described situation is explained in detail. For initial condition it was assumed that product B was being processed, while product A was being collected till it reached economic production run size. Since arrival rate of product $\mathbf{A}$ ( was slower then processing rate of product $\mathbf{B}$, $\lambda<\Pi_{B}$ ) the time required for processing the entire product $B$ was less then time required to reach economic production run size of product $A$. This would result in a steep slope depletion line of processing for product $B$, compared to the slope for line of incrementing product arrival. In second part of the cycle, the processing rate for product A, was faster
then arrival rate of product $B\left(\Pi_{A}<\lambda\right)$. Therefore the time required to process entire production run size of product $A$ was less then time required by to reach economic production run size of product $B$. This resulted in steep slope for product $A$ depletion line compared to the slope of incrementing product $B$ arrival line. With above discussed procedure an inventory profile was developed as shown in fig 3.4.

In the above discussed situation a constraint of limited space would give rise to two possible conditions as explained below.

## Condition A:

Under this condition the total space occupied by both the products was less then space available. This condition would result in no re distribution or disposal of the arriving products as there would always be enough space to receive more products.

## Condition B:

Under this condition the total space occupied by both the products was more then space available. This would lead to re distribution or disposal of products with lower priority (Product A) from the inventory to accommodate high priority product B giving rise to re distribution cost that could be compensated by attaining greater economic advantage with processing of higher priority product $B$.

Under above discussed situation and conditions a cost model was developed considering different costs associated with the situation. The different probabilities associated with the situation were determined using the modified compound poisson process formula and inventory profile diagram. First a single cycle for the particular situation was analyzed. For simplification the probabilities associated with the one cycle of the situation were divided in two parts based on inventory level reached during the
cycle. The part 1 of the cycle was the probability of time required for product A to reach economic production run size of $\mathrm{Q}_{\mathrm{A}}$ before the time required by economic production run size of product $B$ to be processed completely. An analogy of the process involved can be drawn with a compound poisson process with arrival rate $\lambda$ and batch sizes $\mathrm{a}_{\mathrm{j}-\mathrm{k}}$ that were independent and identically distributed as uniformly distributed batch sizes. A time period of observation was set as $t$ and the probabilities were determined as follows. The probability of having $\mathrm{Q}_{\mathrm{A}}$ arriving products and net $\left(\Pi \mathrm{t}-\mathrm{Q}_{\mathrm{B}}\right)$ depleting products in time t was determined by double integrating the modified compound poisson process formula with respect to increment in $Q_{A}$ for arriving products for limit zero to $Q_{A}$ and for depleting products with respect to increment in time $t$ for limit zero to infinity. The formula developed was as shown

$$
\begin{equation*}
p\left\{t_{Q A}>t_{B 0}\right\}=\int_{0}^{t} \int_{0}^{Q_{A}} \frac{\lambda}{Q_{A}} \sum_{k=0}^{Q_{A}-1}\left(Q_{A}-k\right) a_{Q_{A}-k} f_{k}\left(t_{Q A}\right) d Q_{A} *\left[\left(\Pi_{B} t-Q_{B}\right)-k\right]^{*} f\left(t_{\Pi t-Q_{0}}\right) d t \tag{2}
\end{equation*}
$$

Where
$\lambda=$ Arrival rate of the products
$t_{Q A}=$ Time required to reach economic production run size of product A $\mathrm{t}_{\mathrm{B} 0}=$ Time required to process economic production run size of product B $t_{A 0}=$ Time required to process economic production run size of product A $t_{Q B}=$ Time required to reach economic production run size of product $B$ $t=$ up to time $t$
$\mathrm{Q}_{\mathrm{A}}=$ Economic production run size of product A
$\mathrm{Q}_{\mathrm{B}}=$ Economic production run size of product B $\mathrm{a}_{\mathrm{QA}-\mathrm{k}}=$ Batch size distribution
$f_{k}\left(t_{Q A}\right)=$ Recursive function in $\left(\mathrm{Q}_{\mathrm{A}}, \mathrm{t}, \lambda\right)$
$\Pi_{\mathrm{B}}=$ Processing rate of product B
$f_{k}\left(t_{Q A}\right)=$ Recursive function in $\left[\left(\Pi_{\mathrm{A}} \mathrm{t},-\mathrm{Q}_{\mathrm{A}}\right), \mathrm{t}, \lambda\right)$

Similarly probabilities for part 2 of the cycle the probability of time required to reach economic production run size for product $B$ was greater then time required to process all the units in economic production run size of products A , was derived by double integrating the modified compound poisson formula with first order of integration for arriving $B$ products with respect to increment in $Q_{B}$ from zero to $Q_{B}$ and second order of integration of compound poisson process for depleting products A with small increment in $t$ for limit zero to $t$.

$$
\begin{equation*}
p\left\{t_{Q B}>t_{A 0}\right\}=\int_{0}^{t} \int_{0}^{Q_{B}} \frac{\lambda}{Q_{B}} \sum_{k=0}^{Q_{B}-1}\left(Q_{B}-k\right) a_{Q_{B}-k} f_{k}\left(t_{Q B}\right) d Q_{B} *\left[\left(\Pi_{A} t-Q_{A}\right)-k\right] * a_{Q_{B}-k} f_{k}\left(t_{\Pi t-Q_{A}}\right) d t \tag{3}
\end{equation*}
$$

The total probability associated with complete situation 1 can be obtained by multiplying the probabilities associated with two parts of the cycle.

$$
\begin{gather*}
p\left\{t_{Q A}>t_{B 0}\right\} * p\left\{t_{Q B}>t_{A 0}\right\} \\
p\left\{t_{Q A}>t_{B 0}\right\} * p\left\{t_{Q B}>t_{A 0}\right\}=\int_{0}^{t} \int_{0}^{Q A} f\left(t_{Q A}\right) d Q_{A} * f\left(t_{B 0}\right) d t * \int_{0}^{t} \int_{0}^{Q B} f\left(t_{Q B}\right) d Q_{B} * f\left(t_{A 0}\right) d t \tag{4}
\end{gather*}
$$

The probabilities derived of particular situation occurring were used as weighing factors by multiplying probabilities with average inventory to determine total inventory for particular situation in total cost calculation.

### 3.4.3. Average Inventory for Situation 1

The average inventory associated with the situation was determined by the inventory control principle of average number of products is equal to area of inventory triangle. Inventory profile diagram was analyzed to determine number of products associated with a particular situation

Average inventory of product A during the cycle

$$
\begin{equation*}
N_{A}=\left[\frac{Q_{A}}{2} *\left(t_{Q A}+t_{A 0}\right)\right] \tag{5}
\end{equation*}
$$

where

$$
\begin{gather*}
t_{Q A}=\frac{Q_{A}}{\lambda B_{A}}  \tag{6}\\
t_{A 0}=\frac{Q_{A}}{\left(\prod_{A} N \cdot W\right)} \tag{7}
\end{gather*}
$$

Similarly
Average inventory of product B during the cycle

$$
\begin{equation*}
N_{B}=\left[\frac{Q_{B}}{2} *\left(t_{Q A}+t_{Q B}\right)\right] \tag{8}
\end{equation*}
$$

Where

$$
\begin{gather*}
t_{Q B}=\frac{Q_{B}}{\lambda B_{B}}  \tag{9}\\
t_{Q A}=\frac{Q_{A}}{\lambda B_{A}} \tag{10}
\end{gather*}
$$

To determine the total cost associated with situation 1 the average inventory for situation 1 was multiplied with appropriate weighing probabilities of the situation and different cost associated with the situation.

For determining different cost associated with the situation, a detail procedure for cost calculation was developed

### 3.5 Cost Calculation

The analysis of the cost influence diagram concluded that there were four major costs contributing to the total cost of remanufacturing system of reverse logistics viz. holding cost, setup cost, re distribution cost and remanufacturing cost. A cost calculating procedure was developed as follows

Total Cost $=$ Holding Cost + Setup Cost + Re distribution Cost + Re manufacturing Cost

### 3.5.1 Holding Cost: $\mathrm{C}_{\mathrm{h}}$

The costs contributing to holding cost are depreciation cost, material handling cost and opportunity cost
$C_{h}=$ Depriciation $\cos t+$ Materialhanling $\cos t+$ opportunity $\cos t$
(a) Depreciation cost: D

Assuming that the product depreciate only during arrival process (i.e there is no depreciation of product while product being processed)
$\mathrm{C}_{\mathrm{A}}=$ Purchase price pf product A $\$$
$C_{B}=$ Purchase price pf product $B \quad \$$
$D_{A}=0.2$ of purchase cost of prod $A \quad \$ / y r$
$\mathrm{t}_{\mathrm{i}}=$ arrival time of the product during the cycle hrs

$$
\begin{align*}
& \mathrm{t}_{\mathrm{QA}}=\text { time recq for prod } \mathrm{A} \text { to reach } \mathrm{Q}_{\mathrm{A}} \quad \text { hrs } \\
& \mathrm{D}_{\mathrm{B}}=0.3 \text { of purchase cost of prod } \mathrm{B} \text { per unit time }
\end{align*} \quad \$ \mathrm{~h} .
$$

Condition:
$\mathrm{t}_{\mathrm{i}}$ (arrival time of the product during the cycle) $<\mathrm{t}_{\mathrm{QA}}$ (time recq for prod A to reach $\mathrm{Q}_{\mathrm{A}}$ )
(b) Material handling cost: M

Assuming there were fix number of servers available for processing of the products

Utilities required for handling one unit of product $\mathrm{A}=\mathrm{UC}_{\mathrm{A}}$

Utilities required for handling one unit of product $\mathrm{B}=\mathrm{UC}_{\mathrm{B}}$

Material handling cost $=\mathrm{m} \quad \$ / \mathrm{unit} / \mathrm{yr}$

Material handling cost for A

$$
\begin{equation*}
M_{A}=m * U C_{A} \quad \$ / \mathrm{unit} / \mathrm{yr} \tag{13}
\end{equation*}
$$

Material handling cost for B

$$
\begin{equation*}
M_{B}=m^{*} U C_{B} \quad \$ / \mathrm{unit} / \mathrm{yr} \tag{14}
\end{equation*}
$$

(c) Opportunity cost: O

Opportunity cost of product A

$$
\begin{equation*}
O_{A}=\operatorname{MARR}^{*}\left(C_{A}\right)=0.2 * C_{A} \quad \text { \$/unit/yr } \tag{15}
\end{equation*}
$$

Opportunity cost of product B

$$
\begin{equation*}
O_{B}=\operatorname{MARR}^{*}\left(C_{B}\right)=0.2 * C_{B} \quad \text { S/unit/yr } \tag{16}
\end{equation*}
$$

(d) Space occupied cost: S
$\mathrm{U}_{\mathrm{A}}=$ unit space occupied by product $\mathrm{A} \quad$ Sqft
$\mathrm{U}_{\mathrm{B}}=$ unit space occupied by product $\mathrm{B} \quad$ Sqft
$\mathrm{S}=$ cost $\quad \$ / \mathrm{Sqft} / \mathrm{yr}$
Space occupied cost for product A

$$
\begin{equation*}
S_{A}=U_{A} * S \quad \$ / \mathrm{yr} \tag{17}
\end{equation*}
$$

Space occupied cost for product B

$$
\begin{equation*}
S_{B}=U_{B} * S \quad \$ / \mathrm{yr} \tag{18}
\end{equation*}
$$

Condition:
$\mathrm{U}_{\mathrm{A}}>\mathrm{U}_{\mathrm{B}}$ (Unit space occupied by product $\mathrm{A}>$ Unit space occupied by product A )

Total Holding Cost: $\mathrm{C}_{\mathrm{h}}$
$C_{h}=$ Depriciation $\cos t+$ Materialhanling $\cos t+$ opportunity $\cos t+$ spaceoccupied $\cos t$

Holding Cost for A

$$
\begin{equation*}
C_{h A}=D_{A}+O_{A}+M_{A}+S_{A} \tag{19}
\end{equation*}
$$

Holding Cost for B

$$
\begin{equation*}
C_{h B}=D_{B}+O_{B}+M_{B}+S_{B} \tag{20}
\end{equation*}
$$

3.5.2. Setup Cost: $\mathrm{C}_{\mathrm{s}}$

Number of switches $/$ cycle $=2$

Number of cycles $/ \mathrm{yr}=365 /\left(t_{Q A}+t_{Q B}\right)$
$\mathrm{K}=$ setup cost $/$ switch $\$$

$$
\begin{equation*}
\text { Setup Cost }=2 * 1 / T^{*} K \quad \$ / \mathrm{yr} \tag{21}
\end{equation*}
$$

3.5.3 Re manufacturing Cost: $\mathrm{C}_{\mathrm{RM}}$
$\mathbf{M}=$ Material handling cost $\$ /$ unit/yr
Assuming fix number of servers
Labor hrs required to fix unit of product $A=L_{A} \quad$ hrs
Labor hrs required to fix unit of product $B=L_{B} \quad h r s$
Utilities required to fix unit of product $\mathrm{A}=\mathrm{UC}_{\mathrm{A}} \quad$ units
Utilities required to fix unit of product $B=\mathrm{UC}_{\mathrm{B}} \quad$ units
Labor cost $/ \mathrm{hr}=\mathrm{L}_{\mathrm{c}} \quad \$ / \mathrm{hr}$
Number of workers (labor) $=$ N.W

Material handling cost for $\mathrm{A}=\mathrm{C}_{\mathrm{RMA}}=\left(L_{A}\right) *\left(L_{C}\right) *(N . W)+U C_{A}$ \$/unit/yr
Material handling cost for $\mathrm{B}=\mathrm{C}_{\mathrm{RMB}}=\left(L_{B}\right) *\left(L_{C}\right)^{*}(N . W)+U C_{B}$ \$/unit/yr
3.5.4 Re distribution Cost: $\mathrm{C}_{\mathrm{RD}}$

Re distribution Cost $=(\text { Quantity of Disposal })^{*}($ Transportation cost + Opportunity loss cost)

For Condition B:

$$
\begin{equation*}
\left[\lambda U_{A}+\left(\prod t_{Q A}-Q_{B}\right)^{*} U_{B}\right]>S . A \tag{24}
\end{equation*}
$$

Since product $B$ is high priority product we will be disposing product A from inventory to accommodate product $B$.
$C_{d}=($ Quantity of Disposal)*(Transportation cost + Opportunity loss cost $)$

$$
\begin{equation*}
\text { Quantity of disposal }=\left(\Pi_{A} U_{A}-\frac{\lambda U_{B}}{U_{A}}\right) \text { units } \tag{25}
\end{equation*}
$$

(a) Transportation Cost

Transportation cost $=T_{c} \quad \$ /$ cycle

Number of cycles $/ \mathrm{yr}=365 /\left(t_{Q A}+t_{Q B}\right)$
Transportation cost/yr

$$
\begin{equation*}
\mathrm{T}_{\mathrm{c}} * 365 /\left(t_{Q A}+t_{Q B}\right) \quad \$ / \mathrm{yr} \tag{26}
\end{equation*}
$$

(b) Opportunity loss cost:
[Resale value of product A- (purchase cost of product A +holding cost of product A)] \$/unit

$$
\begin{equation*}
\mathrm{OL}=\left[\mathrm{R}_{\mathrm{A}}-\left(\mathrm{C}_{\mathrm{A}}+\mathrm{C}_{\mathrm{hA}}\right)\right] \quad \text { \$/unit } \tag{27}
\end{equation*}
$$

Condition: $\quad \mathrm{R}>\mathrm{C} \quad$ (resale value of product) $>$ (Purchase price of product)
Re distribution Cost

$$
\begin{equation*}
\mathrm{C}_{\mathrm{RD}}=\left(\Pi_{A} U_{A}-\frac{\lambda U_{B}}{U_{A}}\right) *\left\{\mathrm{Tc}+\left[\mathrm{R}_{\mathrm{A}}-\left(\mathrm{C}_{\mathrm{A}}+\mathrm{C}_{\mathrm{hA}}\right)\right]\right\} \quad \$ / \mathrm{yr} \tag{28}
\end{equation*}
$$

### 3.5.5 Total Cost for Situation 1

$$
\begin{aligned}
\text { Total Cost }= & \text { Holding Cost }+ \text { Setup Cost }+ \text { Re distribution Cost }+ \text { Manufacturing } \\
& \text { Cost }
\end{aligned}
$$

$$
\text { Total Cost }=C_{h}+C_{s}+C_{R D}+C_{R M}
$$

Total Cost for Situation 1
$\mathrm{T} . \mathrm{C}=$

$$
\begin{equation*}
p\left\{t_{Q A}>t_{B 0}\right\} \quad p\left\{t_{Q B}>t_{A 0}\right\} *\left(N_{A}+N_{B}\right) *\left(C_{h}+M\right)+C_{R D}+C_{S} \$ / \mathrm{yr} \tag{29}
\end{equation*}
$$

$p\left\{t_{Q A}>t_{B D}\right\} p\left\{t_{Q B}>t_{A D}\right\} *\left\{\left[\frac{Q_{A}}{2} *\left(t_{Q A}+t_{Q B}\right) *\left(C_{h A}+C_{R M A}\right)+\left(\frac{Q_{B}}{2} *\left(t_{Q A}+t_{Q B}\right) *\left(C_{h B}+C_{R M B}\right)\right]+C_{R D}+C_{S}\right.\right.$

Constraints

1. $\lambda>0$ (Arrival rate is grater then zero)
2. $\Pi_{B}, \Pi_{A}>0 \quad$ (Production rate is grater then zero)
3. $S A=$ Cons $\tan t$ (Space available is constant)

### 3.6. Situation 2

### 3.6.1 Description

Under this situation the time required to process the entire economic production run size of product $\mathbf{B}$ was less then time required to reach economic production run size of product A . Also during second part of the cycle, the time required to process entire economic production run size of product A was more then time required to reach economic production run size of product $B$. This situation leads to duration of time where Product $B$ (during first part of cycle) is continuously processed as it is received, since the Product A due to slower arrival rate has not yet reach the economic production run size to be processed economically. During second part of the cycle, product $B$ reached the economic production run size before entire product $A$ is processed this would give rise to a point of time where processing of product $A$ needs to be stopped to switch to product $B$ as it would be economically more advantageous to switch to product $\mathbf{B}$ that has higher priority over product $A$. This would lead to a situation where there would be some product A left in the inventory when production is switched to product B resulting in a residual buffer of products $A$ in the inventory. The schematic diagram of the inventory profile under this situation is shown in fig 3.5.


Figure 3.5 Schematic diagram of Inventory Profile for Situation 2

### 3.6.2. Inventory Profile Diagram for situation 2

A brief procedure for developing an inventory profile of the above described situation is explained in detail as follows. For initial condition it is assumed that product $B$ is being processed, while product $A$ is being collected till it reached economic production run size. Since processing rate of product $B$ is greater then arrival rate of product $A\left(\Pi_{B}>\lambda\right)$. Therefore the time required for processing the entire product $B$ was less then time required by product $A$ to reach economic production run size. This would result in a steep slope for depletion line of processing for product $B$, compared to the slope of incrementing product $A$ arrival line. In second part of the cycle, arrival rate of product $B$ is greater then processing rate of product $A\left(\lambda>\Pi_{A}\right)$ therefore the time required to process entire economic production run size of product A is more then time required to reach economic production run size of product $B$. This resulted in steep slope for
incrementing product $B$ arrival line compared to the slope of product $A$ depletion line. Under this situation for second part of the cycle, processing of product A needs to be discontinued to start more economically advantageous processing of product $\mathbf{B}$ resulting in residual inventory of product A , represented as rectangle at the bottom of profile diagram. With above discussed procedure an inventory profile was developed as shown in figure 3.5.

Condition A:
Under this condition the total space occupied by both the products was less then space available (i.e. $\left.\lambda U_{A}+\left(\Pi t-Q_{B}\right) * U_{B}<S . A\right)$. Under this condition there would be no re distribution of the arriving products as there would always be enough space to receive the products.

Condition B:
Under this condition the total space occupied by both the products was greater then space available (i.e. $\left.\lambda U_{A}+\left(\Pi t-Q_{B}\right) * U_{B}>S . A\right)$. Under this condition there would be re distribution of product A from the inventory to accommodate high priority product B. This condition would give rise to transportation cost for redistribution or disposal of low priority products $A$ in order to attain greater economic advantage by processing of higher priority product $B$. With similar approach as discussed in situation 1, first the weighing probabilities for situation 2 were determined.

The probabilities associated with part 1 of the situation 2 can be determined as follows

$$
\begin{equation*}
p\left\{t_{Q A}>t_{B 0}\right\}=\int_{0}^{t} \int_{0}^{Q_{A}} f\left(t_{Q A}\right) d Q_{A} * f\left(t_{B 0}\right) d t \tag{31}
\end{equation*}
$$

$$
\begin{equation*}
p\left\{t_{Q A}>t_{B 0}\right\}=\int_{0}^{t} \int_{0}^{Q_{A}} \frac{\lambda t}{Q_{A}} \sum_{k=0}^{Q_{A}-1}\left(Q_{A}-k\right) a_{Q_{A}-k} f_{k}\left(t_{Q A}\right) d Q_{A} *\left[\left(\Pi_{B} t-Q_{B}\right)-k\right] * a_{Q_{B}-k} f_{k}\left(t_{\Pi t-Q_{b}}\right) d t \tag{32}
\end{equation*}
$$

The probability associated with part 2 of the situation can be determined as follows

$$
\begin{gather*}
p\left\{t_{Q B}<t_{A 0}\right\}=\int_{0}^{t} \int_{0}^{Q_{B}} f\left(t_{Q B}\right) d Q_{B} * f\left(t_{A 0}\right) d t  \tag{33}\\
p\left\{t_{Q B}<t_{A 0}\right\}=\left(1-p\left\{t_{Q B}>t_{A 0}\right\}\right) \tag{34}
\end{gather*}
$$

$$
\begin{equation*}
p\left\{t_{Q B}<t_{A 0}\right\}=\left\{1-\int_{0}^{t} \int_{0}^{Q_{B}} f\left(t_{Q B}\right) d Q_{B} * f\left(t_{A 0}\right) d t\right\} \tag{35}
\end{equation*}
$$

$$
\begin{equation*}
p\left(t_{Q B}>t_{A 0}\right\}=\left\{1-\int_{0}^{t} \int_{0}^{Q B} \frac{\lambda}{Q_{B}} \sum_{k=0}^{Q_{B}-1}\left(Q_{B}-k\right) a_{Q_{B}-k} f_{k}\left(t_{Q B}\right) d Q_{B} *\left[\left(\Pi_{A} t-Q_{A}\right)-k\right]^{*} a_{Q A-k} f_{k}\left(t_{\Pi t-Q_{A}}\right) * d t\right\} \tag{36}
\end{equation*}
$$

### 3.6.3 Average Inventory Calculation for Situation 2.

The total number of products under the situation 2 can be determined as

Number of products A

$$
\begin{equation*}
N_{A}=\left[\frac{Q_{A}}{2} * t_{Q A}+\frac{\left(Q_{A}-n\right)}{2} * t_{Q A}+n *\left(t_{Q A}+t_{Q B}\right)\right] \tag{37}
\end{equation*}
$$

Number of products B

$$
\begin{equation*}
N_{B}=\left[\frac{Q_{B}}{2} *\left(t_{Q B}+t_{B 0}\right)\right] \tag{38}
\end{equation*}
$$

### 3.6.4. Total Cost for Situation 2

Similar to situation 1 the cost associated with situation 2 can be obtained as

$$
\begin{aligned}
\text { Total Cost }= & \text { Holding Cost }+ \text { Setup Cost }+ \text { Re distribution Cost }+ \text { Manufacturing } \\
& \text { Cost }
\end{aligned}
$$

The total cost associated with situation 2 can be determined as

$$
\mathrm{T} \cdot \mathrm{C}=
$$

$$
\begin{equation*}
\left\{p\left\{t_{Q A}>t_{B 0}\right\} p\left\{t_{Q B}<t_{A 0}\right\} \quad *\left(N_{A}+N_{B}\right)^{*}\left(C_{h}+C_{R M}\right)\right\}+C_{R D}+C_{S} \$ / \mathrm{yr} \tag{39}
\end{equation*}
$$



### 3.7. Situation 3

### 3.7.1 Description

Under this situation the time required to process the entire economic production run size of product $B$ was more then time required to reach economic production run size of product A. Also in second part of the cycle, the time required to process entire economic production run size of product $A$ was less then time required to reach economic production run size of product $B$. This situation resulted in duration of time during which product A is continuously received although economic production run size of product A has been reached as product $B$ is still in processing resulted in re distribution of product A to other remanufacturers with purpose of achieving greater economic benefits by processing higher priority product B. During second part of cycle due to slower arrival
rate of product $B$, the time required to reach economic production run size of product $B$ is greater then time required to process entire economic production run size of product $\mathbf{A}$. This would give rise to duration of time when product $\mathbf{A}$ is processed as it arrived. $\mathbf{A}$ detail schematic of the inventory profile for situation 3 is shown in fig 3.6.


Figure 3.6 Schematic diagram of Inventory profile for situation 3

### 3.7.2 Inventory Profile Diagram for Situation 3

A brief procedure for developing an inventory profile of the above described situation is explained in detail as follows. For initial condition it is assumed that product $B$ is being processed, while product $A$ is being collected till it reached economic production run size. Since the arrival rate of product $A$ is greater then processing rate of product $B\left(\lambda>\Pi_{B}\right)$. The time required for processing the entire economic production run size of product $B$ is more then time required to reach economic production run size of product $\mathbf{A}$. This would result in steep slope for incrementing product $\mathbf{A}$ arrival line, compared to slope of product $B$ depletion line. In second part of the cycle, the arrival rate of product $B$ is less then processing rate of product $A\left(\lambda_{B}<\Pi_{A}\right)$. Therefore time required to process entire economic production run size of product A is less then time required to
reach economic production run size of product $B$. This would result in a steep slope for product A depletion line compared to the slope of incrementing Product $\mathbf{B}$ arrival line. With the above discussed procedure an inventory profile is developed as shown in figure 3.6.

The limited space would give rise to two conditions

## Condition A

Under this condition the total space occupied by both the products was less then space available (i.e. $\left.\lambda U_{A}+\left(\Pi t-Q_{B}\right)^{*} U_{B}<S . A\right)$. Under this condition there would be no re distribution of the arriving products as there would always be enough space to receive the products.

## Condition B

Under this condition the total space occupied by both the products was more then space available (i.e. $\left.\lambda U_{A}+\left(\Pi t-Q_{B}\right) * U_{B}>S . A\right)$. Under this condition there would be re distribution of product A from the inventory to accommodate high priority product B . This condition would give rise to transportation cost for redistribution or disposal of low priority products A in order to achieve greater economic advantage by processing high priority product B first.

With similar approach as discussed in situation 1, first the weighing probabilities for situation 3 were determined as illustrated below.

The probabilities associated with part 1 of the situation 3 can be determined as follows

$$
\begin{gather*}
p\left\{t_{B 0}>t_{Q A}\right\}=\left\{1-P\left\{t_{Q A}>t_{B 0}\right\}\right.  \tag{41}\\
p\left\{t_{B 0}>t_{Q A}\right\}=\left\{1-\int_{0}^{t} \int_{0}^{Q_{A}} f\left(t_{Q A}\right) d Q_{A} * f\left(t_{B 0}\right) d t\right\} \tag{42}
\end{gather*}
$$

$$
\begin{equation*}
p\left\{t_{B 0}>t_{Q A}\right\}=\int_{0}^{t} \int_{0}^{Q_{A}} \frac{\lambda}{Q_{A}} \sum_{k=0}^{Q_{A}-1}\left(Q_{A}-k\right) a_{Q_{A}-k} f_{k}\left(t_{Q A}\right) d Q_{A} *\left[\left(\Pi_{B} t-Q_{B}\right)-k\right] * a_{Q B-k} f_{k}\left(t_{\Pi t-Q_{b}}\right) d t \tag{43}
\end{equation*}
$$

The probability associated with part 2 of the situation 3 can be determined as follows

$$
\begin{gather*}
p\left\{t_{Q B}>t_{A 0}\right\}=\int_{0}^{1} \int_{0}^{Q_{B}} f\left(t_{Q B}\right) d Q_{B} * f\left(t_{A 0}\right) d t  \tag{44}\\
p\left\{t_{Q B}>t_{A 0}\right\}=\left\{\int_{0}^{t} \int_{0}^{Q_{B}} \frac{\lambda}{Q_{B}} \sum_{k=0}^{Q_{B}-1}\left(Q_{B}-k\right) a_{Q B-k} f_{k}\left(t_{Q B}\right) d Q_{B} *\left[\left(\Pi_{A} t-Q_{A}\right)-k\right] * a_{Q A-K} f_{K}\left(t_{\Pi t-Q_{A}}\right) * d t\right\} \tag{45}
\end{gather*}
$$

The total number of products under the situation 3 can be determined as
3.7.3 Average Inventory Calculation for Situation 3

Number of products A

$$
\begin{equation*}
N_{A}=\left[\frac{\lambda_{A}}{2} *\left(t_{B 0}-t_{Q A}\right)+\frac{\left(Q_{A}\right)}{2} * t_{A 0}\right] \tag{46}
\end{equation*}
$$

Number of products B

$$
\begin{equation*}
N_{B}=\left[\frac{Q_{B}}{2} *\left(t_{B 0}+t_{Q B}\right)\right] \tag{47}
\end{equation*}
$$

### 3.7.4 Total Cost for Situation 3

Similar to situation 1 the cost associated with situation 3 can be obtained as
Total Cost $=$ Holding Cost + Setup Cost + Re distribution Cost + Manufacturing Cost

The total cost associated with situation 3 can be determined as

$$
\begin{align*}
& \text { T.C }= \\
& \qquad\left\{p\left\{t_{Q A}<t_{B 0}\right\} \quad p\left\{t_{Q B}>t_{A 0}\right\} *\left(N_{A}+N_{B}\right) *\left(C_{h}+C_{R M}\right)\right\}+C_{R D}+C_{S} \$ / \mathrm{yr}  \tag{48}\\
& p\left\{t_{Q A}<t_{B D}\right\} p\left(t_{Q B}>t_{A 0}\right\} *\left[\left[\xi_{2}^{\lambda}\left(t_{B 0}-t_{Q A}\right)+\frac{Q_{A}}{2}\left(t_{A 0}\right)\right]^{*}\left(C_{R A}+C_{R M A}\right)+\left[\frac{Q_{B}}{2} *\left(t_{B D}+t_{Q B}\right) *\left(C_{h B}+C_{R M B}\right)\right]+C_{R D}+C_{S}\right. \tag{49}
\end{align*}
$$

### 3.8 SITUATION 4

### 3.8.1 Description

Under this situation the time required to process the entire economic production run size of product $B$ is more then time required to reach economic production run size of product A. During the second part of the cycle The time required to process entire economic production run size of product A is more then time required to reach economic production run size of product $B$. In the first part of the cycle, product A reached the production run size before entire economic production run size of product $B$ is processed, this would result in time duration during which product A keeps on arriving even though product $\mathbf{B}$ has not been processed completely leading to re distribution of product A to other remanufacturers to achieve greater economic advantage by processing higher priority product B. Also during second part of cycle since product $\mathbf{B}$ arrival was faster this would lead to a point of time when processing of Product A was stopped as it would be economically more advantageous to switch to product $B$. This would result in residual of inventory $A$ in the system to be processed later.


Figure 3.7 Schematic diagram of Inventory Profile for Situation 4

### 3.8.2. Inventory Profile Diagram for Situation 4

A brief procedure for developing an inventory profile of the above described situation is explained in detail as follows. For initial condition it was assumed that product $B$ is being processed, while product $A$ is being collected till it reached economic production run size. Since the arrival rate of product $A$ is greater then processing rate of product $\mathrm{B}\left(\lambda>\Pi_{B}\right)$ the time required for processing the entire economic production run size of product $\mathbf{B}$ is more then time required to reach economic production run size of product A. This would result in a steep slope for incrementing product A arrival line then slope of depleting product $B$ processing line. In second part of the cycle, the arrival rate of product $B$ is greater then processing rate of product $A\left(\lambda>\Pi_{A}\right)$ therefore the time required to process entire economic production run size of product $\mathbf{A}$ is more then time required to reach economic production run size of product $B$. This resulted in steep slope for incrementing product $\mathbf{B}$ arrival line compared to the slope of product $\mathbf{B}$ depleting line. Since under this situation for the second part of the cycle, the processing of product $A$ is stopped in between due to economic advantage associated with processing of product B
would result in a some residual inventory in the system presented as rectangle block at the bottom of the diagram as shown in figure 3.7.

The limited space constraints would give rise to two conditions

## Condition A:

Under this condition the total space occupied by both the products was less then space available (i.e. $\left.\lambda U_{A}+\left(\Pi t-Q_{B}\right) * U_{B}<S . A\right)$ there would be no re distribution of the arriving products as there would always be enough space to receive more products.

## Condition B:

Under this condition the total space occupied by both the products was more then space available (i.e. $\left.\lambda U_{A}+\left(\Pi t-Q_{B}\right) * U_{B}>S . A\right)$. This would lead to re distribution of product A from the inventory to accommodate high priority product B giving rise to re distribution cost. This extra cost could be compensated by attaining greater economic advantage with processing of higher priority product B. As discussed in situation 1, first the weighing probabilities for situation 4 were determined as illustrated.

The probabilities associated with part 1 of the situation 4 can be determined as follows

$$
\begin{gather*}
p\left\{t_{B 0}>t_{Q A}\right\}=\left\{1-P\left\{t_{Q A}>t_{B 0}\right\}\right\}  \tag{50}\\
p\left\{t_{B 0}>t_{Q A}\right\}=\left\{1-\int_{0}^{t} \int_{0}^{Q_{A}} f\left(t_{Q A}\right) d Q_{A} * f\left(t_{B 0}\right) d t\right\}  \tag{51}\\
p\left\{t_{B 0}>t_{Q A}\right\}=\int_{0}^{1} \int_{0}^{Q_{A}} \frac{\lambda t}{Q_{A}} \sum_{k=0}^{Q_{A}-1}\left(Q_{A}-k\right) a_{Q_{A}-k} f_{k}\left(t_{Q A}\right) d Q_{A} *\left[\left(\prod_{B} t-Q_{B}\right)-k\right] * a_{Q_{B}-k} f_{k}\left(t_{\Pi t-Q_{G}}\right) d t \tag{52}
\end{gather*}
$$

The probability associated with part 2 of the situation 3 can be determined as follows

$$
\begin{equation*}
p\left\{t_{A 0}>t_{Q B}\right\}=\left\{1-P\left\{t_{Q B}>t_{A 0}\right\}\right\} \tag{53}
\end{equation*}
$$

$$
\begin{gather*}
p\left\{t_{A 0}>t_{Q B}\right\}=\left\{1-\int_{0}^{t} \int_{0}^{Q_{B}} f\left(t_{Q B}\right) d Q_{B} * f\left(t_{A 0}\right) d t\right\}  \tag{54}\\
p\left\{t_{A 0}>t_{Q B}\right\}=\left\{\int_{0}^{t} \int_{0}^{Q_{B}} \frac{\lambda t}{Q_{B}} \sum_{k=0}^{Q_{B}-1}\left(Q_{B}-k\right) a_{Q_{B}-k} f_{k}\left(t_{Q B}\right) d Q_{B} *\left[\left(\Pi_{A} t-Q_{A}\right)-k\right] * a_{Q_{A}-k} f_{k}\left(t_{\Pi t-Q_{A}}\right) * d t\right\} \tag{55}
\end{gather*}
$$

The total number of products under the situation 4 can be determined as

### 3.8.3 Average Inventory Calculation for Situation 4

Number of products A

$$
\begin{equation*}
N_{A}=\left[\frac{\lambda_{A}}{2} *\left(t_{B 0}-t_{Q A}\right)+\frac{\left(Q_{A}\right)}{2} *\left(t_{Q A}+t_{Q B}\right)\right] \tag{56}
\end{equation*}
$$

Number of products B

$$
\begin{equation*}
N_{B}=\left[\frac{\left(Q_{B}\right)}{2} *\left(t_{Q A}+t_{Q B}\right)\right] \tag{57}
\end{equation*}
$$

### 3.8.4. Total Cost for Situation 4

Similar to situation 1 the cost associated with situation 4 can be obtained as

Total Cost $=$ Holding Cost + Setup Cost + Re distribution Cost + Manufacturing Cost

The total cost associated with situation 3 can be determined as T.C $=$

$$
\begin{equation*}
\left\{p\left\{t_{Q A}<t_{B 0}\right\} p\left\{t_{Q B}<t_{A 0}\right\} *\left(N_{A}+N_{B}\right) *\left(C_{h}+C_{R M}\right)\right\}+C_{R D}+C_{S} \$ / \mathrm{yr} \tag{58}
\end{equation*}
$$

$p\left\{t_{Q A}<t_{B D}\right\} p\left(t_{Q B}>t_{A B}\right) *\left\{\left[\frac{\lambda}{2}\left(t_{B D}-t_{Q A}\right)+\frac{Q_{A}}{2}\left(t_{Q A}+t_{Q B}\right) *\left(C_{R A}+C_{R M A}\right)+\left[\frac{Q_{B}}{2} *\left(t_{Q A}+t_{Q B} *\left(C_{n B}+C_{R M B}\right]\right)+C_{R D}+C_{S}\right.\right.\right.$

### 3.9 Total Cost

The total cost equation was of the complete system was obtained by adding total cost associated with all four situation. The total cost of the system under all four situations can be represented as

Total Cost of the System

$$
\begin{aligned}
& p\left\{t_{Q A}>t_{B 0}\right\} p\left\{t_{Q B}>t_{A D}\right\} *\left\{\left[\frac{Q_{A}}{2} *\left(t_{Q A}+t_{Q B}\right) *\left(C_{h A}+C_{R M A}\right)+\left(\frac{Q_{B}}{2} *\left(t_{Q A}+t_{Q B}\right) *\left(C_{h B}+C_{R M B}\right]+C_{R D}+C_{S}+\right.\right.\right. \\
& \left.p\left(t_{Q A}>t_{B D}\right\} p\left(t_{Q B}<t_{A D}\right\} *\left\{\left[\frac{Q_{A}}{2} *\left(t_{Q A}\right) \frac{\left(Q_{A}-n\right)}{2} * t_{Q A}+n\left(t_{Q A}+t_{Q B}\right)\right] C_{A A}+C_{R M}\right)+\left[\frac{Q_{B}}{2} *\left(t_{Q B}+t_{B D}\right)\right]^{*}\left(C_{h B}+C_{R M B}\right]\right]+C_{R D}+C_{S}+ \\
& +p\left\{t_{Q A}<t_{B 0}\right\} p\left\{t_{Q B}>t_{A O}\right\} *\left[\left\{\frac{\lambda}{2}\left(t_{B O}-t_{Q A}\right)+\frac{Q_{A}}{2}\left(t_{A O}\right)\right] *\left(C_{h A}+C_{R M A}\right)+\left[\frac{Q_{B}}{2} *\left(t_{B O}+t_{Q B}\right) *\left(C_{h B}+C_{R M B}\right]\right\}+C_{R D}+C_{S}+\right.
\end{aligned}
$$

### 3.10. Equation Solving

To validated the above derived total cost model for production cost control of remanufacturing system in reverse logistics a visual basic program was developed for determining total cost under different situations and conditions. First a visual basic program was developed for determining probabilities associated with each situation using the formula derived. An example of the visual basic program is illustrated in appendix. A stepwise visual basic program for the various cost calculation for all situations under different conditions is developed. An example of visual basic program for the various cost calculation is shown in appendix $B$. The final total cost was determined by
multiplying probabilities obtained for particular situation with various cost associated with that particular situation. The analysis of the model was carried out under different situations and conditions for different production run sizes of both the products.

## CHAPTER 4

## RESULTS AND DISCUSSION

### 4.1 System Parameters

First set of calculations was run for varying production run size under following value of the parameters
$\mathrm{C}_{\mathrm{A}}=$ Purchase price of product $\mathrm{A}=100 \$ /$ unit
$C_{B}=$ Purchase price of product $B=250 \$ /$ unit
$\lambda=$ Arrival rate $=3$ arrivals $/$ day
$\Pi_{\Lambda}=$ Production rate of product $\mathrm{A}=3$ units $/ \mathrm{hr}$
$\Pi_{\mathrm{B}}=$ Production rate of product $\mathrm{B}=3$ units $/ \mathrm{hr}$
$\mathrm{Q}_{\mathrm{A}}=$ Economic production run size of product A
$\mathrm{Q}_{\mathrm{B}}=$ Economic production run size of product B
$\mathrm{n}=$ Buffer size for product $\mathrm{A}=4$ units
$\mathrm{U} \cdot \mathrm{C}_{\mathrm{A}}=$ Utilities required to handle one unit of product $\mathrm{A}=2$
U. $C_{B}=$ Utilities required to handle one unit of product $B=3$
$\mathrm{U}_{\mathrm{A}}=$ Unit space occupied by one product $\mathrm{A}=2 \mathrm{Sq} \mathrm{ft}$
$\mathrm{U}_{\mathrm{B}}=$ Unit space occupied by one product $\mathrm{B}=1 \mathrm{Sq} \mathrm{ft}$
$\mathrm{K}=$ Setup cost $/$ switch $=500 \$ /$ switch
$L_{A}=$ Labor hours required to fix one unit of $A=1 \mathrm{hr} /$ unit
$L_{B}=$ Labor hours required to fix one unit of $B=2 \mathrm{hr}$ /unit
$\mathrm{L}_{\mathrm{c}}=$ Labor cost $/ \mathrm{hr}=6 \$ / \mathrm{hr}$
N.W =Number of workers $=10$
$\mathrm{R}_{\mathrm{A}}=\operatorname{Re}$ sale value of product $\mathrm{A}=300 \$ /$ unit
$R_{B}=$ Re sale value of product $B=650 \$ /$ unit
$\mathrm{m}=$ material handling cost $=25 \$ / \mathrm{unit} / \mathrm{yr}$
$\mathrm{S}=$ Space occupied cost $=20 \$ / \mathrm{Sq} \mathrm{ft} / \mathrm{yr}$
$\mathrm{t}_{\mathrm{B} 0}=$ time required to process entire economic production run size of product $\mathrm{B}=$
$t_{B 0}=\frac{Q_{B}}{\Pi_{B} N \cdot W}$
$t_{A 0}=$ time required to process entire economic production run size of product $\mathrm{A}=$
$t_{A 0}=\frac{Q_{A}}{\prod_{A} N . W}$
$\mathrm{t}_{\mathrm{QA}}=$ time required to reach economic production run size of product $\mathrm{A}=t_{Q A}=\frac{Q_{A}}{\lambda B_{A}}$
$\mathrm{t}_{\mathrm{QB}}=$ time required to reach economic production run size of product $\mathrm{B}=t_{Q B}=\frac{Q_{B}}{\lambda B_{B}}$
Number of cycles/yr for situation 1 and $2=365\left(\mathrm{t}_{\mathrm{QA}}+\mathrm{t}_{\mathrm{QB}}\right)$
Number of cycles/yr for situation 3 and $4=365\left(\mathrm{t}_{\mathrm{QB}}+\mathrm{t}_{\mathrm{B} 0}\right)$
The decision variable of economic production run size is varied for different values to obtain total cost under for different sets of economic production run size. To analyze the effect of varying economic run size of one product on total cost, the run size of second product is set fix. Later in the study other decision variables like production rate and space are varied to analyze their effect on total cost of the system.

### 4.2 Cost Calculation for Equal Production Rate

First trial of analysis is run for fix production run size of product $\mathbf{B}$ with varying production run size of product A from 10 units to 50 units. It should be noted that for the analysis production rate of both the products is consider equal. The results from the analysis are tabulated below in table 4.1.

Table 4.1 Results of Cost Analysis for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for equal production rate for both the products.

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 35 | 0.1783 | 0.2322 | 0.2574 | 0.3337 | 82733.33 | 6153696 | 2041708 |
| 15 | 35 | 0.1874 | 0.2338 | 0.2557 | 0.3212 | 76650 | 6305996 | 2094442 |
| 20 | 35 | 0.1926 | 0.2380 | 0.2546 | 0.3147 | 71672.73 | 6555829 | 2200518 |
| 25 | 35 | 0.1958 | 0.2419 | 0.2514 | 0.3106 | 67525 | 6861027 | 2321712 |
| 30 | 35 | 0.1982 | 0.2449 | 0.2490 | 0.3077 | 64015.38 | 7272005 | 2477159 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 61007.14 | 7759185 | 2656859 |
| 40 | 35 | 0.2015 | 0.2490 | 0.2457 | 0.3037 | 58400 | 8266934 | 2841333 |
| 45 | 35 | 0.2026 | 0.2506 | 0.2444 | 0.3022 | 56118.75 | 8891973 | 3065757 |
| 50 | 35 | 0.2035 | 0.2521 | 0.2431 | 0.3012 | 54105.88 | 9587597 | 3313139 |

A total cost curve was plotted as shown in fig 4.1 for varying production run size of product $A$ and constant production run size of product $B$.


Figure 4.1 Total Cost plot for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for fix and equal production rate for both the products.

Figure 4.1 shows the total cost for fix production run size of product $B$ and varying production run size of product A. It is observed that with increase in production run size of product A the total cost of the system increases. This is due to increase in holding cost with increase in production run size of product A. A more detail analysis of the holding cost is explained below


Figure 4.2 Holding Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for fix and equal production rate for both the products.

Figure 4.2 shows that the holding cost increases with increase production run size of product A . This observation is in line with inventory control principles that as production run size increases, the products A are hold for longer period in the inventory resulting in more system resources required for holding the products in the system ultimately resulting increase in holding cost. Also, since product A is lower priority product, under certain situation their processing is halted in between as soon as higher priority product B reaches its production run size resulting in residual products A in the inventory. This left over inventory of product A results in an extra cost contributing to increase in holding cost.


Figure 4.3 Setup Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for fix and equal production rate for both the products.

It should be noted that the setup cost decrease with increase in production run size of product A as shown in figure 4.3 This is true since with the increase in production run size of product A would result in less frequent switches reducing the setup cost. The complete analysis revealed that since with increase in production run size of product A there is increase in total cost of the system, the two are related linearly. As under linear relation the optimal total cost is the cost associated with the lowest possible production run size, where the holding cost is the least.

With similar set of system parameters a second trial of analysis is run by setting the production run size of product A fix and varying the production run size of product B from 10 units to 50 units. It should be noted that for this trial the production rate of both the products are consider equal. The results of trial are tabulated in table 4.2.

Table 4.2 Results of Cost analysis for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ with equal production rate for both the products.

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | Setup <br> cost | Holding <br> cost | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | 10 | 0.1783 | 0.2574 | 0.2322 | 0.3337 | 145064 | 6223835 | 2892989 |
| 35 | 15 | 0.1874 | 0.2557 | 0.2338 | 0.3212 | 106892 | 6222899 | 2450527 |
| 35 | 20 | 0.1926 | 0.2546 | 0.2380 | 0.3147 | 87531 | 6308412 | 2376208 |
| 35 | 25 | 0.1958 | 0.2514 | 0.2419 | 0.3106 | 77830 | 6644954 | 2424712 |
| 35 | 30 | 0.1982 | 0.2490 | 0.2449 | 0.3077 | 68186. | 7146245 | 2518853 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 61007 | 7759185 | 2656859 |
| 35 | 40 | 0.2015 | 0.2457 | 0.2490 | 0.3037 | 56409 | 8297545 | 2787963 |
| 35 | 45 | 0.2026 | 0.2444 | 0.2506 | 0.3022 | 51599 | 9016727 | 2967426 |
| 35 | 50 | 0.2035 | 0.2431 | 0.2521 | 0.3012 | 47628 | 9777245 | 3161979 |

A total cost curve was plotted for the trial as shown in figure 4.4 for varying production run size of product $\mathbf{B}$ and constant production run size of product $\mathbf{A}$.


Figure 4.4 Total Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for fix and equal production rate for both the products.

Figure 4.4 shows the total cost for fix production run size of product A and varying production run size of product $\mathbf{B}$. It is observed that with increase in production run size of product $B$ the total cost first decreases and then increases. This is observed
due to overriding influence of setup cost on total cost for smaller production run sizes of product $B$ resulting in greater total cost of system. As the production run size is increased the influence of setup cost decreases due to less frequent switches, resulting in decrease in total cost. It is observed that after certain production run size, the influence of holding cost on total cost increases resulting in increase in total cost. Analysis of holding cost for increase in production run size of high priority product $B$ is discussed.


Figure 4.5 Holding Cost plot for fix $Q_{A}$ and varying $Q_{B}$ for fix and equal production rate for both the products.

As shown in figure 4.5 the holding cost increase with increase in production run size of product $\mathbf{B}$. This observation is in accordance with the inventory control principle, that with increase in number of products the holding cost increases as more system resources are required to hold the products in the system. It is also observed that holding cost associated with the system with increase in production run size of higher priority product B is more compared to holding cost associated with the system with increase in production run size of lower priority product $A$. This is true since the product $B$ being higher priority product the cost associated with holding higher priority product is always significant compared to holding lower priority product. From the total cost curve it is
observed that setup cost has dominant influence over total cost for smaller production run size of product B . A detail analysis of the setup cost is explained.


Figure 4.6 Setup Cost plot for fix $Q_{A}$ and varying $Q_{B}$ for fix and equal production rate for both the products.

It should be noted that the setup cost decrease with increase in production run size of product A as shown in figure 4.6. It is also observed that with increase in production run size of higher priority product $B$ there is a steep decrease in setup cost compared to decrease in setup cost for increase in production run size of lower priority product $A$. The reason for this is that product B being higher priority product it is advantageous to process product $B$ for longer period thus avoiding frequent switches.

It is found that varying the production run size of product $B$ will have a prominent influence in determination of optimal total cost of the system as the optimal total cost can be determined at the point on total cost curve where the total cost starts increasing. In the current trial this phenomena is observed for production run size of $B$ at around 25 units. A proper search method should be applied to obtain optimal production run size of high priority product that would result in least total cost.

### 4.3 Cost Calculation for Varying Production Rate

### 4.3.1 Greater Production Rate for Lower Priority Products

In the above trials production rate for both the products type is consider equal. In the next set of trial we tried to vary the production rate and observe the results. The production rate of product A is changed from 3product/hr to 5products/hr. The production rate of product B was kept unchanged at 3products/hr. All other parameters of the system were unchanged.

The trial is run for fix production run size of product $B$ with varying production run size of product A from 10 units to 50 units. The results from the analysis are tabulated below in table 4.3.

Table 4.3 Results of Cost Analysis for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ with greater production rate for lower priority product

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 35 | 0.2278 | 0.1926 | 0.3139 | 0.2654 | 82733 | 6141359 | 1982265 |
| 15 | 35 | 0.1923 | 0.2288 | 0.2643 | 0.3144 | 76650 | 6268547 | 2074728 |
| 20 | 35 | 0.1927 | 0.2379 | 0.2548 | 0.3145 | 71672 | 6480055 | 2171733 |
| 25 | 35 | 0.1957 | 0.2420 | 0.2513 | 0.3108 | 67525 | 6765214 | 2286137 |
| 30 | 35 | 0.1981 | 0.2449 | 0.2490 | 0.3078 | 64015 | 7116953 | 2420235 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 61007 | 7530512 | 2573800 |
| 40 | 35 | 0.2015 | 0.2490 | 0.2457 | 0.3037 | 58400 | 8002674 | 2746261 |
| 45 | 35 | 0.2027 | 0.2506 | 0.2444 | 0.3037 | 56118 | 8531280 | 2937120 |
| 50 | 35 | 0.2036 | 0.2520 | 0.2432 | 0.3010 | 54105 | 9114922 | 3145996 |

A total cost curve is plotted for the trial as shown in fig 4.7 for varying production run size of product $A$ and constant production run size of product $B$.


Figure 4.7 Total Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for higher production rate of low priority product.

Figure 4.7 shows the total cost for fix production run size of product $B$ and varying production run size of product $A$. It can be observed that with increase in production run size of product $A$ the total cost of the system increases. This is due to increase in holding cost with increase in production run size of product $A$. It can be observed that the total cost of the system decreases for the same set of production run size for higher production rate. This is observed due to the fact that as the production rate is increased, inventories are hold for shorter period in the system resulting in decrease in holding cost with consequences of reduced total cost of the system. A more detail analysis of holding cost is explained.


Figure 4.8 Holding Cost plot for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for higher production rate of low priority product.

As shown in figure 4.8 the holding cost increases with increase production run size of product A. This observation is in accordance with inventory control principles that as production run size increases; the products are hold for longer period in the inventory resulting in increase in holding cost. The increase in holding cost is primarily due to increase in number of lower priority products in system. This results in greater utilization of resources resulting in increase in holding cost of the system. It can be observed that the holding cost for the higher production rate system is less compared to system with lower production rate as for systems with higher production rates, due to increase in production rate the products are process faster this results in less amount of time the products are hold in the inventory compared to system with lower production rate. This result in that the products are hold for shorter period as a result the resources involved are less resulting in lower holding cost.


Figure 4.9 Setup Cost plot for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for higher production rate of low priority product.

As shown in figure 4.5 the setup cost decreases with increase in production run sizes of product A . This is true as the production run size increases the frequency of switches decreases resulting in decrease in setup cost. It can be observed that the increase in production rate does not affect the setup cost as setup cost depends on production run size.

From the complete analysis for the system with greater production rate for lower priority product it can be observed that total cost of the system decreases. The varying the production run size of product A will have little influence in determination of optimal total cost of the system as the optimal total cost is the cost associated with the lowest possible production run size, where the holding cost is the least.

The second trial of analysis is run by setting the production run size of lower priority product A fix and varying the production run size of higher priority product B from 10 units to 50 units. For this trial the production rate of product lower priority product A was increase from 3 product/hr to 5 products/ hr while the production rate of product $\mathbf{B}$ was set fix at 3 products $/ \mathrm{hr}$. The results of trial are tabulated in table 4.4.

Table 4.4 Results of Cost analysis for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ with greater production rate of Products with lower priority

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | 10 | 0.2279 | 0.3140 | 0.1927 | 0.2655 | 145064 | 5644325 | 2435022 |
| 35 | 15 | 0.1924 | 0.2643 | 0.2289 | 0.3144 | 106892 | 5678441 | 2258226 |
| 35 | 20 | 0.1927 | 0.2548 | 0.2379 | 0.3146 | 87531 | 5914149 | 2238327 |
| 35 | 25 | 0.1958 | 0.2514 | 0.2421 | 0.3108 | 77830 | 6308671 | 2305098 |
| 35 | 30 | 0.1982 | 0.2490 | 0.2450 | 0.3078 | 68186 | 6874016 | 2420850 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 61007 | 7530512 | 2573793 |
| 35 | 40 | 0.2015 | 0.2457 | 0.2491 | 0.3037 | 56409 | 8089661 | 2711908 |
| 35 | 45 | 0.2027 | 0.2444 | 0.2506 | 0.3037 | 51599 | 8835241 | 2900690 |
| 35 | 50 | 0.2036 | 0.2432 | 0.2520 | 0.3011 | 47628 | 9616208 | 3102693 |

A total cost curve is plotied for the trial as shown in fig 4.10 for varying production run size of product $B$ and constant production run size of product $A$.


Figure 4.10 Total Cost plot for fix $Q_{A}$ and varying $Q_{B}$ for higher production rate of low priority product.

Figure 4.10 shows the total cost for fix production run size of product A and varying production run size of product $B$ for the system with higher production rate of lower priority product. It can be observed that with increase in production run size of
product $B$ the total cost of the system first decreases then increases. This is observed due to predominant influence of setup cost on total cost for smaller production run sizes where there are frequent switches resulting in higher setup cost with consequences of higher total cost of the system. With further increase in production size of product $B$ the dominance of setup cost on total cost decrease with increase in holding cost. It is observed that for higher production run size the affect of holding cost is dominant for total cost calculation. A more detail analysis of holding cost is explained.


Figure 4.11 Holding Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for higher production rate of low priority product.

As shown in figure 4.11 the holding cost increase with increase in production run size of product B. It can be observed that the holding cost for the higher production rate system is less compared to system with lower production rate as for system with greater production rates the products are hold for shorter period compared to system with lower production rate, this results in fewer resources involved are less resulting in lower holding cost compared to system with lower production rate.


Figure 4.12 Setup Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for higher production rate of low priority product.

As shown in figure 4.12 the setup cost decreases with increase in production run sizes of product $B$ due to less frequent switches. Also the setup cost associated with increase in production run size of product $\mathbf{B}$ is more compared to setup cost associated with increase in production run size of product $A$. This is due to the fact that since product $B$ is higher priority product the setup cost associated for switching is more as it is always profitable to continue processing higher priority product $B$.

From the analysis it can be concluded that for system with greater production rate varying the production run size of product $B$ will have a prominent influence in determination of optimal total cost of the system as the optimal total cost can be determined at the point on total cost curve where there is a increase in total cost. In the current trial this phenomena is observed for production run size of $\mathbf{B}$ at around 15.

### 4.3.2 Greater Production Rate for Higher priority Product

In the next set of trial, the production rate of product $B$ is increased from 3 products $/ \mathrm{hr}$ to 5 products $/ \mathrm{hr}$. While the production rate of product A is kept at 3 products / hr. For given parameters the production run size of product A is varied and constant production run size of product $\mathbf{B}$. The results of the trial are tabulated below in table 4.5.

Table 4.5 Results of Cost analysis for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ with greater production rate for higher priority product.

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :--- |
| 10 | 35 | 0.1872 | 0.2334 | 0.2606 | 0.3215 | 92119 | 4921484 | 1691364 |
| 15 | 35 | 0.1881 | 0.2351 | 0.2579 | 0.3202 | 86035 | 5205708 | 1826820 |
| 20 | 35 | 0.1926 | 0.2380 | 0.2547 | 0.3147 | 81058 | 5586610 | 1970694 |
| 25 | 35 | 0.1959 | 0.2420 | 0.2515 | 0.3107 | 76910 | 6013831 | 2125649 |
| 30 | 35 | 0.1982 | 0.2449 | 0.2491 | 0.3078 | 73401 | 6566234 | 2324386 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 70392 | 7206451 | 2553113 |
| 40 | 35 | 0.2015 | 0.2491 | 0.2457 | 0.3037 | 67785 | 7858589 | 2784420 |
| 45 | 35 | 0.2027 | 0.2507 | 0.2444 | 0.3023 | 65504 | 8663510 | 3069012 |
| 50 | 35 | 0.2035 | 0.2521 | 0.2444 | 0.3012 | 63491 | 9556547 | 3383011 |

It is observed from table 4.5 that as production run size of lower priority product increases the total cost of the system increases. It can be observed that there is a increase in holding cost, resulting in higher total cost of the system. A total cost curve is plotted for the trial as shown in fig 4.13 for varying production run size of product A and constant production run size of product $B$.


Figure 4.13 Total Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for higher production rate of higher priority product.

From the figure 4.13 it is observed that the total cost increases with increase in production run size of product A. Also the total cost of system with greater production rate for higher priority product $\mathbf{B}$ is less compared to the total cost of the system with greater production rate for lower priority product A and system with lower and equal production rate for both the products. The decrease in total cost of system compared to other systems is primarily due to increase in production rate of higher priority product, the higher priority products are hold for shorter period, resulting in less amount of resources this lower consumption of resources results in overall less total cost of the system. It is observed that compared to increase in production rate of lower priority product the total cost of production rate of higher priority product is less. This is observe since it is economically more feasible to hold higher priority products for shorter time since it will always consume more resources compared to lower priority products.


Figure 4.14 Holding Cost plot for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for higher production rate of higher priority product.

Figure 4.14 shows that with increase in production run size of product $A$ the holding cost increases. It is also observed holding cost associated with system with greater production rate for higher priority product $\mathbf{B}$ is less compared to the holding cost associated with system with lower and equal production rate for both the products and system with higher production rate for lower priority product A .


Figure 4.15 Setup Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for higher production rate of higher priority product.

It is observed from figure 4.15 that the setup cost decreases with increase in production run size of product A . It is also observed that for system with greater production rate for higher priority product B the setup cost was more then system with lower and equal production rate of 3 product/hr for both the products and system with greater production rate for lower priority product A. This is true since higher production rate would result in more frequent switches.

It can be concluded that for system with greater production rate for higher priority product B with fix production run size of higher priority product B and varying production run size of lower priority product A , the total cost and holding cost of the system decreases while setup cost is increased.

For same set parameters production run size of product $B$ is varied while production run size of product A is set constant. The results of the trial are tabulated below in table 4.6.

Table 4.6 Results of Cost analysis for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ with greater production rate for higher priority products.

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | 10 | 0.1872 | 0.2606 | 0.2334 | 0.3215 | 170333 | 6498095 | 3003401 |
| 35 | 15 | 0.1881 | 0.2579 | 0.2341 | 0.3202 | 127750 | 6509432 | 2698248 |
| 35 | 20 | 0.1926 | 0.2547 | 0.2380 | 0.3147 | 104522 | 6525185 | 2526378 |
| 35 | 25 | 0.1959 | 0.2515 | 0.2420 | 0.3107 | 89425 | 6606022 | 2475182 |
| 35 | 30 | 0.1982 | 0.2491 | 0.2449 | 0.3078 | 78615 | 6855931 | 2493268 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 70392 | 7206451 | 2553113 |
| 35 | 40 | 0.2015 | 0.2457 | 0.2491 | 0.3037 | 63875 | 7622417 | 2640177 |
| 35 | 45 | 0.2027 | 0.2444 | 0.2507 | 0.3023 | 58552 | 8083724 | 2746077 |
| 35 | 50 | 0.2035 | 0.2444 | 0.2521 | 0.3012 | 54105 | 8577947 | 2865514 |

A total cost curve is plotted for varying production run size of product B and fix production run size of product A for greater production rate of higher priority product B as shown in figure 4.16.


Figure 4.16 Total Cost plot for fix $Q_{A}$ and varying $Q_{B}$ for higher production rate of higher priority product.

Figure 4.16 shows that for system with greater production rate for higher priority product, with increase in production run size of product $\mathbf{B}$ the total cost of the system first decreases and then increases. This first decrease and then increase in total cost is due to higher setup cost for lower production run size of product B. It is also worth noting that the total cost of system with production rate of higher priority product $\mathbf{B}$ greater then product A is more then the system with lower and equal production rate of 3 products/hr for both the products and system with greater production rate for lower priority product A for small values of production run size of product B. But for larger value of production run size of product $B$, the total cost decrease due to reason explained as follows. Product B being higher priority product when process at faster rate then lower priority product there is a decrease in holding cost but this effect is override by tremendous increase in
setup cost for smaller production run sizes. A detail analysis of holding cost of system with higher production rate of product $\mathbf{B}$ is explained in detail. The affect of higher production rate for product B with varying production run size of product A on holding cost of system is plotted below in figure 4.17.


Figure 4.17 Holding Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for higher production rate of higher priority product.

From Figure 4.17 it is observed that with increase in production run size of product B for system with higher production rate of product B compared to product A , the holding cost increases with increase in production run size of product $\mathbf{B}$. It is also observed that holding cost of system with greater production rate for higher priority product $B$ is less then holding cost associated with system of equal and lower production rate of 3 products /hr for both the products and system with greater production rate for higher priority product A . This is observed since product B being higher priority product if process at faster rate would result in lower holding cost due to fewer amounts of resources involved.

A different affect of higher production rate for product $B$ on set up cost is observed as shown in figure 4.18 the setup cost curve is plotted for varying production run size of product $B$.


Figure 4.18 Setup Cost plot for fix $Q_{A}$ and varying $Q_{B}$ for higher production rate of higher priority product.

As observed in figure 4.18 the setup cost decreases with increase in production run size of product B. Also, the setup cost of system with greater production rate for higher priority product B is more then system with greater production rate for lower priority product A and system with lower and equal production rate of 3products/hr for both the products. This is observed due to fact that since product B being higher priority product if processed at faster rate would result in frequent changes for smaller production run sizes of product $B$ with consequences of increase in total cost. But the setup cost decreases with increase in production run size.

### 4.3.3 Equal and Greater Production Rate

In above trials only production rate of either product A or product $B$ is increased and production rate of product other product is kept unchanged. In the next trial the production rate of both the products is changed. The production rate of product A is set at 5products/hr while production rate of product B is also set at 5products/hr.

Table 4.7 Results of Cost analysis for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ with simultaneous increase in production rate for both the products.

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 35 | 0.2278 | 0.1926 | 0.3139 | 0.2654 | 92119.05 | 4903860 | 1683080 |
| 15 | 35 | 0.1923 | 0.2288 | 0.2643 | 0.3144 | 86035.71 | 5152210 | 1805733 |
| 20 | 35 | 0.1927 | 0.2379 | 0.2548 | 0.3145 | 81058.44 | 5478362 | 1929719 |
| 25 | 35 | 0.1957 | 0.2420 | 0.2513 | 0.3108 | 76910.71 | 5876956 | 2074731 |
| 30 | 35 | 0.1981 | 0.2449 | 0.2490 | 0.3078 | 73401.1 | 6344732 | 2243045 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 70392.86 | 6879776 | 2434456 |
| 40 | 35 | 0.2015 | 0.2490 | 0.2457 | 0.3037 | 67785.71 | 7481075 | 2537944 |
| 45 | 35 | 0.2027 | 0.2506 | 0.2444 | 0.3022 | 65504.46 | 8148235 | 2658333 |
| 50 | 35 | 0.2036 | 0.2520 | 0.2432 | 0.3010 | 63491.6 | 8881297 | 2791046 |

The total cost curve is plotted for varying production run size of product A as shown in figure 4.1


Figure 4.19 Total Cost plot for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for higher production rate for both the products.

From the figure 4.19 it is observed that for system with increase production rate, the total cost increases with increase in production run size of product A . This affect is due to increase in holding cost of the system. It is also observed that for system with simultaneous increase in production rate of both the products the total cost is less compared to system with increase in production rate of only one product $A$ or $B$ and system with lower production rate for both the products. This is observed because as the production rate of the system increases fewer and fewer products are held in the system resulting in lower total cost of the system.


Figure 4.20 Holding Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for higher production rate for both the products.

As shown in fig 4.20 it is observed that the holding cost of the system with simultaneous increase in production rate of both the products the holding cost increases with increase in production run size of product $A$. As explained earlier this effect is due to increase consumption of the system resources. It is worth noting that the with simultaneous increase in production rate of both the products, the system holding cost decreases compared to system with increase in production rate of only one product A or B and system with lower production rate. This is due to the fact that with increase in production rate for both the products the products are hold for shorter period in the system reducing the resource consumption resulting in lower holding cost compared to other systems.


Figure 4.21 Setup Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for higher production rate for both the products.

As shown in figure it is observed that for system with simultaneous increase in production rate of both the products with increase in production run size of product $A$ the setup cost decreases. It is observed that for the system with simultaneous increase in production rate of both the products the setup cost is higher compared to system with lower equal production rate and system with greater production rate of only product A or product $B$. This true, since with greater production rate for both the products there are more frequent switches resulting in higher setup cost. It is interesting fact that the setup cost for system with greater production rate for both the products and system with greater production rate of higher priority product $\mathbf{B}$ are equal.

Now with same set of parameters the production run size of product $\mathbf{B}$ is varied. The results are tabulated below in table 4.8

Table 4.8 Results of Cost Analysis for Fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ with simultaneous increase in production rate of both the products.

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | 10 | 0.2278 | 0.3139 | 0.1926 | 0.2654 | 170333.3 | 6004733 | 2679719 |
| 35 | 15 | 0.1923 | 0.2643 | 0.2288 | 0.3144 | 127750 | 6007190 | 2441935 |
| 35 | 20 | 0.1927 | 0.2548 | 0.2379 | 0.3145 | 104522.7 | 6003503 | 2326544 |
| 35 | 25 | 0.1957 | 0.2513 | 0.2420 | 0.3108 | 89425 | 6148677 | 2312519 |
| 35 | 30 | 0.1981 | 0.2490 | 0.2449 | 0.3078 | 78615.38 | 6474810 | 2356097 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 70392.86 | 6879776 | 2434454 |
| 35 | 40 | 0.2015 | 0.2457 | 0.2490 | 0.3037 | 63875 | 7336576 | 2537944 |
| 35 | 45 | 0.2027 | 0.2444 | 0.2506 | 0.3022 | 58552.08 | 7829644 | 2658333 |
| 35 | 50 | 0.2036 | 0.2432 | 0.2520 | 0.3010 | 54105.88 | 8349275 | 2791046 |

A total cost curve is plotted for the system with increase in production rate of both the products for varying production run size of product $B$ as shown in figure 4.22 below


Figure 4.22 Total Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for higher production rate for both the products.

It is observed in figure 4.22 that with increase in production run size of product B for system with simultaneous increase in production run size the total cost first decreases with increase in production run size of A then increases. It is observed that for
smaller production run sizes there are frequent switches resulting in higher setup cost with consequences of higher total cost of the system. With further increase in production size of product $B$ the dominance of setup cost on total cost decrease with increase in holding cost. A more detail analysis of holding cost is explained in detail below


Figure 4.23 Holding Cost plot for fix $Q_{A}$ and varying $Q_{B}$ for higher production rate for both the products.

As shown in figure 4.23 the holding cost increases with increase in production run size of product B. As explained earlier this is observed due to increase in resource consumption of the system resulting in increase in holding cost of the system. Another observation is made that the holding cost of the system with simultaneous increase in production rate of both the products, the holding cost is less compared to holding cost of system with increase in production rate of either of one product A or B and system with lower and equal production rate for both products. This is true since for system with higher production rate the products are hold for shorter period in the system resulting in fewer resources utilization with ultimate result of reduction in holding cost compared to other systems.


Figure 4.24 Setup Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for higher production rate for both the products.

As observed in figure 4.24 the setup cost decrease with increase in production run size of product $B$. This is true as increase in production run size decreases number of switches. It is also observed that the setup cost of the system with simultaneous increase in production rate of both the products, is more compared to setup cost associated with system with increase in production rate of only one product A and system with lower and equal production rate for both products. The increase in production rate of the product would result in faster movement of products. Therefore there are frequent switches from one product type to other product type. These frequent switches from one product type to other would result in higher setup cost. As for every switch there is a setup cost involved. Also it is observed that for varying production run size of higher priority product the setup cost is more compared to varying production run size of lower priority product as it is always economically more advantageous to process higher priority product for longer period to avoid higher setup cost involved.
4.3.4 Sensitivity Analysis of Total Cost for Different Production Rate
(a) A sensitivity analysis of total cost under different production rate for varying production run of lower priority product A is shown in figure 4.25 .


Figure 4.25 Total Cost plot for fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for different production rate

It should be noted that compared to the total cost for system with lower equal production rate ( $\mathrm{A}=\mathrm{B}=3$ products $/ \mathrm{hr}$ ) and system with increase production rate of only one product $A$ or product $B$ (system with $A=3$ products/hr and $B=5$ products / hr or system with $\mathrm{A}=5$ products $/ \mathrm{hr}$ and $\mathrm{B}=3$ products $/ \mathrm{hr}$ ), the total cost of the system with increase production rate for both the product $A$ and product $B$ (system with $A=5$ product $/ \mathrm{hr}$ and $\mathrm{B}=3$ products $/ \mathrm{hr}$ ) is less. Thus it can be stated that with simultaneous increase in production rate of two products the total cost of system decreases resulting in greater economic advantage. It is observed that system with greater production rate of higher priority product compared to lower priority products results in lower total cost for smaller production run size of lower priority products but as production run size of lower priority product increases the total cost increases. It is worth noting that for larger production run of lower priority product, the increase in production rate of lower priority
product would result in lower total cost compared to system with greater production rate for higher priority product.
(b) A sensitivity analysis of total cost for different production rate is shown in figure 4.26


Figure 4.26 Total Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for different production rate for both the products.

It is observed that for system with simultaneous increase in production rate of both the products will result in lower total cost with increase in production run size of higher priority product B , compared to increase in production run size of lower priority products. It is also noted that the total cost is least for smaller production run size of higher priority product with lower production rate, but as there is increase in production run size of higher priority product the total cost increases. It should be noted that for larger production run size of higher priority product, the increase in production rate of higher priority product would result in lower total cost as it is always economical advantageous to reduced the number of higher priority product in the system due to greater cost associated with it.

### 4.4 Space Constraint

In the cases studied up to now the space constraint was set as constant. Setting up the space constraint is an important decision variable in designing of the remanufacturing system for reverse logistics. The deficiency of space to stored products would result in re distribution of the product resulting in the redistribution cost. On the other hand if the space is too large it will incur an extra cost for futile space occupied. Therefore an optimal solution is required for determining the value of space constraint in designing the production system for reverse logistics system. A trial calculation is carried out by varying the vale of space constraints and results are analyzed.

The first trial of analysis is run by setting the space constraint at 50 sqft. The production run size of high priority product $B$ is set fix and production run size of Product A is varied from 10 units to 50 units. The production rate of the system is set at 5product/hr for product A and 3products/hr for product B. The results of trial are tabulated in table 4.9.

Table 4.9 Results of Cost Analysis for Fix $\mathrm{Q}_{\mathrm{B}}$ and varying $\mathrm{Q}_{\mathrm{A}}$ for reduced space constraint

| $\mathbf{A}$ | $\mathbf{B}$ | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 10 | 35 | 0.2278 | 0.1926 | 0.3139 | 0.2654 | 82733 | 6141359 | 1982266 |
| 15 | 35 | 0.1923 | 0.2288 | 0.2643 | 0.3144 | 76650 | 6268547 | 2074728 |
| 20 | 35 | 0.1927 | 0.2379 | 0.2548 | 0.3145 | 71672 | 6480055 | 2171733 |
| 25 | 35 | 0.1957 | 0.2420 | 0.2513 | 0.3108 | 67525 | 6765214 | 2286136 |
| 30 | 35 | 0.1981 | 0.2449 | 0.2490 | 0.3078 | 64015 | 7116953 | 2420234 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 61007 | 7530512 | 2573800 |
| 40 | 35 | 0.2015 | 0.2490 | 0.2457 | 0.3037 | 58400 | 8002674 | 2748386 |
| 45 | 35 | 0.2027 | 0.2506 | 0.2444 | 0.3037 | 56118 | 8531280 | 2939245 |
| 50 | 35 | 0.2036 | 0.2520 | 0.2432 | 0.3010 | 54105 | 9114922 | 3148121 |

A total cost curve is plotted for the trial as shown in fig 4.27 for varying production run size of product $A$ and constant production run size of product $B$ under the space constraint of 50 sqft .


Figure 4.27 Total Cost plot for fix $Q_{B}$ and varying $Q_{A}$ for lower space constraints.

Figure 4.27 shows the total cost for fix production run size of product $B$ and varying production run size of product $A$. It can be observed that with increase in production run size of product $A$ the total cost of the system increases. This is due to increase in the holding cost. Also it is observed that after production run size of 35 units of product A the total cost of the system increases compared to total cost of the system with greater space available of 100 sqft due to redistribution cost incurred for excess inventory of lower priority product A. Since the product A is lower priority product it would be economically viable to redistribute the excess of product A instead of switching or holding them in the inventory even after crossing the production run size for product A. The increase in total cost of the system is due to additional redistribution cost incurred.

The second trial of analysis is run by setting the space constraint at 50 sqft . The production run size of product A is set fix and production run size of Product B was varied from 10 units to 50 units. All other parameter of the system is kept unchanged. The results of trial are tabulated in table 4.10.

Table 4.10 Results of Cost Analysis for Fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for reduced space constraint.

| A | B | Situ 1 | Situ 2 | Situ 3 | Situ 4 | SETUP <br> COST | HOLDING <br> COST | TOTAL <br> COST |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 35 | 10 | 0.2278 | 0.3139 | 0.1926 | 0.2654 | 145064.1 | 5644325 | 2435022 |
| 35 | 15 | 0.1923 | 0.2643 | 0.2288 | 0.3144 | 106892.9 | 5678441 | 2258226 |
| 35 | 20 | 0.1927 | 0.2548 | 0.2379 | 0.3145 | 87531.35 | 5914149 | 2238327 |
| 35 | 25 | 0.1957 | 0.2513 | 0.2420 | 0.3108 | 77830.88 | 6308671 | 2305098 |
| 35 | 30 | 0.1981 | 0.2490 | 0.2449 | 0.3078 | 68186.81 | 6874016 | 2420850 |
| 35 | 35 | 0.2000 | 0.2472 | 0.2472 | 0.3055 | 61007.14 | 7530512 | 2573793 |
| 35 | 40 | 0.2015 | 0.2457 | 0.2490 | 0.3037 | 56409.09 | 8089661 | 2711908 |
| 35 | 45 | 0.2027 | 0.2444 | 0.2506 | 0.3037 | 51599.7 | 8835241 | 2900690 |
| 35 | 50 | 0.2036 | 0.2432 | 0.2520 | 0.3010 | 47628.42 | 9616208 | 3102693 |

A total cost curve was plotted for the trial shown in fig 4.28 varying size of product $B$ and constant production run size of product $A$ under space constraint of 50 sqft.


Figure 4.28 Total Cost plot for fix $\mathrm{Q}_{\mathrm{A}}$ and varying $\mathrm{Q}_{\mathrm{B}}$ for lower space constraints

Figure 4.28 shows the total cost for fix production run size of product $B$ and varying production run size of higher priority product $B$. It can be observed that with smaller space constraint the total cost of the system for varying production run size of product B remains unchanged as there is no redistribution of higher priority product B resulting in no extra cost.

## CHAPTER 5

## CONCLUSIONS AND DISCUSSIONS

A stochastic production cost model for remanufacturing system of returned products in reverse logistics is developed, which was one of the objective of this research. The numerical validation and sensitivity analysis of the model under different situations led to several interesting conclusions. It is observed that the products with higher priority have prominent influence in determining an optimal solution for decision parameters like production run size, production rate and space required for the total cost model. The influence of production rate on total cost of the system is evaluated by varying production rate for different types of products. The results from the analysis concluded that for larger production run size of lower priority products, the simultaneous increase in production rate of both the products would result in lower total cost for the system compared to total cost of system with greater in production rate of only one product or lower production rate for both the products. For larger production run size of higher priority products it is found that for system with greater production rate for both the products would result in lower cost compared to other system. Thus it can be concluded that for large production run size of both type of products, the simultaneous increase in production rate of both production run sizes of lower priority products, the increase in production rate of both the
products would result in the lowest cost for the system. It is interesting to find that for smaller production run sizes of higher priority product, the increase in production rate of lower priority product would result in the lesser total cost for the system compared to system with simultaneous increase in production rate of both the products. The optimal solution for total cost of remanufacturing system can be obtained by simultaneous optimization of parameters like production run size, production rate and space. It is recommended to use global search technique for multi parameter optimization to determine optimal total cost of the remanufacturing system for reverse logistics. The future research directions involves following. The model of stochastic product return developed for remanufacturing system is base on probability distribution function of compound poisson process. The formula derived for probability distribution of compound poisson process contains recursive function rendering it almost useless for calculating higher values of the decision parameters. Therefore it is recommended in future research to obtain a simple function that can replace the recursive function in formula for probability distribution of compound poisson process. This will simplify the algorithm for determining probabilities for each situation. Also the total cost model for remanufacturing system of return products is a component of total cost model of complete reverse logistics system. It would be a great boon for remanufacturers if they are made available with total cost model for complete reverse logistics system, right from consumer return to resale of remanufactured product. The total cost model for complete reverse logistics system will provide remanufacturers greater visibility of entire system for cost control. To determine optimal total cost of the
remanufacturing system in reverse logistics, simultaneous optimization of decision parameters of production run size, production rate and space constraint are required. The review of optimization methods divulge that multi parameter simultaneous optimization can be achieved by global search techniques like evolutionary algorithms, tabu search method, gradient surface method, simulated annealing etc. It is found that stochastic search method like genetic search algorithm that is adaptive heuristic search algorithm that operates on principle of the survival of the fittest over consecutive generation to produce best approximation of the solution is best suited for optimizing the production cost model for remanufacturing system in reverse logistics. Also other advantages of using genetic search algorithm includes, G.A optimizes continuous as well as discrete parameters, Provides list of optimal parameters and not just one solution, Simultaneously searches from wide sample of cost surface and deals with large number of parameters. Therefore for multi parameter optimization of the production cost model for remanufacturing systems, genetic search algorithm method is recommended.

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## APPENDIX A

## APPENDIX A

## Visual Basic program for determining probabilities

## Sub mysub1()

Worksheets("Sheet2").Activate
Dim q 2 As Integer
Dim t As Double
Dim t_incr As Double
Dim rate As Double
Dim b As Integer
Dim j As Integer
Dim q_max As Integer
Dim t_max As Integer
Dim QConst As Integer
Dim tb0 As Double
$\mathrm{q}_{\_} \max =\mathrm{Q}_{\mathrm{A} / \mathrm{B}}$
$t \_\max =100$
For $\mathrm{q} 2=0$ To q_max
For $\mathrm{t}=0$ To q_max
Worksheets("Sheet3").Cells(q2 $+2, t+4)$.Value $=-1$
Next t
Next q2
$\mathrm{q} 2=\mathrm{q} \_$max
$\mathrm{t}=1$
t _incr $=\operatorname{Cells}(1,2)$
rate $=\operatorname{Cells}(1,3)$
Dim sum As Double
sum $=0$
For $\mathrm{j}=0$ To q_max
Worksheets("Sheet3").Cells $(j+2,2)$. Value $=\operatorname{probtb}(q 2, t$, rate $)$ ', $b$ )
$t=t+t$ incr
If $\mathrm{j}>=1$ Then
Dim w2 As Double
w2 = Worksheets("Sheet3").Cells $(j+2,2)$.Value
Dim w1 As Double
w1 = Worksheets("Sheet3").Cells $(\mathrm{j}+1,2)$.Value
Dim avg As Double
$\operatorname{avg}=((\mathrm{w} 2+\mathrm{w} 1) / 2) * \mathrm{t}_{-}$incr
End If
sum $=\operatorname{avg}+\operatorname{sum}$
Worksheets("Sheet3").Cells $(j+2,3) \cdot$ Value $=$ sum
Next ${ }^{j}$
End Sub

Function $\operatorname{probtb}(q, t$, rate $)$

Dim a(1000) As Double
For $\mathrm{i}=0$ To 1000
$a(i)=0$
Next i
$a(0)=0$
$\mathrm{a}(1)=0.2$
$\mathrm{a}(2)=0.2$
$a(3)=0.2$
$\mathrm{a}(4)=0.2$
$a(5)=0.2$
$a(6)=0$
$a(7)=0$
If $q=0$ Then
probtb $=\operatorname{Exp}(-1 *$ rate $* \mathbf{t} *(1-a(0)))$
Worksheets("Sheet3").Cells $(q+2, t+4) . V a l u e=p r o b t b$
If probtb $>1$ Or probtb $<0$ Then
MsgBox "problemooo00000"
End If
Else
Dim sum As Double
sum $=0$
Dim k As Integer
For $\mathrm{k}=0$ To $\mathrm{q}-1$

Dim tmp As Double
tmp $=$ Worksheets("Sheet 3 ").Cells $(k+2, t+4)$. Value
If $(\operatorname{tmp}=-1)$ Then
$\mathrm{tmp}=\operatorname{probtb}(\mathrm{k}, \mathrm{t}$, rate $)$
End If
$\operatorname{sum}=\operatorname{sum}+((q-k) * a(q-k)) * \operatorname{tmp}$
Next k
probtb $=($ sum $*$ rate $* t / q)$
Worksheets("Sheet3").Cells( $q+2, t+4)$.Value = probtb
If probtb $>1$ Or probtb $<0$ Then
MsgBox "problemmmmmo"
End If
End If
End Function

Visual Basic code for determining probabilities
Sub mysub5()
Worksheets("Sheet2").Activate
Dim q2 As Integer
Dim t As Double
Dim t_incr As Double
Dim rate As Double
Dim b As Integer

## Dim j As Integer

Dim q_max As Integer
Dim t_max As Integer
Dim QConst As Integer
Dim tb0 As Double
Dim intg As Double
$\mathrm{q} \_\max =\operatorname{Abs}\left((\operatorname{prb} * \mathrm{t})-\mathrm{Q}_{\mathrm{B} / \mathrm{A}}\right)$
$t_{\mathbf{\_}} \max =100$
For $\mathrm{q} 2=0$ To q_max
For $\mathrm{t}=0$ To $\mathrm{t} \_$max
Worksheets("Sheet2").Cells(q2 $+2, \mathrm{t}+4) \cdot$ Value $=-1$
Next t
Next q2
$\mathrm{q} 2=\mathrm{q}$ _ max
$\mathrm{t}=1$
t_incr $=$ Cells $(1,2)$
rate $=\operatorname{Cells}(1,3)$
Dim sum As Double
sum $=0$
For $\mathrm{j}=0$ To t_max
Worksheets("Sheet2").Cells $(\mathrm{j}+2,2)$. Value $=\operatorname{probtb}(\mathrm{q} 2, \mathrm{t}$, rate $)$ ', b$)$

$$
\mathrm{t}=\mathrm{t}+\mathrm{t} \text { incr }
$$

## If $\mathrm{j}>=1$ Then

Dim w2 As Double
$\mathrm{w} 2=$ Worksheets("Sheet2").Cells $(\mathrm{j}+2,2)$.Value
Dim w1 As Double
w1 = Worksheets("Sheet2").Cells( $\mathrm{j}+1,2$ ).Value
Dim avg As Double
$\operatorname{avg}=((w 2+w 1) / 2) * t$ incr
End If
sum $=(\operatorname{avg}+\operatorname{sum})$
Worksheets("Sheet2").Cells(j+2,3).Value = sum
Next ${ }^{j}$
End Sub

Function $\operatorname{probtb}(\mathrm{q}, \mathrm{t}$, rate)
Dim a(1000) As Double
For $\mathrm{i}=0$ To 1000

$$
a(i)=0
$$

Nexti
$a(0)=0$
$\mathrm{a}(1)=0.2$
$\mathrm{a}(2)=0.2$
$\mathrm{a}(3)=0.2$
$\mathrm{a}(4)=0.2$
$a(5)=0.2$
$a(6)=0$
$a(7)=0$
If $q=0$ Then
probtb $=\operatorname{Exp}(-1 *$ rate $* t *(1-a(0)))$
Worksheets("Sheet2").Cells $(q+2, t+4)$.Value $=$ probtb
If probtb $>1$ Or probtb $<0$ Then
MsgBox "problemoooooooo"
End If
Else
Dim sum As Double
sum $=0$
Dim k As Integer
For $\mathrm{k}=0$ To $\mathrm{q}-1$
Dim tmp As Double
$\operatorname{tmp}=$ Worksheets("Sheet2").Cells(k+2,t+4).Value
If ( $\mathrm{tmp}=-1$ ) Then
$\operatorname{tmp}=\operatorname{probtb}(k, t$, rate $)$
End If
$\operatorname{sum}=\operatorname{sum}+((\mathrm{q}-\mathrm{k}) * \mathrm{a}(\mathrm{q}-\mathrm{k})) * \operatorname{tmp}$
Next k

$$
\text { probtb }=(\text { sum } * \text { rate } * t / q)
$$

Worksheets("Sheet2").Cells $(q+2, t+4) \cdot$ Value $=$ probtb
End If
End Function

## APPENDIX B

## APPENDIX B

## Visual Basic Program for Cost Calculations

Sub mysubtc()<br>Worksheets("Sheet1").Activate<br>Dim ca As Integer<br>Dim cb As Integer<br>Dim ar As Double<br>Dim pra As Double<br>Dim prb As Double<br>Dim qa As Double<br>Dim qb As Double<br>Dim buf As Double<br>Dim tqa As Double<br>Dim tqb As Double<br>Dim uta As Integer<br>Dim utb As Integer<br>Dim usa As Integer<br>Dim usb As Integer<br>Dim setco As Integer<br>Dim lbhrsa As Double<br>Dim lbhrsb As Double<br>Dim lbcohrs As Integer

Dim nw As Integer
Dim rsalea As Integer
Dim rsaleb As Integer
Dim tbo As Integer
Dim tao As Integer
Dim utoprco As Integer
Dim spacoccco As Integer
Dim transp As Integer
Dim ncycles 12 As Double
Dim ncycles34 As Double
Dim nc As Double
Dim invt As Double
Dim excesinvcond As Double
$\mathrm{ca}=100$
$\mathrm{cb}=250$
ar $=3$
pra $=5$
$\mathrm{prb}=5$
$\mathrm{qa}=25$
$\mathrm{qb}=35$
buf $=4$
$\operatorname{tqa}=(\mathrm{qa} / \mathrm{ar} * 0.2 * 2)$
$\operatorname{tqb}=(\mathrm{qb} / \mathrm{ar} * 0.2 * 2)$

```
\(u t a=2\)
\(u t b=3\)
usa \(=2\)
usb \(=1\)
setco \(=500\)
lbhrsa \(=1\)
lihrsb \(=2\)
lbcohrs \(=6\)
\(n w=10\)
rsalea \(=300\)
rsaleb \(=650\)
tbo \(=\mathrm{qb} / \mathrm{prb}\)
tao \(=\mathrm{qa} / \mathrm{pra}\)
utoprco \(=25\)
spacoccco \(=20\)
transp \(=50\)
ncycles \(12=365 /(\mathrm{tqa}+\mathrm{tqb})\)
ncycles \(34=365 /(\mathrm{tqb}+\mathrm{tbo})\)
excesinvcond \(=\operatorname{Abs}\left(\left(\left((\operatorname{ar} * \operatorname{tqa} * \mathrm{usa})+(\mathrm{qb}-(\mathrm{prb} * \mathrm{tbo}))^{*} \mathrm{usb}\right)>30\right) \operatorname{Or}(((\mathrm{ar} * \operatorname{tqb} *\right.\)
\(\mathrm{usb})+(\mathrm{qa}-(\mathrm{pra} * \mathrm{tao})) * \mathrm{usa})>30))\)
Worksheets("Sheet1").Cells(2, 2).Value = tqa
Worksheets("Sheet1").Cells(3, 2).Value = tqb
Worksheets("Sheet1").Cells(4, 2).Value = tbo
```

Worksheets("Sheet1").Cells(5, 2).Value = tao
Worksheets("Sheet1").Cells(6, 2).Value = ncycles 12
Worksheets("Sheet1").Cells(7, 2).Value = ncycles34
Worksheets("Sheet1").Cells(8, 2).Value = excesinvcond
Dim detora As Double
Dim detorb As Double
detora $=0.02 * \operatorname{tqa} * \mathrm{ca}$
detorb $=0.03 * \operatorname{tqb} * \mathrm{cb}$
Worksheets("Sheet1").Cells(2, 5).Value = detora
Worksheets("Sheet1").Cells(3, 5).Value = detorb
Dim mtrlhana As Double
Dim mtrlhanb As Double
mtrlhana $=$ uta $*$ utoprco
mtrlhanb $=$ utb * utoprco
Worksheets("Sheet1").Cells(4, 5).Value = mtrlhana
Worksheets("Sheet1").Cells(5, 5).Value = mtrlhanb
Dim opporlossa As Double
Dim opporlossb As Double
opporlossa $=0.2 * \mathrm{ca}$
opporlossb $=0.2 * \mathrm{cb}$
Worksheets("Sheetl").Cells(7, 5).Value = opporlossa
Worksheets("Sheet1").Cells(8, 5).Value = opporlossb

Dim spaceoccosta As Double
Dim spaceoccostb As Double
spaceoccosta $=$ usa $*$ spacoccco
spaceoccostb $=$ usb * spacoccco
Worksheets("Sheet1").Cells(9, 5).Value = spaceoccosta
Worksheets("Sheet1").Cells(10, 5).Value = spaceoccostb
Dim holdcosta As Double
Dim holdcostb As Double
holdcosta $=($ detora + mtrlhana + opporlossa + spaceoccosta $)$
holdcostb $=($ detorb + mtrlhanb + opporlossb + spaceoccostb $)$
Worksheets("Sheet1").Cells(11, 5).Value = holdcosta
Worksheets("Sheet1").Cells(12, 5).Value = holdcostb
Dim manfcosta As Double
Dim manfcostb As Double
manfcosta $=($ lbhrsa $* 6+$ usa $*$ utoprco $)$
manfcostb $=(\mathrm{lbhrsb} * 6+$ usb * utoprco $)$
Worksheets("Sheet1").Cells(13, 5).Value = manfcosta
Worksheets("Sheet1").Cells(14, 5).Value = manfcostb
Dim setupcost12 As Double
setupcost12 $=(2 *$ ncycles $12 *$ setco $)$
Worksheets("Sheet1").Cells(15, 5).Value = setupcost12
Dim setupcost34 As Double
setupcost34 $=(2 *$ ncycles $34 *$ setco $)$

Worksheets("Sheet1").Cells(15, 6).Value $=$ setupcost 34

Dim totalsetupcost As Double
totalsetupcost $=(\operatorname{Cells}(15,5)+\operatorname{Cells}(15,6))$
Worksheets("Sheet1").Cells(15, 7).Value = totalsetupcost
Dim lossopprcost As Double
lossopprcost $=($ rsalea $-(\mathrm{ca}+$ holcosta $))$
Worksheets("Sheet1").Cells(16, 5).Value = lossopprcost
If excesinvcond $=1$ Then
Dim excesinv As Double
excesinv $=\operatorname{Abs}($ pra $*$ usa - ar * usa $/$ usb $)$
Worksheets("Sheet1").Cells(9, 2).Value = excesinv
Else
excesinv $=0$
End If
Dim redistribcost As Double
redistribcost $=\operatorname{excesin} v *(\operatorname{transp}+$ lossopprcost $)$
Worksheets("Sheet1").Cells(17, 5).Value = Abs(redistribcost)
Dim areaoftranga1 As Double
areaoftrangal $=(\mathrm{tqa} *(\mathrm{qa} / 2)+(\mathrm{qa} / 2) * \mathrm{tqb})$
Worksheets("Sheet1").Cells(18, 5).Value = areaoftrangal
Dim areaoftrangb1 As Double
areaoftrangb1 $=((\mathrm{qb} / 2) * \operatorname{tqa}+(q b / 2) *$ tbo $)$

Worksheets("Sheet1").Cells(19, 5).Value = areaoftrangbl
Dim areaoftranga2 As Double
areaoftranga2 $=((\mathrm{qa} / 2) *(\mathrm{tqa}+\operatorname{tqb})+\mathrm{buf} *(\mathrm{tqa}+\mathrm{tqb}))$
Worksheets("Sheet1").Cells(20, 5).Value = areaoftranga2
Dim areaoftrangb2 As Double
areaoftrangb2 $=(\mathrm{qb} / 2 *(\mathrm{tbo}+\mathrm{tqb}))$
Worksheets("Sheet1").Cells(21, 5).Value = areaoftrangb2
Dim areaoftranga3 As Double
areaoftranga3 $=((\operatorname{ar} *(\mathrm{tbo}-\mathrm{tqa})+\mathrm{qa} / 2 *(\mathrm{tao}+\mathrm{tqa})))$
Worksheets("Sheet1").Cells(22, 5).Value = areaoftranga3
Dim areaoftrangb3b As Double
areaoftrangb3b $=(($ tbo +tqb$) *(\mathrm{qb} / 2))$
Worksheets("Sheet1").Cells(23, 5).Value = areaoftrangb3b
Dim areaoftranga4 As Double
areaoftranga4 $=((\operatorname{ar} *(\mathrm{tbo}-\mathrm{tqa})+(\mathrm{qa} / 2) *(\mathrm{tbo}+\mathrm{tqa})+\mathrm{buf} *(\mathrm{tbo}+\mathrm{tqb})))$
Worksheets("Sheet1").Cells(24, 5).Value = areaoftranga4
Dim areaoftrangb4 As Double
areaoftrangb4 $=((\mathrm{qb} / 2) *(\mathrm{tbo}+\mathrm{tqb}))$
Worksheets("Sheet1").Cells(25, 5).Value = areaoftrangb4
Dim holdcostofAforsitu1 As Double
holdcostofAforsitu $1=$ areaoftranga1 $*$ holdcosta $*$ ncycles 12
Worksheets("Sheet1").Cells(28, 5).Value = holdcostofAforsitu1

Dim holdcostofAforsitu2 As Double
holdcostofAforsitu2 $=$ areaoftranga $2 *$ holdcosta $*$ ncycles 12
Worksheets("Sheet1").Cells(29, 5).Value = holdcostofAforsitu2
Dim holdcostofAforsitu3 As Double
holdcostofAforsitu3 $=$ areaoftranga3 $*$ holdcosta $*$ ncycles 34
Worksheets("Sheet1").Cells(30, 5).Value = holdcostofAforsitu3
Dim holdcostofAforsitu4 As Double
holdcostofAforsitu4 $=$ areaoftranga $4 *$ holdcosta $*$ ncycles 34
Worksheets("Sheet1").Cells(31, 5).Value = holdcostofAforsitu4 Dim holdcostofBforsitu1 As Double
holdcostofBforsitu1 $=$ areaoftrangb1 $*$ holdcosta $*$ ncycles 12
Worksheets("Sheet1").Cells(32, 5).Value = holdcostofBforsitu1
Dim holdcostofBforsitu2 As Double
holdcostofBforsitu2 $=$ areaoftrangb2 $*$ holdcosta $*$ ncycles 12
Worksheets("Sheet1").Cells(33, 5).Value = holdcostofBforsitu2
Dim holdcostofBforsitu3 As Double
holdcostofBforsitu3 $=$ areaoftrangb3b * holdcosta $*$ ncycles 34
Worksheets("Sheet1").Cells(34, 5).Value = holdcostofBforsitu3
Dim holdcostofBforsitu4 As Double
holdcostofBforsitu4 $=$ areaoftrangb4 $*$ holdcosta $*$ ncycles34
Worksheets("Sheet1").Cells(35, 5).Value = holdcostofBforsitu4
Dim totalholdcost As Double
totalholdcost $=(\operatorname{Cells}(28,5)+\operatorname{Cells}(29,5)+\operatorname{Cells}(30,5)+\operatorname{Cells}(31,5)+\operatorname{Cells}(32,5)+$ $\operatorname{Cells}(33,5)+\operatorname{Cells}(34,5)+\operatorname{Cells}(35,5))$

Worksheets("Sheet1").Cells(31, 8).Value = totalholdcost
Dim remanufcostforAsitu1 As Double
remanufcostforAsitul $=$ areaoftrangal $*$ manfcosta $*$ ncycles 12
Worksheets("Sheet1").Cells(37, 5).Value = remanufcostforAsitu1
Dim remanufcostforAsitu2 As Double
remanufcostforAsitu2 $=$ areaoftranga $2 *$ manfcosta $*$ ncycles 12
Worksheets("Sheet1").Cells(38, 5).Value = remanufcostforAsitu2
Dim remanufcostforAsitu3 As Double
remanufcostforAsitu3 $=$ areaoftranga $3 *$ manfcosta $*$ ncycles 34
Worksheets("Sheet1").Cells(39, 5).Value = remanufcostforAsitu3
Dim remanufcostforAsitu4 As Double
remanufcostforAsitu4 $=$ areaoftranga $4 *$ manfcosta $*$ ncycles34
Worksheets("Sheet1").Cells(40, 5).Value = remanufcostforAsitu4
Dim remanufcostforBsitu1 As Double
remanufcostforBsitu1 $=$ areaoftrangb1 $*$ manfcostb $*$ ncycles 12
Worksheets("Sheet1").Cells(41, 5).Value = remanufcostforBsitu1
Dim remanufcostforBsitu2 As Double
remanufcostforBsitu2 $=$ areaoftrangb2 $*$ manfcostb $*$ ncycles 12
Worksheets("Sheet1").Cells(42, 5).Value = remanufcostforBsitu2
Dim remanufcostforBsitu3 As Double
remanufcostforBsitu3 $=$ areaoftrangb3b * manfcostb * ncycles34

```
Worksheets("Sheet1").Cells(43, 5).Value = remanufcostforBsitu3
Dim remanufcostforBsitu4b As Double
remanufcostforBsitu \(4 \mathrm{~b}=\) areaoftrangb4 \(*\) manfcostb \(*\) ncycles 34
Worksheets("Sheet1").Cells(44, 5).Value = remanufcostforBsitu4b
Dim subtotalcostforsitu1 As Double
subtotalcostforsitu1 = holdcostofAforsitu1 + holdcostofBforsitu1 + remanufcostforAsitu1
+ remanufcostforBsitu1 + setupcost12 + redistribcost
Worksheets("Sheet1").Cells(46, 5).Value = subtotalcostforsitu1
Dim subtotalcostforsitu2 As Double
subtotalcostforsitu2 \(=\) holdcostofAforsitu2 + holdcostofBforsitu2 + remanufcostforAsitu2
+ remanufcostforBsitu \(2+\) setupcost \(12+\) redistribcost
Worksheets("Sheet1").Cells(47, 5).Value = subtotalcostforsitu2
Dim subtotalcostforsitu3 As Double
subtotalcostforsitu 3 = holdcostofAforsitu \(3+\) holdcostofBforsitu \(3+\) remanufcostforAsitu 3
+ remanufcostforBsitu3 + setupcost34 + redistribcost
Worksheets("Sheet1").Cells(48, 5).Value = subtotalcostforsitu3
Dim subtotalcostforsitu4 As Double
subtotalcostAforsitu4 \(=\) holdcostofAforsitu4 + holdcostofBforsitu \(4+\)
remanufcostforAsitu4 + remanufcostforBsitu4 + setupcost34 + redistribcost
Worksheets("Sheet1").Cells(49, 5).Value = subtotalcostAforsitu4
Dim totalprobability1 As Double
totalprobability \(=((\operatorname{Cells}(2,8)) * \operatorname{Cells}(3,8))\)
Worksheets("Sheet1").Cells(2, 10).Value = totalprobability1
```

Dim probability1forsitu2 As Double
probability1forsitu2 $\left.=(\operatorname{Cells}(2,8))^{\prime} * \operatorname{Cells}(3,8)\right)$
Worksheets("Sheet1").Cells(5, 8).Value = probability1forsitu2
Dim probability2forsitu2 As Double
probability2forsitu2 $\left.=((1-(\operatorname{Cells}(3,8)))){ }^{\prime} * \operatorname{Cells}(3,8)\right)$
Worksheets("Sheet1").Cells(6, 8).Value = probability2forsitu2
Dim totalprobability2 As Double
totalprobability2 $=(\operatorname{Cells}(5,8) * \operatorname{Cells}(6,8))$
Worksheets("Sheet1").Cells(5, 10).Value = totalprobability2
Dim probability1forsitu3 As Double
probability1forsitu3 $=((1-\operatorname{Cells}(2,8)))$
Worksheets("Sheet1").Cells(8, 8).Value = probability1forsitu3
Dim probability2forsitu3 As Double
probability2forsitu3 $=(\operatorname{Cells}(3,8))$
Worksheets("Sheet1").Cells(9, 8).Value = probability2forsitu3
Dim totalprobability3 As Double
totalprobability $3=(\operatorname{Cells}(8,8) * \operatorname{Cells}(9,8))$
Worksheets("Sheet1").Cells(8, 10).Value = totalprobability3
Dim probability1forsitu4 As Double
probability1forsitu4 $=(1-((\operatorname{Cells}(2,8))))$
Worksheets("Sheet1").Cells(11, 8).Value = probability1forsitu4

```
Dim probability2forsitu4 As Double
probability2forsitu4 = ((1-Cells(3, 8)))
Worksheets("Sheet1").Cells(12, 8).Value = probability2forsitu4
Dim totalprobability4 As Double
totalprobability4 =(Cells(11,8)* Cells(12, 8))
Worksheets("Sheet1").Cells(11, 10).Value = totalprobability4
Dim totalcostforsitu1 As Double
totalcostforsitu1 = ((Cells(2,10)*Cells(46,5)) + setupcost12)
Worksheets("Sheet1").Cells(18, 10).Value = totalcostforsitu1
Dim totalcostforsitu2 As Double
totalcostforsitu2 = ((Cells(5,10)*Cells(47, 5)) + setupcost12)
Worksheets("Sheet1").Cells(20, 10).Value = totalcostforsitu2
Dim totalcostforsitu3 As Double
totalcostforsitu3 = ((Cells(8,10) * Cells(48, 5)) + setupcost34)
Worksheets("Sheet1").Cells(22, 10).Value = totalcostforsitu3
Dim totalcostforsitu4 As Double
totalcostforsitu4 =((Cells(11, 10)*Cells(49,5)) + setupcost34)
Worksheets("Sheet1").Cells(24, 10).Value = totalcostforsitu4
Dim finaltotalcost As Double
finaltotalcost = (Cells(18,10)+Cells(20,10) + Cells(22,10)+Cells(24, 10))
Worksheets("Sheet1").Cells(26, 10).Value = finaltotalcost
End Sub
```


## VITA

Gaurang. S. Patel, was born in Ahmedabad, a metro city in India on August $30^{\text {th }} 1979$. He obtained his Bachelors in Mechanical Engineering from University of Bombay, India in year 2002. He worked as industrial engineer in railway track manufacturing firm for sixteen months. His quest for higher education brought him to Graduate school at The University of Texas Pan American in Manufacturing Engineering.

