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CAPM and Irrational Market: Theories and Empirical Studies

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CAPM AND IRRATIONAL MARKET:

THEORIES AND EMPIRICAL STUDIES

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ABSTRACT

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With a new interpretation of the Capital Asset Pricing Model (CAPM), this dissertation explains that the CAPM has an implied assumption of no mispricing. The CAPM should work if mispricing is removed from all assets in the market, leading to the Rational CAPM. The Rational CAPM measures value changes of the market, and then yields value changes of an asset/portfolio by employing value changes of the market in the model. For the Rational CAPM, this dissertation explains that risk, mispricing, irrationality and (investor) sentiment all indicate the same. Using the value changes from the Rational CAPM and historical dividend yields, this dissertation demonstrates how to measure values and bubbles in historical data, leading to the Ex-post Bubble Model (EBM). By the EBM, this dissertation reveals how irrationality forms, develops and reduces bubbles. Based on the Rational CAPM and the EBM, empirical studies are conducted into the U.S. stock market to verify the validity of these models. The results from the U.S. stock market provide the evidence that both Rational CAPM and EBM work with historical data.

DEDICATION

This dissertation is dedicated to my family. Without the love and support of my family, this dissertation could not have been completed. My thanks and appreciations go to my mother, Bakhapja Lee, and my father, Gapchul Koh, for their endless supports. My wife, Jeongyun Jang, my daughters, Kyungyeon and Kyooyeon, and my son, Kyungchan, have never left my side and wholeheartedly supported and encouraged me by all means to complete this degree. My special thanks go to my brothers, Jaeho and Jaesung, and my sister, Mijin. Thank you for your love, support and patience.

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CHAPTER I

INTRODUCTION AND TERMINOLOGY

1.1 Introduction

1.1.1 Research Questions and Objectives

Based on the portfolio selection theory of Markowitz (1952), the Capital Asset Pricing Model (CAPM) of Sharpe (1964) gave birth to asset pricing theories, which led both Markowitz and Sharpe to share the Nobel Prize in Economics with Merton Miller in 1990. The CAPM is theoretically derived and proven. However, "the empirical record of the model is poor – poor enough to invalidate the way it is used in applications" (Fama and French, 2004). Here is my first research question; ***why has the CAPM worked poorly in the empirical studies?*** Is the CAPM a wrong model for reality? Has the CAPM been misused in the empirical studies?

The burst of Internet bubble stunned the market in the early 2000s. A bubble is used to indicate overpricing followed by a sudden collapse in price (Kindleberger, 2000; Brunnermeier, 2007), and to describe boom and crash in the price of a certain sector during a certain period (Siegel, 2003; Simon, 2003). We can then recognize a bubble only after a collapse in price. Here is my second research question; ***why is a bubble identified only after its burst?*** Is there any way to measure a bubble at any time, i.e., before its collapse or during its formation?

These two research questions are seemingly unrelated to each other. However, they are closely related. The first question is about pricing while the second one is related to mispricing. If the CAPM is the correct model, i.e., correctly pricing an asset/portfolio, we can price an asset correctly using the CAPM. Then, we can identify how much an asset is mispriced, i.e., how much an asset contains bubbles in its price.

Unlike previous works testing the validity of the CAPM using historical data, this dissertation elaborates on a new interpretation of the CAPM to find why the CAPM does not work with historical data. Unlike previous studies that define a bubble as overpricing followed by a sudden collapse, this dissertation defines a bubble as the deviation of the price of an asset from its value in order to measure a bubble at any time. Then, measuring a value plays a crucial role in measuring a bubble because prices can be obtained from the market. Unlike previous studies that measures a value using a stream of future dividends and discount rates, this dissertation measures a value using historical data based on the new interpretation of the CAPM.

The objectives of this dissertation are twofold. The first one is to theoretically explain how to use the CAPM in empirical studies, resulting in the Rational CAPM, and how to measure a value and a bubble using historical data, introducing the Ex-post Bubble Model (EBM). The second one is to empirically verify, using the U.S. stock market, that both Rational CAPM and EBM work with historical data.

1.1.2 Contributions and Findings

This dissertation contributes to the existing literature in the following ways. First, this dissertation introduces the Rational CAPM, a new interpretation of the CAPM. Second, this dissertation develops the EBM, the new bubble model with ex-post valuation. Third, this dissertation empirically measures and examines irrationality, value and bubble based on the Rational CAPM and the EBM.

The theoretical approach herein answers two research questions. First, the CAPM has been misused. The CAPM does not hold with the price data that contain bubbles created by investors' irrationality. The CAPM has an implied assumption that investors are perfectly rational, and thus do not misprice any asset. Second, bubbles can be detected before burst if they can be measured at any time. If the Rational CAPM works, value changes can be assessed, and then bubbles can be measured and detected before their burst. The empirical results from the U.S. stock market support both Rational CAPM and EBM.

1.1.3 Dissertation Structure

The rest of this dissertation is organized as follows. The following section, Section 1.2, explains the terminology and definitions used in this dissertation. Chapters II and III explain and examine the Rational CAPM and the EBM, respectively.

Chapters II and III are structured in the same format as follows. Sections 2.1 and 3.1 introduce Chapters II and III, respectively. Sections 2.2 and 3.2 review the literature while Sections 2.3 and 3.3 analyze the literature for each chapter. Sections 2.4 and 3.4 explain the Rational CAPM and the EBM, respectively. For each chapter, Sections 2.5 and 3.5 explain the methodologies and describe the sample, and Sections 2.6 and 3.6 discuss empirical findings. Sections 2.7 and 3.7 summarize and conclude each chapter.

1.2 Terminology and Definitions

1.2.1 Subscript t, Superscripts P and V, and Notation E( )

Subscript t for each variable represents "at the end of Period *t*" for a stock variable such as P denoting price, or "during Period *t*" for a flow variable such as D denoting dividends. For example, P_t means the price at the end of Period *t* while D_t represents the dividends paid out during Period *t*. Superscripts P and V denote variables using price data and value data, respectively. For example, R^P and R^V denote returns measured by price data and value data, respectively (See Sections 1.2.4, 1.2.5, 1.2.7 and 1.2.9).

E( ) denotes an ex-ante expectation in the ex-ante model while an ex-post mean in the ex-post model. For example, with R denoting returns, $E(R_t)$ in the ex-ante model represents ex-ante expected returns weighted by probabilities of expected outcomes during Period *t* assuming any previous information incorporated. In addition, $E_t(R_{t+j})$ denotes an expected returns during Period *t+j* given the information set at the end of Period *t*, and thus $E(R_t)$ can be considered to indicate $E_{t-1}(R_t)$. On the other hand, $E(R_m)$ with ex-post data for Period *t*, where *m* denotes sub-periods of Period *t*, indicates an ex-post mean of R_m weighted by 1/N, where N represents the number of sub-periods, during Period *t*. In the CAPM, $E(R_t)$ represents "systematic" returns, returns expected by the model.

1.2.2 Price and Value

Price is the market price, which can be observed in the market and thus directly obtained from the market. Hence, price is objective, i.e., the same to all investors. Value means fundamental, intrinsic or true value, which cannot be observed in the market and thus should be measured by a model. Hence, value is subjective, i.e., different to investors depending on the model used. Even though the same model is used, the value can be different depending on the input estimates used. Both price and value are assumed to be always positive in this dissertation.

The price of a stock is a good estimate of its value according to the random walk theory and the efficient market hypothesis (Fama, 1965; Malkiel, 2003). However, in reality, there are usually discrepancies between them because it takes time for price to adjust to value when measuring a value is difficult and/or trading an asset is costly (Lee, et al., 1999). When price diverges from value, measuring a value and a bubble is significant in that it allows investors to judge the extent to which a stock is overpriced or underpriced.

1.2.3 Bubble and Relative Bubble

To measure a bubble at any time, this dissertation defines a bubble as the deviation of the price of an asset from its value at a specific time. Then, a bubble includes any mispriced portion of an asset irrespective of magnitude and direction; there can be a negative bubble (i.e., underpriced asset), or a very small bubble. Hence, in this dissertation, a bubble is defined as follows.

$$B_t = P_t - V_t, \tag{1.1}$$

where B, P and V denote bubble, price and value, respectively (See Section 1.2.1 for Subscript *t*, hereafter). In fact, Equation (1.1) has been used to define and measure bubbles in previous studies; for example, the Rational Bubble Model (West, 1987; Flood and Hodrick, 1990) and the Intrinsic Bubble Model (Froot and Obstfeld, 1991) (See Sections 3.2.1.and 3.2.2 for more details). As seen in Equation (1.1), measuring bubbles depends on how to measure values because prices can be obtained from the market as explained in Section 1.2.2. Unlike the Rational and Intrinsic Bubble Models, this dissertation makes no restriction on a bubble; a bubble can be positive, zero or negative in this dissertation.

In addition, a bubble can be expressed by the ratio of bubble to price for comparison, and then,

$$\Omega_t = \frac{B_t}{P_t} = \frac{P_t - V_t}{P_t} = 1 - \frac{V_t}{P_t}, \tag{1.2}$$

where Ω represents a relative bubble. Ω is less than unity at any time by the assumption that price and value are greater than zero. The values of Ω are not symmetric. For upside, Ω is always less than unity. For downside, however, Ω has no limit.

1.2.4 Dividends, Returns and Growth

Some stocks pay dividends while others do not. In this dissertation, dividends mean cash payout to shareholders; stock dividends and stock repurchases are not included in dividends but reflected in the changes in price and value. I assume that values of dividends are the same as their prices because dividends are paid by money, which has no bubble (Tirole, 1985). Then, dividends reduce both price and value by the same amount as dividends paid. Therefore, dividends do not affect the size of a bubble, B, but affect a relative bubble, Ω.

There are two types of measures for changes in price and value: one including dividends and the other excluding dividends, and they are called return and growth, respectively. Returns on price and value are respectively defined as follows.

$$R^P_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}}, \tag{1.3}$$

$$R^V_t = \frac{V_t - V_{t-1} + D_t}{V_{t-1}}, \tag{1.4}$$

where R^P and R^V are named as price return and value return, respectively, and D denotes dividends per share. Equations (1.3) and (1.4) are based on the assumption that for dividends, values are the same as prices. Notice that return measures include dividends. Growth measures can be obtained if dividends are excluded from the return measures. In other words, "growth" means the rate of changes.

$$G^P_t = \frac{P_t - P_{t-1}}{P_{t-1}}, \tag{1.5}$$

$$G^V_t = \frac{V_t - V_{t-1}}{V_{t-1}}, \tag{1.6}$$

where G^P and G^V are named as price growth and value growth, respectively. R^P, R^V, G^P and G^V are greater than minus one (–1) at any time by the assumption that both price and value are greater than zero. R^P and R^V are greater than [the same as] G^P and G^V, respectively, if dividends are paid [not paid].

1.2.5 Dividend Yield

Dividend yield means a dividend income relative to investment, and thus it measures how much cash flows investors earn from their investment in the absence of capital gains. In the dividend-price ratio, dividends are compared to an actual investment, i.e., price. However, if we know the value of assets, investment can be measured by the correct investment without over- or under-payment. Then, the dividend yield can be computed by the dividend-value ratio. This dissertation uses two types of dividend yield:

$$\delta^P_t = \frac{D_t}{P_{t-1}}, \tag{1.7}$$

and

$$\delta^V_t = \frac{D_t}{V_{t-1}}, \tag{1.8}$$

where δ^P and δ^V denote dividend-price ratio and dividend-value ratio, respectively. Equations (1.3) and (1.4) can be related to Equations (1.5) and (1.6), respectively, using dividend yields:

$$R^P_t = G^P_t + \delta^P_t, \tag{1.9}$$

$$R^V_t = G^V_t + \delta^V_t, \tag{1.10}$$

1.2.6 Irrationality: Basis of a Bubble

What allows a bubble to form? Shiller (2005) emphasized that “irrational exuberance” inflated a bubble. Here, “irrational exuberance” results from a composite of various kinds of behavioral or psychological biases, e.g., herd behavior, epidemics, overconfidence, intuitive judgment, etc.

Some previous studies document other features to create or inflate a bubble. Ofek and Rechardson (2003) argue that short-sale constraints were the main cause of the Internet bubble. However, Temin and Voth (2004) argue that bubbles have previously existed when and where no short-sale constraints have been present. Hence, short-sale constraints are not a common basis of all bubbles. Short-sale constraints have been applied not only to the Internet stocks but also to all other stocks. Even with the short-sale constraints, the Internet bubble can be avoided if investors are rational enough to identify values of Internet stocks. Pástor and Veronesi (2003) claim that uncertainty about the firm's average future profitability is positively related to the firm's price. Uncertainty is risk, and an asset with a higher risk should have a lower price to get higher returns than others. Hence, the positive relation between uncertainty and price also represents irrationality.

In conclusion, a bubble is created and developed by investors' irrationality, which implies that there is no bubble if investors are rational. Irrationality is measured for a certain period as seen in Section 1.2.7. Hence, irrationality is defined as investors' behaviors that create a "new" bubble in a certain period.

1.2.7 Measure of Irrationality

Bubbles are created and developed by investors' irrationality. Hence, a bubble (or a relative bubble for comparability) might be a good measure of irrationality. However, a bubble is an accumulated measure of irrationality for the whole life of an asset because it has grown period over period since its inception. In this dissertation, irrationality is measured for a certain period.

If the price of an asset is the same as its value, this asset is rationally priced. For any asset, if the price changes are the same as the value changes in a certain period, it can be said that investors are rational in this period. Hence, the irrationality can be measured by the difference between price changes and value changes during a certain period. There are two types of irrationality measures.

$$\omega^R{}_t = R^P{}_t – R^V{}_t, \tag{1.11}$$

$$\omega^G{}_t = G^P{}_t – G^V{}_t, \tag{1.12}$$

where ω^R and ω^G are named as irrational return and irrational growth, respectively, implying that the value returns [growths] are rational returns [growths]. If return [growth] measures are used to assess prices and values, then irrational returns [growths] will be the measure of the irrationality.

We should be aware of one thing: cause and effect between irrationality and price changes. Equations (1.11) and (1.12) seem to show that price changes affect irrationality, but this is not true. Think in this way;

$$R^P{}_t = R^V{}_t + \omega^R{}_t, \tag{1.13}$$

$$G^P{}_t = G^V{}_t + \omega^G{}_{t,}. \tag{1.14}$$

Equations (1.13) and (1.14) show that price changes are the sum of value changes and irrationality; irrationality affects price changes, and this is true as explained in Section 1.2.6. Price changes can be observed in the market while irrationality cannot, and hence irrationality can be measured by Equations (1.11) and (1.12) if value changes are known; value changes can be measured by the Rational CAPM. Cause and effect is definitely different from observability of data.

1.2.8 Pricing and Mispricing

Following the CAPM, pricing is a measure for a certain period, so is mispricing. In other words, pricing is expressed by a rate of changes in asset prices for a certain period, i.e., returns or growths, and mispricing is measured by a difference between actual pricing and correct pricing for a certain period. If actual pricing is higher [lower] than correct pricing, overpricing [underpricing] occurs. What is the correct pricing? The correct price of an asset should be its value, and thus, the correct pricing is the changes in value, i.e., value returns/growths. Then, mispricing is the same as irrational returns/growths in Section 1.2.7.

Hence, mispricing is a measure of irrationality for a certain period whereas bubble is an accumulated measure of irrationality for the whole life of an asset as explained in Section 1.2.7. Irrationality causes mispricing, which in turn makes the price changes of an asset different from its value changes, creating a new bubble in a certain period. Without mispricing, price changes are the same as value changes, preventing a new bubble.

1.2.9 Risk Premium

Risk premium is defined as returns over risk-free rate. In this dissertation, there are two types of returns. Hence, the risk premium (π) also has two types.

$$\pi^P_t = R^P_t - R_{F,t}$$

$$\pi^V_t = R^V_t - R_{F,t},$$

where π^P and π^V are named as price risk premium and value risk premium, respectively, and R_F denotes a risk-free rate. Then,

$$\pi^P_t - \pi^V_t = \omega^R_t. \tag{1.15}$$

CHAPTER II

RATIONAL CAPM: CAPM WITH RATIONAL INVESTORS

2.1 Introduction

The empirical record of the Capital Asset Pricing Model (CAPM) is very poor even though it is theoretically derived and proven. Here is my research question; ***why has the CAPM worked poorly in the empirical studies?*** This chapter revisits the CAPM to find the answer to this question. This chapter does not depend on the conventional CAPM, the conventional model to test the (original) CAPM with historical data, but elaborates on a new interpretation of the CAPM to find why the CAPM did not work with historical data.

The goals of this chapter are twofold. The first goal is to find why the CAPM did not work in previous empirical studies and how it should be used to work with historical data, resulting in the Rational CAPM. The second goal is to verify that the Rational CAPM works with historical data. To achieve the first goal, this chapter analyzes assumptions made for the CAPM and risks defined in the CAPM. The achievement of the first goal results in the Rational CAPM. To attain the second goal, empirical work is required. However, the Rational CAPM cannot be tested empirically because the Rational CAPM uses value data, which are not observable. Hence, this chapter tests the Rational CAPM indirectly by using irrationality. Irrationality is measured by the Rational CAPM, and thus testing irrationality can verify the validity of the Rational CAPM.

This chapter contributes to the existing literature in the following ways. First, this chapter introduces the Rational CAPM, a new interpretation of the CAPM. The Rational CAPM removes the market irrationality from the conventional CAPM. Second, based on the Rational CAPM, this chapter demonstrates how to measure value changes and irrationalities in historical data. Third, this chapter explains that irrationality means mispricing and risk, and proves that irrationality represents investor sentiments.

The theoretical approach herein answers the research question. The CAPM has been misused. The CAPM does not hold with the price data that contain mispricing created by investors' irrationality. The CAPM has an implied assumption that investors are rational, and thus do not misprice any asset, implying that the CAPM measures value changes. In addition, empirical studies lead to the following results. Price is much more volatile than value, and thus, it is natural that a bubble forms and collapses. The irrationality measured by the Rational CAPM well explains the investor sentiment and the market. In conclusion, empirical results support that the Rational CAPM works with historical data.

The rest of this chapter is organized as follows. Section 2.2 reviews the literature with Section 2.2.1 for the (original) CAPM and Section 2.2.2 for the conventional CAPM. Section 2.3 explains why the conventional CAPM did not work and why the Rational CAPM must be used based on the analyses of assumptions and risks in the CAPM. Section 2.4 explains the Rational CAPM. In Section 2.5, Section 2.5.1 demonstrates how to measure historical value changes and irrationalities. Section 2.5.2 explains how to examine irrationalities in relation to investor sentiments. Section 2.5.3 describes the sample. Section 2.6 discusses empirical findings and Section 2.7 summarizes and concludes this chapter.

2.2 Literature Review

2.2.1 (Original) CAPM

2.2.1.1 Characteristics and assumptions. Based on the mean.variance efficient portfolio theory (Markowitz, 1952), the (original) Capital Asset Pricing Model (CAPM) was developed independently by Treynor (1961), Sharpe (1964), Lintner (1965) and Mossin (1966). Both Markowitz and Sharpe were awarded the Nobel Prize in Economics in 1990 for the theory and the model, respectively. For the CAPM, there are some assumptions (Elton et al., 2007), which can be classified into two categories: perfect market assumptions (perfect competition, no tax, no transaction cost, unlimited lending and borrowing at risk-free rate, unlimited short sales, and infinitely divisible and marketable asset) and homogeneous investors assumptions (the same relevant period, mean-variance frame for decision, identical expectations for stock returns and their variances). The CAPM is based on the mean-variance efficient portfolio theory, which assumes rational investors who know the distribution of expected returns (Markowitz, 1991). Simply speaking, the CAPM assumes rational and homogeneous investors in the perfect market. Is "rational" in the CAPM assumption the same as "rational" in this dissertation (See Sections 1.2.6 to 1.2.8)? The answer to this question will be discussed later in Section 2.3.1.1.

The CAPM is the ex-ante theoretical equilibrium model (Copeland et al., 2005), and most often written in the form

$$E(R_i) = R_F + \beta_i(E(R_M) - R_F), \tag{2.1}$$

where R_i, R_M and R_F denote returns on Asset *i* (hereafter, "asset" represents "asset/portfolio", i.e., an asset can be either an individual asset or a portfolio in the market.), returns on the market (portfolio) and the risk-free rates, respectively, β is the systematic risk of an asset, and E( )

denotes ex-ante expectations. Here, the market (portfolio) must be mean-variance efficient because all investors have the homogeneous expectations (Fama, 1976; Copeland et al., 2005).

The CAPM is the model for a single period. For Period *t*, Equation (2.1) can be written as

$$E(R_{i,t}) = R_{F,t} + \beta_{i,t}(E(R_{M,t}) - R_{F,t}). \tag{2.2}$$

In this single period model, β of an asset is assumed to be constant during Period *t* for which returns are measured. Equation (2.2) implies that the period for which β is measured is the same as that for which returns are measured. β explains the relationship between asset returns and market returns for a certain period, and thus β should be measured for the same period as returns. Equation (2.2) does not imply that β is constant or stable over periods; β changes over time (Bodurtha and Mark. 1991; Jagannathan and Wang, 1996; Ghysels. 1998). If the period for β is different from that for the returns, β might not correctly explain asset returns due to the difference in the period of measurement.

In addition, Equation (2.1) can be expressed as

$E(R_i) - R_F = \beta_i(E(R_M) - R_F)$, and then

$$E(\pi_i) = \beta_i E(\pi_M), \tag{2.3}$$

and thus

$$E(\pi_{i,t}) = \beta_{i,t} E(\pi_{M,t}), \tag{2.4}$$

where π denotes risk premium. Equation (2.4) describes that returns (or risk premia) of an asset are perfectly explained by the market returns (or risk premia) with its β. Hence, the CAPM is regarded as a one-factor model.

2.2.1.2 Risks in the portfolio. The CAPM is the ex-ante model under uncertainty, which can be expected with probabilities by all “rational and homogeneous” investors. This uncertainty is the risk that investors face. Markowitz (1952) measured the risk by variances (or standard deviations) of expected returns. Then, the risk of individual Asset *i* for Period *t* can be measured by

$$VAR(R_{i,t}) = E(R_{i,t} – E(R_{i,t}))^2, \tag{2.5}$$

$$SD(R_{i,t}) = \sqrt{VAR(R_{i,t})}, \tag{2.6}$$

where VAR( ) and SD( ) denote variance and standard deviation, respectively, weighted by the probability of expected outcomes. Hence, the variance represents an expected squared deviation of ex-ante returns. Why is the risk measured by the “squared deviation”? If the simple deviation is used, the expected deviation is always zero, i.e.,

$$E(R_{i,t} – E(R_{i,t})) = 0, \tag{2.7}$$

because positive and negative deviations are perfectly canceled out; this is the reason the squared deviation is used to measure the dispersion of data.

In the same way, the risk of Portfolio *P* can be assessed by

$$\begin{aligned} VAR(R_{P,t}) &= E(R_{P,t} – E(R_{P,t}))^2 = E(\textstyle\sum_{i=1}^{n}[w_{i,t}(R_{i,t} – E(R_{i,t}))])^2 \\ &= \textstyle\sum_{i=1}^{n}(w_{i,t}) \sum_{j=1}^{n}(w_{j,t}COV(R_{i,t}, R_{j,t})) = \sum_{i=1}^{n}(w_{i,t}COV(R_{i,t}, R_{P,t})), \end{aligned} \tag{2.8}$$

where w denotes a weight of an asset in Portfolio *P*, *i* and *j* denote individual assets in Portfolio *P*, *n* is total number of individual assets in Portfolio *P*, and COV( ) denotes covariance between two variables and is measured by

$$COV(R_{i,t}, R_{P,t}) = E([R_{i,t} – E(R_{i,t})][R_{P,t} – E(R_{P,t})]). \tag{2.9}$$

Hence, the risk of a portfolio can be measured by the weighted average of $COV(R_{i,t}, R_{P,t})$ (Fama, 1976). Consequently, the variance risk represents a total risk of a certain asset while the covariance risk measures the systematic risk, the relevant risk of a certain asset to the portfolio in which an asset is included. The difference between them is the unsystematic risk, implying that diversification removes the unsystematic risk.

2.2.1.3 Risks and diversification. $COV(R_{i,t}, R_{P,t})$ can be expressed by

$$COV(R_{i,t}, R_{P,t}) = \sum_{j=1}^{n}(w_{j,t}COV(R_{i,t}, R_{j,t})) \text{ for all } i$$

$$= w_{1,t}COV(R_{i,t}, R_{1,t}) + w_{2,t}COV(R_{i,t}, R_{2,t}) + \cdots$$

$$+ w_{i,t}COV(R_{i,t}, R_{i,t}) + \cdots + w_{n,t}COV(R_{i,t}, R_{n,t}).$$

As seen above, the variance (total risk) of Asset *i* exists in the portfolio because $COV(R_{i,t}, R_{i,t}) = VAR(R_{i,t})$. By separating this variance from the other covariance terms,

$$COV(R_{i,t}, R_{P,t}) = w_{i,t}VAR(R_{i,t}) + \sum_{j=1}^{n}(w_{j,t}COV(R_{i,t}, R_{j,t})) \text{ for all } i \text{ with } j \neq i. \quad (2.10)$$

The variance of returns on Asset *i* is reflected by its weight in its covariance with the portfolio, and thus $w_{i,t}VAR(R_{i,t})$ is a part of the systematic risk (as well as a part of the total risk) of Asset *i* in this portfolio. For any asset, its variance and its covariances with other certain assets are the same regardless of the portfolio. However, the weight of the variance and the relevant (i.e., weighted) covariances with other assets are different for different portfolios (Fama, 1976).

Hence, the risk of an asset is determined by, or measured relative to, the portfolio in which this asset is included. In Portfolio *P* that consists of n assets, each asset has one term of the variance and n-1 terms of the covariances as seen in Equation (2.10). Hence, as the number of assets in the portfolio increases, the weight of Asset *i* decreases and the number of the covariance terms between Asset *i* and the other assets increases. Consequently, the variance will disappear while the weighted average of covariances will remain if the portfolio is well diversified (Copeland et al. 2005; Elton et al., 2007).

2.2.1.4 Risks in the market. An asset's risk is not an absolute measure but a relative measure to the portfolio in which this asset is included. Hence, the risks of Asset *i* in the market (portfolio) can be defined as follows.

$$\text{Total risk} = \frac{VAR(R_i)}{VAR(R_M)}.$$

$$\text{Systematic (market or nondiversifiable) risk} = \beta_i = \frac{COV(R_i, R_M)}{VAR(R_M)}. \tag{2.11}$$

$$\text{Unsystematic (residual, idiosyncratic or diversifiable) risk} = \frac{VAR(R_i) - COV(R_i, R_M)}{VAR(R_M)}.$$

Hence,

$$\frac{VAR(R_i)}{VAR(R_M)} = \frac{COV(R_i, R_M)}{VAR(R_M)} + \frac{VAR(R_i) - COV(R_i, R_M)}{VAR(R_M)}. \tag{2.12}$$

In the CAPM, "systematic" can be interpreted as "related to the market" or "explained by the market", implying that the market is always systematic. The unsystematic risks can be diversified away in the well diversified portfolios. There are no unsystematic risks in the CAPM as seen in Equation (2.1), implying that only the systematic risks are rewarded in the CAPM and that the market is efficient enough to remove all the unsystematic risks. For the market portfolio, from Equations (2.11) and (2.12),

$$\beta_M = \frac{COV(R_M, R_M)}{VAR(R_M)} = 1, \tag{2.13}$$

$$\frac{VAR(R_M)}{VAR(R_M)} = \frac{COV(R_M, R_M)}{VAR(R_M)} + \frac{VAR(R_M) - COV(R_M, R_M)}{VAR(R_M)}, \text{ resulting in } 1=1+0, \tag{2.14}$$

by $COV(R_M, R_M) = VAR(R_M)$. Hence, the market portfolio has no unsystematic risk, and its total risk is the same as its systematic risk of unity.

Therefore, β has two meanings in the CAPM: the relationship between an asset risk premium and the market risk premium as seen in Equation (2.3), and the systematic risk as seen in Equation (2.11), i.e.,

$$\beta_i = \frac{E(\pi_i)}{E(\pi_M)} = \frac{COV(R_i, R_M)}{VAR(R_M)}. \tag{2.15}$$

Consequently, β can be measured by two ways: by the relationship of expected risk premia and by the covariance of expected returns between an asset and the market. In fact, these two measures are the same because the covariance also represents the relationship between asset returns and the market returns.

2.2.2 Conventional CAPM

2.2.2.1 Characteristics. The (original) CAPM is formulated in terms of ex-ante expectations. However, all tests of the CAPM have used ex-post price data (Copeland et al., 2005, Elton et al., 2007), i.e., Equations (2.2) and (2.4) have been used for

$$E(R^P_{i,t}) = R_{F,t} + \beta_{i,t}(R^P_{M,t} - R_{F,t}). \quad (2.16)$$

$$E(\pi^P_{i,t}) = \beta_{i,t}\pi^P_{M,t}, \quad (2.17)$$

where π^P denotes the price risk premium. In Equations (2.16) and (2.17), E( ) at the left-hand side of both equations denote neither ex-ante estimation nor ex-post mean, but simply represent the results from the right-hand side formulae, and thus are called the "systematic" results in this dissertation because they are "explained by or related to the market" through the systematic risk (β) in the CAPM.

Equations (2.16) and (2.17) have been regarded as the same model as Equations (2.2) and (2.4), and thus, used to test the CAPM, implying that Equation (2.1) has been interpreted as

$$E(R^P_i) = R_F + \beta_i(E(R^P_M) - R_F) \quad (2.18)$$

with

$$E(R^P_M) = R^P_M. \quad (2.19)$$

The previous empirical studies have used ex-post price returns of the market as ex-ante returns of the market in the CAPM, computed the systematic price returns of assets using Model (2.16), and then compared these systematic price returns to actual price returns to validate the CAPM. By using ex-post asset price returns in Equations (2.16) and (2.17),

$$R^P_{i,t} = \alpha_{i,t} + R_{F,t} + \beta_{i,t}(R^P_{M,t} - R_{F,t}). \quad (2.20)$$

$$\pi^{P}_{i,t} = \alpha_{i,t} + \beta_{i,t}\pi^{P}_{M,t}, \tag{2.21}$$

where α denotes deviations from the CAPM when price returns are used, and is used to measure abnormal or excess returns called Jensen's α (Jensen, 1968), which is measured by

$$\alpha_{i,t} = R^{P}_{i,t} - E(R^{P}_{i,t}) = \pi^{P}_{i,t} - E(\pi^{P}_{i,t}), \tag{2.22}$$

where $E(R^{P}_{i,t})$ and $E(\pi^{P}_{i,t})$ are the systematic price returns from the Equation (2.16) and the systematic price risk premium from the Equation (2.17), respectively. Hence, α is the price returns that cannot be explained by the market in the CAPM, i.e., the unsystematic (part of) price returns, which is the same as the unsystematic mispricing (See Section 2.3.2.1). In this dissertation, like Equations from (2.16) to (2.21), the ex-post CAPM with price returns is named the conventional CAPM, the conventional interpretation of the CAPM in historical data. The conventional CAPM describes the relationship between systematic price risk premia of assets and those of the market.

2.2.2.2 Systematic risk. With ex-post price data, β is measured differently from the (original) CAPM that uses ex-ante expectations. From Equation (2.11),

$$\beta_{i,t} = \frac{COV(R^{P}_{i,t}, R^{P}_{M,t})}{VAR(R^{P}_{M,t})}. \tag{2.23}$$

To measure the variance and covariance, the conventional CAPM uses sub-period data for Period *t*, e.g., monthly or weekly data for year *t*. Then,

$$COV(R^{P}_{i,t}, R^{P}_{M,t}) = E([R^{P}_{i,m} - E(R^{P}_{i,m})][R^{P}_{M,m} - E(R^{P}_{M,m})]).$$

$$VAR(R^{P}_{M,t}) = E(R^{P}_{M,m} - E(R^{P}_{M,m}))^{2},$$

where *m* denotes sub-periods for Period *t*. Here, the weights used in both equations are not probability but 1/N, where N denotes the number of sub-periods in Period *t*, and thus E( ) measures ex-post means of sub-period data. Hence, annual βs are measured by ex-post monthly or weekly variation in the conventional CAPM while by ex-ante annual variation in the (original)

CAPM. Then, the risk is the volatility of the historical data in the ex-post conventional CAPM while the uncertainty about the future in the ex-ante CAPM.

From Equations (2.17) and (2.23), in the conventional CAPM,

$$\beta_{i,t} = \frac{E(\pi^{P}_{i,t})}{\pi^{P}_{M,t}} = \frac{COV(R^{P}_{i,t}, R^{P}_{M,t})}{VAR(R^{P}_{M,t})}. \tag{2.24}$$

Hence, β has two meanings, and thus can be measured by two ways: by the relationship of ex-post systematic price risk premia and by the covariance of ex-post price returns between an asset and the market. For the first way, the regression is used using Equation (2.21) because the regression, like the covariance, separates the unsystematic components (= α in Equation (2.21)) from asset returns (See Section 2.4.3).

2.2.2.3 Results from the empirical tests. Using the conventional CAPM, some early studies find the evidence to support the CAPM (Sharpe and Cooper, 1972; Black et al., 1972; Fama and MacBeth, 1973 among others). However, most previous studies find that the CAPM works poorly with the historical price data, meaning that the CAPM cannot explain the price returns, leading to the conclusion that the CAPM is dead (Fama and French, 1992 and 1996).

Some suggested the reasons why the CAPM is dead. First, the CAPM misses something: dividend-price ratio (Litzenberger and Ramaswamy, 1979), size (Banz, 1981), price-earnings ratio (Basu, 1983), book-to-market value ratio (Fama and French, 1992), momentum (Carhart, 1997) and/or credit spread (Hwang et al., 2010). Second, the proxy for the market portfolio is wrong; the market portfolio should contain all assets including human capital (Mayers, 1972; Roll, 1977). Third, the market is inefficient due to “noise traders”, who irrationally act on the information to earn abnormal returns (Black, 1986; De Long et al., 1990).

2.2.2.4 Critiques and theories on the empirical tests. Roll (1977) criticized the test of the CAPM. Roll’s critique has two crucial statements. First, testing the CAPM is nothing but testing the efficiency of the market portfolio, meaning that the CAPM and the efficiency of the market portfolio are inseparable (Copeland et al., 2005). Further, if any proxy for the market portfolio is ex-post mean-variance efficient, and βs are calculated using this proxy, then the CAPM must hold even though the true market portfolio is not mean-variance efficient. Second, the CAPM is not testable without the true market portfolio, which is unobservable. The true market portfolio includes all assets.

In addition, Ross (1977) argues that the assumption of homogeneous expectation implies the equality between ex-ante and ex-post mean returns, and thus this assumption is jointly tested with the CAPM. He also explains that "α" in Model (2.21) should be zero for all assets if the market proxy is efficient; if it is not zero, it is due to "either ex-ante inefficiency or sampling variation". Some argue that this α results from investors' heterogeneous expectations about the ex-ante returns (Williams, 1977; Copeland et al., 2005; Gerber and Hens, 2006; Ferson and Lin, 2010).

Roll's second critique negates all the previous empirical tests of the CAPM. However, Gibbons and Ferson (1985) proved that the CAPM can be tested without the true market portfolio if a mean-variance efficient portfolio is used as a market proxy. Let the CAPM with the true market portfolio be

$$E(R_i) = R_F + \beta_{iT}(E(R_T) - R_F), \tag{2.25}$$

where β_{iT} denotes a true β of Asset *i* (β of Asset *i* in the true market portfolio *T*), and R_T denotes returns on the true market portfolio *T*. Equation (2.25) holds for any asset in the CAPM, and thus

$$E(R_P) = R_F + \beta_{PT}(E(R_T) - R_F), \tag{2.26}$$

where β_{PT} denotes the portfolio true β, and R_P denotes returns on Portfolio *P*. Then, rearranging Equation (2.26) yields

$$(E(R_T) - R_F) = \frac{(E(R_P) - R_F)}{\beta_{PT}}. \tag{2.27}$$

By substituting Equation (2.27) into Equation (2.25),

$$E(R_i) = R_F + \frac{\beta_{iT}}{\beta_{PT}}(E(R_P) - R_F). \tag{2.28}$$

Equation (2.28) explains that any portfolio can be used as a market proxy. However, for Equation (2.28) to hold, the portfolio used as the market proxy should be mean-variance efficient as explained in Roll's first critique. β is the relative measure of risk, and thus varying depending on the market proxy used (Fama, 1976; Roll, 1977). If the CAPM holds with the market proxy *P*, then, from Equation (2.25),

$$E(R_i) = R_F + \beta_{iP}(E(R_P) - R_F), \tag{2.29}$$

By comparing Equations (2.28) and (2.29), a new β of Asset *i* with the market proxy *P*, i.e., β_{iP}, should be the same as its true β divided by the market proxy true β, i.e.,

$$\beta_{iP} = \frac{\beta_{iT}}{\beta_{PT}}. \tag{2.30}$$

For the proof of Equation (2.30), see Section 2.3.2.3.

Ben-Horim and Levy (1980) and Bøhren (1997) argue that the standard deviation is a better measure of risk than the variance. Rearranging Equation (2.1) yields

$$E(R_i) - R_F = \left\{\frac{E(R_M) - R_F}{SD(R_M)}\right\} \beta_i SD(R_M). \tag{2.31}$$

They argue that in Equation (2.31), $\{[E(R_M) - R_F]/SD(R_M)\}$ measures the market price of a unit of risk, and thus, $\beta_i SD(R_M)$ determines the equilibrium risk premium. Hence, the total risk and the systematic risk of Asset *i* should be $SD(R_i)$ and $\beta_i SD(R_M)$, respectively. Then,

$$SD^U(R_i) = SD(R_i) - \beta_i SD(R_M), \tag{2.32}$$

where $SD^U(R_i)$ denotes the unsystematic risk of Asset *i*.

2.2.2.5 Risk, mispricing and investor sentiment. Daniel et al. (2001) theoretically examined whether stock returns reflect covariance risk (β) and/or mispricing, and find that expected stock returns are linearly related to both β and mispricing, and that diversification removes the unsystematic mispricing but not systematic mispricing. Bloomfield and Michaely (2004) analyzed professionals' views about β, firm size and book-to-market value ratio in relation to stock returns, and conclude that β represents risk, not mispricing, that book-to-market value ratio indicates mispricing, not risk, that firm size does not imply mispricing but risk, albeit weaker than β, and that both risk and mispricing affect stock returns.

Behavioralists, following the noise trader theory (Black, 1986; De Long et al., 1990), argue that investor sentiments affect mispricing, and hence that pricing models should incorporate investor sentiments (Brown and Cliff, 2005; Baker and Wurgler, 2006). To examine whether investor sentiments have an effect on stock returns, the key is how to measure investor sentiments because "[t]here are no definitive or uncontroversial measures" of (unobserved) investor sentiments (Baker and Wurgler, 2006). Some used direct measures of survey data (Brown and Cliff, 2005; Schmeling, 2009) while others employed indirect measures of financial variables (Baker and Wurgler, 2006). Brown and Cliff (2004) find that indirect measures are related to direct measures, and Schmeling (2009) claims that consumer confidence is a reasonable measure of investor sentiments.

Brown and Cliff (2004) find that investor sentiments explain contemporaneous stock returns but not near-term future stock returns, and that the market performance affects investor sentiments. Brown and Cliff (2005) find that investor sentiments negatively affect returns of large value stocks and the market portfolio over the next several (3+) years using monthly data, and that both statistical and economical significances of the sentiment increase with the horizon.

They also document that investor sentiments are persistent due to a bandwagon effect, that investor sentiments do not explain short-term returns but long-term returns, and that pricing errors are explained by investors' irrational sentiments. Moreover, Kurov (2010) documents that the investor sentiment is affected by the monetary policy, and that the effect of monetary policy on the sentiment is stronger in the bear market than in the bull market.

2.3 Literature Analysis

The purpose of this section is to find why the CAPM has worked poorly in the previous empirical work and how it should be used to work with historical data. For this purpose, this chapter analyzes and reinterprets the assumptions and risks in the CAPM.

2.3.1 Characteristics and Assumptions of the CAPM

As explained in Section 2.2.1.1, the CAPM can be characterized by the ex-ante model, by the single period model, and by the one-factor model. Ex-ante estimations are based on the assumption of rational expectations. In the CAPM, investors are rational, meaning that they "know" asset returns ex-ante. The single period model assumes the constant β during the period for which returns are measured. The one-factor model is based on the assumption of the perfect correlation between asset returns and the market returns.

The perfect correlation between asset returns and the market returns is implicitly assumed in the CAPM. From Equation (2.1),

$$\mathrm{VAR}(R_i) = \beta_i^2 \mathrm{VAR}(R_M) \tag{2.33}$$

Substituting Equation (2.11) into Equation (2.33) yields

$$\mathrm{VAR}(R_i) = \frac{\{\mathrm{COV}(R_i, R_M)\}^2}{\{\mathrm{VAR}(R_M)\}^2}\mathrm{VAR}(R_M)$$

$$= \frac{\{COR(R_i, R_M)SD(R_i)SD(R_M)\}^2}{VAR(R_M)}$$

$$= \frac{\{COR(R_i, R_M)\}^2 VAR(R_i)VAR(R_M)}{VAR(R_M)}$$

$$= \{COR(R_i, R_M)\}^2 VAR(R_i). \tag{2.34}$$

Hence, in the CAPM,

$$COR(R_i, R_M) = 1, \tag{2.35}$$

because β is always positive.

In fact, the assumption of constant β enables the perfect correlation to hold between asset returns and the market returns because Equation (2.33) holds only when β_i is constant. β represents the relationship between asset returns and the market returns, and thus the constant β means the constant relationship between asset returns and the market returns, implying the perfect correlation between asset returns and the market returns. In the CAPM, β can vary over periods, but β remains the same within the period for which β is measured. In addition, by rearranging Equation (2.1),

$$E(R_i) = (1 - \beta_i)R_F + \beta_i E(R_M). \tag{2.36}$$

Equation (2.36) represents the straight line in the $(E(R_M), E(R_i))$ plain for a certain β, meaning the perfect correlation between $E(R_M)$ and $E(R_i)$. The assumption of perfect correlation between asset returns and the market returns allows asset returns to be perfectly explained only by the market returns.

Hence, these two assumptions, rationality and perfect correlation, should be maintained to use and test the CAPM. What do the rationality and the perfect correlation mean in the ex-ante CAPM? How should they be applied to the ex-post data? Ross' arguments and Roll's first critique explained in Section 2.2.2.4 are closely related these two assumptions.

2.3.1.1 Rationality and homogeneity in the ex-ante model. The CAPM assumes that all investors "know" expected returns (rational investors) and have the same expectations (homogeneous investors) as explained in Section 2.2.1.1. What does "know" mean? "Know" means that ex-ante returns will turn out to be true, i.e., to be the same as ex-post returns. If one's estimate is different from an ex-post result, it means that he does not know the result. Hence, "all investors know" automatically implies "all investors have the same expectations" because there is only one ex-post outcome. Hence, "rational" implies "homogeneous". Ross (1977) has already identified "know" by interpreting "homogeneity" as "expectations (…) assumed to coincide with ex post population mean." If investors are not rational about their expectations, they cannot have the homogeneous expectation because it is nearly impossible that all investors have the homogeneous irrationality. Hence, rationality automatically leads to homogeneity in the CAPM.

Whether rational or irrational, investors cannot "know" future price changes that contain investors' irrationalities, which are unpredictable. Consequently, ex-ante returns in the CAPM must not contain investors' irrationalities, implying that ex-ante returns in the CAPM must be value returns. Hence, "know asset returns" means "do not misprice an asset" in the CAPM. If any assets are correctly priced, their ex-ante returns should be the same as their ex-post returns. If investors do not misprice any assets, they should know value returns of all assets. In addition, the market must not be mispriced if all assets in the market are correctly priced. Hence, the assumption of rationality is the same as the assumption of no mispricing, implying that the market is perfectly rational in the CAPM. Consequently, rationality in the CAPM assumption is the same as rationality in this dissertation. There is no mispricing in the CAPM world.

Henceforth, the rationality means no mispricing, and the homogeneity, following Ross (1977), indicates the equality between ex-ante and ex-post returns. Although they are differently defined, they hold simultaneously and they are inseparable in the CAPM. Therefore, they are the same in the CAPM.

2.3.1.2 Perfect correlation and efficiency for the one-factor model. The CAPM implicitly assumes the perfect correlation between asset returns and the market returns. This assumption enables only the market returns to explain returns of all assets by their βs, resulting in the one-factor model. Further, this assumption makes the CAPM always hold, and thus makes the market portfolio efficient. Hence, the perfect correlation between asset returns and the market returns is the same as the efficiency of the market (portfolio) in the CAPM. As reviewed in Section 2.2.1.1, the efficiency of the market portfolio is based on the homogeneous investors. In other words, if investors are homogeneous, the market portfolio must be efficient. Hence, the homogeneity and the efficiency of the market portfolio are inseparable.

In the conventional CAPM, asset returns are not perfectly correlated to the market returns. Compare Equation (2.20) to Equation (2.1); besides the difference between ex-ante and ex-post data, the only difference is α. The existence of α (unsystematic mispricing) prevents the perfect correlation between asset and market returns in the conventional CAPM. For the perfect correlation, α (unsystematic mispricing) should be removed.

The CAPM has only one factor (β) with the same factor loading (the market returns or risk premia) for all assets. The missing factors some argue in the CAPM, as seen in Section 2.2.2.3, are related, to a greater or lesser degree, to previous or current mispricing. Intuitively, mispricing should not affect value returns but price returns. To measure price returns, some factors related to mispricing should be incorporated in the model because prices contain mispricing. In fact, some behavioralists argue that pricing models should include investor sentiments to measure irrational pricing (See Section 2.2.2.5). The CAPM does not have any factors related to mispricing, implying that the CAPM assumes no mispricing; in the CAPM, all assets in the market are correctly priced, and thus the market is correctly priced. Hence, one-factor modeling by β with the market returns confirms the assumption of no mispricing and the use of value returns.

2.3.1.3 Assumptions in the conventional CAPM. As explained in Sections 2.3.1.1 and 2.3.1.2, all four assumptions, (investors') rationality, (investors') homogeneity, perfect correlation (between asset returns and the market returns) and efficiency (of the market portfolio), should hold simultaneously, and thus be inseparable in the CAPM. Hence, homogeneity, perfect correlation and efficiency should hold with rationality (no mispricing) in the CAPM. However, zero α enables the conventional CAPM to achieve the perfect correlation as explained in Section 2.3.1.2. Furthermore, Ross (1977) claims that the efficient market portfolio (or proxy) implies α = 0 in Equation (2.21), and some argue that α results from investors' heterogeneity about returns as explained in Section 2.2.2.4. Why did they argue that only the zero α (= no unsystematic mispricing) can satisfy the homogeneity and the efficiency?

In the conventional CAPM, i.e., in the ex-post price data, no assumptions hold. Only the homogeneity is enforced on the market portfolio, i.e., $E(R^P_M) = R^P_M$, as seen in Equation (2.19), and then, systematic and unsystematic price returns, respectively $E(R^P_i)$ and α, are calculated. By removing α from asset returns, systematic price returns can achieve the perfect correlation and the efficiency. In addition, if systematic price returns are used for both ex-ante and ex-post returns for assets, they satisfy the homogeneity with the enforced homogeneity on the market portfolio. Still, they cannot satisfy the rationality because systematic price returns contain systematic mispricing due to the mispricing of the market (See Section 2.3.2.1). Hence, in the

Table 2.1 Comparison in Assumptions between Original and Conventional CAPMs

This table explains the difference in assumptions between original and conventional CAPMs.

Assumptions		Rational investors	Homogeneous investors	Perfect correlations[1]	Efficiency of the market portfolio
(Original) CAPM		Assumed	Assumed	Implicitly assumed	Must hold
Conventional CAPM	Using raw price data	Not hold	Enforced on the market portfolio Not hold for individual assets	Not hold	Not hold
	Using systematic price data[2]	Not hold	Enforced on the market portfolio Hold for individual assets[2]	Hold	Hold[3]

1. Perfect correlations between asset returns and the market returns
2. Systematic price data means price data that removed unsystematic price returns (= unsystematic mispricing = α).
3. Hold in the conventional CAPM sense, but not hold in the (original) CAPM sense.

conventional CAPM, the rationality is different from the homogeneity. Table 2.1 summarizes the comparison between original and conventional CAPMs for the assumptions discussed above.

The arguments that zero α can satisfy the homogeneity and the efficiency must be from the enforced homogeneity on the market. In other words, the conventional CAPM does not consider the mispricing of the market, and thus assumes (enforces) the equality between ex-ante and ex-post returns on the market. If the heterogeneity were allowed for the market, systematic mispricing as well as unsystematic mispricing should be found as a result of the heterogeneity and removed for the efficiency. Irrationality creates mispricing of all assets and the market, and thus leads to the heterogeneous expectations about the market (portfolio) and the inefficiency of the market portfolio. Due to the enforced homogeneity on the market, the conventional CAPM only separates the unsystematic mispricing (α) from asset price returns (R^P_i) to get the systematic price returns ($E(R^P_i)$), and thus cannot find the systematic mispricing that is contained in the systematic price returns (See Section 2.3.2.1).

It is not reasonable that the irrational market (portfolio) is efficient. It is not possible that all investors have the homogeneous irrationality about the market. The homogeneous expectation and the efficiency of the market portfolio can hold only when the rationality holds. Hence, the efficient market portfolio and the homogeneous expectation achieved by zero α in the conventional CAPM are still inefficient and heterogeneous, respectively, if the market is irrational. However, zero α enables the conventional CAPM to achieve the perfect correlation between asset price returns and market price returns even though the market is mispriced because of the perfect correlation between market mispricing and systematic asset mispricing (See Section 2.3.2.4).

2.3.1.4 Rationality and the CAPM. Rationality, homogeneity, perfect correlation and efficiency all must hold simultaneously, and thus are inseparable in the CAPM. Among these assumptions, however, the key is the rationality because the rationality makes the others always hold; the rationality leads to the homogeneity, which in turn leads to the efficiency that is equivalent to the perfect correlation. Lack of any one makes the CAPM fail. The rationality and the homogeneity imply no mispricing, which makes the equality between ex-ante and ex-post returns. The perfect correlation and the efficiency imply that the CAPM must hold. In fact, the transformation from ex-ante model to ex-post model for the conventional CAPM is based on the assumption that actual returns on assets are the same as their expected returns (Copeland et al., 2005). Without rationality and homogeneity, the ex-post CAPM cannot hold. Further, it is not theoretical that mispricing exists in equilibrium; recall that the CAPM is an equilibrium model.

The conclusion is that the rationality and the CAPM are inseparable; there is no mispricing in the CAPM world, and thus the CAPM must hold with no mispricing. In the real world, however, mispricing exists and affects price returns as explained in Section 2.2.2.5, implying that price returns cannot satisfy the CAPM, and that the CAPM has been ailing from misuse and abuse. This new interpretation of the CAPM can explain why the previous studies using the conventional CAPM conclude that the CAPM is dead, and thus this new interpretation of the CAPM will give the CAPM a new life. The CAPM, unlike other models that try to explain price returns, measures value returns. In this sense, the CAPM can be regarded as the normative model to explain how a value should change in equilibrium, while other models as the descriptive model to find how a price actually changes (Kahneman and Tversky, 2000).

2.3.1.5 Noise and investor sentiment vs. irrationality. According to the "noise trader" theory (Black, 1986; De Long et al., 1990), noise traders irrationally act on the information to earn abnormal returns, resulting in mispricing. Hence, the noise means irrationality and causes mispricing, implying that there would be no mispricing if the market is rational. Behavioralists use the investor sentiment as a measure of noise to explain mispricing. Recall that mispricing is a measure of irrationality as explained in Section 1.2.8, and thus the irrationality perfectly explains mispricing. Consequently, the irrationality would be a perfect measure of the noise. Irrationality and investor sentiment would be closely related because both represent and explain the noise.

For a measure of investor sentiment, many researchers have used survey data. Survey data for investor sentiment reflect future expectation (See Lemmon and Portniaguina (2006) for more details about the survey), which is based on the current market performance, and this explains why the market performance affects the sentiment (Brown and Cliff, 2004). On the other hand, irrationality is measured by market mispricing, independent of future expectation, and affects market performance as explained in Section 1.2.7. Therefore, it can be said that the irrationality influences the market performance, which in turn affects the sentiment.

In addition, survey data are related to the market as a whole, neither a specific portfolio nor a specific industry. However, irrationality can be measured for a specific portfolio or industry. This can be another advantage of irrationality over survey data as a measure of the noise.

2.3.2 Risk and Mispricing in the CAPM

2.3.2.1 Systematic and unsystematic mispricing in historical data. By using Equation (1.13), the conventional CAPM (2.18) yields

$$E(R^P_i) = E(R^V_i + \omega^R_i) = R_F + \beta_i(R^V_M + \omega^R_M - R_F). \tag{2.37}$$

Then,

$$E(R^V_i) + E(\omega^R_i) = R_F + \beta_i(R^V_M - R_F) + \beta_i\omega^R_M. \tag{2.38}$$

Hence, by separating value returns from mispricing

$$E(R^V_i) = R^V_i = R_F + \beta_i(R^V_M - R_F). \tag{2.39}$$

$$E(\omega^R_i) = \beta_i\omega^R_M. \tag{2.40}$$

Equation (2.40) describes the systematic mispricing (or irrationality), which results from the market mispricing through β. Equation (2.39) is the Rational CAPM as seen later in Section 2.4. In the Rational CAPM, ex-ante returns are the same as ex-post returns (See Section 2.4.1 for more details). In addition, Equation (2.37) with Equations (2.39) and (2.40) describes that systematic price returns are the sum of value returns and systematic mispricing.

The unsystematic mispricing (ω^U) is not related to the market, and can be computed by the total mispricing (irrational returns) subtracted by the systematic mispricing (expected irrational returns), i.e.,

$$\begin{aligned}\omega^U_i = \omega^R_i - E(\omega^R_i) &= (R^P_i - R^V_i) - E(\omega^R_i) \text{ (Using Equation (1.11))}\\ &= R^P_i - (R^V_i + E(\omega^R_i))\\ &= R^P_i - E(R^P_i) \text{ (From Equation (2.37))}\\ &= \alpha_i. \text{ (Using Equation (2.22))}\end{aligned}$$

Consequently, the unsystematic mispricing is the same as Jensen's α. Then,

$$\alpha_i = \omega^R_i - E(\omega^R_i) = \omega^R_i - \beta_i\omega^R_M. \text{ (Using Equation (2.40))} \tag{2.41}$$

Thus,

$$\omega^R{}_i = \alpha_i + \beta_i\omega^R{}_M. \tag{2.42}$$

In Equation (2.42), $\omega^R{}_i$, α_i and $\beta_i\omega^R{}_M$ respectively mean (total) mispricing, unsystematic mispricing and systematic mispricing. The conventional CAPM uses price returns, which contain both systematic and unsystematic mispricing. Expected returns from the conventional CAPM, i.e., systematic price returns, exclude unsystematic mispricing (α) but include systematic mispricing.

In addition, Equation (2.41) exactly matches Equation (2.32), implying that the risk is closely related to the mispricing (See Section 2.3.2.3 for Equation (2.32)). In the CAPM, however, the risk is a relative measure to the market portfolio, and thus dividing both sides of Equation (2.42) by $\omega^R{}_M$ yields

$$\frac{\omega^R{}_i}{\omega^R{}_M} = \beta_i + \frac{\alpha_i}{\omega^R{}_M} = \frac{E(\omega^R{}_i)}{\omega^R{}_M} + \frac{\omega^R{}_i - E(\omega^R{}_i)}{\omega^R{}_M}. \text{ (Using (2.40) and (2.41))} \tag{2.43}$$

Compared to Equation (2.12), Equation (2.43) describes that

Total risk = Systematic risk + Unsystematic risk,

implying that the risk is the relative mispricing to the market mispricing.

2.3.2.2 Diversification and unsystematic mispricing in historical data. The market portfolio removes the unsystematic risks in the CAPM as explained in Section 2.2.1.2. Suppose a portfolio *P* that includes *n* stocks for a certain period. From Equation (2.21),

$$\pi^P{}_1 = \alpha_1 + \beta_1\pi^P{}_M,$$

$$\pi^P{}_2 = \alpha_2 + \beta_2\pi^P{}_M,$$

.....

$$\pi^P{}_n = \alpha_n + \beta_n\pi^P{}_M.$$

Then, summing each side with Weight *w* for each stock yields

$$\sum_{i=1}^{n}(w_i\pi^P{}_i) = \sum_{i=1}^{n}(w_i\alpha_i) + \sum_{i=1}^{n}(w_i\beta_i\pi^P{}_M).$$

Then,

$$\pi^P{}_P = \alpha_P + \beta_P\pi^P{}_M. \tag{2.44}$$

Here, the well-diversified portfolio can make the portfolio α equal to zero.

If the portfolio consists of all assets in the market, i.e., if the portfolio is the market portfolio, then Equation (2.44) becomes

$$\pi^P{}_M = \alpha_M + \beta_M\pi^P{}_M. \tag{2.45}$$

In Equation (2.45), $\beta_M = 1$ by Equation (2.13), and thus $\alpha_M = 0$. In other words, the market portfolio removes all the unsystematic mispricing in the portfolio, i.e., the market portfolio has no unsystematic mispricing. Consequently, the unsystematic risk that should be removed by diversification is in fact the unsystematic mispricing, and thus, the risk represents the mispricing. Even though the diversification removes all the unsystematic mispricing in the portfolio, the systematic mispricing still exists if the market is mispriced.

2.3.2.3 β and perfect correlation. As explained in Section 2.3.1, there is the perfect correlation between asset returns and the market returns in the CAPM due to the assumption of constant β. Then,

$$\beta_i = \frac{COV(R_i, R_M)}{VAR(R_M)} = \frac{COR(R_i, R_M)SD(R_i)SD(R_M)}{SD(R_M)SD(R_M)} = \frac{SD(R_i)}{SD(R_M)}.\ (COR(R_i, R_M) = 1) \tag{2.46}$$

Following the notation below, $\beta_i = \beta_{iM}$ (β of Asset *i* in the market portfolio *M*) because Subscript *M* is omitted in the CAPM, where all assets are in the market portfolio. By using Equation (2.46), Equation (2.30) can be proved.

$$\frac{\beta_{iT}}{\beta_{PT}} = \frac{SD(R_i)/SD(R_T)}{SD(R_P)/SD(R_T)} = \frac{SD(R_i)}{SD(R_P)} = \beta_{iP}.\ (COR(R_i, R_T) = COR(R_P, R_T) = 1)$$

Hence, the assumption of perfect correlation allows Equation (2.30) to hold.

The perfect correlation between asset returns and the market returns holds due to the constant β, implying that in Equation (2.46), the relation in the variations between asset returns and market returns is constant, and further that the constant relation in the variations between asset returns and market returns means the perfect correlation between them. Without the perfect correlation between them, Equation (2.46) cannot hold.

If the CAPM holds with price data, Equation (2.18) holds, and thus Equations (2.39) and (2.40) hold. Then, by applying the same way as for Equations (2.33) and (2.34) to Equation (2.18), we can easily find that there is the perfect correlation between asset price returns and market price returns, i.e.,

$$COR(R^P_i, R^P_M) = 1, \tag{2.47}$$

if the conventional CAPM holds.

As reviewed in Section 2.2.2.4, Ben-Horim and Levy (1980) and Bøhren (1997) argue that the total risk and the systematic risk of Asset *i* should be $SD(R_i)$ and $\beta_i SD(R_M)$, respectively. However, both risk measures are the same because $\beta_i SD(R_M) = SD(R_i)$ by Equation (2.46). In addition, as explained in Section 2.2.1.3, the risk of an asset should be measured relative to the portfolio in which the asset is included; risk is a relative measure. Hence, this chapter defines risks in the market in the way explained in Section 2.2.1.4.

2.3.2.4 Rationality and perfect correlation. From Equation (2.40),

$VAR(\omega^R_i) = \beta_i^2 VAR(\omega^R_M)$, and then

$\beta_i^2 = \dfrac{VAR(\omega^R_i)}{VAR(\omega^R_M)}$, and then

$$\beta_i = \frac{SD(\omega^R_i)}{SD(\omega^R_M)}. \tag{2.48}$$

Equation (2.48) shows, if the CAPM holds, that β can be measured by mispricing, and implies that the perfect correlation exists between asset mispricing and market mispricing, i.e.,

$$\mathrm{COR}(\omega^R{}_i, \omega^R{}_M) = 1, \tag{2.49}$$

as explained in Section 2.3.2.3.

The assumption of the perfect correlation between asset returns and the market returns implies that all asset returns perfectly depend on the market returns by β. If the market be mispriced, this market mispricing will affect all asset returns through β, making all assets have the systematic mispricing, the asset mispricing caused by the market mispricing. In the CAPM, the perfect correlation between asset returns and the market returns causes the perfect correlation between asset mispricing and the market mispricing. Hence, in the CAPM

$$\mathrm{COR}(R_i, R_M) = \mathrm{COR}(\omega^R{}_i, \omega^R{}_M)\ (= 1). \tag{2.50}$$

By definition, Equation (2.50) is

$$\frac{\mathrm{COV}(R_i, R_M)}{\mathrm{SD}(R_i)\mathrm{SD}(R_M)} = \frac{\mathrm{COV}(\omega^R{}_i, \omega^R{}_M)}{\mathrm{SD}(\omega^R{}_i)\mathrm{SD}(\omega^R{}_M)}. \tag{2.51}$$

For convenience, squaring both sides of Equation (2.51) yields

$$\frac{\{\mathrm{COV}(R_i, R_M)\}^2}{\mathrm{VAR}(R_i)\mathrm{VAR}(R_M)} = \frac{\{\mathrm{COV}(\omega^R{}_i, \omega^R{}_M)\}^2}{\mathrm{VAR}(\omega^R{}_i)\mathrm{VAR}(\omega^R{}_M)}. \tag{2.52}$$

Then, by substituting Equations (2.5) and (2.9) into Equation (2.52),

$$\frac{\{E([R_i - E(R_i)][R_M - E(R_M)])\}^2}{E(R_i - E(R_i))^2 E(R_M - E(R_M))^2} = \frac{\{E([\omega^R{}_i - E(\omega^R{}_i)][\omega^R{}_M - E(\omega^R{}_M)])\}^2}{E(\omega^R{}_i - E(\omega^R{}_i))^2 E(\omega^R{}_M - E(\omega^R{}_M))^2}. \tag{2.53}$$

For equality in Equation (2.53),

$$R_i - E(R_i) = \omega^R{}_i - E(\omega^R{}_i) \tag{2.54}$$

and

$$R_M - E(R_M) = \omega^R{}_M - E(\omega^R{}_M). \tag{2.55}$$

Equation (2.53) does not always mean Equations (2.54) and (2.55); there might be another way to satisfy Equation (2.53). However, Equations (2.54) and (2.55) are the easiest way to solve Equation (2.53). Then, by rearranging Equations (2.54) and (2.55),

$$R_i - \omega^R_i = E(R_i) - E(\omega^R_i) = E(R_i - \omega^R_i) \tag{2.56}$$

and

$$R_M - \omega^R_M = E(R_M) - E(\omega^R_M) = E(R_M - \omega^R_M). \tag{2.57}$$

Equations (2.56) and (2.57) show that ex-ante and ex-post returns are the same. The equality between ex-ante and ex-post returns implies investors' rationality and homogeneity as explained Section 2.3.1.1, and thus expected returns must be value returns while expected mispricing must be zero. Hence, in the CAPM,

$$E(R_i) = R^V_i \text{ and } E(R_M) = R^V_M. \tag{2.58}$$

$$E(\omega^R_i) = E(\omega^R_M) = 0. \tag{2.59}$$

It is obvious that Equations (2.58) and (2.59) satisfy Equation (2.53) with $R_i = R^P_i$ and $R_M = R^P_M$. Therefore, the rationality satisfies Equation (2.53) and thus Equation (2.50), implying that the rationality leads to the perfect correlations between asset returns and the market returns. In addition, if the perfect correlation should hold simultaneously with the rationality, Equations (2.58) and (2.59) would be the only solution to Equation (2.53).

With actual and expected returns replaced by price and value returns, respectively, Equation (2.5) means that the total risk is the expected squared mispricing, Equation (2.7) describes that the expected mispricing is always zero, and Equation (2.11), which can be simplified to Equation (2.46) with the perfect correlation, implies that the systematic risk is the expected asset mispricing related to the portfolio mispricing. Hence, risk is measured by mispricing, and thus represents fear of mispricing.

2.3.2.5 Systematic risk in the CAPM. If the rationality is maintained, there should be no mispricing in the CAPM. Then, there should be no risk in the CAPM because the risk is measured by mispricing. Why does the risk exist in the CAPM? Recall that the CAPM is the equilibrium model under uncertainty. Even though the expected mispricing is always zero, there still exists the possibility of mispricing; this is the risk. Rational investors know the ex-ante probability of this mispricing. The risk can be quantified by the variance and the covariance as explained in Section 2.2.1.2. Risk represents fear of mispricing; uncertainty means fear. The systematic risk cannot be removed by diversification as far as the market is uncertain as assumed in the CAPM. In the ex-ante model, risk is uncertainty about the future pricing, i.e., the possibility of mispricing. However, the expected mispricing is always zero, i.e., there is no mispricing ex-ante.

In historical data, there is no uncertainty, and thus risk is measured by volatility during a certain period as explained in Section 2.2.2.2. In the conventional CAPM that uses historical price returns, the market portfolio removes the unsystematic mispricing of all assets, but not the systematic mispricing. The systematic mispricing is caused by the mispricing of the market. Therefore, if the mispricing is removed from the market returns, then the systematic returns from the CAPM do not have any mispricing. Hence, we can obtain value returns of an asset if we use the value returns of the market in the CAPM. Value returns are also volatile, meaning that the systematic risk, measured by volatility, still exists even after mispricing is removed; volatility means fear. Systematic risks are the same between conventional and Rational CAPMs (See Section 2.4.3), and thus systematic risks for value returns can be obtained using price returns.

As explained in the previous sections, risk is measured by mispricing; in fact, they indicate the same. However, risk exists even though mispricing does not exist due to the uncertainty or volatility in pricing, which means the possibility or fear of mispricing. In addition, the systematic risk has another function in the CAPM. As seen in Equation (2.4), the systematic risk represents the relationship between returns (or risk premia) of assets and returns (or risk premia) of the market. Hence, the systematic risk explains not only systematic mispricing but also systematic returns.

2.3.2.6 Risk vs. mispricing. Risk and mispricing mean the same as explained in the previous sections. In previous studies, however, risk and mispricing are regarded as different from each other. When they are mentioned together, risk usually indicates β, i.e., systematic risk, while mispricing means unsystematic mispricing, i.e., price changes unrelated to β; for example, Bloomfield and Michaely (2004).

Daniel et al. (2001) explains price changes by three parts: risk, systematic and unsystematic mispricing. Hence, it seems that they consider the risk a factor to affect only value changes. However, their measure of risk is β, which affects systematic price changes, the sum of value changes and systematic mispricing.

2.4 Rational CAPM

2.4.1 Rational CAPM: CAPM with No Mispricing

As analyzed in Section 2.3, the CAPM must hold with no mispricing in any asset in the market. Hence, the CAPM yields value returns of an asset if the value returns of the market are used in the CAPM. Then, Equations (2.1), (2.2) and (2.4), should be rewritten in

$$E(R^V{}_i) = R_F + \beta_i(E(R^V{}_M) - R_F). \tag{2.60}$$

$$E(R^V{}_{i,t}) = R_{F,t} + \beta_{i,t}(E(R^V{}_{M,t}) - R_{F,t}). \tag{2.61}$$

$$E(\pi^V{}_{i,t}) = \beta_{i,t}E(\pi^V{}_{M,t}). \tag{2.62}$$

where π^V denotes value risk premia. By using historical data in Equations (2.61) and (2.62),

$$R^V_{i,t} = R_{F,t} + \beta_{i,t}(R^V_{M,t} - R_{F,t}). \qquad (2.63)$$

$$\pi^V_{i,t} = \beta_{i,t}\pi^V_{M,t}. \qquad (2.64)$$

In this dissertation, the CAPM with value returns such as Models from (2.60) to (2.64) are named as the Rational CAPM while the CAPM with price returns as the conventional CAPM. The Rational CAPM describes the relationship of the value risk premia between assets and the market. The difference between the conventional and Rational CAPMs is in the way to apply the original ex-ante CAPM to ex-post historical data. For expectation, there is no difference between any CAPMs because expected mispricing is zero in any CAPM by the rationality and homogeneity assumptions. βs are the same between ex-ante and ex-post CAPMs and between conventional and Rational CAPMs (See Sections 2.4.2 and 2.4.3). The risk-free rates are also all the same by the definition that the risk-free rate is independent of the market.

Unlike the conventional CAPM that has α in Equations (2.20) and (2.21), the Rational CAPM does not have any deviation measures in Equations (2.63) and (2.64), i.e., value returns are always systematic, since the Rational CAPM always holds with historical value data because ex-ante value returns are the same as ex-post ones as explained in Section 2.3.1.1. In other words,

$$E(R^V_{i,t}) = R^V_{i,t} \text{ and } E(R^V_{M,t}) = R^V_{M,t}. \qquad (2.65)$$

Equation (2.65) means rationality and homogeneity in the CAPM. In addition, by applying the same way as for Equations (2.33) and (2.34) to Equation (2.60), we can verify that there is the perfect correlation between asset value returns and market value returns, i.e.,

$$COR(R^V_i, R^V_M) = 1. \qquad (2.66)$$

2.4.2 βs in Ex-post CAPM vs. Ex-ante CAPM

To apply the ex-ante CAPM to ex-post data, the ex-ante βs measured by the uncertainty (variance in ex-ante data) should be the same as the ex-post βs measured by the volatility (variance in ex-post data). In the CAPM, they are the same due to the perfect correlations between asset returns and the market returns. The perfect correlation allows any changes or expectations in the market returns to affect the changes or expectations in asset returns through β, just like mispricing in Section 2.3.2.4. Hence, the relationship in the volatility is the same as that in the uncertainty between assets and the market for the same period, implying that the ex-post βs are the same as the ex-ante βs. In addition, the assumption of homogeneous expectation is applied not only for the returns but also for the variance in the CAPM. By this assumption, ex-ante βs are the same as ex-post βs in the CAPM.

If the CAPM holds in historical data, Equation (2.1) becomes

$$R_i = R_F + \beta_i(R_M - R_F). \qquad (2.67)$$

For the CAPM to hold, the assumptions of rationality and homogeneity should be maintained as explained in Section 2.3.1.1, which means $R_i = E(R_i)$ and $R_M = E(R_M)$ in the CAPM. Hence, by comparing Equations (2.1) and (2.67), βs in ex-post and ex-ante CAPMs should be the same. Notice that the equality between ex-ante and ex-post βs does not mean invariance in time series but homogeneity for a specific period just like the equality between ex-ante and ex-post returns.

2.4.3 βs in Rational CAPM vs. Conventional CAPM

In the conventional CAPM, based on Equation (2.21), βs are usually measured by the following regression (Bodie et al., 2005).

$$\pi^P{}_{i,m} = c + \beta\pi^P{}_{M,m} + \varepsilon_{i,m}, \qquad (2.68)$$

where c is an intercept, ε represents residuals, and *m* denotes sub-periods for Period *t*. Here, β for Period *t* is measured using sub-period data of Period *t*, and thus it is assumed that β is stable during Period *t* as explained Section 2.2.1.1. Hence, β from Regression (2.68) is not only for Period *t* but also for all sub-periods in Period *t*, i.e., $\beta = \beta_{i,t} = \beta_{i,m}$. In addition, c represents the average sub-period α of all assets, i.e., $E(\alpha_{i,m})$ during Period *t*, and thus, $c + \varepsilon_{i,m}$ is the actual sub-period α, i.e., $\alpha_{i,m}$. Then,

$$c + \varepsilon_{i,m} = \pi^{P}{}_{i,m} - \beta\pi^{P}{}_{M,m} = \alpha_{i,m}. \tag{2.69}$$

By using Equation (1.15), price risk premia can be divided into value risk premia and irrational returns, and then Equation (2.68) becomes

$$(\pi^{V}{}_{i,m} + \omega^{R}{}_{i,m}) = c + \beta(\pi^{V}{}_{M,m} + \omega^{R}{}_{M,m}) + \varepsilon_{i,m}.$$

Then,

$$\pi^{V}{}_{i,m} = c + \beta\pi^{V}{}_{M,m} + \beta\omega^{R}{}_{M,m} - \omega^{R}{}_{i,m} + \varepsilon_{i,m}. \tag{2.70}$$

From Equation (2.42),

$$\omega^{R}{}_{i,m} = \alpha_{i,m} + \beta_{i,m}\omega^{R}{}_{M,m} = \alpha_{i,m} + \beta\omega^{R}{}_{M,m} \tag{2.71}$$

By substituting Equation (2.71) into Equation (2.70),

$\pi^{V}{}_{i,m} = c + \beta\pi^{V}{}_{M,m} + \beta\omega^{R}{}_{M,m} - (\alpha_{i,m} + \beta\omega^{R}{}_{M,m}) + \varepsilon_{i,m}$, and then,

$$\pi^{V}{}_{i,m} = \beta\pi^{V}{}_{M,m} + (c + \varepsilon_{i,m}) + \beta\omega^{R}{}_{M,m} - (\alpha_{i,m} + \beta\omega^{R}{}_{M,m}). \tag{2.72}$$

Then, by using Equation (2.69), $c + \varepsilon_{i,m} = \alpha_{i,m}$, and simplifying Equation (2.72),

$$\pi^{V}{}_{i,m} = \beta\pi^{V}{}_{M,m}. \tag{2.73}$$

Hence, βs in the Rational CAPM from Model (2.73) should be the same as those in the conventional CAPM from Model (2.68), implying that the regression process removes the unsystematic mispricing (α). To get βs for the Rational CAPM, the conventional CAPM (2.68) can be used. Even though both models yield the same βs, there is a big difference between them:

an intercept and residuals, both of which appear due to the mispricing. There are an intercept and residuals in the conventional CAPM (2.68) if mispricing exists in an asset. There are no intercept and residuals in the Rational CAPM (2.73) because no mispricing exists in any asset in the market.

In another way, βs can be computed as defined in Equation (2.23) using ex-post data. For this computation, β is divided into β^P and β^V, respectively; the former β measured by price data for the conventional CAPM and the latter β measured by value data for the Rational CAPM. In the conventional CAPM, β^P can be computed by

$$\beta^P{}_{i,t} = \frac{COV(R^P{}_i, R^P{}_M)}{VAR(R^P{}_M)} = \frac{E(R^P{}_{i,m} - E(R^P{}_{i,m}))(R^P{}_{M,m} - E(R^P{}_{M,m}))}{E(R^P{}_{M,m} - E(R^P{}_{M,m}))^2},$$

where m represents sub-periods of Period t. In the same way, for the Rational CAPM, β^V can be calculated by

$$\beta^V{}_{i,t} = \frac{COV(R^V{}_i, R^V{}_M)}{VAR(R^V{}_M)} = \frac{E(R^V{}_{i,m} - E(R^V{}_{i,m}))(R^V{}_{M,m} - E(R^V{}_{M,m}))}{E(R^V{}_{M,m} - E(R^V{}_{M,m}))^2}$$

$$= \frac{E([(R^P{}_{i,m} - \omega^R{}_{i,m}) - (E(R^P{}_{i,m}) - E(\omega^R{}_{i,m}))][(R^P{}_{M,m} - \omega^R{}_{M,m}) - (E(R^P{}_{M,m}) - E(\omega^R{}_{M,m}))])}{E((R^P{}_{M,m} - \omega^R{}_{M,m}) - [E(R^P{}_{M,m}) - E(\omega^R{}_{M,m})])^2}$$

$$= \frac{E([(R^P{}_{i,m} - E(R^P{}_{i,m})) - (\omega^R{}_{i,m} - E(\omega^R{}_{i,m}))][(R^P{}_{M,m} - E(R^P{}_{M,m})) - (\omega^R{}_{M,m} - E(\omega^R{}_{M,m}))])}{E((R^P{}_{M,m} - E(R^P{}_{M,m})) - (\omega^R{}_{M,m} - E(\omega^R{}_{M,m})))^2}. \tag{2.74}$$

For the left bracket, $[(R^P{}_{i,m} - E(R^P{}_{i,m})) - (\omega^R{}_{i,m} - E(\omega^R{}_{i,m}))]$, in the numerator of Equation (2.74),

$$R^P{}_{i,m} - E(R^P{}_{i,m}) = \{\alpha_{i,m} + (R_{F,m} + \beta^P{}_{i,t}(R^P{}_{M,m} - R_{F,m})\} - \{E(\alpha_{i,m}) + (R_{F,m} + \beta^P{}_{i,t}(E(R^P{}_{M,m}) - R_{F,m})\}$$

$$= \alpha_{i,m} - E(\alpha_{i,m}) + \beta^P{}_{i,t}(R^P{}_{M,m} - E(R^P{}_{M,m})). \text{ (Using (2.16) and (2.20))}$$

$$\omega^R{}_{i,m} - E(\omega^R{}_{i,m}) = (\alpha_{i,m} + \beta^P{}_{i,t}\omega^R{}_{M,m}) - \{E(\alpha_{i,m}) + \beta^P{}_{i,t}E(\omega^R{}_{M,m})\}$$

$$= \alpha_{i,m} - E(\alpha_{i,m}) + \beta^P{}_{i,t}(\omega^R{}_{M,m} - E(\omega^R{}_{M,m})). \text{ (Using Equation (2.42))}$$

Then,

$$(R^{P}_{i,m} - E(R^{P}_{i,m})) - (\omega^{R}_{i,m} - E(\omega^{R}_{i,m})) = \beta^{P}_{i,t}((R^{P}_{M,m} - E(R^{P}_{M,m})) - (\omega^{R}_{M,m} - E(\omega^{R}_{M,m}))). \tag{2.75}$$

By substituting Equation (2.75) into Equation (2.74),

$$\beta^{V}_{i,t} = \frac{\beta^{P}_{i,t}E((R^{P}_{M,m} - E(R^{P}_{M,m})) - (\omega^{R}_{M,m} - E(\omega^{R}_{M,m})))^2}{E((R^{P}_{M,m} - E(R^{P}_{M,m})) - (\omega^{R}_{M,m} - E(\omega^{R}_{M,m})))^2} = \beta^{P}_{i,t}. \tag{2.76}$$

As a result, β of the Rational CAPM is the same as that of the conventional CAPM, implying that the covariance process removes the unsystematic mispricing. To get βs for the Rational CAPM, price data can be used, just like the conventional CAPM.

A little explanation should be mentioned for Equation (2.75). In the conventional CAPM, $E(R^{P}_{i,m})$ and $E(\omega^{R}_{i,m})$ measure systematic returns and systematic mispricing as explained in Sections 2.2.2.1 and 2.3.2.1, respectively. In the calculations above, however, $E(R^{P}_{i,m})$ and $E(\omega^{R}_{i,m})$ represent ex-post mean returns and ex-post mean mispricing, respectively. Hence, $R^{P}_{i,m} - E(R^{P}_{i,m}) \neq \alpha_{i,m}$ and $\omega^{R}_{i,m} - E(\omega^{R}_{i,m}) \neq \alpha_{i,m}$, different from (2.22) and (2.41), respectively.

Why are βs the same between the conventional and Rational CAPMs? From the conventional CAPM using Equations (2.17) and (2.40),

$$\beta_i = \frac{E(\pi^{P}_{i})}{\pi^{P}_{M}} = \frac{E(\omega^{R}_{i})}{\omega^{R}_{M}}. \tag{2.77}$$

Then,

$$E(\pi^{P}_{i})\omega^{R}_{M} = \pi^{P}_{M}E(\omega^{R}_{i}), \tag{2.78}$$

Subtracting $E(\pi^{P}_{i})\pi^{P}_{M}$ from both sides of Equation (2.78) yields

$E(\pi^{P}_{i})(\omega^{R}_{M} - \pi^{P}_{M}) = \pi^{P}_{M}(E(\omega^{R}_{i}) - E(\pi^{P}_{i}))$, and then by using Equation (1.15)

$E(\pi^{P}_{i})(-\pi^{V}_{M}) = \pi^{P}_{M}(-\pi^{V}_{i})$ because $E(\pi^{V}_{i}) = \pi^{V}_{i}$, and thus

$$\frac{E(\pi^{P}_{i})}{\pi^{P}_{M}} = \frac{\pi^{V}_{i}}{\pi^{V}_{M}}. \tag{2.79}$$

From Equations (2.77) and (2.79),

$$\beta_i = \frac{E(\omega^R{}_i)}{\omega^R{}_M} = \frac{E(\pi^P{}_i)}{\pi^P{}_M} = \frac{\pi^V{}_i}{\pi^V{}_M}. \tag{2.80}$$

The right-hand side of Equation (2.80) can be obtained from the Rational CAPM (2.64). Equation (2.80) implies that all the same are the βs measured by systematic mispricing, systematic price returns and value returns. Both regression and covariance using price returns remove (or separate) the unsystematic mispricing (or risk) from the price returns, finding the systematic mispricing (or risk) of the price returns. Therefore, β measured by price returns is the same as β measured by value returns; β from the conventional CAPM is the same as β from the Rational CAPM.

2.4.4 Rational vs. Conventional CAPM

Rearranging Equation (2.63) with Equation (1.11) yields

$$R^V{}_{i,t} = R_{F,t} + \beta_{i,t}(R^P{}_{M,t} - \omega^R{}_{M,t} - R_{F,t}), \tag{2.81}$$

and then,

$$R^V{}_{i,t} = R_{F,t} + \beta_{i,t}(R^P{}_{M,t} - R_{F,t}) - \beta_{i,t}\omega^R{}_{M,t}. \tag{2.82}$$

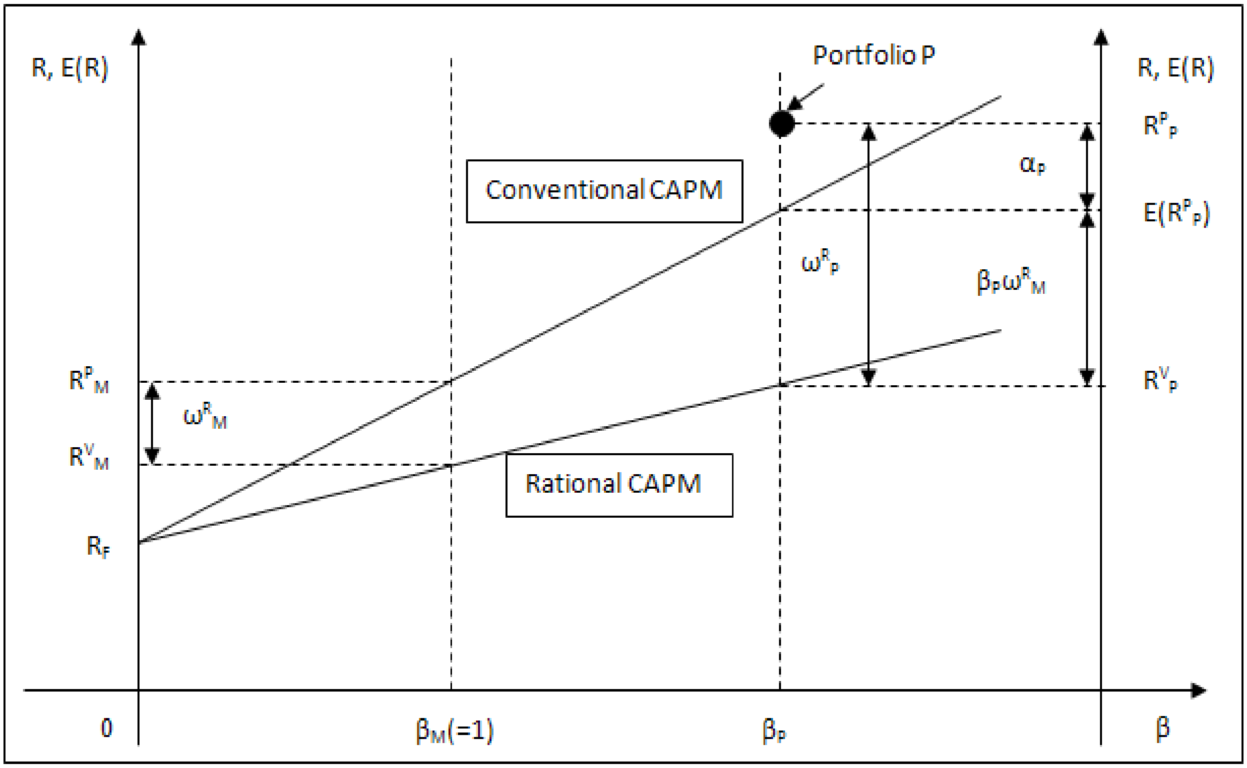
Equation (2.82) describes that the market irrationality (mispricing), if not removed from the market price returns, is conveyed to asset returns through β, causing assets to have the systematic mispricing. Hence irrational returns of the market should be removed from its price returns to measure value returns of an asset. By substituting Equation (2.16) into Equation (2.82),

$$R^V{}_{i,t} = E(R^P{}_{i,t}) - \beta_{i,t}\omega^R{}_{M,t}. \tag{2.83}$$

$E(R^P)$ is systematic price returns from the conventional CAPM. The Rational CAPM removes irrational returns of the market from the conventional CAPM, and thus yields value returns of an asset. Hence, even if the market is irrational, the Rational CAPM produces the rational equilibrium returns or value returns of an asset using the rational or value returns of the efficient market portfolios. Equations (2.82) and (2.83) show the difference between conventional and

Figure 2.1 Conventional and Rational CAPMs

This figure illustrates the difference between conventional and Rational CAPMs. R^P, R^V and ω^R (= $R^P - R^V$) denote price returns, value returns and irrational returns, respectively. Subscripts *M* and *P* indicate the market (portfolio) and a portfolio in the market, respectively. α and β represent unsystematic mispricing and systematic risk, respectively. R_F denotes risk-free rate, and E(   ) denotes an expectation.



Rational CAPMs. If the market is rational, i.e., irrational returns of the market (ω^R_M) are zero, then both models yield the same returns, but their meanings are still different: price returns from the conventional CAPM vs. value returns from the Rational CAPM.

Figure 2.1 illustrates the difference between conventional and Rational CAPMs. As seen in Figure 2.1, if the irrational return of the market exists, the higher the β, the larger is the gap between conventional and Rational CAPMs, implying that the market irrationality is conveyed to asset returns through β.

Substituting Equation (2.83) into Equation (1.11) yields

$\omega^R_{i,t} = R^P_{i,t} - R^V_{i,t} = R^P_{i,t} - [E(R^P_{i,t}) - \beta_{i,t}\omega^R_{M,t}]$, and then,

$$\omega^R_{i,t} = [R^P_{i,t} - E(R^P_{i,t})] + \beta_{i,t}\omega^R_{M,t}. \tag{2.84}$$

Then, by substituting Equation (2.22) for Equation (2.84),

$$\omega^R_{i,t} = \alpha_{i,t} + \beta_{i,t}\omega^R_{M,t}. \tag{2.85}$$

Equation (2.85) is the same as Equation (2.42). For the market portfolio,

$$\omega^R_{M,t} = \alpha_{M,t} + \beta_{M,t}\omega^R_{M,t}.$$

$\beta_{M,t} = 1$ by Equation (2.13), and thus $\alpha_{M,t} = 0$ regardless of irrational returns of the market; the market (portfolio) has no unsystematic mispricing, but it can have (systematic) mispricing. In the same way as for the mispricing in Section 2.3.2.1, Equation (2.85) can be interpreted as

Irrationality = Unsystematic irrationality + Systematic irrationality,

with

Unsystematic irrationality = Jensen's α, and

Systematic irrationality = Systematic risk × Irrationality of the market.

Irrationality is the same as mispricing as explained in Section 1.2.8, and thus indicates risk as explained in Section 2.3.2.

2.4.5 Value Returns of the Market and an Asset

2.4.5.1 Value returns of the market. As seen in the Rational CAPM, Equations (2.60) to (2.64), value returns of the market should be first measured to compute value returns of assets. From Equation (2.64) with π^V_i replaced by $\pi^P_i - \omega^R_i$ using Equation (1.15),

$$\pi^P_{i,t} - \omega^R_{i,t} = \beta_{i,t}\pi^V_{M,t}, \text{ and then}$$

$$\pi^P_{i,t} = \omega^R_{i,t} + \pi^V_{M,t}\beta_{i,t}. \tag{2.86}$$

Hence, by Equation (2.86), the following regression yields the value risk premium of the market for Period *t*.

$$\pi^P_{i,t} = m + p\beta_{i,t} + \xi_{i,t}, \tag{2.87}$$

where βs can be measured by the conventional CAPM (2.68) as explained in Section 2.4.3. In Regression (2.87), $m = E(\omega^R_{i,t})$, $p = \pi^V_{M,t} = R^V_{M,t} - R_{F,t}$ and $\xi_{i,t} = \omega^R_{i,t} - E(\omega^R_{i,t})$; $E(\omega^R_{i,t})$ is average total mispricing here, different from Equation (2.40) in which $E(\omega^R_{i,t})$ is the systematic mispricing (See Section 1.2.1). Hence, m captures the simple average irrational returns of the sample (because $E(\xi_{i,t}) = 0$), and p measures value risk premium of the market estimated by the sample. To measure the value risk premia of the market, the sample should be the market portfolio. Value returns of the market can be computed from the coefficient (p) of βs in Regression (2.87), i.e.,

$$R^V_{M,t} = p + R_{F,t}. \tag{2.88}$$

$$\omega^R_{M,t} = R^P_{M,t} - R^V_{M,t}. \text{ (From Equation (1.11))} \tag{2.89}$$

2.4.5.2 Comparison to conventional CAPM. Based on Equation (2.21), Model (2.87) has been used in the conventional CAPM. However, interpretations on intercept (m) and coefficient (p) of βs are different between Rational and conventional CAPMs. In the conventional CAPM, $m = E(\alpha_{i,t}) = E(\omega^R_{i,t}) - \beta_{i,t}\omega^R_{M,t}$, $p = \pi^P_{M,t} = R^P_{M,t} - R_{F,t}$ and $\xi_{i,t} = \alpha_{i,t} - E(\alpha_{i,t})$; E(   ) denotes average. This difference between Rational and conventional CAPMs is caused by the different interpretation on the (original) CAPM. The conventional CAPM implies that price data should work while the Rational CAPM explains that value data should be used. In other words, the conventional CAPM does not consider mispricing or irrationality while the Rational CAPM incorporates it.

By replacing π^V_M by $\pi^P_M - \omega^R_M$ in Equation (2.86),

$$\pi^P_{i,t} = \omega^R_{i,t} + \beta_{i,t}(\pi^P_{M,t} - \omega^R_{M,t}), \tag{2.90}$$

and then by rearranging,

$$\pi^P_{i,t} = (\omega^R_{i,t} - \beta_{i,t}\omega^R_{M,t}) + \beta_{i,t}\pi^P_{M,t}. \tag{2.91}$$

Equations (2.90) and (2.91) show the difference between Rational and conventional CAPMs when tested with historical data. Equation (2.90) explains what is captured by m and p in the Rational CAPM while Equation (2.91) describes what is expected by m and p in the conventional CAPM. The difference is where the systematic mispricing ($\beta_{i,t}\omega^{R}_{M,t}$) belongs. The Rational CAPM explains that p will capture the value risk premium of the market that reflects irrational returns of the market, and that m will measure total mispricing of assets. On the other hand, the conventional CAPM describes that p measures the price risk premium of the market, and that m represents the unsystematic mispricing that reflects the systematic mispricing.

When Regression (2.87) is performed, which one captures irrational returns of the market, intercept (m) or coefficient (p) of βs? Which grouping indicates the results from Regression (2.87), Equation (2.90) or (2.91)? The irrational returns of the market must be captured by p, not by m. Equation (2.90) is better than Equation (2.91) to explain the results from Regression (2.87). In Model (2.87), p captures the value, not price, risk premium of the market, and m represents average total, not unsystematic, mispricing of assets. This explains why and how the conventional CAPM does not work in empirical studies.

2.4.5.3 Value returns of an asset. If value returns of the market are measured, value returns of other assets can be assessed by the Rational CAPM (2.60). Hence, the method for the market portfolio, called the market method, is a one-step method whereas the method for other assets, called Dow Jones method because Dow Jones portfolio is used to explain this method in Section 2.5.1.2, is a two-step method. To measure value returns, the market or one-step method uses only Model (2.87) while the Dow Jones or two-step method uses Equation (2.60) after Model (2.87) for the market value returns. (For more details about these methods, see Section 2.5.1.)

2.4.6 Ex-ante Rational CAPM

The Rational CAPM is an application of the (original) ex-ante CAPM to ex-post historical data with the different interpretation from the conventional CAPM; the Rational CAPM uses value data whereas the conventional CAPM uses price data. Hence, both Rational and conventional CAPMs are ex-post models while the (original) CAPM is ex-ante model. For future estimation, there is no difference among all CAPMs because expected mispricing is zero for all CAPMs, which assume the investors' rationality and homogeneity on the future. For future estimation, therefore, value returns of the market are the same as price returns of the market in the Rational CAPM.

2.5 Methodology and Sample

2.5.1 Methodology: Measurement of Historical Value Returns and Irrationalities

This section demonstrates how to measure value returns and irrational returns of the market and other assets step by step. As explained in Section 2.4.5, there are two ways to measure value returns: market method and Dow Jones method. The market method is a one-step method while the Dow Jones method is a two-step method. Value returns can be measured for any portfolio by using one or both of these methods.

2.5.1.1 Market (portfolio): market method. The market portfolio is indispensable in the CAPM. The market portfolio consists of all stocks in the market for every period, and thus its components are frequently changing, not restricted in numbers. The first thing to do in the Rational CAPM is to measure value returns of the market, which can be measured using Model (2.87) as explained in Section 2.4.5.

For Model (2.87), annual βs of all assets for every Year *t* are measured by Model (2.68) using monthly returns of Year *t*. After performing Model (2.87) using annual data for the market portfolio, the following measures of the market can be computed;

$R^V_{M,t} = p + R_{F,t}$, where p is the coefficient of β in Model (2.87) (from Equation (2.88)).

$\omega^R_{M,t} = R^P_{M,t} - R^V_{M,t}$ (from Equation (2.89)).

2.5.1.2 Dow Jones portfolio: Dow Jones method. The Dow Jones portfolio is the portfolio that consists of constituents of the Dow Jones Industrial Average (DJIA) index. The DJIA index is one of representative stock market indices. It is composed of relatively small number of stocks with a relatively low frequency of changes like an equity fund with a small number of stocks. Hence, the way to measure value returns and irrationalities of the Dow Jones portfolio is different from those of the market (portfolio).

To measure value returns and irrationalities of the DJIA index, value returns of the market should be first measured; this is why this method is called a two-step method. The following steps show how to measure value returns and irrationalities of the DJIA index. In Step 7, value returns of the market are used. Hereafter, Subscript *D* denotes the Dow Jones portfolio.

1. Obtain the Dow Jones portfolio each year.
2. Compute monthly and annual price growths of the Dow Jones portfolio, $G^P_{D,m}$ and $G^P_{D,t}$, respectively. Growth means price appreciation, and thus does not include dividends. Price growths are computed by % changes in the DJIA indices. For annual price growths,

   $G^P_{D,t} = \dfrac{DJIA_{D,t} - DJIA_{D,t-1}}{DJIA_{D,t-1}}$, where DJIA denotes DJIA index at the end of year.
3. Get monthly dividend-price ratios of the Dow Jones portfolio, $\delta_{D,m} = TD_{D,m}/TP_{D,m-1}$, where δ denotes dividend-price ratio while TD and TP denote total dividends and total price of the portfolio. $TD_{D,m} = \sum_{j=1}^{30}(D_{j,m})$ and $TP_{D,m-1} = \sum_{j=1}^{30}(P_{j,m-1})$, where *j* and *m*

represent each component of the Dow Jones portfolio and each month of the year *t*, respectively.

4. Compute monthly price returns of the Dow Jones portfolio, $R^P_{D,m} = G^P_{D,m} + \delta_{D,m}$.
5. Compute annual price returns of the Dow Jones portfolio, $R^P_{D,t} = \prod_{m=1}^{12}(1 + R^P_{D,m}) - 1$.
6. Measure the annual βs of the Dow Jones portfolio, $\beta_{D,t}$, by Equation (2.68) in Section 2.4.3; individual stock (Subscript *i*) should be replaced by the Dow Jones portfolio (Subscript *D*). Annual βs of Dow Jones portfolio for Year *t* are measured using monthly returns of Year *t* in this chapter.
7. Compute annual value returns of the Dow Jones portfolio, $R^V_{D,t}$, using the Rational CAPM (2.61) with the value returns of the market from Equation (2.88).

   $R^V_{D,t} = R_{F,t} + \beta_{D,t}(R^V_{M,t} - R_{F,t})$.
8. Compute annual irrational returns of the Dow Jones portfolio.

   $\omega^R_{D,t} = R^P_{D,t} - R^V_{D,t}$. (From Equation (1.11))

2.5.1.3 Internet portfolio. The Internet bubble (= bubble in the Internet firms) is not only most recent but also biggest of all stock market bubbles. Hence, measuring irrationalities of the Internet portfolio will reveal how significant the Internet bubble was. The Internet portfolio is similar to the market portfolio; it consists of all Internet stocks in the market, not restricted in numbers, and frequently changing. However, the Internet portfolio is a part of the market portfolio, like the Dow Jones portfolio. Consequently, value returns and irrationalities of the Internet portfolio can be measured either by the market method (the method used for the market portfolio) if all Internet firms can be identified every year or by the Dow Jones method (the method used for the Dow Jones portfolio) if the Internet portfolio can be constructed by some fixed number of representative stocks with low frequency of changing.

2.5.1.3.1 Market method. For the market method, the Internet portfolio should include all Internet firms traded publicly (See Section 2.5.3.1), and then should be treated in the same way as the market portfolio; in other words, suppose that the market consists of only the Internet firms and all Internet firms consist in the market. Hence, to measure value returns and irrational returns of the Internet portfolio, follow the same way as for the market portfolio in Section 2.5.1.1; perform Model (2.87) using the Internet portfolio, and then compute value returns and irrational returns of the Internet portfolio using Equations (2.88) and (2.89) with the market portfolio (Subscript *M*) replaced by the Internet portfolio (Subscript *N*). Hereafter, Subscript *N* denotes the Internet portfolio.

$R^V_{N,t} = p + R_{F,t}$. (From Equation (2.88))

$\omega^R_{N,t} = R^P_{N,t} - R^V_{N,t}$. (From Equation (2.89))

2.5.1.3.2 Dow Jones method. For the Dow Jones method, the Internet portfolio can be constructed to include majority of representative internet firms through the whole sample period. To exactly reflect the Internet bubble (both boom and crash) and to maintain the consistency through the sample period, the components of the Internet portfolio should experience the whole sample period. If the Internet portfolio is made in this way, then the Dow Jones method can be used to measure value returns and irrationalities of the Internet portfolio.

1. Obtain the Internet portfolio (See Section 2.5.3.1).
2. Get monthly and annual price returns of the Internet portfolio, $R^P_{N,m}$ and $R^P_{N,t}$.
3. Follow all the steps from Step 6 in Section 2.5.1.2; replace “the Dow Jones portfolio” by “the Internet portfolio”.

2.5.2 Methodology: Examination of Irrationalities

The purpose of this section is to verify the validity of the Rational CAPM. To use and test the Rational CAPM, we need value data as seen in Section 2.4.1, but value data are unobservable. However, the Rational CAPM produces value returns and irrational returns as explained in Section 2.5.1. Hence, this chapter examines the Rational CAPM indirectly by examining irrational returns. As explained in Section 1.2.7, the irrational returns are the measure of irrationality. Section 2.3.1.5 explains that irrationality would be closely related to the investor sentiment because both represent the noise. To verify this argument, this research compares the irrationality in various ways to the consumer confidence index (CCI), which is a widely used measure of investor sentiment. If the irrationality well explains the investor sentiment and the market, it can be said that the Rational CAPM works with historical data.

The Internet portfolio has only twelve observations or measurements from 1998 to 2009 for each variable. This sample size is too small to get meaningful results from the empirical work. Hence, all empirical work in this section excludes the Internet portfolio, and thus is performed only for the market and Dow Jones portfolios.

2.5.2.1 Relationship among irrationality, price returns and CCI. As a preliminary test for the relationship among irrationality, price returns, CCI and CCI^G, correlations are measured and examined among them; CCI and CCI^G respectively denote level and growth of consumer confidence index (CCI). Then, to verify their relationships mentioned in Section 2.3.1.5, which explains that irrationality affects price changes, and price changes affect sentiment, ordinary least squares (OLS) regressions are used for contemporaneous relationships while a vector autoregression (VAR) model for dynamic relationships. For the OLS regressions,

$$CCI^G_t = p_1 + q_1\omega^R_{P,t} + r_1R^P_{P,t} + u_{1,t}, \tag{2.92}$$

$$R^P_{P,t} = p_2 + q_2\omega^R_{P,t} + r_2CCI^G_t + u_{2,t}, \tag{2.93}$$

$$\omega^R_{P,t} = p_3 + q_3R^P_{P,t} + r_3CCI^G_t + s\omega^R_{P,t-1} + u_{3,t}. \tag{2.94}$$

CCI^G is replaced by CCI for all regressions to examine which one is more closely related to irrationality and price returns. One lagged term is included in Model (2.94) due to persistence of irrationality.

For the VAR model,

$$\mathbf{L_t} = \mathbf{C} + \sum_{l=1}^{l}[M_l\mathbf{L_{t\text{-}l}}] + \mathbf{N_t}, \tag{2.95}$$

where $\mathbf{L_t} = [\omega^R_t \;\; R^P_t \;\; CCI^G_t]'$, 3×1 vector of variables at year t, **C** is the 3×1 vector of constants, M_l is the 3×3 matrix of coefficients for $\mathbf{L_{t\text{-}l}}$, $\mathbf{N_t}$ is the 3×1 vector of shocks in $\mathbf{L_t}$, and l indicates the lag length, which is determined by lag selection criteria. All variables are stationary (See Table 2.5). To determine the lag length, the sequential modified likelihood ratio (LR) test is used because LR is better for the small sample size than others (Hatemi-J and Hacker, 2009). LR test with maximum lags =5 yields two lags (l =2) for the market portfolio and three lags (l =3) for the Dow Jones portfolio. This VAR is performed using the Cholesky decomposition, which solves the possible serial correlation in error terms (Stock and Watson, 2001).

2.5.2.2 Irrationality vs. CCI: effect on subsequent stock performance. To compare irrationality to CCI, the following popular regression in the sentiment literature is conducted for each portfolio.

$$APR_{P,t+i} = a_1 + b_1X_t + c_1L^1APR_{P,t+i} + d_1\mathbf{ME}_t + e_{1,t+i}, \tag{2.96}$$

where APR denotes average price returns for the next i years, i.e., $APR_{P,t+i} = \sum_{\tau=t+1}^{t+i}\{R^P_{P,\tau}\}/i$, L^1 denotes one-year lagged term, i.e., $L^1APR_{P,t+i} = \sum_{\tau=t}^{t+i-1}\{R^P_{P,\tau}\}/i$, X denotes one of three sentiment measures; ω^R_P is used for each portfolio P while CCI and CCI^G are used for both market and Dow Jones portfolios, and **ME** represents a vector of macro-economic control variables of inflation (= Consumer Price Index growth), real activity (= Industrial Production Index growth) and interest rate (= risk-free rate), all of which are expected to affect, or interact with, stock returns (Fama, 1981; Geske and Roll, 1983; Ram and Spencer, 1983; James et al., 1985; Lee, 1992). For both portfolios, the lagged term is excluded when i = 1 whereas included when i ≥ 2 due to the persistence in APR. To check the robustness to the control variables, all regressions are performed both with and without macro-economic control variables.

In addition, to examine whether the sentiment affects value changes, the following regression is performed in the same way as for Model (2.96).

$$AVR_{P,t+i} = a_2 + b_2X_t + c_2L^1AVR_{P,t+i} + d_2\mathbf{ME}_t + e_{2,t+i}, \qquad (2.97)$$

where AVR denotes average value returns for the next *i* years, i.e., $AVR_{P,t+i} = \sum_{\tau=t+1}^{t+i}\{R^V_{P,\tau}\}/i$, and all other variables are defined in the same way as in Model (2.96).

For CCI, this research employs the University of Michigan survey index, which is available from 1952. Hence, Models (2.96) and (2.97) are performed for the period from 1953 (due to CCI^G) to 2009 to compare the results between irrationality and CCI. Then, both models are performed for the full sample period, 1926 to 2009 for the market and 1929 to 2009 for the Dow Jones, only for irrationality to generalize the findings on the relationship between irrationality and future stock performance.

2.5.2.3 Investor sentiment and monetary policy. As mentioned in Section 2.2.2.5, Kurov (2010) claims that the investor sentiment is affected by the monetary policy and that the effect of monetary policy on the sentiment is stronger in the bear market than in the bull market. To examine this claim, this research employs the following regression:

$$X_t = a_0 + b_0UP\times R_{FF,t} + c_0(1–UP)\times R_{FF,t} \ (+ d_0X_{t-1}) + e_{0,t}. \qquad (2.98)$$

In Model (2.98), X denotes one of three sentiment measures; ω^R_P for each portfolio *P* while CCI and CCI^G for both market and Dow Jones portfolios, and R_{FF} denotes the U.S. Federal Reserve (Fed) fund rates. UP is a dummy variable to indicate positive returns of each portfolio, and thus unity when $R^P_{P,t} \geq 0$ and zero otherwise for each portfolio *P*. One lagged term is included only for CCI due to its persistence; CCI represents levels while both CCI^G and ω^R measure changes during a certain period. In Model (2.98), Fed fund rates represent the monetary policy (Bernanke and Blinder, 1992) while the dummy variable (UP) separates the market into the bull and bear markets.

Model (2.98) is performed for the period from 1955 to 2009 by three sentiment measures because annual Fed fund rates are available from 1955. Moreover, it is well documented that the Fed more aggressively responds to inflation since 1979 than before 1979, when Volcker was appointed as Fed Chairman (Mehra, 1999, Clarida, et al., 2000; Dennis, 2006; Berument and Froyen, 2006; Boivin and Giannoni, 2006). Thus, using three sentiment measures, Model (2.98) is also performed for two sub-periods, one from 1955 to 1978 and the other from 1979 to 2009, to investigate whether the changes in monetary policy have any effect on the forecasting power of the investor sentiment over future stock performance.

2.5.3 Sample

2.5.3.1 Sample selection. All firms in the Center for Research in Security Prices (CRSP) data will be included in the market portfolio. For the market portfolio, the sample period is from 1926 to 2009 because the full year data are available from 1926 in the CRSP. For the Dow Jones portfolio, each year-end historical components of the DJIA index will be used. I assume that year-end components of DJIA index had existed from the beginning of the same year. For the Dow Jones portfolio, the sample period is from 1929 to 2009 because the number of Dow Jones components became 30 in 1928.

The Internet portfolio for the market method will include the following firms: firms in the List of Publicly Traded Internet Firms maintained by Internet.com's Wall Street Research Net as

Table 2.2 Internet Portfolio for the Dow Jones Method

This Table illustrates 17 components of the Internet portfolio for the Dow Jones method to measure a bubble. Only these firms had existed from 1998 till 2009 without missing data.

	Internet portfolio for the Dow Jones method
1	CISCO SYSTEMS INC
2	TIME WARNER INC (A O L)
3	IAC INTERACTIVECORP
4	MODUSLINK GLOBAL SOLUTIONS INC (C M G I)
5	OPEN TEXT CORP
6	I D T CORP
7	EARTHLINK INC (Mindspring enterprises)
8	YAHOO INC
9	S 1 CORP
10	FACTSET RESEARCH SYSTEMS INC
11	E TRADE FINANCIAL CORP
12	TERREMARK WORLDWIDE INC (Amtec)
13	T D AMERITRADE HOLDING CORP
14	BLUEFLY INC
15	ONSTREAM MEDIA CORP (Visual Data)
16	TRADESTATION GROUP INC (Omega Research)
17	REALNETWORKS INC

of August 31, 2000 from Davis (2002), and Nasdaq internet index components as of Nov. 27, 2007 and Jan. 31, 2010. I tried to find all Internet firms during the sample period from 1998 to 2009. Davis' list includes almost all Internet firms before 2000. For the Internet firms which began to start after 2000, I used Nasdaq Internet Index because this index includes the greatest number of Internet firms among all Internet Indices. However, the launching date of this index is Nov. 27, 2007. Consequently, the Internet portfolio in this chapter does not include the Internet firms which had existed only between August 31, 2000 and Nov. 27, 2007. This portfolio includes 394 firms, and I assume that this portfolio includes all Internet firms from 1998 to 2009.

The Internet portfolio for the Dow Jones method includes 17 firms (See Table 2.2), which have no missing data in any month from 1998 till 2009. To measure value returns and irrationalities of the Internet portfolio, the Dow Jones method can be used only if the Internet portfolio should be made of some Internet firms that had experienced the Internet boom and bust to exactly measure both Internet boom and bust and are still surviving in the market to measure the current returns. Further, this portfolio should be representative of the Internet world. However, only 17 firms have experienced the whole sample period from 1998 to 2009.

The following firms are excluded from the sample each year, not from the whole sample period.

1. Firms with missing data in any month.
2. Firms that have one of the followings.
    a. Negative beta
    b. Extreme positive returns or betas
        i. Greater than or equal to 20
        ii. Greater than or equal to 10 if greater than the assumed outlier, which is more-than-30 % greater than the next highest one
3. Firms that have a R^V less than -1.

The following data are collected from the CRSP: price returns of the market (R^P_M) weighted by the market capitalization, price returns of the Internet portfolio (R^P_N) weighted by the market capitalization, all individual stock price returns (R^P_i), and monthly prices ($P_{j,m}$) and dividends ($D_{j,m}$) of firms in the Dow Jones portfolio. DJIA indices are obtained from Yahoo! (http://finance.yahoo.com/q?s=^DJI&ql=1), and risk-free rates (R_F) from the French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/).

Consumer Confidence Index (CCI), Consumer Price Index (CPI), Industrial Production Index (IPI) and Fed fund rates (R_{FF}) are collected from the website of Federal Reserve Bank of St. Louis (http://research.stlouisfed.org/fred2/). Inflation is measured by CPI growth (CPI^G),

$$CPI^G_t = \frac{CPI_t - CPI_{t-1}}{CPI_t}.$$

Table 2.3 Descriptive Statistics of Price and Macro-Economy Data

This table reports the descriptive summary statistics of the annual price and macro-economy data. The sample periods are from 1926 to 2009 for the market portfolio and macro-economy except R_{FF}, CCI and CCI^G, from 1929 to 2009 for the Dow Jones portfolio, and from 1998 to 2009 for the Internet portfolio. The sample period of R_{FF} is from 1955 to 2009. The sample periods of CCI and CCI^G are from 1952 and 1953, respectively, to 2009. R^P denotes price returns, and subscripts *M*, *D* and *N* indicate the market (portfolio), the Dow Jones portfolio and the Internet portfolio, respectively. R_F, R_{FF}, IPI, CPI and CCI denote risk-free rates, Fed fund rates, industrial production index, consumer price index, and consumer confidence index, respectively. Superscript *G* means growth rate (= rate of changes).

		n	Minimum	Maximum	Mean	Std. Dev.	t-values[2]
Price returns	R^P_M	84	-.4436	.5750	.116110	.2060868	5.164**
	R^P_D	81	-.4943	.7096	.107612	.2032064	4.766**
	R^P_N[1]	12	-.8284	2.1042	.467886	.9612670	1.686
Macro-economy	R_F	84	-.0004	.1472	.037081	.0308720	11.008**
	R_{FF}	55	.0016	.1638	.055647	.0328050	12.580**
	IPI^G	84	-.2704	.2422	.036003	.0969646	3.403**
	CPI^G	84	-.1027	.1813	.030934	.0420279	6.746**
	CCI^G	57	-.2762	.3343	.004343	.1227389	.267
	CCI	58	59.50	105.40	86.2759	11.87771	55.319**

1. Internet portfolio from the market method
2. Two-tailed mean test (test value = 0): ** = p<0.01, the others = p>0.05

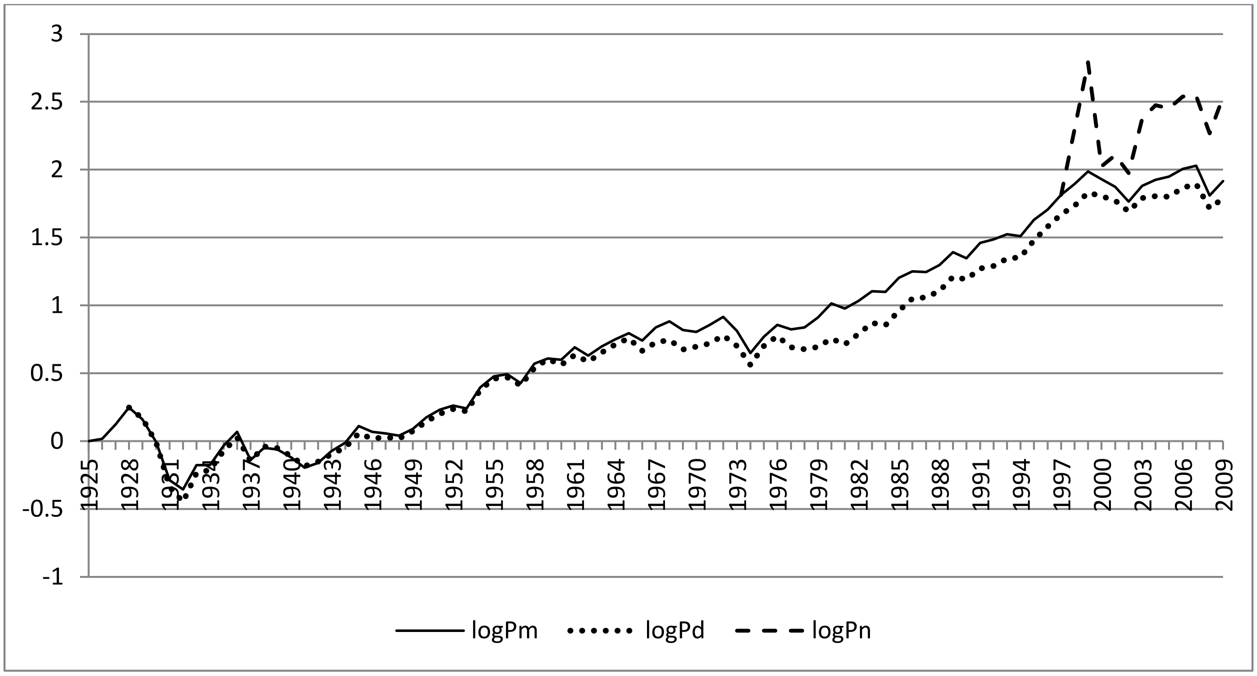
IPI growth (IPI^G) and CCI growth (CCI^G) are calculated in the same way as CCI^G. For CCI, this research uses year-end indices of the University of Michigan survey, which is available from 1952, and has been released on a monthly basis since 1978 while on a quarterly basis for months 2, 5, 8 and 11 until 1978; hence, indices for month 11 are used as the year-end indices before 1978. CCI of the first quarter beginning February 1966 is set to 100.

2.5.3.2 Descriptive statistics. Table 2.3 shows the descriptive summary statistics of annual price returns from 1926 to 2009 for the market portfolio, from 1929 to 2009 for the Dow Jones portfolio, and from 1998 to 2009 for the Internet portfolio. On average, the price returns are 11.6% for the market while 10.8% for the Dow Jones; the market earns more than the Dow Jones. The Internet firms on average earn the highest returns and show the highest volatility.

Table 2.3 also shows the descriptive summary statistics of annual macro-economy data: risk-free rate (R_F), IPI^G and CPI^G from 1926 to 2009, CCI and CCI^G from 1952 and 1953, respectively, to 2009, and Fed fund rate (R_{FF}) from 1955 to 2009. R_F is highly correlated to R_{FF}

Figure 2.2 Price Movements: Market vs. Dow Jones and Internet Portfolios

This figure illustrates historical price movements of the market (portfolio), the Dow Jones Industrial Average (DJIA) index and the Internet portfolio. The prices are scaled by the common logarithm (the logarithm with base 10). The market price for 1925 is fixed at 1. The prices of both Dow Jones and Internet portfolios are adjusted for comparison; they are adjusted to have the same level as the market price in 1928 and 1997, respectively. Pm (= P_M), Pd (= P_D) and Pn (= P_N) denote prices of the market, the DJIA index and the Internet portfolio, respectively. Year represents the end of each year.



with the correlation of .991, and on average a little higher than inflation (= CPI^G). IPI^G is nearly at the same level as, but more volatile than, risk-free rates. The average CCI is 86.3, and the average growth of CCI is 0.43%, which is not significantly different from zero. CCI was highest (105.4) at the end of 1999 while lowest (59.5) at the end of 1974, and CCI^G was highest (33.43%) at the end of 1992 while lowest (-27.62%) at the end of 1990. For inflation, its average is 3.1%, and it was highest (18.13%) in 1946 while lowest (–10.27%) in 1932. For a longer term period, it was –23.8% during the period from 1930 to 1932 in Great Depression whereas 51.4% during the period from 1978 to 1981 around Volcker's appointment as Fed Chairman.

Figure 2.2 plots the price movements of the market, DJIA index and the Internet portfolio.

The prices are scaled by the common logarithm (the logarithm with base 10). The market price for 1925 is fixed at 1. The prices of both Dow Jones and Internet portfolios are adjusted for comparison; they are adjusted to have the same level as the market price in 1928 and 1997, respectively. Figure 2.2 reveals that both the market and the DJIA index had moved very closely until mid-1960s, but since then the market price has been always higher than the DJIA index relative to their 1928 prices. This gap between them was made late 1960s and widened late1970s (this will be discussed later in Section 3.6.1.3.4), and then has been kept relatively stable since then. Figure 2.2 also shows that Internet stocks on average have earned higher than others relative to their 1997 prices, and that Internet stocks are more volatile than others in terms of price movements.

2.6 Empirical Results

2.6.1 Measurement of Value Changes and Irrationalities

2.6.1.1 Summary statistics. For the Internet portfolio, measurements from the market method are used for the following reasons. First, the whole Internet portfolio is better to represent the Internet world than the partial Internet portfolio constructed by only 17 firms. Second, the results from the market method are better to explain the Internet boom and bust and post-Internet bubble periods than those from the Dow Jones method as seen later in Chapter III (See Figure 3.4).

According to Table 2.4, compared to Table 2.3, for all three portfolios, prices on average have earned more than values, and thus their irrational returns have all positive means, which are not significantly different from zero. Table 2.4, compared to Table 2.3, reports that for all three portfolios, price returns have larger standard deviations than value returns, meaning that prices

Table 2.4 Descriptive Statistics of Value and Irrationality Data

This table reports the descriptive summary statistics of the annual value returns and irrational returns of three portfolios: market, Dow Jones and Internet portfolios. Section 2.5.1explains how to measure them. The sample periods are from 1926 to 2009 for the market portfolio, from 1929 to 2009 for the Dow Jones portfolio, and from 1998 to 2009 for the Internet portfolio. R^V and ω^R denote price returns and irrational returns, respectively. Subscripts *M*, *D* and *N* denote the market portfolio, Dow Jones portfolio and Internet portfolio, respectively. For the Internet portfolio, R^V and ω^R are obtained from the market method.

	n	Minimum	Maximum	Mean	Std. Dev.	t-values[1]
R^V_M	84	-.1429	.6033	.105180	.1383994	6.965**
ω^R_M	84	-.3796	.3667	.010930	.1529118	.655
R^V_D	81	-.1482	.4878	.100182	.1265471	7.125**
ω^R_D	81	-.3460	.3759	.007430	.1575095	.425
R^V_N	12	-.2037	2.0216	.281696	.5951320	1.640
ω^R_N	12	-.9451	1.9927	.186189	.8024967	.804

1. Two-tailed mean test (test value = 0): ** = p<0.01, the others = p>0.05

Table 2.5 Augmented Dickey-Fuller Unit Root Test

This table reports results of the augmented Dicky-Fuller unit root tests. All tests are performed without both constant and trend. Lag length is determined based on Akaike Information criterion (AIC) with maximum lags = 5 for the market and the Dow Jones portfolios and macro-economy data while maximum lags = 2 for the Internet portfolio. R^P and ω^R denote price returns and irrational returns, respectively. CCI and CCI^G denote level and growth (= rate of changes) of consumer confidence index, respectively.

	Variable	Lag length	t-statistic
Market Portfolio	ω^R	2	-4.775542**
	R^P	2	-4.030900**
Dow Jones Portfolio	ω^R	0	-6.992416**
	R^P	2	-3.608579**
Internet Portfolio	ω^R	0	-4.081514**
	R^P	0	-3.206709**
Macro-Ecomomy	CCI	2	-0.507959
	CCI^G	1	-8.198178**

Null Hypothesis: There is a unit root, ** = p<0.01; the others = p>0.1

are more volatile than values (See Section 2.6.1.2.1 for explanation), and further implying that it is natural for bubbles to form and collapse. The Internet portfolio has not only the highest value returns and irrationality but also the highest volatility in both value returns and irrationality.

Table 2.5 reports results from the Augmented Dickey-Fuller unit root test. These unit root tests are performed for the VAR model (2.95). For the VAR, all variables should be stationary. CCI is non-stationary, but its growth (CCI^G) is stationary, and thus used in the VAR models.

Table 2.6 Correlations and Covariances of Three Portfolios

This table reports correlations among R^P, R^V and ω^R and covariances of ω^R with R^P and R^V of three portfolios. R^P, R^V and ω^R denote price returns, value returns and irrational returns, respectively. Subscripts *M*, *D* and *N* indicate the market (portfolio), the Dow Jones portfolio and the Internet portfolio, respectively. For the Internet portfolio, all results are from the market method.

Panel A: Correlations

	The market portfolio			The Dow Jones portfolio			Internet portfolio		
	R^P_M	R^V_M	ω^R_M	R^P_D	R^V_D	ω^R_D	R^P_N	R^V_N	ω^R_N
R^P_M									
R^V_M	.670**								
ω^R_M	.741**	-.002							
R^P_D	.963**	.618**	.748**						
R^V_D	.689**	.992**	.032	.632**					
ω^R_D	.690**	.000	.939**	.782**	.012				
R^P_N	.757**	.836**	-.018	.769**	.831**	-.020			
R^V_N	.318	.756**	-.588*	.442	.716**	-.410	.554		
ω^R_N	.671*	.441	.414	.593*	.464	.280	.787**	-.078	

Two-tailed test, pairwise test: ** = P<0.01 and * = p<0.05

Panel B: Covariances with irrationalities

	Market	Dow Jones	Internet
$COV(R^P, \omega^R)$	0.02335	0.02503	0.60710
$COV(R^V, \omega^R)$	-0.00004	0.00024	-0.03725

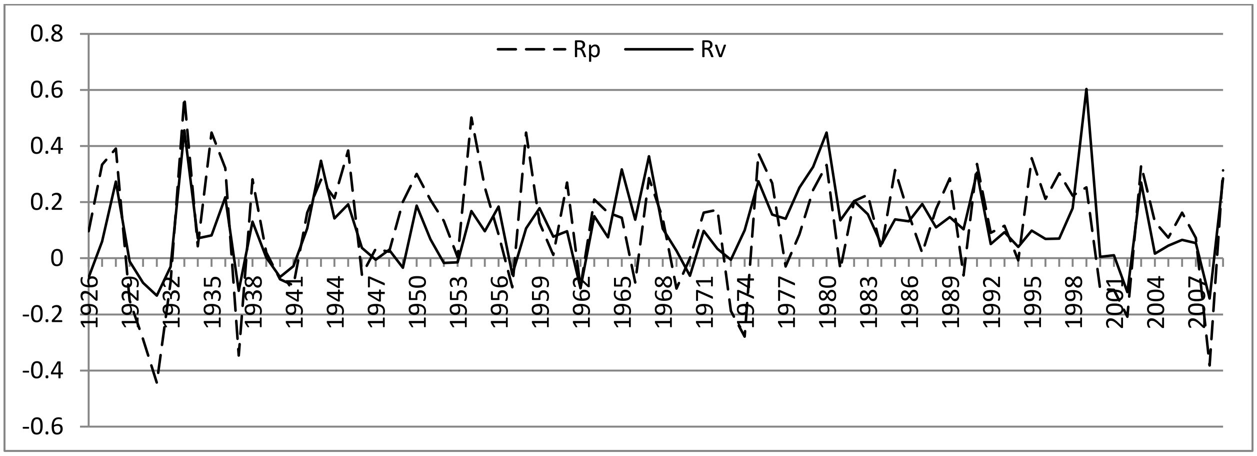
In Table 2.6, Panel A reports the correlations between variables while Panel B shows the covariances with irrationalities. According to Panel A, irrationalities (or mispricing) are closely related to price changes while not related to value changes. The Dow Jones portfolio is almost perfectly correlated to the market; all correlations between variables of the market and their counterparts of the Dow Jones portfolio are greater than 0.93. The value and price changes of the Internet portfolio are also highly correlated to their counterparts of the market and Dow Jones portfolios with correlations greater than 0.7. However, irrationalities of the Internet portfolio are not significantly correlated to those of the market and Dow Jones, implying that mispricing of the Internet stocks is quite different from other stocks in the market.

Panel B in Table 2.6 reveals that the Internet portfolio has a very high covariance between price/value returns and irrational returns compared to other portfolios. This is because the Internet portfolio has very high standard deviations for both price/value returns and irrational

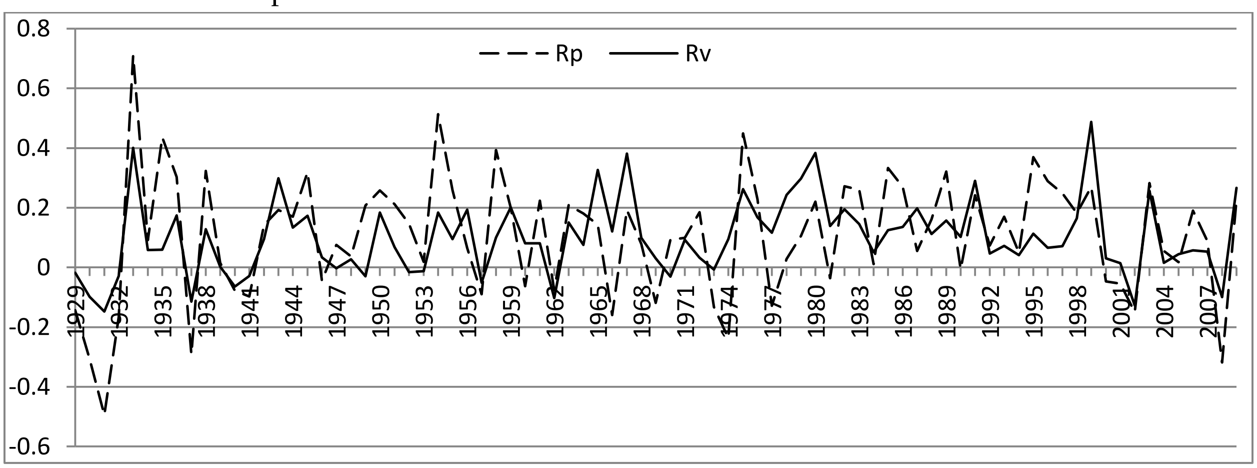
Figure 2.3 Value Returns vs. Price Returns

This figure illustrates the comparison between price returns and value returns of three portfolios: the market (portfolio), the Dow Jones portfolio (DJIA index) and the Internet portfolio. Rp (= R^P) and Rv (= R^V) denote price returns and value returns, respectively. Year represents the end of each year.

Panel A: Market portfolio



Panel B: Dow Jones portfolio



Panel C: Internet portfolio

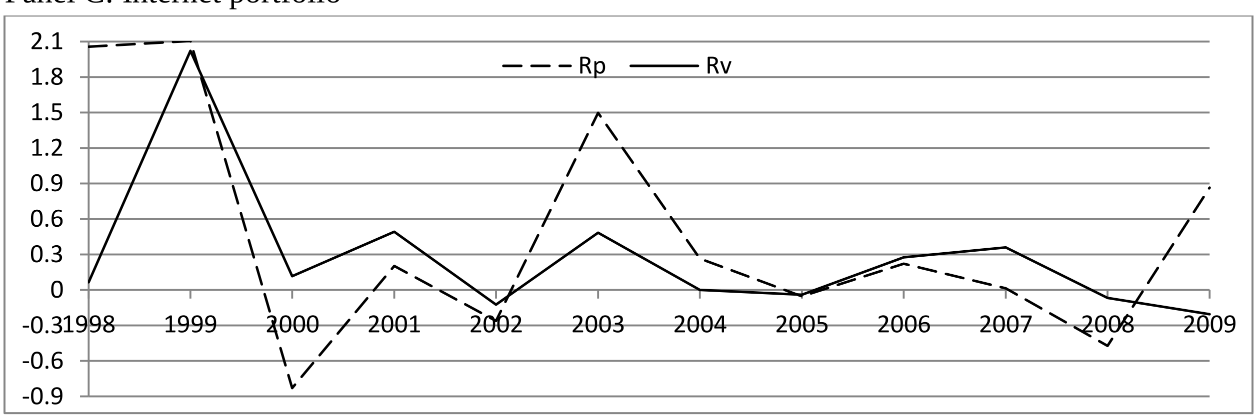
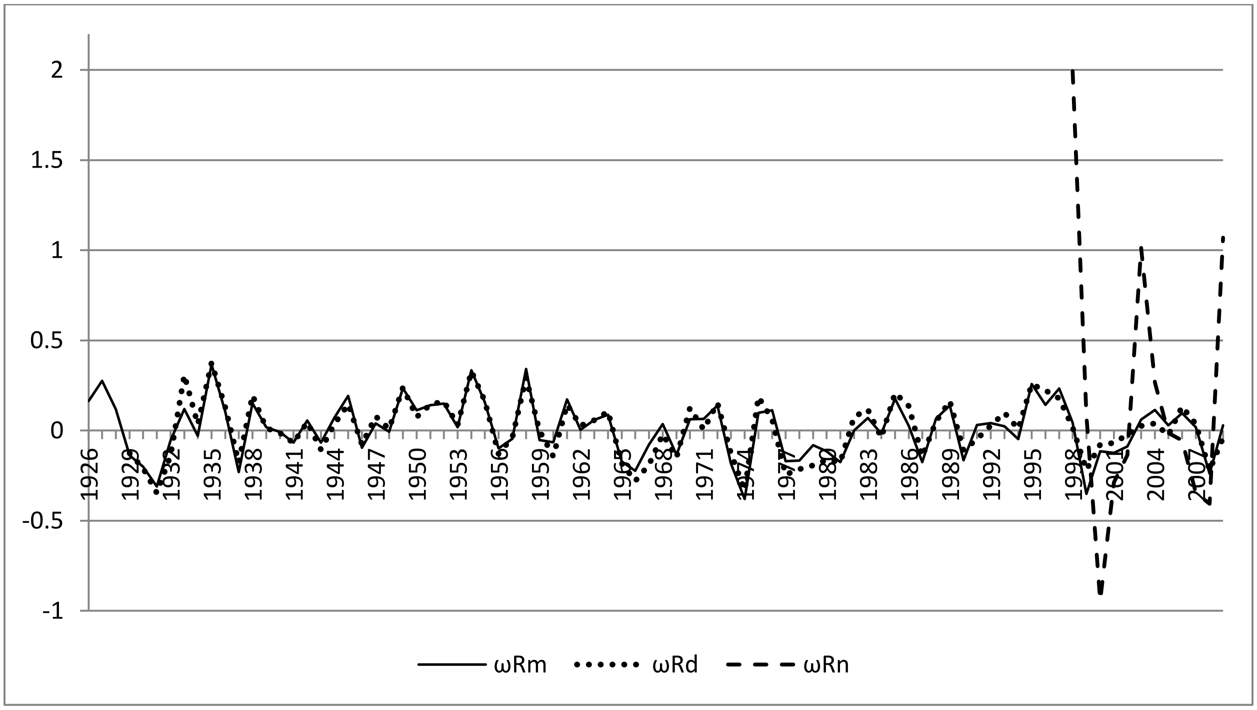


Figure 2.4 Irrationalities: Market vs. Dow Jones and Internet Portfolios

This figure illustrates irrational returns of three portfolios: the market (portfolio), the Dow Jones portfolio (DJIA index) and the Internet portfolio. ωRm (= ω^R_M), ωRd (= ω^R_D) and ωRn (= ω^R_N) denote irrational returns of the market, the Dow Jones portfolio and the Internet portfolio, respectively. Year represents the end of each year.



returns compared to other portfolios as seen in Tables 2.5 and 2.6. In fact, covariance cannot be compared between portfolios. Instead, a standardized measure, correlation, should be used for comparison. Correlations are not significantly different for all three portfolios.

Figure 2.3 plots value returns and price returns of three portfolios. The difference between them is irrational returns, which are plotted in Figure 2.4. Figure 2.3 illustrates that price returns are more volatile than value returns.

2.6.1.2 Analyses.

2.6.1.2.1 Irrationality. As explained in Section 2.6.1.1, for all three portfolios, price returns are more volatile than value returns. The relationship in the variances between value returns and price returns can be explained as follows.

$$VAR(R^V) = VAR(R^P - \omega^R) = VAR(R^P) + VAR(\omega^R) - 2COV(R^P, \omega^R). \quad (2.99)$$

$$VAR(R^P) = VAR(R^V + \omega^R) = VAR(R^V) + VAR(\omega^R) + 2COV(R^V, \omega^R). \quad (2.100)$$

The covariance can be computed by

$$COV(R^P, \omega^R) = COR(R^P, \omega^R)SD(R^P)SD(\omega^R), \quad (2.101)$$

$$COV(R^V, \omega^R) = COR(R^V, \omega^R)SD(R^V)SD(\omega^R), \quad (2.102)$$

where COR( ) and SD( ) denote the correlation and the standard deviation, respectively.

As seen in Table 2.6, irrationalities are closely related to price changes while not related to value changes. Hence, $COV(R^V, \omega^R)$ in Equation (2.100) can be regarded as zero. Then, from Equation (2.100),

$$VAR(R^P) = VAR(R^V) + VAR(\omega^R). \quad (2.103)$$

Equation (2.103) explains why prices are more volatile than values; irrationality does not affect value changes but affect price changes.

2.6.1.2.2 Ex-ante estimation. Figure 2.4 shows that historical irrational returns are mean-reverting for all three portfolios. Furthermore, Table 2.5 describes that irrational returns are stationary without both intercept and trend for all three portfolios, and Table 2.4 describes that irrational returns have means not significantly different form zero for all three portfolios. Hence, zero can be used for expected irrational returns, and thus expected price returns can be used as the estimation of the value returns. Hence, the assumption of no mispricing in expectation, explained in Section 2.4.6, holds in the CAPM.

2.6.2 Examination of Irrationalities

2.6.2.1 Irrationality vs. CCI: correlation. For irrationality to be a measure of investor sentiment, it should be closely related to CCI because CCI is a popular measure of investor sentiment. Table 2.7 reports correlations of CCI measures to irrationality and price returns. CCI^G

Table 2.7 Correlations of Sentiments

This table reports correlations among measures of investor sentiments (CCI, CCI^G and ω^R of the market or Dow Jones portfolio) and between the sentiment measures and R^P of the market or Dow Jones portfolio. CCI and CCI^G denote level and growth (= rate of changes) of consumer confidence index, respectively. R^P and ω^R denote price returns and irrational returns, respectively. Subscripts *M* and *D* indicate the market (portfolio) and the Dow Jones portfolio, respectively.

	CCI	CCI^G	ω^R_M	ω^R_D
CCI		.377**	.359**	.376**
CCI^G	.377**		.494**	.465**
R^P_M	.279*	.609**	.741**	.690**
R^P_D	.315*	.605**	.748**	.782**

Two-tailed test, pairwise test: ** = P<0.01 and * = p<0.05

is more correlated to irrationalities than CCI, and further, CCI^G is more correlated to irrationalities than to CCI. In addition, CCI^G is much more closely correlated to price returns than CCI, but less than irrationality. If investor sentiment is used to explain price changes, CCI^G is a better measure for the sentiment than CCI, and irrationality is better than CCI^G. Both irrationality and CCI^G capture changes in sentiment while CCI represents only the level of sentiment.

2.6.2.2 Relationship among irrationality, price returns and CCI.

2.6.2.2.1 Contemporaneous relationship. Table 2.8 reports the contemporaneous relationship among irrationality, price returns and CCI changes. Both portfolios show almost the same results. Both irrationality and CCI^G have a significant and positive relation to price returns, consistent with Table 2.7. However, the results from Models (2.92) and (2.94) reveal that irrationality and CCI^G are not related to each other, implying that their significant correlation shown in Table 2.7 is not causal but spurious. Irrationality and CCI^G are associated through price returns. Without a lagged term in Model (2.94), the results, not reported, for other coefficients remain almost the same, but DW statistics decreases to 1.444 for the market portfolio and 1.377 for the Dow Jones portfolio.

Table 2.8 Contemporaneous Relationship among Irrationality, Price Returns and CCI

This table reports the contemporaneous relationship among R^P, ω^R and CCI^G for the market and Dow Jones portfolios using the following regressions.

$$CCI^G_t = p_1 + q_1\omega^R_{P,t} + r_1R^P_{P,t} + u_{1,t}, \quad (2.92)$$

$$R^P_{P,t} = p_2 + q_2\omega^R_{P,t} + r_2CCI^G_t + u_{2,t}, \quad (2.93)$$

$$\omega^R_{P,t} = p_3 + q_3R^P_{P,t} + r_3CCI^G_t + v\omega^R_{P,t-1} + u_{3,t}. \quad (2.94)$$

CCI^G denotes growth (= rate of changes) of consumer confidence index. R^P and ω^R denote price returns and irrational returns, respectively. Subscript *P* indicates the portfolio used in the models. The sample periods are from 1953 to 2009 for all regressions.

P		M (Market portfolio)			D (Dow Jones portfolio)		
Model		(2.92)	(2.93)	(2.94)	(2.92)	(2.93)	(2.94)
Dep. Var.		CCI^G	R^P	ω^R	CCI^G	R^P	ω^R
Intercept		-.036*	.120**	-.065**	-.043*	.117**	-.074**
		(.018)	(.012)	(.018)	(.019)	(.014)	(.017)
Ind. Var.	ω^R	.110	.643**		.039	.655**	
		(.122)	(.125)		(.128)	(.107)	
	R^P	.343**		.514**	.402**		.610**
		(.099)		(.098)	(.112)		(.098)
	CCI^G		.527**	.136		.479**	.052
			(.153)	(.147)		(.134)	(.139)
	$\omega^R(-1)^1$			.169			.210*
				(.094)			(.088)
Adj. R^2		.358	.562	.495	.344	.611	.557
F stat		16.581**	36.989**	19.266**	15.683**	44.930**	24.489**
DW stat		1.864	2.019	1.730	1.903	2.133	1.734

* = p<0.05; ** = p<0.01, Standard errors in ( )

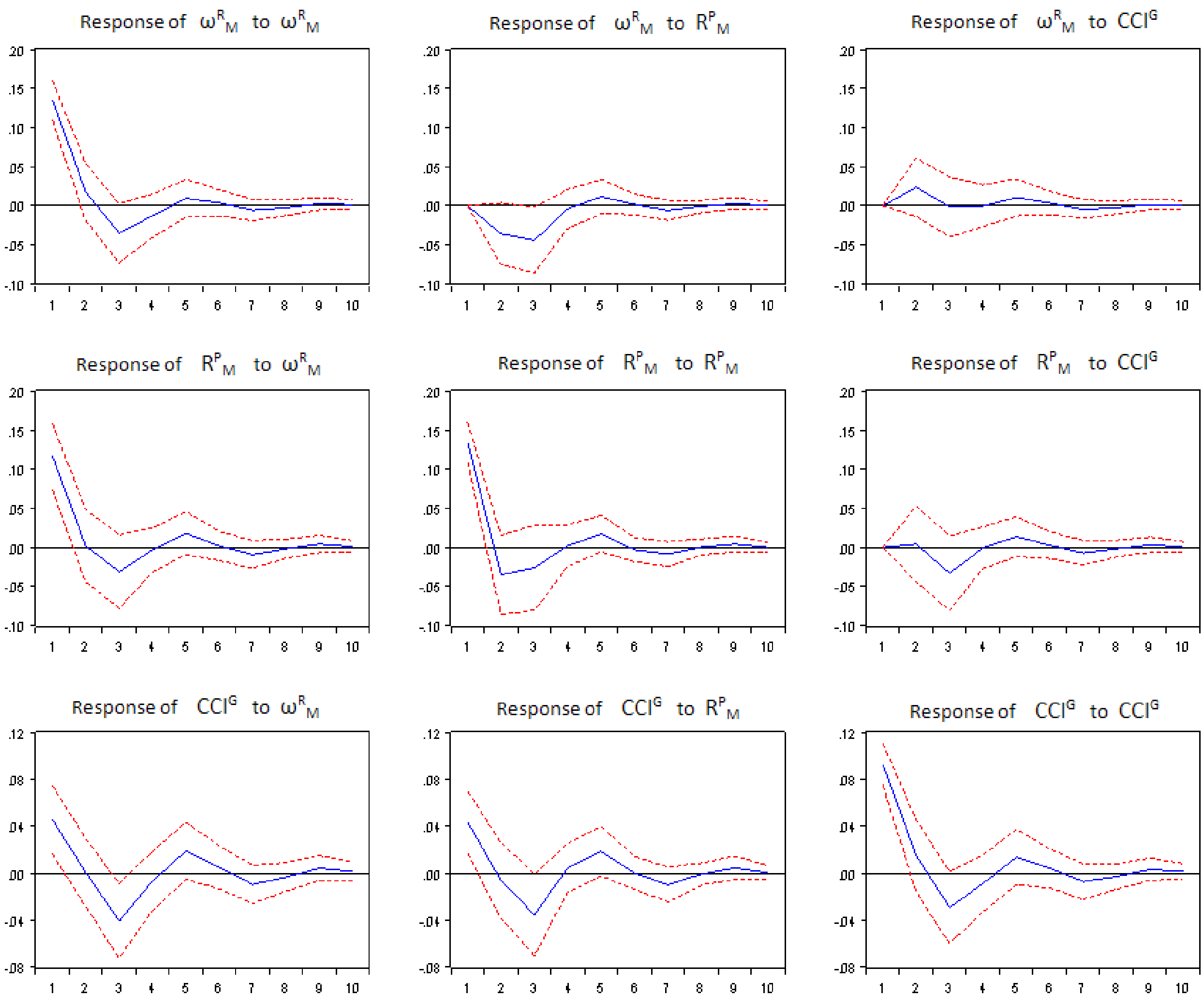
1. $\omega^R(-1)$ denotes ω^R in the previous period (one year lagged term of ω^R).

When CCI level is used instead of CCI changes in the same models, results show, albeit not reported, that there is no significant relation between CCI and price changes, and between CCI and irrationality at 5% significance level, implying that CCI^G is a better measure of sentiment than CCI if the relation between CCI measures and price changes are critical in the empirical work.

2.6.2.2.2 Dynamic interactions. Figure 2.5 shows the impulse responses of the market portfolio from the VAR with three variables of ω^R_M, R^P_M and CCI^G. The impulse responses of the Dow Jones portfolio from the same VAR as the market are almost the same, and thus not reported. Figure 2.5 reveals the following results. The shocks on irrationality have significant, positive and persistent effects on price changes and CCI changes from the subsequent year. The

Figure 2.5 Impulse Responses from the VAR of Irrationality, Price Returns and CCI for the Market Portfolio

This figure illustrates impulse responses to Cholesky one standard deviation innovations ± two standard errors from the VAR of ω^R_M, R^P_M and CCI^G for the market portfolio. ω^R_M, R^P_M and CCI^G denote market irrationality, market price returns and changes in consumer confidence index. Lag length = 2 is determined by the sequential modified likelihood ratio (LR) test with maximum legs = 5. The sample periods are from 1953 to 2009.



shocks on price changes have significant, positive and persistent effects on CCI changes from the subsequent year while insignificant on irrationality in any subsequent year. The shocks on CCI changes have no significant effects on both irrationality and price changes in any subsequent year. These results indicate that irrationality affects price changes, which in turn affect CCI, and this relation is confirmed by the variance decompositions in Table 2.9.

Table 2.9 Variance Decomposition from the VAR of Irrationality, Price Returns and CCI

This table reports the variance decomposition from the VAR of R^P, ω^R and CCI^G for the market and Dow Jones portfolios. R^P and ω^R denote price returns and irrational returns, respectively. CCI^G denotes growth (= rate of changes) of consumer confidence index. The sample periods are from 1953 to 2009.

Variables explained	By shocks on		
	ω^R	R^P	CCI^G
Panel A: Market portfolio			
ω^R	91.05	6.22	2.73
R^P	41.16	58.78	0.06
CCI^G	16.08	15.10	68.82
Panel B: Dow Jones portfolio			
ω^R	87.27	5.21	7.52
R^P	43.07	53.83	3.10
CCI^G	18.39	19.98	61.63

% of error variance explained by shocks in 2 years by Cholesky decomposition

Table 2.9 reports the variance decomposition in two years from the VAR of ω^R, R^P and CCI^G. Both portfolios show almost the same results. In two years, about 90% of error variance in irrationality is explained by its own current shocks, meaning that current shocks on both price and CCI changes do not affect irrationality in two years. By the same way, price changes in two years are explained by current shocks on irrationality (more than 40%) and themselves (about 55%), not affected by CCI changes. CCI changes in two years are explained mainly by their own shocks (more than 60%), but also by irrationality shocks (more than 15%) and price shocks (more than 15%).

In dynamic interactions, CCI does not affect stock price performance; instead, stock market performance influences CCI. On the other hand, irrationality significantly explains stock performance while stock performance does not have an impact on irrationality. Hence, irrationality is a better measure than CCI to explain future stock performance.

Table 2.10 Comparison between Irrationality and CCI with Price Returns from 1953 to 2009

This table reports the results from the following regression using one of three measures of investor sentiments: CCI, CCI^G and ω^R of the market or Dow Jones portfolio. The results from CCI are not significantly different from those from CCI^G, and thus not reported.

$$APR_{P,t+i} = a_1 + b_1X_t + c_1L^1APR_{P,t+i} + d_1\mathbf{ME}_t + e_{1,t+i}, \quad (2.96)$$

where $APR_{P,t+i} = \sum_{\tau=t+1}^{t+i}\{R^P_{P,\tau}\}/i$, $L^1APR_{P,t+i} = \sum_{\tau=t}^{t+i-1}\{R^P_{P,\tau}\}/i$, X denotes sentiment measures; ω^R_P is used for each portfolio *P* while CCI and CCI^G are used for both market and Dow Jones portfolios, and **ME** represents a vector of macro-economic control variables of CPI^G, IPI^G and R_F. R^P and ω^R denote price returns and irrational returns, respectively. CCI, CPI, IPI and R_F indicate consumer confidence index, consumer price index, industrial production index and risk-free rate, respectively. Superscript *G* means rate of changes. No lagged term ($L^1APR_{P,t+i}$) is included when i = 1. The sample period is from 1953 to 2009.

Panel A: P = M (Market portfolio)

A1: without **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_M	b_1	-.003	-.317**	-.115	-.159**	-.181**	-.078**	-.075**	-.058**
	$A.R^2$	-.019	.199	.293	.534	.559	.749	.821	.866
	DW	2.100	1.556	1.775	1.849	2.131	1.812	1.990	1.693
CCI^G	b_1	-.135	-.460**	-.132	-.103	-.131*	-.093**	-.047*	-.041*
	$A.R^2$	-.011	.272	.315	.476	.437	.742	.741	.786
	DW	2.056	1.542	1.838	1.724	1.947	1.864	1.903	2.007

A2: with **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_M	b_1	.141	-.257	-.071	-.164*	-.167**	-.051	-.051*	-.052**
	$A.R^2$	.059	.169	.263	.514	.552	.745	.840	.864
	DW	1.998	1.557	1.728	1.877	2.154	1.784	1.887	1.731
CCI^G	b_1	.117	-.396*	-.083	-.067	-.061	-.065	-.014	-.028
	$A.R^2$	.053	.243	.292	.459	.458	.747	.799	.817
	DW	1.917	1.560	1.827	1.770	2.016	1.854	1.873	2.023

Panel B: P = D (Dow Jones portfolio)

B1: without **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_D	b_1	.067	-.287*	-.105	-.163**	-.181**	-.068**	-.078**	-.056**
	$A.R^2$	-.015	.187	.365	.600	.644	.844	.908	.932
	DW	2.118	1.582	1.832	2.147	2.284	1.996	2.026	1.930
CCI^G	b_1	-.089	-.389**	-.096	-.101	-.133*	-.081**	.048*	-.045*
	$A.R^2$	-.015	.231	.351	.535	.555	.837	.844	.884
	DW	2.011	1.537	1.730	1.963	2.038	1.910	1.817	1.951

B2: with **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_D	b_1	.179	-.204	-.052	-.164*	-.159**	-.038	-.055**	-.047**
	$A.R^2$	.117	.181	.349	.587	.653	.847	.919	.932
	DW	2.038	1.591	1.781	2.121	2.283	1.979	1.987	1.931
CCI^G	b_1	.164	-.321*	-.052	-.063	-.066	-.056	-.002	-.026
	$A.R^2$	.108	.224	.347	.532	.585	.853	.898	.908
	DW	1.925	1.568	1.751	1.995	2.109	2.027	2.033	2.010

* = p<0.05; ** = p<0.01

2.6.2.3 Irrationality vs. CCI: effect on subsequent stock price performance.

2.6.2.3.1 From 1953 to 2009. Table 2.10 reports the relationship between investor sentiment and subsequent stock performance for the period from 1953 to 2009. The results using CCI are not significantly different from those using CCI^G as X (investor sentiments) in the regression, and thus not reported. According to Panel A that reports the results from the market portfolio, irrationality has a higher forecasting ability about the future stock performance than CCI^G with or without macro-economic control variables. Without control variables, irrationality is significantly and negatively related to next two year returns and to long-horizon returns over four years. CCI^G shows similar results to irrationality, but its statistical significance of the coefficients is much lower than that of irrationality. With control variables, irrationality still has a significant explanatory power on mid- to long-horizon returns while CCI^G lose almost all its forecasting power over future price returns. Hence, irrationality is more powerful and more robust measure of investor sentiment to predict future returns than CCI and CCI^G.

When macro-economic control variables are included, the explanatory power of the model (adjusted R^2) is not weaker than when they are not included, even though the significance of investor sentiment measures (irrationality or CCI^G) decreases. This is attributable to the lagged term (not reported in the table), which is almost always significant at 1% level. With control variables, both CCI and CCI^G cannot explain future stock price performance, which is mostly explained by the lagged term.

The Dow Jones portfolio, as seen in Panel B in Table 2.10, has the same results as the market portfolio, implying that any measure of investor sentiment, one of irrationality, CCI and CCI^G, has the same forecasting power between aggregate market (=market portfolio) and large

Table 2.11 Irrationality and Subsequent Price Returns for the Full Sample Period

This table reports the results from the following regression using ω^R of the market or Dow Jones portfolio.

$$APR_{P,t+i} = a_1 + b_1X_t + c_1L^1APR_{P,t+i} + d_1\mathbf{ME}_t + e_{1,t+i}, \qquad (2.96)$$

where $APR_{P,t+i} = \sum_{\tau=t+1}^{t+i}\{R^P_{P,\tau}\}/i$, $L^1APR_{P,t+i} = \sum_{\tau=t}^{t+i-1}\{R^P_{P,\tau}\}/i$, X denotes ω^R_P for each portfolio *P*, and **ME** represents a vector of macro-economic control variables of CPI^G, IPI^G and R_F. R^P and ω^R denote price returns and irrational returns, respectively. CCI, CPI, IPI and R_F indicate consumer confidence index, consumer price index, industrial production index and risk-free rate, respectively. Superscript *G* means rate of changes. No lagged term ($L^1APR_{P,t+i}$) is included when i = 1. The sample period is from 1926 to 2009 for the market and from 1929 to 2009 for the Dow Jones portfolio.

Panel A: P = M (Market portfolio)

A1: without **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_M	b_1	.136	-.467**	-.216**	-.256**	-.208**	-.088**	-.078**	-.058**
	$A.R^2$	-.002	.329	.430	.587	.655	.757	.839	.854
	DW	2.066	1.404	1.606	1.653	1.874	1.905	1.898	1.826

A2: with **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_M	b_1	.243	-.490**	-.202**	-.262**	-.203**	-.079**	-.080**	-.065**
	$A.R^2$	-.005	.320	.432	.587	.672	.760	.835	.861
	DW	1.983	1.447	1.581	1.737	1.983	1.881	1.928	2.048

Panel B: P = D (Dow Jones portfolio)

B1: without **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_D	b_1	.131	-.392**	-.153*	-.234**	-.196**	-.078**	-.082**	-.057**
	$A.R^2$	-.002	.300	.379	.558	.624	.817	.879	.901
	DW	2.064	1.579	1.499	1.789	1.822	2.090	1.879	1.901

B2: with **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_D	b_1	.233	-.411**	-.142*	-.243**	-.178**	-.067**	-.085**	-.064**
	$A.R^2$	-.011	.288	.383	.562	.655	.823	.875	.903
	DW	2.002	1.639	1.567	1.892	1.959	2.015	1.929	2.091

* = p<0.05; ** = p<0.01

stocks (= Dow Jones portfolio). Irrationality forecasts future stock returns for both aggregate market and large stocks over long-horizon with or without macro-economic control variables.

2.6.2.3.2 Full sample period. Table 2.11 reports the relationship between irrationality and subsequent stock performance for the full sample period: 1926 to 2009 for the market and 1929 to 2009 for the Dow Jones portfolio. For both portfolios, the results are the same between with and without control variables.

Table 2.12 Comparison between Irrationality and CCI with Value Returns from 1953 to 2009

This table reports the results from the following regression using one of three measures of investor sentiments: CCI, CCI^G and ω^R of the market or Dow Jones portfolio. The results from CCI are not significantly different from those from CCI^G, and thus not reported.

$$AVR_{P,t+i} = a_2 + b_2X_t + c_2L^1AVR_{P,t+i} + d_2\mathbf{ME}_t + e_{2,t+i}, \quad (2.97)$$

where $AVR_{P,t+i} = \sum_{\tau=t+1}^{t+i}\{R^V_{P,\tau}\}/i$, $L^1AVR_{P,t+i} = \sum_{\tau=t}^{t+i-1}\{R^V_{P,\tau}\}/i$, X denotes sentiment measures; ω^R_P is used for each portfolio *P* while CCI and CCI^G are used for both market and Dow Jones portfolios, and **ME** represents a vector of macro-economic control variables of CPI^G, IPI^G and R_F. R^P and ω^R denote price returns and irrational returns, respectively. CCI, CPI, IPI and R_F indicate consumer confidence index, consumer price index, industrial production index and risk-free rate, respectively. Superscript *G* means rate of changes. No lagged term ($L^1APR_{P,t+i}$) is included when i = 1. The sample period is from 1953 to 2009.

Panel A: P = M (Market portfolio)

A1: without **ME**

X	i =	1	2	3	4	5	10	15	20
$\omega^R{}_M$	b_2	-.162	.034	.067	.051	.022	.015	-.002	.005
	$A.R^2$	.016	.327	.439	.663	.623	.761	.836	.865
	DW	1.909	1.511	1.810	1.335	1.794	1.530	1.604	1.746
CCI^G	b_2	-.212	-.186*	-.040	-.019	-.027	-.037	-.005	.015
	$A.R^2$	.019	.377	.428	.652	.623	.772	.836	.869
	DW	1.810	1.404	1.735	1.227	1.727	1.477	1.593	1.696

A2: with **ME**

X	i =	1	2	3	4	5	10	15	20
$\omega^R{}_M$	b_2	-.078	.077	.109	.068	.048	.032	-.017	.001
	$A.R^2$	.002	.339	.445	.651	.613	.758	.837	.871
	DW	1.964	1.571	1.840	1.348	1.819	1.556	1.815	2.158
CCI^G	b_2	-.172	-.236*	-.057	-.036	-.011	-.034	-.016	-.014
	$A.R^2$	.015	.398	.419	.637	.603	.755	.836	.876
	DW	1.958	1.463	1.795	1.199	1.751	1.488	1.804	2.154

Panel B: P = D (Dow Jones portfolio)

B1: without **ME**

X	i =	1	2	3	4	5	10	15	20
$\omega^R{}_D$	b_2	-.177	.014	.061	.056	.036	.014	001	.003
	$A.R^2$	.033	.333	.431	.678	.627	.760	.843	.870
	DW	1.946	1.476	1.842	1.343	1.854	1.667	.572	.652
CCI^G	b_2	-.199	-.181*	-.035	-.013	-.027	-.037	-.004	-.012
	$A.R^2$	.022	.393	.419	.660	.621	.774	.843	.873
	DW	1.819	1.365	1.751	1.179	1.722	1.609	1.539	1.622

B2: with **ME**

X	i =	1	2	3	4	5	10	15	20
$\omega^R{}_D$	b_2	-.119	.060	.107	.079	.068	.032	-.009	.001
	$A.R^2$	.009	.343	.443	.674	.625	.762	.840	.879
	DW	1.993	1.554	1.920	1.411	1.912	1.672	1.704	2.082
CCI^G	b_2	-.172	-.226*	-.053	-.032	-.012	-.034	-.013	-.012
	$A.R^2$	.015	.414	.413	.649	.600	.758	.841	.882
	DW	1.964	1.431	1.823	1.144	1.742	1.625	1.712	2.081

* = p<0.05; ** = p<0.01

Table 2.13 Irrationality and Subsequent Value Returns for the Full Sample Period

This table reports the results from the following regression using ω^R of the market or Dow Jones portfolio.

$$AVR_{P,t+i} = a_2 + b_2X_t + c_2L^1AVR_{P,t+i} + d_2\mathbf{ME}_t + e_{2,t+i}, \quad (2.97)$$

where $AVR_{P,t+i} = \sum_{\tau=t+1}^{t+i}\{R^V_{P,\tau}\}/i$, $L^1AVR_{P,t+i} = \sum_{\tau=t}^{t+i-1}\{R^V_{P,\tau}\}/i$, X denotes ω^R_P for each portfolio *P*, and **ME** represents a vector of macro-economic control variables of CPI^G, IPI^G and R_F. R^P and ω^R denote price returns and irrational returns, respectively. CCI, CPI, IPI and R_F indicate consumer confidence index, consumer price index, industrial production index and risk-free rate, respectively. Superscript *G* means rate of changes. No lagged term ($L^1APR_{P,t+i}$) is included when i = 1. The sample period is from 1926 to 2009 for the market and from 1929 to 2009 for the Dow Jones portfolio.

Panel A: P = M (Market portfolio)

A1: without **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_M	b_2	-.053	-.072	-.013	-.030	-.019	.002	-.009	-.005
	$A.R^2$	-.009	.325	.443	.606	.660	.835	. 902	.936
	DW	1.830	1.475	1.626	1.453	1.625	1.651	1.559	1.647

A2: with **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_M	b_2	.021	-.051	.010	-.023	-.005	.011	-.010	-.002
	$A.R^2$	.010	.307	.432	.593	.657	.835	.900	.941
	DW	1.886	1.449	1.546	1.423	1.606	1.565	1.622	1.842

Panel B: P = D (Dow Jones portfolio)

B1: without **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_D	b_2	-.105	-.092	-.013	-.014	-.005	.004	-.008	-.006
	$A.R^2$	.005	.347	.426	.625	.651	.843	.901	.939
	DW	1.768	1.479	1.592	1.371	1.642	1.828	1.523	1.569

B2: with **ME**

X	i =	1	2	3	4	5	10	15	20
ω^R_D	b_2	-.058	-.083	.003	-.014	.014	.013	-.010	-.004
	$A.R^2$	.024	.334	.419	.613	.655	.844	.899	.942
	DW	1.884	1.473	1.544	1.354	1.614	1.738	1.616	1.798

* = p<0.05; ** = p<0.01

Irrationality can explain subsequent long horizon returns over 2~20 years while cannot explain returns of the following one year for the aggregate market and large stocks, consistent with Brown and Cliff (2005).

2.6.2.4 Irrationality vs. CCI: effect on subsequent stock value performance. Tables 2.12 and 2.13 with value returns are respectively counterparts of Tables 2.10 and 2.11 with price returns. Like in Table 2.10, in Table 2.12, the results using CCI are not significantly different from those using CCI^G as X (investor sentiments) in the regression, and thus not reported; the only difference is that CCI^G is significant over two year returns with and without macro-economic control variables while CCI is not. Table 2.12 demonstrates that investor sentiment, either irrationality or CCI^G, has not been related to future value changes since 1953; the only exception is CCI^G over two year returns. Table 2.13 confirms no relation between irrationality and future value changes for the full sample period. Still, adjusted R^2 is high due to the lagged term as explained in Section 2.6.2.3.1.

2.6.2.5 Investor sentiment and monetary policy. Table 2.14 reports the effect of Fed fund rates on the investor sentiment. Fed fund rates are known to be a good measure of the monetary policy (Bernanke and Blinder, 1992). All the results are almost the same between market and Dow Jones portfolios. The results for the monetary policy are not significantly different among three measures of investor sentiment. Adjusted R^2 is always higher for the models with irrational returns than those with CCI growths except one for the Dow Jones portfolio from 1955 to 1978. The models with CCI show the highest explanatory power, but their adjusted R^2 cannot be directly compared to the other two because they include one lagged terms, which are always significant at 1% level and seem to explain most portion of the dependent variable.

The results from Panel A in Table 2.14 are consistent with Kurov (2010), who documents that the effect of monetary policy on the investor sentiment is stronger in the bear market than in the bull market. When the sample period is divided by 1979, Panels B and C show different results. The significance of Fed fund rates is much weaker in the post-1979 period than in the pre-1979 period for both bull (UP) and bear (1–UP) markets. For the post-1979 period, the effect

Table 2.14 Investor Sentiment and Monetary Policy

This table reports the results from the following regression using one of three measures of investor sentiments: CCI, CCI^G and ω^R of the market or Dow Jones portfolio.

$$X_t = a_0 + b_0 UP\times R_{FF,t} + c_0(1–UP)\times R_{FF,t} \ (+ d_0 X_{t-1}) + e_{0,t}. \quad (2.96)$$

X denotes sentiment measures; $\omega^R{}_P$ is used for each portfolio *P* while CCI and CCI^G are used for both market and Dow Jones portfolios, and R_{FF} denotes Fed fund rates. UP is a dummy variable to indicate positive returns of each portfolio, and thus unity when $R^P{}_{P,t} \geq 0$ and zero otherwise for each portfolio. R^P and ω^R denote price returns and irrational returns, respectively. CCI and CCI^G denote level and growth (= rate of changes) of consumer confidence index, respectively. One lagged term is included only for CCI due to its persistence.

	P = M (Market portfolio)			P = D (Dow Jones Portfolio)		
X	$\omega^R{}_M$	CCI^G	CCI	$\omega^R{}_D$	CCI^G	CCI
Panel A. 1955-2009						
a_0	.039	.025	37.418**	.029	.024	36.787**
	(.034)	(.031)	(9.341)	(.035)	(.031)	(9.220)
b_0	-.242	.081	-51.803	-.075	.122	-47.631
	(.575)	(.527)	(39.320)	(.605)	(.520)	(38.776)
c_0	-2.501**	-1.554**	-166.678**	-2.262**	-1.617**	-171.025**
	(.604)	(.554)	(39.431)	(.637)	(.548)	(38.901)
d_0			.619**			.625**
			(.096)			(.095)
Adj. R^2	.269	.155	.579	.219	.175	.591
F stat	10.952**	5.943**	25.780**	8.587**	6.733**	27.043**
DW stat	1.858	1.812	1.617	1.543	1.781	1.601
Panel B. 1955-1978						
a_0	.130*	.089	69.466**	.094	.093	68.063**
	(.060)	(.051)	(13.396)	(.069)	(.049)	(13.249)
b_0	-2.271	-1.262	-300.612**	-1.642	-1.250	-291.311**
	(1.387)	(1.183)	(76.346)	(1.576)	(1.128)	(73.616)
c_0	-4.463**	-3.064**	-343.158**	-4.006**	-3.144**	-343.881**
	(1.030)	(.879)	(51.658)	(1.209)	(.866)	(51.015)
d_0			.372*			.386**
			(.132)			(.132)
Adj. R^2	.472	.376	.754	.346	.398	.759
F stat	11.286**	7.928**	24.547**	7.082**	8.598**	25.183**
DW stat	1.769	1.480	1.469	1.461	1.479	1.486
Panel C. 1979-2009						
a_0	.013	.006	29.174*	.021	.001	28.679*
	(.045)	(.044)	(12.282)	(.045)	(.044)	(12.091)
b_0	.134	.336	-10.281	.098	.414	-3.223
	(.670)	(.650)	(48.879)	(.670)	(.651)	(48.610)
c_0	-1.637*	-.831	-102.702	-1.463	-.901	-109.432
	(.781)	(.758)	(55.627)	(.770)	(.748)	(54.541)
d_0			.684**			.685**
			(.127)			(.125)
Adj. R^2	.129	.022	.553	.092	.042	.568
F stat	3.212	1.343	13.396**	2.514	1.666	14.152**
DW stat	1.738	1.846	1.677	1.424	1.820	1.669

* = p<0.05; ** = p<0.01, Standard errors in (   )

of the monetary policy is almost insignificant even in the bear market. This result can be explained by one or both of the following arguments.

First, by focusing on investors, i.e., how they react to the monetary policy, the effect of monetary policy on the investor sentiment becomes weaker, because investors are more sophisticated, in recent three decades than in earlier decades. In recent three decades, investors have become more and more sophisticated due to increases in institutional investors' holdings (French, 2008). Institutional investors are regarded as more sophisticated than individual investors (Hand, 1990; Utama and Cready, 1997; Walther, 1997; El-Gazzar, 1998). Further, even individual investors are likely to be more sophisticated in recent three decades than in earlier decades because the revolution of new information technology allows them to easily access all relevant information. Sophisticated investors trade stocks on the reasonable or rational basis of fundamental values, and thus, investors are likely to be more rational as they are more sophisticated. As investors become more sophisticated and thus more rational, movements of stock price are more based on their fundamental values, and thus investors' sentiments are less affected by the external shock, the monetary policy, in recent three decades than earlier decades.

Second, by focusing on the central bank, i.e., how it conducts the monetary policy, the effect of monetary policy on the investor sentiment becomes more effective, because the Fed responds to inflation more aggressively as explained in Section 2.5.2.3, in recent three decades than in earlier decades. Stock market signals inflation because stock returns are a leading indicator for the real sector (Fama, 1981; Geske and Roll, 1983). Theoretically, positive relations are expected between stock performance and inflation, but empirically found negative (Fama, 1981; Geske and Roll, 1983; Ram and Spencer, 1983; James et al., 1985). This empirical negative relation between stock performance and inflation is not causal but spurious (Fama, 1981;

Geske and Roll, 1983) due to the intervention of the monetary authority for the stable inflation. The aggressive monetary policy to expected inflation implies aggressive responses to the stock market; if not to stock price, at least but to signals of the market (Bernanke and Gertler, 1999). Stock price movements are more controlled, at least indirectly if not directly, by the monetary policy in recent three decades than earlier decades in which the Fed allows high inflation. Thus, the aggressive monetary policy will prevent the stock market from going higher, and thus lower the irrationality (or investor sentiment). Less irrationality by higher interest rates yields the seemingly weaker effect of the monetary policy on irrationality, but in fact means the more effective conduct of the policy to reduce irrationality. Monetary policy since 1979 has been not responsive but proactive to inflation, reducing non-fundamental impacts on the economy (Clarida, et al., 2000; Boivin and Giannoni, 2006).

2.7 Summary and Conclusion

This chapter explains that the Capital Asset Pricing Model (CAPM) has the implied assumption that no assets are mispriced in the market, and that this assumption should be maintained for the CAPM to work. Hence, the CAPM does not work with the ex-post price data that contain mispricing, but work with the ex-post value data that have no mispricing, leading to the Rational CAPM, a new interpretation of the CAPM. If the market is irrational, the market irrationality should be removed from the CAPM to measure the value changes of assets because the market irrationality, if not removed, is conveyed to asset returns through β, systematic risk of an asset.

The difference between the Rational and conventional CAPMs is in the application of the ex-ante (original) CAPM to the historical data; the Rational CAPM uses value data whereas the conventional CAPM uses price data. For future estimation, there is no difference among all CAPMs because expected mispricing is zero for all CAPMs due to the rationality and homogeneity assumptions on the future.

In the CAPM, the risk represents the fear of mispricing, and mispricing is the measure of investors' irrationality; hence, risk, mispricing and irrationality all indicate the same thing. The noise trader theory emphasizes the effect of the noise, i.e., irrational response to the information, on stock returns. Some behavioralists use the investor sentiment as a measure of the noise to explain mispricing. Mispricing is a measure of irrationality, and thus irrationality perfectly explains mispricing. Hence, irrationality is a perfect measure of the noise, and closely related to the investor sentiment.

Theoretical analysis and explanation for the Rational CAPM give the answer to the research question in the beginning. ***Why does the CAPM work poorly in the empirical studies?*** It is because price returns contain mispricing caused by investors' irrationality. For the CAPM to hold, mispricing should be removed from the model. In other words, not price returns but value returns should be used in the CAPM. Investors' irrationality makes ex-post price returns different from ex-ante price returns, in turn making ex-post market proxies inefficient, causing the CAPM not to work. In the CAPM, however, rational investors "know" rational (value) returns, implying that ex-post value returns are the same as ex-ante value returns.

Based on the Rational CAPM, value returns and irrationalities are measured for the market portfolio, the Dow Jones Industrial Average (DJIA) index and the Internet portfolio. Then, irrationalities are analyzed in relation to investor sentiments for the market portfolio and the DJIA index in order to examine whether the Rational CAPM works with historical data. The empirical studies reveal the following results. First, price is much more volatile than value because mispricing (or irrationalities) does not affect value changes but price changes. Hence, it is normal that a bubble forms and collapses. Second, irrational returns of all three portfolios are stationary without trend, and their historical means are not significantly different from zero, and thus ex-ante value returns can be regarded as the same as ex-ante price returns. Third, irrationality has a forecasting power over subsequent long-horizon price returns of 2-20 years while not over the following one-year price returns. Moreover, irrationality has nothing to do with subsequent value returns. Fourth, the effect of monetary policy on irrationality is strong in the bear market while weak in the bull market, consistent with Kurov (2010). Furthermore, the effect of monetary policy on irrationality is weaker in the post-1979 period than in the pre-1979 period; this can be explained by the investor sophistication and/or by the proactive execution of monetary policy. All in all, irrationality explains the investor sentiment and the market very well.

No direct empirical research is possible for the Rational CAPM because the Rational CAPM uses value data that are not available in the market. However, this chapter has performed indirect empirical work for the Rational CAPM by comparing irrationalities to investor sentiments. Empirical findings from the U.S. stock market prove that irrationality from the Rational CAPM well explains the investor sentiment and the market, and thus verify that the Rational CAPM works with historical data. In this chapter, the Rational CAPM is tested using irrational returns. In the following chapter, the Rational CAPM will be tested using value returns, another outcome of the Rational CAPM.

CHAPTER III

EX-POST BUBBLE MODEL: EX-POST VALUATION WITH RATIONAL CAPM

3.1 Introduction

A bubble is used to explain overpricing followed by a sudden collapse in price. We can then recognize a bubble only after a collapse in price. Here is my research question; ***why is a bubble identified only after its burst?*** This chapter defines a bubble as the deviation of the price of an asset from its value in order to measure a bubble at any time. Then, measuring a value plays a crucial role in measuring a bubble because prices can be obtained from the market. Unlike previous studies, this chapter measures a value using historical data based on the Rational CAPM explained in Chapter II.

The Rational CAPM enables one to measure historical value changes of assets using historical data. Using the Rational CAPM, therefore, we can measure historical values of assets if we can identify the value of assets at any specific time. This chapter determines the initial value, the value at the beginning of the sample period, and all subsequent values of assets simultaneously using their historical dividend yields and historical value changes. Historical value changes are measured by the Rational CAPM. Hence, this chapter is a continuing study of the Rational CAPM, and thus, the results from this chapter can verify the validity of the Rational CAPM.

The goals of this chapter are twofold. The first goal is to theoretically explain how to measure values and bubbles using historical data, introducing the Ex-post Bubble Model (EBM).

The second goal is to empirically verify that the EBM works, which means that the Rational CAPM works with historical data. The valuation in the EBM is based on the Rational CAPM. Hence, if the EBM works, it can be said that the Rational CAPM works with historical data.

This chapter contributes to the existing literature in the following ways. First, this chapter develops the EBM, the new bubble model with ex-post data. The EBM enables one to measure a value and a bubble at any time by using historical data. Second, for the EBM, this chapter introduces ex-post valuation, valuation by historical data, using historical value changes measured by the Rational CAPM and the initial value determined by historical dividend-price ratios. Third, from the EBM, this chapter reveals the behavior of a bubble. Fourth, based on the EBM, this chapter measures historical values and bubbles of the U.S stock market, and examines dynamic interactions among dividends, irrationality, price and bubble in the U.S stock market.

The theoretical approach herein answers the research question. Bubbles can be detected before burst if they can be measured at any time. If the Rational CAPM works, value changes can be assessed, and then bubbles can be measured and detected before their burst using historical data. In addition, the empirical work leads to the following results. Irrationality persists, and thus it takes time for assets to be rationally priced. Both irrationalities and bubbles are independent of price changes in the previous period. Bubble measurements of the EBM are not quite different from previous findings. In conclusion, empirical results from the U.S stock market verify that the Rational CAPM works with historical data because the valuation in the EBM is based on the Rational CAPM.

The rest of this chapter is organized as follows. Section 3.2 reviews literature while Section 3.3 analyzes literature. Section 3.4 explains the EBM: how to measure bubbles in Section 3.4.1 and how bubbles form and behave in Section 3.4.2. Based on the theories in

Section 3.4 empirical studies are conducted. In Section 3.5, Section 3.5.1 explains how to measure historical bubbles. Section 3.5.2 explains how to examine bubbles with Section 3.5.2.1 for rational pricing, Section 3.5.2.2 for the self-control of bubbles. Section 3.5.3 describes the sample. Section 3.6 discusses empirical findings and Section 3.7 summarizes and concludes this chapter.

3.2 Literature Review

3.2.1 Rational Bubble Model

According to a standard efficient market model, a stock price can be measured by Equation (3.1) (West, 1987; Flood and Hodrick, 1990).

$$P_t = \frac{E_t(P_{t+1} + D_{t+1})}{(1+r)}, \tag{3.1}$$

where r denotes a constant discount rate. Solving Equation (3.1) recursively yields

$$P_t = \sum_{i=1}^{\infty} \frac{E_t(D_{t+i})}{(1+r)^i} + \frac{E_t(P_{t+\infty})}{(1+r)^{\infty}}. \tag{3.2}$$

Under the transversality condition that

$$\frac{E_t(P_{t+\infty})}{(1+r)^{\infty}} = 0, \tag{3.3}$$

the unique market fundamental solution to Equation (3.2), F_t, is given by

$$F_t = \sum_{i=1}^{\infty} \frac{E_t(D_{t+i})}{(1+r)^i}. \tag{3.4}$$

If Equation (3.3) does not hold, then the general solution to Equation (3.2) has the following form.

$$P_t = F_t + B_t, \tag{3.5}$$

where F denotes (the value implied by) the market fundamental, and B denotes a bubble, a deviation of the current price from the market fundamental, and satisfies

$$B_t = \frac{E_t(B_{t+1})}{(1+r)}. \tag{3.6}$$

This Equation (3.6) can be rewritten as

$$E_t(B_{t+1}) = (1+r)B_t, \tag{3.7}$$

and thus,

$$B_{t+1} - (1+r)B_t = z_{t+1}, \tag{3.8}$$

where z is the innovation in the bubble which has a mean zero, and defined as

$$z_{t+1} = B_{t+1} - E_t(B_{t+1}). \tag{3.9}$$

From Equation (3.7), Diba and Grossman (1988b) argue that a bubble in the price can never be negative, and that a bubble cannot start again if it ever is zero, implying that if a bubble exists currently, it must have started at the initiation of the market.

West (1987) performed a specification test to check whether Equation (3.1) is consistent with the data, and finds that Equation (3.1) is acceptable. He compared two sets of estimates of parameters necessary to compute the expected present value of a dividend stream. One set was obtained by regressing the stock price on a set of lagged dividends, and the other set from a pair of equations: one for discount rate and the other for dividend process. One possible explanation of any difference between the two sets of estimates is a bubble. Since he finds a large difference between two sets, he rejects the null hypothesis of no bubbles. Flood and Hodrick (1990), however, contend that no one has yet certainly proved the existence of a bubble, but that misspecification of the model seems to contribute to the findings of a bubble.

Diba and Grossman (1988a) launched cointegration tests, which only assume that the first difference of unobservable variables is stationary. With an unobservable variable, μ, Equation (3.4) can be written as follows:

$$F_t = \sum_{i=1}^{\infty} \frac{E_t(\alpha D_{t+i} + \mu_{t+i})}{(1+r)^i}, \tag{3.10}$$

where, α is a positive constant that adjust expected dividend to expected capital gain, and μ is a variable that the researcher does not observe. If α is equal to unity and μ to zero for all *t+i*, then Equation (3.10) is the same as Equation (3.4).

By assuming that the process generating dividends is nonstationary in levels but stationary in first differences, they argue that the first differences of stock prices are stationary if rational bubbles do not exist but nonstationary if rational bubbles do exist. From Equation (3.8), the first differences of a bubble can be expressed as

$$[1 - (1+r)L](1 - L)B_t = (1 - L)z_t, \tag{3.11}$$

where L is a lag operator. Therefore, if z_t is white noise, then the first differences of rational bubbles follow an autoregressive and moving average (ARMA) process that $(1 - L)B_t$ is nonstationary. In other words, if the stock price contains a rational bubble, its differenced form can never be stationary. Therefore, the way to detect bubbles is to test whether or not the first difference of stock prices and dividends is stationary, and then, the result that rejects the null hypothesis of nonstationarity means nonexistence of bubbles.

Furthermore, Diba and Grossman (1988a) argue that prices and dividends are cointegrated, i.e., their linear combination is stationary, if the first differences of dividends are stationary and if rational bubbles do not exist. Substituting Equation (3.10) into Equation (3.5) and rearranging terms yields

$$P_t - \alpha r^{-1} D_t = B_t + \alpha r^{-1} \sum_{i=1}^{\infty} \frac{E_t(\Delta D_{t+i})}{(1+r)^{i-1}} + \sum_{i=1}^{\infty} \frac{E_t(\mu_{t+i})}{(1+r)^{i}}, \tag{3.12}$$

where Δ is a difference operator. If μ_{t+i}, ΔD_{t+i} and B_t are stationary, then the right hand side of Equation (3.12) is stationary, and then, the left-hand side of Equation (3.12), $P_t - \alpha r^{-1} D$, is stationary although P_t and D_t are not stationary. In other words, if the process generating ΔD_t and μ_t are stationary, and if B_t is equal to zero, then P_t and D_t are cointegrated of order (1,1) with cointegrating vector $(1, -\alpha r^{-1})$. They find that $P_t - \alpha r^{-1} D_t$ was stationary, indicating no evidence for the existence of rational bubbles, during 1871 to 1986. Evans (1991), however, claims that conventional unit root and cointegration tests cannot detect a bubble if a bubble is periodically collapsing.

3.2.2 Intrinsic Bubble Model

Froot and Obstfeld (1991) introduced intrinsic bubbles. Unlike rational bubbles deviated from the fundamental in the RBM, intrinsic bubbles are driven by the fundamental in the Intrinsic Bubble Model (IBM). In the IBM, they use "present value" instead of "fundamental" in Equation (3.4) with continuous rates for discount and growth rates. Then,

$$P^{PV}{}_t = \sum_{s=t+1}^{\infty} e^{-r(s-t)} E_t(D_s), \tag{3.13}$$

where P^{PV} is the present value solution for Equation (3.1) with the transversality condition, Equation (3.3). They assume that log dividends follow the geometric martingale, i.e.,

$$\log(D_{t+1}) = g + \log(D_t) + \xi_{t+1}, \tag{3.14}$$

where g is the trend growth in dividends, and ξ is a normal random variable with conditional mean zero and variance σ^2. Then,

$$P^{PV}{}_t = kD_{t+1}, \tag{3.15}$$

where $k = (e^{r} - e^{g+\sigma^2/2})^{-1}$. For the sum in Equation (3.13) to converge, $r > g + \sigma^2/2$. They also define a bubble, B_t, as

$$B_t = cD_{t+1}^{\lambda}, \tag{3.16}$$

where λ is the positive root of the quadratic equation

$$\lambda^2\sigma^2/2 + \lambda g - r = 0 \tag{3.17}$$

and c is an arbitrary constant. In Equation (3.16), B_t is a function of fundamental (dividends) only, and thus called the intrinsic bubble. λ is always greater than 1 because $r > g + \sigma^2/2$, and c is always positive so that stock prices cannot be negative. Hence, the intrinsic bubble, like the rational bubble, is always positive. Equation (3.16) satisfies the following Equation (3.18), the continuous version of Equation (3.6).

$$B_t = e^{-r}E_t(B_{t+1}). \tag{3.18}$$

Now, by summing the present-value price and the intrinsic bubble, the stock price equation in the IBM is

$$P_t = P^{PV}_t + B_t = kD_{t+1} + cD_{t+1}^{\lambda}. \tag{3.19}$$

Therefore, the empirical model of the IBM is

$$P_t = c_0D_{t+1} + cD_{t+1}^{\lambda} + \varepsilon_t, \tag{3.20}$$

where $c_0 = k = (e^{r} - e^{g+\sigma^2/2})^{-1}$, ε is residuals and represents a present value of single-period excess returns. Hence, the estimated price, $E(P_t)$, from the IBM is

$$E(P_t) = c_0D_{t+1} + cD_{t+1}^{\lambda}. \tag{3.21}$$

The model (3.20) implies that there exists intrinsic bubbles and nonlinear relationship between price and dividends unless c = 0. By applying this model to U. S. stock market, Froot and Obstfeld (1991) find significant evidence to support their model.

3.2.3 Near-Rational Bubble

In the IBM, it is assumed that the price-dividend ratio has a positive drift, and thus it is ever-growing and non-stationary because $r > g + \sigma^2/2$; this assumption is also applied to the typical RBM. However, this assumption is far from reality. Lansing (2010) introduced the near-rational bubble (NRB) solution based on the assumption that the bubble component of the price-dividend ratio develops as a geometric random walk without a drift so that the bubble growth is mean-reverting (with a mean zero) and stationary. Hence, the near-rational bubble expands and collapses irregularly, even allows the price to dip below the value. He finds evidence to support the near-rational bubble in the U.S. stock market.

3.2.4 Other Measures of Bubbles

Some researchers look for measures closely related to the fundamentals, and then assess asset bubbles by evaluating these measures relative to the assumed level for the fundamental asset values. To identify a bubble, for example, De Long and Shleifer (1991) examined the market value of closed-end funds relative to their net asset value while Rappoport and White (1993) used the premium demanded on loans collateralized by the purchase of stocks. De Long and Shleifer (1991) argue that the S&P composite were priced at least 30% and possibly up to 80% above fundamentals in the summer of 1929; these can be converted to the relative bubbles of the market (Ω_M) of 0.23 and 0.44, respectively, if the S&P bubble was assumed to be at the same level as the market bubble. Rappoport and White (1993), however, find no bubbles in 1929.

Wu (1997), by using a stochastic version of rational bubble model with a S&P 500 index, finds that bubbles estimated by his model can explain most part of bubbles in the stock market, and that bubbles accounts for a substantial part of stock prices. Lee et al. (1999) measured the intrinsic value of the DJIA using the residual income valuation model. With monthly data for the period from January 1979 to June 1996, they find that P/V ratios for the DJIA had a mean 1.35 and almost always stayed within the two-standard-deviation range, i.e., rarely exceeded 1.8 or fell below 0.9. These can be converted into relative bubbles: mean 0.26 and the range between –0.11 and 0.44. Siegel (2003) proposed an operational definition of a bubble: a bubble existed when realized asset returns are more than two standard deviations from their expected returns during asset duration, and conclude that there have been no bubbles since 1900, that the low points of 1932 and 1982 were negative bubbles, and that the Internet bubble indeed existed. Koh et al. (2010) compared de facto abnormal returns with de jure abnormal returns to examine the Internet bubble, and find that the Internet bubble indeed existed and that the Internet bubble formed due to investors' risk-seeking behavior, i.e., asymmetric reactions to good and bad information.

3.2.5 Innovation and Value

According to Schumpeter (1934), a new value is created through the technological development from innovation, and there are several sources of innovation including the introduction of new goods or new production methods, the creation of new markets, the discovery of new supply sources, and the reorganization of industries. In Schumpeter's theory, innovation is the key to create a value, and further, this innovation creates a discontinuous change (jump) in value. Schumpeter (1947) also argues that innovation is promoted by large firms and by imperfect competition (Schumpeterian hypothesis) because economies of scale and barriers to entry are essential to succeed in innovative activities.

However, some studies find that this Schumpeterian hypothesis is not always true. Acs and Audretsch (1987) find that large firms have some innovative advantages in imperfect competition while small firms in the competitive markets. Rothwell (1989) finds that the innovative advantages of large firms are mainly material, i.e., from relatively greater financial and technological resources, whereas those of small firms are behavioral, i.e., from entrepreneurial dynamism, internal flexibility and responsiveness to changing circumstances.

3.3 Literature Analysis

3.3.1 RBM vs. IBM

The IBM was developed based on the RBM. The only difference between them is the relationship between bubbles and dividends. In the RBM, rational bubbles are said to be independent of dividends. In the IBM, on the other hand, intrinsic bubbles are dependent only on dividends. From equation (3.4),

$$F_{t+1} = \sum_{i=1}^{\infty} \frac{E_{t+1}(D_{t+i+1})}{(1+r)^{i}}. \tag{3.22}$$

If the expectations are the same between Period *t* and *t+1*, then

$$F_t = \frac{E_t(D_{t+1})}{(1+r)} + \frac{F_{t+1}}{(1+r)}. \tag{3.23}$$

If $E_t(D_{t+1}) = D_{t+1}$, then

$$F_t = \frac{D_{t+1}}{1+r} + \frac{F_{t+1}}{1+r} = \frac{F_{t+1} + D_{t+1}}{1+r}. \tag{3.24}$$

Rearranging Equation (3.24) yields

$$r = \frac{F_{t+1} + D_{t+1}}{F_t} - 1 = \frac{(F_{t+1} - F_t) + D_{t+1}}{F_t}. \tag{3.25}$$

In Equation (3.25), r, discount rate in the RBM, is the value return if F means the value (See Equation (1.4)). In addition, Equations (3.1) and (3.6) imply that both price return and bubble growth are the same as the value return in the RBM, implying that bubbles in the RBM are dependent on the fundamental. From Equations (3.13) and (3.18), the bubble growth are the same as the value return if P^{PV} means the value, and thus as the price return in the IBM, too.

To sum up, the fundamental in the RBM is the same as the present value in the IBM, and rational bubbles are also the same as intrinsic bubbles; rational bubbles are implicitly, while intrinsic bubbles are explicitly, dependent on the fundamental. The IBM incorporated stochastic dividends in the RBM, and then finds the nonlinear relationship between bubbles and dividends, and thus between prices and dividends.

3.3.2 RBM and No Irrationality

As seen in Section 3.3.1, both bubbles and prices grow at the same rate as values in the RBM (including the IBM, hereafter), implying no irrationality. A bubble growth can be defined and computed as follows.

$$G^B_t = \frac{B_t - B_{t-1}}{B_{t-1}} = \frac{(P_t - V_t) - (P_{t-1} - V_{t-1})}{B_{t-1}} = \frac{(P_t - P_{t-1}) - (V_t - V_{t-1})}{B_{t-1}}, \tag{3.26}$$

where G^B denotes a bubble growth. From Equations (1.3) and (1.4),

$$P_t - P_{t-1} = P_{t-1}R^P_t - D_t, \tag{3.27}$$

$$V_t - V_{t-1} = V_{t-1}R^V_t - D_t. \tag{3.28}$$

Thus, dividends do not affect the size of a bubble. Substituting Equations (3.27) and (3.28) into Equation (3.26) yields

$$G^B_t = \frac{P_{t-1}R^P_t - V_{t-1}R^V_t}{B_{t-1}} = \frac{P_{t-1}R^P_t - (P_{t-1} - B_{t-1})R^V_t}{B_{t-1}} = \frac{P_{t-1}(R^P_t - R^V_t) + B_{t-1}R^V_t}{B_{t-1}}$$

$$= \frac{P_{t-1}}{B_{t-1}}(R^P_t - R^V_t) + \frac{B_{t-1}}{B_{t-1}}R^V_t, \text{ and then,}$$

$$G^B_t = \frac{P_{t-1}\omega^R_t}{B_{t-1}} + R^V_t = \frac{\omega^R_t}{\Omega_{t-1}} + R^V_t \left(= \frac{\omega^G_t}{\Omega_{t-1}} + G^V_t\right). \tag{3.29}$$

In Equation (3.29), the formula in ( ) is derived using $P_t - P_{t-1} = P_{t-1}G^P_t$ and $V_t - V_{t-1} = V_{t-1}G^V_t$ in Equation (3.26).

Equation (3.29) indicates that the bubble growth is the same as the value return only if no irrational return exists. The RBM, as its name implies, assumes no irrationality, and thus the bubble growth is the same as the value returns in the RBM. Further, the measure of bubble growth in Equation (3.29) is not good to use when the previous bubble is zero or negative, but bubbles are always positive in the RBM.

3.3.3 NRB vs. IBM

Dividing Equation (3.19) by Dt yields

$$\frac{P_t}{D_t} = \frac{P^{PV}{}_t}{D_t} + \frac{B_t}{D_t} = \frac{kD_{t+1}}{D_t} + \frac{cD_{t+1}{}^{\lambda}}{D_t}. \tag{3.30}$$

Then, the first and second terms in the right hand side of Equation (3.30) can be regarded as a fundamental and bubble components, respectively, of the price-dividend ratio. In the IBM, from Equation (3.16),

$$B_{t-1} = cD_t^{\lambda}. \tag{3.31}$$

From Equation (3.14),

$$E(D_{t+1}) = e^{g}D_t, \text{ because } E(\xi_{t+1}) = 0.$$

In other words, g is the continuous growth rate of dividends. Hence, from Equation (3.16),

$$E(B_t) = cE(D_{t+1}{}^{\lambda}) = c(e^{g}D_t)^{\lambda} = c(e^{g\,\lambda}D_t^{\lambda}) = e^{g\,\lambda}(cD_t^{\lambda}) = e^{g\,\lambda}B_{t-1}. \tag{3.32}$$

The expected bubble growth is $e^{g\,\lambda}$ in the IBM while zero in the NRB.

In the NRB, a bubble is intrinsic, i.e., dependent on dividends and thus the same as in the IBM, meaning $\lambda > 1$. Hence, $E(g) = 0$ in the NRB. Therefore, in the NRB solution, from Equations (3.32) and (3.16),

$$E(B_t) = B_{t-1}= cD^{\lambda}.$$

$$c = \frac{B_{t-1}}{D^{\lambda}}. \tag{3.33}$$

Hence, in the NRB, c is dependent on the previous bubble, and thus neither constant nor always positive if the previous bubble can be negative. The bubble component is always positive in the IBM (and the same in the RBM) whereas possibly negative in the NRB solution. In other words, c is always positive in the IBM whereas possibly negative in the NRB solution. As a result, bubbles are always positive and ever-growing in the IBM while possibly negative and sometimes collapsing in the NRB. In the IBM, bubbles move together along with the fundamentals. In the NRB, however, bubbles hover around the fundamentals.

By substituting Equation (3.33) and zero for c and g (in k), respectively, into Equation (3.30), and using constant D (from g = 0), the NRB can be described as

$$\frac{P_t}{D} = k + \frac{B_{t-1}}{D},$$

$$P_t = kD + B_{t-1},$$

where $k = (e^r - e^{\sigma^2/2})^{-1}$. Hence, in the NRB solution, the current price consists of discounted constant dividends and the previous bubble.

3.4 Ex-post Bubble Model

The purpose of this section is to explain how to measure values and bubbles of assets using historical data and how bubbles behave in historical data. The EBM measures historical values using historical value returns, which are measured by the Rational CAPM (See Section 2.5.1). Hence, the valuation in the EBM is based on the Rational CAPM.

3.4.1 Measurement of Bubbles

3.4.1.1 Ex-post valuation and Ex-post Bubble Model. As seen in Equation (1.1), a bubble can be measured by the deviation of the price of an asset from its value. Asset prices can be obtained in the market while asset values cannot be observed. Therefore, the key to measure a bubble is to measure a value. Previous studies measure a value by the present value of all future income flows to shareholders – dividends, free cash flows or net incomes (Penman and Sougiannis, 1998). As seen in Equation (3.4), the RBM (in fact, the dividend discount model) measures a value by a stream of expected dividends with a constant discount rate. To value a firm by Equation (3.4), therefore, one needs correct estimates of all future dividends and a constant future discount rate, implying that no one can measure a value correctly because it is neither possible to correctly estimate all future dividends nor reasonable to assume the constant future discount rate.

If historical value data are available, a value can be measured directly using historical data. By Equation (1.5), we can compute a current price by using the previous price and its growth in the current period.

$$P_t = P_{t-1}(1 + G^P{}_t) \tag{3.34}$$

By the same way, from Equation (1.6),

$$V_t = V_{t-1}(1 + G^V{}_t). \tag{3.35}$$

The EBM uses growth measures because it measures price and value by ex-post historical data that excluded dividends. Section B in Appendix explains how to measure a bubble by return measures with ex-post data. By solving Equations (3.34) and (3.35) recursively up to the first period (t = 1) with an initial price or an initial value (P_0 or V_0, respectively), which is the price or value at the beginning of the first period,

$$P_t = P_0(1 + G^P{}_1)(1 + G^P{}_2)(1 + G^P{}_3) \cdots\cdots (1 + G^P{}_t), \tag{3.36}$$

$$V_t = V_0(1 + G^V{}_1)(1 + G^V{}_2)(1 + G^V{}_3) \cdots\cdots (1 + G^V{}_t). \tag{3.37}$$

Equation (3.37) explains ex-post valuation; one can measure a value using ex-post data if knowing an initial value (V_0) and value growths of all subsequent periods ($G^V{}_t$, t = 1 to t).

From Equations (1.1) and (1.2) with Equations (3.36) and (3.37),

$$B_t = P_0(1 + G^P{}_1)(1 + G^P{}_2) \cdots\cdots (1 + G^P{}_t) - V_0(1 + G^V{}_1)(1 + G^V{}_2) \cdots\cdots (1 + G^V{}_t), \quad (3.38)$$

$$\Omega_t = 1 - \frac{V_0(1 + G^V{}_1)(1 + G^V{}_2) \cdots\cdots (1 + G^V{}_t)}{P_0(1 + G^P{}_1)(1 + G^P{}_2) \cdots\cdots (1 + G^P{}_t)}. \quad (3.39)$$

If the current price is known, then,

$$B_t = P_t - V_0(1 + G^V{}_1)(1 + G^V{}_2) \cdots\cdots (1 + G^V{}_t), \quad (3.40)$$

$$\Omega_t = 1 - \frac{V_0(1 + G^V{}_1)(1 + G^V{}_2) \cdots\cdots (1 + G^V{}_t)}{P_t}. \quad (3.41)$$

Hence, to measure a bubble, Models (3.38) to (3.41) use ex-post data; for this reason, these are called the Ex-post Bubble Model (EBM). A bubble can be zero or negative in the EBM, different from the RBM. The EBM measures not only the existence but also the size of a bubble. To measure a value and a bubble in the EBM, both initial value (V_0) and all subsequent value growths (from $G^V{}_1$ to $G^V{}_t$) should be obtained.

3.4.1.2 Value growth in the ex-post valuation. From Equations (1.8) and (1.10),

$$R^V{}_t - G^V{}_t = \delta^V{}_t = \frac{D_t}{V_{t-1}}. \quad (3.42)$$

Then,

$$G^V{}_t = R^V{}_t - \frac{D_t}{V_{t-1}}. \quad (3.43)$$

With dividends, Equation (3.43) is used to compute value growths. Without dividends, dividends can be easily calculated using price data. From Equations (1.7) and (1.9),

$$R^P{}_t - G^P{}_t = \delta^P{}_t = \frac{D_t}{P_{t-1}}, \quad (3.44)$$

Then,

$$D_t = P_{t-1}(R^P{}_t - G^P{}_t). \quad (3.45)$$

By substituting Equation (3.45) into Equation (3.43),

$$G^V{}_t = R^V{}_t - \frac{P_{t-1}}{V_{t-1}}(R^P{}_t - G^P{}_t). \tag{3.46}$$

If value returns and previous values are measured, then value growths can be calculated by Equation (3.43) or (3.46). Value returns can be measured using the Rational CAPM. Hence, the valuation in the EBM is based on the Rational CAPM. In Equation (3.43) or (3.46), if value returns are known, only the previous value is unknown to get the value growth. Hence, the value growths depend on the previous values, which depend on the initial value. The EBM measures all historical values and value growths simultaneously by determining the initial value.

3.4.1.3 Initial value and dividend trap in the ex-post valuation. If we know the value of an asset at any specific time, we can compute all its subsequent values. Equation (3.43) or (3.46) produces value growths, and Equation (3.35) yields values using value growths for the following period. However, values are unobservable, and thus must be measured. The EBM measures all historical value growths and values simultaneously by determining an initial value. The initial value means the value at the beginning of the first period of the sample data.

From Equations (3.44) and (3.42),

$$G^P{}_t = R^P{}_t - \frac{D_t}{P_{t-1}} = R^P{}_t - \delta^P{}_t. \tag{3.47}$$

$$G^V{}_t = R^V{}_t - \frac{D_t}{V_{t-1}} = R^V{}_t - \delta^V{}_t. \tag{3.48}$$

Without dividends, growths are the same as returns. With dividends, these ratios are different. Equations (3.47) and (3.48) imply the followings.

1. If the dividend-price ratio is greater [less] than the price return, then $G^P{}_t < [>] 0$, i.e., the price decreases [increases].
2. If the dividend-value ratio is greater [less] than the value return, then $G^V{}_t < [>] 0$, i.e., the value decreases [increases].

Suppose a huge positive [negative] bubble in the previous period, $P_{t-1} >> [<<] V_{t-1}$, and a dividends payment with positive price and value returns in the current period *t*. Then, dividends are relatively small [large] to the previous price but relatively large [small] to the previous value, making big difference between dividend-price ratio and dividend-value ratio, in turn making $\delta^P_t < [>] R^P_t$ and $\delta^V_t > [<] R^V_t$, resulting in $G^P_t > [<] 0$ and $G^V_t < [>] 0$, i.e., the price increases [decreases] while the value decreases [increases], extending the positive [negative] bubble by dividends paid. In other words, if huge positive [negative] bubbles existed in the previous period, the current dividends extend positive [negative] bubbles in the current period. Then, with the similar situation in the following period, the value [price] will decrease and the price [value] will increase faster in the following period than in the current period, and this will continue in all subsequent periods, amplifying the positive [negative] bubble; a vicious cycle of a bubble. This phenomenon is called a dividend trap in this chapter.

If the dividend trap continues, either value or price will finally turn out to be negative theoretically (See Panels A and C in Table 3.1), but prices are directly obtained from the market, where prices are always positive. Even though neither negative value nor negative price is created, the dividend trap can happen. If there was a huge positive [negative] bubble in the previous period, the gap between dividend-price and dividend-value ratios is so significant that the growth of the price is much higher [lower] than that of the value, magnifying the previous positive [negative] bubbles (See Panels B and D in Table 3.1).

This dissertation assumes that both price and value of an asset is always positive, and also assumes that in the market as a whole, firms on average have appropriately paid dividends compared to their prices and values, and thus they would not be caught in the dividend trap. Hence, when the dividend trap occurs, values should be adjusted to avoid the dividend trap because prices cannot be adjusted; prices are obtained directly from the market. To prevent a dividend trap with a positive [negative] bubble at any period, the initial value should be raised [lowered]. However, adjustments to the initial value affect all the subsequent historical values. Furthermore, Equation (3.43) or (3.46) explains that changes in values affect value growths of the following period, implying that adjustments to the initial value affect all the subsequent value growths. If the dividend trap happens, the EBM adjusts the initial value, which will affect all the subsequent values not only by changes in initial value but also by changes in value growths caused by changes in values.

As a result, the dividend trap can be used to identify the range of the initial value and bubble. To use the dividend trap to identify the range of the initial value, however, a firm or firms on average in the portfolio should pay dividends and have such a long history that it can get caught in the dividend trap with some changes in the initial value. The longer the history, the narrower is the range of the initial value.

3.4.1.4 Dividend trap and dividend-value ratio in the ex-post valuation. The dividend trap occurs due to the previous huge bubble that makes a big difference in the denominators between dividend-price and dividend-value ratios; both ratios have the same numerator of dividends payments.

Table 3.1 explains the dividend trap. Dividends of $1 per share are assumed for Panels A and C while dividends of $0.30 per share for Panels B and D, and both price and value are assumed to earn 10% returns every year for all panels. Prices and values for the period *t-1* are given for all panels. Panel A shows the dividend trap when the dividend-value ratio is greater than the value return in the huge positive bubble situation. In this case, values decrease while prices increase, and thus bubbles grow fast. Finally, the value becomes negative. These results

Table 3.1 Dividend Trap: Examples

This table shows the examples of the dividend trap. δ, D, R, G, P, V, B and Ω denote dividend yield, dividends, returns, growth, price, value, bubble and relative bubble, respectively. Superscript *P* and *V* indicate variables using price data and value data, respectively.

Year	δ^P	δ^V	D	R^P	G^P	R^V	G^V	P	V	B	Ω
Panel A: Positive bubble in the previous period and $\delta^V > R^V$											
t-1			$1.00					$20.00	$4.00	$16.00	0.800
t	0.050	***0.250***	$1.00	0.100	0.050	0.100	-0.150	$21.00	$3.40	$17.60	0.838
t+1	0.048	***0.294***	$1.00	0.100	0.052	0.100	-0.194	$22.10	$2.74	$19.36	0.876
-	-	-	-	-	-	-	-	-	-	-	-
t+4	0.041	***0.823***	$1.00	0.100	0.059	0.100	-0.723	$26.11	$0.34	$25.77	0.987
t+5	0.038	***2.968***	$1.00	0.100	0.062	0.100	-2.868	$27.72	***-$0.63***	$28.34	1.023
t+6	0.036	NA	$1.00	0.100	0.064	0.100	NA	$29.49	***-$1.69***	$31.18	1.057
t+7	0.034	NA	$1.00	0.100	0.066	0.100	NA	$31.44	***-$2.86***	$34.30	1.091
t+8	0.032	NA	$1.00	0.100	0.068	0.100	NA	$33.58	***-$4.15***	$37.73	1.124
t+9	0.030	NA	$1.00	0.100	0.070	0.100	NA	$35.94	***-$5.56***	$41.50	1.155
t+10	0.028	NA	$1.00	0.100	0.072	0.100	NA	$38.53	***-$7.12***	$45.65	1.185
Panel B: Positive bubble in the previous period and $\delta^V < R^V$											
t-1			$0.30					$20.00	$4.00	$16.00	0.800
t	***0.015***	0.075	$0.30	0.100	0.085	0.100	0.025	$21.70	$4.10	$17.60	0.811
t+1	***0.014***	0.073	$0.30	0.100	0.086	0.100	0.027	$23.57	$4.21	$19.36	0.821
-	-	-	-	-	-	-	-	-	-	-	-
t+4	***0.011***	0.067	$0.30	0.100	0.089	0.100	0.033	$30.38	$4.61	$25.77	0.848
t+5	***0.010***	0.065	$0.30	0.100	0.090	0.100	0.035	$33.12	$4.77	$28.34	***0.856***
t+6	***0.009***	0.063	$0.30	0.100	0.091	0.100	0.037	$36.13	$4.95	$31.18	***0.863***
t+7	***0.008***	0.061	$0.30	0.100	0.092	0.100	0.039	$39.44	$5.14	$34.30	***0.870***
t+8	***0.008***	0.058	$0.30	0.100	0.092	0.100	0.042	$43.09	$5.36	$37.73	***0.876***
t+9	***0.007***	0.056	$0.30	0.100	0.093	0.100	0.044	$47.09	$5.59	$41.50	***0.881***
t+10	***0.006***	0.054	$0.30	0.100	0.094	0.100	0.046	$51.50	$5.85	$45.65	***0.886***
Panel C: Negative bubble in the previous period and $\delta^P > R^P$											
t-1			$1.00					$4.00	$20.00	-$16.00	-4.000
t	***0.250***	0.050	$1.00	0.100	-0.150	0.100	0.050	$3.40	$21.00	-$17.60	-5.176
t+1	***0.294***	0.048	$1.00	0.100	-0.194	0.100	0.052	$2.74	$22.10	-$19.36	-7.066
-	-	-	-	-	-	-	-	-	-	-	-
t+4	***0.823***	0.041	$1.00	0.100	-0.723	0.100	0.059	$0.34	$26.11	-$25.77	-76.477
t+5	***2.968***	0.038	$1.00	0.100	-2.868	0.100	0.062	***-$0.63***	$27.72	-$28.34	45.037
t+6	NA	0.036	$1.00	0.100	NA	0.100	0.064	***-$1.69***	$29.49	-$31.18	18.424
t+7	NA	0.034	$1.00	0.100	NA	0.100	0.066	***-$2.86***	$31.44	-$34.30	11.986
t+8	NA	0.032	$1.00	0.100	NA	0.100	0.068	***-$4.15***	$33.58	-$37.73	9.096
t+9	NA	0.030	$1.00	0.100	NA	0.100	0.070	***-$5.56***	$35.94	-$41.50	7.461
t+10	NA	0.028	$1.00	0.100	NA	0.100	0.072	***-$7.12***	$38.53	-$45.65	6.413
Panel D: Negative bubble in the previous period and $\delta^P < R^P$											
t-1			$0.30					$4.00	$20.00	-$16.00	-4.000
t	0.075	***0.015***	$0.30	0.100	0.025	0.100	0.085	$4.10	$21.70	-$17.60	-4.293
t+1	0.073	***0.014***	$0.30	0.100	0.027	0.100	0.086	$4.21	$23.57	-$19.36	-4.599
-	-	-	-	-	-	-	-	-	-	-	-
t+4	0.067	***0.011***	$0.30	0.100	0.033	0.100	0.089	$4.61	$30.38	-$25.77	-5.589
t+5	0.065	***0.010***	$0.30	0.100	0.035	0.100	0.090	$4.77	$33.12	-$28.34	***-5.940***
t+6	0.063	***0.009***	$0.30	0.100	0.037	0.100	0.091	$4.95	$36.13	-$31.18	***-6.301***
t+7	0.061	***0.008***	$0.30	0.100	0.039	0.100	0.092	$5.14	$39.44	-$34.30	***-6.668***
t+8	0.058	***0.008***	$0.30	0.100	0.042	0.100	0.092	$5.36	$43.09	-$37.73	***-7.041***
t+9	0.056	***0.007***	$0.30	0.100	0.044	0.100	0.093	$5.59	$47.09	-$41.50	***-7.419***
t+10	0.054	***0.006***	$0.30	0.100	0.046	0.100	0.094	$5.85	$51.50	-$45.65	***-7.799***

t+2 and t+3 years are omitted for space. NA: not applicable due to negative denominators are due to the high dividend-value ratio that is much greater than the dividend-price ratio (before turning into negative values). To prevent this type of dividend trap, the upper limit should be set on the dividend-value ratios. Panel B shows the dividend trap when the dividend-value ratio is less than the value return in the huge positive bubble situation. In this case, both prices and values increase, but prices increase much faster than values, accelerating huge positive bubbles. This phenomenon results from the low dividend-price ratio that is much lower than the dividend-value ratio. To prevent this type of dividend trap, the lower limit should be imposed on the dividend-price ratios. Panels C and D show the dividend trap when the negative bubbles exist, and can be explained in the opposite way to Panels A and B. To avoid the Panel C [D] type of dividend trap, the upper [lower] limit should be put on the dividend-price [value] ratios.

To sum up, the upper and lower limits should be set on both dividend-price and dividend value ratios to avoid the dividend trap. In the dividend trap with the positive [negative] bubble, dividend-price [value] ratio is too low and/or dividend-value [price] ratio is too high to escape from the dividend trap. To avoid the dividend trap, these ratios should be neither excessive nor trivial. However, firms on average are assumed to pay dividends appropriately compared to their prices, and thus the situations seen in Panels B and C in Table 3.1 would not happen. Further, dividend-price ratios cannot be adjusted because they are obtained from the market. Hence, the dividend trap should be avoided by using dividend-value ratios, which can vary by adjusting the initial value.

3.4.1.5 Dividend-price ratio and dividend-value ratio. Dividend-price ratio is one indicator for the valuation; the higher the dividend price ratios, the more the firms are underpriced, and vice versa (Litzenberger and Ramaswamy, 1979; Fama and French, 1988; Cole et al., 1996), implying that dividend-price ratios are stable within a certain range if investors seek for a correct price using dividend-price ratios. According to the Gordon Growth Model (Gordon, 1962), under the constant cost of capital (r) and the constant dividend growth (g),

$$P_t = \frac{D_{t+1}}{r - g}, \text{ where } r > g, \text{ and thus}$$

$$\delta_{t+1} = \frac{D_{t+1}}{P_t} = r - g, \text{ where } r > g. \tag{3.49}$$

Equation (3.49) describes that dividend-price ratios can be approximated by the difference between cost of capital and dividend growth, and thus will stay within a certain range.

Prices fluctuate around values according to the Efficient Market Hypothesis and the Random Walk theory (Fama, 1965; Malkiel, 2003). Then, it also looks as if values are hover around prices. In other words, for the relatively stable dividends, dividend-price ratios fluctuate, so do dividend-value ratios. If firms on average are correctly priced historically, the historical range of dividend-value ratios is not significantly different from that of the dividend-price ratios. Hence, the upper and lower limits of the allowable historical dividend-value ratios can be assumed to be the same as, or very similar to, those of the historical dividend-price ratios.

The range of initial values can be determined by using these limits of the dividend value ratios; the initial value should satisfy these limits for all periods. The lower and upper limits of the dividend-value ratios can be used to find the upper and lower limits of the initial value, respectively. Then, within the range of the initial value, one point can be used as the initial value if it best describes the market, or the median value of the range might be the best measurement of the initial value unless it contradicts other evidence.

The range of the dividend-value ratios that is assumed to be the same as that of the historical dividend-price ratios might be too narrow to find the range of the initial value. If this situation happens, the range of the dividend-value ratios can be extended beyond that of the dividend-price ratios, say, by 5% or 10% of each limit. This can be justified because dividends are related to prices (recall that dividends-price ratios can be used to check the adequacy of pricing) but independent of values because values are unknown. Hence, dividend-value ratios are likely to be more volatile than dividend-price ratios because the relationship of dividends is weaker to values than to prices.

In conclusion, the EBM uses dividends to measure values and bubbles; this approach can be justified by existing literature. First, dividends play an important role in valuation literature. Dividends have been used to value firms in dividend discount model (See Sections 3.2.1 and 3.4.1.1) and Gordon Growth Model (See Section 3.4.1.5). These models measure a value at a specific time by discounting ex-ante dividends while the ex-post valuation finds all end-of-period values at the same time by using ex-post dividend-value ratios. Second, dividends play a crucial role in bubble literature. Representative bubble models have used the relationship between dividends and price to verify and/or measure bubbles (See Sections 3.2 and 3.3). To measure a bubble, the EBM uses dividend-price ratios whose historical range is assumed to be the same as (or similar to) the range of historical dividend-value ratios.

3.4.1.6 Steps to determine an initial value in the ex-post valuation.

3.4.1.6.1 Dividend-trap method. If dividends have been paid during a long period, an initial value of an asset can be determined using dividend value ratios as explained in Sections 3.4.1.3 to 3.4.1.5. The steps to determine an initial value are as follows.

1. Assign any number to an initial value; this number is called a preliminary initial value. Before determining an initial value, value growths should be computed for all periods using Equation (3.43) or (3.46), which cannot be used without an initial value. The preliminary initial value is necessary only for convenience to compute value growths. To

be more convenient, for a preliminary initial value, use the same number as the initial price.

2. Find the historical range of dividend-price ratios during the sample period.
3. Determine the historical range of dividend-value ratios during the sample period. Assume that the historical range of dividend-value ratios is the same as that of dividend-price ratios during the sample period.
4. Find the range of initial values that satisfy the range of dividend-value ratios for all periods. The lower and upper limits of the dividend-value ratios can be used to find the upper and lower limits of the initial value, respectively. Notice that changing initial value leads to changes of value growths of all subsequent periods (See Equation (3.43) or (3.46)), causing all subsequent dividend-value ratios to change.
5. If no values for an initial value satisfy the range of dividend-value ratios, extend the range of the dividend-value ratios by some % (say, 5% or 10%) of each limit. Then, repeat Steps 4 and 5 until any value for an initial value can satisfy the new range of dividend-value ratios.
6. Determine the initial value by appropriate value within the range of initial values if it best fits, or by the midpoint of the range of initial values unless it contradicts other evidence.

3.4.1.6.2 No-dividend method. If dividends are not paid or their history is too short to find an initial value, we cannot use the dividend trap, and thus we need to use a different approach as follows.

1. Determine an initial value by one of the following methods (b is preferred to a).
   a. Use the initial price as the initial value, based on the assumption that the initial value is the same as the initial price, i.e., the initial bubble is zero.

b. Use the value with the same relative bubble as that of the market if the market bubble is known, based on the assumption that the market bubble represents the average of all asset bubbles and that the bubble of an asset is the same as the average bubble in the market.

2. Check the validity of the initial value, and then adjust it if necessary, by comparing the results to the previous findings and evidence.

3.4.2 Behavior of a Bubble

3.4.2.1 Anatomy of a bubble. By substituting Equations (3.34) and (3.35) into Equation (1.1),

$$B_t = P_{t-1}(1 + G^P_t) - V_{t-1}(1 + G^V_t)$$

$$= P_{t-1} + P_{t-1}G^P_t - V_{t-1} - V_{t-1}G^V_t$$

$$= P_{t-1} - V_{t-1} + P_{t-1}G^P_t - V_{t-1}G^V_t$$

$$= P_{t-1} - V_{t-1} + P_{t-1}(G^V_t + \omega^G_t) - V_{t-1}G^V_t$$ (by Equation (1.12)), and then,

$$B_t = (P_{t-1} - V_{t-1}) + (P_{t-1} - V_{t-1})G^V_t + P_{t-1}\omega^G_t. \quad (3.50)$$

Thus,

$$B_t = B_{t-1} + B_{t-1}G^V_t + P_{t-1}\omega^G_t = B_{t-1}(1+ G^V_t) + P_{t-1}\omega^G_t. \quad (3.51)$$

Then,

$$B_t = B_{t-1} + B^{NV}_t + B^{N\omega} = B^{GV}_t + B^{N\omega}_t, \quad (3.52)$$

with

$$B^{NV}_t = B_{t-1}G^V_t,\ B^{N\omega}_t = P_{t-1}\omega^G_t \text{ and } B^{GV}_t = B_{t-1}(1+ G^V_t) = B_{t-1} + B^{NV}_t. \quad (3.53)$$

where B^{NV}, $B^{N\omega}$ and B^{GV} denote new (or net) evolving bubble (NEB), new irrational bubble (NIB) and (gross) evolving bubble (GEB), respectively. Equation (3.51) describes that the GEB results from the previous bubble grown by the rate of value growth while the NIB forms by the irrational growth. Hence, the NIB forms newly each period, whereas the GEB evolves every period. In addition,

$$B^N_t = B_t - B_{t-1} = B^{NV}_t + B^{N\omega}_t \text{ (from Equation (3.52),} \qquad (3.54)$$

where B^N denotes a new bubble. Equations (3.52) and (3.54) depict that a bubble can exist and even grow without irrational growths, because evolving bubbles grow at the value growth irrespective of the irrational growth. For a certain period, no irrational growth does not mean no bubble but mean no NIB. Bubbles exist and grow without irrationality if they existed in the previous period; this is the same as rational bubbles in the RBM.

A bubble can be computed by the following Formula (3.55) (See Equation (A3) in Appendix).

$$\begin{aligned} B_t = {} & B_0(1+ G^V{}_1)(1+ G^V{}_2) \cdots\cdots (1+ G^V{}_{t-1})(1+ G^V{}_t) \\ & + \textstyle\sum_{i=1}^{t-1}\{P_{i-1}\omega^G{}_i(1+ G^V{}_{i+1})(1+ G^V{}_{i+2}) \cdots\cdots (1+ G^V{}_{t-1})(1+ G^V{}_t)\} \\ & + P_{t-1}\omega^G{}_t, \ (i+1 \le t \text{ in the second term}). \end{aligned} \qquad (3.55)$$

As explained in Section 3.3.2, the RBM implies the assumption of no irrational growth. In Equation (3.55), hence, the first term in the right hand side is conceptually the same as the rational bubble in the RBM, and thus called the rational bubble (RB). Then, the sum of the other two terms represents an irrational bubble from the perspective of the RBM. The second term describes the sum of all bubbles that are created by the irrational growth but grow at the value growth, along with rational bubbles, from the subsequent period, and thus is called the rationalized irrational bubble (RIB). The last term is the NIB. Then,

$$B_t = B^{IV}_t + B^{\omega V}_t + B^{N\omega}_t, \qquad (3.56)$$

$$B_t = B^{IV}_t + B^{A\omega}_t, \qquad (3.57)$$

with

$B^{IV}{}_t = B_0(1+ G^V{}_1)(1+ G^V{}_2) \cdots\cdots (1+ G^V{}_{t-1})(1+ G^V{}_t)$ (3.58)

$B^{\omega V}{}_t = \sum_{i=1}^{t-1}\{P_{i-1}\omega^G{}_i(1+ G^V{}_{i+1})(1+ G^V{}_{i+2}) \cdots\cdots (1+ G^V{}_{t-1})(1+ G^V{}_t)\}$ (3.59)

where B^{IV}, $B^{\omega V}$ and $B^{A\omega}$ denote RB (grown from the initial bubble by the value growth), RIB (created by the irrational growth but developed by the value growths) and accumulated irrational bubble (AIB, bubbles related to all irrational growths), respectively.

As explained in Section 3.3.2, the RBM assumes no irrationality, implying $B^{A\omega} = 0$. In the RBM, thus, there is only the RB expressed in Equation (3.58), and hence, the sign of current bubble depends on the initial bubble because $G^V{}_t > -1$ as explained in Section 1.2.4. If the initial bubble is positive [negative, zero], then the RB is positive [negative, zero] at any time. However, in the EBM, the irrational growth creates the NIB even though no bubbles existed in the previous period, and this is the major difference between the RBM and the EBM. Further, the NIB is absorbed to the GEB, and then they grow together, from the subsequent period. (See Section A in Appendix for more details).

3.4.2.2 Irrationality for rational pricing. Equations (3.52) and (3.56) show that irrational growths not only create a bubble but also develop it. If no bubble existed in the previous period, the irrational growth is indeed irrational because it creates a bubble in an asset. However, once a bubble is created, irrational growths can increase or decrease it, implying that an irrational growth can be rational in that it can reduce an existing bubble. If an asset is to be rationally priced, irrational growths should reduce the previous bubble. If not, a bubble persists or grows in the portfolio. Irrationality can lead to rational pricing.

By setting $B_t = 0$ in Equation (3.51),

$B_{t-1}(1 + G^V{}_t) + P_{t-1}\omega^G{}_t = 0.$

$$\omega^G{}_t = \frac{-B_{t-1}(1 + G^V{}_t)}{P_{t-1}} = -\Omega_{t-1}(1 + G^V{}_t). \quad (3.60)$$

As explained in Section B in Appendix, all equations from (3.50) to (3.60) in Sections 3.4.2.1 and 3.4.2.2 hold even though G^P, G^V and ω^G are replaced by R^P, R^V and ω^R, respectively, because dividends do not affect the size of bubbles; however, this rule cannot be applied to some equations in Section 3.4.2.4 because dividends do affect prices and thus relative bubbles. Then, we can get

$$\omega^R{}_t = \frac{-B_{t-1}(1 + R^V{}_t)}{P_{t-1}} = -\Omega_{t-1}(1 + R^V{}_t). \tag{3.61}$$

Therefore, to eliminate or at least reduce a bubble (i.e., to rationally price an asset) in the subsequent period or in a short-term period, irrational growths or returns ($\omega^G{}_t$ or $\omega^R{}_t$) should be negatively related to previous relative bubbles (Ω_{t-1}) because both G^V and R^V are always greater than –1 as assumed in Section 1.2.4.

3.4.2.3 "Invisible hand": self-control mechanism of bubbles in the market. Price changes directly affect dividend-price ratios, which also affect prices if dividend-price ratios are stable within a certain range. Hence, prices and dividend-price ratios interact mutually. The market tries to find a correct price (= value) of an asset through this interaction. In addition, irrationalities affect price changes through bubbles as explained in Sections 1.2.6 and 1.2.7. Therefore, there is the causal relationship that dividend-price ratio affects irrationality, which in turn affects bubble changes, which in turn affects price changes, which in turn affects dividend-price ratio, and this sequence continues.

The effect of dividend-price ratio on irrationality is expected to be positive. High [Low] dividend-price ratio implies low [high] prices for assets at the beginning of period. This information of low [high] price will affect investor sentiments, leading to increases [decreases] in demand and/or decreases [increases] in supply, and then resulting in increases [decreases] in price. These changes in price can be either rational or irrational. Recall that irrationality is measured by mispricing, the difference between price changes and value changes, as explained in Sections 1.2.7 and 1.2.8. Whenever price changes are different from value changes, there exists irrationality. However, irrationality can be rational in pricing as explained in Section 3.4.2.2. If the dividend-price ratio is higher [lower] than normal, price changes are likely to be higher [lower] than value changes due to positive [negative] irrationality. Hence, irrationality represents investor sentiment as explained in the Rational CAPM (See Sections 2.3.1.5 and 2.6.2).

The effects of irrationality and bubble changes on bubble changes and price changes, respectively, are both expected to be positive; positive [negative] irrationality increases [decreases] bubbles, resulting in increases [decreases] in price. The effect of price changes on dividend-price ratio will be negative in the subsequent period if dividends are not significantly changed because the denominator in the dividend-price ratio is a beginning price.

By the causal relations explained above, increases in dividend-price ratios lead to positive irrationality, causing increases in bubbles, making prices increase. Then, in the next period, dividend-price ratios will decrease, leading to negative irrationality, causing decreases in bubbles, making prices decrease, and then increasing dividend-price ratios in the following period, and this will continue. Therefore, if the market works, bubbles will be self-controlled by the "invisible hand" in the market. This "invisible hand" can make the market rationally priced in the end.

3.4.2.4 Relative bubble and irrational growth. From Equations (1.2) and (3.51),

$$\Omega_t = \frac{B_t}{P_t} = \frac{B_{t-1}}{P_t} + \frac{B_{t-1}G^V{}_t}{P_t} + \frac{P_{t-1}\omega^G{}_t}{P_t}. \quad (3.62)$$

Relative new bubbles (Ω^N) and difference in relative bubbles (Ω^d) are respectively defined as

$$\Omega^N{}_t = \frac{B^N{}_t}{P_t}, \quad (3.63)$$

$$\Omega^d_t = \Omega_t - \Omega_{t-1}. \tag{3.64}$$

Ω^N is different from Ω^d when prices change as seen in Equation (3.65).

$$\Omega^N_t = \frac{B^N_t}{P_t} = \frac{B_t - B_{t-1}}{P_t} = \frac{B_t}{P_t} - \frac{B_{t-1}}{P_t} = \frac{B_t}{P_t} - \frac{B_{t-1}}{P_{t-1}(1+G^P t)} = \Omega_t - \frac{\Omega_{t-1}}{1+G^P t}. \tag{3.65}$$

From Equations (3.63) with Equations (3.53) and (3.54),

$$\Omega^N_t = \frac{B^N_t}{P_t} = \frac{B_{t-1}G^V_t}{P_t} + \frac{P_{t-1}\omega^G_t}{P_t}. \tag{3.66}$$

Following Equation (3.53),

$$\Omega^{NV}_t = \frac{B^{NV}_t}{P_t} = \frac{B_{t-1}G^V_t}{P_t} = \frac{B_{t-1}G^V_t}{P_{t-1}(1+G^P_t)} = \frac{\Omega_{t-1}G^V_t}{1+G^P_t}, \tag{3.67}$$

$$\Omega^{N\omega}_t = \frac{B^{N\omega}_t}{P_t} = \frac{P_{t-1}\omega^G_t}{P_t} = \frac{P_{t-1}\omega^G_t}{P_{t-1}(1+G^P_t)} = \frac{\omega^G_t}{1+G^P_t}, \tag{3.68}$$

where Ω^{NV} and $\Omega^{N\omega}$ denote relative NEB and relative NIB, respectively. Then,

$$\Omega^N_t = \Omega^{NV}_t + \Omega^{N\omega}_t. \tag{3.69}$$

From Equation (3.62),

$$\Omega_t = \frac{B_{t-1}(1+G^V_t)}{P_t} + \frac{P_{t-1}\omega^G_t}{P_t} = \frac{B_{t-1}(1+ G^V_t)}{P_{t-1}(1+ G^P_t)} + \frac{P_{t-1}\omega^G_t}{P_{t-1}(1+ G^P_t)}$$

$$= \frac{\Omega_{t-1}(1+ G^V_t) + \omega^G_t}{(1+ G^P_t)} = \frac{\Omega_{t-1}(1+ G^P_t - \omega^G_t) + \omega^G_t}{(1+ G^P_t)} \text{ (Using Equation (1.12))}$$

$$= \frac{\Omega_{t-1}(1+ G^P_t) + (1 - \Omega_{t-1})\omega^G_t}{(1+ G^P_t)} = \Omega_{t-1} + (1 - \Omega_{t-1})\Omega^{N\omega}_t \text{ (By Equation (3.68)).}$$

Then,

$$\Omega_t = \Omega^{N\omega}_t(1 - \Omega_{t-1}) + \Omega_{t-1}. \tag{3.70}$$

Hence,

$$\Omega^d_t = \Omega_t - \Omega_{t-1} = \Omega^{N\omega}_t(1 - \Omega_{t-1}). \tag{3.71}$$

i.e., Ω remains the same when $\Omega^{N\omega}{}_{t} = 0$, i.e., when the irrational growth is zero. Furthermore, Ω increases [decreases] when the irrational growth is positive [negative] because Ω is always less than unity as explained in Section 1.2.3.

Rearranging Equation (3.70) yields

$$\Omega_t = (1 - \Omega^{N\omega}{}_t)\Omega_{t-1} + \Omega^{N\omega}{}_t. \tag{3.72}$$

In Equation (3.72), $(1 - \Omega^{N\omega}{}_t) = (1+G^V{}_t)/(1+G^P{}_t)$ is the growth rate of the previous Ω, and thus Equation (3.72) explains that Ω consists of two parts: growth of the previous Ω and $\Omega^{N\omega}$. The previous Ω (= B/P) grows at the ratio of $(1+G^V{}_t)$ to $(1+ G^P{}_t)$ because the previous bubble grows at the rate of value growth while the previous price grows at the rate of price growth. In addition, by using Equation (3.72), one can calculate a relative bubble recursively if a bubble is determined in the initial year and if irrational growths of all subsequent years are measured (See Section C in Appendix). Irrational growths can be easily computed by Equation (1.12) if value growths are measured, or directly from irrational returns (See Section A in Appendix).

3.4.2.5 Levels of bubbles. The values of Ω are not symmetric. For upside, Ω is always less than unity. For downside, however, Ω has no limit. Table 3.2 shows some values of relative bubbles according to the level of bubbles. To judge the level of a bubble, Level 2 and 4 can be used as thresholds for severe and extreme bubbles. For level 2 (severe) bubbles,

$$\Omega_t = \frac{B_t}{P_t} = \frac{P_t - V_t}{P_t} = \frac{2V_t - V_t}{2V_t} = \frac{V_t}{2V_t} = 0.5 \text{ for positive bubbles when } P_t = 2V_t\text{, and}$$

$$\Omega_t = \frac{B_t}{P_t} = \frac{P_t - V_t}{P_t} = \frac{P_t - 2P_t}{P_t} = \frac{-P_t}{P_t} = -1 \text{ for negative bubbles when } V_t = 2P_t.$$

For level 4 (extreme) bubbles,

$$\Omega_t = \frac{B_t}{P_t} = \frac{P_t - V_t}{P_t} = \frac{4V_t - V_t}{4V_t} = \frac{3V_t}{4V_t} = 0.75 \text{ for positive bubbles when } P_t = 4V_t\text{, and}$$

Table 3.2 Levels of Bubble and Relative Bubbles

This table shows the levels of bubble using the range of price and value. Level 2 and Level 4 can be used as the threshold of severe bubbles and extreme bubbles, respectively.

Level	Positive bubbles		Negative bubbles	
	Range of price	Ω	Range of value	Ω
1	P = V	0.0000	V = P	0.0000
1.5	P = 1.5V	0.3333	V = 1.5P	-0.5000
2	P = 2V	0.5000	V = 2P	-1.0000
3	P = 3V	0.6667	V = 3P	-2.0000
4	P = 4V	0.7500	V = 4P	-3.0000
5	P = 5V	0.8000	V = 5P	-4.0000
8	P = 8V	0.8750	V = 8P	-7.0000
10	P = 10V	0.9000	V = 10P	-9.0000

$$\Omega_t = \frac{B_t}{P_t} = \frac{P_t - V_t}{P_t} = \frac{P_t - 4P_t}{P_t} = \frac{-3P_t}{P_t} = -3 \text{ for negative bubbles when } V_t = 4P_t.$$

3.4.2.6 Estimation of bubbles in the EBM. The EBM, as its name implies, is an ex-post model. However, it can be used to estimate future bubbles. The valuation in the EBM is based on the Rational CAPM. In the Rational CAPM, irrational returns are expected to be zero, i.e., value returns are expected to be the same as price returns by the assumption of rational investors (See Sections 2.4.6 and 2.6.1.2.2). Then, $E(R^P{}_t) = E(R^V{}_t)$. From Equations (1.9) and (1.10),

$E(G^P{}_t + \delta^P{}_t) = E(G^V{}_t + \delta^V{}_t)$, and thus

$$E(G^P{}_t) + E(\delta^P{}_t) = E(G^V{}_t) + E(\delta^V{}_t). \tag{3.73}$$

In Equation (3.73), $E(\delta^P{}_t) = E(\delta^V{}_t)$ because, as explained in Section 3.4.1.5, prices are assumed to be around values historically, i.e., E(P) = E(V), and dividends are the same for both prices and values. Thus, $E(G^P{}_t) = E(G^V{}_t)$.

In the EBM, value growths are expected to be equal to price growths, and thus irrational growths are expected to be zero. Therefore, bubbles are expected to grow at the rate of value changes as seen in Equation (3.29), leading to the same assumption as in the RBM, and relative bubbles are expected to remain the same as the previous ones as seen in Equation (3.71) with Equation (3.68).

3.5 Methodology and Sample

The purpose of this section is to illustrate how to measure values and bubbles of assets and to explain how to examine bubbles measured by the EBM. If bubbles assessed by the EBM give a good explanation of the market and/or give a fresh insight into the market, it can be said that the EBM works, and thus that the Rational CAPM works with historical data because the valuation in the EBM is based on the Rational CAPM.

3.5.1 Methodology: Measurement of Historical Bubbles

To measure bubbles using the EBM, one needs to know prices and values. Prices can be directly obtained from the market or can be easily computed using price growths as seen in Equation (3.36). Values are assessed by the ex-post valuation using value returns and the dividend trap as explained in Section 3.4.1. Value returns can be obtained by the Rational CAPM (See Section 2.5.1 for how to measure value returns and irrational returns). When it comes to the value returns, the EBM is based on the Rational CAPM, and thus follows the Rational CAPM. This chapter uses the same data and the same methodology as Chapter II (the Rational CAPM) to measure value returns and irrational returns, and thus both value returns and irrational returns from this chapter are the same as those from Chapter II.

All bubbles can be measured by the following steps, which should be done recursively, by year by year from the first year ($t = 1$); this chapter uses annual data. Hereafter, Subscripts P denotes one of three portfolios: the market (portfolio) when $P = M$, the Dow Jones portfolio when $P = D$, and the Internet portfolio when $P = N$.

1. Get price returns and growths of the portfolio, $R^P_{P,t}$ and $G^P_{P,t}$, respectively, and then measure value returns and irrational returns, $R^V_{P,t}$ and $\omega^R_{P,t}$, respectively (irrational returns are not needed to measure bubbles but used in the empirical work in Section 3.5.2). See the Rational CAPM (Section 2.5.1) for all these variables of the three portfolios.
2. Compute prices of the portfolio, $P_{P,t} = P_{P,t-1}(1+ G^P_{P,t})$. Unity is assigned for the price of the initial year ($P_{P,0} = 1$).
3. Select a preliminary initial value of the portfolio, $V_{P,0}$; this is the same as selecting a preliminary initial bubble of the market, $B_{P,0}$. Zero bubble can be assigned to the initial year, i.e., $V_{P,0} = P_{P,0}$,. In this step, any positive number can be used for an initial value. In Step 10, the initial value will be determined. This step is only for convenience because all the subsequent value growths cannot be computed without an initial value (See Section 3.4.1.6).
4. Compute value growths of the portfolio, $G^V_{P,t}$, using Equation (3.46)

   $$G^V_{P,t} = R^V_{P,t} - \frac{P_{P,t-1}}{V_{P,t-1}}(R^P_{P,t} - G^P_{P,t}),$$

   and irrational growths of the portfolio, $\omega^G_{P,t}$ (= $G^P_{P,t} - G^V_{P,t}$) (irrational growths are not needed to measure bubbles but could be used in the empirical work in Section 3.5.2).
5. Compute values of the portfolio, $V_{P,t} = V_{P,t-1}(1 + G^V_{P,t})$.
6. Compute bubbles of the portfolio, $B_{P,t} = P_{P,t} - V_{P,t}$.
7. Compute relative bubbles of the portfolio, $\Omega_{P,t} = B_{P,t}/P_{P,t}$.
8. Compute annual dividend-price ratios of the portfolio, $\delta^P_{P,t} = R^P_{P,t} - G^P_{P,t}$, using Equation (3.44). Dividend-price ratios will be used for the dividend trap in Step 9.

9. Determine the initial value and bubble by using the dividend trap explained in Section 3.4.1.6.1, leading to bubbles for all the subsequent periods.

For the Internet portfolio, no dividends are assumed (See Section 3.5.3), i.e., $R^{P}_{N,t} = G^{P}_{N,t}$ and $R^{V}_{N,t} = G^{V}_{N,t}$. Then, the dividend trap cannot be used for the Internet portfolio. In Step 9, hence, the no-dividend method explained in Section 3.4.1.6.2 should be used for the Internet portfolio. The Internet portfolio for the Dow Jones method can be regarded as the blue chips in the Internet world, like Dow Jones components in the market, because they have experienced and survived the whole sample period as publicly traded firms. For the initial value of the Internet portfolio, hence, it can be assumed that $\Omega_{N,1997} = \Omega_{M,1997}$ when the market method is used while $\Omega_{N,1997} = \Omega_{D,1997}$ when the Dow Jones method is used to measure value returns of the Internet portfolio (See Section 2.5.1.3). If these initial values are not appropriate for the real market, those can be adjusted to fit the real market.

3.5.2 Methodology: Examination of Bubbles

As explained in Section 1.2.7, the irrationality is measured by irrational returns or irrational growths. These two measures of irrationality show little difference in their behaviors (See Figure 3.1 and Table 3.5). In fact, the difference between them is caused by the previous bubbles and current dividends as seen in Equation (A9) in Appendix. If positive [negative] bubbles exist in the previous period, irrational growths are slightly greater [less] than irrational returns when dividends are paid.

For the empirical tests, either return measures or growth measures can be used. However, it must be consistent; do not mix both measures in one model. Growth measures are better than return measures conceptually because bubbles are measured by after-dividends prices and values as seen in the Ex-post Bubble Model. However, return measures are popular in practice. All empirical tests show little difference between results with growth measures and those with return measures. In fact, as seen in Equations (A2) and (B6) in Appendices A and B, respectively, the same for both measures is the relationship among bubbles, irrationalities and value changes (and thus price changes, which are sums of irrationalities and value changes from Equations (1.11) and (1.12)). Henceforth, return measures are used for the empirical work in this section.

The Internet portfolio has only twelve observations or measurements from 1998 to 2009 for each variable. This sample size is too small to get meaningful results from the empirical work. Hence, all empirical work in this section excludes the Internet portfolio, and thus is performed only for the market and Dow Jones portfolios.

3.5.2.1 Rational pricing. Section 3.4.2.2 explained that irrational returns (ω^R) should be negatively related to previous relative bubbles (Ω) in the subsequent period to rationally price an asset. To examine this rational pricing, the following regression is performed for each portfolio.

$$\omega^R_{P,t+1} = a_1 + b_1\Omega_{P,t} + c_1\omega^R_{P,t} + e_{1,t+1}, \tag{3.74}$$

where subscript *P* denotes the portfolio (P = M or D for the market portfolio or the Dow Jones portfolio, respectively, hereafter.). Irrationality persists (Brown and Cliff, 2005), and this is true for the market and Dow Jones portfolios (See Section 3.6.2.2.2). Thus, one lagged term is included in Model (3.74).

In addition, to measure how much bubbles are eliminated with time horizons, the following regression is performed for each portfolio.

$$TBC_{P,t+i} = a_2 + b_2\Omega_{P,t} + \sum_{j=1}^{j}\{c_j L^j TBC_{P,t+i}\} + e_{2,t+i}, \tag{3.75}$$

where TBC denotes total bubble changes in the next *i* years, i.e., $TBC_{P,t+i} = \Omega_{P,t+i} - \Omega_{P,t}$, and L^j denotes j-th lagged term, i.e., $L^j TBC_{P,t+i} = \Omega_{P,t+i-j} - \Omega_{P,t-j}$. For both market and Dow Jones portfolios, one lagged term (j = 1) is included when i = 1, but two lagged terms (j = 2) when i ≥ 2 due to the increase in persistence for longer horizons.

3.5.2.2 "Invisible hand": dynamic interactions for the self-control of bubbles. To examine whether bubbles are self-controlled in the market by the "invisible hand" explained in Section 3.4.2.3, the following VAR model is performed in the same way as Model (2.95) (See Section 2.5.2.1 for more details).

$$\mathbf{\Gamma_t} = \mathbf{\Lambda} + \sum_{l=1}^{l}[\mathbf{\Phi}_{\boldsymbol{l}}\mathbf{\Gamma}_{\mathbf{t}-\boldsymbol{l}}] + \mathbf{\Psi_t}, \tag{3.76}$$

where $\mathbf{\Gamma_t} = [\Delta\delta^P_t \quad \omega^R_t \quad \Omega^d_t \quad R^P_t]'$. Dividend-price ratios are not stationary for both market and Dow Jones portfolios as seen later in Table 3.6. Hence, the first differences of dividend-price ratios are used, i.e., $\Delta\delta^P_t = \delta^P_t - \delta^P_{t-1}$. The LR test with maximum lags =5 yields one lag length, i.e., $l = 1$, for both market and Dow Jones portfolios.

3.5.3 Sample

The sample of this chapter is the same as that of Chapter II (See Section 2.5.3.1). The CRSP cannot yield the price growths of the Internet portfolio. I assume that Internet firms had not paid dividends during the sample period based on the fact that new firms usually do not pay dividends, and thus the price and value growths are the same as the price and value returns, respectively, for the Internet portfolio,

Table 3.3 shows the descriptive summary statistics of annual price data from 1926 to 2009 for the risk-free rate and the market portfolio, from 1929 to 2009 for the Dow Jones portfolio, and from 1998 to 2009 for the Internet portfolio. The statistics confirms Equation (3. 47). On average, the price returns and price growths are 11.6% and 7.4%, respectively, for the market while 10.8% and 6.4% for the Dow Jones; the market earns and grows more than the Dow Jones. The Internet firms on average earn, and thus grow by, 46.8%, but this is neither

Table 3.3 Descriptive Statistics of Price Data

This table shows the descriptive summary statistics of the annual price data. The sample periods are from 1926 to 2009 for the market portfolio and risk-free rates, from 1929 to 2009 for the Dow Jones portfolio, and from 1998 to 2009 for the Internet portfolio. R^P, G^P and δ^P denote price returns, price growth and dividend-price ratio, respectively.

		n	Minimum	Maximum	Mean	Std. Dev.	t-values[2]
Market Portfolio	R^P	84	-.4436	.5750	.116110	.2060868	5.164**
	G^P	84	-.4766	.5108	.074238	.1993951	3.412**
	δ^P	84	.0098	.0868	.041872	.0167288	22.940**
Dow Jones Portfolio	R^P	81	-.4943	.7096	.107612	.2032064	4.766**
	G^P	81	-.5267	.6374	.064499	.1970972	2.945**
	δ^P	81	.0145	.0838	.043113	.0157942	24.567**
Internet Portfolio[1]	$R^P = G^P$	12	-.8284	2.1042	.467886	.9612670	1.686
	δ^P	12	.0000	.0000	.000000	.0000000	N/A
Risk-free	R_F	84	-.0004	.1472	.037081	.0308720	11.008**

1. Internet portfolio: portfolio for the market method, and no dividends assumed
2. Two-tailed mean test (test value = 0): ** = $p < 0.01$, the others = $p > 0.05$

significantly different from zero due to its excessive volatility nor meaningful due to its short history. The historical ranges of the dividend-price ratios are from 0.98% to 8.68% for the market portfolio and from 1.45% to 8.38% for the Dow Jones portfolio. No dividends are assumed for the Internet portfolio.

The price movements of the market, DJIA index and the Internet portfolio are plotted in Figure 2.1. Value returns from the Rational CAPM are compared to price returns for three portfolios in Figure 2.2. Irrational returns of three portfolios are compared in Figure 2.3.

3.6 Empirical Results

3.6.1 Measurement of Bubbles

3.6.1.1 Initial values. The market price for 1925 (the initial price of the market) is fixed at 1, and the market value for 1925 (the initial value of the market) is determined as 1.17235 by using the dividend trap explained in Section 3.4.1.6. Fortunately, the market has such a long history that the initial market value can be narrowed by the dividend trap. The upper and lower limits of the dividend-price ratio of the market are 0.98% and 8.68%, respectively, as seen in Table 3.3. Hence, 1% and 9% are used as the upper and lower limits of the dividend-value ratio of the market to determine the initial value of the market by the dividend trap. The initial market value results in the range from 1.1575 to 1.1872, leading to the median value of 1.17235.

The bubble in the DJIA index (simply Dow Jones bubble) is measured from 1929 since the number of components of the index became 30 in 1928. The Dow Jones price for 1928 (the initial price of the Dow Jones) is set as 1, and the Dow Jones value for 1928 (the initial value of the Dow Jones) is determined as 0.74845 by using the dividend trap explained in Section 3.4.1.6. The lower and upper limits of the dividend-price ratio of the Dow Jones portfolio are 1.45% and 8.38%, respectively, as seen in Table 3.3. Hence, 1.5% and 8.5% are used as the upper and lower limits of the dividend-value ratio of the Dow Jones portfolio to determine the initial value of the Dow Jones portfolio by the dividend trap. The initial Dow Jones value results in the very narrow range from 0.7483 to 0.7486, leading to the median value of 0.74845.

The Internet bubble is measured from 1997, because it is commonly said that the Internet bubble began to form in 1998 and only a few firms have data available before 1997. No dividends are assumed for the Internet portfolio, implying that the price and value growths are the same as the price and value returns, respectively, for the Internet portfolio, and that the dividend trap cannot be used for the Internet portfolio. To determine the initial Ω_N (= $\Omega_{N,1997}$), it is assumed that the Internet bubble is the same as the market bubble when the market method is used while as the Dow Jones bubble when the Dow Jones method is used to measure value returns of the Internet portfolio, i.e., $\Omega_{N,1997} = \Omega_{M,1997} = 0.39685$ when the market method is used while $\Omega_{N,1997} = \Omega_{D,1997} = 0.16355$ when the Dow Jones method is used (See Section 2.5.1.3).

Table 3.4 Descriptive Statistics of Value and Bubble Data

This table reports the descriptive summary statistics of the annual value and bubble data. The sample periods are from 1926 to 2009 for the risk-free rate and the market portfolio, from 1929 to 2009 for the Dow Jones portfolio, and from 1998 to 2009 for the Internet portfolio. R^V and ω^R respectively denote price returns and irrational returns while G^V and ω^G respectively denote price growth and irrational growth. Ω^d denotes changes in relative bubbles (Ω = B/P), and δ^V denotes dividend-value ratio. Section 2.5-1explains how to measure R^V and ω^R while Section 3.5-1 explains how to measure Ω for each portfolio. Ω excludes the initial Ω for each portfolio.

		n	Minimum	Maximum	Mean	Std. Dev.	t-values[2]
Market Portfolio	R^V	84	-.1429	.6033	.105180	.1383994	6.965**
	ω^R	84	-.3796	.3667	.010930	.1529118	.655
	G^V	84	-.1645	.5751	.060580	.1342246	4.137**
	ω^G	84	-.3500	.3576	.013658	.1483758	.844
	Ω	84	-.8656	.5938	-.010126	.4127573	-.225
	Ω^d	84	-.6215	.4381	.001375	.1550191	.081
	δ^V	84	.0133	.0835	.044600	.0174287	23.454**
Dow Jones Portfolio	R^V	81	-.1482	.4878	.100182	.1265471	7.125**
	ω^R	81	-.3460	.3759	.007430	.1575095	.425
	G^V	81	-.1773	.4666	.059187	.1238211	4.302**
	ω^G	81	-.3494	.3514	.005312	.1526977	.313
	Ω	81	-1.3557	.5963	-.186873	.4563052	-3.686**
	Ω^d	81	-.8233	.4930	-.007241	.1898784	-.343
	δ^V	81	.0151	.0849	.040994	.0184067	20.044**
Internet Portfolio[1]	$R^V = G^V$	12	-.2037	2.0216	.281696	.5951320	1.640
	$\omega^R = \omega^G$	12	-.9451	1.9927	.186189	.8024967	.804
	Ω	12	-1.3043	.7956	-.158925	.6192980	-.889
	Ω^d	12	-1.1261	1.3205	-.031722	.6842441	-.161
	δ^V	12	.0000	.0000	.000000	.0000000	N/A

1. Internet portfolio: the market method used, and no dividends assumed
2. Two-tailed mean test (test value = 0): ** = $p < 0.01$, the others = $p > 0.05$

Hence, the initial value is 0.60315 for the market method while 0.83645 for the Dow Jones method when the initial price is set as 1 for 1997. These initial values are good enough to explain the real market, and thus no adjustment is necessary.

For the Internet bubble, the market method looks better than the Dow Jones method for the following reasons. First, the whole Internet portfolio is better to represent the Internet world than the partial Internet portfolio with only 17 firms (See Table 2.2). Second, the results from the market method are better to explain the Internet boom and bust and post-Internet bubble periods than those from the Dow Jones method, as explained later in Section 3.6.1.3.3. Henceforth, measurements from the market method are used for the Internet portfolio.

3.6.1.2 Summary statistics. In Table 3.4, relative bubbles (Ωs) show different results for each portfolio (See Section 3.6.1.3 for more details about the bubbles). The market portfolio has on average negative 1% bubbles, which is not significantly different from zero, while the DJIA index negative 19%, significantly different from zero. Hence, on average, the DJIA index has been underpriced while the market as a whole has been correctly priced. The Internet portfolio also has negative 16% bubbles, but this is neither significantly different from zero due to its excessive volatility nor meaningful due to its short history.

According to Table 3.4, compared to Table 3.3, for all three portfolios, prices on average have grown faster and earned more than values since their initial years, and thus their irrational growths and irrational returns have all positive means, but their means are not significantly different from zero. Between irrational returns and irrational growths, the former is lower than the latter for the market while higher for the Dow Jones, and they are the same for the Internet portfolio due to the assumption of no dividends. Figure 3.1 illustrates historical irrational returns and growths; for all portfolios, there is little difference between irrational returns and growths.

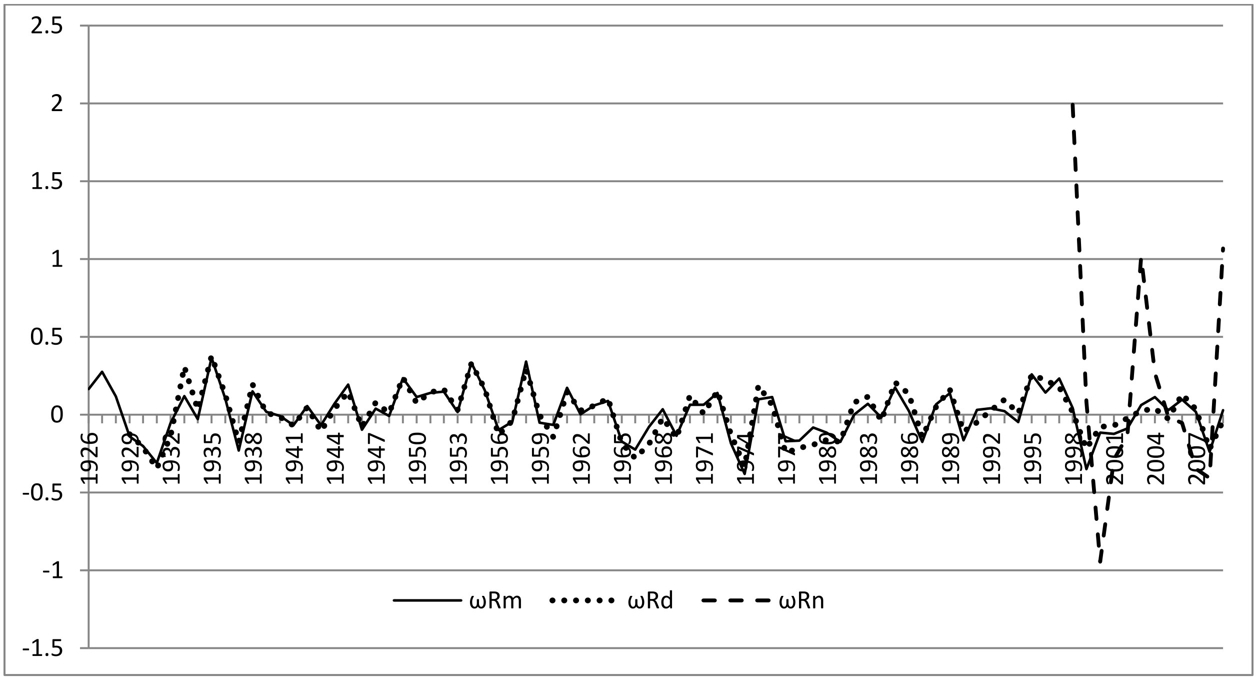
Table 3.5 reports the correlations between variables among three portfolios. Irrationalities (or mispricing) are closely related to price changes while not related to value changes (See Section 2.6.1.2.1 for more explanation). Irrational returns and growths are very highly correlated for all portfolios (perfectly correlated for the Internet portfolio due to the assumption of no dividends). See Section 2.6.1.1 for more explanations about correlations.

Table 3.6 reports results from the Augmented Dickey-Fuller unit root test. These unit root tests are performed for the VAR model (3.76). For the VAR, all variables should be stationary.

Figure 3.1 Irrationalities: Market vs. Dow Jones and Internet Portfolios

This figure illustrates historical irrational returns and growths. ωRm (= ω^R_M), ωRd (= ω^R_D), ωRn (= ω^R_N) denote irrational returns, while ωGm (= ω^G_M), ωGd (= ω^G_D), ωGn (= ω^G_N) denote irrational growths, of the market, Dow Jones and Internet portfolios, respectively. Year represents the end of each year.

Panel A: Irrational returns



Panel B: Irrational growths

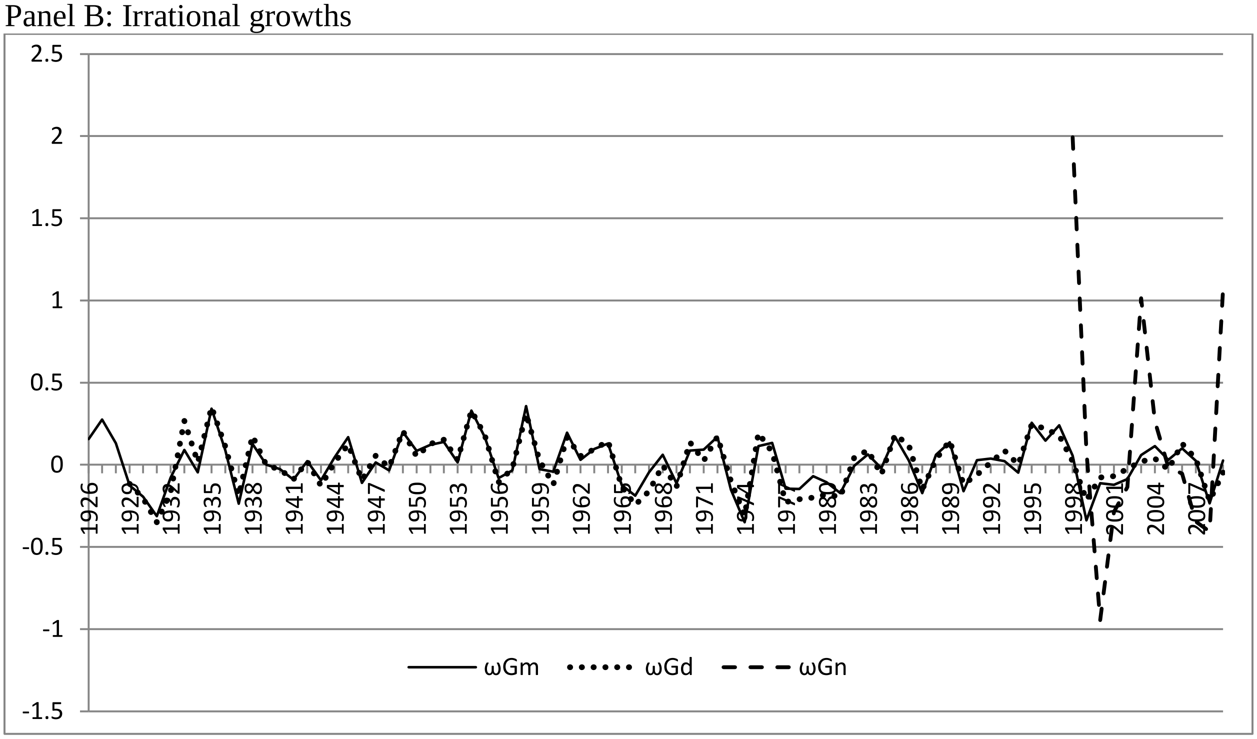


Table 3.5 Correlations of Three Portfolios

This table reports correlations among R^P, R^V, ω^R, G^P, G^V and ω^G of and between three portfolios. R^P, R^V, ω^R, G^P, G^V and ω^G denote price returns, value returns, irrational returns, price growth, value growth, and irrational growth, respectively. Subscripts *M*, *D* and *N* indicate the market (portfolio), the Dow Jones portfolio and the Internet portfolio, respectively. For the Internet portfolio, no dividends are assumed.

	The market portfolio (n = 84)						The Dow Jones portfolio (n = 81)					
	R^P_M	R^V_M	ω^R_M	G^P_M	G^V_M	ω^G_M	R^P_D	R^V_D	ω^R_D	G^P_D	G^V_D	ω^G_D
R^P_M												
R^V_M	.670**											
ω^R_M	.741**	-.002										
G^P_M	.997**	.676**	.732**									
G^V_M	.659**	.992**	-.010	.668**								
ω^G_M	.744**	.011	.992**	.740**	-.007							
R^P_D	.963**	.618**	.748**	.963**	.617**	.746**						
R^V_D	.689**	.992**	.032	.694**	.980**	.048	.632**					
ω^R_D	.690**	.000	.939**	.685**	.009	.924**	.782**	.012				
G^P_D	.958**	.616**	.742**	.963**	.619**	.745**	.997**	.631**	.780**			
G^V_D	.680**	.987**	.025	.689**	.992**	.030	.630**	.989**	.017	.632**		
ω^G_D	.685**	-.005	.938**	.685**	-.005	.937**	.777**	.012	.993**	.778**	.005	
R^P_N	.757**	.836**	-.018	.762**	.831**	-.004	.769**	.831**	-.020	.775**	.828**	-.013
R^V_N	.318	.756**	-.588*	.329	.763**	-.578*	.442	.716**	-.410	.461	.727**	-.400
ω^R_N	.671*	.441	.414	.669*	.430	.424	.593*	.464	.280	.586*	.453	.281

Internet portfolio[1] (n = 12)		
	R^P_N	R^V_N
R^V_N	.554	
ω^R_N	.787**	-.078

Two-tailed test, pairwise test: ** = P < 0.01 and * = p < 0.05; not applied for the covariance

1. For the Internet portfolio, no dividends are assumed. Hence, $G^P_N = R^P_N$, $G^V_N = R^V_N$ and $\omega^G_N = \omega^R_N$.

Table 3.6 Augmented Dickey-Fuller Unit Root Test

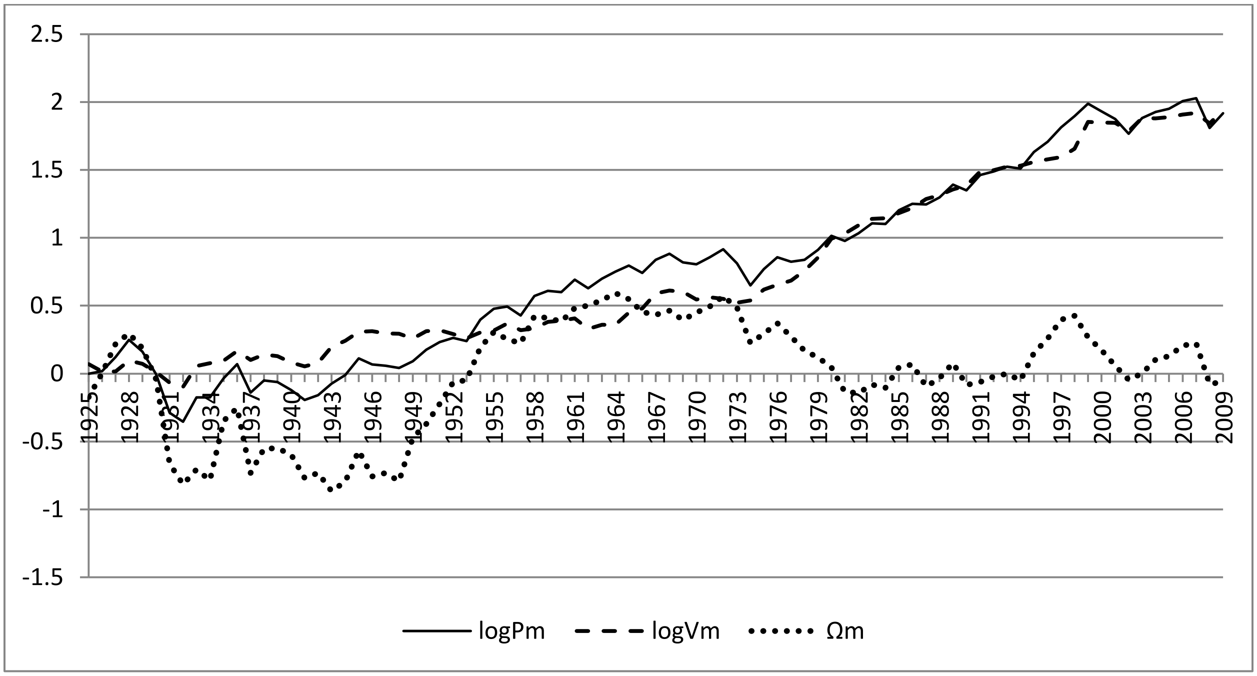
This table reports results of the augmented Dicky-Fuller unit root tests. All tests are performed without both constant and trend. Lag length is determined based on Akaike Information criterion (AIC) with maximum lags = 5 for the market and the Dow Jones portfolios while maximum lags = 2 for the Internet portfolio. ω^R, ω^G, Ω^d, R^P, δ^P and $\Delta\delta^P$ denote irrational returns, irrational growth, changes in relative bubbles (Ω), price returns and dividend-price ratio and changes in dividend-price ratio, respectively. For the Internet portfolio, no dividends are assumed.

Portfolio	Variable	Lag length	t-statistic
Market Portfolio	ω^R	2	-4.775542**
	ω^G	3	-5.104245**
	Ω^d	3	-5.508404**
	R^P	2	-4.030900**
	δ^P	2	-1.102785
	$\Delta\delta^P$	1	-8.977909**
Dow Jones Portfolio	ω^R	0	-6.992416**
	ω^G	0	-7.067378**
	Ω^d	3	-6.872191**
	R^P	2	-3.608579**
	δ^P	2	-0.607005
	$\Delta\delta^P$	1	-9.194147**
Internet Portfolio	$\omega^R = \omega^G$	0	-4.081514**
	Ω^d	0	-3.083066**
	R^P	0	-3.206709**

Null Hypothesis: There is a unit root, ** = p<0.01; the others = p>0.1

Figure 3.2 Market Bubble

This figure illustrates historical Ωm (= Ω_M) along with the common logarithms (the logarithms with base 10) of Pm (= P_M) and Vm (= V_M). Ωm (= Ω_M), Pm (= P_M) and Vm (= V_M) denote relative bubbles, prices and values, respectively, of the market (portfolio). The market price for 1925 is fixed at 1. The initial relative bubbles of the market for 1925 ($\Omega_{M,1925}$) is –0.17235. Year represents the end of each year.



Dividend-price ratios are non-stationary, but their differences are stationary, and thus used in the VAR models.

3.6.1.3 Bubbles.

3.6.1.3.1 Market bubble. Figure 3.2 plots the annual relative bubbles of the market (Ω_M's) from 1925 to 2009 along with common logarithms of the market prices and the market values with the initial Ω_M of –0.17235. The results are consistent with DeLong and Shleifer (1991), who arguing that as explained in Section 3.2.4, the relative market bubble, Ω_M, measured by the S&P composite were at least 0.23 and possibly up to 0.44. For the S&P indices, the summer of 1929 was a peak; the late summer of 1929 was sharply up above the end of 1928 but the end of 1929 was far below the end of 1928. Hence, for the Ω_M, the summer of 1929 was probably greater than the end of 1928, which was likely to be greater than the end of 1929, consistent with the results from this research ($\Omega_{M,1928}$ = 0.296 and $\Omega_{M,1929}$ = 0.188), which implies that the market was overpriced at least 40% above its fundamentals (Ω_M = 0.286) in the summer of 1929.

Wu (1997), using S&P 500 index, shows a very similar graph to Figure 3.2 except mid-1930s, around-1945, 1987 and early-1990s; however, bubbles from Wu (1997) are much more volatile and values from Wu (1997) are lower than those from the EBM in this chapter. Both Wu (1997) and the EBM find significant bubbles in 1960s and early 1970s, and near zero-bubbles between late 1970s and early 1990s except mid 1980s for Wu. The results from Siegel (2003) are also very similar to those from the EBM except the period from early 1970s to early 1990s (Siegel's results are almost the same as Dow Jones bubble in this chapter; See Figure 3.3) if Level 2 measures of the market bubble (Ω_M of 0.5 and -1) are matched to 2 standard deviations from the required rate of returns of 7% for the real stock returns. During the period from early 1970s to early 1990s, the EBM shows the positive or around-zero bubbles while Siegel (2003) finds the negative bubbles; hence, the EBM values the market lower than Siegel (2003). When it comes to valuation, the EBM is between Wu (1997) and Siegel (2003). In addition, Both Wu and Siegel as well as the EBM find huge bubbles in 1960s, which are well explained in Baker and Wurgler (2006).

Table 3.7 reports jumps and crashes in the market price and value. There have been several jumps in the market value since 1930. In 1933, 1943, 2003 and 2009, jumps can be considered rebounds from the previous crashes. In these rebounds, jumps are greater than the previous crashes for the values while less for the prices. Usually, the values recover the previous crashes much faster than the prices. Without the previous crashes, there are noticeable jumps in

Table 3.7 Discontinuous Changes in the Market Price and Value

This table reports the discontinuous changes in the market price and value since 1926.

Jumps			Crashes		
periods	% increases in value	% increases in price	periods	% decreases in value	% decreases in price
1933	42.01%	51.08%	1929-1932	-35.84%	-75.13%
1943	31.04%	21.67%	1937-1941	-23.11%	-45.56%
1965-1967	70.67%	22.27%			
1979-1980	72.63%	50.37%			
1998-1999	81.11%	49.03%			
2003	24.79%	30.79%	2000-2002	-14.37%	-40.03%
2009	25.52%	28.10%	2008	-16.15%	-39.67%

Period from 1975 to 1999 (25 years): 1,959% and 2,075% increases in value and price, respectively.
Period from 1949 to 1972 (24 years): 80% and 648% increases in value and price, respectively.

the market value in the following periods: 1965-1967, 1979-1980 and 1998-1999. These jumps in the market value could be explained by the Schumpeterian innovation.

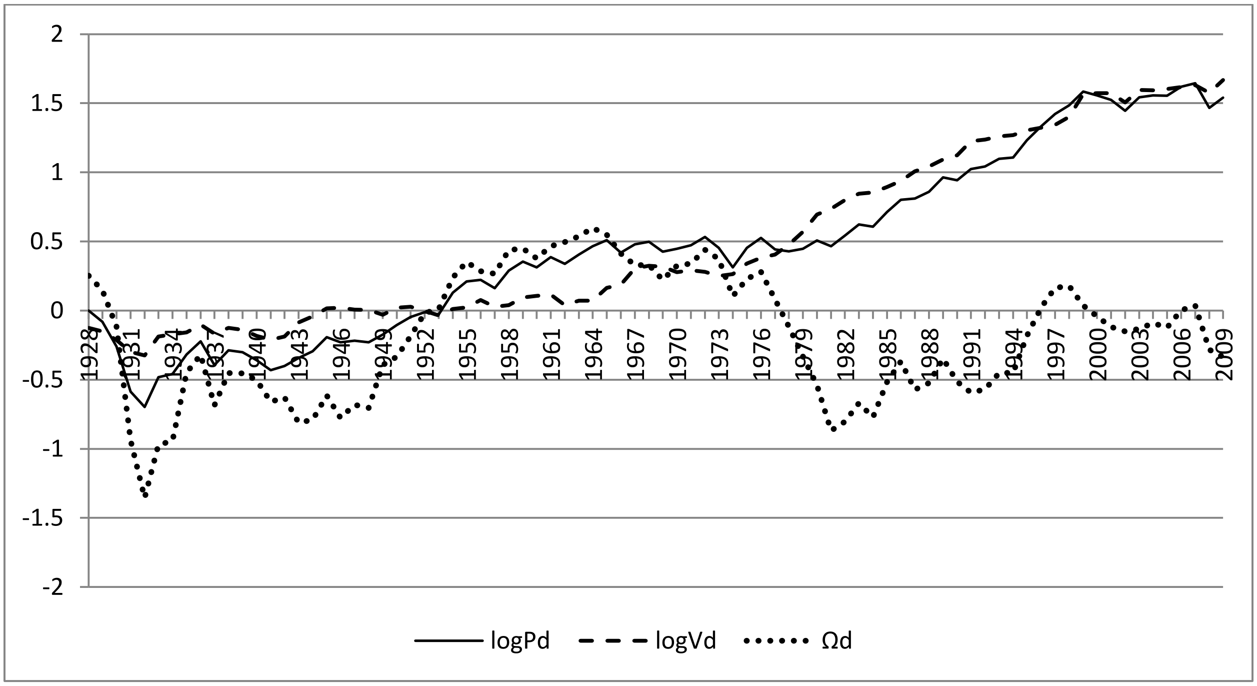
The jump in 1965-1967 can be explained by big businesses such as GM, Standard Oil of New Jersey (Exxon today), Ford and IBM – reorganizing the industry, and by globalized firms such as IBM, Campbell's, Heinz, Proctor and Gamble's and Coca Cola – creating the new market (Encyclopedia.com, 2001a). The jump in 1979-1980 was made by the small service-oriented businesses such as Apple, Microsoft, Wal-Mart, and Nike – introducing new goods and new supply channels, since the U.S. economy became dependent more on the service-oriented businesses and less on the manufacturing companies (Encyclopedia.com, 2001b). These new economies began or grew rapidly during the 1970s, setting the stage for a long-run value enhancement until the Internet bubble burst in early 2000. The jump in 1998-1999 was clearly from the Internet revolution, which not only created new industry but also affected all existing industries. As seen in Table 3.7, in these three innovations, values jumped more than prices because new technologies enhance values by allowing the firms to innovate but simultaneously decrease prices by rendering old knowledge and equipment obsolete (Laitner and Stolyarov, 2003). Further, the Internet innovation was led by the then-small firms with no or little barriers to entry (Recall that Internet is cheap and easy to equip), contrary to the Schumpeterian hypothesis.

As seen in Figure 3.2, there have been two long market rallies since 1930: the one from 1949 to 1972 (24 years) and the other from 1975 to 1999 (25 years). During the first rally from 1949 to 1972, the market value increased by 80% (annual average 3.00%) whereas the market price by 648% (annual average 9.78%). As mentioned above, there was a jump in the market value in the period 1965-1967. Before this jump, the market value increased by 16% (annual average 1.30%) whereas the market price by 412% (annual average 11.85%) from 1949 to 1964. However, during the second rally from 1975 to 1999, the market value increased by 1,959% (annual average 13.46%) whereas the market price by 2,075% (annual average 13.89%). Hence, it can be said that the first rally was not supported by value changes, especially before the jump in the market value in 1965-1967, while the second rally was. This can be explained by investor sophistication; investors become more sophisticated and thus more rational in recent three decades than earlier decades (See Section 2.6.2.5 for more discussion about investor sophistication).

3.6.1.3.2 Bubble in the DJIA index. Figure 3.3 plots the annual Ω_D's from 1928 to 2009 along with common logarithms of the Dow Jones prices and values with the initial Ω_D of 0.25155. The results can be compared to Lee et al. (1999); the overall shape is similar each other, especially since 1980; however, during the period from 1980 to 1996, Figure 3.3 shows the negative bubbles while Lee et al. (1999) reveals the positive bubbles in most times. The EBM values the DJIA index higher than Lee et al. (1999). As mentioned above for the market bubble, Dow Jones bubbles in Figure 3.3 resemble the results from Siegel (2003). Combined with the results from the market portfolio, the EBM values the market lower than Siegel (2003) and

Figure 3.3 Bubble in the DJIA Index

This figure illustrates historical Ωd (= Ω_D) along with the common logarithms (the logarithms with base 10) of Pd (= P_D) and Vd (= V_D). Ωd (= Ω_D), Pd (= P_D) and Vd (= V_D) denote relative bubbles, prices and values, respectively, of the Dow Jones portfolio. The price of the Dow Jones portfolio for 1928 is fixed at 1. The initial relative bubbles of the Dow Jones portfolio for 1928 ($\Omega_{D,1928}$) is 0.25155. Year represents the end of each year.



higher than Wu (1997), and the DJIA index higher than Lee et al. (1999).

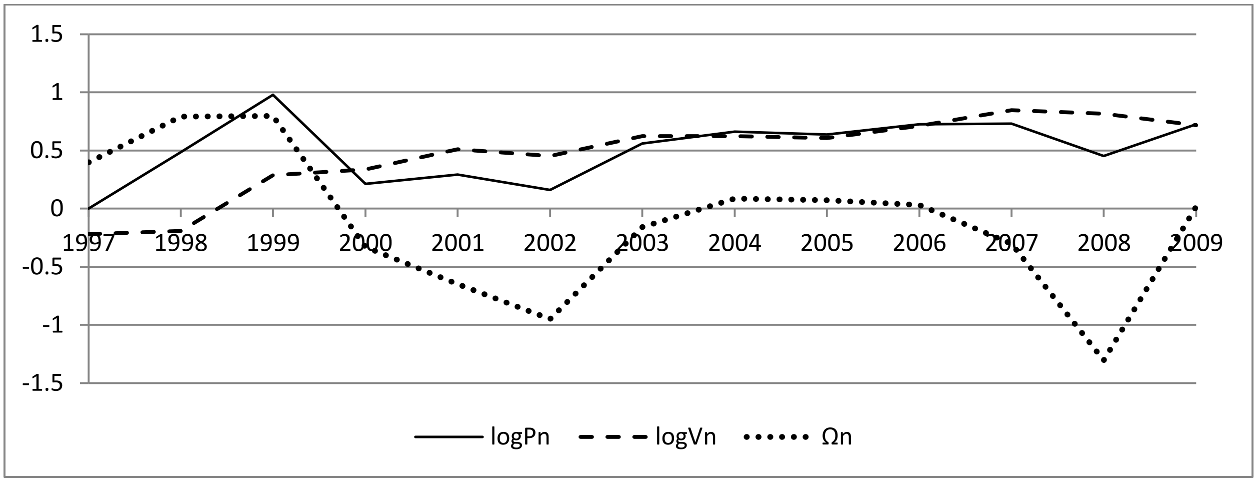
3.6.1.3.3 Internet bubble. Figure 3.4 plots the annual relative Internet bubbles from 1997 to 2009 along with common logarithms of the prices and values of the Internet portfolio: Panel A for the market method and Panel B for the Dow Jones method. As explained in Section 3.6.1.1, the market method looks better than the Dow Jones method to explain the Internet bubble.

According to Panel A, the Internet bubble was very extreme, say, $\Omega_N > 0.75$, in 1998 and 1999 and burst in 2000, and then the Internet portfolio began to find the reasonable prices from 2003 after experiencing the transition period of 2001 and 2002. The price of 2009 is much higher than that of 1997 but less than that of 1999. The value has been increasing steadily since 1997. The jump and crash in the prices of Internet firms in 1998 and 2000, respectively, were too much

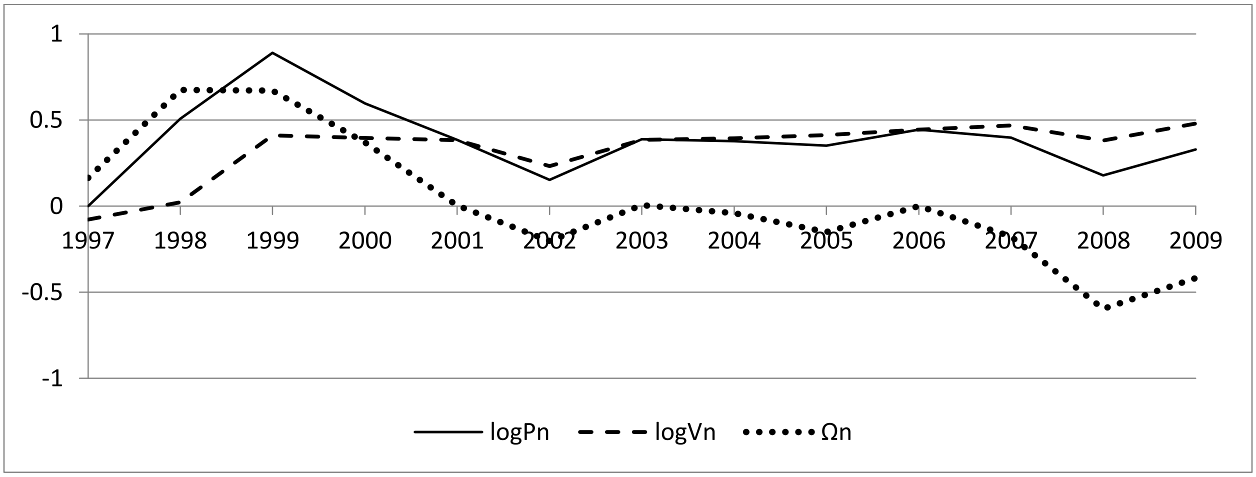
Figure 3.4 Internet Bubble

This figure illustrates historical Ωn (= Ω_N) along with the common logarithms (the logarithms with base 10) of Pn (= P_N) and Vn (= V_N). Ωn (= Ω_N), Pn (= P_N) and Vn (= V_N) denote relative bubbles, prices and values, respectively, of the Internet portfolio. The price of the Internet portfolio for 1997 is fixed at 1. The initial relative bubbles of the Internet portfolio for 1997 ($\Omega_{N,1997}$) is 0.39685 by the market method while 0.16355 by the Dow Jones method. Year represents the end of each year.

Panel A: Market method



Panel B: Dow Jones method

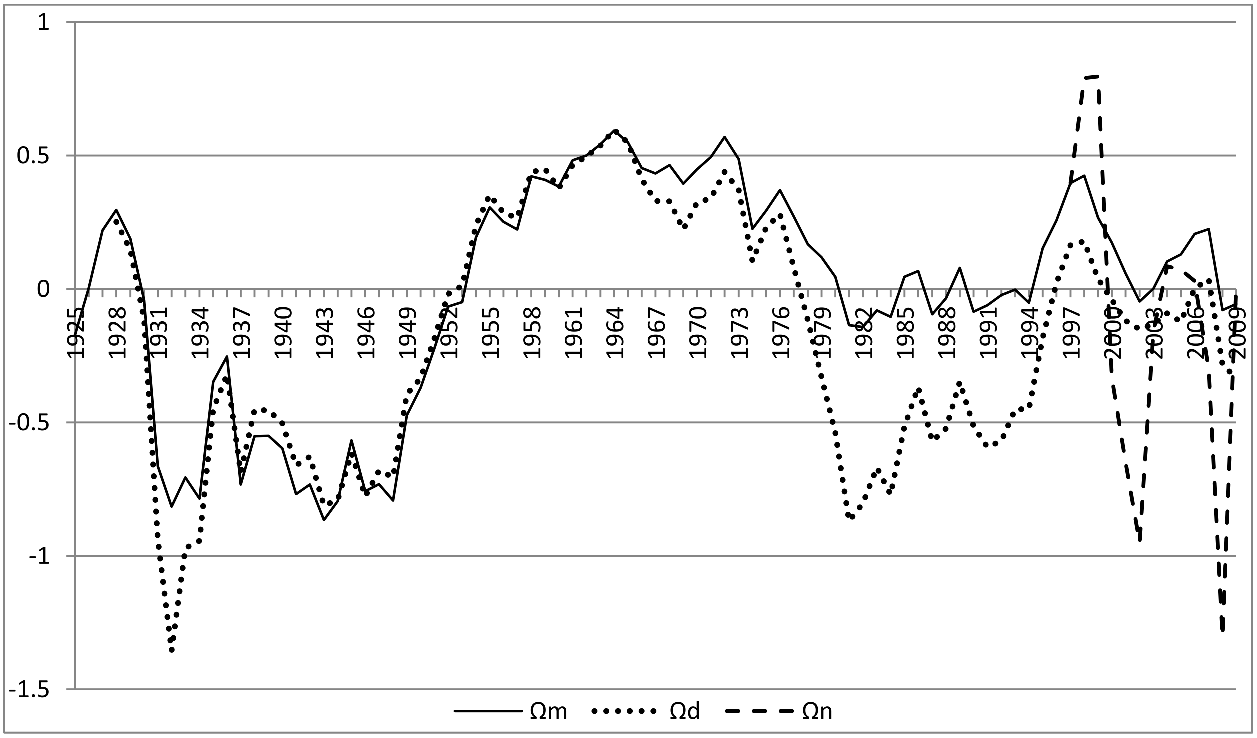


relative to their values.

3.6.1.3.4 Comparisons among bubbles. Figure 3.5 plots the annual relative bubbles of the market, the Internet portfolio and the Dow Jones portfolio. Dow Jones relative bubbles show very similar movements to the market relative bubbles until 1965. Since then, however,

Figure 3.5 Comparisons among Bubbles

This figure illustrates historical Ωm (= Ω_M), Ωd (= Ω_D) and Ωn (= Ω_N): Ωn (= Ω_N) from the market method. Ωm (= Ω_M), Ωd (= Ω_D) and Ωn (= Ω_N) denote relative bubbles of the market, the Dow jones portfolio and the Internet portfolio, respectively. Year represents the end of each year.



Dow Jones relative bubbles have been always less than the market relative bubbles. This gap between them opened up late 1960s and widened from late 1970s through early 1980s. This gap can be explained by the bubbles in 1967-1968 and 1977-1982, which are well summarized in Baker and Wurgler (2006). Characteristics of the bubble in 1967-1968 are focus on growth potential, ignorance of the quality, and escape from the old industrial giants. The bubble in 1977-1982 formed by gambling mania (Dreman, 1998) and hot issue mania on natural resource stocks (Ritter, 1984). As explained by Baker and Wurgler (2006), bubbles are likely to form in new small growth stocks, and thus usually make a gap between market and Dow Jones bubbles because the Dow Jones does not include those stocks while the market does; this gap narrows after bubbles burst. However, market-wide bubbles, e.g., the bubble in early 1960s and the bubble in late 1990s, do not make a gap between market and Dow Jones bubbles because the market-wide bubbles influence both the market and Dow Jones at the same time.

Figure 3.5 also reveals that the Internet bubble has been much more volatile than other bubbles and that Internet firms seem to be in the middle of searching correct prices. Baker and Wurgler (2006) attribute the Internet bubble to high investor sentiments.

3.6.1.4 Ex-ante estimation. According to Table 3.6, both irrational returns and growths are stationary without intercept and trend for all three portfolios. As seen in Table 3.4, both irrational returns and growths have means not significantly different form zero for all three portfolios. Hence, zero can be used for expected irrational returns and growths, and thus expected price returns and growths can be used as the estimation of the value returns and growths, respectively. In addition, changes in relative bubbles (Ω^d) also have a mean not significantly different form zero (Table 3.4) and are stationary without intercept and trend (Table 3.6) for all three portfolios, and thus expected changes in relative bubbles are also zero. Equation (3.71), along with Equation (3.68), confirms that expected changes in relative bubble are zero if expected irrationalities are zero. Hence, the assumption on the future bubble changes in the EBM in Section 3.4.2.6 holds.

3.6.2 Examination of Bubbles

3.6.2.1 Rational pricing. Panel A in Table 3.8 reports the results for the relationship between bubble and subsequent irrationality. For both portfolios, the relationship is negative and significant, meaning that assets are rationally priced as explained in Section 3.4.2.2. Equation (3.61) shows that coefficients of Ω should be $-(1+R^V)$ for bubbles to entirely disappear, and thus -1 even if $R^V = 0$; however, the results are much lower (in absolute terms) than -1, meaning that bubbles do not completely disappear although they decrease in the subsequent year.

Table 3.8 Rational Pricing

This table reports the results from the regressions to examine whether assets are rationally priced.

Panel A: Bubble vs. subsequent irrationality

$$\omega^R_{P,t+1} = a_1 + b_1\Omega_{P,t} + c_1\omega^R_{P,t} + e_{1,t+1}, \quad (3.74)$$

where ω^R and Ω denote irrational returns and relative bubbles (= B/P), respectively, and subscript *P* indicates a portfolio used in the regression.

P (portfolio)	M (market)	D (Dow Jones)
a_1	.006	-.015
	(.016)	(.018)
b_1	-.113**	-.117**
	(.038)	(.036)
c_1	.200	.248*
	(.104)	(.103)
Adj. R^2	.109	.151
F-stat	6.003**	8.036**
DW stat	1.938	1.983

* = p<0.05; ** = p<0.01. Standard errors are in ( )

Panel B: Bubble vs. subsequent bubble changes

$$TBC_{P,t+i} = a_2 + b_2\Omega_{P,t} + \sum_{j=1}^{j}\{c_j L^j TBC_{P,t+i}\} + e_{2,t+i}, \quad (3.75)$$

where $TBC_{P,t+i} = \Omega_{P,t+i} - \Omega_{P,t}$, and $L^j TBC_{P,t+i} = \Omega_{P,t+i-j} - \Omega_{P,t-j}$. Ω denotes relative bubbles (= B/P), and subscript *P* indicates a portfolio used in the regression. j= 1 when i = 1and j = 2 when i $\geq$ 2.

P	M (market portfolio)					D (Dow Jones portfolio)				
i	n	b_2	Adj. R^2	F	DW	n	b_2	Adj. R^2	F	DW
1	82	-.079^	.033	2.4^	1.993	79	-.106*	.085	4.6*	1.895
2	80	-.109*	.368	16.3**	1.849	77	-.147**	.369	15.8**	1.613
3	79	-.099*	.649	49.0**	2.023	76	-.136**	.654	48.3**	1.984
4	78	-.164**	.633	45.4**	2.026	75	-.229**	.609	39.5**	1.558
5	77	-.169**	.676	53.8**	1.884	74	-.199**	.719	63.2**	1.786
10	72	-.255**	.891	194.3**	1.801	69	-.270**	.880	166.7**	1.632
15	67	-.397**	.924	267.2**	1.673	64	-.486**	.925	261.5**	1.467
20	62	-.641**	.956	441.6**	1.473	59	-.662**	.946	340.0**	1.196

^ = p<0.10; * = p<0.05; ** = p<0.01

Panel B in Table 3.8 reports the results for the relationship between bubble and subsequent bubble changes in various time horizons. In the following year, bubbles decrease by 7.9% for the market portfolio and by 10.6% for the Dow Jones portfolio. Bubbles decrease a little more rapidly for the Dow Jones portfolio than for the market, but their patterns are very similar to each other; they decrease by around 10~15% in next three years, around 15~20% in next five years, around 25~30% in next ten years, around 40~50% in next fifteen years, and more than 60% in twenty years. All in all, assets are rationally priced. Bubbles decrease from the subsequent period, but it takes a long time to eliminate bubbles due to the persistence of irrationality (See Section 3.6.2.2.2).

For longer horizons, Durbin-Watson statistics (DW) tend to be lower. For example, DW from 20 year horizon for the Dow Jones portfolio is 1.196 with two lagged terms. Both coefficient (b_2) and DW depend on the number of lagged terms. For comparability between portfolios and among various horizons, the number of lagged terms in the model is maintained at two except one-year horizon with one lagged term.

3.6.2.2 "Invisible hand": dynamic interactions for the self-control of bubbles.

3.6.2.2.1 Impulse responses. Figure 3.6 shows impulse responses from Model (3.76) for the market portfolios, and the results for the Dow Jones portfolio are almost the same, and thus not reported. The results from VAR models reveal the followings.

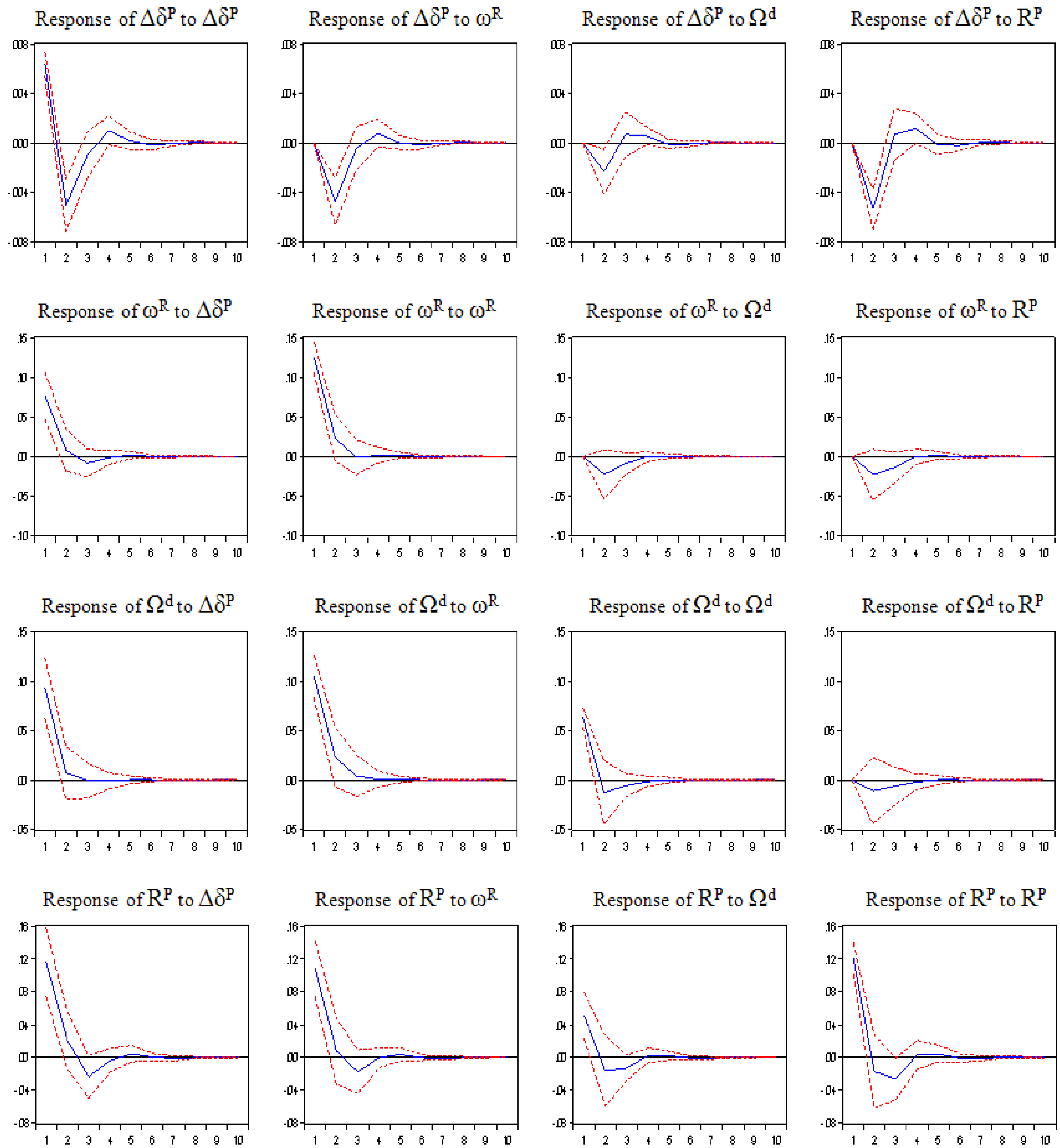
First, the shocks on irrationality have significant, positive and persistent effects on themselves from the subsequent year. This result supports that irrationalities are persistent (See Section 3.6.2.2.2).

Second, the effect of dividend-price ratio on itself is significantly positive in the following first year, but significantly negative in the following second year, and thus the net effect in two years are almost zero. This result means that changes in dividend-price ratio are cancelled out in two years, and thus supports the assumption that dividend-price ratios are stable within a certain range.

Third, the shocks on price returns do not have significant effects on both irrationality and bubbles. This result implies that both irrationality and bubbles are independent of the previous price changes.

Figure 3.6 Impulse Responses from the VAR of Dividend-Price Ratio, Irrationality, Bubble Changes and Price Returns for the Market Portfolio

This figure illustrates impulse responses to Cholesky one standard deviation innovations ± two standard errors from the VAR of $\Delta\delta^P$, ω^R, Ω^d and R^P for the market portfolio. $\Delta\delta^P$, ω^R, Ω^d and R^P denote changes in dividend-price ratio (δ^P), irrational returns, changes in relative bubbles (Ω) and price returns. Lag length = 1 is determined by the sequential modified likelihood ratio (LR) test with maximum legs = 5.



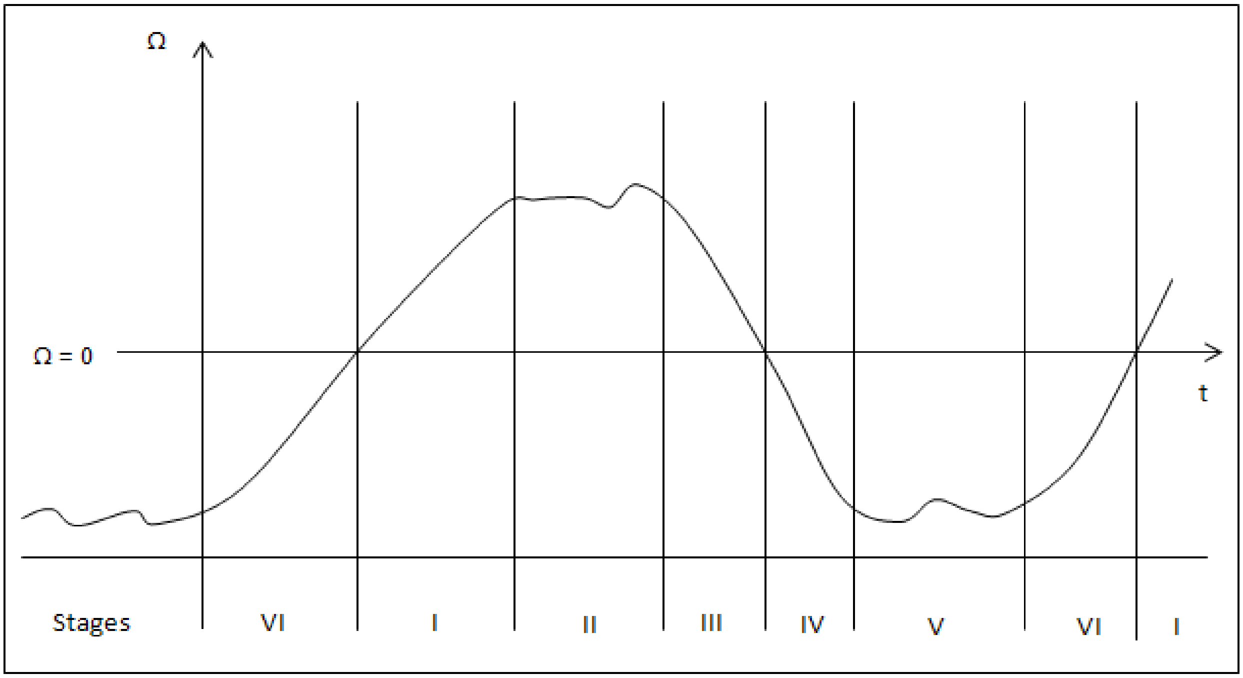
Fourth, the shocks on dividend-price ratio, irrationality and bubble changes have significant, positive and persistent effects on irrationality, bubble changes and price changes, respectively, from the subsequent year. However, the shocks on price changes have significant, negative and persistent effects on dividend-price ratio from the following second year, not in the immediate subsequent year. This lagged response of dividend-price ratio to price changes makes a gap in the causal relations explained in Section 3.5.2.2. This gap might cause the "invisible hand" (for the self-control of bubbles in the market) not to work on time, and thus the irrationality to persist, allowing bubbles to exist for a long time (See Section 3.6.2.2.3).

3.6.2.2.2 Persistence of irrationality. The persistence of irrationalities does not always mean irrational pricing. As explained in Section 3.4.2.2, irrationalities can be rational in pricing. Figure 3.7 plots the example of the behavior of a bubble. The height and depth of a bubble and the length of each stage are varying as seen in Figure 3.5. In general, the movements of a bubble can be seen as the repetition of these six stages if a bubble fluctuates around zero. When irrationality is negative [positive] with positive [negative] bubbles, i.e., when bubbles decrease [increase] but are still positive [negative] in the previous period in Stage III [VI], the negative [positive] irrationality in the subsequent period is rational in pricing because it further reduces the previous positive [negative] bubble. On the other hand, when irrationality is positive [negative] with positive [negative] bubbles in Stage I [IV], the positive [negative] irrationality in the subsequent period is irrational because it amplifies the previous positive [negative] bubble.

Hence, the persistence of irrationality does not mean irrational pricing but may explain the movements of a bubble, repetition of rational and irrational pricing. Persistent irrationality means rational pricing in Stages III and VI while irrational pricing in Stages I and IV. In Stages II and V, irrationality is not persistent (i.e., repetition of positive and negative irrationality),

Figure 3.7 Behavior of Bubbles

This figure illustrates a typical behavior of bubbles, which consists of six stages from I to VI. The bubble behavior can be described by the repetition of six stages from I to VI.



allowing bubbles to stay at the same level. The persistent irrationality does not mean that assets are irrationally priced, but implies that it takes time for an asset to be rationally priced, or for the "invisible hand" to work, as seen in Figure 3.5.

3.6.2.2.3 Lagged response of dividend–price ratios to the shock in the price returns. Another interesting result is the response of dividend–price ratios to the shock in the price returns; no response in the subsequent year, but negative responses in the following year. This lagged response of dividend–price ratios to the shock in the price returns might be the cause of the persistent irrationality. It is reasonable that dividend-price ratios decrease in the subsequent year if price returns increase. What makes dividend-price ratios not respond to the shock in the price returns in the immediate subsequent year? This can be explained by the discount-rate effect (Fama and French, 1988).

Suppose the dividend-price ratio ($\delta^P_t = D_t/P_{t-1}$) is high. Then, the price returns (R^P_t) are likely to increase as explained in Section 3.4.2.3. The expected response of price (P_t) to the shock in the price returns (R^P_t) is positive. On the other hand, the price returns persist; in other words, the expected response of price returns in the following year (R^P_{t+1}) to the shock in the price returns (R^P_t) is also positive, and then stable thereafter as seen in Figure 3.6. Expected price returns are in fact the discount rates to price assets, and thus, the increase in the expected price returns lowers the expected price (P_t). Therefore, the net effect of price returns (R^P_t) on the expected price (P_t) is nearly zero, and thus, the effect of price returns (R^P_t) on the subsequent dividend-price ratios ($\delta^P_{t+1} = D_{t+1}/P_t$) is insignificant if dividends stay at the same level as before. However, the expected price returns will be stable after *t+1*, and the firms will be still underpriced at the beginning of *t+1* because their dividend-price ratios (δ^P_{t+1}) will be still high (their prices are not increased to the expected price due to the discount-rate effect); hence, the prices will increase and dividend-price ratios will decrease in the following year (*t+1*).

3.6.2.2.4 Variance decomposition. Table 3.9 reports the variance decomposition in two years from the VAR of $\Delta\delta^P$, ω^R, Ω^d and R^P. This table confirms the causal relation among variables explained in Section 3.4.2.3. Panel A reveals the results from the market portfolio. This table shows that price changes are least affected by bubble changes, which seems different from the causal relation explained in Section 3.4.2.3. However, the shocks on bubble changes explain price changes by 6.6% while the shocks on price changes have little effect (0.5%) on bubble changes; hence, the relation of cause and effect is the same as explained in Section 3.4.2.3. Then, what makes the effect of bubble changes so small on price changes? Bubble changes are largely affected by dividend-price ratio and irrationality (83% together), both of which also have strong

Table 3.9 Variance Decomposition from the VAR of Dividend-Price Ratio, Irrationality, Bubble Changes and Price Returns

This table reports the variance decomposition from the VAR of $\Delta\delta^P$, ω^R, Ω^d and R^P for the market and Dow Jones portfolios. $\Delta\delta^P$, ω^R, Ω^d and R^P denote changes in dividend-price ratio (δ^P), irrational returns, changes in relative bubbles (Ω) and price returns.

Variables explained	By shocks on			
	$\Delta\delta^P$	ω^R	Ω^d	R^P
Panel A: Market portfolio				
$\Delta\delta^P$	54.00	18.28	4.43	23.29
ω^R	25.87	69.76	2.14	2.23
Ω^d	35.75	46.85	16.94	0.46
R^P	32.02	26.90	6.60	34.48
Panel B: Dow Jones portfolio				
$\Delta\delta^P$	52.58	21.72	2.55	23.16
ω^R	22.73	72.82	2.23	2.22
Ω^d	25.46	57.85	16.21	0.47
R^P	26.67	35.71	6.78	30.84

% of error variance explained by shocks in 2 years by Cholesky decomposition

effects (59% together) on price changes. This very strong power of dividend-price ratio and irrationality limits the power of bubble changes over price changes.

Panel B reports the variance decomposition for the Dow Jones portfolio, and shows almost the same results as Panel A. The major difference from the market portfolio is that price changes are affected more by irrationality (35.71%) than changes in dividend-price ratio (26.67%). The results from both portfolios reveal that stock performance in two years is about evenly explained by dividend-price ratio, irrationality and itself.

3.7 Summary and Conclusion

Based on the Rational CAPM, this chapter explains ex-post valuation, which measures a value using ex-post data of an initial value and subsequent value changes, different from others which use ex-ante data such as a stream of estimated future dividends and discount rates. In the ex-post valuation, the key to measure a value is to measure an initial value because all value changes can be obtained from the Rational CAPM. The initial value can be approximated based on the assumption that no firms get caught in the dividend trap, which describes that either excessive or trivial dividends compared to the value amplify the previous bubble when the huge bubble existed in the previous period. If the dividend trap happens, the initial value should be adjusted. To avoid the dividend trap, the range of the allowable dividend-value ratios should be determined; it is assumed to be the same as (or very similar to) the range of the historical dividend-price ratios.

If a value is measured, then a bubble can be simply computed because it is defined as a deviation of the price from the value, and prices can be directly obtained from the market. In this chapter, a value is measured by ex-post data, so is a bubble; for this reason, this model is named the Ex-Post Bubble Model (EBM). The EBM reveals the behavior of a bubble; the previous bubble grows at the rate of value growth while a new bubble forms by the irrational growth, and thus, bubbles exist and even grow without irrationality if they existed in the previous period. This chapter assumes the causal relations among dividend-price ratio, irrationality, bubble changes and price changes. If this causal relationship works in the market, bubbles will be self-controlled by the "invisible hand" in the market.

There might be confusion between "initial value" and IPO (initial public offering) price (or value). The initial value in the EBM means the value at the beginning of the first year of the historical sample, and thus is different from the IPO price. IPO firms cannot be valued by the EBM because they do not have the historical data publicly available. The EBM needs a relatively long history of data to use the dividend trap as explained above. For SEO (seasoned, or secondary, equity offering) firms, the EBM can be used if they have a relatively long data. However, the initial value is not the SEO price but the value at the beginning of the first year of the data. If the initial value is determined, the SEO price can be assessed by the EBM. The initial value in the EBM is different from the IPO or SEO price.

Theoretical analysis and explanation for the EBM give the answer to the research question in the beginning. ***Why is a bubble identified only after its burst?*** It is because no one can measure a bubble at a specific time. During the formation and inflation of a bubble, some can suspect a bubble but cannot measure and verify it. If a bubble can be measured at any time, it can be detected before its burst. If the Rational CAPM works, value changes can be assessed, and then we can measure a bubble at any time with an initial value approximated, as explained in the EBM.

Based on the EBM, bubbles are measured for the market portfolio, the Dow Jones Industrial Average (DJIA) index and the Internet portfolio, and analyzed for the market portfolio and the DJIA index. The empirical work leads to the following results. First, irrationality persists, implying that it takes time for assets to be rationally priced. Second, both irrationality and bubbles are independent of the previous price changes. Third, the lagged response of dividend-price ratio to price changes might cause the "invisible hand" (for the self-control of bubbles in the market) not to work on time, allowing bubbles to exist for a long time. Fourth, irrational returns and growths of all three portfolios are stationary without trend, and their historical means are not significantly different from zero, and thus ex-ante value returns and growths can be regarded as the same as ex-ante price returns and growths, respectively. Fifth, the market rally from 1975 to 1999 was supported by value changes while the market rally from 1949 to 1972 was not; this might be explained by the investor sophistication. Sixth, in terms of the relative bubbles, the DJIA had been at the nearly same level as the market before 1965 but stayed below the market since then; this might be caused by bubbles in new small growth stocks. Seventh, the overall effect of the Internet on the market is positive; in 2007, the market and the DJIA both recovered both prices and values of 1999, when the Internet bubble was at its peak. Eighth, for the Internet portfolio, the jump and crash in the prices in 1998 and 2000, respectively, were too much relative to its values. The Internet portfolio has not recovered the price of 1999 yet but already recovered the value of 1999 right after the burst of the Internet bubble.

The EBM in this chapter is a continuing study of the Rational CAPM in Chapter II. No direct empirical research is possible for the Rational CAPM because the Rational CAPM uses value data that are not observable. However, this chapter has performed indirect empirical studies for the Rational CAPM using the EBM whose valuation is based on the Rational CAPM. Empirical findings from the EBM can verify that the Rational CAPM works with historical data if those findings give a good explanation of the market and/or if those findings give a fresh insight into the market. Based on the empirical findings coupled with the theoretical explanation, it is found that the Rational CAPM works with U.S. historical data.

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APPENDIX

APPENDIX

MATHEMATICAL MEASUREMENTS OF BUBBLES

A. Measurement of a Bubble by Growth Measures

This section explains how a bubble forms and develops using growth measures. From Equation (3.51),

$$B_t = B_{t-1}\,(1+ G^V{}_t) + P_{t-1}\omega^G{}_t. \qquad \text{(A1)}$$

Then,

$$B_1 = B_0(1+ G^V{}_1) + P_0\omega^G{}_1.$$

$$B_2 = B_1\,(1+ G^V{}_2) + P_1\omega^G{}_2 = \{B_0(1+ G^V{}_1) + P_0\omega^G{}_1\}(1+ G^V{}_2) + P_1\omega^G{}_2$$

$$= B_0(1+ G^V{}_1)(1+ G^V{}_2) + P_0\omega^G{}_1(1+ G^V{}_2) + P_1\omega^G{}_2$$

$$B_3 = B_2\,(1+ G^V{}_3) + P_2\omega^G{}_3$$

$$= [B_0(1+ G^V{}_1)(1+ G^V{}_2) + P_0\omega^G{}_1(1+ G^V{}_2) + P_1\omega^G{}_2](1+ G^V{}_3) + P_2\omega^G{}_3$$

$$= B_0(1+ G^V{}_1)(1+ G^V{}_2)(1+ G^V{}_3) + P_0\omega^G{}_1(1+ G^V{}_2)(1+ G^V{}_3) + P_1\omega^G{}_2(1+ G^V{}_3) + P_2\omega^G{}_3$$

$$B_4 = B_3(1+ G^V{}_4) + P_3\omega^G{}_4$$

$$= [B_0(1+ G^V{}_1)(1+ G^V{}_2)(1+ G^V{}_3) + P_0\omega^G{}_1(1+ G^V{}_2)(1+ G^V{}_3) + P_1\omega^G{}_2(1+ G^V{}_3)$$
$$+ P_2\omega^G{}_3](1+ G^V{}_4) + P_3\omega^G{}_4$$

$$= B_0(1+ G^V{}_1)(1+ G^V{}_2)(1+ G^V{}_3)(1+ G^V{}_4) + P_0\omega^G{}_1(1+ G^V{}_2)(1+ G^V{}_3)(1+ G^V{}_4)$$
$$+ P_1\omega^G{}_2(1+ G^V{}_3)(1+ G^V{}_4) + P_2\omega^G{}_3(1+ G^V{}_4) + P_3\omega^G{}_4.$$

Then, in general,

$$B_t = B_0(1+ G^V{}_1)(1+ G^V{}_2) \cdots\cdots (1+ G^V{}_{t-1})(1+ G^V{}_t) \qquad \text{: growth of the initial bubble}$$

$+ P_0\omega^G_1(1+ G^V_2)(1+ G^V_3) \cdots\cdots (1+ G^V_t)$: growth of NIB created in Period *1*

$+ P_1\omega^G_2(1+ G^V_3)(1+ G^V_4) \cdots\cdots (1+ G^V_t)$: growth of NIB created in Period *2*

$+ P_2\omega^G_3(1+ G^V_4)(1+ G^V_5) \cdots\cdots (1+ G^V_t)$: growth of NIB created in Period *3*

$+ \cdots\cdots$

$+ P_{j-1}\omega^G_j(1+ G^V_{j+1})(1+ G^V_{j+2}) \cdots\cdots (1+ G^V_t)$: growth of NIB created in Period *j*

$+ \cdots\cdots$

$+ P_{t-2}\omega^G_{t-1}(1+ G^V_t)$: growth of NIB created in Period *t-1*

$+ P_{t-1}\omega^G_t$: NIB created in Period *t*,

where NIB denotes an new irrational bubble. Hence,

$$B_t = B_0(1+ G^V_1)(1+ G^V_2) \cdots\cdots (1+ G^V_{t-1})(1+ G^V_t)$$
$$+ \sum_{i=1}^{t}\{P_{i-1}\omega^G_i(1+ G^V_{i+1})(1+ G^V_{i+2}) \cdots\cdots (1+ G^V_{t-1})(1+ G^V_t)\},\ (i+1 \leq t). \quad \text{(A2)}$$

By separating the current period from the second term in the right hand side of Equation (A2),

$$B_t = B_0(1+ G^V_1)(1+ G^V_2) \cdots\cdots (1+ G^V_{t-1})(1+ G^V_t)$$
$$+ \sum_{i=1}^{t-1}\{P_{i-1}\omega^G_i(1+ G^V_{i+1})(1+ G^V_{i+2}) \cdots\cdots (1+ G^V_{t-1})(1+ G^V_t)\}$$
$$+ P_{t-1}\omega^G_t,\ (i+1 \leq t \text{ in the second term}). \quad \text{(A3)}$$

Irrational Growth and Irrational Return

An irrational growth can be computed from an irrational return. From Equations (1.11) and (1.12),

$$\omega^R_t = R^P_t - R^V_t. \quad \text{(A4)}$$

$$\omega^G_t = G^P_t - G^V_t. \quad \text{(A5)}$$

Subtracting Equation (A4) from Equation (A5),

$$\omega^G_t - \omega^R_t = (G^P_t - R^P_t) - (G^V_t - R^V_t). \quad \text{(A6)}$$

Then,

$$\omega^G{}_t = \omega^R{}_t + (R^V{}_t - G^V{}_t) - (R^P{}_t - G^P{}_t). \quad \text{(A7)}$$

From Equation (3.46),

$$R^V{}_t - G^V{}_t = \frac{P_{t-1}(R^P{}_t - G^P{}_t)}{V_{t-1}}. \quad \text{(A8)}$$

Substituting Equation (A8) into Equation (A7),

$$\omega^G{}_t = \omega^R{}_t + (R^P{}_t - G^P{}_t)(\frac{P_{t-1}}{V_{t-1}} - 1) = \omega^R{}_t + (R^P{}_t - G^P{}_t)(\frac{P_{t-1} - V_{t-1}}{V_{t-1}}), \text{ and then,}$$

$$\omega^G{}_t = \omega^R{}_t + \frac{B_{t-1}}{V_{t-1}}(R^P{}_t - G^P{}_t). \quad \text{(A9)}$$

Hence, irrational growths can be computed from irrational returns. Equation (A9) implies the followings.

1. If there is no previous bubble, then $\omega^G{}_t = \omega^R{}_t$.
2. If there is no dividends, or $R^P{}_t = G^P{}_t$, then $\omega^G{}_t = \omega^R{}_t$.
3. If there is positive previous bubble and current dividends, then $\omega^G{}_t > \omega^R{}_t$.
4. If there is negative previous bubble and current dividends, then $\omega^G{}_t < \omega^R{}_t$.

B. Measurement of a Bubble by Return Measures

This section explains how to measure a bubble by using return measures. By using Equations (1.3) and (1.4), we can compute a current price and value by Equations (B1) and (B2), respectively.

$$P_t = [P_{t-1}(1+R^P{}_t) - D_t]. \quad \text{(B1)}$$

$$V_t = [V_{t-1}(1+R^V{}_t) - D_t]. \quad \text{(B2)}$$

Then, by definition in Equation (1.1),

$$B_t = P_t - V_t$$

$$= [P_{t-1}(1+R^P{}_t) - D_t] - [V_{t-1}(1+R^V{}_t) - D_t]$$

$$= P_{t-1}(1+R^P_t) - V_{t-1}(1+R^V_t)$$

$$= P_{t-1} + P_{t-1}R^P_t - (V_{t-1} + V_{t-1}R^V_t)$$

$$= P_{t-1} - V_{t-1} + P_{t-1}R^P_t - V_{t-1}R^V_t$$

$$= (P_{t-1} - V_{t-1}) + P_{t-1}(R^V_t + \omega^R_t) - V_{t-1}R^V_t, \text{ and then,}$$

$$B_t = (P_{t-1} - V_{t-1}) + (P_{t-1} - V_{t-1}) R^V_t + P_{t-1}\omega^R_t, \tag{B3}$$

and then,

$$B_t = B_{t-1} + B_{t-1}R^V_t + P_{t-1}\omega^R_t. \tag{B4}$$

Equation (B4) can be obtained by substituting R^V and ω^R for G^V and ω^G, respectively, in Equations (3.51). In fact, all Equations from (3.50) to (3.60) in Sections 3.4.2.1 and 3.4.2.2 hold even though G^P, G^V and ω^G are replaced by R^P, R^V and ω^R, respectively, because dividends do not affect the size of a bubble. Just like Equation (3.51), Equation (B4) tells that irrational returns create a new bubble and the previous bubble grows at the rate of value returns.

From Equation (B4),

$$B_t = B_{t-1}(1 + R^V_t) + P_{t-1}\omega^R_t, \tag{B5}$$

By developing Equation (B5) in the same way as Equation (A1) in Section A, or by simply replacing G^V and ω^G by R^V and ω^R, respectively, in Equation (A2),

$$B_t = B_0(1+ R^V_1)(1+ R^V_2) \cdots\cdots (1+ R^V_{t-1})(1+ R^V_t)$$

$$+ \sum_{i=1}^{t}\{P_{i-1}\omega^R_i(1+ R^V_{i+1})(1+ R^V_{i+2}) \cdots\cdots (1+ R^V_{t-1})(1+ R^V_t)\}. \ (i+1 \le t) \tag{B6}$$

Hence, a bubble can be measured by return measures in the same way as by growth measures in Section A. However, in Equation (B6), a new irrational bubble is defined differently from the other parts of this chapter so that irrational returns, not irrational growths, create new bubbles, and bubbles grow at the same rate as value return, not value growth, once they are created. In this sense, Equation (B6) is conceptually wrong because a bubble should be measured after excluding dividends in the ex-post model (See Section A for comparison). However, Equation (B6) is the easier way to compute a bubble than Equation (A2) because value returns and irrational returns need not be converted into value growths and irrational growths.

C. Measurement of a Relative Bubble

This section explains how to directly measure a relative bubble. From Equation (3.72),

$$\Omega_t = (1 - \Omega^{N\omega}{}_t)\Omega_{t-1} + \Omega^{N\omega}{}_t\,, \tag{C1}$$

Then,

$$\Omega_1 = (1 - \Omega^{N\omega}{}_1)\Omega_0 + \Omega^{N\omega}{}_1.$$

$$\Omega_2 = (1 - \Omega^{N\omega}{}_2)\Omega_1 + \Omega^{N\omega}{}_2 = (1 - \Omega^{N\omega}{}_2)\{(1 - \Omega^{N\omega}{}_1)\Omega_0 + \Omega^{N\omega}{}_1\} + \Omega^{N\omega}{}_2$$

$$= (1 - \Omega^{N\omega}{}_2)(1 - \Omega^{N\omega}{}_1)\Omega_0 + (1 - \Omega^{N\omega}{}_2)\Omega^{N\omega}{}_1 + \Omega^{N\omega}{}_2$$

$$\Omega_3 = (1 - \Omega^{N\omega}{}_3)\Omega_2 + \Omega^{N\omega}{}_3$$

$$= (1 - \Omega^{N\omega}{}_3)[(1 - \Omega^{N\omega}{}_2)(1 - \Omega^{N\omega}{}_1)\Omega_0 + (1 - \Omega^{N\omega}{}_2)\Omega^{N\omega}{}_1 + \Omega^{N\omega}{}_2] + \Omega^{N\omega}{}_3$$

$$= (1 - \Omega^{N\omega}{}_3)(1 - \Omega^{N\omega}{}_2)(1 - \Omega^{N\omega}{}_1)\Omega_0 + (1 - \Omega^{N\omega}{}_3)(1 - \Omega^{N\omega}{}_2)\Omega^{N\omega}{}_1 + (1 - \Omega^{N\omega}{}_3)\Omega^{N\omega}{}_2$$

$$+ \Omega^{N\omega}{}_3$$

As explained in Section 3.4.2.4, $(1 - \Omega^{N\omega}{}_t)$ is the growth of the previous Ω, which is the same as the ratio of $(1+ G^V{}_t)$ to $(1+ G^P{}_t)$. Then, in general,

$$\Omega_t = \Omega_0(1 - \Omega^{N\omega}{}_1)(1 - \Omega^{N\omega}{}_2) \cdots\cdots (1 - \Omega^{N\omega}{}_{t-1})(1 - \Omega^{N\omega}{}_t) \text{ : growth of the initial } \Omega$$

$$+ \Omega^{N\omega}{}_1(1 - \Omega^{N\omega}{}_2) \cdots\cdots (1 - \Omega^{N\omega}{}_{t-1})(1 - \Omega^{N\omega}) \quad \text{: growth of } \Omega^{N\omega} \text{ created in Period } \mathit{1}$$

$$+ \Omega^{N\omega}{}_2(1 - \Omega^{N\omega}{}_3) \cdots\cdots (1 - \Omega^{N\omega}{}_{t-1})(1 - \Omega^{N\omega}{}_t) \quad \text{: growth of } \Omega^{N\omega} \text{ created in Period } \mathit{2}$$

$$+ \cdots\cdots$$

$$+ \Omega^{N\omega}{}_j(1 - \Omega^{N\omega}{}_{j+1})(1 - \Omega^{N\omega}{}_{j+2}) \cdots\cdots (1 - \Omega^{N\omega}{}_t) \quad \text{: growth of } \Omega^{N\omega} \text{ created in Period } j$$

$$+ \cdots\cdots$$

$+ \Omega^{N\omega}{}_{t-1}(1 - \Omega^{N\omega}{}_{t})$: growth of $\Omega^{N\omega}$ created in Period *t-1*

$+ \Omega^{N\omega}{}_{t}$: $\Omega^{N\omega}$ created in Period *t*.

Hence,

$$\Omega_t = \Omega_0(1 - \Omega^{N\omega}{}_{1})(1 - \Omega^{N\omega}{}_{2}) \cdots\cdots (1 - \Omega^{N\omega}{}_{t-1})(1 - \Omega^{N\omega}{}_{t})$$

$$+ \sum_{i=1}^{t}\{\Omega^{N\omega}{}_{i}(1 - \Omega^{N\omega}{}_{i+1})(1 - \Omega^{N\omega}{}_{i+2}) \cdots\cdots (1 - \Omega^{N\omega}{}_{t-1})(1 - \Omega^{N\omega}{}_{t})\}.\ (i+1 \le t) \qquad \text{(C2)}$$

In other words,

Current Ω = Ω grown from the initial Ω

+ Sum of {Ω grown from $\Omega^{N\omega}$ created in each period}

As seen in Equation (C2), $\Omega^{N\omega}$, created by the irrational growth, affects Ω by combining with all the subsequent $\Omega^{N\omega}$'s.

Alternate Way to Measure a Relative Bubble

From Equation (C1),

$$\Omega_t = \Omega_{t-1} - \Omega^{N\omega}{}_{t}\Omega_{t-1} + \Omega^{N\omega}{}_{t}$$

$$= 1 - (1 - \Omega_{t-1} - \Omega^{N\omega}{}_{t} + \Omega^{N\omega}{}_{t}\Omega_{t-1}).$$

Hence,

$$\Omega_t = 1 - (1 - \Omega_{t-1})(1 - \Omega^{N\omega}{}_{t}). \qquad \text{(C3)}$$

Then, from Equation (C3),

$$\Omega_1 = 1 - (1 - \Omega_0)(1 - \Omega^{N\omega}{}_{1}). \qquad \text{(C4)}$$

$$\Omega_2 = 1 - (1 - \Omega_1)(1 - \Omega^{N\omega}{}_{2})$$

$$= 1 - \{1 - [1 - (1 - \Omega_0)(1 - \Omega^{N\omega}{}_{1})]\}(1 - \Omega^{N\omega}{}_{2}) \text{ (by using Equation (C4))}$$

$$= 1 - (1 - \Omega_0)(1 - \Omega^{N\omega}{}_{1})(1 - \Omega^{N\omega}{}_{2}). \qquad \text{(C5)}$$

$$\Omega_3 = 1 - (1 - \Omega_2)(1 - \Omega^{N\omega}{}_{3})$$

$$= 1 - \{1 - [1 - (1 - \Omega_0)(1 - \Omega^{N\omega}{}_{1})(1 - \Omega^{N\omega}{}_{2})]\}(1 - \Omega^{N\omega}{}_{3}) \text{ (by using Equation (C5))}$$

$$= 1 - (1 - \Omega_0)(1 - \Omega^{N\omega}{}_1)(1 - \Omega^{N\omega}{}_2)(1 - \Omega^{N\omega}{}_3).$$

Then, in general,

$$\Omega_t = 1 - (1 - \Omega_0)(1 - \Omega^{N\omega}{}_1)(1 - \Omega^{N\omega}{}_2) \cdots\cdots (1 - \Omega^{N\omega}{}_{t-1})(1 - \Omega^{N\omega}{}_t). \qquad \text{(C6)}$$

and

$$1 - \Omega_t = (1 - \Omega_0)(1 - \Omega^{N\omega}{}_1)(1 - \Omega^{N\omega}{}_2) \cdots\cdots (1 - \Omega^{N\omega}{}_{t-1})(1 - \Omega^{N\omega}{}_t). \qquad \text{(C7)}$$

Equation (C6) implies that the initial bubble and (new irrational bubbles created from) the irrational growth in each period contribute to the current bubble, and that bubbles increase [decrease] if the initial bubble or (new irrational bubbles created from) the irrational growth in each period are positive [negative].

BIOGRAPHICAL SKETCH

Jaehan Koh received a Ph.D. in Business Administration with emphasis in Finance at the University of Texas-Pan American in the U.S.A in 2012. He earned two Master's degrees in the U.S.A: one in Accounting at Indiana University Bloomington in 2005 and the other in Finance at the University of Illinois at Urbana-Champaign in 2007. He completed his Bachelor's degree in Business Administration at Yonsei University in South Korea in 1991.

Jaehan's research interests include capital asset pricing, mispricing, investor behaviors, market efficiency, and overinvestment. He has published three articles on financial topics in peer-reviewed journals: one, joint with another faculty member, on innovation and breakthrough markets in *International Journal of Business Innovation and Research* in 2012, another on agency costs of free cash flow in *Journal of Business and Behavioral Sciences* in 2011, and the other, joint with other faculty members, on investors' asymmetric responses and the Internet bubble in *International Journal of Electronic Finance* in 2010. He has also presented four different papers in four conferences.

Jaehan had worked for SsangYong Investment and Securities Co. Ltd. (Shinhan Investment Corp. now) from March, 1991 to May, 1996, and for his own business from July, 1996 to June, 2001 in South Korea. He passed the CPA exam in California, U.S.A. in 2003, and the CFA exam Levels I and II in 2005 and 2006, respectively, in the U.S.A.

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