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Analyzing Related Strategic Behavior Through Strictly Alternating Interactions with Two-Memory Length

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Abstract: Reciprocal altruism can often be modeled through the iterated Prisoner's Dilemma game in which players take turns in the roles of donor and recipient. Several late studies were based on memory alteration in the repeated Prisoner's Dilemma game. This prompted us to study this alteration in a strictly alternating iterated Prisoner's Dilemma game. In our work, we represented the repeated games played by finite states of automata. Also, we supposed that there is relatedness between the players in this game. A relatedness average degree r considered between players, where $0 \leq r \leq 1$. The effect of noise on the relatedness degree among players can be examined with regard to the behavior of the strategies in their competitions.

Keywords: Alternative game, Iterated Prisoner's Dilemma Game (IPD), Payoff Matrix, Relatedness, Two Memory, Two-Player Game, Transition Matrix.

1 Introduction

Game theory is a set of ideas for analyzing conflict models with mathematical techniques. It is a formal method for analyzing the strategic interaction between rational agents acting strategically [1]. Repetitive game theory is a method more relevant to the biological and behavioral sciences. It gives the main pattern to explain how selfish players can still cooperate in a long-term relationship. Therefore, it is often cited by anthropologists, political scientists, economists [2,3] and scholars interested in human cooperation. There are multiple game models, including the simultaneous model, in which the competitors choose without each of them knowing the other's choice. As well as the alternating model, in which the players choose their decisions in turns, such as in the game of chess. There are two types of this model: strictly alternating models, which are the focus of this work, and random alternating models. In strictly alternating models, we consider two (or more) players and two options each for a game. In each round, the player who starts with his choice in the round is called the leader and the other is called the recipient. In the random model, each player has a chance to be a leader with a certain probability.

The prisoner's dilemma can be used to make a decision in a number of areas in one's personal life, such as competition between people, buying a car, negotiation skills, and so on. In game theory, the prisoner's dilemma is the famous form, but in biology and economics, there are many different recurring games such as: Hawk-Dove, location games,...etc. In these games, they explain why two sane people don't cooperate, even if it seems that way to be in their interest [4]. For example, a well-known example that is an important motif for the cooperating game is the vampire bat. In this example, they bond with each other and feed on the blood of livestock in their nocturnal habitats. Depending on the amount of blood they receive, bats may alternate between the roles of donors and recipients [5].

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The prisoner's dilemma is a complex example of mutual cooperation. Both players can be in one of two possible motions C, which stands for cooperate, and D which stands for defect [5,6]. If both players cooperate, each earns $\mathcal{R}$ (reward), while if both players defect, each earns $\mathcal{P}$ (the penalty), which is smaller than $\mathcal{R}$. If one is defective and the opponent cooperates, the cooperator will get the lowest payoff $\mathcal{S}$ (Sucker's) while the defector will receive the highest payoff $\mathcal{T}$ (the temptation to defect). Obviously, defection is the only best option for one round. The defective player will get $\mathcal{T}$ if the other player cooperates, and the cooperative player will get $\mathcal{R}$ if the other player cooperates, where $\mathcal{R}$ is less than $\mathcal{T}$. The defective player will get $\mathcal{P}$ if the opponent defects, while the cooperative player will get $\mathcal{S}$ if the other player has defects, where $\mathcal{S}$ is less than $\mathcal{P}$. There is a potential difference that occurs if the game is repeated (repeated prisoners' dilemma game, i.e. IPD). Therefore, we have four outgoing transitions [both players cooperate (C, C), first player cooperates and the other defects (C, D), first player defects and the other cooperates (D, C), and both players defect (D, D)] where the first place is the player under investigation, while the other place represents the opponents. Moreover, the payoff matrix for each round of the Prisoner's Dilemma game [5] is presented by

$$\begin{array}{c} \\ C \\ D \end{array}\begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{bmatrix} \mathcal{R} & \mathcal{S} \\ \mathcal{T} & \mathcal{P} \end{bmatrix} \end{array}, \tag{1}$$

where

$$2\mathcal{R} > \mathcal{T} + \mathcal{S} \;\; and \;\; \mathcal{T} > \mathcal{R} > \mathcal{P} > \mathcal{S}. \tag{2}$$

Therefore, there are a total of 16 different strategies made out of 24 probability transition arrows denoted by $S_0, S_1, ..., S_{15}$ which can be categorized by (y_1, y_2, y_3, y_4) of ones and zeros. Here y_i = 1 or 0; if the player plays C or D respectively. For instance, $S_9 = (1, 0, 0, 1)$, which is known as the Pavlov strategy [5], which represented as in Fig. 1

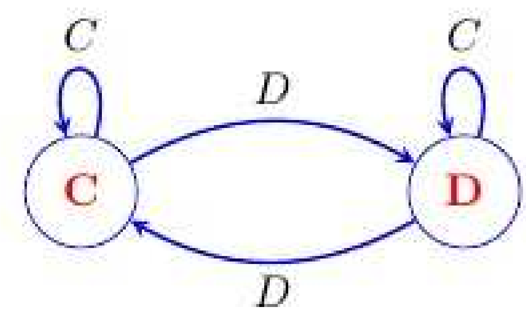


Fig. 1: Automaton of Pavlov Strategy

In Prisoner's Dilemma game, we suppose that a player with the $P = (p_{CC}, p_{CD}, p_{DC}, p_{DD})$ strategy matches the opponent using the $Q = (q_{CC}, q_{CD}, q_{DC}, q_{DD})$ strategy. The Markov chain transition matrix for simultaneous games (M_S) is given by [5,7,8]

$$M_S = \begin{array}{c} \\ \mathcal{R} \\ \mathcal{S} \\ \mathcal{T} \\ \mathcal{P} \end{array}\begin{array}{c} \begin{array}{cccc} \mathcal{R} & \mathcal{S} & \mathcal{T} & \mathcal{P} \end{array} \\ \begin{bmatrix} p_{CC}q_{CC} & p_{CC}(1-q_{CC}) & (1-p_{CC})q_{CC} & (1-p_{CC})(1-q_{CC}) \\ p_{CD}q_{DC} & p_{CD}(1-q_{DC}) & (1-p_{CD})q_{DC} & (1-p_{CD})(1-q_{DC}) \\ p_{DC}q_{CD} & p_{DC}(1-q_{CD}) & (1-p_{DC})q_{CD} & (1-p_{DC})(1-q_{CD}) \\ p_{DD}q_{DD} & p_{DD}(1-q_{DD}) & (1-p_{DD})q_{DD} & (1-p_{DD})(1-q_{DD}) \end{bmatrix} \end{array}, \tag{3}$$

The simultaneous Iterated Prisoner's Dilemma game has been examined in most previous studies for two [5,15] and three players [7,9,10], whether with memory one or two. Furthermore, the effect of the relationship between players on the behavior of the strategies was studied in all the previously mentioned cases [7,10,11,12,13]. Then some recent studies looked at games as an alternative to repeated prisoner's dilemma but with memory one [8,14]. Recently, in our recent research, we studied the alternative repeated prisoner's dilemma, but with memory two, and its effect on the competition between strategies. Now, in this paper, we will study the effect of the relationship between the players on the behavior of the strategies using r as a relationship coefficient in the alternative repeated prisoner's dilemma with memory two.

2 Alternating Prisoner's Dilemma Game

In the alternating Prisoner's Dilemma game, the leader in each round is one of the players who can control the outcome. If the leader chooses C, this means that he receives A_1, and the opponent receives B_1. D option means that the leader gets A_2, points and the opponent gets B_2.

In a single round, the leader's option D is better than C. The loss is A_2 - A_1 and the benefit is B_1 - B_2, then we have

$$0 < A_2 - A_1 < B_1 - B_2 \tag{4}$$

We consider the players take turns leading in two consecutive rounds. They get A_1 + B_1 which is denoted by $\mathcal{R}$ if both play C. Also, both of them get A_2 + B_2, which is denoted by $\mathcal{P}$ if both play D. But the cooperator gains A_1 + B_2, which is what we denote by $\mathcal{S}$, and the defector gains A_2 + B_1 which is denoted by $\mathcal{T}$ if one plays C, and the opponent D. Therefore, we obtain

$$2\mathcal{R} > \mathcal{T}+\mathcal{S} \ and \ \mathcal{T} > \mathcal{R} > \mathcal{P} > \mathcal{S} \tag{5}$$

Inequality (2) is considered the condition of the payoff for simultaneous Prisoner's Dilemma games. While, in the alternating Prisoner's Dilemma games, the condition of payoff becomes as in both (2) and (6).

$$\mathcal{T} + \mathcal{S} = \mathcal{R} + \mathcal{P} \tag{6}$$

Also, the matrix M_s, which is shown in (3) represents the Markov chain transition matrix for simultaneous games. Where, if we want to transform state $\mathcal{R}$ to $\mathcal{T}$, it means we will transform from (C,C) to (D,C), so player I will change his decision and player II will insist on his decision. Therefore, the transformation probability from state $\mathcal{R}$ to $\mathcal{T}$ is equal to $(1-p_{CC})q_{CC}$. But, the Markov chain transition matrix for alternating games is different as shown [8,14]

$$M_A = \begin{array}{c} \\ \mathcal{R} \\ \mathcal{S} \\ \mathcal{T} \\ \mathcal{P} \end{array} \begin{array}{c} \begin{array}{cccc} \mathcal{R} & \mathcal{S} & \mathcal{T} & \mathcal{P} \end{array} \\ \begin{bmatrix} p_{CC}q_{CC} & p_{CC}(1-q_{CC}) & (1-p_{CC})q_{CD} & (1-p_{CC})(1-q_{CD}) \\ p_{CD}q_{DC} & p_{CD}(1-q_{DC}) & (1-p_{CD})q_{DC} & (1-p_{CD})(1-q_{DD}) \\ p_{DC}q_{CC} & p_{DC}(1-q_{CC}) & (1-p_{DC})q_{CD} & (1-p_{DC})(1-q_{CD}) \\ p_{DD}q_{DC} & p_{DD}(1-q_{DC}) & (1-p_{DD})q_{DD} & (1-p_{DD})(1-q_{DD}) \end{bmatrix} \end{array} \tag{7}$$

Where, if we want to transform state $\mathcal{R}$ to $\mathcal{T}$, it means player I will change his decision and play D after (C,C) and player II will insist on his decision and play C after (C,D) as shown in Table 1. Therefore, the probability of the transformation from state $\mathcal{R}$ to $\mathcal{T}$ is equal to $(1-p_{CC})q_{CD}$

Table 1

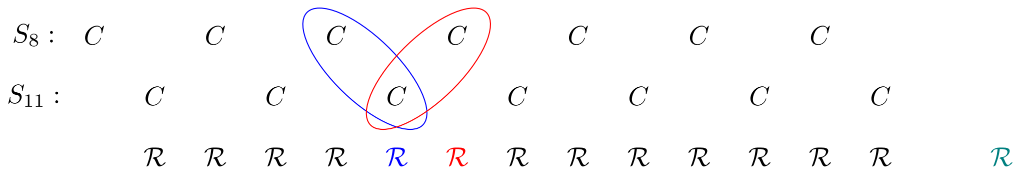
Number of round	$2n-2$	$2n-1$	$2n$	$2n+1$
P-player	C		D	
Q-player		C		C

unit $2n-2$ (rounds $2n-2$, $2n-1$); unit $2n-1$ (rounds $2n-1$, $2n$); unit $2n$ (rounds $2n$, $2n+1$)

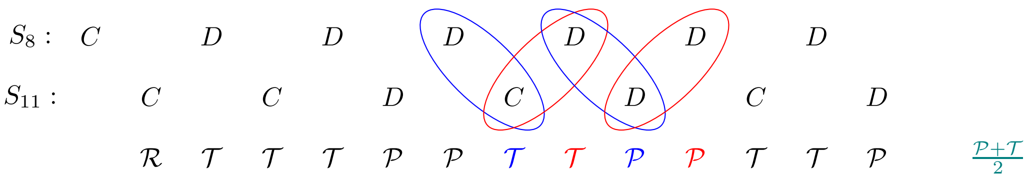
3 Contention between the strategy S_8 against the strategy S_{11}

We will suppose, game for two players, the first one plays with strategy S_8 against the other player who plays with S_{11}. We will get the following eight sequences as follows:

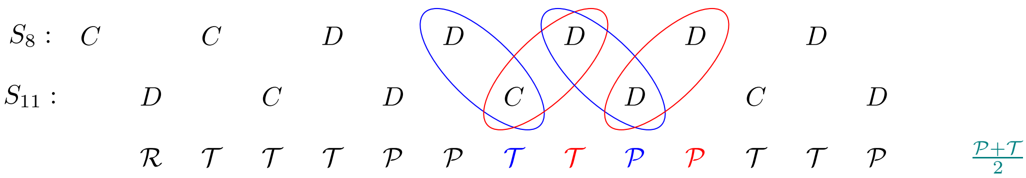
1) If we suppose that players play C in the first three rounds, then causing players to play C in each round. So that the repetition period of this sequence is two rounds with payoffs $(\mathcal{R}, \mathcal{R})$. This produces an average payoff with a value of $\mathcal{R}$ and is called approach A.



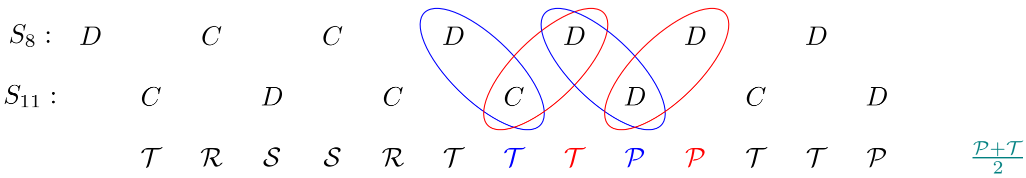
2) If S_8 and S_{11} are assumed to play C in their first round but S_8 plays D in his second round, then causing S_8 to play D in each round and S_{11} to switch between C and D in order. So that the repetition period of this sequence is four rounds with payoffs $(\mathcal{T}, \mathcal{T}, \mathcal{P}, \mathcal{P})$. This produces an average payoff with a value of $\frac{\mathcal{P}+\mathcal{T}}{2}$ and is called approach B.



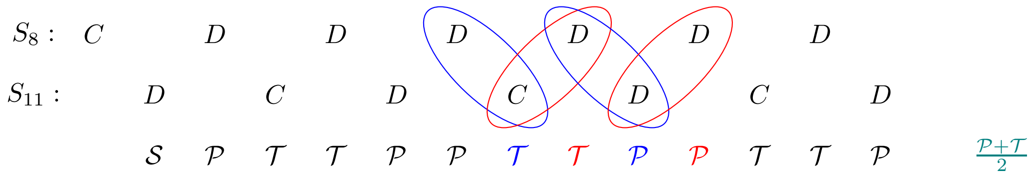
3) If S_8 starts with C in his first two rounds while S_{11} starts with D in his first round, then causing S_8 to play D in each round and S_{11} to switch between C and D in order. So that the repetition period of this sequence is four rounds with payoffs $(\mathcal{T}, \mathcal{T}, \mathcal{P}, \mathcal{P})$. This produces an average payoff with a value of $\frac{\mathcal{P}+\mathcal{T}}{2}$ and is called approach B.



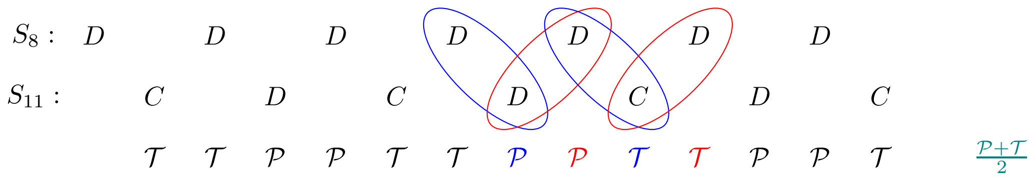
4) If S_8 starts with D in his first round while S_{11} play C in first round and S_8 plays C in second round. Then the repetition period of this sequence is four rounds with payoffs $(\mathcal{T}, \mathcal{T}, \mathcal{P}, \mathcal{P})$. This produces an average payoff with a value of $\frac{\mathcal{P}+\mathcal{T}}{2}$ and is called approach B.



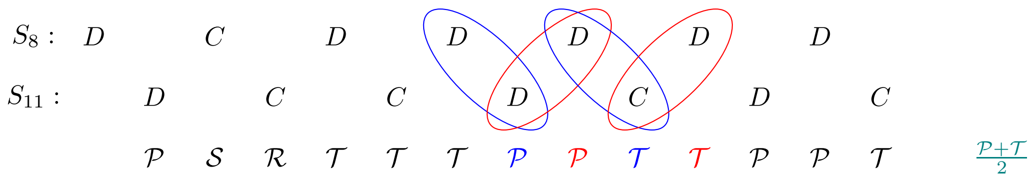
5) If S_8 starts with C in his first round while S_{11} plays D in first round and S_8 plays D in second round, then causing S_8 to play D in each round and S_{11} to switch between C and D in order. So that the repetition period of this sequence is four rounds with payoffs $(\mathcal{T},\mathcal{T},\mathcal{P},\mathcal{P})$. This produces an average payoff with a value of $\frac{\mathcal{P}+\mathcal{T}}{2}$ and is called approach B.



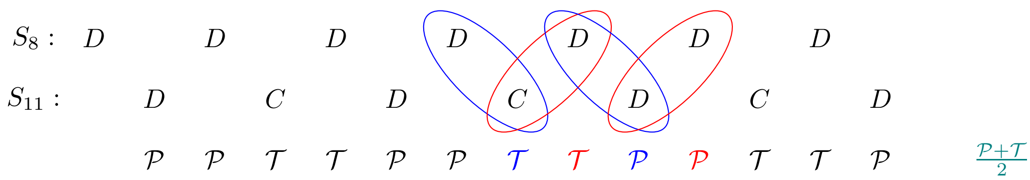
6) If S_8 starts with C in his first two rounds while S_{11} starts with D in his first round, then causing S_8 to play D in each round and S_{11} to switch between D and C in order. So that the repetition period of this sequence is four rounds with payoffs $(\mathcal{T},\mathcal{T},\mathcal{P},\mathcal{P})$. This produces an average payoff with a value of $\frac{\mathcal{P}+\mathcal{T}}{2}$ and is called approach B.



7) If S_8 and S_{11} are assumed to play D in their first round but S_8 plays C in his second round, then causing S_8 to play D in each round. So that the repetition period of this sequence is four rounds with payoffs $(\mathcal{T},\mathcal{T},\mathcal{P},\mathcal{P})$. This produces an average payoff with a value of $\frac{\mathcal{P}+\mathcal{T}}{2}$ and is called approach B.



8) If we suppose that players play D in the first three rounds, then causing S_8 to play D in each round and S_{11} to switch between C and D in order. So that the repetition period of this sequence is four rounds with payoffs $(\mathcal{T},\mathcal{T},\mathcal{P},\mathcal{P})$. This produces an average payoff with a value of $\frac{\mathcal{P}+\mathcal{T}}{2}$ and is called approach B.



In the previous eight sequences, there are two approaches resulting from S_8 competing with S_{11}, which is called A and B. Let's make a little random noise as in the next section, where every decision in the repetition period will be tested under possible error (playing C instead of D or vice versa). This study will only consider implementation flaws.

4 Perturbed Payoff

If one of the two players makes an incorrect move or decision (when the transition rule specifies D, he plays C, or when the transition rule specifies C, he plays D), then there is an implementation error. In regime A, one of the two players plays D due to a rare perturbation cause and thus leads after little steps to regime B: Each element in the repetition period will be changed individually and new states will be tracked to determine the approach and the payoff. Now, if the first element of the iteration period changes i.e. S_8 plays D instead of C, then approach A is changed to approach B:

$$\begin{array}{lccccccccccccccc}
S_8: & C & & C & & D & & D & & D & & D & & D & & \\
S_{11}: & & C & & C & & C & & D & & C & & D & & C & \\
 & & \mathcal{R} & \mathcal{R} & \mathcal{R} & \mathcal{T} & \mathcal{T} & \mathcal{T} & \mathcal{P} & \mathcal{P} & \mathcal{T} & \mathcal{T} & \mathcal{P} & \mathcal{P} & \mathcal{T} & \frac{\mathcal{P}+\mathcal{T}}{2}
\end{array}$$

Also, If the second element of the repetition period changes, i.e., S_{11} plays D instead of C, then approach A is changed to approach B:

$$\begin{array}{lccccccccccccccc}
S_8: & C & & C & & C & & D & & D & & D & & D & & \\
S_{11}: & & C & & C & & D & & D & & C & & D & & C & \\
 & & \mathcal{R} & \mathcal{R} & \mathcal{R} & \mathcal{R} & \mathcal{S} & \mathcal{P} & \mathcal{P} & \mathcal{P} & \mathcal{T} & \mathcal{T} & \mathcal{P} & \mathcal{P} & \mathcal{T} & \frac{\mathcal{P}+\mathcal{T}}{2}
\end{array}$$

It is clear that when a wrong decision occurs in approach A, it will be changed to approach B in all possible mutations with a probability of 100 %. Similarly, when a wrong decision occurs in approach B, it will not change in all possible mutations. Then, the corresponding transition matrix will be in the form

$$\begin{array}{cc} & \begin{array}{cc} A & B \end{array} \\ \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \end{array} \tag{8}$$

The first row in the above matrix represents the probabilities of approach A, where it has probability zero in the first column as it will not be converted to itself but with probability one in the second column because it will be changed to approach B in all possible mutations. The second row represents the probabilities of approach B, where it has probability zero in the first column as it will not be converted to approach A but with probability one in the second column because it will be changed to itself in all possible mutations. By using the following equation, we can calculate the corresponding stationary distribution of contention

$$\Pi M = \Pi, \tag{9}$$

where the eigenvector $\Pi = (\pi_1, \pi_2, \pi_3, \pi_4)$ is the stationary distribution of transition matrix M and

$$\sum_{i=1}^{4} \pi_i = 1. \tag{10}$$

Then,

$$\begin{pmatrix}\pi_1 & \pi_2\end{pmatrix} \times \begin{pmatrix}0 & 1\\ 0 & 1\end{pmatrix} = \begin{pmatrix}\pi_1\\ \pi_2\end{pmatrix} \tag{11}$$

The payoff for S_8-player against S_{11}-player is

$$H(P,Q) = \pi_1\mathcal{R} + \pi_2\mathcal{S} + \pi_3\mathcal{T} + \pi_4\mathcal{P}$$

$$H(S_8, S_{11}) = 0\,\mathcal{R} + 1\,\frac{\mathcal{P}+\mathcal{T}}{2} = \frac{1}{2}\mathcal{P} + \frac{1}{2}\mathcal{T} \tag{12}$$

Then, the payoff vector $(0,0,1,1)$. By repeating this method, we get 16^2 payoff values of S_i strategy for player I against the S_j opponents strategies for player II are obtained, where i,j take the value from 0 to 15. The conflict payoff between any two strategies used in this paper under the influence of approach repetition can be represented in the following table

Table 2: Payoff Vectors

	S_0	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
S_0	0010	1010	0010	1010	1020	1000	1010	1000	0010	1010	0010	1010	1010	1000	2010	1000
S_1	0011	1121	1111	1111	0011	2110	2121	2110	0011	2121	1111	1111	2011	1000	1000	1000
S_2	0010	1111	2011	1111	0010	1111	2121	1111	0010	2121	1021	2111	1120	1100	1100	1100
S_3	0011	1111	1111	1111	0011	1111	1111	1111	0011	1111	1111	1111	1111	1100	1100	1100
S_4	0021	1010	0010	1010	1021	1000	1010	1000	0021	1010	0010	1010	3021	1000	2010	1000
S_5	0001	0112	1111	1111	0001	1111	1111	2110	0001	1111	1111	1111	1001	1000	1000	1000
S_6	0011	1122	1122	1111	0011	1111	1111	2211	0011	1111	1111	2211	1111	1100	1100	1100
S_7	0001	0122	1111	1111	0001	0112	1212	1211	0001	1212	1111	1111	1102	1100	1100	1100
S_8	0010	1010	0010	1010	1020	1000	1010	1000	0010	1010	0010	1010	2130	1000	2110	2100
S_9	0011	1122	1122	1111	0011	1111	1111	2211	0011	1111	1111	2211	1111	1100	1100	1100
S_{10}	0010	1111	1021	1111	0010	1111	1111	1111	0010	1111	1111	1201	0110	0100	0100	0100
S_{11}	0011	1111	1112	1111	0011	1111	1212	1111	0011	1212	1201	1201	0211	0100	0100	0100
S_{12}	0011	1012	0121	1111	1023	1001	1111	2101	0132	1111	0110	1210	1111	3201	2310	0100
S_{13}	0001	0001	0101	0101	0001	0001	0101	0101	0001	0101	0100	0100	1203	1201	1200	1200
S_{14}	0012	0001	0101	0101	0012	0001	0101	0101	0112	0101	0100	0100	0312	0201	0100	0100
S_{15}	0001	0001	0101	0101	0001	0001	0101	0101	0102	0101	0100	0100	0100	0201	0100	0100

5 Relatedness Between S_8 and S_{11}

We will investigate the effect of relatedness between players on the behavior of strategies by using different numerical values for $\mathcal{T}, \mathcal{R}, \mathcal{P}$ and $\mathcal{S}$ and with different relatedness averages among the two players.

- Case 1: For $\mathcal{T} = 5$, $\mathcal{R} = 3$, $\mathcal{P} = 1$ and $\mathcal{S} = 0$, we get three sub cases for different r as in Tables 3:5

Table 3: Payoff Values with $r = 0.0001$

	S_0	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
S_0	1.00	3.00	1.00	3.00	2.33	5.00	3.00	5.00	1.00	3.00	1.00	3.00	3.00	5.00	3.67	5.00
S_1	0.50	2.00	2.25	2.25	0.50	3.50	2.50	3.50	0.50	2.50	2.25	2.25	2.75	5.00	5.00	5.00
S_2	1.00	2.25	2.75	2.25	1.00	2.25	2.50	2.25	1.00	2.50	1.75	2.80	2.50	4.00	4.00	4.00
S_3	0.50	2.25	2.25	2.25	0.50	2.25	2.25	2.25	0.50	2.25	2.25	2.25	2.25	4.00	4.00	4.00
S_4	0.67	3.00	1.00	3.00	1.75	5.00	3.00	5.00	0.67	3.00	1.00	3.00	2.83	5.00	3.67	5.00
S_5	0	1.00	2.25	2.25	0	2.25	2.25	3.50	0	2.25	2.25	2.25	2.50	5.00	5.00	5.00
S_6	0	1.00	2.25	2.25	0	2.25	2.25	3.50	0	2.25	2.25	2.25	2.50	5.00	5.00	5.00
S_7	0	1.00	2.25	2.25	0	1.00	2.00	2.40	0	2.00	2.25	2.25	2.00	4.00	4.00	4.00
S_8	1.00	3.00	1.00	3.00	2.33	5.00	3.00	5.00	1.00	3.00	1.00	3.00	2.67	5.00	3.50	4.33
S_9	0.50	1.67	1.67	2.25	0.50	2.25	2.25	2.83	0.50	2.25	2.25	2.83	2.25	4.00	4.00	4.00
S_{10}	1.00	2.25	1.75	2.25	1.00	2.25	2.25	2.25	1.00	2.25	2.25	2.75	2.00	3.00	3.00	3.00
S_{11}	0.50	2.25	1.80	2.25	0.50	2.25	2.00	2.25	0.50	2.00	2.75	2.75	1.75	3.00	3.00	3.00
S_{12}	0.50	1.50	1.25	2.25	1.17	2.50	2.25	3.25	1.00	2.25	2.00	3.00	2.25	3.50	3.33	3.00
S_{13}	0	0	1.50	1.50	0	0	1.50	1.50	0	1.50	3.00	3.00	1.83	2.75	3.67	3.67
S_{14}	0.33	0	1.50	1.50	0.33	0	1.50	1.50	1.00	1.50	3.00	3.00	1.67	2.00	3.00	3.00
S_{15}	0	0	1.50	1.50	0	0	1.50	1.50	1.00	1.50	3.00	3.00	3.00	2.00	3.00	3.00

Table 4: Payoff Values with $r = 0.5$

	S_0	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
S_0	1.50	3.25	1.50	3.25	2.67	5.00	3.25	5.00	1.50	3.25	1.50	3.25	3.25	5.00	3.83	5.00
S_1	2.00	3.00	3.38	3.38	2.00	4.00	3.33	4.00	2.00	3.33	3.38	3.38	3.50	5.00	5.00	5.00
S_2	1.50	3.38	3.50	3.38	1.50	3.38	3.33	3.38	1.50	3.33	2.63	3.70	3.13	4.75	4.75	4.75
S_3	2.00	3.38	3.38	3.38	2.00	3.38	3.38	3.38	2.00	3.38	3.38	3.38	3.38	4.75	4.75	4.75
S_4	1.83	3.25	1.50	3.25	2.63	5.00	3.25	5.00	1.83	3.25	1.50	3.25	3.42	5.00	3.83	5.00
S_5	2.50	2.75	3.38	3.38	2.50	3.38	3.38	4.00	2.50	3.38	3.38	3.38	3.75	5.00	5.00	5.00
S_6	2.00	2.92	2.92	3.38	2.00	3.38	3.38	3.83	2.00	3.38	3.38	3.83	3.38	4.75	4.75	4.75
S_7	2.50	2.75	3.38	3.38	2.50	2.75	3.42	3.60	2.50	3.42	3.38	3.38	3.63	4.75	4.75	4.75
S_8	1.50	3.25	1.50	3.25	2.67	5.00	3.25	5.00	1.50	3.25	1.50	3.25	3.17	5.00	4.00	4.83
S_9	2.00	2.92	2.92	3.38	2.00	3.38	3.38	3.83	2.00	3.38	3.38	3.83	3.38	4.75	4.75	4.75
S_{10}	1.50	3.38	2.63	3.38	1.50	3.38	3.38	3.38	1.50	3.38	3.38	4.13	3.00	4.50	4.50	4.50
S_{11}	2.00	3.38	3.20	3.38	2.00	3.38	3.42	3.38	2.00	3.42	4.13	4.13	3.25	4.50	4.50	4.50
S_{12}	2.00	2.88	2.50	3.38	2.58	3.75	3.38	4.25	2.33	3.38	3.00	3.88	3.38	4.42	4.17	4.50
S_{13}	2.50	2.50	3.50	3.50	2.50	2.50	3.50	3.50	2.50	3.50	4.50	4.50	3.58	4.13	4.67	4.67
S_{14}	2.17	2.50	3.50	3.50	2.17	2.50	3.50	3.50	2.75	3.50	4.50	4.50	3.33	3.83	4.50	4.50
S_{15}	2.50	2.50	3.50	3.50	2.50	2.50	3.50	3.50	3.17	3.50	4.50	4.50	4.50	3.83	4.50	4.50

Table 5: Payoff Values with $r = 0.999$

	S_0	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
S_0	2.00	3.50	2.00	3.50	3.00	5.00	3.50	5.00	2.00	3.50	2.00	3.50	3.50	5.00	4.00	5.00
S_1	3.50	4.00	4.50	4.50	3.50	4.50	4.17	4.50	3.45	4.17	4.50	4.50	4.29	5.00	5.00	5.00
S_2	2.00	4.50	4.25	4.50	2.00	4.50	4.17	4.50	2.00	4.17	3.50	4.60	3.75	5.50	5.50	5.50
S_3	3.50	4.50	4.50	4.50	3.50	4.50	4.50	4.50	3.50	4.50	4.50	4.50	4.50	5.50	5.50	5.50
S_4	3.00	3.50	2.00	3.50	3.50	5.00	3.50	5.00	3.00	3.50	2.00	3.50	4.00	5.00	4.00	5.00
S_5	5.00	4.50	4.50	4.50	5.00	4.50	4.50	4.50	5.00	4.50	4.50	4.50	5.00	5.00	5.00	5.00
S_6	3.50	4.16	4.16	4.50	3.50	4.50	4.50	4.83	3.50	4.50	4.50	4.83	4.50	5.50	5.50	5.50
S_7	5.00	4.50	4.50	4.50	5.00	4.50	4.83	4.80	5.00	4.83	4.50	4.50	5.25	5.50	5.50	5.50
S_8	2.00	3.50	2.00	3.50	3.00	5.00	3.50	5.00	2.00	3.50	2.00	3.50	3.67	5.00	4.50	5.33
S_9	3.50	4.16	4.16	4.50	3.50	4.50	4.50	4.83	3.50	4.50	4.50	4.83	4.50	5.50	5.50	5.50
S_{10}	2.00	4.50	3.50	4.50	2.00	4.50	4.50	4.50	2.00	4.50	4.50	5.50	4.00	6.00	6.00	6.00
S_{11}	3.50	4.50	4.60	4.50	3.50	4.50	4.83	4.50	3.50	4.83	5.50	5.50	4.75	6.00	6.00	6.00
S_{12}	3.50	4.25	3.75	4.50	4.00	5.00	4.50	5.25	3.66	4.50	4.00	4.75	4.50	5.33	5.00	6.00
S_{13}	5.00	5.00	5.50	5.50	5.00	5.00	5.50	5.50	5.00	5.50	6.00	6.00	5.33	5.50	5.66	5.66
S_{14}	4.00	5.00	5.50	5.50	4.00	5.00	5.50	5.50	4.50	5.50	6.00	6.00	5.00	5.66	6.00	6.00
S_{15}	5.00	5.00	5.50	5.50	5.00	5.00	5.50	5.50	5.33	5.50	6.00	6.00	6.00	5.66	6.00	6.00

- Case 2: For $\mathcal{T} = 1, \mathcal{R} = 0, \mathcal{P} = -10$ and $\mathcal{S} = -1$, we get three sub cases for different r as in Tables 6:8

Table 6: Payoff Values with $r = 0.0001$

	S_0	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
S_0	-1.00	-0.50	-1.00	-0.50	-0.67	0	-0.50	0	-1.00	-0.50	-1.00	-0.50	-0.50	0	-0.33	0
S_1	-5.50	-2.20	-2.50	-2.50	-5.50	-0.00	-1.83	-0.00	-5.50	-1.83	-2.50	-2.50	-2.75	0	0	0
S_2	-1.00	-2.50	-2.75	-2.50	-1.00	-2.50	-1.83	-2.50	-1.00	-1.83	-3.00	-2.00	-0.25	0.50	0.50	0.50
S_3	-5.50	-2.50	-2.50	-2.50	-5.50	-2.50	-2.50	-2.50	-5.50	-2.50	-2.50	-2.50	-2.50	0.50	0.50	0.50
S_4	-4.00	-0.50	-1.00	-0.50	-3.00	0	-0.50	0	-4.00	-0.50	-1.00	-0.50	-2.00	0	-0.33	0
S_5	-10.00	-5.00	-2.50	-2.50	-10.00	-2.50	-2.50	0	-10.00	-2.50	-2.50	-2.50	-5.00	0	0	0
S_6	-5.50	-3.50	-3.50	-2.50	-5.50	-2.50	-2.50	-1.50	-5.50	-2.50	-2.50	-1.50	-2.50	0.50	0.50	0.50
S_7	-10.00	-5.00	-2.50	-2.50	-10.00	-5.00	-3.17	-1.80	-10.00	-3.17	-2.50	-2.50	-4.75	0.50	0.50	0.50
S_8	-1.00	-0.50	-1.00	-0.50	-0.67	-0.00	-0.50	-0.00	-1.00	-0.50	-1.00	-0.50	-0.33	-0.00	-0.00	0.33
S_9	-5.50	-3.50	-3.50	-2.50	-5.50	-2.50	-2.50	-1.50	-5.50	-2.50	-2.50	-1.50	-2.50	0.50	0.50	0.50
S_{10}	-1.00	-2.50	-3.00	-2.50	-1.00	-2.50	-2.50	-2.50	-1.00	-2.50	-2.50	-2.00	-0.00	1.00	1.00	1.00
S_{11}	-5.50	-2.50	-4.00	-2.50	-5.50	-2.50	-3.17	-2.50	-5.50	-3.17	-2.00	-2.00	-2.25	1.00	1.00	1.00
S_{12}	-5.50	-5.25	-2.75	-2.50	-5.33	-5.00	-2.50	-2.25	-3.67	-2.50	0.00	0.25	-2.50	-1.33	0.33	1.00
S_{13}	-10.00	-10.00	-4.50	-4.50	-10.00	-10.00	-4.50	-4.50	-10.00	-4.50	1.00	1.00	-4.67	-2.00	0.67	0.67
S_{14}	-7.00	-10.00	-4.50	-4.50	-7.00	-10.00	-4.50	-4.50	-5.00	-4.50	1.00	1.00	-3.00	-2.67	1.00	1.00
S_{15}	-10.00	-10.00	-4.50	-4.50	-10.00	-10.00	-4.50	-4.50	-6.33	-4.50	1.00	1.00	1.00	-2.67	1.00	1.00

Table 7: Payoff Values with $r = 0.5$

	S_0	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
S_0	-1.50	-3.25	-1.50	-3.25	-2.67	-5.00	-3.25	-5.00	-1.50	-3.25	-1.50	-3.25	-3.25	-5.00	-3.83	-5.00
S_1	-5.75	-3.30	-3.75	-3.75	-5.75	-2.50	-3.58	-2.50	-5.75	-3.58	-3.75	-3.75	-5.38	-5.00	-5.00	-5.00
S_2	-1.50	-3.75	-5.38	-3.75	-1.50	-3.75	-3.58	-3.75	-1.50	-3.58	-4.50	-4.00	-1.63	-1.75	-1.75	-1.75
S_3	-5.75	-3.75	-3.75	-3.75	-5.75	-3.75	-3.75	-3.75	-5.75	-3.75	-3.75	-3.75	-3.75	-1.75	-1.75	-1.75
S_4	-4.33	-3.25	-1.50	-3.25	-4.50	-5.00	-3.25	-5.00	-4.33	-3.25	-1.50	-3.25	-4.67	-5.00	-3.83	-5.00
S_5	-10.00	-5.00	-3.75	-3.75	-10.00	-3.75	-3.75	-2.50	-10.00	-3.75	-3.75	-3.75	-7.50	-5.00	-5.00	-5.00
S_6	-5.75	-4.42	-4.42	-3.75	-5.75	-3.75	-3.75	-3.08	-5.75	-3.75	-3.75	-3.08	-3.75	-1.75	-1.75	-1.75
S_7	-10.00	-5.00	-3.75	-3.75	-10.00	-5.00	-3.92	-2.70	-10.00	-3.92	-3.75	-3.75	-5.88	-1.75	-1.75	-1.75
S_8	-1.50	-3.25	-1.50	-3.25	-2.67	-5.00	-3.25	-5.00	-1.50	-3.25	-1.50	-3.25	-2.17	-5.00	-2.50	-2.83
S_9	-5.75	-4.42	-4.42	-3.75	-5.75	-3.75	-3.75	-3.08	-5.75	-3.75	-3.75	-3.08	-3.75	-1.75	-1.75	-1.75
S_{10}	-1.50	-3.75	-4.50	-3.75	-1.50	-3.75	-3.75	-3.75	-1.50	-3.75	-3.75	-3.00	0	1.50	1.50	1.50
S_{11}	-5.75	-3.75	-5.00	-3.75	-5.75	-3.75	-3.92	-3.75	-5.75	-3.92	-3.00	-3.00	-2.13	1.50	1.50	1.50
S_{12}	-5.75	-6.63	-2.88	-3.75	-6.33	-7.50	-3.75	-4.63	-3.83	-3.75	0	-0.88	-3.75	-3.67	-1.17	1.50
S_{13}	-10.00	-10.00	-10.00	-4.25	-10.00	-10.00	-4.25	-4.25	-10.00	-4.25	1.50	1.50	-5.33	-3.00	-0.67	-0.67
S_{14}	-7.17	-10.00	-4.25	-4.25	-7.17	-10.00	-4.25	-4.25	-5.00	-4.25	1.50	1.50	-2.83	-2.33	1.50	1.50
S_{15}	-10.00	-10.00	-4.25	-4.25	-10.00	-10.00	-4.25	-4.25	-6.17	-4.25	1.50	1.50	1.50	-2.33	1.50	1.50

Table 8: Payoff Values with $r = 0.999$

	S_0	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}
S_0	-2.00	-5.99	-2.00	-5.99	-4.66	-9.99	-5.99	-9.99	-2.00	-5.99	-2.00	-5.99	-5.99	-9.99	-7.33	-9.99
S_1	-6.00	-4.40	-5.00	-5.00	-6.00	-5.00	-5.33	-5.00	-6.00	-5.33	-5.00	-5.00	-7.99	-9.99	-9.99	-9.99
S_2	-2.00	-5.00	-7.99	-5.00	-2.00	-5.00	-5.33	-5.00	-2.00	-5.33	-6.00	-6.00	-3.00	-4.00	-4.00	-4.00
S_3	-6.00	-5.00	-5.00	-5.00	-6.00	-5.00	-5.00	-5.00	-6.00	-5.00	-5.00	-5.00	-5.00	-4.00	-4.00	-4.00
S_4	-4.67	-5.99	-2.00	-5.99	-6.00	-9.99	-5.99	-9.99	-4.67	-5.99	-2.00	-5.99	-7.33	-9.99	-7.33	-9.99
S_5	-10.00	-5.00	-5.00	-5.00	-10.00	-5.00	-5.00	-5.00	-10.00	-5.00	-5.00	-5.00	-10.00	-9.99	-9.99	-9.99
S_6	-6.00	-5.33	-5.33	-5.00	-6.00	-5.00	-5.00	-4.66	-6.00	-5.00	-5.00	-4.66	-5.00	-4.00	-4.00	-4.00
S_7	-10.00	-5.00	-5.00	-5.00	-10.00	-5.00	-4.67	-3.60	-10.00	-4.67	-5.00	-5.00	-7.00	-4.00	-4.00	-4.00
S_8	-2.00	-5.99	-2.00	-5.99	-4.67	-9.99	-5.99	-9.99	-2.00	-5.99	-2.00	-5.99	-3.99	-9.99	-5.00	-6.00
S_9	-6.00	-5.33	-5.33	-5.00	-6.00	-5.00	-5.00	-4.66	-6.00	-5.00	-5.00	-4.66	-5.00	-4.00	-4.00	-4.00
S_{10}	-2.00	-5.00	-6.00	-5.00	-2.00	-5.00	-5.00	-5.00	-2.00	-5.00	-5.00	-4.00	0	-2.00	-2.00	-2.00
S_{11}	-6.00	-5.00	-6.00	-5.00	-6.00	-5.00	-4.67	-5.00	-6.00	-4.67	-4.00	-4.00	-2.00	-2.00	-2.00	-2.00
S_{12}	-6.00	-8.00	-3.00	-5.00	-7.33	-10.00	-5.00	-7.00	-4.00	-5.00	0	-2.00	-5.00	-6.00	-2.67	-2.00
S_{13}	-10.00	-10.00	-4.00	-4.00	-10.00	-10.00	-4.00	-4.00	-10.00	-4.00	2.00	2.00	-6.00	-4.00	-2.00	-2.00
S_{14}	-7.33	-10.00	-4.00	-4.00	-7.33	-10.00	-4.00	-4.00	-5.00	-4.00	2.00	2.00	-2.67	-2.00	2.00	2.00
S_{15}	-10.00	-10.00	-4.00	-4.00	-10.00	-10.00	-4.00	-4.00	-6.00	-4.00	2.00	2.00	2.00	-2.00	2.00	2.00

6 Results

In this section, we will discuss the domination between the 16 strategies for (2P-IPD). We note that S_i is outcompeted by S_j if both $a_{ji} > a_{ii}$ and $a_{jj} > a_{ij}$, where a_{ii}, a_{ij}, a_{ji} and a_{jj} are elements of the payoff matrix as in (12). If the strategy S_i is outcompeted by S_j, we can write $S_i << S_j$.

$$\begin{array}{cc} & \begin{array}{cc} S_i & S_j \end{array} \\ \begin{array}{c} S_i \\ S_j \end{array} & \begin{bmatrix} a_{ii} & a_{ij} \\ a_{ji} & a_{jj} \end{bmatrix} \end{array} \tag{13}$$

From Case 1, we get the following

1. When the relatedness average among the two players was $r = 0.0001$ (small value), we see that the strategy S_2 can not be defeated by any other strategies, so it is the strongest one. We also note that the strategies S_0 (All D) and S_2 can outcompete the largest number of strategies.

2. When the relatedness average among players was $r = 0.5$, we found no other strategy can defeat the strategies S_2 and S_4, so they are the strongest strategies. Also, we noticed that the strategy S_6 and S_9 (Grim) are outcompeted by the same unique strategy S_{11}.

3. When the relatedness average among players was $r = 0.999$ (large value), we notice that S_2 retreated from its strength, and the strategies S_{14} and S_{15} (All C) become the strongest strategies, as there is no competitor. Also, we see that The strategies S_7, S_{11} and S_{13} are outcompeted by the same strategies, rather they are the strongest strategies, S_{14} and S_{15} (All C) .

From Case 2, we get the following

1. When the relatedness average among players was r = 0.0001 (small value), the strategies S_0 (All D), S_8, S_{14} and S_{15} (All C) are the strongest strategies because no other strategy can defeat them and they defeat most of the strategies. We also note that the strategies S_{10} (TFT) and S_{13} cannot defeat any other strategy.

2. When the relatedness average among players was r = 0.5, the strategies S_0 (All D), S_8, S_{14} and S_{15} (All C) are the strongest strategies because no other strategy can defeat them, and they defeat most of the strategies. We also note that the strategies S_2, S_{12} and S_{13} cannot defeat any other strategy. Therefore, we see that The strategies S_6 and S_9 (Grim) are outcompeted by the same strategies.

3. When the relatedness average among players was r = 0.999 (large value), the strategies S_0 (All D), S_1, S_7, S_8, S_{14} and S_{15} (All C) are the strongest strategies because no other strategy can defeat them. We also note that the strategies S_2, S_{12} and S_{13} cannot defeat any other strategy. Therefore, we see that The strategies S_6 and S_9 (Grim) are outcompeted by the same strategies.

Domination table of strategies S_i

	$T=5, R=3, P=1, S=0$			$T=1, R=0, P=-10, S=-1$		
	$r=0.0001$	$r=0.5$	$r=0.999$	$r=0.0001$	$r=0.5$	$r=0.999$
S_0	S_2, S_{10}	S_2, S_3, S_6, S_9, $S_{10}, S_{11}, S_{12}, S_{14}$	$S_1, S_2, S_3, S_4, S_6, S_9$, $S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15}$			
S_1	S_0, S_2, S_3, S_4, S_8, S_{10}, S_{11}	S_2, S_3, S_4, S_{10}, S_{11}	S_3, S_6, S_7, S_9, $S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15}$	S_0, S_4, S_8	S_0, S_4, S_8	
S_2			S_3, S_5, S_7, S_{11}, S_{14}, S_{15}	$S_0, S_1, S_3, S_5, S_7, S_8$	$S_0, S_1, S_3, S_5, S_7, S_8$, $S_{10}, S_{11}, S_{14}, S_{15}$	$S_0, S_1, S_3, S_5, S_6, S_7, S_8, S_9$, $, S_{10}, S_{11}, S_{14}, S_{15}$
S_3	S_0, S_2, S_4, S_7, S_8, S_{11}	S_2, S_7, S_2, S_7, S_{11}	S_7, S_{11}, S_{14}, S_{15}	S_0, S_1, S_4, S_7, S_8	S_0, S_1, S_4, S_7, S_8, S_{11}	S_1, S_7, S_{11}, S_{14}, S_{15}
S_4	S_0, S_8		$S_{12}, S_{13}, S_{14}, S_{15}$	S_0, S_8	S_0, S_8	S_0, S_8
S_5	S_0, S_1, S_2, S_4, S_8, S_{11}	S_1, S_2, S_4, S_{11}	$S_{11}, S_{13}, S_{14}, S_{15}$	S_0, S_1, S_4, S_8, S_{11}	S_1, S_{11}	S_1, S_{11}
S_6	S_0, S_1, S_2, S_4, S_8	S_{11}	S_{11}, S_{14}, S_{15}	S_0, S_1, S_2, S_4, S_8	S_0, S_1, S_4, S_8	S_7, S_{11}, S_{14}, S_{15}
S_7	$S_0, S_1, S_4, S_5, S_6, S_8, S_9$, S_{12}	S_1, S_4, S_5	S_{14}, S_{15}	$S_0, S_1, S_4, S_5, S_6, S_8, S_9$	S_1, S_5	
S_8	S_2, S_{10}	S_2, S_3, S_6, S_9, $S_{10}, S_{11}, S_{12}, S_{14}$	$S_1, S_2, S_3, S_4, S_6, S_9$, $S_{10}, S_{11}, S_{12}, S_{13}, S_{14}, S_{15}$			
S_9	S_0, S_1, S_2, S_4, S_8	S_{11}	S_{11}, S_{14}, S_{15}	S_0, S_1, S_2, S_4, S_8	S_0, S_1, S_4, S_8	S_7, S_{11}, S_{14}, S_{15}
S_{10}	S_7, S_{11}, S_{14}, S_{15}	S_7, S_{11}, S_{14}, S_{15}	S_7, S_{11}, S_{14}, S_{15}	S_0, S_1, S_7, S_8, S_{11}, S_{14}, S_{15}	S_0, S_1, S_7, S_8, S_{11}, S_{14}, S_{15}	S_0, S_1, S_7, S_8, S_{11}, S_{14}, S_{15}
S_{11}	$S_0, S_2, S_4, S_6, S_8, S_9$, S_{12}, S_{14}, S_{15}	S_{14}, S_{15}	S_{14}, S_{15}	S_0, S_4, S_6, S_8, S_9, S_{14}, S_{15}	S_{14}, S_{15}	S_{14}, S_{15}
S_{12}	S_0, S_1, S_2, S_4, S_{15}	S_1, S_4, S_{15}	$S_{11}, S_{13}, S_{14}, S_{15}$	S_0, S_2, S_4, S_8, S_{15}	S_0, S_8, S_{14}, S_{15}	S_8, S_{14}, S_{15}
S_{13}	$S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9$, S_{12}	S_1, S_2, S_4, S_5, S_7	S_{14}, S_{15}	$S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_8, S_9$, S_{12}	S_3, S_6, S_7, S_9, S_{14}, S_{15}	S_7, S_{14}, S_{15}
S_{14}	$S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_9$, S_{12}, S_{13}	S_1, S_2, S_5, S_7, S_{13}				
S_{15}	$S_0, S_1, S_2, S_3, S_4, S_5, S_6, S_7, S_9$, S_{13}	S_1, S_2, S_4, S_5, S_7, S_{13}				

Conclusion

Since the game is infinitely repeated, we consider any game strategy represented by finite states of automata (two states). We obtained the 16 x 16 payoff matrix for strategies by using the possibility of a small error in the automata implementation. We have taken various values of $\mathcal{T}, \mathcal{R}, \mathcal{P}$ and $\mathcal{S}$ for this game. The effect of noise on the degree of relatedness among players regarding the behavior of strategies and their payoff was studied using various average relatedness values. We concluded that, for Axelrod's values ($\mathcal{T} = 5, \mathcal{R} = 3, \mathcal{P} = 1, \mathcal{S} = 0$) with $r = 0.0001$ and $r = 0.5$, all strategies are defeated by at least two others except S_2. Also, for $r = 0.999$, no strategy can outcompete the strategies S14 and S15. In the case of the Chicken game ($\mathcal{T} = 1, \mathcal{R} = 0, \mathcal{P} = -10, \mathcal{S} = -1$), whatever the strength of the relationship between players, no strategy can out compete the strategies S_0 (All D), S_8, S_{14} and S_{15} (All C).

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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