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Investigating the Mathematical Dispositions and Self-Efficacy for Teaching Mathematics of Preservice Teachers

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INVESTIGATING THE MATHEMATICAL DISPOSITIONS AND SELF-EFFICACY FOR TEACHING MATHEMATICS OF PRESERVICE TEACHERS

A Thesis

by

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ABSTRACT

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The study of the individual's beliefs and the role and influence they have on the individual's actions and behaviors, have long been examined and investigated by educators and psychologists. Moreover, researchers have overwhelmingly claimed and demonstrated that the beliefs held by teachers significantly influences their behavior and educational practices in the classroom. This thesis study investigates the mathematical disposition and self-efficacy for teaching mathematics of preservice teachers. The study's primary goals are to discover if there is a relationship or association between a teacher's mathematical disposition(MD) and his/her self-efficacy for teaching mathematics (SEFTM), and if there are significant differences between the mathematical disposition of one group of teachers over another group; likewise, for self-efficacy for teaching mathematics. In effort to answer these questions, data was collected in the fall semester 2016 at a medium-sized public university in the south western United States through a survey and a total of 238 responses (all from preservice teachers) were collected. Data analysis produced statistically significant evidence that there was an association between MD and SEFTM of the study's preservice teachers. Moreover, it was discovered that elementary preservice teachers scored statistically significantly lower on the MD scale than both middle and high school preservice teachers, and significantly lower on the SEFTM scale than high school preservice teachers.

DEDICATION

This thesis is dedicated to my parents. Thank you for always believing in me; for raising me to value hard work, determination, and perseverance; and for instilling within me a moral compass and the desire to be a positive influence in my community and country.

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CHAPTER I

INTRODUCTION

The education of American children has been under the spotlight for roughly half a century. This intense interest and scrutiny began in the late 1950s when Russia launched the Sputnik satellite, causing the American government and general public to fear that American students lacked the necessary academic knowledge, especially in the STEM (Science, Technology, Engineering, and Mathematics) fields, in order to successfully compete with other first world nations (Fritzberg, 2012). A few decades later, in 1983, a report titled A Nation at Risk was commissioned by the Secretary of Education, who was Terrell Bell at the time (Klein, 2003, p. 188). This report described the education of American students in bleak terms, with a special focus on the deficiencies in mathematics education. To understand the tone of this report, one has only to look at the following quote contained within it, which warmed that "If an unfriendly foreign power had attempted to impose on America the mediocre educational performance that exists today, we might well have viewed it as an act of war" (Klein, 2003, p. 188). It is no wonder, why this report, unlike the ones before, captured the attention of the American public. This disquieting publication was the catalyst for the federal accountability movement, which is still present today and still holds great influence over the educational policies and "plans of action" produced and promoted by the government.

1.1 Educational Context of the Study

Since the 1980s, there have been many reforms in mathematics education with the aim of increasing students' academic success and achievement. As a result of these reforms, the ways in which the mathematics education community viewed the nature of mathematics itself changed. A national example of this educational shift is evidenced by the Common Core State Standards in Math (2010), while a local example can be seen by the increased emphasis on process standards in the Texas Mathematics TEKS published (and consistently revised) by the Texas Education Agency (2007-2015). Traditionally, mathematics teaching was focused on the learning and application of algorithms to solve a problem. Specific attention was paid to rote memorization, with students having little conceptual understanding of the concepts they were learning. Reformed mathematics teaching, which is currently promoted by the majority of mathematics educators and education programs around the country, instead focuses on the processes and procedures of understanding a given problem and developing ways to solve it. This new model of the nature of mathematics promotes problem solving strategies and conceptual thinking that leads to the answer, with decreased emphasis on "the correct answer" itself.

Moreover, the pedagogies that have guided our education system have changed over the past few decades, as new research shed light on numerous important topics. Some reforms have shone a spotlight on the student, investigating the ways in which children learn mathematics best and the most effective methods to teach mathematics, the ideology of Piaget and other development psychologists (Van de Walle, 2001). Other reforms have placed the spotlight on the teacher and sought to understand the role a teacher plays in increasing/decreasing, promoting/discouraging academic success in his/her students as well as how significant this role is. Research studies spawned by these reforms examined what it means to be an effective teacher, what are the qualities exhibited by the effective teacher, and what the elements of effective mathematics instruction are (Jacobs & Morita, 2002; Corey, Peterson, Lewis, & Bukarau, 2010; Doabler, Fien, Nelson-Walker, & Baker, 2012).

In 2007, a review of research was published which investigated, discussed, and critiqued the previous and current methods and approaches of how to measure a teacher's mathematical knowledge (Hill, Ball, Sleep, & Lewis, 2007). The aim of this article (and others like it), was to show that in order for students to be successful in mathematics, they must have effective teachers. This means that the effective teacher possesses not only a deep, conceptual understanding of the content itself, but also ample pedagogical knowledge; that one without the other is inadequate.

Additionally, in recent years mathematics educators have conducted and published research studies that investigated the beliefs of mathematics teachers, both in-service and preservice, and their connection and influence on effective teaching instruction and student academic success. A significant number of publications have established that teachers' belief and conceptions play a dominant role in the process of teaching and learning (Freeman & Porter, 1989; Thompson, 1992; Pehkonen, 1994; McLeod, 1994). Moreover, many mathematics educators have researched and established that teacher beliefs influence not just their behavior in the classroom but also how they perform their educational practices and define and achieve learning objectives. Thus, an extensive amount of research has focused on studying preservice teachers in attempt to provide teacher preparation programs and teacher educators with the information they need to ensure that future possess the beliefs and conceptions that lead to increased student academic success.

In 1992, M. Frank Pajares made a case for the studying of teachers' beliefs in his publication titled Teachers' Beliefs and Educational Research: Cleaning up a Messy Construct. In this article, he showed that "the beliefs teachers hold influence their perceptions and judgments, which, in turn, affect their behavior in the classroom, or that understanding the belief structures of teachers and teacher candidates is essential to improving their professional preparation and teaching practices" (p. 307). In regards to the question of whether knowledge (content or pedagogical) or beliefs are more influential on a teacher's actions and practices, he claimed that beliefs are "far more influential than knowledge in determining how individuals organize and define tasks and problems and are stronger predictors of behavior" (p. 311).

Michael Battista highlights the importance of teachers' beliefs, especially in regards to a reformed view of the nature of mathematics and mathematics teaching, when he said, "these beliefs play a critical role not only in what teachers teach but in how they teach it" (1994, p. 462). In 1997, Anne Raymond explored the relationship between a elementary school teacher's beliefs and his/her practice. Her study's results caused her to propose that "teacher education programs would have a stronger indirect effect on practice if they focused on influencing the beliefs of prospective teachers" (p. 572). Evidently, teacher beliefs are composed of numerous subdomains, which are intertwined and related to each other (Nespor, 1987; Lester, Garofalo & Croll, 1989; McLeod, 1992). These subdomains, or components, include beliefs about the learning and teaching of mathematics, mathematics assessment, attitudes or dispositions towards mathematics, pedagogical beliefs, beliefs about the teacher's role in the classroom, and beliefs about one's own ability to do mathematics or to effectively teach mathematics. Although the topic is large and encompasses a multitude of subdomains, the scope of this thesis focuses primarily on two chief elements of teacher beliefs: self-efficacy for teaching mathematics and mathematical dispositions/attitudes.

A well-known researcher, Dona Kagan, published a review of extant research on teacher beliefs in 1992. In this article, she highlighted research that alluded to what she called "special forms of teacher belief" which "have provided most of what we know about important correlates of teacher belief" (p. 67). The first component is a teacher's sense of self-efficacy, which Kagan describes as "a teacher's generalized expectancy concerning the ability of teachers to influence students, as well as the teacher's beliefs concerning his or her own ability to perform certain professional tasks"; the second component is content-specific beliefs, which she defines as "a teacher's orientation to specific academic content" (p. 67). In this thesis, content-specific beliefs are labeled as mathematical dispositions and attitudes, which include beliefs about the nature and usefulness of mathematics, the learning of mathematics, and perseverance in mathematics.

1.2 Purpose and Significance of the Study

The purpose of this study is to investigate the association between preservice teachers' self-efficacy for teaching mathematics and one's attitude, orientation, or disposition towards mathematics. Although research studies have examined self-efficacy for teaching and mathematical dispositions, there is a shortage of research which investigates the relationship between these two important components that comprise teachers' beliefs. Since preservice teachers will be the teachers of tomorrow, it is important to know how their attitudes about mathematics are related to their beliefs about their own ability to teach mathematics effectively.

As mathematics educators, our goal is for our students to achieve both a love for the subject and a deep understanding of its concepts. Since it has already been established that teacher beliefs directly affect student learning outcomes, the examination of how these two components are related, may provide valuable information on how to increase student performance and achievement. Moreover, it would also provide mathematics educators and teacher education programs a motive to work on enhancing and promoting positive mathematics dispositions and self-efficacy beliefs of preservice teachers, as well as in-service teachers, with the goal of enhancing the academic achievement of their future (or current) students.

CHAPTER II

LITERATURE REVIEW

The study of the individual's beliefs and the role and influence they have on the individual's actions and behaviors, have long been examined and investigated by educators and psychologists. According to Pajares, in research literature there has been some confusion as to what constitutes belief and what constitutes knowledge (1992). He highlighted the distinction between beliefs and knowledge when he said, "Belief is based on evaluation and judgment; knowledge is based on objective fact" (p. 313). Here we can infer that while knowledge lends itself to be more objective and structured in nature, beliefs are based on the individual's attitudes, values, and experiences.

Additionally, Nespor (1987) alluded to the difference and superiority of beliefs (compared to knowledge) when she said, "knowledge of a domain can be conceptually distinguished from feelings about a domain" and "beliefs serve as means of defining goals and tasks, whereas knowledge systems come into play where goals and the paths to their attainment are well-defined" (p. 319). Philippou and Christou provide another lens in which to view belief and its complexity. They suggest that "beliefs may be defined as one's amalgamated mixture of subjective knowledge and feelings about a certain object or person" (1998, p. 190).

2.1 Teacher Beliefs

There is little doubt that beliefs play an important and influential role in the creation, development, and progression of one's attitudes, actions, and practices. This is certainly true for teachers. The study of teachers' beliefs has been the focus of many research studies worldwide for the last few decades. Some studies focused primarily on the in-service teachers' beliefs (Raymond, 1997), while others focused on preservice teachers' beliefs (Wilcox, Schram, Lappan, & Lanier, 1991). Still other studies were designed to investigate the interaction between the various and distinct beliefs held by teachers, as well as to classify the most prominently held beliefs (Nespor, 1987). Throughout the myriad of teacher beliefs-related studies, researchers have overwhelmingly claimed and demonstrated that the beliefs held by teachers significantly influences their behavior and educational practices in the classroom (Fenstermacher, 1979, 1986; Munby, 1982; Nespor, 1987).

In the article titled *The Role of Beliefs in the Practice of Teaching*, Jan Nespor conducted a research study to examine the belief systems of eight in-service teachers. As a result, she was able to define the four components that an individual's belief system is comprised of: existential presumption, alternatively, affective and evaluative loading, and episodic structure (1987, p. 318). By examining beliefs through the lens of these four components, Nespor explained, that teachers' beliefs play a crucial role in defining teaching tasks and organizing knowledge and relevant information because the environments/contexts that teachers typically find themselves in are "ill-defined and deeply tangled" and as such "beliefs are peculiarly suited to making sense of such contexts" (p. 324).

While Nespor explored the belief systems of teachers and sought to define what types of beliefs define these systems, other researchers have investigated the connections between teacher beliefs and their practices. For instance, Anne Raymond published an article describing her examination of the relationship between a teacher's mathematics beliefs and his/her practices in the classroom (1997). She defines mathematics beliefs as "personal judgments about mathematics formulated from experiences in mathematics, including about the nature of mathematics, learning mathematics, and teaching mathematics" (p. 552). In this study, Raymond provides a model that illustrates the direct influence of mathematics beliefs on a teacher's mathematics teaching practice. She uses this model to explain how "mathematical beliefs are central to the beliefs-practice relationship" (p. 552). Thus, it is reasonable to conclude that by targeting the beliefs of prospective teachers, teacher education programs could produce more successful and productive teachers.

Linda Behar-Horenstein, Frank Pajares, and Paul George conducted a research study aimed at discovering the extent of the influence of teachers' beliefs on their instructional practices and student academic outcomes when a curricular innovation was supported/promoted by the teachers or unsupported by the teachers (1996). This study resulted in the authors noticing that the students' academic grades were shown to increase or decrease depending on whether the teachers had welcomed/supported the curriculum innovation or not.

Similarly, Sandra Wilcox, Pamela Schram, Glenda Lappan and Perry Lanier conducted an intervention-style study with participants of the elementary teacher education program (1991). The goal of the study was to provide prospective teachers with a deeper, conceptual understanding of knowledge about mathematics, and about the teaching and learning of mathematics. The rationale behind the intervention was to change preservice teachers' attitudes and beliefs about what being a mathematics educator really is. To change the commonly held beliefs that consider teaching "as a matter of technical competence rather than reflection and decision making based on what children are coming to know" (p. 31).

In an article called *Learning to Teach Hard Mathematics: Do Novice Teachers and Their Instructors Give up Too Easily?*, the authors describe a case study of a student teacher in an elementary/middle school preservice program (Borko, Eisenhart, Brown, Underhill, Jones, & Agard, 1992). The aim of the study was to explore the student teacher's emergent beliefs, thinking, and actions regarding the teaching of mathematics and students learning of mathematics. Interestingly, the authors found that there were discrepancies between the student teacher's beliefs and her knowledge base, which were evident when the student teacher tried to make mathematics meaningful. Hence, the authors hint that while student teachers may believe in a valid concept/attribute of good mathematics teaching, they are often unable to successfully apply it to their teaching because of a lack of content knowledge (superficial or fragmented knowledge), limited experience, inadequate conceptual knowledge, and a lack of feeling that it is the teacher's responsibility to consistently improve his/her understanding of mathematics.

2.2 Teachers' Mathematical Beliefs and Practices

Other studies have sought to identify emergent teachers' beliefs and how they are constructed in regards to the learning and teaching of mathematics. In 1998, Thomas Cooney, Barry Shealy, and Bridget Arvold published an article in the attempt to provide an in-depth understanding of how preservice secondary teachers' beliefs are structured. The authors explore four preservice secondary teachers' beliefs about mathematics, and about the learning and teaching of mathematics through the lens of constructivism. They also use the multidimensional perspective developed by Green (1971) to conduct an analysis of beliefs, where Green argues there are "three dimensions of belief systems" (p. 47).

In the same year, George Philippou and Constantinos Christou produce a paper that described the results of a longitudinal investigation into "a program which was designed to improve the attitudes of prospective teachers towards mathematics" implemented by the University of Cyprus (1998, p. 189). In regards to defining beliefs, Philippou and Christou state, "Beliefs may be defined as one's amalgamated mixture of subjective knowledge and feelings about a certain object or person. Beliefs are seen as distinct from knowledge; the latter must involve a certain degree of objectivity and validation vis-a-vis reality." (p. 190).

Moreover, the Philippou and Christou (1998) discuss the vital role that efficacy beliefs play in influencing a person's efforts, persistence, and self-confidence when pursuing goals, facing challenges, and performing new tasks. By conducting a thorough analysis of the pre-test, treatment, post-test data over the course of three years (which included surveys and interviews), the authors were able to classify, measure, and discuss the prospective teachers' changes in their conceptions about mathematics, as well as identify some predictors for attitude change.

On the other hand, a research study on teachers' beliefs conducted by Ronald Beghetto, examined the test-taking ability of 87 preservice teachers with the intent of discovering the extent of the influence of past experiences, existing opinions of testing, and views on the future use of testing on the self-judgments of the preservice teachers' own testing ability (Beghetto, 2005). Throughout the study, Beghetto looked at the "differences between preservice teachers who held positive self-judgments of their test-taking ability (positive self-judgers) and preservice teachers who held negative self-judgments of their test-taking ability (negative self-judgers)" (p. 377).

In 1999, Paul Andrews and Gillian Hatch describe a statistical study they conducted in England, the aim of which was to discover the common conceptions of mathematics and mathematics teaching held by British mathematics teachers. To obtain this information they developed a research-based, four-section questionnaire, which was initially piloted to 54 teachers. After the validity of the instrument had been established, teachers from 200 secondary schools from three regions of England were sent this survey, with a total of 577 responses received for factor analysis. By analyzing these responses, the authors identified five chief conceptions of mathematics and five chief conceptions of mathematics teaching. The conceptions of mathematics are: mathematics as an economic tool, a diverse and pleasurable activity, an essential life tool, a service provider to other careers, and that curriculum determination should be the sanction of teachers, not government; while, the conceptions of mathematics teaching are: process-oriented, skills-oriented, the establishment of individualized working practices for students, the creation of a cooperative and collaborative classroom, and the creation of a mathematically enriched classroom.

Evidence from the factor analysis showed that in regards to the five conceptions of mathematics teaching, almost all the conceptions were found to correlate with each other. In addition, all conceptions of mathematics were found to correlate with at least two conceptions of mathematics teaching, except for the conception of mathematics as an economic tool. Moreover, these four conceptions all correlated with math teaching as both skills-oriented and process-oriented. In conclusion, the authors revealed that "teachers hold simultaneously a variety of not necessarily consistent conceptions of, and beliefs about, mathematics and its teaching" (p. 221).

2.3 Methods/Designs of Studying Teacher Beliefs

While some researchers, like many of those mentioned above, have sought to discover the attributes, relationships, characteristics, and effects of teachers' beliefs on student achievement, teacher instruction, and implementation of curriculum, others have dedicated their efforts to examining the methods and designs used by the educational research community to study teacher beliefs. Research overviews and analysis have been conducted to find new and improve procedures, designs, and instruments to more precisely study teacher beliefs and to ensure that the conclusions and theories made by these studies accurately portray the existing beliefs of the teachers' in the study.

One such researcher is Natasha Speer, who in 2005 published a paper that examines the commonly used methods and designs of studies whose primary focus is on teacher beliefs, as well as the typical classifications used by researchers to categorized teacher beliefs (Speer, 2005). Speer discusses the meaning and usage of "professed" and "attributed" teacher beliefs, terms which have a strong hold in extant literature, when she said, "In research on teachers, professed beliefs are defined as those stated by teachers, while attributed beliefs are those that researchers infer based on observational or other data. This distinction has become a fixture of research on teachers' beliefs." (p. 361). She also provides a sound argument and rationale for why teacher beliefs should not be classified as purely professed, because to some extent these claims are "to greater or lesser extents, attributed to teachers by researchers" (p. 362).

Moreover, in this article, one of Speer's primary goals is to "provide an examination of this classification's strengths and limitations and an exploration of how it might be influencing research in teacher beliefs and practices" (p. 362). Her work is important and necessary to the expanding literature on teacher beliefs because it notifies current and prospective researchers in this area to the typical pitfalls, limitations, and shortcomings of some of the presently used methodology, designs, and theoretical perspectives.

2.4 Studies Involving Changing Teacher Beliefs

Alternatively, there have been numerous studies that have sought to observe changes in professional teachers' beliefs, attitudes, and conceptions about mathematics, and the learning and teaching of mathematics, as well as the reasons or factors behind these changes (Prawat & Anderson, 1989; Peterson, Fennema, Carpenter, & Loef, 1989; Litt & Turk, 1985). Furthermore, many studies have aimed to discover relationships and/or associations between teachers' beliefs and their observed classroom practices/instruction and self-reported appraisals. Several studies have investigated the impact of teacher beliefs' that fall under a certain theoretical construct (like traditional/behaviorist methods) versus those that fall in the theoretical construct of inquiry-oriented methods/constructivism.

A team of mathematics educators conducted a two-year long intervention program (three nontraditional mathematics courses that explored number theory, geometry, probability and statistics) at the Michigan State University for the expressed purpose of "creating in new teachers a more conceptual level of knowledge about mathematics and teaching and learning mathematics" (Wilcox, Schram, Lappan, & Lanier, 1991, p. 32). The reason they focused their efforts on increasing conceptual knowledge in prospective elementary teachers is because conceptual understanding is linked to a deeper comprehension of a subject as it emphasizes meaning making and classroom discourse. In addition, once a person has learned a concept conceptually, instead of merely by remote memorization or by replicating the algorithm described by the teacher, he/she can grasp the true nature of mathematics, a subject that is multifaceted, ever changing, creative, logical, reflective, and surprising, along with the identification of ideal approaches to use to enhance the learning and teaching of mathematics.

The results of this study showed that because of the intervention, the beliefs and conceptions about mathematics and mathematics learning and teaching of the preservice elementary teachers changed noticeably over the two years. Where initially, they considered small-group work was only necessary for "slow learners"; however, by the end, every prospective teacher commented "about the value of group work in their experiences as learners of mathematics" (p. 36). Moreover, the discourse and reflective nature of the intervention allowed participants to appreciate the need for "nonroutine problem situations and multiple representations as powerful ways to explore mathematics and construct mathematical knowledge" (p. 36).

Staying in a similar train of thought, mathematics teacher educators, John Lannin and Kathryn Chval, published a paper that discusses the challenges of trying to change prospective teachers' beliefs, as well as some tried-and-proven strategies that can be used to "confront assumptions about the teaching and learning of mathematics" (2013, p. 508). The ultimate goal of the authors (and their mathematics methods courses) is to change the commonly held notions of elementary school teachers of mathematics as "a static body of disconnected facts", where students simply memorize and regurgitate facts and procedures, to where they instead "view mathematics learning as sense making and reasoning and to view teaching mathematics as problem solving and decision making" (p. 509).

Lannin and Chval suggest that it is possible to challenge the preconceived beliefs of elementary preservice teachers by incorporating student interviews and online discussions, where participants are required to discuss and reflect on their own or others beliefs; introducing student artifacts and classroom videos, where participants can observe and understand interactions between students and their teachers in elementary school; and the investigation of mathematical tasks, which "challenge the perception that teaching elementary school mathematics is "easy" and does not require a teacher to possess deeper mathematical understanding" (p. 509).

Furthermore, the National Council of Teachers of Mathematics has repeatedly advocated and promoted the necessity for teachers of mathematics to teach the subject as a dynamic, evolving tool instead of a set of static knowledge, procedures, and operations that need to be memorized and applied (NCTM, 1991).

An article titled *Teachers' beliefs and practices related to mathematics instruction* sheds light on how teacher beliefs' and practices are related (Stipek, Givvin, Salmon, & MacGyvers, 2001). This study involved the participation of 21 teachers (of fourth to sixth grade) in Los Angles, California. The duration of the data collection lasted an entire school year and teacher completed a pre-test and post-test designed to measure their beliefs concerning the nature, learning, and teaching of mathematics.

Remarkably, the analysis of the data showed that there existed "substantial coherence among teachers' beliefs and consistent associations between their beliefs and their practices" as well as "teachers' self-confidence as mathematics teachers was also significantly associated with their students' self-confidence as mathematical learners" (p. 213). Other notable findings of this study include evidence that scores on traditional beliefs were positively correlated with an emphasis on precision and speed (rather than conceptual learning). Furthermore, belief in the importance of teacher control and correctness were negatively associated with effort and creativity, and scoring high on traditional beliefs was negatively associated with enjoying mathematics and exhibiting enthusiasm in the classroom (p. 223).

Therefore, it seems reasonable to conclude that to change teachers' classroom practices and instruction and evaluation procedures, it is essential to understand, identify, influence, and shape the beliefs of these teachers. As Dona Kagan said, "As we learn more about the forms and functions of teacher belief, we are likely to come a great deal closer to understanding how good teachers are made" (1992).

In summary, the study of teachers' beliefs has interested the educational community, especially the mathematics education community, for decades. Over this time, numerous research studies, publications, and discussions have investigated a variety of aspects of this broad and multilayered topic. Many of these studies have defined teacher beliefs, explored the relationship between a teacher's mathematical beliefs and his/her practices, investigated the methods and designs commonly used to study teacher beliefs, and examined interventions used by mathematics educators to change teacher beliefs.

Despite the diverse wealth of information regarding teachers' beliefs, I have not found any research study or publication that discusses the relationship and/or association between two principal components of a teachers' beliefs: a teacher's mathematics disposition and his/her self-efficacy for teaching mathematics. There are several reasons why discovering a relationship between these two components would be important. Firstly, if such a connection was discovered between mathematical disposition (MD) and self-efficacy for teaching mathematics (SEFTM), for example, that one's MD helped predict one's SEFTM, then teachers (novice and veteran) would be able to strengthen their self-efficacy beliefs (which, per research, would increase their students' academic achievement) by working on having a more positive/productive mathematical attitude/disposition. Moreover, if university educational programs and professional development organizers knew about, and had measured, the connection between these two components, they would have an alternative (quantifiable) way of understanding and influencing the way teachers view mathematics and their capacity to teach it. Thus, they could design courses/seminars with the specific goal of increasing teachers' mathematical disposition and affecting teachers' self-efficacy. Lastly, mathematics educators could investigate specific strategies and interventions that would help teachers (both in-service and preservice) increase their mathematical disposition and in turn self-efficacy for teaching mathematics. For these reasons, investigating whether an association exists between these two chief elements of a teacher's beliefs is one of the primary purposes of this thesis study.

CHAPTER III

THEORETICAL FRAMEWORK

As the previous chapter illustrated, the importance and influence of teachers' beliefs on teacher practice, instruction, and student success is well-documented in the mathematics education community. Moreover, the study of teacher beliefs' is exceedingly broad, as beliefs are multi-dimensional, interrelated, and complex. Whenever one chooses to study teachers' mathematics beliefs there are a multitude of viewpoints or perspectives she/he could take. It all depends on what the purpose of the study is and what the research desires to do with the learned information.

Some researchers are interested in adding to the current wealth of knowledge and understanding that defines mathematics beliefs and their structures (Nespor, 1987), while others are interested in finding a means of measuring teacher mathematics beliefs, identifying specific beliefs that make an "effective, high-quality" teacher and the ones that do not (Jacobs & Morita, 2002; Kloosterman & Stage, 1992). Some studies target understanding the theory of mathematics beliefs (Speer, 2005), while others aim at finding ways to apply knowledge concerning mathematics beliefs to achieve increased student academic success, greater involvement of reform mathematics agendas by teachers, and produce more qualified and able teachers of mathematics in elementary, middle school, and high school (Battista, 1994; Thompson, 1992).

This thesis study takes a close look at the beliefs of preservice mathematics teachers at a medium-sized public university in the south western United States. Since the study of mathematics beliefs is widespread in the mathematics communities around the world, it is no surprise that there are numerous and varied definitions of this concept. Hence, for this study, I employ Anne Raymond's definition of mathematics beliefs as my own. Raymond defines mathematics beliefs as "personal judgments about mathematics formulated from experiences in mathematics, including about the nature of mathematics, learning mathematics, and teaching mathematics" (Raymond, 1997, p. 552). Moreover, she alludes to the merit of studying preservice teachers' beliefs when she says, "teacher education programs would have a stronger indirect effect on practice if they focused on influencing the beliefs of prospective teachers" (p. 572).

As the previous chapter has shown, there is much discussion as to what are the most important (or influential) components of a teacher's mathematics beliefs. The purpose of this thesis is not to claim that a certain aspect/dimension/factor of mathematics beliefs is more important than the others. Nor is my goal to rate/rank the elements of mathematics beliefs in a specific order. The aim of my thesis study was to explore, understand, and investigate two research-based, critical categories of teachers' mathematics beliefs: mathematical disposition (MD) and self-efficacy for teaching mathematics (SEFTM). My hope was to determine if there is a relationship or association between these two important aspects of a teacher's mathematics beliefs, as well as how these components vary in regards to specific groups of teachers. Since these two categories are broad and wide-ranging, I have further divided each main category into subcategories.

Figure 1 provides a snapshot view of the transition from the broad spectrum of teacher beliefs to the narrowed focus of this thesis. This illustration depicts mathematical disposition as comprised of three chief components, and self-efficacy for teaching mathematics as comprised of two key components.

3.1 Mathematical Disposition

Over the last few decades, the mathematics community (both nationally and internationally) has advocated and promoted the need to nurture and develop students' mathematical dispositions. Evidence of this topic's popularity in mathematics education can be seen by the multitude of articles published on the topic (National Research Council, 2001; Singh, Granville & Dika, 2002; Nicolaidou & Philippou, 2003; Sanchez, Zimmerman & Ye, 2004). Many of these studies have sought to understand how a mathematics disposition develops in a person, how students' attitudes towards mathematics affect their mathematics academic achievement, and whether learning environments and teacher-related factors relate to a grade school student's mathematical disposition.

On the other hand, several mathematics educators have dedicated their research efforts to the understanding and examining of the mathematical dispositions of teachers, whether prospective or professional. An article published by C. Adam Feldhaus investigated how the mathematical dispositions of four elementary preservice teachers were formed, with a focus on their past experiences (positive or negative) in their learning mathematics (Feldhaus, 2014). The study highlights the need for elementary preservice teachers to have (or cultivate) positive mathematical dispositions because "when these preservice teachers who have an unproductive mathematical disposition are charged with teaching mathematics to early-grade elementary students, their anxieties about mathematics may be transmitted to their students" (p. 92).

An article titled *Attitudes towards Mathematics: Effects of Individual, Motivational, and Social Support Factors* describes a research study conducted in Portugal which endeavored to understand the attitudes towards mathematics of 1719 students, ranging from grade 5 to 12 (Mata, Monteiro & Peixoto, 2012). One of the primary goals of this study was to understand how several diverse yet interconnected factors (including motivation, background, gender, and social environment) could be used to explain/interpret student mathematical attitudes. In reference to categorizing mathematical dispositions as positive or negative, the authors explain, "Attitudes

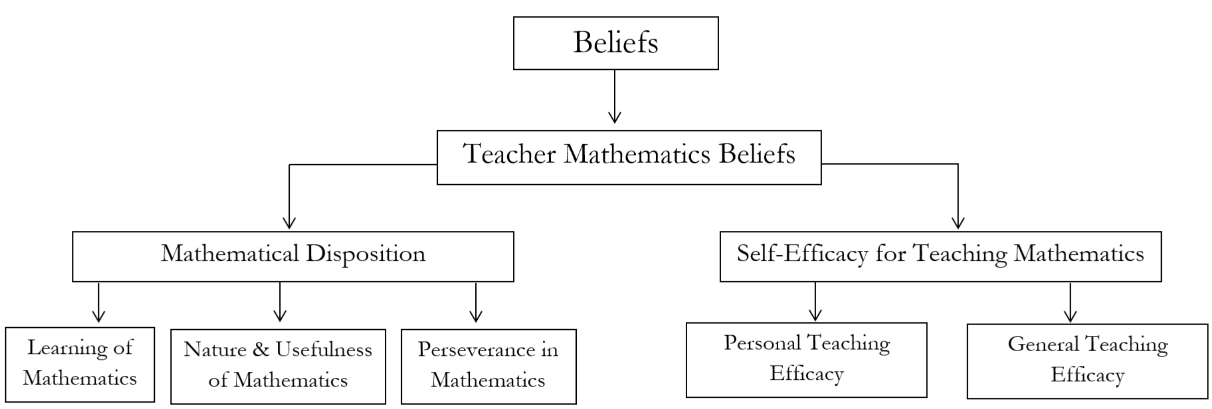

Figure 1: Components of Teacher Beliefs

can be seen as more or less positive. A positive attitude towards mathematics reflects a positive emotional disposition in relation to the subject and, in a similar way, a negative attitude towards mathematics relates to a negative emotional disposition" (p. 2). While the above-mentioned study involves grade school students as participants, as opposed to this thesis study which involves adult, preservice teachers, the explanation of how attitudes or dispositions can be viewed as positive or negative is important to this study since one of the primary aims is to measure mathematical disposition.

As stated earlier, mathematical disposition is one of the primary components of a teacher's beliefs that is being investigated in this thesis study. Therefore, we begin the discussion of mathematical disposition and its components (as specified by this thesis) by defining what this term generally means in the mathematics educational community as well as its working definition in this thesis.

An early usage of the term "disposition" and a general definition is provided by Lauren Resnick in her book titled *Education and Learning to Think*, published in 1987. She argues that "the disposition to higher order thinking" is necessary if students are to view themselves as capable, active, independent learners (p. 41). Furthermore, she explains that "The term disposition should not be taken to imply a biological or inherited trait. As used here, it is more akin to a habit of thought, one that can be learned and, therefore, taught" (p. 41).

In 1989, the National Council of Teachers of Mathematics (NCTM) published a book titled *Curriculum and Evaluation Standards for School Mathematics*, with a section devoted to the discussion and assessment of students' mathematical dispositions (pp. 233-237). The following quote from this report provides insight into what the term "mathematical disposition" means as well as how it is revealed in our students' mathematics learning experiences:

> Mathematical disposition is much more than a liking for mathematics $\cdots$ Disposition refers not simply to attitudes but to a tendency to think and act in positive ways. Students' mathematical dispositions are manifested in the way they approach tasks - whether with confidence, willingness to explore alternatives, perseverance, and

> interest - and in their tendency to reflect on their own thinking. (p. 233).

This passage clearly alludes to the importance of observing, fostering, and assessing a positive and productive disposition in K-12 students of mathematics.

Two years later, *Professional Standards for Teaching Mathematics* (1991) was produced by the NCTM. This report differs from the one mentioned previously in a significant way. Instead of shining a spotlight on how to nurture and assess students' mathematical dispositions, this publication stresses the value and necessity of "assessing the teacher's fostering of students' mathematical dispositions" (pp. 104-109). In this section, the teacher is in the spotlight and advice is given on how the teacher can ensure that he/she is positively developing/influencing students' mathematical dispositions. We can see evidence of this when the authors state:

> Assessing the teacher's fostering of students' mathematical disposition should focus on whether the teacher facilitates students' flexibility, inventiveness, and perseverance in engaging mathematical tasks and on whether students demonstrate confidence in doing mathematics. (p. 104).

Furthermore, NCTM explains that in order to cultivate students' mathematical dispositions, teachers of mathematics must demonstrate a love for the subject, engage students in mathematical discourse, encourage students to question and challenge ideas, and present the subject material with various, diverse approaches and methods.

Exactly a decade later, the National Research Council presented the five strands of mathematics proficiency, one of which is a productive disposition, in chapter four of the report called *Adding it up: Helping children learn mathematics* (2001). In this text, a productive disposition is defined as "the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, to believe that steady effort in learning mathematics pays off, and to see oneself as an effective learner and doer of mathematics" (p. 131). Moreover, this chapter not only argues that a productive disposition is essential to the development of the other strands of mathematical proficiency, but also provides examples of how this occurs in each one.

In the effort to construct a working definition of a positive/productive mathematical disposition for this thesis, the researcher of this study identified key components of disposition that were present in most of the articles/books (including those referenced above) on the topic. Additionally, attention was given to detecting the components that are consistent with her own understanding of what mathematical disposition is comprised of. Hence, this thesis study defines a person's mathematical disposition as a set of beliefs consisting of three components: beliefs about the learning of mathematics, beliefs about the nature and usefulness of mathematics, and beliefs about the importance of perseverance in becoming successful in mathematics. The ensuing passages present research-based evidence of the importance of each of these elements to teachers' beliefs.

3.1.1 Beliefs about the Learning of Mathematics

Beliefs about the learning of mathematics generally fall under two broad categories: behaviorist perspectives and constructivist perspectives. Briefly stated, the behaviorist mindset is one that views learning as something that can be observed, with little emphasis on the thought processes and memory. It focuses on stimulus, the responses, and reinforcement of desired behavior, actions, or habits. (Skinner, 1974; Ertmer & Newby, 1993). Additionally, the teacher plays the central and active role in the students' learning and the students' play the passive/secondary role. The teacher possesses the subject knowledge and imparts this knowledge into the students. As Ertmer and Newby explain, "The learner is characterized as being reactive to conditions in the environment as opposed to taking an active role in discovering the environment." (1993, p. 55).

The problem with this mindset is that students are not engaged in the learning process. Because they are viewed as passive learners, they depend on the teacher to learn instead of taking a portion of the responsibility of learning on their own shoulders. Moreover, "it is generally agreed that behavioral principles cannot adequately explain the acquisition of higher level skills or those that require a greater depth of processing" (Ertmer & Newby, 1993, p. 56). Since this outlook does not allow for higher level processes such as problem solving and critical thinking (which increases conceptual understanding and adaptive reasoning), it goes directly against the advocacy of the National Council of Teachers of Mathematics (NCTM, 1989; NCTM, 1991), the National Research Council (2001), and numerous mathematics educators (Schafer, 1974; McCormack, 1984; Gelven & Stewart, 2001; Bonotto, 2013), all of whom promote the use and necessity of critical thinking and problem solving in the learning process.

On the other hand, constructivism is an alternative (and markedly different) perspective on the nature of knowledge and learning. Constructivists view knowledge as something that is created by the individual; that the individual uses his/her own experiences to give meaning to a situation, and by doing so creates knowledge relating to that experience. In other words, "Constructivists do not deny the existence of the real world but contend that what we know of the world stems from our own interpretations of our experiences. Humans create meaning as opposed to acquiring it." (Ertmer & Newby, 1993, p. 62).

Constructivists view the student as having an active role in his/her learning, as opposed to a passive one. A teacher whose beliefs are aligned with this mindset understands that "Knowledge is not received from the outside or from someone else; rather, it is the individual learner's interpretation and processing of what is received through the senses that creates knowledge. The learner is the center of the learning, with the instructor playing an advising and facilitating role." (Ally, 2004, p. 30). Thus, the teacher's job is to facilitate learning: to be the guide, the navigator of the subject and to allow the students opportunities to explore, investigate, and discover the subject's concepts, tenets, and procedures.

Another key aspect of the constructivist perspective is that learning should not occur using only one method of instruction (Can, 2006). This outlook encourages teachers to depart from the one-size-fits-all method of instruction and instead use multiple perspectives, examples and explanations, meaningful and challenging activities, and real-world illustrations in the instruction of the content. Moreover, emphasis is placed on situated learning, or learning as a contextual process (Ally, 2004). In order for students to develop a deep understanding of a subject, they must be allowed (and encouraged) to inspect, experiment, and apply the knowledge they are learning in various and diverse situations and disciplines. As Jean Piaget, the founder of constructivism, once said:

> Children should be able to do their own experimenting and their own research. Teachers, of course, can guide them by providing appropriate materials, but the essential thing is that in order for a child to understand something, he must construct it himself, he must re-invent it. Every time we teach a child something, we keep him from inventing it himself. (1972, p. 27).

In recent years, mathematics educators and researchers have used constructivist theory in their efforts to understand the learning of mathematics (Cobb, Yackel, & Wood, 1993; Steffe & Gale, 1995; Alsup, 2005). Consequently, several mathematics educational reforms have been initiated by The National Council of Teachers of Mathematics in 1989 and 1991, which promote mathematics learning under the banner of constructivism (Klein, 2003). In fact, Klein explains that the NCTM's *Curriculum and Evaluation Standards for School Mathematics* (1989) emphasize specific constructivist-based goals for student learning of mathematics, including but not limited to the use of manipulatives, hands-on inquiry methods, mental computations, pattern recognition and description, and operation sense. This document also contained a detailed list of learning goals to be de-emphasized, including rote memorization of rules and algorithms, tedious paper-and-pencil computations, and relying unduly on the teacher to complete tasks/assignments. In other words, "The NCTM Standards reinforced the general themes of progressive education, dating back to the 1920s, by advocating student centered, discovery learning" (Klein, 2003, p. 192).

Constructivism not only highlights the goals for student mathematics learning but also the responsibilities of the teacher. As Martin Simon stated in his article *Reconstructing Mathematics Pedagogy from a Constructivist Perspective*, "The teacher has the dual role of fostering the development of conceptual knowledge among her or his students and of facilitating the constitution of shared knowledge in the classroom community" (Simon, 1995, p. 119). He also argues that traditional views of learning which involve focusing on one skill or concept in isolation to other related concepts/skills coupled with memorization and excessive practice of the topic are not as effective as the constructivist approach. In regards to the latter, Simon remarks that "Learning is likely to be fostered by challenging the learner's conceptions using a variety of contexts" (p. 139).

Drawing upon the constructivist nature of the learning of mathematics presented above, this study holds that beliefs regarding the learning of mathematics (which positively influence one's mathematical disposition) should include the following:

- There are multiple ways to learn mathematics and to teach mathematics.
- Memorization and mastery of mathematical algorithms do not signify learning.
- All students can learn mathematics.
- The role of students of mathematics is an active one; one where the student is an independent and reflective explorer.

3.1.2 Nature and Usefulness of Mathematics

As can be inferred from the discussion on the learning of mathematics component of mathematical disposition, the lens through which this thesis views the nature and usefulness of mathematics is also a constructivist one. As the name implies, this component is comprised of two subcomponents: beliefs about the nature of mathematics and beliefs about the usefulness of mathematics.

Discussions and research about the nature of mathematics have been around for thousands of years, beginning, if not earlier, with Plato and Aristotle in Greece (Dossey, 1992). It is valuable for mathematicians and mathematics educators to research and examine conceptions about the nature of mathematics because "Perceptions of the nature and role of mathematics held by our society have a major influence on the development of school mathematics curriculum, instruction, and research." (Dossey, 1992, p. 39). Likewise, research studies have shown that teacher conceptions about the nature of mathematics are especially critical to the way a teacher approaches mathematical concepts and ideas in the classroom (Cooney, 1985).

In the article *The Nature of Mathematics: Its Role and Its Influence*, John Dossey explains that "The conception of mathematics held by the teacher may have a great deal to do with the way in which mathematics is characterized in classroom teaching. The subtle messages communicated to children about mathematics and its nature may, in turn, affect the way they grow to view mathematics and its role in their world." (1992, p. 42). This passage implies that it is possible for a teacher to unknowingly project his/her own beliefs about mathematics, its nature and usefulness, onto his/her students. The problem with this is if a teacher's beliefs about the nature of mathematics are less than positive or belong to a school of thought that is no longer promoted and encouraged by the mathematics education community, his/her students are in danger of acquiring the same beliefs, which may negatively affect their progress and success in mathematics.

Dossey goes on to explain that there are two prominent views about the nature of mathematics, which he terms as external and internal conceptions. External conceptions involve those that consider mathematics as "an externally existing, established body of concepts, facts, principles, and skills available in syllabi and curricular materials" (p. 43). Teachers who lean towards this view of the nature of mathematics, believe that for them to be successful mathematics teachers, they must convey this knowledge - that mathematics is a fixed and static body of knowledge - to their students.

In contrast, internal conceptions of the nature of mathematics consider mathematics to be a dynamic, fluid, problem-motivated, multilayered subject. Teachers who hold this outlook believe that the student should have the primary, active role in the doing of the mathematics, as opposed to passively receiving the mathematics from the teacher. Internal conceptions lean towards a student-centered, discovery-based curriculum and instruction and as such are closely aligned with constructivist beliefs, agendas, and reforms. As Dossey explains, in regards to internal conceptions, "This emphasis on students doing mathematics is the hallmark of this conceptualization of mathematics. It is the "doing"- the experimenting, abstracting, generalizing, and specializing-that constitutes mathematics, not a transmission of a well-formed communication." (1992, p. 44).

Another way to consider external and internal conceptions about the nature of mathematics is as traditional and nontraditional conceptions respectively. We see evidence of this from the following quote: "Traditionally, mathematics has been taught as a static discipline, that is, as a set of preexisting facts and procedures that is passed along from teacher to student in an authoritarian manner (i.e., rote memorization and practice)" (Wilkins & Ma, 2003, p. 61). Yet, regardless of which of these two conceptions (external or internal/traditional or nontraditional) is held by the teacher, its influence on the teacher's disposition towards mathematics, practice, and instruction is significant. As Reuben Hersh, an American mathematician best known for his work examining the nature and social impact of mathematics, once wrote, "One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it." (1979, p. 33).

In addition, beliefs about the usefulness of mathematics are intertwined with beliefs about the nature of mathematics. The way a person views mathematics has much to do with how one considers it to be useful or useless to one's daily life and activities. This reality is adeptly expressed in the ensuing excerpt: "A person's mathematical disposition related to her or his beliefs about and attitude toward mathematics may be as important as content knowledge for making informed decisions in terms of willingness to use this knowledge in everyday life." (Wilkins & Ma, 2003, p. 52). This quote comes from a publication titled *Modeling Change in Student Attitude toward and Beliefs about Mathematics*, which describes a longitudinal research study that examined the beliefs of high school students about and towards mathematics. One of the specific beliefs that was emphasized and investigated in this study was "a recognition of the societal impact and utility of mathematics" (Wilkins & Ma, 2003, p. 52). Interestingly, the study's results showed the attitudes of parents, peers, and teachers significantly affected a student's beliefs in the usefulness of mathematics. In fact, the study found that "Attitude toward and beliefs about the social importance of mathematics declined at a significantly slower rate for students who experienced positive teacher push than for those who experienced negative push from teachers" (p. 58). Moreover, and here is how we see the connection between perceived usefulness of mathematics and conceptions about the nature of mathematics, the researchers of this study, Wilkins and Ma, concluded that "If teachers choose activities that portray mathematics as static, boring, and unchallenging, students may view the subject as unimportant, and they may not perceive the usefulness of it" (p. 61).

Along the same lines, in 1984, Laurie Hart Reyes penned an article called *Affective Variables and Mathematics Education*. The intention of the article was to provide a synopsis of the extant knowledge about affective variables in regards to mathematics education. The article specifically discussed four affective variables that Reyes claimed were critical to the learning of mathematics, one of which was the "perceived usefulness of mathematics" (p. 559). Reyes goes on to explain that how useful mathematics appears to a student in regards to his/her current and future needs directly affects whether they will take advanced mathematics courses in high school. As a result, those who don't view mathematics as very useful are narrowing career opportunities in their future. Therefore "A better understanding of the importance of mathematics in a wide range of careers and in education beyond high school is important for students as they make decisions about how much mathematics to take in high school" (p. 571). Lastly, Reyes explains that fostering students' positive attitudes regarding the usefulness of mathematics is the teacher's responsibility, as he/she has ample time, influence, and information.

> Teachers are in a good position both to assess how useful their students view mathematics to be and to give students information about the importance of studying mathematics. Of the four affective variables discussed here, usefulness may be the easiest to change, and teachers are in a good position to bring about change in students' views. (p. 572).

Consequently, the extent to which teachers (preservice or in-service) view mathematics as useful and important in their daily lives and career can help shape (positively or negatively) their students' conceptions on the usefulness of mathematics. Since conceptions of the usefulness of mathematics have been found to impact students' academic motivation and success in mathematics, as well as students' choice of mathematics courses and the number of mathematics courses taken (Sherman & Fennema, 1977; Perl, 1979; Reyes, 1984), it is crucial for teachers of mathematics to have positive attitudes regarding mathematics usefulness as well as to promote these attitudes and outlooks to their students.

In summary, the overall concepts and perspectives used to view and measure the nature and usefulness of mathematics component in this study of mathematical disposition are based on the previous literature. Hence, beliefs regarding the nature and usefulness of mathematics should subscribe to the subsequent notions:

- The nature of mathematics is dynamic, adaptable, applicable, problem-motivated, surprising, relative, and continually expanding.
- Mathematics is not merely a collection of unrelated and fixed facts, rules, and skills.
- Mathematics can be used in many ways to benefit society and one's daily life.
- Mathematics is necessary and relevant to an array of academic subjects, scientific fields and careers.

3.1.3 Perseverance in Mathematics

The third and last component of mathematical disposition, as defined by this thesis study, is perseverance in mathematics. This element comprises of beliefs regarding the necessity and importance of perseverance and hard work to becoming successful in mathematics. In recent years, the mathematics educational community has stressed the need for students to engage in problem-solving and higher-level thinking processes (NCTM 1989; 2000). For instance, NCTM published a book titled Principles and Standards for School Mathematics in 2000 which reiterated the importance of problem solving because "problem solving is an integral part of all mathematics learning" (p. 52). The authors go on to explain that problem solving is not only a process or method one can use to learn mathematics but also a way to build new mathematical knowledge and understanding. Furthermore, the NCTM echo the benefits of problem solving and exploring challenging mathematics tasks to students' everyday lives in the following passage:

> Students should have frequent opportunities to formulate, grapple with, and solve complex problems that require a significant amount of effort $\cdots$ By learning problem solving in mathematics, students should acquire ways of thinking, habits of persistence and curiosity, and confidence in unfamiliar situations that will serve them well outside the mathematics classroom. (2000, p. 52).

The message of the above passage is evident. It is beneficial to students academically to struggle and grapple with unfamiliar and complex tasks. Moreover, through this struggle, students develop tenacity and perseverance, which are essential to obtaining a deep understanding of mathematics as well as developing the ability to adapt to a variety of real world situations.

Hiroko K. Warshauer, a mathematics educator and professor, rationalizes the role of perseverance in mathematics to deepen a student's understanding and knowledge of the subject in an article titled *Strategies to Support Productive Struggle* (2015). In this article, the author describes how teachers can approach the instruction of challenging mathematical problems. The teacher can spoon-feed the procedures, steps, and answers to the students in effort to save the students from "struggling"; however, in this type of interaction "the student's work is reduced to numerical manipulations" (p. 390). Alternatively, the teacher can allow students to engage in a "productive struggle" to gain a below-the-surface understanding of a mathematical concept or idea. As Warshauer explains, "An episode, however, in which students encounter difficulty while working on a challenging task can be viewed as an opportunity for them to grapple with important mathematical ideas." (pp. 390-391). Lastly, the author highlights the need for teachers to integrate struggle into their classroom instruction/practice by "acknowledging students' consternation, encouraging perseverance, asking questions, and offering time to work through problems" to demonstrate to their students that "struggling to make sense of mathematics is an important and natural part of learning" (p. 393).

Other mathematics educators have written on the topic of productive struggle and its importance to the learning of mathematics. For example, James Hiebert and Douglas Grouws published an article called *The Effects of Classroom Mathematics Teaching on Students' Learn-*

ing, in which they clarify and define what the term "struggle" means and why it is so important to students and teachers of mathematics (2007). "We use the word struggle to mean that students expend effort to make sense of mathematics, to figure something out that is not immediately apparent. We do not use struggle to mean needless frustration or extreme levels of challenge created by nonsensical or overly difficult problems." (p. 387). Here we can make a connection between perseverance and productive struggle: both represent the effort and hard work students need to spend when formulating, investigating, and solving a mathematical problem or task.

Hiebert and Grouws provide another way of viewing struggle and its specific link to the subject of mathematics and its understanding, when they state, "If understanding is defined as the mental connections among mathematical facts, ideas, and procedures, then struggling is viewed as a process that reconfigures these things." (2007, p.388). Additionally, they explain that "When students struggle (within reason), they must work more actively and effortfully to make sense of the situation, which, in turn, leads them to construct interpretations more connected to what they already know and/or to reexamine and restructure what they already know. This yields content and skills learned more deeply." (p. 389, 2007). Consequently, it is vital for teachers to incorporate mathematical challenges in their instruction to provide the opportunities to learn for their students. Yet, for teachers to do this, they themselves must hold with the viewpoint that perseverance and struggle is essential to the learning and teaching of mathematics. This is why perseverance has been included into this thesis's definition of mathematical disposition. It is needed not only to facilitate students' mathematics learning, but perhaps even more necessary for the teachers to possess (as it is the teachers' responsibility to incorporate opportunities for productive struggle in their instruction and curriculum).

Doug and Barbara Clarke provide another perspective on perseverance in mathematics (Clarke & Clarke, 2003). They conducted a research study in Victoria, Australia that involved 350 K-2 teachers and their prospective students. The study consisted of the teachers using problem solving strategies and approaches with their students to model the necessity and "normalness" of perseverance/struggle in solving or understanding mathematics ideas and concepts. The authors explain that most students, from elementary to high school, hold the beliefs that to be "good" in mathematics one should be able to solve a given mathematics problem in very little time and that "if problems can’t be solved almost immediately, they are impossible" (p. 214). Another common, perturbing belief is that the average student should not be expected to obtain a deep understanding of the concept, but rather they are only capable of memorizing and replicating what they have already learned or seen the teacher do. Therefore, they argue that "teachers in all grade levels have a responsibility, by their words and actions to present a different view" (Clarke & Clarke, 2003, p. 205).

Another research study used metaphor theory to investigate high school students’ beliefs about mathematics (Schinck, Neale, Pugalee, & Cifarelli, 2008). The results of this study revealed that perseverance was one of two dominant themes in students’ beliefs about mathematics, "represented by codes referring to math being challenging, requiring effort to overcome, and mathematics being rewarding" (p. 596). In addition, the authors found that "Students consistently expressed the belief that math is challenging and requires effort, but half the students that found math challenging found it rewarding as well" (p. 598). This result lead the authors to conclude that "these students recognize the important link between perseverance and success in mathematics" (p. 598).

In conclusion, the National Council of Teachers of Mathematics and many mathematics educators have advocated and promoted the use of problem solving in mathematics. Many of these studies have shown a link between perseverance and success in problem solving and challenging mathematical tasks that increase students’ conceptual understanding of the idea or concept. This implies that a student’s beliefs regarding the importance of persistence, hard work, and meticulousness are often linked to his/her success in mathematics, and a teacher’s beliefs about the importance of perseverance in mathematics are often linked to his/her inclusion (or exclusion) of challenging mathematical assignments and deeper, conceptual knowledge of the topic. Hence, this thesis subscribes to the following notions as beliefs about perseverance in mathematics:

- Ability in mathematics increases when one studies hard.
- "Regular/ordinary" students can learn and understand mathematics at a deep, conceptual level.
- Hard work and diligence are keys to success in mathematics regardless of perceived ability in mathematics.
- At times, mathematical problems/tasks take a long time to solve; this "productive struggle" is often necessary for students to gain a meaningful understanding of a new concept.

3.2 Self-Efficacy for Teaching Mathematics

As was explained in the introduction chapter, this thesis has defined teachers' beliefs to be comprised of two primary components: mathematical disposition and self-efficacy for teaching mathematics. Since we have already discussed the first component and its subcomponents, we now turn to self-efficacy beliefs specifically related to the teaching of mathematics. In this section, we will provide a research-based, working definition of self-efficacy; discuss the crucial role self-efficacy plays in influencing teachers' beliefs and practice; and explore the two subcomponents that encompass self-efficacy for teaching mathematics as determined by the thesis author.

In order to begin the discussion on self-efficacy, we must first define it. We do this by presenting a compilation of definitions and descriptions of the term from a variety of research articles on the topic. A person's self-efficacy is:

- "beliefs in one's capabilities to organize and execute the courses of action required to manage prospective situations" (Bandura, 1997, p. 2).
- "defined as people's beliefs about their capabilities to produce designated levels of performance that exercise influence over events that affect their lives. Self-efficacy beliefs determine how people feel, think, motivate themselves and behave." (Bandura, 1994, p. 71).

- "defined in terms of individuals' perceived capabilities to attain designated types of performances and achieve specific results" (Pajares, 1996, p. 546).

In other words, a person's self-efficacy can be understood as how one views their own ability to affect change or control in a given situation (often a challenging or new one). In regards to this thesis, the situation is a mathematics classroom and the person is a teacher. Thus, a teacher's self-efficacy beliefs fall under the same frame of thought. One way a teacher's self-efficacy has been defined is "as teachers' judgment about whether or not they are capable of promoting students' learning" (Guo, Connor, Yang, Roehrig, & Morrison, 2012, p. 4). Teachers' self-efficacy has also been referred to as "beliefs about confidence to affect students' performance $\cdots$ and about confidence to perform specific tasks" (Pajares, 1992, p. 316). On a similar tune, Dembo and Gibson defined teaching efficacy beliefs as "the extent to which teachers believe they can affect student learning" (1985, p. 173). To summarize the previous definitions, teaching efficacy describes the way teachers perceive their own ability to influence: student learning/achievement and teaching practices and instructional strategies.

There are many reasons why self-efficacy beliefs have received so much attention from the educational community, especially in mathematics education. Perhaps, some of the most important reasons are that self-efficacy has been linked to student academic achievement and performance (Guo, Connor, Yang, Roehrig, & Morrison, 2012; Ashton & Webb, 1986) and has been found to greatly effect one's level of effort and perseverance in demanding tasks (Pajares, 1996). As Pajares remarked, regarding the importance and influence of self-efficacy beliefs, "Efficacy beliefs help determine how much effort people will expend on an activity, how long they will persevere when confronting obstacles, and how resilient they will prove in the face of adverse situations-the higher the sense of efficacy, the greater the effort, persistence, and resilience. Efficacy beliefs also influence individuals' thought patterns and emotional reactions." (1996, p. 544). Moreover, he claims that "self-efficacy beliefs are strong determinants and predictors of the level of accomplishment that individuals finally attain." (1996, p. 545).

Albert Bandura, a distinguished psychologist from Stanford University, is credited with developing the concept of self-efficacy and for "providing the theoretical framework for studying teacher efficacy" (Coladarci,1992, p. 323). In an article titled *Self-efficacy: Toward a Unifying Theory of Behavioral Change*, Bandura argues that human behavior is influenced by one's beliefs about two different kinds of expectations: efficacy expectations and outcome expectations (1977, p. 193). He goes on to explain that efficacy expectations are "the conviction that one can successfully execute the behavior required to produce the outcomes" and that outcome expectations are "defined as a person's estimate that a given behavior will lead to certain outcomes" (1977, p. 193). Theodore Coladarci sheds some light on how to translate these two dimensions of self-efficacy into the context of teaching. He explains that "an outcome expectation is illustrated by the teacher who believes that skillful instruction can offset the effects of an impoverished home environment", while "an efficacy expectation, in contrast, would be reflected by the teacher's confidence that he or she personally is capable of such instruction, that the individual possesses personal agency with respect to the task of pedagogy" (Coladarci,1992, p. 324).

Consequently, this thesis's definition of self-efficacy as well as the primary components that it is comprised of, is based on Bandura's and Coladarci's perceptions and descriptions. As such, self-efficacy for teaching mathematics is defined as having two components: general teaching efficacy (which refers to outcome expectations) and personal teaching efficacy (which refers to efficacy expectations).

3.2.1 General Teaching Efficacy

As noted previously, teaching efficacy (used interchangeably with general teaching efficacy) describes a teacher's beliefs regarding his/her confidence in his/her ability to effectively teach mathematics and deal with the challenges that it entails. Another way to look at teaching efficacy is "the belief that effective teaching will have a positive effect on student learning" (Enochs, Smith, & Huinker, 2000, p. 195). It is clear to see why this is an essential component to a teacher's overall beliefs as student learning and performance is one of the primary goals of teaching.

It is interesting to discover the ways in which teaching efficacy influences student learning. Research has shown that teaching self-efficacy beliefs can influence a teacher's selection of curriculum and instructional activities. As Philippou and Christou explain, "Efficacy beliefs may enhance or undermine performance and influence selection of activities and environments; they are a key factor in the self-regulation of one's motivation to pursue a task or meet a challenge, such as the teaching of mathematics." (1998, p. 190). Additionally, these authors provide some insight into the link between one's self-efficacy and his/her teaching performance, when they state, "The individual's conceptions and self-perceived relationship to mathematics are of primary importance in the formation of their learning and teaching behavior" (1998, p. 189). As such, teachers who possess high teaching efficacy are more inclined to select instructional strategies and activities that are not only challenging (both for the teacher to teach and for the students to learn) but that increase problem solving ability, cognitive skills, and conceptual understanding of the mathematics concept.

Another way self-efficacy is connected to student learning and performance is by influencing the teacher's desire or motivation to engage in challenging, complex, or multilayered teaching practices. In essence, "The self-efficacy mechanism is a central determinant of a person's ability to exert power, action, and influence $\cdots$ The motivation to pursue a task or challenge (such as teaching in an urban school) arises from individuals' internalized goals, needs, and aspirations, which are dependent on the self-efficacy mechanism." (Chester & Beaudin, 1996, p. 235). Moreover, Chester and Beaudin explain that the level of self-efficacy a teacher has (high or low) directly influences the selection of effective teaching strategies (group or whole class), their interaction with students (both high achievers and low achievers), and teacher feedback procedures. We find evidence of this, when they state, "Compared with teachers who have low self-efficacy beliefs, high-efficacy teachers conduct more large-group and/or whole-class instruction, are better able to keep other students engaged while instructing small groups, assist low-achieving students during failure situations, and praise low-achieving students more and criticize them less than teachers with low self-efficacy beliefs." (1996, p. 236).

Furthermore, teaching efficacy has been linked to specific classroom behavior, such as the amount of time devoted to small group and whole class instruction and the type of feedback provided by the teacher to students who answered a question incorrectly. In an article titled *Teachers' Sense of Efficacy: An Important Factor in School Improvement*, Myron Dembo and Sherri Gibson discuss the results yielded by a research study they conducted the previous year with 208 elementary teachers using a 30-item teacher efficacy survey instrument (Dembo & Gibson, 1985). Some interesting results are that "high-efficacy teachers allocated twice the amount of time to whole-class instruction than did low-efficacy teachers" and that "although both high- and low-efficacy teachers provided students further opportunities to correct their responses (persistence), high-efficacy teachers were more effective in leading students to correct responses through their questioning, while low-efficacy teachers called on other students or went on to another question" (1985, p. 176). These findings hint to the notion that teachers with high self-efficacy exhibit instructional characteristics that are student-centered and geared to developing and nurturing knowledge in their students, instead of merely giving them the knowledge and the answers. Here the students are active participants in the learning environment and the teacher is the coach, leader, and guide. Consequently, self-efficacy promotes constructivist ideology and practices.

In summary, teaching efficacy refers to "a teacher's belief that his or her ability to bring about change is limited because of external factors" (Yeh, 2006, p. 515). As such, researchers have explored the notion of teaching efficacy and its influence and connection to teaching practices and desired teaching outcomes. Many studies have found that teaching efficacy directly influences student learning and performance, effective instructional strategies, teacher behavior in regards to student feedback, the amount of time spent in group work (small and whole class), and the extent to which the teacher feels responsibility for student learning. Thus, in this thesis, the teaching efficacy component signifies beliefs that subscribe to the following notions:

- The teacher is generally responsible for student learning/achievement in mathematics.
- When the teacher exerts extra effort, the students can do better in mathematics.
- Student achievement in mathematics is directly related to effective instruction.

- Teachers are a powerful influence on student motivation.
- When students are underachieving in mathematics, most likely the teacher is not teaching the subject effectively.

3.2.2 Personal Teaching Efficacy

In this section, we turn from the discussion of general teaching efficacy to personal teaching efficacy. Although both components relate to self-beliefs or self-perceptions, there is a notable difference between the two. The distinction between teacher efficacy and personal efficacy is adeptly explained in the following quote, "teaching efficacy, which identifies beliefs about the consequences of teaching in general, is distinguished from personal efficacy, which defines an individual's beliefs about his or her ability to effect desired results through the execution of personal action." (Chester & Beaudin, 1996, p. 238). Other definitions of personal teaching efficacy include: "a belief in one's ability to teach effectively" (Enochs, Smith, & Huinker, 2000, p. 195) and "an integration of teaching efficacy and personal efficacy (a more general sense of effectiveness not specific to a particular situation" (Dembo & Gibson, 1985, p. 175). Therefore, personal teaching efficacy refers to beliefs in one's own ability to bring about a desired result, as opposed to the belief that teachers (as a group) can or cannot accomplish or produce a desired result. For example, a teaching efficacy belief might be 'all teachers have little influence in the motivation of their students', while a personal teaching efficacy belief refers to a teacher believing that 'he or she cannot motivate his/her students'.

Personal teaching self-efficacy has been found to be a factor in determining the level of persistence, hard work, and willpower one exerts to complete a challenging or unfamiliar task/activity. In an article discussing self-efficacy beliefs of women in mathematics/science fields, Amy Zeldin and Frank Pajares remark:

> The self-efficacy beliefs that people hold influence the choices they make, the amount of effort they expend, their resilience to encountered hardships, their persistence in the face of adversity, the anxiety they experience, and the level of success they ul-

timately achieve. Individuals with strong self-efficacy beliefs work harder and persist longer when they encounter difficulties than those who doubt their capabilities. (2000, p. 218).

Here we find evidence supporting a connection or link between one of the subcomponents of mathematical disposition (perseverance) and personal teaching self-efficacy.

There are numerous reasons why personal teaching efficacy is significant to the overall umbrella of teacher beliefs. Its influence is far-reaching in regards to a variety of domains: including student success, confidence, commitment to teaching, and differentiation of instruction to high and low achievers alike. Many researchers have investigated, studied, and written about personal teaching efficacy, its importance and how it is manifested in teachers' behaviors and practices. For example, the aforementioned research study conducted by Dembo and Gibson discovered that "personal teaching efficacy was positively related to a secure, accepting climate that supported student initiative and was concerned with meeting the needs of individual students" (1985, p. 176). This quote has implications for the individual teacher's behavior and classroom practices because developing a good rapport with the students and working towards ensuring that all the students' needs are meet are decidedly (though usually challenging) responsibilities of every teacher.

In addition, personal teaching efficacy is important for teacher's beliefs because it has been linked to the level of commitment a teacher has to the teaching profession. As Coladarci revealed, "general and personal efficacy significantly predicted commitment to teaching" (1992, p. 332). Alternatively, some research studies have discovered that personal teaching efficacy influences the teacher to use more (or less) student-centered approaches and learning strategies in classroom instruction. One such research study was conducted in Canada by university professors, John Ross and Catherine Bruce (2007). The findings of this study lead the authors to conclude that "high-efficacy teachers use classroom management approaches that stimulate student autonomy ··· and have positive attitudes toward low achieving students, build friendly relationships with them, and set higher academic standards for this group than do low-efficacy teachers"

(2007, p. 50-51). Moreover, teachers with high personal efficacy agreed more with the notion that teacher effort has a substantial influence on student success or failure. "Teachers with highly perceived efficacy view student failure as an incentive for greater teacher effort rather than conclude that the causes of failure are beyond teacher control and cannot be reduced by teacher action" (2007, p. 51).

In a nutshell, personal teaching efficacy "refers to a teacher's belief that he or she does have the competence and skills to bring about student learning" (Yeh, 2006, p. 515). Beliefs of personal teaching efficacy deal with notions relating to whether a teacher has confidence in his/her own ability to effectively teach the content and positively affect student learning and achievement. Moreover, "evidence has been found that personal teaching efficacy is a stronger predictor of teacher effectiveness than is teaching efficacy" (Yeh, 2006, p. 515)

The factors that influence personal teaching efficacy are found within the control of teacher, as opposed to teaching efficacy, where the factors are typically out of the control or influence of the teacher. Accordingly, this thesis endorses the subsequent self-perspectives as beliefs of mathematics teachers with high personal teaching efficacy:

- I should and can consistently explore new and better ways to teach mathematics.
- I will welcome student questions and promote activities that challenge student thinking and encourage conceptual understanding of the concept.
- I can teach mathematics effectively so that my students can become proficient and confident in their abilities to do mathematics.
- I have the necessary skills to effectively monitor mathematics discourses and activities.

3.3 Research Questions

The argument that mathematical disposition and self-efficacy is central to teacher beliefs (which are directly linked to student success) has been established in this thesis through the literature and theoretical framework. However, little is currently known about the interaction between these two critical components of teacher beliefs. Hence, the primary purpose of this thesis study is to investigate whether such an interaction or association exists between a teacher's mathematical disposition and his/her self-efficacy for teaching mathematics. Another key area of interest in this study is whether there exists variation between mathematical disposition (and separately self-efficacy for teaching mathematics) and specific categories of mathematics teachers, such as elementary, middle school, and high school teachers. To gain insight into these areas of interest, this thesis study has converged on the ensuing research questions:

1. To what extent does the mathematical disposition of elementary, middle school, and high school preservice teachers vary?

2. How does the self-efficacy for teaching mathematics differ between elementary, middle school, and high school preservice teachers?

3. What association exists between the mathematical disposition and self-efficacy for teaching mathematics of preservice teachers?

CHAPTER IV

METHODOLOGY

This chapter is dedicated to the discussion of the research methods the thesis author used to answer the research questions below. Specifically, this chapter is divided into three main sections. The first section describes the survey instrumentation, including an explanation of how the survey was constructed, examples of survey items, and a conversation regarding the validity and reliability of the instrument used. The next section describes the data collection process, including the setting and participants, with the help of descriptive data. The third and final section summarizes the data analysis tests used to answer the research questions.

4.1 Survey Instrumentation

When one is conducting a research search, there are two general types of research methods to choose from: qualitative and quantitative designs. Often, the specific type of method chosen reflects the research questions and the specific results the researcher is looking for. Since this thesis study is investigating a relationship between two elements or factors with numerical data, we decided to employ the use of quantitative designs. In effort to answer the research questions presented in the previous chapter, a survey was developed to provide the quantifiable data necessary to employ statistical analysis to draw conclusions and answer these questions.

The survey instrument consisted of 50 items; 10 items for each component that comprised mathematical disposition and self-efficacy for teaching mathematics. In a book titled Engaging in the Scholarship of Teaching and Learning, the authors explain that typically survey questions fall into one of the following groups: "open-ended, closed-ended, partially open-ended, and Likert rating scales" (Bishop-Clark & Dietz-Uhler, 2012, p. 54). The survey used in this thesis to conduct data collection utilized a Likert rating scale. Participants were given a statement and then asked to select one of five options that most closely reflected their views towards the statement: 1 = strongly disagree, 2 = disagree, 3 = uncertain, 4 = agree, and 5 = strongly agree. Moreover, of the 50 statements, 25 were positively worded statements and 25 were negatively worded statements. Of the 10 items in each component, 5 were positively worded and 5 were negatively worded. This is a common practice with researchers who develop survey instruments and essential to establishing the validity of the survey instrument. As Robert DeVellis, in his book titled Scale Development: Theory and Applications, adeptly explains:

> The intent of wording items both positively and negatively within the same scale is usually to avoid an acquiescence, affirmation, or agreement bias...If, for example, a scale consists of items that express a high degree of self-esteem, then an acquiescence bias would result in a pattern of responses appearing to indicate very high esteem. If the scale is made up of equal numbers of positively and negatively worded items, on the other hand, then an acquiescence bias and an extreme degree of self-esteem could be differentiated from one another by the pattern of responses. (2012, pp. 83-84).

While positively/negatively worded questions can prevent/reduce response bias and in turn increase overall validity of the instrument, there are two specific types of validity that also contribute to making the instrument valid: content validity and construct validity. Content validity refers to "the extent to which a specific set of items reflects a content domain" (DeVellis, 2012, p. 59), while construct validity of the instrument is related to "the extent to which a measure "behaves" the way that the construct it purports to measure should behave with regard to established measures of other constructs" (DeVellis, 2012, p. 64). In effort to ensure both content and construct validity, all survey items/statements were either taken directly (without modification) from past research studies or minimally modified (change of verb tense or negation of a positively worded item) versions of the original question of previous research studies. The thesis author gleaned items from past research on each component of mathematical disposition and self-efficacy for teaching mathematics. Various publications already had survey instruments the corresponding authors had created to measure the specific domains of this thesis study. Moreover, most of these research studies had already determined that their instruments were reliable and valid through a variety of means, including corresponding Cronbach's alpha values of the survey domains.

Accordingly, we will provide evidence that the 50 survey items from both mathematical disposition and self-efficacy components are indeed based on past research and the theory outlined, in detailed, in chapters 2 and 3. The more closely the instrument is based on theory, the more confidence we have that it measures what we want it to measure. First, we explore the components of mathematical disposition. The ten statements of the Nature and Usefulness of Mathematics component were taken from two publications: one by Anne Raymond (1997), where the proposed items for this domain were supported by a variety of cited research studies, and one by Doepken, Lawsky, and Padwa (2004), which presented modified statements of the Fennema-Sherman attitude scale, which was published in 1976 and had a Cronbach alpha of 0.86. Examples of some statements include: "Mathematics is an unrelated collection of facts, rules, and skills", "Mathematics is a worthwhile, necessary subject", and "Taking mathematics is a waste of time". Notice that the first and second statements are positively worded, while the third statement is negatively worded. All statements from the Learning of Mathematics component were taken from the above-mentioned publication by Anne Raymond (1997) and include questions like "As a student of mathematics, I passively receive knowledge from the teacher", "many students are just not able to learn mathematics", and "As a student of mathematics, my role is that of an autonomous explorer". Observe that the first two are negatively worded statements, while the third one is positively worded. The last component of mathematical disposition, Perseverance in Mathematics, contained ten statements from the Indiana Mathematics Belief Scales by Kloosterman and Stage (1992) and had an average Cronbach's alpha of 0.805. It was comprised of questions such as "If I can't do a mathematics problem in a few minutes, I probably can't do it at all", "Ability in mathematics increases when one studies hard", and "I can get smarter in mathematics if I try hard". Clearly, the first statement is negatively worded and the latter two are positively worded.

Now we look at the components of Self-Efficacy of Teaching Mathematics and discuss from where the statements were obtained and provide examples of items from each component. Survey questions from the Personal Teaching Efficacy component were taken entirely from published statements from the Personal Mathematics Teaching Efficacy subscale by Enochs, Smith, and Huinker (2000), which had a Cronbach's alpha of 0.88. Examples include the following positively and negatively worded statements: "When teaching mathematics, I will usually welcome student questions", "I will continually find better ways to teach mathematics", and "I do not know what to do to turn students on to mathematics". On the other hand, questions from the General Teaching Efficacy component were taken from two sources: the Mathematics Teaching Outcome Expectancy subscale by Enochs, Smith, and Huinker (2000), which had a Cronbach's alpha of 0.77, and the Measure of Self-Efficacy Beliefs by Chester and Beaudin (1996), which had a Cronbach's alpha of 0.78. An example of a positively worded question in this category is "When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort", while a negatively worded question is "Teachers are not a very powerful influence on student achievement when all factors are considered". Appendix A provides a synopsis of each component in the survey, what publication(s) items were taken from, and whether the survey item was positively or negatively worded. Additionally, to view the survey in its entirety, and in exactly the same format as it was given to participants, see Appendix A.

4.1.1 Pilot Study

To ensure reliability of the survey instrument a pilot study was conducted in the beginning of the fall semester 2016. A total of 38 preservice teachers at a medium-sized public university in the south western United States participated in the survey. One participant only filled out one side of the two-page survey, and as such, only answered questions regarding mathematical disposition and none from the self-efficacy for teaching mathematics component. As such, one survey with incomplete responses was removed from analysis. Thus, 37 responses were used to calculate the

Table 1:
Pilot Study Cronbach's Alpha (N=37)

Components	Cronbach Alpha	Number of Items
Mathematical Disposition (MD)	.835	30
Self-Efficacy of Teaching Mathematics (SEFTM)	.769	20
Entire Survey	.875	50

Cronbach's alpha of the survey instrument.

The Cronbach's alpha is "a test reliability technique that requires only a single test administration to provide a unique estimate of the reliability for a given test" (Gliem & Gliem, 2003, p. 84). In other words, the Cronbach's alpha test is a way to determine if the items of a survey are internally consistent with the intended classification and therefore, reliable. Moreover, a high Cronbach's alpha value supports the construct validity of the instrument or the extent to which the items on the survey actually measure the proposed components. The Cronbach's alpha (α) values range from 0 to 1; the higher the value, the more internally consistent and reliable the items in the instrument are. Specifically, if $\alpha < 0.5$, reliability is unacceptable; if $0.5 < \alpha < 0.6$, reliability is poor; if $0.6 < \alpha < 0.7$, reliability is questionable; if $0.7 < \alpha < 0.8$, reliability is acceptable; if $0.8 < \alpha < 0.9$, reliability is good; and if $\alpha > 0.9$, reliability is excellent (George & Mallery, 2003, p. 231). Table 1 depicts the results of the pilot study's Cronbach's alpha values computed by the statistics program SPSS. Observe that the overall Cronbach's alpha is .835, which implies that the survey instrument's level of internal consistency is good and reliable. Therefore, no changes were made to the survey instrument and the exact survey that was administered to the pilot study's participants was administered to the participants in the thesis study. In fact, because no changes were made to the instrument, the 37 participants responses were incorporated into the total results of the data collection of the official thesis study (which were 201), making the total number of participant responses for the thesis study 238.

4.2 Data Collection

The research thesis study was conducted in the fall semester of 2016 at a medium-sized public university in the south western United States. The participants were all preservice teachers from elementary, middle school, and high school degree programs. Once IRB approval was granted by the university IRB Board (IRB 2016-194-10), professors that taught mathematics courses where all students were preservice teachers were emailed and asked for permission to recruit their students to participate in the research study. After seven university professors granted permission to enlist their students as participants, the thesis author visited these respective classrooms about 15 minutes before class ended (as per the professors' request) and solicited volunteers to take the mathematical disposition and self-efficacy for teaching mathematics survey. Students who volunteered (all were at least 18 years old) were given a paper survey (see Appendix A), which included an implied consent form which the participant could keep if so desired, and asked to return 50-item survey once they had completed it. On average the survey took 6-8 minutes to complete and the data were collected in roughly three weeks in November, 2016. No identifying information was collected from participants (such as name, student ID number, or email address). As a result, there was no follow up with the participants, and their confidentiality was secured. Moreover, to ensure that no participant turned in more than one survey, and the beginning of each recruitment session, the thesis author asked all students who had already taken the survey in another class/course to exit the room, leaving behind only those who had not yet taken the survey.

In total, 238 surveys (including the 37 from the pilot study) were completed and turned in to the thesis author; this included participants from all three teacher preparation programs: elementary, middle school, and high school. As mentioned in the previous section, apart from containing the 50 items measuring MD (mathematical disposition) and SEFTM (self-efficacy for teaching mathematics), the survey also solicited some biodata, including type of preservice teacher program (elementary, middle school, or high school), gender, age, classification or college level (freshman, sophomore, junior, or senior), subject participant desired to teach (generalist, mathematics, or science), the influence of previous mathematics teachers (positive, negative, or neutral), and whether the participant played a musical instrument. The second to the last variable, the influence of previous mathematics teachers, was included in the survey because we wanted to investigate if former mathematics teachers affect their students current mathematical disposition and self-efficacy. The last item was included in the survey out of the thesis author's curiosity of the suggested relation between mathematics and music by some mathematics educators and a side note to the purpose of the thesis study. Since two participants failed to select the type of teacher they were studying to become, and because two of the research questions explicitly relate to the type of teacher by group (elementary, middle school, and high school), these two participants' surveys were excluded from the descriptive statistics and data analysis of the thesis study. Hence, the total number of participant responses used in the data analysis was 236.

4.2.1 Participants: Descriptive Statistics

The descriptive statistics representing the study's 236 participants are illustrated in detail in Table 2. As can be observed, Table 2 describes all participants using the previously mentioned categories (gender, teacher, subject, classification, age, previous mathematics teacher influence, and musical ability). Notice, that out of the 236 total survey participants, 187 were females, 42 were males, and 7 participants failed to report their gender. Tables describing only the female participants and male participants respectively, excluding the categories of musical ability and influence of previous mathematics teachers as these variables are expected to have no specific relation to gender, can be found in Appendix B.

Moreover, of the total participants, 139 (roughly 59%) were elementary preservice teachers, 62 (roughly 26%) were middle school preservice teachers, and 35 (roughly 15%) were high school teachers. Out of the 139 elementary preservice teachers, 133 were studying to become generalists, while 6 of them were studying to teach mathematics in elementary. Of the 62 middle school participants, 54 were studying to teach mathematics, while only 8 were studying to teach science. Also, of the 35 high school participants, 24 were studying to become mathematics teachers, while only 11 were studying to become science teachers. Hence, in total, out of all the

Table 2:
Summary of All Preservice Teacher Participants (N=236)

		All Students			
	Elementary	Middle School	High School	Total	Total %
Subject					
Mathematics	6	54	24	84	36%
Generalist	133	0	0	133	56%
Science	0	8	11	19	8%
Gender					
Male	5	25	12	42	18%
Female	131	34	22	187	79%
Unknown	3	3	1	7	3%
Classification					
Freshman	5	0	0	5	2%
Sophomore	37	9	4	50	21%
Junior	66	26	16	108	46%
Senior	25	26	15	66	28%
Unknown	6	1	0	7	3%
Age					
18-21	112	29	21	162	68.6%
22-25	16	20	12	48	20%
26-30	5	8	0	13	6%
31+	6	5	1	12	5%
Unknown	0	0	1	1	0.4%
Influence					
Positive	91	43	23	157	67%
Negative	8	0	2	10	4%
Neutral	40	19	10	69	29%
Musical					
Yes	45	26	18	89	38%
No	94	36	17	147	62%

preservice teacher groups, there were 133 participants (roughly 56%) studying to become generalists, 84 participants (roughly 36%) studying to become mathematics teachers, and 19 participants (roughly 8%) studying to become science teachers.

The data collected from the 236 responses was initially transferred to an Excel file along with the personal questions found at the end of the survey. These personal questions refer to the information including but not limited to the participant's gender, school classification, and teacher type (a complete list of categories and coding can be found in Appendix B), which were considered as independent variables in the study's analysis. Additionally, missing values from the 50-item questionnaire were substituted with the participant's mean score, based on the number of questions the participant did answer. This is a common approach researchers use to deal with missing values and sometimes called the "person mean substitution approach (PMS)" (Downey & King, 2010, p. 117). Once the missing values were accounted for, the mean scores for each component of MD and SEFTM were calculated for each participant, as well as the overall MD mean score and overall SEFTM score. These scores were used as dependent variables in the various statistical procedures used in effort to answer the research questions. Notice, that the higher the MD score, the more positive or productive the participant's mathematical attitude or disposition is. Similarly, the higher the participant's SEFTM score, the more positive their self-efficacy for teaching mathematics is.

4.3 Data Analysis

Once the data collection had been completed and the descriptive data described, the thesis author began to analyze the data with the goal of answering or shedding some light on the study's research questions. Note that all analysis procedures were conducted by the statistical software SPSS (Statistical Package for the Social Sciences). Firstly, however, the Cronbach's alpha values were calculated using the entire data collection responses, to verify again if the instrument used was reliable and had satisfactory construct validity. To further validate the instrument's reliability and to ensure that the survey indeed measured what the thesis author intended it to measure, exploratory factor analysis (principle component analysis method) was conducted. One of primary reasons researchers use factor analysis is to help them "in determining how many latent variables underlie a set of items" (DeVellis, 2012, p. 116). In this case, the factor analysis can help establish how many constructs, in actuality, underlie the 50 items in the thesis study's survey.

In an effort to answer the thesis's research questions, various statistical tests were conducted and scrutinized. The first research question asks, "to what extent does the mathematical disposition of elementary, middle school, and high school preservice teachers vary?". To answer this question, a one-way ANOVA (analysis of variance) was performed, with the dependent variable being the participant's average mathematical disposition (MD) score, while the independent variables were the type of teacher the participant was studying to become (elementary, middle school, high school). This test allows the researcher to determine if there exists a significant difference between the mean scores of the three groups of teachers in regard to their MD score, which is exactly what the research question is trying to discover. Moreover, other statistical procedures, including comparisons of the group means and mean plot, robust tests (Welch and Brown-Forsythe), and a univariate analysis of variance with post hoc tests were conducted in attempt to explain and determine the statistically significant difference between the three groups' mean MD score.

To answer the second research question, "how does the self-efficacy for teaching mathematics differ between elementary, middle school, and high school preservice teachers?", the thesis author again applied the use of the one-way ANOVA test. Except, this time, the dependent variable was the self-efficacy for teaching mathematics (SEFTM) average score, while the independent variables remained the type of teacher the participant desired to become. Since the one-way ANOVA test determined that there was a meaningful difference between the mean SEFTM scores of these three teacher groups, we employed the follow-up statistical procedures mentioned in answering research question 1 (with SEFTM as the dependent variable) to pinpoint where the difference(s) between the three teacher groups' SEFTM mean scores occurred.

Lastly, the third research question asks, "what association exists between the mathematical disposition and self-efficacy for teaching mathematics of preservice teachers?". To produce results that could answer this question, the thesis author conducted a multivariate analysis of variance (MANOVA), which is similar to the ANOVA. Whereas, ANOVA has one primary dependent variable, MANOVA has several dependent variables. In the MANOVA test, the dependent variables were mathematical disposition (MD) and self-efficacy for teaching mathematics (SEFTM), while the type of teacher (elementary, middle school, high school) was the independent variable. Also, a paired t-test was conducted to determine if the difference between the participants' MD and SEFTM scores are significantly different. Once an association between these two primary components of teacher beliefs was determined to be statistically significant by the paired samples correlations test, linear regression was conducted in effort to explain the relationship between these two dependent components and to make a prediction as to which component (MD or SEFTM) acts as a predictor of the other component. Finally, multiple linear regression was conducted between each dependent variable and the study's seven independent variables to determine which independent variables have a marked influence on the corresponding dependent variable.

CHAPTER V

RESULTS

After describing the methodology of the underlying study, including the pilot study's results, and after presenting the descriptive statistics of the study's measurements, we performed statistical analyses to the data in effort to answer the study's research questions. Before we delve into the results that pertain to the research questions, we are going to provide the instrumentation validity analysis and present our findings.

5.1 Results of Instrumentation: Internal Consistency and Validity

In the previous chapter, we presented the theoretical basis behind the selection of the 50 items of the survey: ten for each of the three subcomponents of mathematical disposition and of the two subcomponents of self-efficacy for teaching mathematics. However, to further demonstrate this, a number of statistical procedures were run in order to provide additional, quantifiable evidence of the instrument's internal consistency and validity.

5.1.1 Cronbach's Alpha

As was mentioned before in the discussion of the pilot study, the Cronbach alpha (sometimes called Cronbach's coefficient alpha, α, is a standard measure of the internal consistency of an instrument, also interpreted as its reliability of an instrument. The Cronbach's alpha was computed for the data, which had a total of 236 responses. As can be observed from Table 3, the value of alpha for the entire survey is .858, which is considered strong, while the alpha values for mathematical disposition and self-efficacy for teaching mathematics are in a good and acceptable range respectively (George & Mallery, 2003). As such, the Cronbach's alpha test offers evidence of the internal consistency and reliability of the thesis study instrumentation; it provides

Table 3:
Thesis Study Cronbach's Alpha (N=236)

Components	Cronbach's Alpha	Number of Items
Mathematical Disposition (MD)	.820	30
Self-Efficacy of Teaching Mathematics (SEFTM)	.720	20
Entire Survey	.858	50

evidence that the instrument had adequate fidelity in measuring different levels of the underlying constructs.

5.1.2 KMO and Bartlett's Test

The method of factor analysis was used to assess the underlying latent variable structure of the survey, with the aim of finding further evidence that the survey indeed measured what it was designed to measure: i.e. three aspects of mathematical dispositions and two aspects of self-efficacy for teaching mathematics. Before we run the Principle Component Analysis, there are some preliminary tests that can help us determine whether a factor/dimension reduction procedure like the PCA can provide meaningful results. For instance, if it is discovered that none of the items in the survey are correlated, running the PCA would be meaningless as we would need 50 factors to summarize 50 items if there are no correlations between the items. The Kaiser-Meyer-Olkin Measure of Sampling Adequacy (KMO Index) and Bartlett's Test of Sphericity are two commonly used procedures to assist investigators in ascertaining if they can efficiently use PCA to reduce the number of factors in the original data set. While these two test share the same goal, they use different methods.

For example, the KMO Index analyzes the correlation matrix and determines the partial correlations that influence the correlation between two variables. Specifically, the KMO index ranges from 0 to 1; the closer the value is to 1, the more PCA is suited to the data set and can act effectively, the closer the value is to 0, the more irrelevant running a test like the PCA becomes. As can be observed by Table 4, the KMO index is 0.829, which indicates that the data sampling is adequate for factor analysis and that there is ample correlation between survey items (Cerny

Table 4:
Results of KMO and Bartlett's Tests

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.829
Bartlett's Test of Sphericity	Approx. Chi-Square	4141.862
	df	1225
	Sig	.000

Note: df = degrees of freedom & Sig. = level of significance

& Kaiser, 1977). On the other hand, Bartlett's test of sphericity assesses the adequacy of the correlation matrix, which means it determines whether correlation matrix is significantly different from the identity matrix. Mathematically speaking, "if test value is large and the significant level is small (<0.05), the hypothesis that the variables are independent can be rejected" (Ho, 2006, p. 218). In the current analysis, as can be observed from Table 4, the test value generated by Bartlett's test of sphericity is 4141.862 and the level of significance associated with this value is less than 0.001. Hence, we can reject the hypothesis that the correlation matrix is the identity matrix and that there is sufficient correlations between items to employ factor analysis.

5.1.3 Factor Analysis: Principal Component Analysis

As mentioned before, the survey is comprised of 50 items; however, the theoretical framework of the study suggests that there are five primary constructs (also called latent variables) that underlie these 50 items. Now, factor analysis is a statistical tool that allows the investigator/researcher to reduce the dimensions of a survey by extracting the clusters of highly intercorrelated items, where each cluster represents a latent variable or factor. In other words, "factor analysis is the orderly simplification of a large number of intercorrelated measures to a few representative constructs or factors" (Ho, 2006, p. 203).

Initially, using SPSS, factor analysis was conducted on the 50-item data set using Principal Component Analysis (PCA) extraction method, using Varimax rotation, and suppressing the absolute value of the factor loadings less than 0.33. The PCA extracted components/factors using the default method, which extracts components only if they have an eigenvalue greater than 1. Additionally, suppressing factor loadings that are less than 0.33 is a common statistical practice

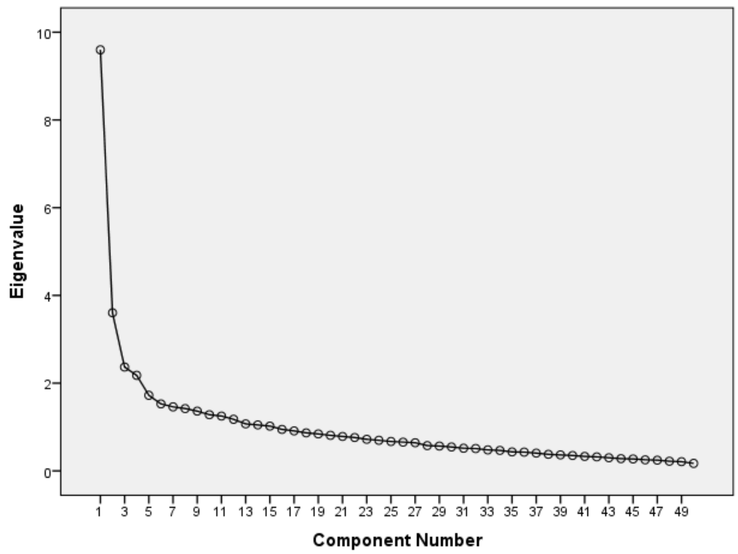

Figure 2: The Scree Plot from the Factor Analysis

as "this indicates that approximately 10% or more of the variance in that item is accounted for by its common factor" (Ho, 2006, p. 213). The PCA extracted 15 components using the criteria mentioned above, which suggests that a 15-factor model can represent the 50-item data set. Also, according to the Total Variance Explained table (see Appendix B), these 15 components represent almost 65% of the total variance that is credited to these 15 factors. However, the Scree plot, which is depicted by Figure 2, suggests that perhaps a 5-factor model would be able to represent the data set sufficiently. Moreover, after examining the Rotated Component Matrix (see Appendix B), which depicts the factor loadings of the 50 items on these 15 components, the thesis author noticed that 32 of the 50 questions of the survey (64%) loaded highly on just 5 components. Also, it was observed that the rotated matrix contains various significant cross-loadings. One typical way to deal with multiple cross-loadings is to "rerun factor analysis, stipulating a smaller number of factors to be extracted" (Ho, 2006, p. 221). Hence, considering the findings of the theoretical framework, the Scree Plot, and Rotated Component Matrix, the thesis author decided to repeat the factor analysis but this time, specifying the number of factors to be extracted based on the theoretical framework of the study.

The second factor analysis employed all the same options/selections as the first run, ex-

Table 5:
Rotated Component Matrix: Factor Loadings with Varimax Rotation

	Components				
	1	2	3	4	5
Q37	.646				
Q33	.632				
Q38	.593				
Q29	.584				
Q32	.582				
Q34	.577				
Q35	.568				
Q40	.558				
Q22	.446				
Q25	.441				
Q36	.435				
Q39	.364				
Q9		.735			
Q10		.681			
Q8		.675			
Q7		.651			
Q5		.634			
Q16		.522			
Q31		.506			
Q4		.453			
Q26		.422			
Q15		.402			
Q20		-.344			
Q24			.664		
Q27			.606		
Q23			.582		
Q30			.563		
Q21			.558		
Q19			.523		
Q14			.499		
Q28			.425		
Q2			.357		
Q1					
Q45				.572	
Q49				.561	
Q46				.518	
Q44				.495	
Q41				.494	
Q43				.473	
Q17				-.382	
Q18				-.372	
Q11				-.367	
Q42					
Q47					.600
Q48					.596
Q50					.425
Q12					-.360
Q13					-.351
Q6					
Q3					

Extraction Criteria: 5 Factors

cept the PCA extracted 5 components. The factor loadings on these 5 components with the cross-loadings can be seen in Appendix B. Since, there are relatively few cross-loadings in the second factor analysis run, the thesis author employed another convention that is typically applied when the number of cross-loadings are few, and deleted these cross-loadings (Ho, 2006, p. 221). In addition, to make for easier viewing, the factor loadings for the 5 components have been sorted so that the items are ordered in reference to how high they load on a particular factor. Table 5 presents the sorted, Rotated Component Matrix of the second PCA test, without cross-loadings. Notice that the 46 of the 50 items have significant loadings on these 5 factors. Moreover, looking closely at the items that loaded to component 1, we see that these items mostly come from the Personal Teaching Efficacy component, while the items that load on component 2 are mainly from the Learning of Mathematics and Nature/Usefulness of Mathematics component. Component 3 has factor loadings primarily from the Perseverance in Mathematics component. Lastly, both components 4 and 5 consist of factor loadings from the General Teaching Efficacy and the Learning of Mathematics components. As a result, after combining the results of both factor analysis runs, there seems to be adequate evidence to support a 5-factor model representation of the 5 theoretically-based components that comprised teacher beliefs.

5.2 Results of the Research Questions

In this section, we will present the results that allow us to answer the study's research questions. To answer each question, various statistical tests and procedures were run and analyzed. However, before we describe the tests and present these interesting results, we must first check for normality in the data set. In other words, one of the basic assumptions we must guarantee before running statistical tests like ANOVA, MANOVA, t-tests, and linear (and multiple) regression is that the data is approximately normally distributed. First, we conducted a well-known test of normality called the Shapiro-Wilk test. This test investigates how likely is it that the MD and SEFTM scores are normally distributed and has a null hypothesis that assumes that the data set is normally distributed. The results of the Shapiro-Wilk test show a significance level of $p=.00012$ for the MD score and a significant level of $p=.672$ for the SEFTM score.

However, due to the fact the data set is based on a Likert-scale (which is ordinal and not continuous), we also tested for normality using the non-parametric test, the Kolmogorov-Smirnov. This test has the same null hypothesis as the Shapiro-Wilk test and showed that in both cases, we should accept the null hypothesis that the data is approximately normally distributed, as the level of significance was p=0.073 for MD score and p=0.200 for SEFTM score. To see the complete table of both tests of normality see Appendix C. To summarize, both tests of normality showed that SEFTM data was approximately normally distributed but differed in their conclusions as to whether the MD data was approximately distributed. Hence, we examined other commonly used methods (examination of histograms and Q-Q plots and skewness and kurtosis values) to access whether the ANOVA, MANOVA, and t-test normality assumptions are satisfied.

The histograms depicted in Figure 3 display the mean scores of participants' mathematical disposition (MD) and their self-efficacy for teaching mathematics (SEFTM). Observe that the data is approximately bell-shaped for both MD and SEFTM scores. To further verify the assumption that our data set is approximately normal, we look at the skewness and kurtosis values. Skewness refers to the "measure of the asymmetry of the distribution of a variable", while kurtosis refers to the "measure of the peakedness of a distribution" (Kim, 2013, pp. 52-53). Furthermore, if the skewness value is 0, we have a symmetric, normal distribution; it is commonly accepted that if the absolute skewness value is greater than 2.0, then we have a significant departure from normality. On the other hand, if a distribution has an absolute kurtosis value that is greater than 7.0, the normality assumption will fail or not be satisfied. Since the skewness value for the mathematical disposition variable is -.706, and the corresponding kurtosis value is 1.624, our data supports a approximately normal distribution. Similarly, for the self-efficacy for teaching mathematics variable, the skewness value is 0.005, while the kurtosis value is -.305.

Finally, we look at the Normal Q-Q plots (Quantile-Quantile plots) to again verify if the distribution of our data is roughly normal (see Appendix F). Ideally, in a normal, symmetric distribution, the data points would fall compactly on a straight line. Notice that the Normal Q-Q plot of the MD score shows a slightly positively skewed distribution with the majority of the

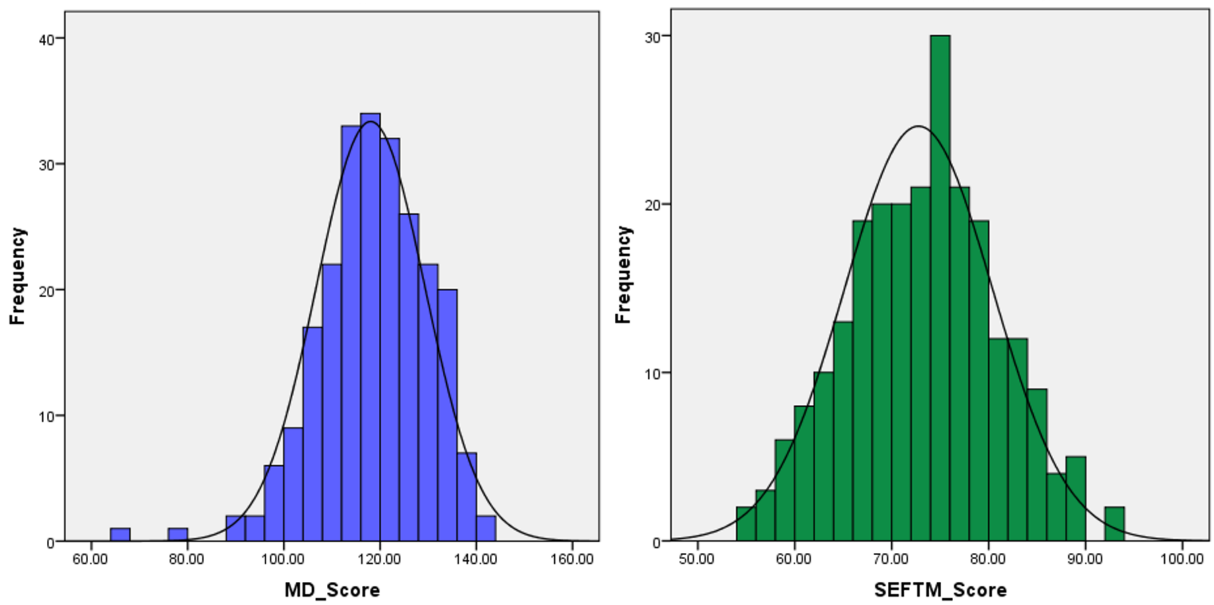

Figure 3: Mathematical Disposition (MD) and Self-Efficacy for Teaching Mathematics (SEFTM) Histograms

data points falling on the straight line, while the Normal Q-Q plot of the SEFTM score show a symmetric distribution with all of the data points falling on the straight line. Hence, taking the various tests of normality and graphs/plots results, we can say with confidence that the data is approximately normally distributed. Thus, we conducted the necessary statistical methods to answer our research questions.

5.2.1 Research Question 1: Mathematical Disposition and Teacher Level

Recall that the first research question explores whether there is a difference between the mathematical disposition scores of elementary, middle school, and high school preservice teachers. To do this we run a one-way ANOVA test, a univariate analysis of the variance, with mathematical disposition as the dependent variable, the type of teacher as the independent variable, and the level of significance at $\alpha = 0.05$. Results from the ANOVA showed that there is a significant difference between the mean MD scores of at least one pair of teacher groups, $F(2,233) = 13.207$, $p < 0.001$, $\eta^2 = 0.10$. Additionally, Levene's test for the homogeneity of variances, $F(2,233) = .621$ with $p = 0.538$, indicated that we could accept the null hypothesis that

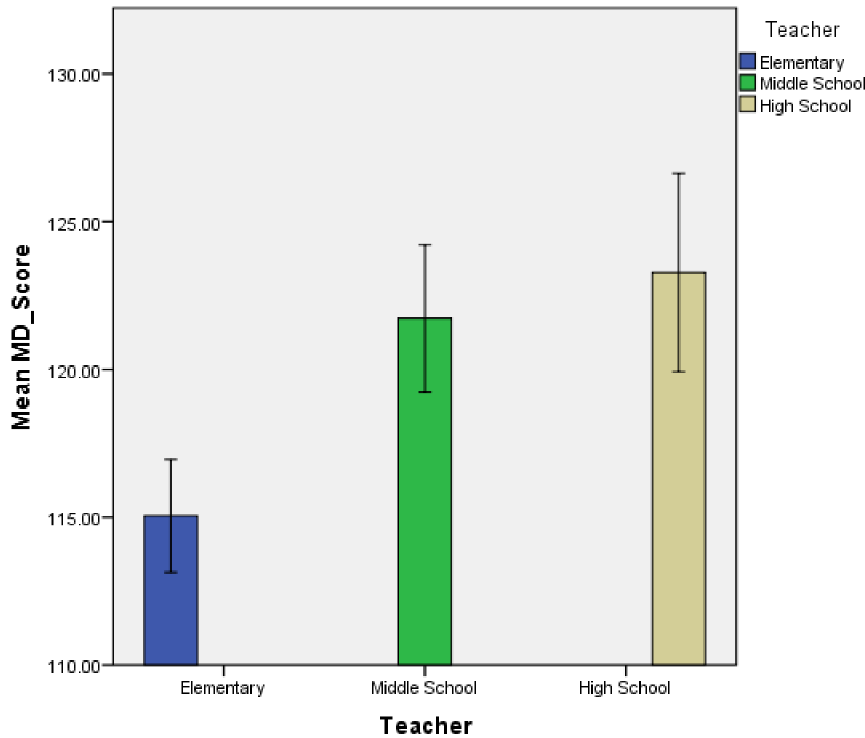

Figure 4: Mathematical Disposition Score categorized by Teacher Type

the variance of the dependent variable is equal across groups. Thus, the homogeneity of variance assumption is satisfied. Moreover, since the participants' MD scores (computed by summing up values on a Likert-scale and substituting missing values with the participant's mean) are not measures on a continuous scale, we ran a nonparametric test to confirm the ANOVA with post hoc tests results. We have also used the independent samples Kruskal-Wallis to test the null hypothesis that the distribution of MD scores is the same across categories of teacher type. The result showed a test statistic of 24.523 with two degrees of freedom and a p-value of less than 0.001, implying that we should reject the null hypothesis (see Appendix F). As such, the nonparametric test, Kruskal-Wallis, echoes the results of the one-way ANOVA that there is a statistically significant difference between the teacher type categories.

To discover where this difference occurs between the three teacher groups, we analyzed the mean scores and plots and ran post hoc tests. Figure 4 illustrates the mean MD scores by teacher type, with error bars of 95% confident levels. Observe that the average elementary pre-

Table 6:
Post Hoc Tests for MD and Teacher Type

Multiple Comparisons					Estimated Marginal Means	
		Mean Difference			95% Confidence Interval	
(I) Teacher	(J) Teacher	(I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
Elementary	Middle School	-6.683*	1.641	**.000**	113.251	116.841
	High School	-8.228*	2.032	**.000**		
Middle School	Elementary	6.683*	1.641	.000	119.041	124.417
	High School	-1.546	2.271	.794		
High School	Elementary	8.228*	2.032	.000	119.697	126.852
	Middle School	1.546	2.271	.794		

Based on observed means. The error term is Mean Square(Error) =115.417.
* The mean difference is significant at the .05 level.

service teacher ($M = 115.05$), scored almost 7 points lower than middle school ($M = 121.73$) and 8 points lower than the high school ($M = 123.27$) preservice teacher. Next we ran the post hoc tests to explore exactly which teacher type differs from the others regarding the dependent variable MD.The results of the multiple comparison test, depicted in Table 6, confirms that the mathematical disposition of elementary teachers is statistically significantly different from the mathematical disposition of both middle school and high school teachers. Furthermore, the table shows that the mathematical disposition of middle school and high school teachers are statistically the same, which implies that there is a no significant difference between mean MD scores of these two groups. Now, if we observe the estimated marginal means, also present in Table 6, we note that there is almost a 3-point difference between the upper bound of C.I. of the mean MD scores of elementary teachers compared to the lower bound of the C.I. of mean MD scores of middle school and high school teachers. We also see that there is major overlapping between the lower and upper bounds of the middle school and high school preservice teachers' C.I. of MD scores, which supported the post hoc test that there is no significant difference between the MD scores of these two teacher type groups.

Results of Participants' Mathematical Disposition Compared with Other Factors

After discovering the notable role teacher type plays to influence a preservice teachers' mathematical disposition, we wanted to investigate further in the attempt to find out if other factors play apart or explain this difference. To begin, the mean MD scores of participants were computed in respect to the other six independent factors: subject, gender, age, classification, influence of previous mathematics teachers, and if the participant has musical ability (a table of these results can be viewed in Appendix C). After analyzing the means, we decided to run a one-way ANOVA on each one of these six factors to discover if there was a statistically significant difference of mean MD scores regarding these independent variables. Results from the ANOVA showed that there was a statistically significant difference in MD scores compared to all independent factors except musical ability (see Appendix C), implying that participants' who had or did not have musical ability scored the same on the mathematical disposition scale. Moreover, we checked for the homogeneity of variances by looking at Levene's test from the ANOVA output (see Appendix C), and the necessary assumption was satisfied in all cases.

Table 7 depicts the results of the multiple comparison tests, the combined outputs from the univariate analysis of variance with post hoc tests using MD as the dependent variable and the remaining five factors as independent variables. This table was used to determine whether there was a statistically significant difference in mean MD scores regarding each corresponding independent variable and its categories (note that statistically significant values are in bold). Following is a list of notable findings produced from these tests using MD as the dependent variable and the remaining four factors (subject, age, classification, and influence) as independent variables. The independent variable gender could not be subject to post hoc test because it contains only two categories and three or more are required to make a multiple comparison test. However, the results from the one-way ANOVA tests (see Appendix C) provide sufficient information to judge if there is a statistically significant difference between the mean MD scores of males and females.

1. Subject and Mathematical Disposition

   On average, preservice mathematics teachers ($M = 123.82$), scored 9 points higher than preservice generalists ($M = 114.84$) and preservice science teachers ($M = 114.66$) on

Table 7:
Multiple Comparisons Tests (Scheffe) with dependent variable MD

(I) Subject	(J) Subject	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Mathematics	Generalist	8.9866*	1.45941	**.000**	5.3912	12.5819
	Science	9.1628*	2.66021	**.003**	2.6092	15.7164
Generalist	Mathematics	-8.9866*	1.45941	.000	-12.5819	-5.3912
	Science	.1762	2.56822	.998	-6.1507	6.5032
Science	Mathematics	-9.1628*	2.66021	.003	-15.7164	-2.6092
	Generalist	-.1762	2.56822	.998	-6.5032	6.1507
(I) Age	(J) Age	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
18-21 yrs.	22-25 yrs.	-5.9581*	1.82035	**.015**	-11.0846	-.8316
	26-30 yrs.	-4.8671	3.19311	.509	-13.8595	4.1254
	31+ yrs.	.8640	3.31398	.995	-8.4688	10.1969
22-25 yrs.	18-21 yrs.	5.9581*	1.82035	.015	.8316	11.0846
	26-30 yrs.	1.0910	3.46335	.992	-8.6625	10.8445
	31+ yrs.	6.8221	3.57510	.305	-3.2461	16.8903
26-30 yrs.	18-21 yrs.	4.8671	3.19311	.509	-4.1254	13.8595
	22-25 yrs.	-1.0910	3.46335	.992	-10.8445	8.6625
	31+ yrs.	5.7311	4.43436	.644	-6.7570	18.2192
31+ yrs.	18-21 yrs.	-.8640	3.31398	.995	-10.1969	8.4688
	22-25 yrs.	-6.8221	3.57510	.305	-16.8903	3.2461
	26-30 yrs.	-5.7311	4.43436	.644	-18.2192	6.7570
(I) Classification	(J) Classification	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Freshman	Sophomore	-7.6936	5.16172	.529	-22.2330	6.8458
	Junior	-10.0663	5.03414	.264	-24.2463	4.1137
	Senior	-14.9364*	5.10453	**.038**	-29.3146	-.5581
Sophomore	Freshman	7.6936	5.16172	.529	-6.8458	22.2330
	Junior	-2.3727	1.88241	.662	-7.6750	2.9296
	Senior	-7.2428*	2.06327	**.007**	-13.0545	-1.4310
Junior	Freshman	10.0663	5.03414	.264	-4.1137	24.2463
	Sophomore	2.3727	1.88241	.662	-2.9296	7.6750
	Senior	-4.8700*	1.71939	**.048**	-9.7132	-.0269
Senior	Freshman	14.9364*	5.10453	.038	.5581	29.3146
	Sophomore	7.2428*	2.06327	.007	1.4310	13.0545
	Junior	4.8700*	1.71939	.048	.0269	9.7132
(I) Influence	(J) Influence	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Positive	Negative	4.3572	3.59242	.480	-4.4929	13.2074
	Neutral	5.7906*	1.59096	**.002**	1.8711	9.7100
Negative	Positive	-4.3572	3.59242	.480	-13.2074	4.4929
	Neutral	1.4333	3.72707	.929	-7.7486	10.6152
Neutral	Positive	-5.7906*	1.59096	.002	-9.7100	-1.8711
	Negative	-1.4333	3.72707	.929	-10.6152	7.7486

Based on observed means. The error term is Mean Square(Error) = 121.327.
*. The mean difference is significant at the .05 level.

the MD scale. There was a statistically significant difference (p <001) between the mean MD scores of mathematics preservice teachers and both generalists and science preservice teachers. There was no statistically significant difference (p =.998) between the mean MD scores of generalists (all of whom are elementary teachers) and science preservice teachers.

2. Gender and Mathematical Disposition

   On average, male preservice teachers (M=121.88) scored 4 points higher on the MD scale than female preservice teachers (M=117.18) did. There was a statistically significant difference (p=0.015) between the mean MD scores of male and female preservice teachers.

3. Age and Mathematical Disposition

   On average, preservice teachers in the 22-25 yrs. ($M = 122.57$) and 26-30 yrs. ($M = 121.48$) age range, scored 5-6 points higher on the MD scale than did preservice teachers in the 18-21 yrs. ($M = 116.61$) and 31^{+} yrs. ($M = 115.75$) age range. There was a statistically significant difference between the mean MD scores of preservice teachers in the 22-25 yrs. age range and 18-21 yrs. age range. No other category interaction was found to be statistically significant.

4. Classification and Mathematical Disposition

   On average, freshman preservice teachers ($M = 107.21$) scored the lowest in the MD scale, scoring 15 points lower than senior preservice teachers ($M = 122.14$), 10 points lower than junior preservice teachers ($M = 117.27$), and 7 points lower than sophomore preservice teachers ($M = 114.90$). Senior preservice teachers scored statistically significantly higher than freshman, sophomore, and junior preservice teachers. There was no statistically significant difference between the mean MD scores of freshman, sophomore, and junior preservice teachers.

5. Previous Mathematics Teachers' Influence and Mathematical Disposition

   On average, preservice teachers who had been positively influence by former math teachers

($M = 119.90$) scored higher in the MD scale than teachers who were negatively influence by former math teachers ($M = 115.54$) or felt that their former math teachers had a neutral influence ($M = 114.11$). Surprisingly, there was a statistically significant difference between the mean MD scores of preservice teachers who selected 'positive influence' compared to those who selected 'neutral influence'; yet, there was no statistically significant difference between the MD scores of those who selected 'positive influence' to those who selected 'negative influence'. There was no significant difference between the MD scores of pre-service teachers who considered former math teachers to have had a negative or neutral influence on them.

5.2.2 Research Question 2: Self-Efficacy for Teaching Mathematics and Teacher Level

The second research question explores whether self-efficacy for teaching mathematics (SEFTM) differs between elementary, middle school, and high school pre-service teachers. Since we are trying to determine if there is a statistically significant difference between the SEFTM and teacher category, we ran the same tests as in the previous question, except this time SEFTM was the dependent variable while teacher type remained the independent variable.

A one-way ANOVA found that there was a statistically significant difference between the mean SEFTM scores between at least one of the teacher groups, $F(2,233) = 6.135, p = 0.003, \eta^2 = 0.05$. To satisfy the homogeneity of variance assumption needed to be confident that the results of the ANOVA are accurate to the data set, we analyzed Levene's test. Since $F(2,233) = 1.970$ with $p = 0.142$, a value notably larger than the standard alpha level of 0.05, we accept the null hypothesis that the variance of the dependent variable is equal across groups. In addition, because the participants' mean SEFTM scores do not belong to the scale of real numbers, we ran a nonparametric independent samples Kruskal-Wallis test to confirm the ANOVA results (see Appendix F). The Kruskal-Wallis tests the null hypothesis that the distribution of SEFTM scores is the same across categories of teacher type. The resulting test statistic of 10.356 with two degrees of freedom and $p = 0.006$, allowed us to reject the null hypothesis assumption and accept the ANOVA conclusions.

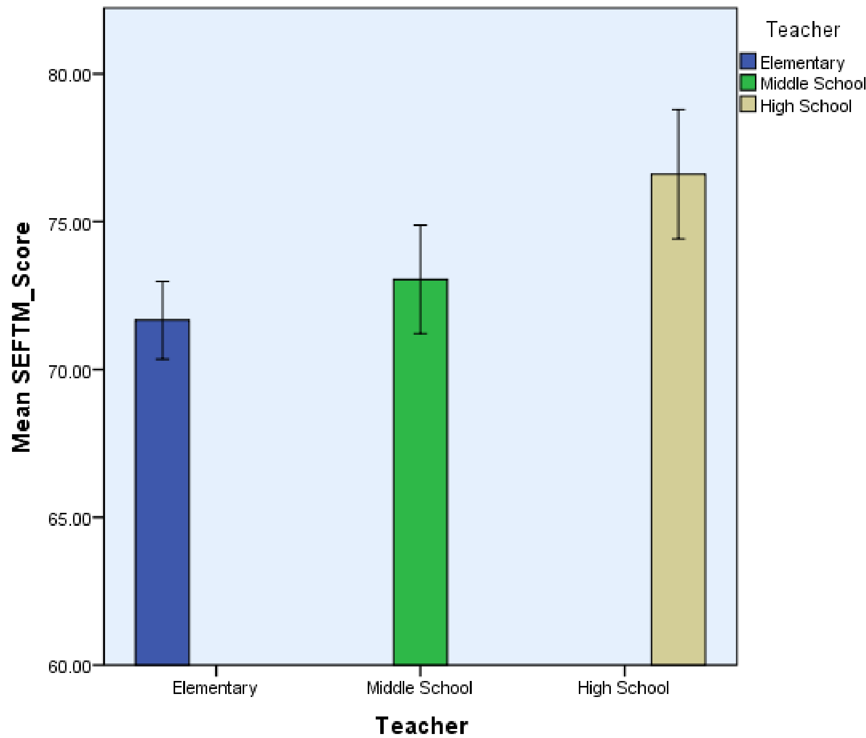

Figure 5: Self-Efficacy for Teaching Mathematics Score categorized by Teacher Type

Now that we were sure that there is was a difference between the self-efficacy for teaching mathematics of at least one pair of teacher groups, we first noticed the variance in mean scores of the three teacher groups by observing the mean plots, including the error bars of 95% confident levels, depicted in Figure 5. Notice that on average, elementary preservice teachers ($M = 71.67$) scored roughly 2 points lower than middle school preservice teachers ($M = 73.04$) and 5 points lower than high school teachers ($M = 76.60$). However, observing the variance in means does not provide sufficient evidence to make conclusions about where the statistically significant difference occurs between teacher type and SEFTM. Thus, we ran a univariate analysis of variance with post hoc tests to statistically and accurately pinpoint where the deviation occurs. The results of the multiple comparisons (post hoc) test, presented in left side of Table 7, provide numerous interesting conclusions. These findings show that self-efficacy for teaching mathematics of preservice elementary teachers is statistically the same as the self-efficacy for teaching mathematics of middle school preservice teachers. Moreover, we find that there is a

Table 8:
Post Hoc Tests for SEFTM and Teacher Type

	Multiple Comparisons				Estimated Marginal Means	
		Mean Difference			95% Confidence Interval	
(I) Teacher	(J) Teacher	(I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
Elementary	Middle School	-1.377	1.143	.485	70.415	72.917
	High School	-4.934*	1.416	**.003**		
Middle School	Elementary	1.377	1.143	.485	71.171	74.916
	High School	-3.557	1.583	.082		
High School	Elementary	4.934*	1.416	.003	74.107	79.093
	Middle School	3.557	1.583	.082		

Based on observed means. The error term is Mean Square(Error) =56.029.
*. The mean difference is significant at the .05 level.

statistically significant different between the mean SEFTM scores of elementary and high school preservice teachers, yet there is no statistically significant difference between the SEFTM mean scores of middle and high school preservice teachers. It is curious to note that although we could not accept a conclusion of a significant difference between the SEFTM scores of middle and high school teachers as p=0.082, it is close to the standard value of significant p=0.05.

To provide additional evidence that the results of the post hoc test is true, we observe the results of the estimated marginal means, included in Table 7. Notice how the 95% confidence intervals of elementary and middle school preservice teachers clearly overlap, and the 95% confidence intervals of middle school and high school overlap only slightly. Yet, the 95% confidence intervals of elementary and high school preservice teachers do not overlap.

Results of Participants' SEFTM Compared with Other Factors

While it was insightful and informative to discover that teacher type seems to influence the SEFTM of preservice teachers, we were intrigued to discover the role the other independent factors played in regard to a preservice teachers' self-efficacy for teaching mathematics. To do this, the average SEFTM scores were calculated in respect to the six remaining independent factors: subject, gender, age, classification, influence of previous mathematics teachers, and if the participant has musical ability (see Appendix D). Furthermore, a one-way ANOVA was conducted for each one of these factors to identify if there was a statistically signification difference between the mean scores between the groups of each factor regarding SEFTM. If there was, then further tests were run to determine where this difference specifically occurred. If there was not, the factor was excluded from further analysis.

Again, the ANOVA resulted in confirming that there was statistically significant difference in SEFTM scores compared to all independent factors except musical ability (see Appendix D). We also checked the homogeneity of variances assumption for each independent variable/factor compared to SEFTM. Levene's test from the ANOVA output (see Appendix D) showed that homogeneity of variances was satisfied in all cases except for gender, implying that the corresponding ANOVA outcomes might be biased and unreliable. As a result, the independent variables gender and musical ability were excluded from additional analysis.

Table 8 depicts the detailed results from the multiple comparison tests from the univariate analysis of variance with post hoc tests using SEFTM as the dependent variable and the remaining four factors (subject, age, classification, and influence) as independent variables. Below is a summary of the post hoc tests' findings.

1. Subject and Self-Efficacy for Teaching Mathematics

   On average, preservice mathematics teachers ($M = 75.21$), scored roughly 4 points higher than preservice generalists ($M = 71.49$) and 5 points higher than preservice science teachers ($M = 70.79$) on the SEFTM scale. There was a statistically significant difference ($p = .003$) between the mean SEFTM scores of mathematics preservice teachers and generalist preservice teachers. There was no statistically significant difference ($p = .930$) between the mean SEFTM scores of generalists and science preservice teachers and between the mean SEFTM scores of mathematics and science preservice teachers, though it was very close to being significant ($p = 0.068$).

2. Age and Self-Efficacy for Teaching Mathematics

   On average, preservice teachers in the 22-25 yrs. age range ($M = 77.15$), scored 5-6 points

Table 9:
Multiple Comparisons Tests (Scheffe) with SEFTM

(I) Subject	(J) Subject	Mean Difference (I-J)	Std.Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Mathematics	Generalist	3.7166*	1.03906	**.002**	1.1568	6.2764
	Science	4.4158	1.89399	.068	-.2501	9.0818
Generalist	Mathematics	-3.7166*	1.03906	.002	-6.2764	-1.1568
	Science	.6992	1.82850	.930	-3.8054	5.2039
Science	Mathematics	-4.4158	1.89399	.068	-9.0818	.2501
	Generalist	-.6992	1.82850	.930	-5.2039	3.8054

(I) Age	(J) Age	Mean Difference (I-J)	Std.Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
18-21 yrs.	22-25 yrs.	-5.6000*	1.21017	**.000**	-9.0081	-2.1919
	26-30 yrs.	-.9137	2.12277	.980	-6.8919	5.0645
	31+ yrs.	.2209	2.20313	1.000	-5.9836	6.4254
22-25 yrs.	18-21 yrs.	5.6000*	1.21017	.000	2.1919	9.0081
	26-30 yrs.	4.6864	2.30243	.249	-1.7978	11.1705
	31+ yrs.	5.8209	2.37672	.115	-.8724	12.5143
26-30 yrs.	18-21 yrs.	.9137	2.12277	.980	-5.0645	6.8919
	22-25 yrs.	-4.6864	2.30243	.249	-11.1705	1.7978
	31+ yrs.	1.1346	2.94796	.985	-7.1675	9.4366
31+ yrs.	18-21 yrs.	-.2209	2.20313	1.000	-6.4254	5.9836
	22-25 yrs.	-5.8209	2.37672	.115	-12.5143	.8724
	26-30 yrs.	-1.1346	2.94796	.985	-9.4366	7.1675

(I) Classification	(J) Classification	Mean Difference (I-J)	Std.Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Freshman	Sophomore	.1826	3.55076	1.000	-9.8191	10.1842
	Junior	-2.9012	3.46300	.873	-12.6557	6.8532
	Senior	-4.5404	3.51141	.644	-14.4312	5.3505
Sophomore	Freshman	-.1826	3.55076	1.000	-10.1842	9.8191
	Junior	-3.0838	1.29492	.132	-6.7313	.5637
	Senior	-4.7229*	1.41932	**.013**	-8.7208	-.7250
Junior	Freshman	2.9012	3.46300	.873	-6.8532	12.6557
	Sophomore	3.0838	1.29492	.132	-.5637	6.7313
	Senior	-1.6391	1.18277	.590	-4.9707	1.6924
Senior	Freshman	4.5404	3.51141	.644	-5.3505	14.4312
	Sophomore	4.7229*	1.41932	.013	.7250	8.7208
	Junior	1.6391	1.18277	.590	-1.6924	4.9707

(I) Influence	(J) Influence	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval Lower Bound	Upper Bound
Positive	Negative	2.8681	2.45142	.505	-3.1711	8.9074
	Neutral	3.3757*	1.08565	**.009**	.7011	6.0503
Negative	Positive	-2.8681	2.45142	.505	-8.9074	3.1711
	Neutral	.5076	2.54331	.980	-5.7580	6.7732
Neutral	Positive	-3.3757*	1.08565	.009	-6.0503	-.7011
	Negative	-.5076	2.54331	.980	-6.7732	5.7580

Based on observed means. The error term is Mean Square(Error) = 56.496.
*. The mean difference is significant at the .05 level.

higher on the SEFTM scale than did preservice teachers in the 18-21 yrs. ($M = 71.55$), 26-30 yrs. ($M = 72.47$), and 31^{+} yrs. ($M = 71.33$) age range. There was a statistically significant difference between the mean SEFTM scores of preservice teachers in the 18-21 yrs. age range and those in the 22-25 yrs. age range. No other category interaction was found to be statistically significant.

3. Classification and Self-Efficacy for Teaching Mathematics
   On average, sophomore preservice teachers ($M = 69.90$) scored the lowest in the SEFTM scale, closely followed by freshman preservice teachers ($M = 70.08$); junior preservice teachers ($M = 72.98$) trailed behind senior preservice teachers ($M = 74.62$). Senior preservice teachers scored statistically significantly higher than sophomore preservice teachers. No other category interaction was found to be statistically significant.

4. Previous Mathematics Teachers' Influence and Self-Efficacy for Teaching Mathematics

   On average, preservice teachers who had been positively influence by former math teachers ($M = 73.87$) scored higher in the SEFTM scale than teachers who were negatively influence by former math teachers ($M = 71.00$) or felt that their former math teachers had a neutral influence ($M = 70.49$). Again, there was a statistically significant difference between the mean SEFTM scores of preservice teachers who selected 'positive influence' compared to those who selected 'neutral influence'; yet, there was no statistically significant difference between the SEFTM scores of those who selected 'positive influence' to those who selected 'negative influence'. There was no significant difference between the SEFTM scores of preservice teachers who considered former math teachers to have had a negative or neutral influence on them.

5.2.3 Research Question 3: Association between Mathematical Disposition (MD) and Self-Efficacy for Teaching Mathematics (SEFTM)

The third and final research question investigated if there exists an association or relationship between the mathematical disposition and self-efficacy for teaching mathematics of preservice teachers. In other words, if a preservice teacher has a high mathematical disposition, does that imply that she/he also has a high self-efficacy for teaching mathematics, and vice versa? In order to test this association, three statistical procedures were conducted: a MANOVA test, or multivariate analysis of variance test, independent paired samples t-test, and linear regression. The reason why we used the MANOVA test even though we had already used two one-way ANOVAs for each dependent variable, MD and SEFTM, was to reduce the probability of having a Type I error. Recall that in hypothesis testing, a Type I error is equated with a "false positive"; meaning that we reject the true null hypothesis when in fact we should accept it. The benefit of using the MANOVA over multiple ANOVAs is that MANOVA maintains the probability of a Type I error at 0.05; whereas, with each additional ANOVA test, the probability of making a Type I error increases.

The MANOVA was run with two dependent variables, mathematical disposition and self-efficacy for teaching mathematics, and the independent, categorical variable, teacher type. The first output of the MANOVA was Box's Test of Equality of Covariance Matrices, which has the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups. The results showed that the assumption of homogeneity of covariances across the teacher groups was satisfied, as $F(6,100,927) = 0.658$, $p = 0.684$, a p-value much larger than the standard $p < 0.05$ needed to reject the null hypothesis. Moreover, Levene's test of Equality of Error Variances showed that for both MD and SEFTM, the error variance was equal across groups, as the level of significant for MD was $p = 0.538$ and for SEFTM was $p = 0.142$. These tests provide evidence that Wilk's lambda test is suitable to be used and interpreted.

As can be observed in detail in Appendix E, the multivariate tests of the MANOVA provide evidence that there was a statistically significant difference between teacher type when considered jointly on the two dependent variables, mathematical disposition and self-efficacy for teaching mathematics, Wilk's $\Lambda = .879$, $F(4,464) = 7.697$, $p < 0.001$, multivariate $\eta^2 = 0.06$. This indicates that the elementary, middle school, and high school preservice teachers differ when considered jointly with MD and SEFTM. Note that the multivariate $\eta^2 = 0.06$ indicated that approximately 6% of multivariate variance of the two dependent variables is associated with teacher type.

Given that the results of the MANOVA were significant, we examined the ANOVA tests that considered teacher type jointly with MD and SEFTM. These results showed a prominent level of significance, $F(2,233) = 13.207, p < 0.001$, multivariate $\eta^2 = 0.102$ and $F(2,233) = 6.135$, $p = 0.003$, multivariate $\eta^2 = 0.050$ respectively. In other words, there was significant differences between elementary, middle school, and high school participants on a linear combination of MD and SEFTM, the two dependent variables.Also, the outcome of the multivariate η^2 implied that teacher type accounts for roughly 10% of the multivariate variance associated with preservice teachers' MD score, and roughly 5% of the multivariate variance associated with their corresponding SEFTM score. Complete tables for the multivariate analyses are presented in Appendix E.

To determine whether the preservice teachers scored statistically the same on the MD scale as they scored on the SEFTM score, an independent paired samples t-test was conducted. Additionally, a nonparametric version of the t-test, the Wilcoxon signed rank test, was also conducted to verify the results, since data collected using the Likert-scale is not on a continuous scale. Since the MD scale ranged from 30 to 150 points (30 points being the lowest a participant could score and 150 being the highest) and SEFTM scale ranged from 20 to 100 (20 being the lowest a participant could score and 100 being the highest), the participants MD and SEFTM scores needed to be standardized before either of these tests could be used. To standardize the MD and SEFTM, z-scores were used as follows:

$$\frac{X - \mu_X}{\sigma_X} \qquad \text{and} \qquad \frac{Y - \mu_Y}{\sigma_Y},$$

where X and Y represent individual scores on the MD, SEFTM scales respectively, with their associated μ and σ being their mean and variances.

After the standardization was completed, the independent paired samples t-test was con-

Table 10:
Paired Samples T-Test: Statistics, Correlations, and Differences

Statistics		Mean	N	Std. Deviation	Std.Error Mean
Pair 1	MD_Score	-.0094	236	.99621	.06485
	SEFTM_Score	-.0000000000763	236	1.00000	.06509

Correlations		N	Correlation	Sig.
Pair 1	MD_Score & SEFTM_Score	236	.553	.000

Differences Pair 1	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference Lower	Upper	t	df	Sig. (2-tailed)
MD_Score - SEFTM_Score	-.00938	.94327	.06140	-.13035	.11158	-.153	235	.879

ducted to compare how participants scored in the mathematical disposition scale and in the self-efficacy for teaching mathematics scale. As can be substantiated by Table 10, there was a lack of sufficient evidence to support the claim that there was a significant difference in the participants' MD scores and SEFTM scores, $t(235) = -.153$, $p = .879$. These findings suggest that we must accept the null hypothesis that preservice teachers scored the same on the MD scale as they did on the SEFTM scale. Plainly speaking, if a teacher scored a 75% on the MD scale, it would be reasonable to conclude that he/she would score roughly 75% on the SEFTM scale. Additional support for these findings, comes in the form of the paired samples correlation results which showed that there is a positive correlation of 0.553 between a participants' MD and SEFTM score. In other words, the statistically significant ($p < 0.001$) correlation of 0.553 indicates that preservice teachers who scored low on the MD scale tended to score low on the SEFTM scale. Conversely, higher MD scores were associated with higher SEFTM scores.

As mentioned earlier, the Wilcoxon signed rank nonparametric test was also conducted to verify the paired samples t-test results (see Appendix F). The null hypothesis for this test was that the median of differences between the MD score and SEFTM score equals zero. The results of this test echoed those of the t-test, as the Wilcoxon signed rank test found that there was no statistically significant difference, with $p = 0.702$, and as such we are to retain or accept the null hypothesis.

Lastly, we attempted to build a simple linear regression model in which we could use the mathematical disposition scores to predict the self-efficacy for teaching scores. In order to conduct a linear regression and be confident that our results are statistically valid, we must ensure that the basic linear regression assumptions are satisfied. First, we must check for linearity, or that there is a linear relationship between the two variables. We can do this by using the scatter plot, with the outcome variable (SEFTM) on the y-axis and predictor variable (MD) on the x-axis, which can be viewed in Appendix F. Notice that the scatter plot shows a medium, positive relationship between the variables. In other words, as the participants' MD score increases so does his/her SEFTM score. Thus, generally, we do have a linear relationship which we can model using regression.

Other assumptions we must check are that the residuals are normally distributed and that we generally have a random distribution of scores. To verify these assumptions, we analyze a histogram of the residuals and a scatter plot of the regression standardized predicted value compared to the regression standard residual in regard to the outcome variable, SEFTM. Both graphs can be viewed in Appendix F; the histogram shows that the residuals follow a roughly normal distribution and the scatter plot confirms that we have a random distribution of scores, which is want we want. Finally, we must ensure that there is little to no autocorrelation in the data and we do this by the Durbin-Watson test (see model summary table in Appendix E). Since the Durbin-Watson statistic is 1.883, we can assume that there is no autocorrelation in the data.

We then used a linear regression model to predict SEFTM scores based on participants' MD scores. A significant regression equation was discovered ($F(1,234) = 104.338$, $p < 0.001$), with $R^2 = 0.308$. Since the R Squared value tells us how much variance in the SEFTM scores are explained by the predictor variable (MD), the results suggest that approximately 31% of all the variance in a participants' SEFTM score can be predicted by the MD score. Another important

Table 11:
Linear Regression Coefficients[a] Test

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	28.357	4.367		6.494	.000
	MD_Score	.376	.037	.555	10.215	.000

a. Dependent Variable: SEFTM_Score

output of the linear regression is the coefficients test which allows us to come up with an equation that we can use to predictor SEFTM scores if we know the MD score. These results, depicted in Table 11, show a y-intercept value of 28.357 and a gradient value of .376. This implies that for each one point increase in a participants' MD score, we have a corresponding increase of 0.376 points in his/her SEFTM score.

Hence, the linear regression equation to model the association of MD and SEFTM is:

$$\hat{Y} = .376X + 28.357,$$

where $\hat{Y}$ is the predicted value of the participants' SEFTM score given "X", the participants' MD score. Similarly, if we use SEFTM score as the predictor variable and MD score as the outcome, or dependent variable, we get the following regression equation:

$$\hat{Y} = .820X + 58.382,$$

where $\hat{Y}$ is the predicted value of the participants' MD score given "X", the participants' SEFTM score (to view the detailed results of the coefficients test with MD as the outcome variable see Appendix E).

Multiple Linear Regression Results

To acquire more information that could shed light on the influence of the independent variables on a participants' mathematical disposition or self-efficacy, a multiple linear regression was conducted. In the first run, mathematical disposition was the dependent variable with the

Table 12:
Multiple Linear Regression: Model Summary with MD Score

Model Summary[e]					
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.353[a]	.125	.121	10.70212	
2	.422[b]	.178	.171	10.39186	
3	.468[c]	.219	.208	10.15434	
4	.500[d]	.250	.236	9.97711	1.925

a. Predictors: (Constant), Subject
b. Predictors: (Constant), Subject, Classification
c. Predictors: (Constant), Subject, Classification, Influence
d. Predictors: (Constant), Subject, Classification, Influence, Teacher
e. Dependent Variable: MD_Score

previously mentioned 7 independent factors, and in the second run self-efficacy for teaching mathematics was the dependent variable with the same 7 independent variables. The regression used the stepwise method, which means that a predictor variable would only be entered into the model if it met the criteria of making a statistically significant contribution to the model. This process repeats until there are no remaining predictor variables that qualify to be entered into the model.

Table 12 depicts the four predictor variables that were removed by the regression stepwise method as being statistically significant to respondents' MD score: subject, classification, influence, and teacher. Recall that the *R* Squared determines how much of the total variance of MD scores can be explained by variance in the independent variables. Hence, the summary model suggests that 25% of the variance in the dependent variable, MD, is attributable to the four independent variables mentioned above. This implies that the subject the participant is studying to teach and the type of teacher he/she is studying to become, the participant's classification, and previous mathematics teachers' influence, greatly affect his/her mathematical disposition.

Similarly, Table 13 depicts the three predictor variables that were added by the regression stepwise method as being statistically significant to the model based on the dependent variable self-efficacy for teaching mathematics: teacher, influence, and classification. Notice how these

Table 13:
Multiple Linear Regression: Model Summary with SEFTM Score

Model Summary[d]					
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.210[a]	.044	.040	7.52989	
2	.291[b]	.085	.076	7.38661	
3	.329[c]	.108	.096	7.30745	1.988

a. Predictors: (Constant), Teacher
b. Predictors: (Constant), Teacher, Influence
c. Predictors: (Constant), Teacher, Influence, Classification
d. Dependent Variable: SEFTM_Score

three predictor variables were also found to be important to participants' mathematical disposition, MD. The model summary suggests that approximately 11% of the variance of the dependent variable, SEFTM, is explained by what type of teacher the participant is, what he/she is studying to teach, and his/her previous mathematics teachers' influence. Although not as influential as in the case of MD, these findings do indicate that these three independent variables are associated with a preservice teacher's self-efficacy for teaching mathematics. To view the complete outputs of the multiple linear regression models for both dependent variables see Appendix E.

CHAPTER VI

DISCUSSION

6.1 Conclusions

Over the last few decades, teacher beliefs have been a popular subject of study for the mathematics education community. Furthermore, numerous of these studies have targeted the mathematical disposition and self-efficacy of teachers, both in-service and preservice. However, there is a lack of studies or literature that has examined the relationship or interaction between these two components, as well as how each one of these components is influenced by whether the person is (or studying to become) an elementary, middle school, or high school teacher. As a result, the primary purpose of this study was two-fold: to examine whether the type of teacher the participant is studying to become influences his/her mathematical disposition and self-efficacy for teaching mathematics, and to investigate the relationship between these two key components of preservice teachers' beliefs.

The results of the first research question showed that mathematical disposition of middle school and high school teachers was statistically significantly higher than the mathematical disposition of their elementary preservice teacher (PST) counterparts. In fact, middle and high school teachers scored, on average, 7 to 8 points higher than the elementary teachers. A probable explanation for this difference could be the fact that elementary teachers have a weaker mathematics background than middle and high school teachers. Many studies have shown that elementary teachers' mathematics knowledge is weak or that they lack a deep, conceptual understanding of even basic mathematics concepts (Becker, 1986; Grootenboer & Zevenbergen, 2008). In fact, a study conducted by mathematics educators/professors of over 25,000 elementary teachers in the state of North Carolina found that only 25% of the teachers had positive attitudes towards teaching mathematics and that roughly only 28% felt most qualified to teach mathematics (Berenson, Hodgin, Ward, Andrews, & Rudin, 1991).

The weak mathematics background of elementary teachers can likely be explained by the relatively few mathematics courses required by most universities' elementary education programs, with almost none of them requiring higher level mathematics courses like Calculus, Linear Algebra, or Modern Algebra, which require students to think about mathematics abstractly and conceptually. Additionally, middle school and high school teachers teach only one subject and are required to specialized in that subject. Thus, middle and high school mathematics teachers have chosen to both study higher mathematics and to teach it; whereas, elementary teachers are generalists and are not required to have deep knowledge in any one subject. This may explain why middle school and high school teachers had statistically the same scores on the mathematical disposition (MD) scale.

Additional tests involving MD and other independent factors showed a number of interesting results. For instance, mathematics PSTs scored a shocking 9 points higher on the MD scale than generalist or science PSTs; moreover, the average generalist PST scored not even half a point difference than the average science PST. Clearly, the subject a PST desires to teach influences his/her mathematical disposition. The apparent explanation for why mathematics PSTs scored highest on the MD scale is because if one has the desire to study and eventually teach mathematics, it is highly likely that they have a positive or productive attitude towards the subject. On the other hand, it was surprising that generalists scored, on average, almost exactly as the science PSTs did. Since mathematics is incorporated into many branches of science, it seemed more likely that science PSTs would score similar to the mathematics PSTs than generalist PSTs. However, since we had a small number of science PSTs compared to generalist or mathematics PSTs, future studies with a larger science PST sample size would need to examine if these results repeat.

Another interesting result was that male PSTs had a statistically significantly higher mathematical disposition than the female PSTs. In effort to provide an explanation or this, we look to various research studies that have investigated gender differences in mathematics. In 1990, a meta-analysis consisting of 100 published studies relating to mathematics performance and gender differences of school aged and college students discovered that females scored the same and sometimes higher in mathematics than males in the elementary and middle school grades, even in problem solving, but scored lower than males in high school and college, especially in problem solving tasks (Hyde, Fennema, & Lamon, 1990). Another study of 100 adults, males and females, discovered that "gender was found to moderate the relation between anxiety and math performance, and this moderating effect differed depending on the type of math performance" (Miller & Bichsel, 2004, p. 604). For example, males had higher math anxiety in basic math performance skills than females did, yet females had higher math anxiety than males in applied math performance. Thus, perhaps the female PSTs have a mixture of math anxiety and low confidence in math that may have caused their scores to be lower than their male counterparts. Another explanation could be that female PSTs were overwhelming generalists, and generalists, overall, scored low on the MD scale.

Age was also found to influence a PSTs mathematical disposition in a surprising way. PSTs in the 22-25 yrs. and 26-30 yrs. age ranged scored 5-6 points higher than those in the 18-21 and 31+ age range. It was curious to see how the youngest and oldest PSTs scored similarly and the PSTs in the "middle" age ranges scored similar. This seems to imply that perhaps there is a "peak" of mathematical disposition, the middle age ranges. Perhaps this is the time when the PST is most confident in his/her math abilities and positive towards teaching a challenging subject like mathematics. Whereas the young PSTs might be less confident or persevering in math (due to their lack of knowledge or experience in learning and succeeding in math) and the older PSTs might be in a similar boat as they are likely returning to college after many years and lack the confidence in their ability to succeed in math, and might be less appreciate of the usefulness and challenges of mathematics.

While the influence of age seemed to pattern an inverted parabola (low - high - low) in regard to MD, the classification level of the PST in relation to MD followed a chronological, linear pattern. As PSTs progressed through the classification levels (freshman, sophomore, junior, and senior) their MD increased, with seniors scoring statistically significantly higher than all three other classification categories. A plausible explanation for this is the fact that seniors have the most content and pedagogical knowledge, are likely in an education certification program, and feel more positively towards math because they have experienced it more, persevered through their math classes, and as a result better understand what it means to the learn math and grasp its nature and usefulness.

Lastly, it was unexpected to find that PSTs who stated that their previous mathematics teachers had a positive influence on them scored significantly different than those who had stated that their previous mathematics teachers had no influence (neutral) on them, but no different than those who stated their previous mathematics teachers influenced them negatively. I am unsure of why this occurred as, intuitively, it seems like those who claimed "positive influence" would differ from those who stated "negative influence". It would be interesting to discover if this outcome repeats in future studies with different student populations.

The results of the second research question showed that elementary preservice teachers scored significantly lower (roughly 5 points lower) than their high school preservice teacher counterparts on the SEFTM scale. However, and this contrasts from the case of mathematical disposition, the elementary PSTs scored statistically the same as the middle school PSTs. Furthermore, there was no statically significant difference between the mean SEFTM scores of middle school PSTs and high school PSTs, though the p-value was 0.08 in the post hoc test that compared these two categories, which is fairly close to the required level of significance ($p < 0.05$) that would make the two categories statistically different.

One possible explanation for why elementary and middle school PSTs had, on average, lower self-efficacy for teaching mathematics than high school PSTs is provided by the literature review and theoretical framework of this study, which revealed that self-efficacy is connected to curriculum choice and a teacher's desire to engage in challenging teaching practices (Chester & Beaudin, 1996). Thus, the literature confirms our study's findings that suggest that self-efficacy is mediated by grade-level (type of teacher) and by subject. It would therefore seem natural to conclude that elementary and middle school PSTs, overall, may be exactly those teachers that are more prone to be limited by their own self-efficacy in their curriculum choice and in their personal aspirations to engage in difficult teaching choices. Another difference between high school PSTs and elementary and middle school PSTs is that high school PSTs are required to take a number of advanced courses in abstract and conceptual mathematics, so perhaps they have more confidence in their ability to teach the subject because of their advanced content knowledge.

The results of tests investigating SEFTM and subject found that PSTs of mathematics scored higher than generalist PSTs (same as results involving MD and subject) but not statistically significantly higher than science PSTs (though it was very close to being significant as $p = 0.068$). This was an interesting and unexpected result. Perhaps since there were only 19 science PSTs responses collected, we have an incomplete picture of how these teachers scored in SEFTM compared to math PSTs. Perhaps with a larger sample science PSTs we would find that they did score significantly different than the math PSTs.

In comparison to MD and age, SEFTM and age results showed that PSTs from the 22-25 age range scored the highest, on average, in the SEFTM scale and that they scored statistically significantly difference from PSTs in the 18-21 age range. It is curious why PSTs in the 22-25 age range scored highest in both MD and SEFTM, and why the youngest and oldest age categories again scored almost the same in the SEFTM and MD scales. One possible explanation for this is that young PSTs have not yet acquired a lot of college-level content knowledge and as such perceive their own ability to teach the subject as insufficient; while older PSTs likely have been out of school for numerous years and as such also have a weak content knowledge and lack the confidence in the own teaching ability and ability to influence student academic success in the subject.

In contrast, PSTs classified as seniors, on average, scored highest out of all classification categories in the SEFTM scale, though senior PST's SEFTM scores were only found to be statistically significantly greater than sophomore PST's scores. It seems reasonable that seniors would have the most self-efficacy for teaching because they have the most content knowledge and have the advantage of being enrolled in an educational certification program which teaches pedagogy, instructional strategies, and requires students to conduct live observations of classrooms and student teaching. Why senior PSTs were found to be statistically different from only sophomore PSTs in regard to the SEFTM scale is unknown.

Furthermore, we again found that while PSTs who claimed to have been positively influenced by previous math teachers scored higher than the other PSTs on the SEFTM scale, they only scored statistically different (higher) than those who said their former math teachers had no influence on them. Again, we are baffled by this result and curious to discover why this would be the case. Perhaps, further studies and analysis will provide the answer.

The results from research questions 1 and 2 showed many patterns repeating with the same independent variables and the two dependent variables, MD and SEFTM. This provides even more cause to wonder about the relationship or association between these two variables, which leads us to the results of the third research question. These results showed that there were statistically significant differences between elementary, middle school, and high school teachers when considered jointly on the MD and SEFTM scale. Moreover, it was discovered that PSTs scored in a statistically equivalent manner on the MD scale as they did on the SEFTM scale and that there is a positive association between the two variables. This finding has great importance because it tells us that there is a direct association between the two dependent variables; if one goes up the other goes up, if one goes down, the other goes down.

The implications for these findings are many. As the literature review showed, high self-efficacy teachers engage in challenging teaching practices, have higher expectations of their students, and place more responsibility of student learning on themselves. As a result, students are more likely to succeed in their mathematics classes if their teachers have high self-efficacy for teaching the subject. Hence, if university education programs and professional development workshops focus on increasing or enhancing teachers' (preservice and in-service) mathematical disposition than this would directly increase their self-efficacy for teaching, which would in turn make their students more successful in mathematics.

Furthermore, we were able to develop a linear regression model to predict a PSTs self-efficacy if we know their mathematical disposition from the data collected in this study. This suggests that if we were to measure a teacher's mathematical disposition, we would be able to predict how positive (or negative) their self-efficacy for teaching is. If future studies show that this model holds, it would imply that we may only have to measure a PST's mathematical disposition to predict their self-efficacy for teaching mathematics. In other words, we may only have to use the mathematical disposition scale of the survey to answer research questions relating to self-efficacy.

6.2 Study Limitations

As with all research studies, this study too had limitations. Though our sample size ($N = 236$) is not considered small, it was not large enough to make strong conclusions regarding PSTs across the country. The benefit of having a larger sample size is that it would enable us to generalize our conclusions. A larger sample size could have been possible if the principal researcher had more time to gather data and administer surveys. Since the data collection had to be completed in just one semester in order for the thesis to be completed in time, we did not have sufficient time to acquire a larger sample size. Additionally, we would have liked to have more male participants in the study compared to female. This way we would be more certain that our gender-related conclusions are statistically sound and not a product of chance. Also, since we had very few science PSTs, we were not able to make any strong conclusions regarding this group in relation to the other groups (math and generalist) and the dependent variables, MD and SEFTM.

Another limitation of this study (one that is common to most studies) is the fact that participation in the study was voluntary. This could imply that there were some participants whose answers were biased or who mechanically answered the survey, without giving each answer sufficient thought and consideration. We tried to account for this by incorporating both positively and negatively worded questions in the survey, to avoid agreement bias, and by making the survey relatively short, to account for participants' who may have been in a hurry to finish the survey.

Lastly, a limitation of the study was that it was conducted in only one university with a population that is overwhelmingly Hispanic. Ideally, we would like to repeat this study in universes around Texas and the country to check if the results and conclusions of this particular study are echoed by the results from repeated studies at various universities with more diverse student populations.

6.3 Future Research Possibilities

We see this study as the first step to understanding the relationship between mathematical disposition, a quality stressed by the National Council of Teachers of Mathematics as one of the five strands of mathematical proficiency (NCTM, 2001), and self-efficacy for teaching mathematics, which has been linked to student success (Ross & Bruce, 2007). We are especially interested in how distinct types of teacher categories (elementary, middle school, and high school) have lower or higher self-efficacy and more positive or negative mathematical disposition. In the future, ideally, we would like to repeat this study with PSTs at a variety of universities around Texas and the country and compare the results to see if they match the findings of this initial study.

Additionally, we are very interested in discovering whether the results of this study based on preservice teachers would be comparable to those of a study involving in-service, practicing teachers instead. For example, would elementary in-service teachers have a significantly lower (or less positive/productive) mathematical disposition than in-service middle and high school teachers? Moreover, does an in-service teacher's years of teaching play a role in influencing his/her mathematical disposition or self-efficacy for teaching mathematics?

Furthermore, future studies could examine whether preservice or in-service teachers of the same category (i.e. preservice and in-service elementary teachers) score the same on the mathematical disposition (MD) scale, and similarly on the self-efficacy for teaching mathematics (SEFTM) scale. If additional studies find that relationship between MD and SEFTM actually exists, and is not a phenomenon of this study, and that by increasing a teacher's MD, we can also increase his/hers SEFTM, it would behoove mathematics educators and professors to conduct research studies to investigate how we can increase a PST's MD in a university education program, so that we can in turn increase their SEFTM so as to make them more effective and successful once they become in-service teachers.

Finally, future studies that investigate this association between MD and SEFTM, have important applications for professional development workshops and seminars. Organizers of these events involving professional teachers could bring this association (between MD and SEFTM) to the teachers' attention and work with them to increase their mathematical disposition in effort to become more effective teachers of high-quality mathematics instruction.

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APPENDIX A

APPENDIX A

SURVEY INSTRUMENTATION

Consent Form

Study Title: Investigating the Mathematical Dispositions and Self-Efficacy for Teaching Mathematics of Pre-service Teachers

This research is being conducted by Jasmine Cruz from the University of Texas Rio Grande Valley. The research study aims to examine the mathematical disposition and self-efficacy for teaching mathematics of pre-service teachers. The survey should take about 15 minutes to complete.

If you would prefer not to participate in this study, simply return the blank survey. Your responses are anonymous; you should not include any identifying information on this survey. We ask that you try to answer all questions. However, if there are any questions that you would prefer to skip, simply leave the answer blank. You must be at least 18 years old to participate. *If you are not 18 or older, please inform the researcher and do not complete the survey.*

Researcher contact information:

Name: Jasmine Cruz
Title: Graduate Student
Dept: Mathematics
The University of Texas Rio Grande Valley
Phone: (956) 665-3362
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This research has been reviewed by the Institutional Review Board for the Protection of Human Subjects (IRB). If you have any questions about your rights as a participant, or if you feel that your rights have been violated, please contact the IRB at (956) 665-2889 or irb@utrgv.edu.

Please keep this sheet for your reference.

Mathematical Disposition & Self-Efficacy for Teaching Mathematics Scales

- <u>Nature and Usefulness of Mathematics</u>

Teachers' Beliefs About the Nature of Mathematics by Anne Raymond (1997)

– Mathematics is an unrelated collection of facts, rules, and skills.
– Mathematics is fixed, predicable, absolute, certain, and applicable.
+ Mathematics is dynamic, problem driven, and continually expanding.
+ Mathematics can be surprising, relative, doubtful, and aesthetic.

Modified Fennema-Sherman Attitude Scales by Doepken, Lawsky, & Padwa (2004)

+ I study mathematics because I know how useful it is.
+ Mathematics is a worthwhile, necessary subject.
+ I will use mathematics in many ways as an adult.
– Doing well in mathematics is not important for my future.
– Taking mathematics is a waste of time.
– I don't expect to use much mathematics when I graduate.

- <u>Learning of Mathematics</u>

Teachers' Beliefs About the Learning of Mathematics by Anne Raymond (1997)

– As a student of mathematics, I passively receive knowledge from the teacher.
– There is only one way to learn mathematics.
– Memorization and mastery of algorithms signify learning.
– Many students are just not able to learn mathematics.
– My learning of mathematics depends solely on the teacher.
+ As a student of mathematics, my role is that of an autonomous explorer.
+ Mathematics can be learned without textbook or paper-and-pencil activities.
+ All students can learn mathematics.
+ As a mathematics student, I am an active learner.
+ There are many ways to learn mathematics.

- <u>Perseverance in Mathematics</u>

Indiana Mathematics Belief Scales by Kloosterman & Stage (1992)

+ By trying hard, I can become smarter in mathematics.
+ Ability in mathematics increases when one studies hard.
+ I feel I can do mathematics problems that take a long time.
+ Hard work can increase my ability to do mathematics.
+ I can get smarter in mathematics if I try hard.
– If I can't do a mathematics problem in a few minutes, I probably can't do it at all.
– If I can't solve a mathematics problem quickly, I quit trying.
– I'm not very good at solving mathematics problems that take a while to figure out.
– Ability in mathematics is not influenced by hard work.
– I can't get smarter in mathematics, even if I try.

- Personal Mathematics Teaching Efficacy

Personal Mathematics Teaching Efficacy Subscale by Enochs, Smith, & Huinker (2000)

+ I will continually find better ways to teach mathematics.
+ I know how to teach mathematics concepts effectively.
+ I understand mathematics concepts well enough to be effective in teaching elementary mathematics.
+ I will typically be able to answer students' questions.
+ When teaching mathematics, I will usually welcome student questions.

– Even if I try very hard, I will not teach mathematics as well as I will most subjects.
– I will not be very effective monitoring mathematics activities.
– I do not know what to do to turn students on to mathematics.
– I will find it difficult to use manipulatives to explain to students why mathematics works.
– I wonder if I have the necessary skills to teach mathematics.

- Mathematics Teaching Outcome Expectancy

Mathematics Teaching Outcome Expectancy Subscale by Enochs, Smith, & Huinker (2000)

+ When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.
+ If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.
+ The inadequacy of a student's mathematics background can be overcome by good teaching
+ When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.
+ Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.

– The teacher is generally not responsible for the achievement of students in mathematics.

Measure of Self-Efficacy Beliefs by Chester & Beaudin (1996).

– A teacher can have only limited influence on a student's motivation.
– There is a limited amount that teachers can do to raise the performance of students who begin school with low abilities.
– Teachers are not a very powerful influence on student achievement when all factors are considered.
– If parents would do more with their children, I could do more.

Mathematical Disposition & Self-Efficacy for Teaching Mathematics Survey

This survey is voluntary and your responses are anonymous. We ask that you try to answer all questions. However, if there are any questions that you would prefer to skip, simply leave the answer blank. By participating in this survey, you are giving your implied consent to be a participant in this study.

Instructions: Read each item carefully and circle the response (Strongly Disagree, Disagree, Uncertain, Agree, Strongly Agree) which most closely describes your feeling toward each statement.

Item #	Section I	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
1	Mathematics is an unrelated collection of facts, rules, & skills.	1	2	3	4	5
2	Mathematics is dynamic, problem motivated, & continually expanding.	1	2	3	4	5
3	Mathematics can be surprising, relative, doubtful, and aesthetic.	1	2	3	4	5
4	Doing well in mathematics is not important for my future.	1	2	3	4	5
5	Mathematics is a worthwhile, necessary subject.	1	2	3	4	5
6	Mathematics is fixed, predicable, absolute, and applicable.	1	2	3	4	5
7	I will use mathematics in many ways as an adult.	1	2	3	4	5
8	Taking mathematics is a waste of time.	1	2	3	4	5
9	I study mathematics because I know how useful it is.	1	2	3	4	5
10	I don't expect to use much mathematics when I graduate.	1	2	3	4	5
Item #	Section II	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
11	Memorizing and mastering algorithms is how people learn math.	1	2	3	4	5
12	As a student of math, my role is that of an independent explorer.	1	2	3	4	5
13	Mathematics can be learned without textbook or paper-and-pencil activities.	1	2	3	4	5
14	There is only one way to learn mathematics.	1	2	3	4	5
15	All students can learn mathematics.	1	2	3	4	5
16	As a mathematics student, I am an active learner.	1	2	3	4	5
17	Many students are just not able to learn mathematics.	1	2	3	4	5
18	My learning of math depends solely on the teacher.	1	2	3	4	5
19	There are many ways to learn mathematics.	1	2	3	4	5
20	As a student of math, I passively receive knowledge from the teacher.	1	2	3	4	5
Item #	Section III	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
21	By trying hard, I can become smarter in mathematics.	1	2	3	4	5
22	If I can't solve a math problem quickly, I quit trying.	1	2	3	4	5
23	Ability in mathematics increases when one studies hard.	1	2	3	4	5
24	I can get better in mathematics if I make the effort.	1	2	3	4	5
25	If I can't do a mathematics problem in a few minutes, I probably can't do it at all.	1	2	3	4	5
26	I feel I can do mathematics problems that take a long time.	1	2	3	4	5
27	Hard work can increase my ability to do mathematics.	1	2	3	4	5
28	I can't get smarter in mathematics, even if I try.	1	2	3	4	5
29	I'm not very good at solving mathematics problems that take a while to figure out.	1	2	3	4	5

30	Ability in mathematics is not influenced by hard work.	1	2	3	4	5
Item #	Section IV	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
31	I will continually find better ways to teach mathematics.	1	2	3	4	5
32	I do not know what to do to turn students on to math.	1	2	3	4	5
33	I will find it difficult to use manipulatives to explain to students why mathematics works.	1	2	3	4	5
34	I know how to teach mathematics concepts effectively.	1	2	3	4	5
35	I wonder if I have the necessary skills to teach mathematics	1	2	3	4	5
36	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	1	2	3	4	5
37	I will typically be able to answer students' questions.	1	2	3	4	5
38	Even if I try very hard, I will not teach mathematics as well as I will most subjects.	1	2	3	4	5
39	When teaching math, I will usually welcome student questions.	1	2	3	4	5
40	I will not be very effective at monitoring math activities.	1	2	3	4	5
Item #	Section V	Strongly Disagree	Disagree	Uncertain	Agree	Strongly Agree
41	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	1	2	3	4	5
42	If parents would do more to support their children in school, I could do more to help them as their teacher.	1	2	3	4	5
43	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	1	2	3	4	5
44	The teacher is generally not responsible for the achievement of students in mathematics.	1	2	3	4	5
45	The inadequacy of a student's mathematics background can be overcome by good teaching.	1	2	3	4	5
46	When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.	1	2	3	4	5
47	A teacher can have only limited influence on a student's motivation.	1	2	3	4	5
48	There is a limited amount that teachers can do to raise the performance of students who begin school with low abilities.	1	2	3	4	5
49	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	1	2	3	4	5
50	Teachers are not a very powerful influence on student achievement when all factors are considered.	1	2	3	4	5

Tell us a little about yourself.

Gender: M F Age: 18-21 22-25 26-30 31+ Estimated math GPA (4-pt scale): ________

Are you a: freshman sophomore junior senior

What kind of teacher are you studying to become (circle one): elementary middle school high school none

What subject will you teach (circle one): Mathematics Science All Subjects (Generalist) Other

Most of my math teachers have had a: 1) positive 2) negative 3) neutral influence on me.

Lastly, out of curiosity, do you play a musical instrument? Yes or No

APPENDIX B

APPENDIX B

DESCRIPTIVE STATISTICS AND FACTOR ANALYSIS RESULTS

Table 14:
Summary of Female Preservice Teacher Participants (N=187)

	Females			
	Elementary	Middle School	High School	Total
Subject				
Mathematics	4	27	17	48
Generalist	127	0	0	127
Science	0	7	5	12
Total	131	34	22	187
Classification				
Freshman	5	0	0	5
Sophomore	35	5	4	44
Junior	61	12	9	82
Senior	24	17	9	50
Unknown	6	0	0	6
Total	131	34	22	187
Age				
18-21	108	16	14	138
22-25	13	8	8	29
26-30	5	5	0	10
31+	5	5	0	10
Total	131	34	22	187

Table 15:
Summary of Male Preservice Teacher Participants (N=42)

	Males			
	Elementary	Middle School	High School	Total
Subject				
Mathematics	1	24	6	31
Generalist	4	0	0	4
Science	0	1	6	7
Total	5	25	12	42
Classification				
Freshman	0	0	0	0
Sophomore	2	4	0	6
Junior	2	12	7	21
Senior	1	8	5	14
Unknown	0	1	0	1
Total	5	25	12	42
Age				
18-21	3	12	7	22
22-25	2	11	3	16
26-30	0	2	0	2
31+	0	0	1	1
Unknown	0	0	1	1
Total	5	25	12	42

Table 16:
List of Variables

Variables	Explanation	Values/key
Dependent Variables (primary components)		
Mathematical Disposition (MD) Score	The numerical score of mathematical disposition	30 - 150
Self-Efficacy for Teaching Mathematics (SEFTM) Score	The numerical score of self-efficacy for teaching mathematics	20 - 100
Dependent Variables (subcomponents)		
Nature/Usefulness of Mathematics (MD)	The numerical score of nature/usefulness component	10 - 50
Learning of Mathematics (MD)	The numerical score of learning of mathematics component	10 - 50
Perseverance in Mathematics (MD)	The numerical score of perseverance component	10 - 50
Personal Teaching Efficacy (SEFTM)	The numerical score of personal teaching efficacy component	10 - 50
General Teaching Efficacy (SEFTM)	The numerical score of general teaching efficacy component	10 - 50
Independent Variables		
Teacher	The type of teacher the participant is studying to become	0 = Elementary 1 = Middle School 2 = High School
Subject	The subject the participant desires to teach	0 = Mathematics 1 = Generalist 2 = Science
Gender	Whether the participant is male or female	0 = Male 1 = Female
Classification	The participant's school status	0 = Freshman 1 = Sophomore 2 = Junior 3 = Senior
Influence	The influence of previous mathematics teachers	0 = Positive 1 = Negative 2 = Neutral
Musical	The ability to play a musical instrument	0 = Yes 1 = No

Table 17:
Principal Component Analysis: Total Variance Explained

	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
Component	Total	% of Var.	Cum. %	Total	% of Var.	Cum. %	Total	% of Var.	Cum. %
1	9.598	19.197	19.197	9.598	19.197	19.197	5.189	10.379	10.379
2	3.604	7.207	26.404	3.604	7.207	26.404	3.861	7.721	18.100
3	2.366	4.733	31.137	2.366	4.733	31.137	3.366	6.733	24.832
4	2.177	4.354	35.491	2.177	4.354	35.491	2.094	4.188	29.021
5	1.723	3.446	38.937	1.723	3.446	38.937	1.777	3.553	32.574
6	1.527	3.054	41.991	1.527	3.054	41.991	1.753	3.506	36.080
7	1.459	2.918	44.909	1.459	2.918	44.909	1.733	3.466	39.546
8	1.420	2.839	47.748	1.420	2.839	47.748	1.733	3.466	43.012
9	1.362	2.723	50.472	1.362	2.723	50.472	1.673	3.345	46.358
10	1.281	2.561	53.033	1.281	2.561	53.033	1.653	3.305	49.663
11	1.250	2.500	55.534	1.250	2.500	55.534	1.630	3.260	52.923
12	1.176	2.352	57.885	1.176	2.352	57.885	1.456	2.912	55.835
13	1.071	2.142	60.027	1.071	2.142	60.027	1.442	2.885	58.720
14	1.049	2.097	62.124	1.049	2.097	62.124	1.377	2.753	61.474
15	1.021	2.042	64.167	1.021	2.042	64.167	1.347	2.693	64.167
16	.945	1.890	66.056						
17	.912	1.823	67.880						
18	.866	1.732	69.612						
19	.842	1.684	71.296						
20	.810	1.620	72.916						
21	.785	1.571	74.487						
22	.761	1.523	76.010						
23	.719	1.439	77.448						
24	.697	1.394	78.843						
25	.670	1.340	80.183						
26	.656	1.311	81.494						
27	.642	1.283	82.777						
28	.576	1.152	83.929						
29	.566	1.133	85.062						
30	.546	1.091	86.153						
31	.516	1.033	87.186						
32	.509	1.018	88.204						
33	.479	.958	89.162						
34	.467	.934	90.095						
35	.432	.863	90.958						
36	.428	.855	91.814						
37	.404	.808	92.622						
38	.377	.753	93.375						
39	.364	.728	94.103						
40	.351	.701	94.805						
41	.330	.660	95.465						
42	.319	.639	96.103						
43	.300	.600	96.703						
44	.279	.558	97.261						
45	.273	.545	97.807						
46	.252	.505	98.311						
47	.244	.488	98.799						
48	.221	.442	99.241						
49	.209	.418	99.659						
50	.170	.341	100.000						

Note: Var. = Variance & Cum. = Cumulative

Table 18:
Rotated Component Matrix: Factor Loadings with Cross-Loadings (Run 1)

	Component														
Items	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Q10	.743														
Q9	.741														
Q8	.721														
Q7	.720														
Q5	.657														
Q16	.563														
Q31	.496														
Q4	.488														
Q26	.468	.333													
Q33		.789													
Q32		.761													
Q35		.520													
Q29	.456	.509													
Q34	.428	.490													
Q22		.478													
Q37		.470				.365									
Q40		.447	.366												
Q38	.354	.435				.348									
Q27			.719												
Q24			.716												
Q23			.701												
Q21			.600												
Q30			.534		.364										
Q28		.393	.411												
Q45				.666											
Q44				.638											
Q50				.602											
Q39	.351			.369											
Q18					.750										
Q43					-.624										
Q17		.384			.386	-.338									
Q11						-.663									
Q36						.518									
Q42							.810								
Q41							-.542								
Q25							.339								
Q14								.737							
Q19								.679							
Q47									.820						
Q48									.771						
Q3										.835					
Q2										.682					
Q46											.668				
Q49											.474				
Q15	.331										-.435				
Q13												.825			
Q20													-.657		
Q12											.336		.653		
Q6														.781	
Q1															.816

Extraction Criteria: Eigenvalue greater than 1 with Varimax Rotation

Table 19:
Rotated Component Matrix: Factor Loading with Cross-Loadings (Run 2)

	Component				
	1	2	3	4	5
Q37	.646				
Q33	.632				
Q38	.593				
Q29	.584	.396			
Q32	.582				
Q34	.577	.440			
Q35	.568				
Q40	.558		.434		
Q22	.446				
Q25	.441		.424		
Q36	.435			.363	
Q39	.364		.363		
Q9		.735			
Q10		.681			
Q8		.675	.332		
Q7		.651			
Q5		.634			
Q16	.346	.522			
Q31		.506			
Q4		.453			
Q26	.399	.422			
Q15		.402			
Q20		-.344			
Q24			.664		
Q27			.606		
Q23			.582		
Q30			.563		
Q21			.558		
Q19			.523		
Q14			.499		
Q28	.422		.425		
Q2			.357		
Q1					
Q45				.572	
Q49				.561	
Q46				.518	
Q44				.495	
Q41				.494	
Q43				.473	
Q17				-.382	
Q18				-.372	
Q11				-.367	
Q42					
Q47					.600
Q48					.596
Q50			.365		.425
Q12					-.360
Q13					-.351
Q6					
Q3					

Extraction Criteria: Extraction of 5 Components with Varimax Rotation

APPENDIX C

APPENDIX C

RESEARCH QUESTION 1 TESTS

Table 20:
Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
MD_Score	.056	236	.073	.972	236	.00012
SEFTM_Score	.047	236	.200*	.995	236	.672

*.This is a lower bound of the true significance.
a. Lilliefors Significance Correction

Table 21:
Mathematical Disposition (MD) Mean Score by the Seven Factors

Factor		N	Mean MD Score	SD
Teacher	Elementary	139	115.0460	1.34882
	Middle School	62	121.7289	9.80486
	High School	35	123.2744	9.78293
Subject	Mathematics	84	123.8241	8.84693
	Generalist	133	114.8376	11.42651
	Science	19	114.6613	10.05158
Gender	Male	42	121.8787	9.14171
	Female	187	117.1791	11.58692
Classification	Freshman	5	107.2076	3.95947
	Sophomore	50	114.9012	9.84490
	Junior	108	117.2739	10.77385
	Senior	66	122.1440	12.41394
Age	18-21	162	116.6140	10.88472
	22-25	48	122.5721	12.76579
	26-30	13	121.4811	10.32551
	30+	12	118.0561	5.47930
Influence	Positive	157	119.8996	9.92040
	Negative	10	115.5424	12.16738
	Neutral	69	114.1091	13.05200
Musical	Yes	89	119.1989	11.27045
	No	147	117.3094	11.27632

Note: N = number of participants & SD = standard deviation

Table 22:
One-way ANOVA of MD and the Seven Independent Factors

Factors		Sum of Squares	df	Mean Square	F	Sig.
Teacher	Between Groups	3048.545	2	1524.272	13.207	**.000**
	Within Groups	26892.062	233	115.417		
	Total	29940.606	235			
Subject	Between Groups	4391.133	2	2195.567	20.023	**.000**
	Within Groups	25549.473	233	109.654		
	Total	29940.606	235			
Gender	Between Groups	757.467	1	757.467	6.055	**.015**
	Within Groups	28398.167	227	125.102		
	Total	29155.633	228			
Age	Between Groups	1532.126	3	510.709	4.162	**.007**
	Within Groups	28343.854	231	122.701		
	Total	29875.980	234			
Classification	Between Groups	2251.994	3	750.665	6.198	**.000**
	WithinGroups	27248.893	225	121.106		
	Total	29500.887	228			
Influence	Between Groups	1671.449	2	835.724	6.888	**.001**
	Within Groups	28269.158	233	121.327		
	Total	29940.606	235			
Musical Ability	Between Groups	197.908	1	197.908	1.557	.213
	Within Groups	29742.698	234	127.106		
	Total	29940.606	235			

Table 23:
Levene's Test of Equality of Variances with Mathematical Disposition

Independent Variable	F	df1	df2	Sig.
Teacher	.621	2	233	.538
Subject	1.262	2	233	.285
Gender	1.704	1	227	.193
Age	1.236	3	231	.297
Classification	1.374	3	225	.252
Influence	1.881	2	233	.155

* Tests the null hypothesis that the variance of the dependent variable (MD) is equal across groups.

APPENDIX D

APPENDIX D

RESEARCH QUESTION 2 TESTS

Table 24:
Levene's Test of Equality of Variances with SEFTM

Independent Variable	F	df1	df2	Sig.
Teacher	1.970	2	233	.142
Subject	2.096	2	233	.125
Gender	10.244	1	227	**.002**
Age	1.075	3	231	.360
Classification	.286	3	225	.836
Influence	.283	2	233	.754

* Tests the null hypothesis that the variance of the dependent variable (SEFTM) is equal across groups.

Table 25:
SEFTM Mean Score by the Seven Factors

Factor		N	Mean SEFTM Score	SD
Teacher	Elementary	139	71.6660	7.84397
	Middle School	62	73.0435	7.22415
	High School	35	76.6000	6.37181
Subject	Mathematics	84	75.2097	7.01329
	Generalist	133	71.4931	7.85145
	Science	19	70.7938	6.37499
Gender	Male	42	75.0146	5.79559
	Female	187	72.0388	7.86803
Classification	Freshman	5	70.0809	9.41596
	Sophomore	50	69.8983	7.16301
	Junior	108	72.8983	7.28981
	Senior	66	74.6212	8.17080
Age	18-21	162	71.5542	7.47303
	22-25	48	77.1543	7.03722
	26-30	13	72.4679	7.95470
	30+	12	71.3333	6.38654
Influence	Positive	157	73.8681	7.56356
	Negative	10	71.0000	8.57645
	Neutral	69	70.4924	7.25305
Musical	Yes	89	72.9878	7.91900
	No	147	72.6215	7.64704

Note: N = number of participants & SD = standard deviation

Table 26:
One-way ANOVA of SEFTM and the Seven Independent Factors

Factors		Sum of Squares	df	Mean Square	F	Sig.
Teacher	Between Groups	687.433	2	343.716	6.135	**.003**
	Within Groups	13054.732	233	56.029		
	Total	13742.165	235			
Subject	Between Groups	791.008	2	395.504	7.115	**.001**
	Within Groups	12951.157	233	55.584		
	Total	13742.165	235			
Gender	Between Groups	303.707	1	303.707	5.348	**.022**
	Within Groups	12891.643	227	56.791		
	Total	13195.350	228			
Age	Between Groups	1187.800	3	395.933	7.301	**.000**
	Within Groups	12526.786	231	54.229		
	Total	13714.586	234			
Classification	Between Groups	678.911	3	226.304	3.949	**.009**
	WithinGroups	12894.423	225	57.309		
	Total	13573.334	228			
Influence	Between Groups	578.556	2	289.278	5.120	**.007**
	Within Groups	13163.609	233	56.496		
	Total	13742.165	235			
Musical Ability	Between Groups	7.440	1	7.440	.127	.722
	Within Groups	13734.724	234	58.695		
	Total	13742.165	235			

APPENDIX E

APPENDIX E

RESEARCH QUESTION 3 TESTS

Table 27:
Tests of Homogeneity

Box's Test of Equality of Covariance Matrices[a]				
Box's M	F	df1	df2	Sig.
4.020	.658	6	100927.117	.684
Levene's Test of Equality of Error Variances[b]				
	F	df1	df2	Sig.
MD_Score	.621	2	233	.538
SEFTM_Score	1.970	2	233	.142

a. Tests the null hypothesis that the observed covariance matrices of the dependent variables are equal across groups

b. Tests the null hypothesis that the error variance of the dependent variable is equal across groups.

Table 28:
MANOVA Results: Multivariate Tests

Multivariate Tests[a]									
Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[d]
Intercept	Pillai's Trace	.991	12588.439[b]	2.000	232.000	.000	.991	25176.877	1.000
	Wilks' Lambda	.009	12588.439[b]	2.000	232.000	.000	.991	25176.877	1.000
	Hotelling's Trace	108.521	12588.439[b]	2.000	232.000	.000	.991	25176.877	1.000
	Roy's Largest Root	108.521	12588.439[b]	2.000	232.000	.000	.991	25176.877	1.000
Teacher	Pillai's Trace	.123	7.613	4.000	466.000	.000	.061	30.452	.997
	Wilks' Lambda	.879	7.697[b]	4.000	464.000	.000	.062	30.787	.997
	Hotelling's Trace	.135	7.780	4.000	462.000	.000	.063	31.119	.998
	Roy's Largest Root	.114	13.243[c]	2.000	233.000	.000	.102	26.487	.997

a. Design: Intercept + Teacher
b. Exact statistic
c. The statistic is an upper bound on F that yields a lower bound on the significance level.
d. Computed using alpha = .05

Table 29:
MANOVA Results: Tests of Between-Subjects Effects

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared	Noncent. Parameter	Observed Power[c]
Corrected Model	MD_Score	3048.545[a]	2	1524.272	13.207	.000	.102	26.413	.997
	SEFTM_Score	687.433[b]	2	343.716	6.135	.003	.050	12.269	.886
Intercept	MD_Score	2498048.090	1	2498048.090	21643.755	.000	.989	21643.755	1.000
	SEFTM_Score	943793.469	1	943793.469	16844.764	.000	.986	16844.764	1.000
Teacher	MD_Score	3048.545	2	1524.272	13.207	.000	.102	26.413	.997
	SEFTM_Score	687.433	2	343.716	6.135	.003	.050	12.269	.886
Error	MD_Score	26892.062	233	115.417					
	SEFTM_Score	13054.732	233	56.029					
Total	MD_Score	3317229.380	236						
	SEFTM_Score	1263117.328	236						
Corrected Total	MD_Score	29940.606	235						
	SEFTM_Score	13742.165	235						

a. R Squared = .102 (Adjusted R Squared = .094)
b. R Squared = .050 (Adjusted R Squared = .042)
c. Computed using alpha = .05

Table 30:
Linear Regression Outputs

Model Summary[a]					
Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Durbin-Watson
1	.555[b]	.308	.305	6.37312	1.883

ANOVA[a]						
Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	4237.855	1	4237.855	104.338	.000[b]
	Residual	9504.309	234	40.617		
	Total	13742.165	235			

a. Dependent Variable: SEFTM_Score, b. Predictors: (Constant), MD_Score

Table 31:
Linear Regression Coefficients ^{b} Test

Model		Unstandardized Coefficients B	Std. Error	Standardized Coefficients Beta	t	Sig.
1	(Constant)	58.382	5.871		9.945	.000
	SEFTM_Score	.820	.080	.555	10.215	.000

b. Dependent Variable: MD_Score

Table 32:
Multiple Linear Regression Model for Dependent Variable MD

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	122.896	1.136		108.221	**.000**	120.658	125.134
	Subject	-6.648	1.191	-.353	-5.583	**.000**	-8.995	-4.301
2	(Constant)	115.709	2.199		52.620	**.000**	111.375	120.042
	Subject	-6.228	1.162	-.331	-5.362	**.000**	-8.517	-3.939
	Classification	3.408	.902	.233	3.778	**.000**	1.630	5.186
3	(Constant)	116.949	2.180		53.644	**.000**	112.652	121.246
	Subject	-5.793	1.142	-.308	-5.071	**.000**	-8.044	-3.541
	Classification	3.448	.882	.236	3.911	**.000**	1.710	5.185
	Influence	-2.520	.749	-.203	-3.364	**.001**	-3.996	-1.044
4	(Constant)	115.885	2.172		53.355	**.000**	111.604	120.166
	Subject	-4.720	1.179	-.251	-4.002	**.000**	-7.044	-2.395
	Classification	2.788	.894	.191	3.117	**.002**	1.025	4.550
	Influence	-2.585	.736	-.208	-3.511	**.001**	-4.037	-1.134
	Teacher	2.936	.991	.190	2.963	**.003**	.983	4.889

Table 33:
Multiple Linear Regression Model for Dependent Variable SEFTM

Model		Unstandardized Coefficients		Standardized Coefficients			95.0% Confidence Interval for B	
		B	Std. Error	Beta	t	Sig.	Lower Bound	Upper Bound
1	(Constant)	71.284	.636		112.043	**.000**	70.031	72.538
	Teacher	2.186	.686	.210	3.186	**.002**	.834	3.539
2	(Constant)	72.382	.718		100.826	**.000**	70.967	73.797
	Teacher	2.175	.673	.209	3.232	**.001**	.849	3.502
	Influence	-1.675	.541	-.201	-3.095	**.002**	-2.742	-.609
3	(Constant)	69.463	1.409		49.284	**.000**	66.686	72.241
	Teacher	1.735	.691	.167	2.512	**.013**	.374	3.097
	Influence	-1.681	.536	-.201	-3.138	**.002**	-2.736	-.625
	Classification	1.570	.655	.159	2.398	**.017**	.279	2.861

APPENDIX F

APPENDIX F

GRAPHS

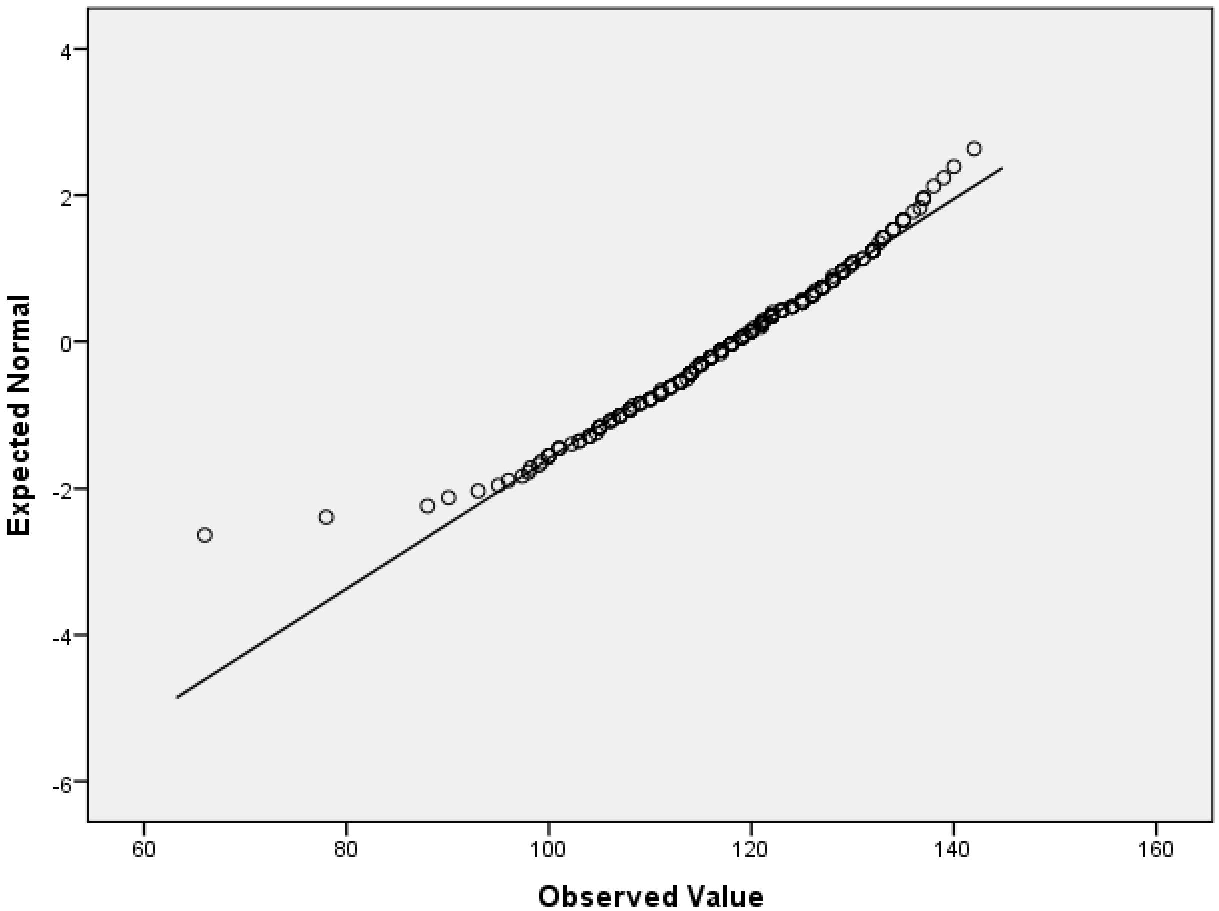


Figure 6: The Normal Q-Q Plots with Mathematical Disposition (MD)

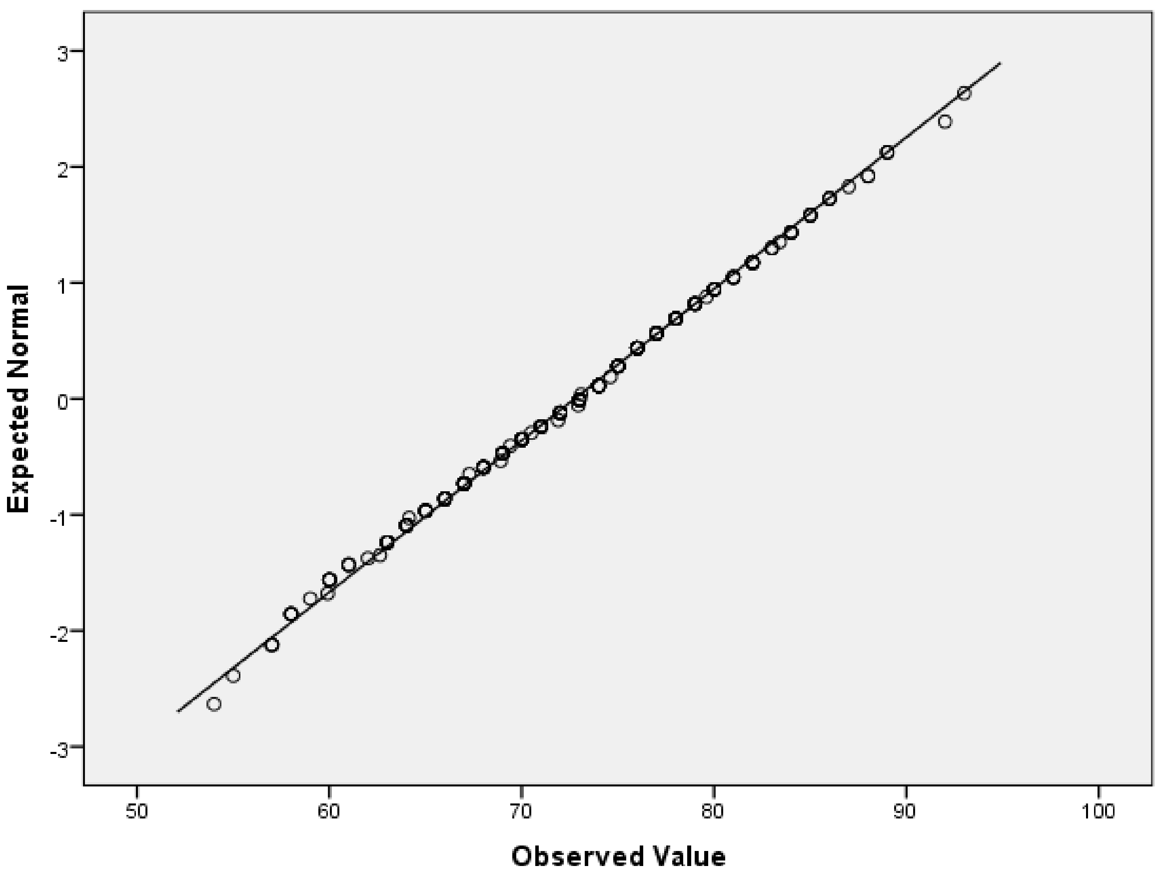
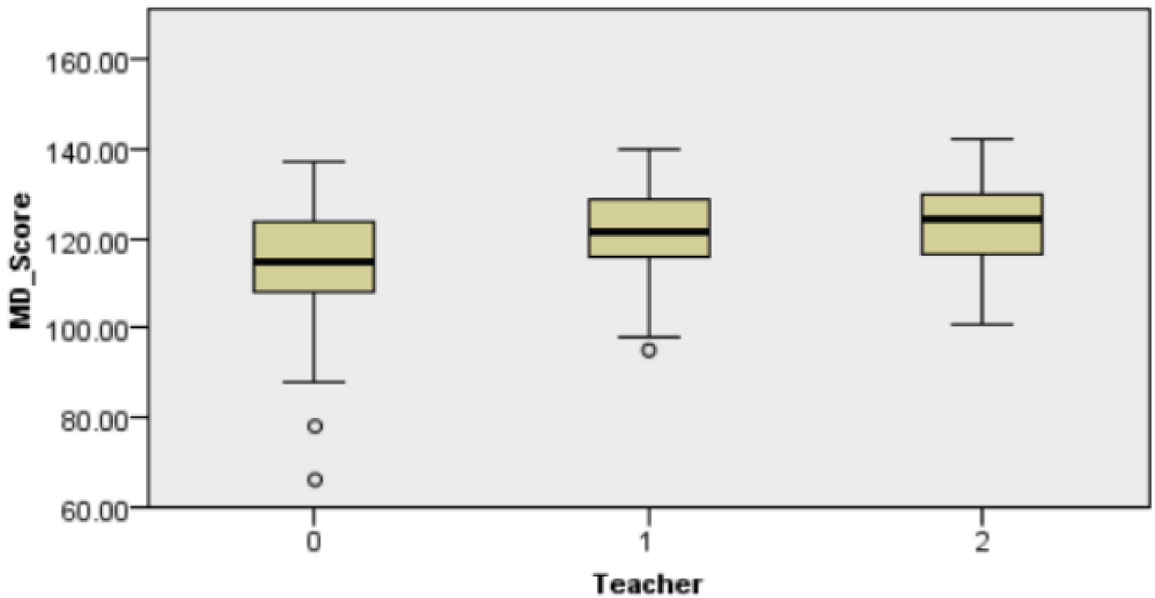


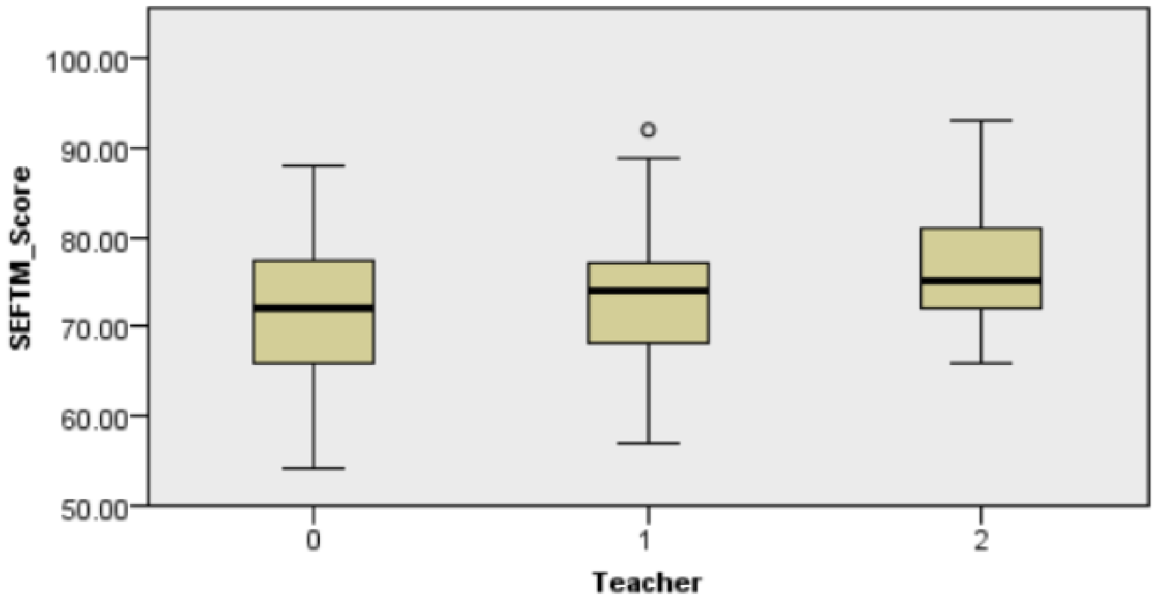
Figure 7: The Normal Q-Q Plots with Self-Efficacy for Teaching Math (SEFTM)



Total N	236
Test Statistic	24.523
Degrees of Freedom	2
Asymptotic Sig. (2-sided test)	.000

1. The test statistic is adjusted for ties.

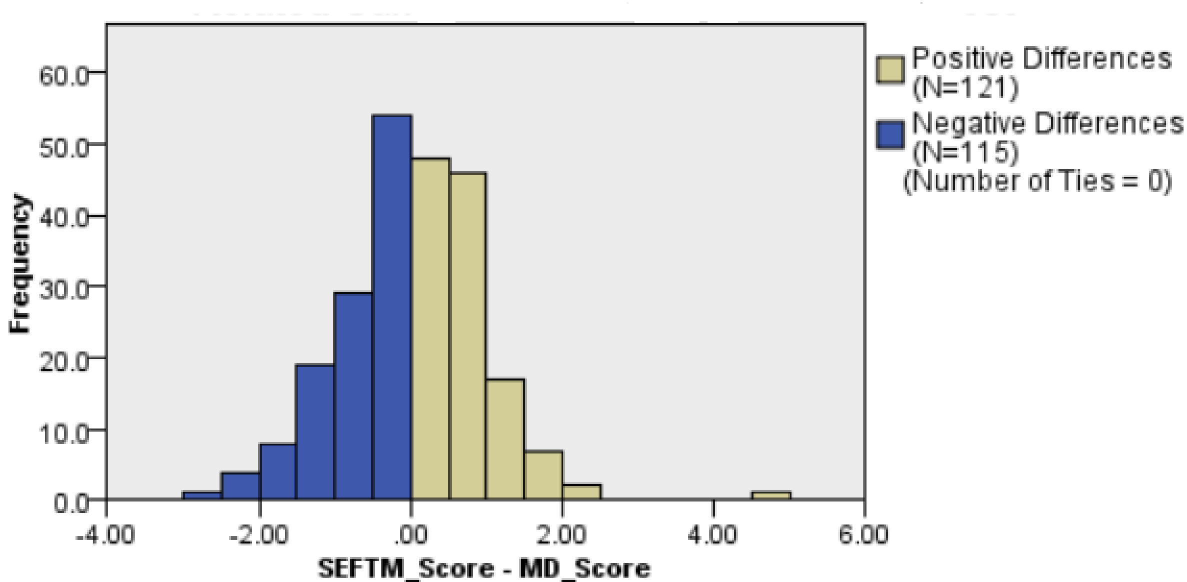
Figure 8: Independent-Samples Krustal-Wallis Test for MD



Total N	236
Test Statistic	10.356
Degrees of Freedom	2
Asymptotic Sig. (2-sided test)	.006

1. The test statistic is adjusted for ties.

Figure 9: Independent-Samples Krustal-Wallis Test for SEFTM



Total N	236
Test Statistic	14,385.000
Standard Error	1,049.916
Standardized Test Statistic	.383
Asymptotic Sig. (2-sided test)	.702

Figure 10: Related-Samples Wilcoxon Signed Rank Test

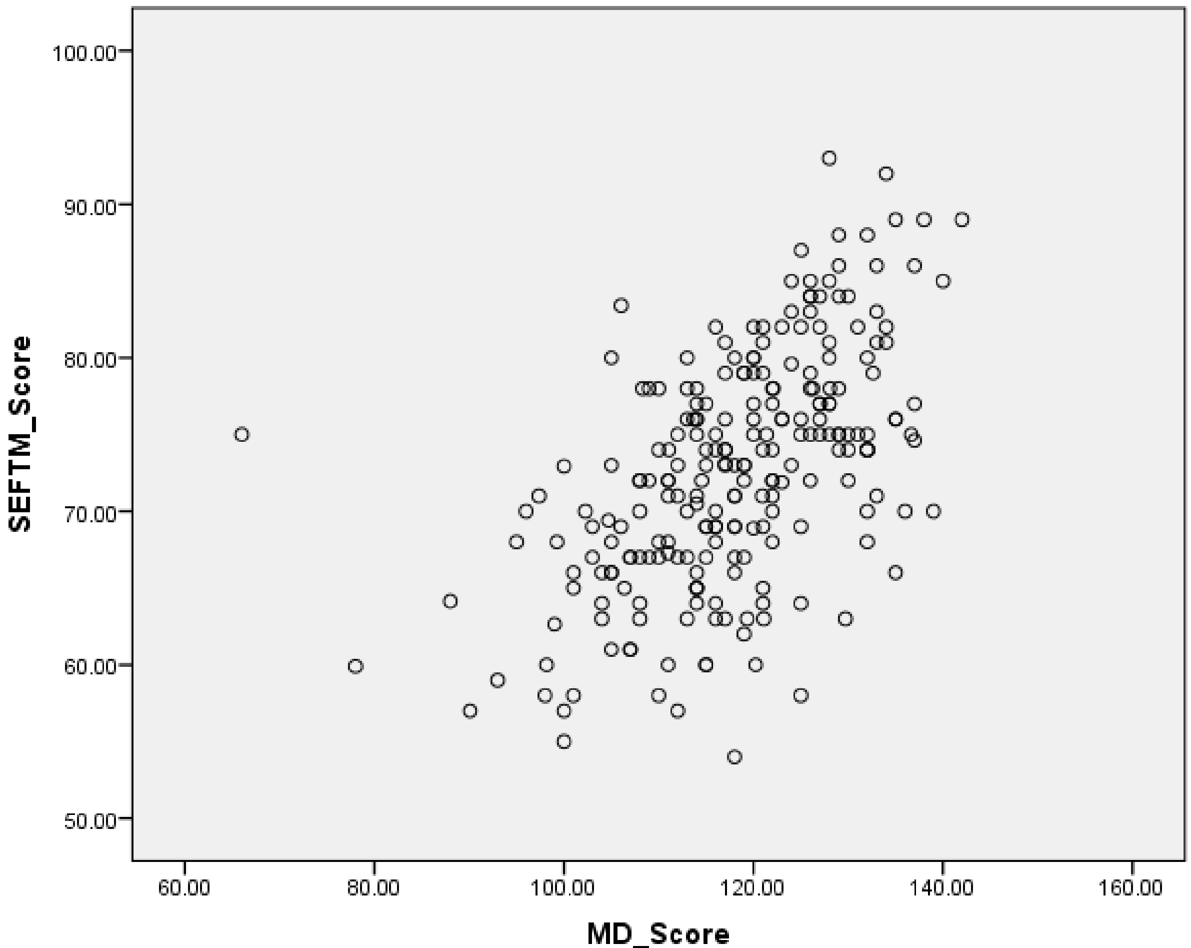


Figure 11: Scatter Plot to check for Linearity

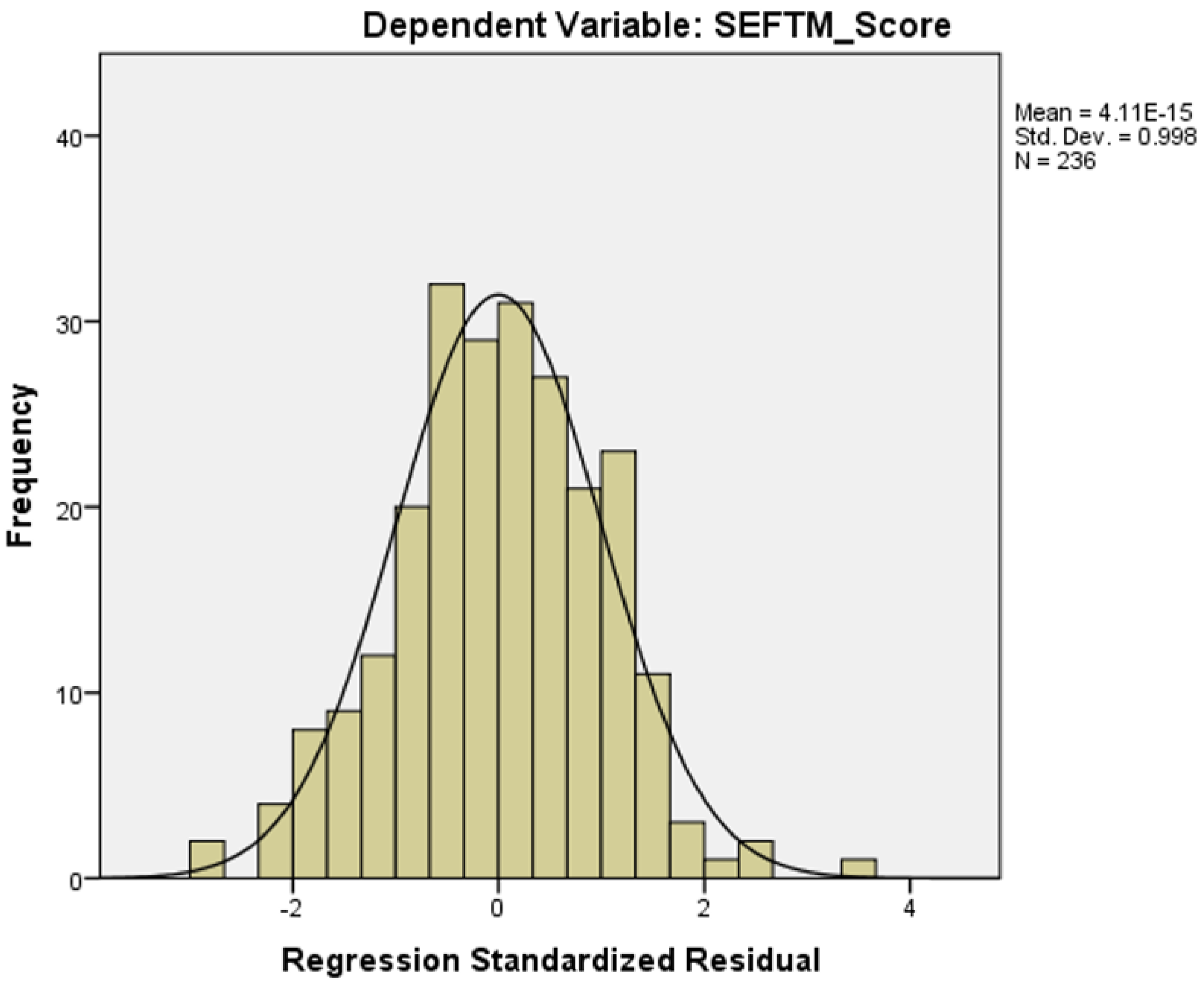


Figure 12: Histogram of Residuals

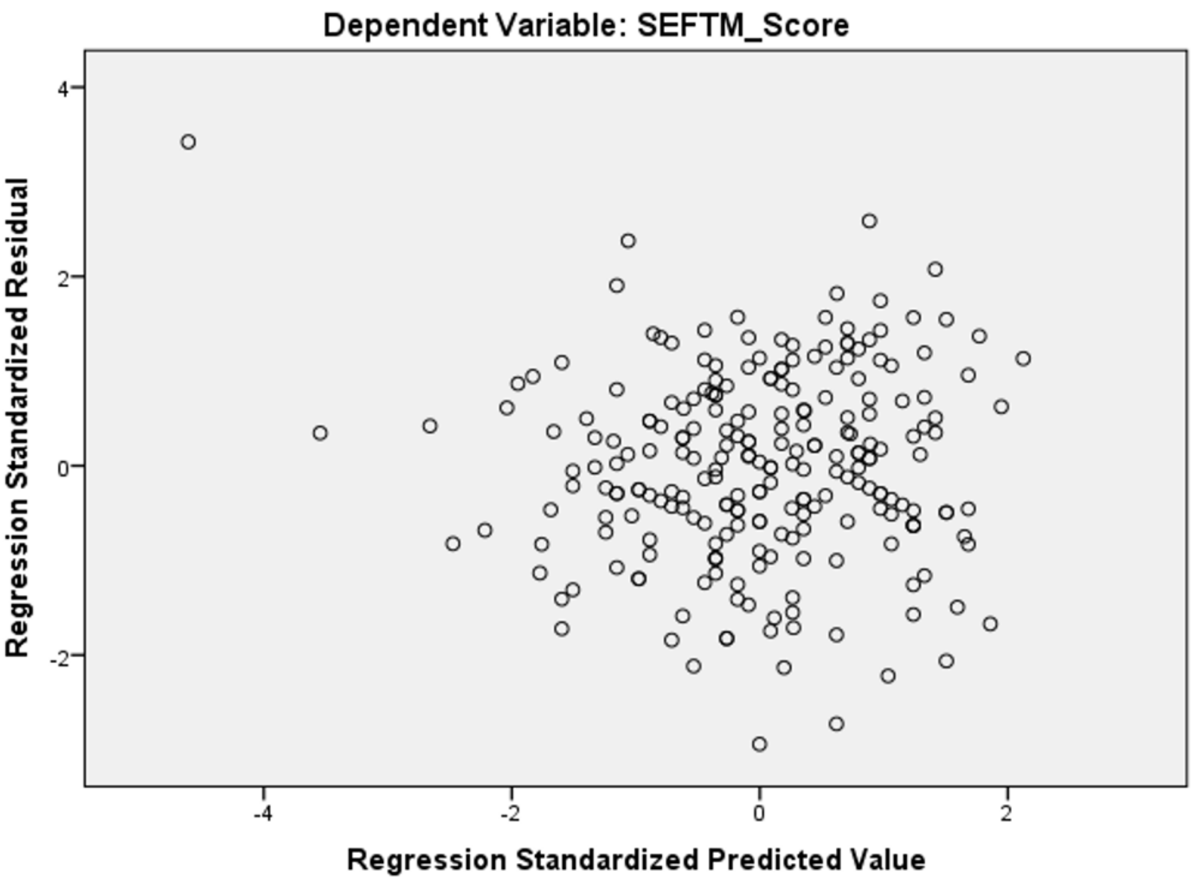


Figure 13: Scatter Plot from Linear Regression

BIOGRAPHICAL SKETCH

Jasmine M Cruz was born in Bangalore, India in 1989 and spent her grade school years in several countries with her American parents, who were missionaries and social workers. Ms. Cruz completed a homeschool correspondence course to receive her high school diploma in 2008. After that, she spent roughly three years in Mexico doing volunteer work before returning to the US and completing an Associate of Arts degree in elementary teaching at South Texas College in 2012, graduating with honors as a member of Phi Theta Kappa. In May 2015, Ms. Cruz completed a Bachelor of Science in Mathematics specializing in Interdisciplinary studies with Magna Cum Laude honors at the University of Texas - Pan American.

Ms. Cruz began the Master of Science in Mathematics program at the School of Mathematical and Statistical Sciences at the University of Texas Rio Grande Valley in August 2015. During her graduate education, she had the opportunity to work as a graduate teaching assistant for the Mathematics department and became actively involved with the Society of Industrial and Applied Mathematics (SIAM) chapter at UTRGV, first as a member, and later as the chapter's Treasurer. The SIAM chapter organized food drives, conducted Calculus I and II review sessions for students of Calculus, and planned Pi Day and mathematics competition events to promote mathematics and the STEM fields.

Jasmine Cruz received a Master of Science in Mathematics, specializing in Mathematics Education from the University of Texas Rio Grande Valley in May 2017. She plans to teach high school students and inspire within them a love for learning and mathematics. She is also considering pursuing a PhD program in Mathematics Education in the near future. Her email address is jasmine_cruz06@hotmail.com and current address is 1121 W. Iris Ave. Apt B, McAllen, TX 78501.