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The bid generation problem in combinatorial auctions for transportation service procurement

Fang Yang[a,b], Sheng-Zhu Li[b], Yao-Huei Huang[c*]

[a]*School of Economics and Management, Chongqing University of Posts and Telecommunications, Chongqing, 400065, China*
[b]*Chongqing University of Posts and Telecommunications E-commerce and Modern Logistics, Municipal Key Laboratory Postdoctoral Research Workstation, Chongqing, 400065, China*
[c]*Department of Information Management, Fu Jen Catholic University, New Taipei City, Taiwan*

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A B S T R A C T

In this work, a probabilistic bid generation problem with the pricing of a bundle of lanes and carrier's vehicle routing is considered as it is an importation in transportation service procurement. Depending on the network of the vehicle, there exist multiple lanes for traveling between two locations. To solve the bid generation problem efficiently, a two-phase method approach is presented. At the core of the procedure a feasible vehicle routing problem on a multidigraph is solved by an exhaustive search algorithm to enumerate all routes concerning routing constraints and treat each route as a decision variable in the set partitioning formulation. We examine our model both analytically and empirically using a simulation-based analysis.

1. Introduction

The internet market developed rapidly at the end of the 20th century. The electronic transportation market has emerged and offers a relatively cheap way to apply auction mechanisms designed to match the buyers' demand with the sellers' transportation services, with the benefit of reducing information collection, participation and transaction costs, while increasing geographical and temporal convenience (Goldsby & Eckert, 2003; Lucking, 1999). Caplice (2007) and Sheffi (2004) provided a good overview of shippers using the electronic market to purchase truckload (TL) transportation services from carriers who typically handle freight that is picked up from an origin and shipped directly to a destination. Further, Caplice (2007) proposed that for the procurement of transportation services, the predominant way is to use a single-round sealed bid combinatorial auction. Since 1997, hundreds of companies have used combinatorial auctions to procure transportation services, including Sears Logistics Services, K-Mart Corp., Wal-Mart Stores, Inc., International Paper, The Home Depot Inc., Ford Motor Company, Compaq Computer Corp., Staples, Inc., Limited Brands, Inc. and many others (Caplice & Sheffi, 2003; Ledyard et al., 2002).

An important benefit of a combinatorial auction is that it allows carriers to express their true preferences and can submit a bundled bid on a set of lanes to produce more efficient movements. This is the case in TL shipping, because after the carrier arrives at the destination of one load it must drive to the origin of the subsequent load which is called an empty move because the carrier is not paid by any shipper for driving this distance. Large TL carriers have an average empty mile of between 6% and 12%, while private fleets have an average of more than 24% (Caplice, 2007). The cost of servicing a lane is strongly

* Corresponding author Tel: +886 917-623052
E-mail: yaohuei.huang@gmail.com (Y.-H. Huang)
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influenced by a follow-on service lane out of that destination which is called as economies of scope (Sheffi, 2004). Therefore, the carrier may offer a lower price for transporting a load from B to A if it already transports a load from A to B. Economies of scope explain why combinatorial auctions were applied in the TL transportation services procurement.

There are two major challenges for carriers to generate bundles of lanes to submit as bids in combinatorial auctions. The first is the expression of bundles. To adequately express economies of scope between all auction lanes, carriers must construct and submit bids for an exponential number of subsets of these lanes ($2^n - 1$ for n lanes). This is intractable because it has been proposed by Lee et al. (2007) and Song & Regan (2005) that the bid generation problem (BGP) is an NP-hard problem. The second is the price of bundles. Although we already know that the cost of servicing a lane is affected by other lanes in the carrier's transportation network because of economies of scope (also called as synergy), some studies handled bundle price as the sum of the ask prices of the auction lanes (Lee et al., 2007; Song & Regan, 2005). An et al. (2005) and Wang & Xia (2005) proposed methods for computing synergies that were not implemented in their models. To simplify the calculation, Triki et al. (2014) considered that there was only one load in each lane, simplified the synergy between the lanes into the synergy between the loads, and calculated the synergy between the loads based on the method proposed in An et al. (2005).

To generate the most profitable bundle, carriers must first generate possible transportation routes, then calculate the actual price for each auctioned lane to form a bundle price. Lee et al. (2007) and Song & Regan (2005) proposed that carriers already have booked lanes before the auction. The addition of auctioned lanes is usually integrated into the carrier's current transportation network. As a result, carriers must find an effective way to integrate these auction lanes with the booked lanes, and to reflect the economies of scope from the transportation routes. This is normally modeled as a vehicle routing problem (Lee et al., 2007; Song & Regan, 2005; Wang & Xia, 2005) which is generally treated via the representation of the transportation network as a directed complete graph. Each arc of the graph represents the lane for an origin-destination connection with different attributes, such as booked lane, auctioned lane or empty lane without shipment as illustrated in Fig. 1. Hence, for the same transportation route of a vehicle, the lanes passed by may have the same origin-destination but different attributes. Due to synergies, even the same auctioned lane will have different prices on different transportation routes. Garaix et al. (2010) first pointed out that when several attributes are defined on arcs, there may be several arcs between each pair of nodes, so one cannot transform a vehicle routing problem on a road network into a standard VRP. To explore this issue, they introduced a multidigraph representation and developed two specialized solution approaches. In this paper, we consider these different types of lanes through a multidigraph representation of the transportation network.

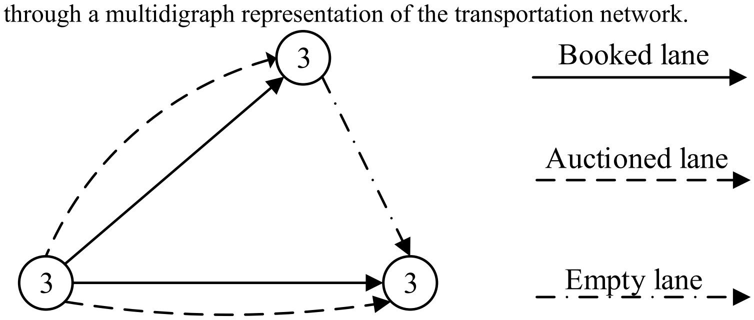

Fig. 1. Illustrative transportation network

In summary, our work features the following contributions to the BGP literature: (i) In most relevant BGP studies, unit demand in each lane has generally been assumed to reduce the complexity of the problem (Chang, 2009; Guo et al., 2006; Song & Regan, 2003, 2005; Triki et al., 2014; Wang & Xia, 2005). Moreover, numerous researchers proposed that there is multi-unit demand in each lane which is estimated based on demand history data (Caplice, 2007; Caplice & Sheffi, 2003). Therefore, this study considers multi-unit demand in each lane which is different from relevant BGP studies and increases the complexity of the problem. (ii) This study considers the bundle price as a decision variable, the carrier first selects possible transportation routes, then with the consideration of each auctioned lane suffering different synergies in different transportation routes to calculate the bidding price for each auctioned lane to form a bundle price, this is the first time to obtain the auctioned lane with its price in the submitted bundle. In most relevant BGP studies, they considered bundle price as a parameter equal to the sum of the ask prices of the bundle's loads (Chang, 2009; Lee et al., 2007) or Triki et al. (2014) based on the assumption unit load in each lane to determine the whole bundle price. (iii) This study first considers the carrier's transportation network as a multidigraph network instead of considering it by a simple graph as in (Lee et al., 2007; Song & Regan, 2005; Wang & Xia, 2005).

The rest of this study is organized as follows. Section 2 first describes the problem including assumptions and notations, then formulates the model. In Section 3 we present our solution approaches. Section 4 develops the method for solving the problem. Section 5 presents numerical examples and discusses the numerical results. Section 6 provides the conclusions and future research.

2 Literature review

2.1 Compute synergies

Carriers bidding in bundles need to take into account synergies of trucking routes and capacity, especially when the service cost of a bundle is significantly smaller than the sum of the service cost of individual lanes in that bundle, then carriers can give a more realistic and competitive bidding bundle and prices, which can improve the probability of winning the bids (Chang, 2009; Hammami et al., 2021; Triki, 2016; Triki et al., 2014).

An et al. (2005) thought that the value of a bundle increases linearly with the size of the bundle, so they proposed a calculation method to calculate the synergies between the lanes. But because the real data from combinatorial auctions are generally not made public, their model was not verified.

Wang & Xia (2005) defined first-order synergy as the complementarity between a set of auctioned lanes and a set of booked lanes, and second-order synergy as the complementarity between a pair of sets of auctioned lanes and booked lanes. Then demonstrated that the synergy of a bid bundle may depend on other bundles that will be won. But their elaborate definitions of different synergies are not implemented in their proposed model.

Triki et al. (2014) designed a probabilistic optimization model to help TL carriers in making bidding decisions in a CA for spot transportation markets. The TL carrier will apply such a model to generate the desirable bid and its associated price by solving an integrated bid generation and pricing problem. They construct the model based on the space-time extended network considering the average synergies overall pair of auctioned loads and also between the booked and auctioned loads.

Different from Triki et al. (2014) 's method of computation synergy between loads, Triki (2016) develops an optimization approach based on the use of the location technique to calculate the synergy between auctioned loads and booked loads. They show the validity of their approach by applying it to solve a real-life problem.

Hammami et al. (2021) determined synergy factors by generating pairwise synergies between each pair of auctioned lanes and adapting the formula presented in An et al. (2005), Triki et al. (2014) and Triki (2016).

2.2 Bundle price

Because of the synergy, the bidding price of the carriers' submitted bundles should be a decision variable. Lee et al. (2007) proposed that lanes in different routes can be of different value and we account for this by having the distinct routes that a lane can be involved in to have in general different bid prices reflecting the different sets of lanes in the respective routes. Each route will have a different physical cost even if routes have lanes in common with each other. Thus, we do not need to have separate lane prices corresponding to each possible route. We capture the different values of packages with lanes in common by the cost of servicing the bundles as determined by the carrier model and these costs will be influenced by the other lanes not in common.

Triki et al. (2014) proposed a probabilistic optimization model integrating both bid generation and pricing problems allowing the generation of one load's bundle. A chance constraint is defined for each possible bundle of auctioned loads that could form a bundled bid and linearized based on the probability distribution of loads' clearing prices.

Kuyzu et al. (2015) and Olcaytu & Kuyzu (2018) proposed that carriers place bids in multiple auctions simultaneously without knowing the lanes they win until the end of the auctions. This leads to uncertainty in the calculation of the lane bid price quotes at the beginning of the auctions. Kuyzu et al. (2015) proposed a stochastic bid price optimization model for addressing the uncertainty, their model is a non-concave maximization problem that requires historical data and solving an exponential number of NP-Hard optimization problems. However, Olcaytu & Kuyzu (2018) proposed that in real-life situations, several auctions may need to be priced simultaneously. So they developed an efficient synergy-based method to calculate a carrier's bid prices so that the carrier can compete with other carriers and win the tendered lanes in a profitable way, where the bid prices can be determined simultaneously with less computational effort.

Hammami et al. (2021) addressed the BGP with stochastic clearing prices taking into account uncertainty on other competing carriers' offers. lanes' selection and pricing decisions are integrated to generate multiple bundle bids. They thought their model is more complex than Triki et al. (2014) because no carrier knows in advance the prices at which lanes will be allocated when the auction is cleared.

2.3 Vehicle routing

Wang and Xia (2005) took the winning probability into account and show that the optimal solution to a vehicle routing problem may lead to inferior bid packages. Even though they demonstrate the drawback of employing vehicle routing models to generate bids, they eventually model their bid generation problem as a generic vehicle routing problem with time windows. The routing problem assumes that all auctioned lanes must be served, and the objective is to minimize the total transportation cost. As pointed out by Lee et al. (2007), minimizing the total empty repositioning cost may not generate the right set of bid packages. They develop a TL vehicle routing model to maximize the profit in order to simultaneously, instead of sequentially such as that in Song & Regan (2005), solve the bid generation and selection problems. Hammami et al. (2019) proposed an arc-based vehicle routing formulation to solve the bid generation problem for heterogeneous truckload operations. While Hammami et al. (2021) assumed that the carrier has a homogeneous fleet, and each vehicle route must start and end at the depot.

2.4 Multigraph

The main advantage of studying the transportation network as a multigraph is that such a network maintains multiple attributes on the arcs based on Garaix et al. (2010), while simple graph representation is unable to handle the problem and many available solutions may be discarded. Reinhardt et al. (2016) considered VRP with TWs in which a fixed charge should be paid for accessing a set of arcs. They analyzed the problem in both simple and multigraph networks. Ticha et al. (2017) investigated the VRP with multiple-attribute parallel arcs and incorporated the TW constraint into the problem. They implemented a branch-and-price algorithm. Tikani and Setak (2019) studied a reliable distribution problem in urban environments. The authors considered multi-attribute parallel links in transportation networks in disaster response operations. Their investigations showed that multigraph yields a faster and more reliable distribution process. Soriano et al. (2020) proposed to consider alternative paths with different distances between visit locations to explore a larger set of feasible route options in the cash-in-transit planning problem. The resulting multigraph network better captures the characteristics of urban networks. Moreover, the extra flexibility achieved with the alternative paths helps find better routing plans while meeting time constraints. Tikani et al. (2021) proposed a new cash-in-transit model involving deterministic and stochastic time-varying traffic congestion. They covered the travel time and robbery risk of previous cash-in-transit routing models in multigraph networks, where multigraph representation maintains a set of non-dominated parallel arcs.

3. Problem formulation

3.1 Problem formulation

Generally, carriers prefer to bid on the auctioned lanes that are profitable under their current transportation network. As a result, carriers must find an effective way to integrate these auctioned lanes with the booked lanes, and to reflect the economies of scope from the transportation routes which are normally modeled as a vehicle routing problem. Therefore, this section shows how to formulate a carrier's bid generation with its bidding price in combinatorial auctions with the consideration of transportation routes.

Let $G = (V, A)$ be the multidigraph induced by a carrier's transportation network.
$V = \{0,1,\cdots,n\}$ is a set of nodes where node 0 represents the depot which is served as the home garage for all trucks the carrier owns. For $(j,k) \in V \times V$, let e be the type of arcs, e.g., $e = 1$ means the booked lanes, $e = 2$ represents the auctioned lanes, $e = 3$ represents the empty lanes. An arc$(j,k)^e \in A, e = 1,2,3$. A route is a sequence of nodes starting and ending at the depot 0 that satisfies all operational constraints. A_i represents the set of arcs in route i. Next, we present the following common notation, definitions and assumptions.

Assumptions

(i) All routes start and end at depot 0.
(ii) The carrier has a fixed number of trucks, all trucks are homogeneous, and each truck has one unit capacity.
(iii) The carrier is to generate a single optimal bidding bundle of a set of auctioned lanes, with its price and the corresponding transportation routes.
(iv) The same auctioned lane in different routes can be a different value, because in a different route, there exist different lanes, so there has a different synergy to auctioned lanes.
(v) Empty lanes exist only between depot 0 and nodes.

Parameters

I: Number of routes.

e: Number of lanes' type. $e = \begin{cases} 1 & \textit{booked lanes} \\ 2 & \textit{auctioned lanes} \\ 3 & \textit{empty lanes} \end{cases}$

n: Number of cities.

T: Number of trucks.
ω: The given maximum length allowed for a route.
v: The unit distance cost for a truck.
$\beta \in (0,1)$: The probability threshold.
$f_{(j,k)^e}$: The flow of lane $(j,k)^e$.
$t^i_{(j,k)^e}$: The number of trucks needed by lane $(j,k)^e$ in route i.
t_i: The number of trucks in route i, $t_i = max_{\alpha^i_{(j,k)^e}}\{t^i_{(j,k)^e} | \forall (j,k)^e \in A_i\}$.
$Y_{(j,k)^2}$: Random variable denoting the clearing price offered by the competitors for auctioned lane $(j,k)^2$.
d_i: The distance of route i.

Variables

$x^i_{(j,k)^e}$: Binary variable, where $x^i_{(j,k)^e} = 1$ if node j to node k with the lane of type e in route i is chosen to be submitted; $x^i_{(j,k)^e} = 0$, otherwise.
y_i: Binary variable, where $y_i = 1$ if route i is chosen to be submitted; $y_i = 0$, otherwise.
$p^i_{(j,k)^2}$: Bidding price for auctioned lane $(j,k)^2$ in tour i.

Based on the above-mentioned descriptions and definitions, a BGP model is formulated to maximize the carrier's profit, which is defined as the total revenue from both booked lanes and auctioned lanes minus the total routing cost computed with (1). Constraints (2) give the probability for the carrier's bid price to win the auctioned lanes. Constraints (3) guarantee that the number of trucks needed to serve all submitted routes does not exceed the total truck capacity T of the carrier. Constraints (4) and (5) ensure that if the lane with any type in route i is selected to be served then the route must be served. Constraints (6) force that the booked lanes must be covered exactly once in the submitted bundle. Constraints (7) make sure that the auctioned lanes can be covered at most once in the submitted bundle. Constraints (8) impose that the flow of each lane in route i must be satisfied. Constraints (9) ensure that each route must satisfy the route length limit. Constraints (10)-(12) are binary variables or integral variables for the decision variables.

Proposed BGP Model

$$\text{Maximize } R + \sum_{i \in I} \sum_{(j,k)^2 \in A_i} p^i_{(j,k)^2} x^i_{(j,k)^2} - \sum_{i \in I} v d_i t_i y_i \tag{1}$$

$$\text{s.t. } P\left(p^i_{(j,k)^2} x^i_{(j,k)^2} \le Y^i_{(j,k)^2}\right) \ge 1 - \beta \quad \forall (j,k)^2 \in A_i \ and \ \forall i \in I \tag{2}$$

$$\sum_{i \in I} y_i t_i \le T \tag{3}$$

$$M y_i \ge \sum_{(j,k)^1 \in A_i} x^i_{(j,k)^1} + \sum_{(j,k)^2 \in A_i} x^i_{(j,k)^2} \quad \forall i \in I \tag{4}$$

$$y_i \le \sum_{(j,k)^1 \in A_i} x^i_{(j,k)^1} + \sum_{(j,k)^2 \in A_i} x^i_{(j,k)^2} \quad \forall i \in I \tag{5}$$

$$\sum_{i \in I} x^i_{(j,k)^1} = 1 \quad \forall (j,k)^1 \in A_i \tag{6}$$

$$\sum_{i \in I} x^i_{(j,k)^2} \le 1 \quad \forall (j,k)^2 \in A_i \tag{7}$$

$$f_{(j,k)^e} x^i_{(j,k)^e} \le t_i \quad b \quad \forall (j,k)^2 \in A_i \ and \ \forall i \in I \tag{8}$$

$$d_i y_i \le \omega \quad \forall i \in I \tag{9}$$

$$x^i_{(j,k)^e} \in \{0,1\} \quad \forall (j,k)^e \in A_i \ and \ \forall i \in I \tag{10}$$

$$y_i \in (0,1) \quad \forall i \in I \tag{11}$$

$$p^i_{(j,k)^2} \ge 0 \quad \forall (j,k)^2 \in A_i \ and \ \forall i \in I \tag{12}$$

The Proposed BGP Model results to be nonlinear because both the objective function (1) and constraints (2). In the next section, we will linearize this model step by step.

4 linearize the Proposed BGP model

4.1 Normal distribution for the probabilistic constraints

Constraints (2) are stochastic probability constraints ensure that the carrier has a chance of eventually winning the auctioned lane in route while the bidding price is when competing with other carriers. The challenge for the carrier to determine auctioned lanes' bid prices is uncertain since that is not known until the end of the auctions. To overcome the uncertainty and nonlinearity related to the Proposed BGP Model, the auction clearing price for each lane is modeled as a continuous random variable. Here we study from Kuyzu et al. (2015) and Triki et al. (2014). Kuyzu et al. (2015) considered a uniform distribution of each lane individually under a simultaneous auction. Triki et al. (2014) considered the clearing price from the perspective of unit load, but in real life, there is always no unit load in each lane, and the clearing price is always for the lane not for one load. In our paper, we assume that although the same auctioned lane will have different bidding prices in different routes in the combinatorial auction and model the clearing price for each auctioned lane as a continuous random variable according to the independent normal distribution.

Here, we denote the random variable $X_{j,k}$ representing the clearing price of each auctioned lane (j,k). We assume that each $X_{j,k}$ follows a normal distribution with mean $\mu_{j,k}$ and variance $\sigma_{j,k}^2$: $X_{j,k} = N(\mu_{j,k}, \sigma_{j,k}^2)$. Note that mean and variance are estimable from historical data.

Because we consider the transportation routes, in each route, there must exist synergies between lanes. The synergies mean that the traveling and other operating costs associated with running a set of lanes are lower than the sum of the traveling and other operating costs per lane associated with running each lane alone in TL transportation. We introduce a corrective symbol to express the synergy among the lanes in route i. Then we model the random variable $Y_{j,k}^i$ representing the clearing price of the auctioned lane (j,k) in route i as $Y_{j,k}^i = s_{j,k}^i X_{j,k}$. Where $s_{j,k}^i$ is the parameter quantifying the synergy level within route i. The way of computing $s_{j,k}^i$ will be discussed in the next subsection.

Then the constraints (2) can be expressed as follows:

$$p_{(j,k)^2}^i x_{(j,k)^2}^i \le s_{j,k}^i\left[\mu_{j,k} + \Phi^{-1}(\beta)\sigma_{j,k}\right] \quad \forall (j,k)^2 \in A_i \ \ and \ \forall i \in I \tag{13}$$

where Φ^{-1} is the inverse function of the cumulative distribution function for a standard normal distribution.

4.2 Route synergy computation

The synergies between loads are considered in few studies (An et al., 2005; Chang, 2009; Triki et al., 2014; Wang & Xia, 2005), but there is no study formulate the synergies between lanes among transportation routes related to BGP model under combinatorial auction. We develop a different approach for incorporating the synergies between lanes within transportation routes. Triki et al. (2014) proposed a generic synergy method to generate the bundle's price. Their model calculated the synergy as a formulation of the average of the synergies over all pairs of loads within submitting bundle and also those between the booked and submitting bundle's loads. In this paper, we want to calculate the bidding price of each auctioned lane selected to be submitted within transportation routes i then form the bundle and its price. The synergy $s_{j,k}^i$ is related to two factors, one is the absolute difference values between auctioned lane $(j,k)^2$'s flow $f_{(j,k)^2}$ and the max flow in route i, another is the number of empty lanes in route i. We assume that each truck has unit capacity, so the flow on each lane represents the number of trucks needs to serve the lanes. The total number to serve route i is determined by the max flow volume on the lane in the route. In route i, the less difference in flow values between the two lanes, the stronger the synergy. The conception of the hop is defined as the number of empty lanes in route i. Therefore, the fewer the number of hops in route i, the stronger the synergy. As we defined before, the empty lanes only exist between depot 0 and nodes, so in route i, at most two empty lanes.

4.3 linearize the model

We construct the BGP model as a mixed integer non-linear model, so we can't obtain the global optimal solution directly. Now, we use the linearization techniques, the nonlinear model can be approximated to a mixed integer linear program solvable to obtain a global optimum.

Proposition 1 For a set of binary variables $\left\{x_{(j,k)^2}^i \in (0,1) | (j,k)^2 \in A_i\right\}$, a set of non-negative variables $\left\{p_{(j,k)^2}^i \ge 0 | (j,k)^2 \in A_i\right\}$ and $\left\{\pi_{(j,k)^2}^i \ge 0 | (j,k)^2 \in A_i\right\}$, we have $\pi_{(j,k)^2}^i = p_{(j,k)^2}^i x_{(j,k)^2}^i$ if the following linear system is satisfied:

$$-M x_{(j,k)^2}^i \le \pi_{(j,k)^2}^i \le M x_{(j,k)^2}^i \quad \forall (j,k)^2 \in A_i \ \ and \ \ \forall i \in I \tag{14}$$

$$p_{(j,k)^2}^i + M\left(x_{(j,k)^2}^i - 1\right) \le \pi_{(j,k)^2}^i \le p_{(j,k)^2}^i + M\left(1 - x_{(j,k)^2}^i\right) \quad \forall (j,k)^2 \in A_i \ \ and \ \ \forall i \in I \tag{15}$$

Proof: After checking (14) and (15), we have the following situations:
Putting (i) and (ii) together proves that the nonlinear expression of $p_{(j,k)^2}^i x_{(j,k)^2}^i$ can be linearized as $\pi_{(j,k)^2}^i$ via the above linear system.

Following Proposition 1, the expressions (13) can be expressed as:

$$\pi_{(j,k)^2}^i \le s_{j,k}^i\left[\mu_{j,k} + \Phi^{-1}(\beta)\sigma_{j,k}\right] \ \forall (j,k)^2 \in A_i \ \ and \ \ \forall i \in I \tag{16}$$

According to the linearization techniques mentioned above, this Proposed BGP Model can be approximated to a mixed integer linear program as follows:

Reformed BGP Model:

$$\max R + \sum_{i\in I}\sum_{(j,k)^2\in A_i} \pi^i_{(j,k)^2} - \sum_{i\in I} v d_i t_i y_i$$
$$\text{subject to } -Mx^i_{(j,k)^2} \le \pi^i_{(j,k)^2} \le Mx^i_{(j,k)^2} \quad \forall (j,k)^2 \in A_i \ and \ \forall i \in I$$
$$p^i_{(j,k)^2} + M\left(x^i_{(j,k)^2} - 1\right) \le \pi^i_{(j,k)^2} \le p^i_{(j,k)^2} + M\left(1 - x^i_{(j,k)^2}\right)$$
$$\forall (j,k)^2 \in A_i \ and \ \forall i \in I$$
(3)-(12), (16)

5 Solution Methodology

We already know the BGP is an NP-hard problem, to solve the problem, we proposed a two-phase method. The first phase is enumerating all possible routes concerning routing constraints and treating each route as a decision variable y_i in the Reformed BGP model. The steps as detail:
(i) According to all lanes $(j,k)^e \in A, e = 1,2,3$ with the flow volume $f_{(j,k)^e}$, generate a multidigraph to represent the carrier's transportation network;
(ii) Generating all possible routes i as follow:
(a) A route must start and end at the depot,
(b) A route visits each node at most once,
(c) The maximum route distance d_i is restricted by parameter ω,
(d) The total demand in each route must greater than 0 (e.g. no route with only two empty lanes);
(iii) Obtain the max demand (because the vehicle's capacity is unit, so we can use t_i to represent this) and the number of empty lanes in route i to calculate the synergies $s^i_{j,k}$ of auctioned lane $(j,k)^2$ in route i.

The second phase associates a binary variable with each candidate bid auctioned lane $x^i_{(j,k)^2}$, then solves the Reformed BGP model by Gurobi to determine desirable auctioned lanes with related prices and the corresponding transportation routes.

6. Numerical Experiments

In this section, we will construct several numerical experiments to illustrate our Reformed BGP Model. Since there is always no real data available for combinatorial auctions, we only have to construct experiments by ourselves. All the experiments were conducted on a PC equipped with 2.3 GHz Intel Core i5, 8 GB RAM. The model has been implemented and compiled by using Python 3.6 integrated with Gurobi for solving all the arising MILP models.

The synergy values associated with each auctioned lane are produced basis on the difference values between the flow of the auctioned lane and the max flow in route i and the number of empty lanes in route i, as described in Section 4.2. Eleven levels have been chosen to characterize the synergy defined in Table 1, the small the values, the stronger synergy between lanes in route i.

Table 1
Synergy values

The difference values between auctioned lane's flow and the max flow volume in route i	Hop 0	Hop 1	Hop 2
0	0.50	0.70	0.85
5	0.51	0.71	0.86
10	0.52	0.72	0.87
15	0.53	0.73	0.88
20	0.54	0.74	0.89
25	0.55	0.75	0.90
30	0.56	0.76	0.91
35	0.57	0.77	0.92
40	0.58	0.78	0.93
45	0.59	0.79	0.94
50	0.60	0.80	0.95

6.1 An illustrative example

Let us consider a simple example to illustrate a BGP instance having 7 cities, 7 booked lanes, 9 auctioned lanes and 14 empty lanes. In Fig. 2, we posed the problem on a multigraph having 8 nodes: 7 nodes for the cities and the depot is located at node

0. For each lane, the demand and distance are provided in parentheses, in this order (demand, distance).

Traveling a unit distance incurs a fixed transportation cost of two to all the carriers regardless of the amount of flow volume on the truck. The maximal length of each route is limited to 140. The carrier has 250 trucks and all trucks are identical. Set the revenue from booked lanes to 100000. The clearing price matrix for each auctioned lane is given in Table 2. Set the probabilistic threshold β to 0.05.

Table 2
Clearing price

Auctioned lanes	(2,3)	(3,4)	(4,3)	(1,4)	(1,0)	(5,1)	(0,5)	(7,0)	(5,7)
Clearing price	604.8	1404	2371.2	3711.1	709.8	1755.6	2167.2	1184.4	2833.6

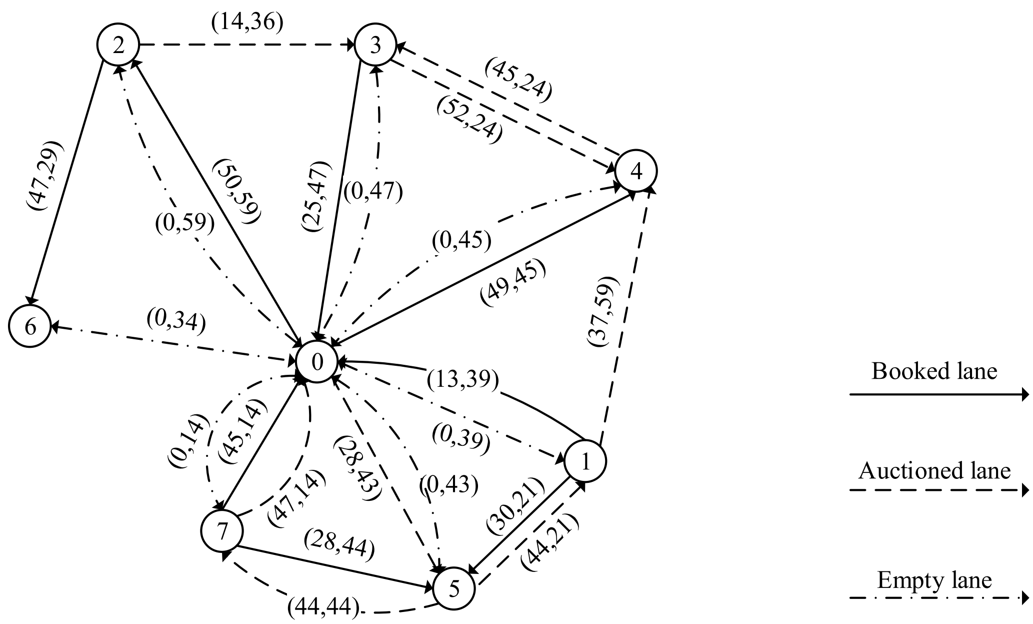

Fig. 2. Illustrative transportation network

First, enumerate all routes as described in Section 5. Hence, we obtain 28 routes, a max demand t_i and the number of empty lanes in each route i. Then we get the synergy $s^i_{j,k}$ of each auctioned lane in different route i. After treating each route i as one decision variable y_i, then we solve the Reformed BGP Model and obtain the optimal solution.

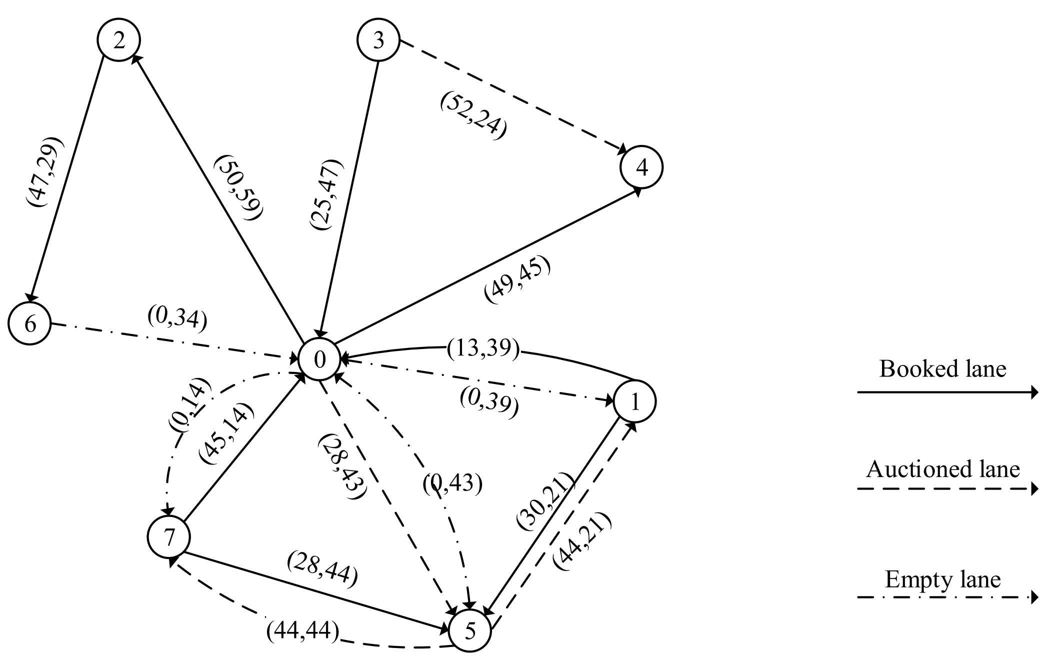

Fig. 3. The result of worked example transportation network

From the results reported in Fig.3, we notice all booked lanes are selected to be severed, while 5 auctioned lanes are selected to be severed, all the severed lanes are included in 6 routes. The bidding price for each auctioned lane is shown in Table 3 to form a bundle price of 6482.2. We can obtain the optimal objective 10988.6 within 0.44s.

Table 3
Bidding price

Selected auctioned lane	(4,3)	(1,0)	(5,1)	(0,5)	(5,7)
Bidding price	1189.7	545.7	1234.7	1522.8	1989.3

Moreover, we do another test to calculate the profit of the carrier to bid on no auctioned lanes. To satisfy all booked lanes, the carrier needs 6 routes and 249 vehicles. as shown in Fig. 4. The revenue from booked lanes is 100000, and the total cost of vehicles to serve all routes is 4330.8, so the carrier's profit is 5669.2. By comparing the results of bidding with or without auctioned lanes, it is obvious that the carrier has a higher profit to bid on auctioned lanes.

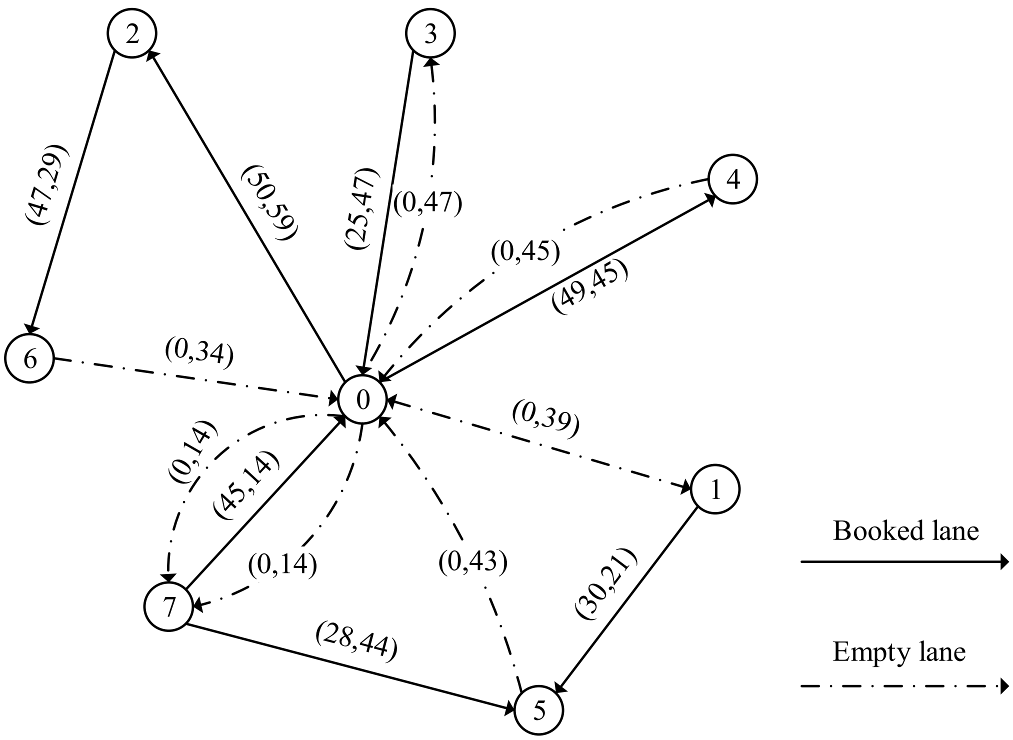

Fig. 4. The result of the carrier to bid no auctioned lanes

6.2 Configuration of test problems

We generate distance matrix, booked lanes flow, auctioned flow, clearing price matrix, the number of vehicles, and limit on the length of tours as follows:
(i) Distance matrix $l_{j,k}$ derived from the actual distance between cities in China.
(ii) The flow matrix $f_{(j,k)^e}$ are randomly generated according to a uniform distribution in a range of (0,50], especially, since each empty lane's flow is set to 0.
(iii) The clearing prices of auctioned lanes are randomly generated according to a uniform distribution in a range of [300, 3000].
(iv) The number of trucks is determined randomly and ranged between 80% and 100% of the sum of all booked lanes' flow, which is followed by Lee et al. (2007).
(v) The limit of the total route length is set as the product of truck working time and average speed in a period, but the length will increase as the number of city nodes increases (Lee et al., 2007).
(vi) For problem instances with 9, 12, 15, 18, 20, 22, 25, 28, 30 and 35 nodes without calculating the depot 0, the number of booked lanes, auctioned lanes and empty lanes are shown in Table 4.

Table 4
The number of different types of lanes

Instance	N	Number of booked lanes	Number of auctioned lanes	Number of empty lanes
1	9	12	27	18
2	12	20	47	24
3	15	22	98	30
4	18	24	109	36
5	20	26	127	40
6	22	28	136	44
7	25	30	154	50
8	28	37	160	56
9	30	41	165	60
10	35	50	221	70

6.3 Computational results

Table 5 presents the main computational results, where the first column denotes the problem scale, the second column denotes the total routes, and the third column denotes the CPU time. Our BGP model is MILP, so we obtain the global optimal solution. Take instance 5 as an example, 20 cities need the carrier to serve, the carrier has his depot 0. After solving the BGP model, there are 27 serving routes in the carrier's transportation, including the 26 booked lanes, 127 auctioned lanes and 40 empty lanes. The CPU time is 405.3s and the simplex iteration numbers are 2729. From the results reported in Table 5, we know the CPU time is increasing rapidly with the number of nodes, so we may need to develop heuristic approaches to the BGP problem to help a carrier make quick bidding strategies.

Table 5
The computational results

Instance	N	Number of served routes	CPU time(s)	Iterations
1	9	14	4.65	197
2	12	20	101.26	4010
3	15	26	488.81	8421
4	18	32	265.37	3216
5	20	27	405.30	2729
6	22	30	640.69	4356
7	25	32	2947.53	7895
8	28	35	1680.25	11900
9	30	40	870.90	6521
10	35	43	3188.76	12916

7. Conclusions and future research

This paper represents a novel model that integrates the bid package generation and pricing problems with the carrier's routing considering multidigraph transportation networks, which has never been considered before. In addition, we first formulate a mixed nonlinear integer program, then we use the linearization technique to let the model be a linear one so that we can solve this model by Gurobi to obtain a global optimal solution. The model represents a utility maximizing diction problem that carriers can use to determine the best package of lanes to bid for with corresponding routing in the combinatorial auction. The model can handle hundreds of lanes.

We have considered instances with up to 341 lanes, and the computational results reflect that our model works well in obtaining a global optimal solution. One of the challenges in solving our model is to generate all the possible routes that satisfy the conditions, which is a huge work and takes a long time; The second challenge is to solve the real-world problem. The possible future research direction is to develop heuristic algorithms to generate routes and solve the model, which can solve larger scale problems.

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