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An Iterative Method for Solving the Dispersive Partial Differential Equations

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Abstract: The Daftardar-Gejji and Jafari method (DJM) was utilized in a recent study to propose a novel numerical solution for dispersive partial differential equations. The study showcased the remarkable effectiveness of DJM by analyzing a diverse set of test cases. In addition, the study conducted a thorough comparison between DJM and the exact solution, which was presented to illustrate the accuracy and robustness of the proposed method. This research breakthrough highlights the significance of DJM in advancing the field of numerical analysis and its potential to be applied to a wide range of complex problems.

Keywords: Third-order dispersive PDE; Daftardar-Gejji and Jafari method (DJM); Numerical method.

1 Introduction

Numerical analysis has played a crucial role in the development of realistic mathematical models that are widely used in science and engineering. This has been made possible by the increased power and accessibility of digital computers over the past five decades. Numerical techniques used to solve ordinary differential equations (ODEs) can also be applied to partial differential equations (PDEs). Many problems can be solved by using numerical techniques to address initial value concerns, as demonstrated in examples such as, [1,2,3,4,5,6,7,8,9].

Numerical analysis is the field that focuses on developing practical methods for solving complex computational problems. Many mathematical problems in science and engineering are difficult and do not have straightforward solutions. To make these challenges more tractable, it is important to have accurate and efficient numerical methods. In recent years, numeracy has become an essential tool for scientists and engineers due to the tremendous advances in computing technology. As a result, there are now numerous software packages available, such as Matlab, Mathematica, and Maple, that allow users to quickly and easily solve even the most complex problems. These programs utilize traditional numerical techniques and allow users to obtain results with just a single command, without having to input any additional parameters. Numerical analysis involves developing algorithms for solving numerical problems in continuous mathematics, which are commonly encountered in various fields such as mathematics, computer science, the sciences, engineering, healthcare, and business. These problems often arise when algebra, geometry, and calculus are applied to continuous variables. Numerical analysis involves the creation, analysis, and application of these algorithms. [10], [11], [12], [13], [14], [15], [16], [17], [18] and [19].

Partial differential equations (PDEs) are an excellent tool for describing various models in real-world issues, [20,21, 22,27]. Djidjeli and Twizell [23] devised numerical techniques for solving third-order dispersive equations. Wazwaz [24] employed the Adomian decomposition approach to solve various third-order dispersive PDEs. To solve fractional dispersive equations, Kanth and Aruna [25] used the fractional differential transform technique (FDTM) and a modified version of FDTM. The predictor-corrector approach and a linearized implicit method were both used by Djidjeli et al [26] to solve the dispersive equations. Rui et al [28] employed the integral bifurcation approach to solve the dispersive PDE family.

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In order to demonstrate the effectiveness of DJM, we will solve third-order dispersive PDEs using DJM and give a comparison between DJM and precise solutions. The results obtained by DJM are a complete match with the exact solution.

2 The method of solution

Daftardar and Jafari successfully presented the DJM in 2006 [29]. This technique was used to solve many types of nonlinear differential equations [30,31,32,33,34,35,36,37,38,39,40,41,42]. Using DJM, a new predictor-corrector method was developed [43,44]. Using DJM, Noor et al developed new numerical ways of dealing with algebraic equations [45].

In this section, we will present the DJM numerical method as follows:

$$u = f + L(u) + N(u), \tag{1}$$

In the equation above, f is a known function, and L and N are linear and nonlinear operators, respectively. The NIM solution for Eq. (1) has the form

$$u = \sum_{i=0}^{\infty} u_i. \tag{2}$$

Since L is linear then

$$L\left(\sum_{i=0}^{\infty} u_i\right) = \sum_{i=0}^{\infty} L(u_i). \tag{3}$$

The nonlinear operator N in Eq. (1) is decomposed as below

$$\begin{aligned} N\left(\sum_{i=0}^{\infty} u_i\right) &= N(u_0) + \sum_{i=1}^{\infty} \left\{ N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\}. \\ &= \sum_{i=0}^{\infty} A_i, \end{aligned} \tag{4}$$

where

$$\begin{aligned} A_0 &= N(u_0) \\ A_1 &= N(u_0+u_1) - N(u_0) \\ A_2 &= N(u_0+u_1+u_2) - N(u_0+u_1) \\ &\vdots \\ A_i &= \left\{ N\left(\sum_{j=0}^{i} u_j\right) - N\left(\sum_{j=0}^{i-1} u_j\right) \right\}, \quad i \geq 1. \end{aligned}$$

Using Eqs.(2), (3) and (4) in Eq. (1), we get

$$\sum_{i=0}^{\infty} u_i = f + \sum_{i=0}^{\infty} L(u_i) + \sum_{i=0}^{\infty} A_i. \tag{5}$$

The solution of Eq. (1) can be expressed as

$$u = \sum_{i=0}^{\infty} u_i = u_0 + u_1 + u_2 + \ldots + u_n + \ldots, \tag{6}$$

where

$$\begin{aligned} u_0 &= f \\ u_1 &= L(u_0) + A_0 \\ u_2 &= L(u_1) + A_1 \\ &\vdots \\ u_n &= L(u_{n-1}) + A_{n-1} \\ &\vdots \end{aligned} \tag{7}$$

Algorithm

$$
\begin{aligned}
INPUT\ :\ & Read\ M(Number\ of\ iterations);\\
& Read\ L(u);\ N(u);\ f\\
& Step-1\ : u_{-1}=0,\ u_0=f\\
& Step-2:\ For(n=0,\ n\leq M, n++)\\
& \{\\
& Step-3: A_n = f(u_n)-f(u_{n-1});\\
& Step-4: u_{n+1} = f+L(u_n)+A_n;\\
& Step-5: u = u_{n+1}\\
& \}\ end\\
OUTPUT\ :\ & u
\end{aligned}
$$

3 The convergence of the DJM

Theorem 1: For any n and for some real $L>0$ and $||u_i|| \leq M < \frac{1}{e}, i=1,2,...,$ if N is $C^{(\infty)}$ in the neighborhood of u_0 and $||N^{(n)}(u_0)|| \leq L$, then $\sum_{n=0}^{\infty} H_n$ is convergent absolutely and $||H_n|| \leq LM^n e^{n-1}(e-1),\ \ n=1,2,....$

Proof:

$$||H_n|| \leq LM^n \sum_{i_n=1}^{\infty}\sum_{i_{n-1}=0}^{\infty}\cdots\sum_{i_1=0}^{\infty}\left(\prod_{j=1}^{n}\frac{1}{i_j!}\right) = LM^n e^{n-1}(e-1).$$

Thus the series $\sum_{n=1}^{\infty}||H_n||$ is dominated by the convergent series $LM(e-1)\sum_{n=1}^{\infty}(Me)^{n-1}$, where $M<1/e$. Hence, $\sum_{n=0}^{\infty} H_n$ is absolutely convergent, due to the comparison test.

As it is difficult to show boundedness of u_i, for all i, a more useful result is provedin the following theorem, where conditions on $N^{(k)}(u_0)$ are given which are sufficient to guarantee convergence of the series.

Theorem 2: The series $\sum_{n=0}^{\infty} H_n$ is convergent absolutely if N is $C^{(\infty)}$ and $||N^{(n)}(u_0)|| \leq M \leq e^{-1}, \forall n.$

Proof: Consider the recurrence relation

$$\varepsilon_n = \varepsilon_0 exp(\varepsilon_{n-1}),\ \ n=1,2,3,..., \tag{8}$$

where $\varepsilon_0 = M$. Define $\eta_n = \varepsilon_n - \varepsilon_{n-1}, n=1,2,3,\cdots$. We observe that

$$||H_n|| \leq \eta_n,\ n=1,2,3,\cdots. \tag{9}$$

Let

$$\sigma_n = \sum_{i=1}^{n}\eta_i = \varepsilon_n - \varepsilon_0. \tag{10}$$

Not that $\varepsilon_0 = e^{-1} > 0$, $\varepsilon_1 = \varepsilon_0 exp(\varepsilon_0) > \varepsilon_0$ and $\varepsilon_2 = \varepsilon_0 exp(\varepsilon_1) > \varepsilon_0 exp(\varepsilon_0) = \varepsilon_1$. In general, $\varepsilon_n > \varepsilon_{n-1} > 0$. Hence $\sum\eta_n$ is a series of positive real numbers. Note that

$$
\begin{aligned}
0 < \varepsilon_0 = M = e^{-1} < 1,\\
0 < \varepsilon_1 = \varepsilon_0 exp(\varepsilon_0) < \varepsilon_0 e^1 = e^{-1}e^1 = 1,\\
0 < \varepsilon_2 = \varepsilon_0 exp(\varepsilon_1) < \varepsilon_0 e^1 = 1.
\end{aligned} \tag{11}
$$

In general $0<\varepsilon_n<1$. Hence, $\sigma = \varepsilon_n - \varepsilon_0 < 1$. This implies that $\{\sigma_n\}_{n=1}^{\infty}$ is bounded above by 1, and hence convergent. Therefore, $\sum H_n$ is absolutely convergent by comparison test.

4 Applications

Here, we will employ DJM to dispersive PDEs in one, two and three dimensions.

Example 1: In this example, we will apply DJM to one demension:

$$u_{\mathbf{t}}(x,t)+2u_{\mathbf{x}}(x,t)+u_{\mathbf{xxx}}(x,t)=0. \tag{12}$$

With:

$$u_0=\sin\mathbf{x}. \tag{13}$$

With exact solution:

$$u=\sin(\mathbf{x}-\mathbf{t}). \tag{14}$$

We can solve eq. (12) and (13) via DJM as follows:

$$u=\sin\mathbf{x}-\int_0^{\mathbf{t}} 2u_{\mathbf{x}}+u_{\mathbf{xxx}}\;d\mathbf{t}. \tag{15}$$

By using algorithm (8) we have:

$$\begin{aligned}
u_0 &= \sin\mathbf{x},\\
u_1 &= -\mathbf{t}\cos(\mathbf{x}),\\
u_2 &= -\frac{1}{2}\mathbf{t}^2\sin(\mathbf{x}),\\
u_3 &= \frac{1}{6}\mathbf{t}^3\cos(\mathbf{x}),\\
&\vdots
\end{aligned}$$

Thus,

$$\begin{aligned}
\sum_{i=0}^{5}u_i &= \sin(\mathbf{x})-\cos(\mathbf{x})\mathbf{t}-\frac{1}{2}\sin(\mathbf{x})\mathbf{t}^2+\frac{1}{6}\cos(\mathbf{x})\mathbf{t}^3\\
&+\frac{1}{24}\sin(\mathbf{x})\mathbf{t}^4-\frac{1}{120}\cos(\mathbf{x})\mathbf{t}^5.
\end{aligned} \tag{16}$$

The comparison between DJM u_5 and the exact solution is displayed in table (1) and figure (1) showing the good results agreement.

Example 2: We shall apply DJM to the third- ordered dispersive PDEs in two dimensions as follows:

$$u_{\mathbf{t}}+u_{\mathbf{xxx}}+u_{\mathbf{yyy}}=0,\quad \mathbf{t}>0, \tag{17}$$

with:

$$u_0=\cos(\mathbf{x}+\mathbf{y}), \tag{18}$$

and the exact solution is:

$$u=\cos(\mathbf{x}+\mathbf{y}+2\mathbf{t}). \tag{19}$$

By integrating equation (17) and applying equation (18), we obtain:

$$u=\cos(\mathbf{x}+\mathbf{y})-\int_0^{t} u_{\mathbf{xxx}}+u_{\mathbf{yyy}}\;d\mathbf{t}. \tag{20}$$

By using algorithm (8) we get:

$$\begin{aligned}
u_0 &= \cos(\mathbf{x}+\mathbf{y}),\\
u_1 &= -2\mathbf{t}\sin(\mathbf{x}+\mathbf{y}),\\
u_2 &= -2\mathbf{t}^2\cos(\mathbf{x}+\mathbf{y}),\\
u_3 &= \frac{4}{3}\mathbf{t}^3\sin(\mathbf{x}+\mathbf{y}),\\
&\vdots
\end{aligned}$$

Thus,

$$\sum_{i=0}^{5} u_i = \cos(\mathbf{x}+\mathbf{y}) - 2t\sin(\mathbf{x}+\mathbf{y}) - 2\mathbf{t}^2\cos(\mathbf{x}+\mathbf{y}) + \frac{4}{3}\mathbf{t}^3\sin(\mathbf{x}+\mathbf{y}) + \frac{2}{3}\mathbf{t}^4\cos(\mathbf{x}+\mathbf{y}) - \frac{4}{15}\mathbf{t}^5\sin(\mathbf{x}+\mathbf{y}). \tag{21}$$

Table (1) and Figure (2) present a comparison between the DJM u_5 and the exact solution, which clearly demonstrate the high level of accuracy achieved by the method.

Example 3: Finally, we employ DJM to the third- ordered dispersive PDEs in three dimensions:

$$u_{\mathbf{t}} + u_{\mathbf{xxx}} + \frac{1}{8}u_{\mathbf{yyy}} + \frac{1}{27}u_{\mathbf{zzz}} = \cos t\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) - 3\sin\mathbf{t}\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z}), \tag{22}$$

with:

$$u_0 = 0, \tag{23}$$

with the exact solution:

$$u = \sin t\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z}). \tag{24}$$

In this case, we integrate eq. (22) and apply the initial condition eq. (23) to obtain:

$$u = -\int_0^t u_{xxx} + \frac{1}{8}u_{\mathbf{yyy}} + \frac{1}{27}u_{\mathbf{zzz}} + F \;\; d\mathbf{t}, \tag{25}$$

where,

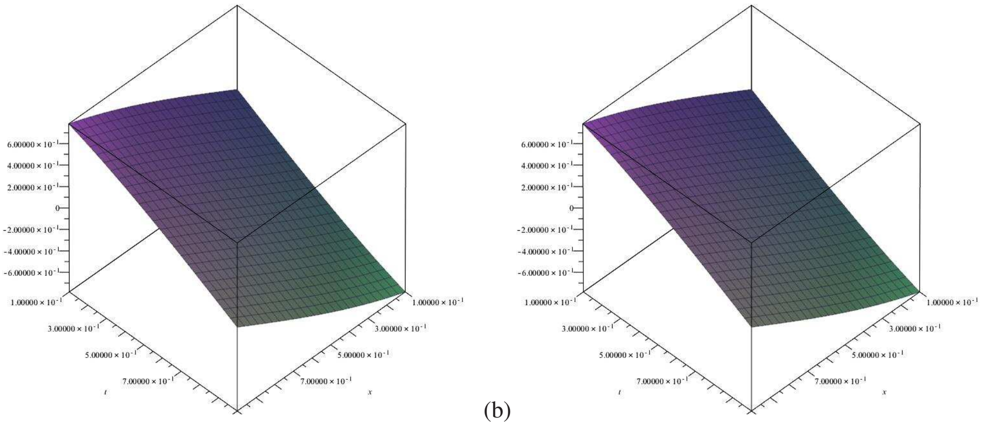
$$F = 3\sin(\mathbf{t})\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) - \cos(\mathbf{t})\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) \tag{26}$$

By applying (8) we have:

$$\begin{aligned}
u_0 &= 0,\\
u_1 &= -3\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) + 3\cos(\mathbf{t})\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) + \sin(\mathbf{t})\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z}),\\
u_2 &= 9\mathbf{t}\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) - 9\sin(\mathbf{t})\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) + 3\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\\
&\quad - 3\cos(\mathbf{t})\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z}),\\
u_3 &= -27\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) + \frac{27}{2}\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\mathbf{t}^2 + 27\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\cos(\mathbf{t})\\
&\quad + 9\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\sin(\mathbf{t}) - 9\sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\mathbf{t}.\\
&\;\;\vdots
\end{aligned}$$

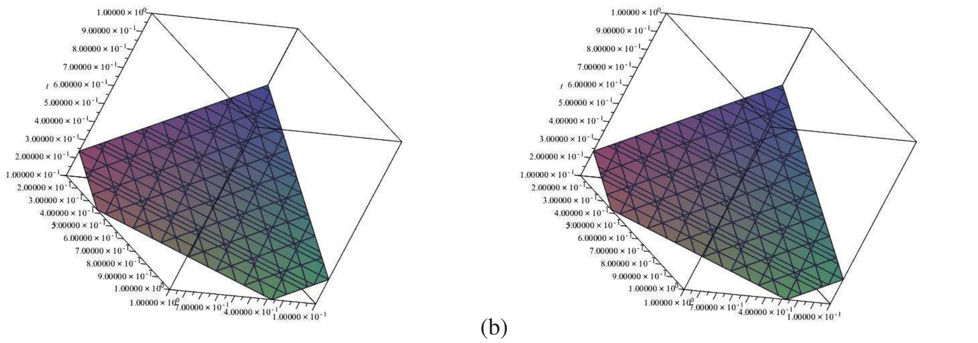
So,

$$\sum_{i=0}^{5} u_i = -243\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z}) + \frac{243}{2}\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\mathbf{t}^2 - \frac{81}{8}\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\mathbf{t}^4 + 243\cos(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\cos(\mathbf{t}) + \sin(\mathbf{x}+2\mathbf{y}+3\mathbf{z})\sin(\mathbf{t}) \tag{27}$$



(a) (b)

Fig. 1: The numerical results for DJM to the third- ordered dispersive PDEs in one dimension.



(a) (b)

Fig. 2: The numerical results for DJM to the third- ordered dispersive PDEs in two dimensions.

The comparison between DJM u_5 and the exact solutionIn is displayed in table (1), which shows the good accuracy of the method.

Table 1: Absolute errors between the DJM u_5, and the exact solutions for One-dimensional, Two-dimensional and Three-dimensional.

t	x	One-dimensional	y	Two-dimensional	z	Three-dimensional
0.1	0.1	1.1996×10^{-10}	0.1	8.64000×10^{-8}	0.1	3.00000×10^{-7}
0.5		6.19900×10^{-7}		1.29829×10^{-3}		4.33300×10^{-3}
1.0		5.84954×10^{-5}		7.63875×10^{-2}		2.73632×10^{-1}
0.1	0.5	7.0000×10^{-10}	0.5	4.59000×10^{-8}	0.5	4.00000×10^{-7}
0.5		9.00221×10^{-6}		5.72507×10^{-4}		5.19740×10^{-3}
1.0		4.82382×10^{-4}		2.45188×10^{-2}		3.28222×10^{-1}
0.1	1.0	1.50000×10^{-9}	1.0	3.93000×10^{-8}	1.0	2.88050E-7
0.5		1.73454×10^{-5}		7.45707×10^{-4}		5.04089×10^{-4}
1.0		1.04234×10^{-3}		5.63184×10^{-2}		3.18334×10^{-1}

5 Conclusions

The article focuses on the solution of dispersive partial differential equations using DJM in one, two, and three dimensions. Through a comparison of the results with the exact solution, the effectiveness of DJM has been established. The method has proven to be highly accurate and reliable, making it a strong tool for finding solutions to partial differential equations in a straightforward manner. The use of DJM has opened up new avenues for exploring the behavior of dispersive PDEs and has potential applications in various fields, including physics, engineering, and mathematics. The study serves as a valuable contribution to the field of numerical analysis and highlights the importance of developing innovative techniques for solving complex mathematical problems.

Conflict of Interest

Authors declare no conflict of interest as regards to publication of this paper.

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