Auditory and haptic feedback to train basic mathematical skills of children with visual impairments

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Abstract

Physical manipulatives, such as rods or tiles, are widely used for mathematics learning, as they support embodied cognition, enable the execution of epistemic actions and foster conceptual metaphors. Counting them, children explore, rearrange and reinterpret the environment through the haptic channel. Vision generally complements physical actions, which makes the use of traditional manipulatives limited for children with visual impairments (VIs). Digitally augmenting manipulatives with feedback through alternative modalities might alleviate them. We specifically discuss conveying number representations to children with VIs using haptic and auditory channels, within an environment encouraging exploration and supporting active touch counting strategies, while promoting reflection. This paper presents LETSMath, a tangible system to train basic mathematical skills for children with VIs developed through Design-Based Research with three iterations in which we involved 19 children with VIs and their educators. We discuss how the system may support training in number composition skills, and the impact of the different system features in slowing down the interaction pace to trigger reflection, in understanding, and in incorporation.

1.Introduction

Traditional manipulatives such as (physical) counting rods or tiles exploit the haptic channel allowing users to scan, grasp and count them. These objects (e.g. Cuisenaire Rods [12]) support embodied cognition [3,72], the execution of epistemic actions [31] and foster conceptual metaphors [49] during the processes common within the acquisition of mathematical skills. Vision typically complements haptics: a quick view allows users to perceive the state of the system and possible (inter)actions. Visual information can be perceived, processed in parallel and decoded very quickly [16,35]. Classical manipulatives are limited for children with visual impairments (CVIs), who only access information through (sequential) physical manipulation, taking longer and demanding more cognitive resources. Augmenting the sensory channels could increase the opportunities of embodied cognition for children with (and without) visual impairments (VIs). Complementing haptic with auditory feedback provides an alternative strategy to scan, grasp or count different elements in one go, instead of sequential one at a time.

The objective of our research was to build an innovative interactive mathematics learning tool for CVIs while generating design knowledge. We conducted Design-Based Research, realising an iterative, respectful of the context and theory-oriented design process [36]. We drew on previous research with the same population [52], and deepened on cognitive and learning theories, resulting in an initial concrete model [47] of augmented manipulatives, followed by three (iterative) designs and user studies to test different aspects of the system, the role played by specific features (particularly, different feedback modalities providing number representation in “one gesture") and the potentiality of being used autonomously for training math skills.

19 children aged from 6 to 12 years old and their teachers, recruited within special educational programs for CVIs, participated in the studies (one of them participated in the first and second user study). Children had VIs varying from low vision to blind, and many presented cognitive disabilities, labelled as Pervasive Developmental Disorder (PDD). They were in the process of developing basic mathematical skills, and most showed delays in their accomplishment, particularly in counting (or cardinality) skills.

The study of the initial prototype followed a Wizard-of-Oz approach, and involved six children and their teachers. Its outcomes led to a re-design of the prototype and a second test with six children, three activities (*Find a block, Composition and Broken Blocks*) and interviewing the school director. The third stage full system LETSMath (Learning Environment for Tangible Smart Mathematics) [43] incorporates Logarin [52], a narrative video game that challenges children to solve problems by composing numbers on a wooden number line using the tangible blocks. Eight children and their educators took part in this final evaluation, where two researchers and one educator undertook structured observations and conducted a short questionnaire after each session.

Each of our three research questions deals with a specific evaluation aspect along the problem-solving process:

RQ1: Understanding, i.e., do children understand the concrete model, specifically the physical static properties of the blocks such as shape and size, the digital feedback they provide and their informational relations (mapping)?
RQ2: Incorporation, is the digital feedback incorporated during: **a)** the block recognition process, **b)** the composition task?
RQ3: Reflection, to what extent the use of a structured input space can slow down the interaction pace and support children's more reflective strategies for solving composition tasks?

Our results suggest that the participants *understood* the concrete model, as they were able to train number composition skills in a potentially unsupervised composition task. The *incorporation* of digital feedback took place in a more nuanced way: the results suggest that all children incorporated digital feedback for recognition of the blocks, but only blind ones needed to use it during the composition task. Children complemented their different levels of limited vision with physical scanning, spatial memory and digital feedback when needed. *Reflection* is promoted by guiding the interaction in the input space as performing the composition directly in it may lead to more errors.

The iterative design and test, allows us to discuss detailed implications for digital feedback design through multiple modalities, which represents a substantial contribution in terms of design knowledge for digital/computationally augmented [56] manipulatives for CVIs.

The rest of this article is organized as follows: Section 2 presents background concepts related to interaction and cognitive processes, and on children with VIs. Section 3 discusses an overview of related work. In Section 4 we present our conceptual design rationale. In Section 5 we present the methods of the three user studies. In Section 6 we describe how the system evolved as part of the Design-Based Research approach, taking results from one study as the input for re-design the prototype for the next one. We explain the methodology used to validate the concrete model and the results of the first two user studies. In Section 7, we present the final design and implementation of the system and the final user study. In Section 8 we discuss findings and design implications. In Section 9 we present conclusions and in Section 10 we discuss limitations and future directions.

2. Background

In this section we present key concepts related to CVIs, of developing counting skills, and (embodied) cognitive processes that have been taken into account in the Design-Based Research.

2.1 Special needs of children with VIs in acquiring basic mathematical skills

The *International Classification of Diseases* differentiates VIs as moderate, severe and blindness, depending on the visual acuity [50]. Our studies involved children belonging to the three conditions, and we refer to these conditions as *blind* and *low vision*.

Haptic interaction is a natural strategy to compensate VIs, and has been exploited in ***Tangible User Interfaces*** (TUIs), based on physical objects and environments augmented with digital information becoming interaction devices [25].

Abstract information, like numbers, is usually represented by spatial structures typically perceived through vision. Number concept acquisition benefits from perceptual processes, as it can be regarded as the passage of a perceptual construal (I can see three things in front of me) towards an abstract one (3 = * * *) [17]. In this process, it is key to perceive the whole group and its components at the same time. Children start by understanding that sets have different magnitudes (cardinality) [17], then learn that each magnitude relates to an exact quantity, and finally that these quantities can be represented with symbolic expressions (numbers) [19].

Sighted children as young as three months can automatically detect exact quantities of one to three items without counting, which is known as *subitizing [13,30]*. It is understood that subitizing is one of the first forms of chunking faced by children and prepares them for number conceptualization [8]. The abstraction for acquiring cardinality, or the size of a group of objects, is more difficult for blind children, who do not generally perceive objects simultaneously but sequentially (touching them one by one) [35]. CVIs usually detect structures by touch [34], and develop strategies for **active touch** to successfully perform counting [63]. Active touch consists of three main stages: after a preliminary scan (analog to a glimpse) they search for perceptual keys for counting (e.g., detecting dots), and, finally, they usually partition the space by setting aside already checked elements [63]. However, this sensory system is slower than vision, requiring more working memory and cognitive load [47].

Enabling the simultaneous perception of a group of objects would be key to foster blind children's capabilities to acquire the number concept, through an immediate understanding of the cardinality of a group. Specifically, the task of additive composition has been claimed to favour cardinality acquisition [18,70], implying number recognition, grouping and awareness of the composition result. Especially, digitally enhanced manipulatives are a valuable assistance for this activity [51].

Non-speech audio can be used [22] to represent abstract concepts. Auditory information may be a crucial complement [35], although sequential auditory feedback cannot convey as much information as the visual channel [7,16]. This is one of the reasons why we propose an interactive system that combines tactile and auditory feedback, reinforcing the complementary perceptual channels when vision is not available. The cardinality of a set can be represented as a group of sounds. Indeed, aligned with Leuders [35], rapid sequences of sounds as representations of quantities, could be an alternative for people with VIs, beyond counting slower sequences of beats, as fast auditory patterns may be perceived as a unique event but include a rapid counting of components at the same time [57]. The experience could be closer to the simultaneous sight of a group of objects; what in numerical cognition theory is called auditory subitizing [29]. On the contrary, subitizing is difficult for the haptic modality and limited to specific actions [54].

2.2 Physical interaction and cognitive processes

2.3.1 Physically distributed learning theory

The application of embodiment theories to interaction design for educational purposes has been summarized in the *physically distributed learning* (PDL) theory [44]. PDL [44] stresses the importance of allowing children to rearrange the environment in order to represent the solution to a problem. Reinterpreting the environment allows children to reveal the abstract structure of the underlying operation. These operations foster *conceptual metaphors* [33], i.e., analogies that enable the understanding of abstract concepts in terms of more familiar and understood concrete ones. For instance, a typical **spatial representation** reflecting number knowledge is the *number line* standing for ordinality and making explicit relationships among cardinals [26].

2.3.2 Pragmatic and Epistemic Actions

When solving a problem, actions performed to get closer to the goal are called *pragmatic [31]*. *Epistemic actions* are those performed to reveal information that might be partially hidden or hard to detect - thus, improving cognition by reducing the memory load, the number of steps and probability of error in mental computation [31]. In the context of manipulatives for mathematics learning, epistemic actions might allow children to save cognitive resources and discover more or better strategies to solve a problem [31].

2.3.3 Concrete Models

Designing an interactive learning experience with manipulatives implies elaborating a concrete model [47], by proposing a certain placement of objects and rules about how they interact and respond; the model involves interconnected knowledge of physical objects, actions performed on them and symbolic representations [44], besides their material features (as shape and size). The concrete model should reflect in a directly perceivable way the abstract relationships to be learned.

Within the stages of active touch counting strategies of CVIs*,* **concrete models** should foster the possibility of arranging the space in order to facilitate *preliminary scanning*, giving salient keys for *counting*, and provide a spatial structure that favors *partitioning*. Moreover, these concepts are especially relevant to frame the presentation of physical models for abstract ideas, and this is crucial for CVIs.

2.3.4 Tangible Learning Design Framework

Physical objects, digital objects, actions, informational relations and learning activities are the elements of the taxonomy proposed within the *Tangible Learning Design Framework* (TLDF) [5]. This framework provides a structure to design TUIs for learning purposes and design guidelines, approaching the design from a cognitive perspective, which is very relevant to our research. We apply the TLDF to describe the design of LETSMath and discuss the results. This way, the connection of our work with previous research [4,5,42] becomes more explicit and intelligible. In particular, the design implications resulting from our study are classified under the TLDF categories, with the goal of making them more accessible to other researchers.

3. Related Work

This section introduces and discusses relevant previous work on manipulatives for mathematics learning, both traditional and augmented ones. Then, we focus on TUIs specifically for children with VIs, first in general, and then, in particular for mathematics learning.

3.1 Manipulatives for Mathematics Learning

For many years mathematical manipulatives have been used in schools for children with and without VIs. Building blocks and tiles, such as the Cuisenaire rods or the abacus have been used to instruct mathematics in early stages [28]. They afford physical exploration but mostly rely on visual abilities for an optimal comprehension of the metaphors behind them. Currently, besides them, VI children use other ones specifically designed for them, such as the Taylor Frame for arithmetic calculations[11] or braille math blocks [75]. However, none of them exploit the auditory channel as an external representation or to provide guidance during the learning task.

3.2 Digital manipulatives and reflection in learning

Traditional manipulatives enhanced with digital technologies are known as *digital manipulatives* [56]. Those focused on modeling abstract structures (such as the number composition) are known as "Montessori-inspired Manipulatives" (MiMs)[74]. Digital manipulatives and MiMs in particular, are key for children with VIs, as they afford haptic and audio feedback compensating for the lack of visual information. They also represent an opportunity to engage multiple senses in a constructive process [5,6,74]. However, exploration and engagement of different senses does not always shape a reflective activity or ensure reflective thinking, which is important in the meaning construction process [15]. Ackerman [2] proposes a model of cognitive growth combining exploration and reflection stages in an “ongoing dance”. Digital manipulatives and embodied interaction learning environments, should afford both easy and fast exploration, and encourage reflection to balance the “ongoing dance”. Previous research has pointed out the importance of mechanisms to promote reflection within embodied interactive learning experiences [6,37,39,42].

In interactive systems, feedback can be an important behaviour regulator. Two kinds of feedback are considered in TUIs: *process feedback* refers to the real-time coupling between user actions and system augmentations, i.e., users get continuous feedback about their actions [21,23]; *task feedback* is about the correctness of the solution of a proposed task, e.g., the evaluation of a number composition. Previous research suggests that providing continuous feedback at the process level encouraged students to dive into action and proceed with trial and error strategies [24,42]. Thus, it might result in more intuitive interaction or that solutions become easier to find, but this does not mean that reflective thinking is taking place. Actually, students who did not have immediate process level feedback reflected more and reached a higher learning gain [24]. Thus, different feedbacks and delays might slow down the interaction pace, having an impact on the availability to reflect. Actually, several studies suggest that slowing down the interaction pace may trigger reflection [5,39,42,55]. In a similar vein, the physical disposition of inputs and outputs may also cause delays favouring reflection or not [6].

Previous work explored digital manipulatives for mathematics learning [42,62], including ours, CETA, a system that enables the use of tangibles, similar to Cuisenaire rods, to solve additive tasks embedded in a digital game [42]. This approach is promising for younger children: they improved at mathematics after playing 13 sessions of the game that required the use of manipulatives to solve addition and subtraction tasks [51]. Other examples are the FlowBlocks, SystemBlocks, SmartBlocks [20,74]. However, none of these approaches adapts the haptic and audio feedback for CVIs, for instance, the digital feedback provided in SmartBlocks takes place through a GUI.

3.3 TUI as learning tools for children with VIs

The development of interactive tangible maps for CVIs has attracted increasing interest in recent years. MapSense [9] is a multi-sensory interactive map specifically designed for CVIs. It incorporates passive haptic feedback as a tactile raised-line map overlay over a touch-screen, auditory, olfactory and gustatory feedback. After conducting several user studies, the authors propose valuable design guidelines stressing the importance of an **inclusive and collaborative design** using multi-sensory interactions, and the efficacy of storytelling to stimulate engagement and reflection. Other TUI for CVIs that stress on the relevance of inclusive and collaborative design are [16,48].

Recently, several studies have been also focused on teaching Computational Thinking to CVIs combining TUIs and audio feedback [32,48], some of them affording CVIs to spatially move mainstream robots [1,10,58].

Jafri et al. [27] developed a tangible tabletop, as a low-cost (based on 3D printed objects, a lamp and a USB camera) system for teaching the tactile shape perception and spatial awareness for CVIs. Such tabletop TUIs (also in [40]) require a non-trivial setup and dedicated classroom space, i.e., are not portable. Rühmann et al. [59] present a tangible system for CVIs for geometry learning based on an Android app and *Appccesories*, which is more portable than previous examples.

Other examples of TUI for CVI are [53] and our iCETA [52], which used tangibles inspired in Cuisenaire rods. Children compose blocks to solve basic addition and subtraction operations and the experience is shaped within a narrative game. However, the digital feedback was only provided by the computer, i.e., the physical blocks are passive. Thus, tangibles could be enhanced to provide multisensorial information to help in the abstract representation of numbers. Most of these cases employ sound as verbal feedback. Other developments have exploited non-speech audio to convey graphic information implying some kind of “auditory substitution” [45,71] as intended in our approach.

To the best of our knowledge, there is no tangible and portable system providing active haptic and acoustic feedback to specifically train additive composition and number line representation in the VIs context. To address this gap we designed LETSMath (Learning

Environment for Tangible Smart Mathematics) consisting of a set of tangibles electronic blocks with tactile and auditory feedback, a working area, and a computer-mediated narrative game for mathematics learning. The use of multisensory feedback provides opportunities for the integration of the system in a classroom involving students with different visual abilities, thus promoting inclusion. Metatla et al. [46] , found that this kind of proposal affords collaborative activities that are engaging for both sighted and VI children, something important given the growing trend to include CVIs in mainstream schools.

4. Design rationale

In this section, we first introduce a set of general design requirements. Next, we provide the conceptual design of LETSMath which was initially implemented for the first user study and iteratively evolved until the final design introduced in section 7.

4.1. General Design Requirements

Based on the related work, theories discussed in the background section and on previous research [42,52] , we identified the following design requirements (DR) that have driven our design: **DR1)** ***portability and size****:* design suitable technology in terms of portability and size that could be incorporated in classrooms [16] (see 3.3), **DR2)** ***inclusivity****:* design inclusive and collaborative environments where CVIs and sighted children can work together having shared experiences [16,64,71] (see 3.3), **DR3) storytelling**: as a powerful tool to stimulate engagement and reflection [9], and favouring to train mathematical skills autonomously through a computer game (see 3.1 and 3.3).

In addition to these wider principles, the following requirements are specific for CVIs in a mathematics learning context: **DR4)** ***suitable for active touch strategies:*** design systems where children can easily understand spatial structures and are able to organize the space in order to perceive informational relationships (see 2.3.3) **DR5)** ***continuity:*** build on previous tangible manipulatives (see 3.1), **DR6)** ***digital enhancement for number recognition:*** provide digital feedback, such as vibration and/or sound [16,41], into digital manipulatives to provide abstract number representations [35].

4.2. Conceptual system design

In this section we describe the system in terms of the TLDF.

Learning Activity

The goal of the system is to help children acquire the concept of numbers and cardinality. Two key concepts in the cardinality acquisition are [57]: additive composition - understand how numbers can be composed by smaller numbers - and the number line representation - understand that numbers can be represented on a line either horizontal or vertical -.

The learning activity is conceptually a narrative game conveying information through the auditory channel. To fulfill the inclusivity requirement (DR2), it has a GUI suitable for sighted and low vision children. It incorporates storytelling elements (DR3) to increase the engagement as well as provide guidance to help children playing autonomously. During the game, children will be challenged to compose numbers using tangible objects.

Physical Objects

The system is conceptually a Token+constraint interface [68], where the tokens are physical **blocks** used to compose numbers on a stable region called **working area**.

Blocks (tokens): We observed that educators make their own adaptations to classical manipulatives, for example, adding physical marks to traditional Cuisenaire rods [12] to enable physical exploration and counting. Our augmented blocks design is inspired (DR5, continuity) in (augmented) Cuisenaire rods [42,52]. Blocks represent numbers, conveying such information through multiple modalities: haptic, visual and auditory, in particular, through its respective size, colour, physical cues and braille symbols.
Working Area (input stable space): The working area has two roles, it is the input space and, at the same time, a stable structure. While blocks allow physical rearrangement of the environment and active exploration as PDL theory suggests [44], the (representation of the) number line is the stable structure which scaffolds the construction of the composition [6,68]. This fixed stable area decreases the demand of visual attention.

Digital Objects

Digital information has a key role in the learning experience since it is through it that we intend to enhance the number representations and therefore the composition training (*digital enhancement for number recognition* (DR6)) . Thus, we conceptually distinguish between two main pieces of information to be conveyed through digital strategies: the number to be composed (N) and the number represented by each physical block.
Composition sound: This sound will represent the number to be composed (N), i.e, the challenge.
Unit feedback: This is the digital feedback that blocks should individually provide using sound and vibrations to represent its cardinality. The design of acoustic and vibrational representations favours number units perception contributing to the *digital enhancement for number recognition* (DR6).

Actions

The actions that can be taken on the physical blocks have to be suitable for active touch strategies (DR4) and inclusivity (DR2). Thus, the implementation of the system should afford the execution of relevant actions in a mathematics learning context.

When it specifically comes to the composition task, joining and grouping objects are relevant actions that allow children to modify the environment and reduce cognitive load. Such actions have been observed as relevant epistemic actions in previous work [40] and originally emerge from traditional manipulatives such as Cuisenaire rods, which contributes to fulfilling continuity (DR5).

Children should also be able to intuitively trigger the unit feedback of the blocks to perceive the digital representation of their cardinality, taking advantage of the digital enhancement for number recognition (DR6).

Informational Relations

Informational relations make reference to the mapping between physical objects, digital objects and actions. For example, how children do in order to trigger the *unit feedback* of the physical blocks. As this category depends on the actual implementation of the system, it will be discussed in the context of the iterative development (Section 6) and in the final system design (Section 7).

5. Methods

Context: The studies took place in schools or centers for children with special needs in Uruguay (Study 1 and 2) and Spain (Study 3) (see Appendix A - Table A1). The studies were conducted by three to five researchers and one educator (Study 3) who acted as participant observers in each case and interviewed participants at the end of their session (Studies 1 and 3),(Appendix A, Figure A2 (study 1) and A14 (study 3)).

Participants: The studies included 19 participants (6, study 1, 6 study 2, 8 study 3) aged 6-12 (7-12 studies 1 and 2, 6-8 study 3), 8 with low vision (2 study 1, 2 study 2, 4 study 3), the rest were blind. 8 of them had PDD (6 study 1, 3 study 2, 0 study 3), (see Appendix A - Table A1). We also had an interview with the director of one of the schools (study 2) and the pedagogical technical director of the center and two educators (study 3).

Data Collection and Analysis:
We recorded videos of all the activities (study 1 and 2) and we conducted structured observations of the videos (study 1 and 2) and directly during the study (Study 3),(see Appendix A, Figures A3, A4, A5 and A13).

We were not allowed to record study 3. All the data had to be observed and annotated in real-time. As a consequence, in some cases we missed the observed events. In particular, we missed 8,44% of the observations of the composition strategy and 25.9% (see Table 3) of the answers justifications during the Broken Blocks game. After the sessions, the two researchers reviewed all the observation sheets and discussed until agreement was reached.

The researchers of each study used content analysis for the interpretation of the structured observations, until agreement on the topic of each study [60]. Thus, we integrated triangulation of participants and observers in each study for an in-depth understanding of the study [14].

6. Iterative Design-Based Research

Within our Design-Based Research approach we carried out three iterations of prototyping and corresponding user studies.
Our first prototype introduced *active feedback* of the blocks, through sound and vibration instead of using only the computer for feedback and was followed by an exploratory study (1) to validate the physical shape, and whether the active feedback allowed CVIs to understand the number representation.
The improvements resulted in a more complete second prototype, and in the user study 2 where number composition with blocks was tested (different recognition and strategies emerged), as well the understanding of the informational relations through a Broken Blocks activity.
The third and final prototype incorporated the wooden number line as input space and the narrative game, and the user study could better help to answer more completely the research questions. The following table describes more precisely the different learning activities carried out, and relates them to the research questions.
Within the user studies, activities were individual. Parents or tutors were previously informed of the activity and they provided consent for children's participation. Children gave oral assent to participate and understood that they could stop anytime. Video recordings (studies 1 and 2) were only accessible to the researchers for video analysis.

For further information regarding activities, participants, data collection methods and system status at the time of each user test see *Appendix A - User studies detailed information*.

Learning Activity	Description	RQ
Exploration	The system provides the composition sound according to the blocks placed on the working area. It is an introductory activity where there are no right or wrong answers. The objective is to warm-up and address RQ1.	RQ1
Composition (Study 1 guided by the system. Study 2 guided by the experimenter)	The system or the experimenter says aloud a number and the child has to compose it on the working area (table or number line depending on the study). If the solution is correct, the child receives positive feedback (from the system or experimenter) and another number is said, continuing the challenge. It addresses (RQ2-b).	RQ2-b
Order	Children are instructed to order three blocks of different values (values one, two and three) in increasing or decreasing order, as they prefer. It addresses the comprehension of the concrete model (RQ1) in terms of physical representation.	RQ1
Broken Blocks	The experimenter explains that some blocks are broken without specifying that the feedback is the dysfunctional element.	RQ1

	Children are asked to identify the broken blocks and also to explain their answer. It addresses the comprehension of the concrete model (RQ1) in terms of informational relations.	
Find a Block	In this activity, the participants are asked to find and give to the experimenter a specific block. The four blocks are on the table and, for example, the experimenter asks “could you give me block number 3?”. This activity was designed to observe the incorporation of digital feedback during the block’s recognition process, i.e., to address RQ2-a.	RQ2-a
Logarin (composition computer game)	The Composition activity is shaped by the narrative videogame Logarin. Children have to perform compositions on the wooden number line working area (see section 4 for a detailed description). It addresses RQ1, RQ2-b and RQ3.	RQ1 RQ2-b RQ3

Table 1. Description of the learning activities

6.1. First User Studies

Prototype and User Study 1

Procedure
The individual activities were carried out in a classroom followed by an interview (see Appendix A - Figure A2 for the interview’s script). The interview took the form of a radio program in order to make it more engaging for the children. During this interview an educator from the school was present.
In four cases, the blocks were switched ON at the beginning, thus playing the *unit feedback*. In the other two cases, the blocks were OFF to focus on the identification of physical aspects: size and unit markers. First, we presented blocks 1 and 2 to the pupils and we guided them to explore size, unit markers and unit feedback if available. We also guided them to identify the relationship between the value of a block and its size. After this brief introduction, children were guided through the learning activities Exploration, Composition and Order (see Table 1).

System
Physical objects: For this study, we resorted to a wizard-of-oz strategy, manually sending Open Sound Control (OSC) [73] messages to the system to simulate touch and block placement on the working area detection. We tested the system using the PARALLEL unit feedback (see Figure 1) in a loop, i.e., blocks played *sound and vibrations* as long as they were being touched. A rectangular wooden working area was used during this study (see Figure 2), such area had no detection capabilities but it was simulated by the wizard strategy.

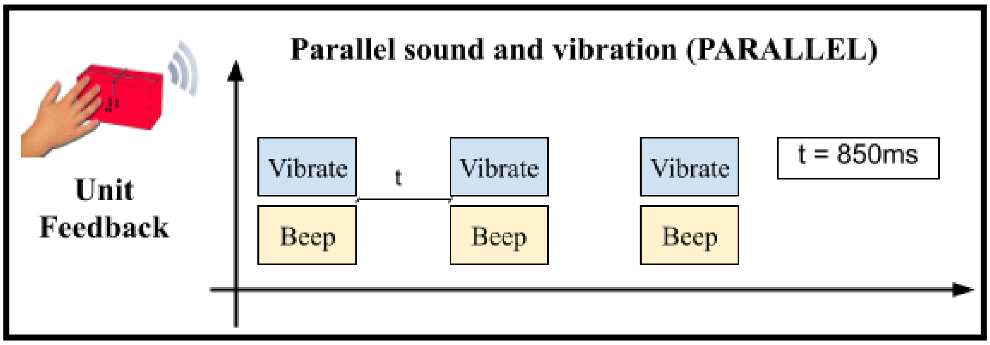

Figure 1. Parallel sound and vibration *Unit Feedback*

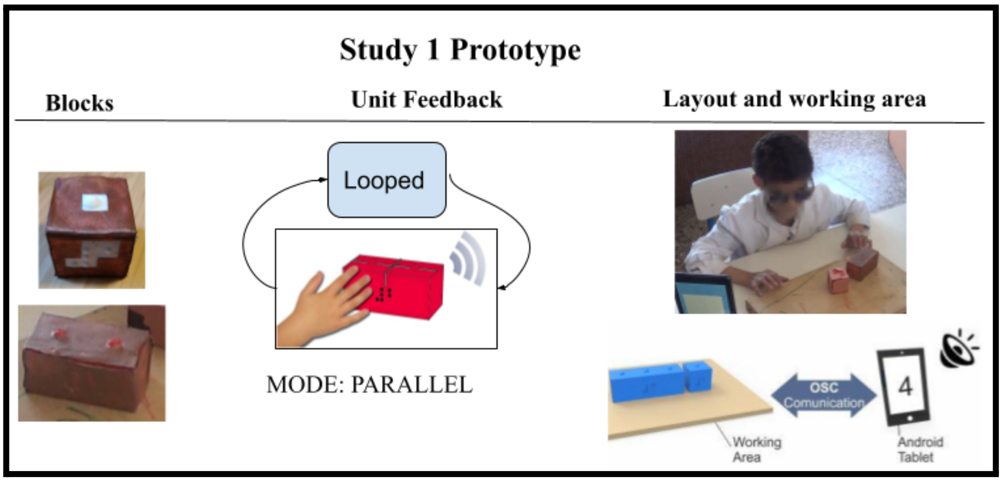

Figure 2. Initial prototype used during the Study 1

Results

This study allowed us to validate that children partially understood the concrete model (RQ1), and to gain insights to improve the design. Next, we analyze the main results through the TLDF categories. For an extended version of the results see Appendix A - Study 1 - Observations and Results.

Learning Activity: The activities were quite straightforward and all the participants were able to understand their goals. However, they had no storytelling that should engage the children.

Physical Objects and Actions: Only two participants used the **unit markers for counting** to discover the block value. We observed that the big distance between the unit markers made it difficult to discover them all. An educator suggested adding the guiding lines between unit markers to lead the exploration to the next point. The other participants recognized the blocks through size and then by spatial location.

Digital Objects and Informational Relations:

We observed that many children did not take advantage of the *unit feedback* to recognize the number represented in each block, although all of them showed enthusiasm with this feature. Despite the sound, they took the effort of measuring the size of the blocks or used the unit markers.

In addition, each block reproduced the unit feedback in a loop while being touched by the user. We observed that despite the lapse of time between repetitions the loop was confusing their counting. Thus, for the following prototypes and studies, we provided one-time unit feedback until blocks are released and touched again.
A **composition sound** was played by a tablet when a block was placed on the working area, i.e., if the block number 2 was placed on the working area then the tablet reproduced a sequence of 2 sounds. Such feedback is independent of the **unit feedback** played by the blocks when they are touched. When both events happen at the same time, i.e., a block placed on the working area is also being touched, the composition sound and unit feedback are played simultaneously. We observed that multiple simultaneous feedback can hinder the understanding and make the situation hard to interpret. Thus, in the prototypes following, blocks only give unit feedback when the working area is empty.

In sum, this study allowed us to partially answer RQ1, i.e., children understood the physical model, they associated the size of the blocks with the units, but it showed some design drawbacks which were improved and tested in the second and/or third study.

Prototype and User Study 2

Besides validating the physical design improvements (RQ1), and that the informational relations were understood by children, we aimed to observe whether the different available representations support the number/block recognition (RQ2-a); and observe which elements of the model were used during the composition task, specifically if digital feedback was incorporated (Q2-b).

Procedure
At the beginning of the session the experimenters explained the relationship between the size and unit markers. Additionally, when unit feedback was enabled, the experimenters explained the mapping, e.g., “this block represents a number 3 and vibrates three times”. After a brief introduction to the blocks, participants completed the activities always in the same order: Find a block, Composition and Broken Blocks.

System
Blocks: Based on the drawbacks of study 1, we incorporated the guiding lines in the blocks and tested three different *unit feedback* modes: OFF (no feedback provided), one-time PARALLEL (same as in previous study but not looped) and VIBRATION, a new modality consisting of *fast vibration* stimuli (see Figure 3).
Learning activities: We used three learning activities: Find a Block, Composition (guided by the experimenter) and Broken blocks (see Table 1).

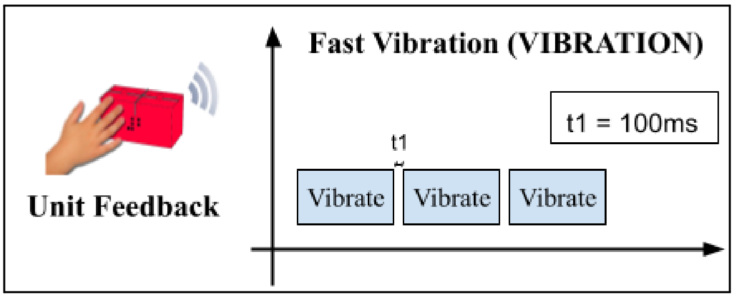

Figure 3. Fast vibration Unit Feedback

Working area: The two blind children in this study used the number line working area (see Figure 4). Although the activities do not require using the working area, we still tested it to gain insights about its affordances to guide blind children during the composition activity. For the rest of the activities and participants, the problems were solved on a regular desk.

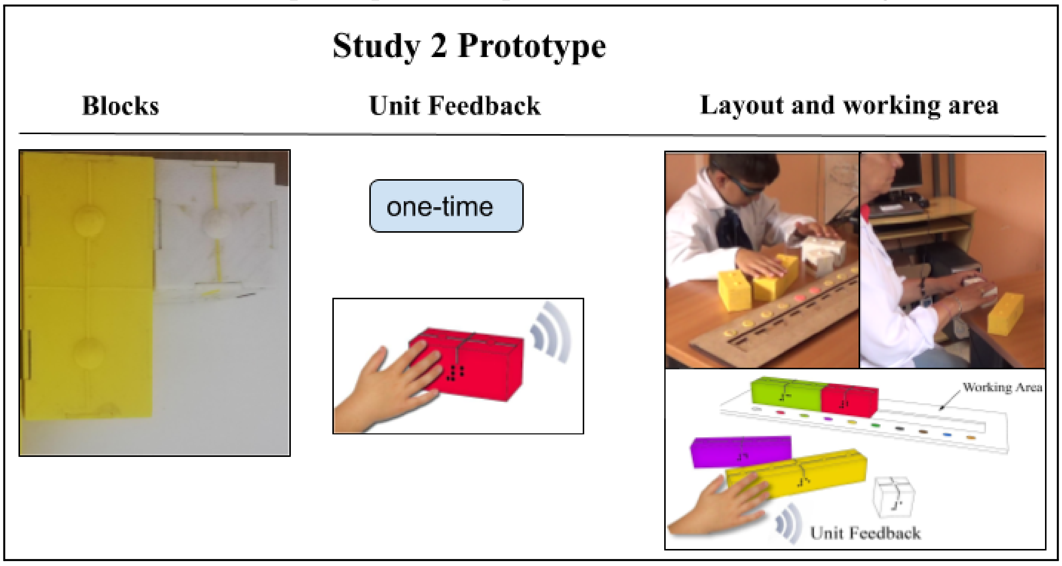

Figure 4. Prototype used during the Study 2

Results

In general terms, children varied the actions depending on their visual impairment and the specific task. In the composition task, most showed understanding on how to use the blocks to compose bigger numbers and followed coherent strategies rather than trial and error approaches. Next, we present the main results for each activity. The extended version of the results and the quantitative analysis are available in In Appendix A - Observations and Results - Study 2 - Results.

Find a Block

During this activity we focused on the observation of the incorporation of digital feedback, thus providing significant insights towards addressing RQ2-a. In general, children were able to *find a block* without difficulties. They understood the proposed model including the availability of different external representations to recognize the blocks through multiple modalities. Their different visual abilities shaped the preference for certain representations and perceptual channels.

Children with low vision privileged the visual modality to recognize the blocks, they also used their hands to count the number of dots (*unit counting*) when no digital feedback was available. However, when other sensorial representations become available, we observed a change in the block recognition pattern. They still greatly relied on visual cues but partially replaced *unit counting* with *fast vibration* or *sound and vibration*. That is to say, when unit feedback becomes available, children with low vision incorporate it to *find a block*. This could be due to the novelty effect, real usefulness for block recognition, or both.

For **blind children**, *unit counting* was the most employed strategy to recognize the blocks, followed by *size estimation* using both hands. When sound and vibration representations became available they tended to exploit such new representations to recognize the block:

feeling vibrations or listening to the sounds of each block. Thus, for those who were blind, we observed a more distributed use of the sensorial representations whereas children with low vision showed a strong tendency to use vision in detriment of other sensorial representations. In addition, when digital feedback became available, blind children diminished the use of unit counting.

To sum up we might say that **children tended to adopt digitally enhanced strategies to *find a block***. Interestingly, **fast vibration emerged as a valid strategy for number recognition**, even when children had little time to get used to it.

Composition
Compared to the recognition task, during composition **children with low vision** tended to decrease observation and make more use of unit counting. Interestingly, children with low vision showed a higher use of the observation strategy under the three feedback conditions during the find block activity than in the composition activity. Additionally, they abandoned the use of the *unit feedback*. They did not make use at all of *fast vibration* and barely used *sound and vibration*.
Our hypothesis is that **for low vision children, when it comes to a composition task, *unit feedback* does not afford any benefit or cognition offloading**. That is to say, *unit feedback* does not help to compute the additive composition result since it only provides feedback representing each individual block. On the contrary, physical unit markers allow children to touch and count, aiding them to compute the composition result. It is worth noting that **unit counting allows a larger cognitive offloading than the transient auditory/vibratory stimulation**. For this more demanding task, it seems reasonable that children relied on a slower but more reliable resource as haptic counting.
For **children with low vision, results show that the horizontal line is the most used arrangement** (see Figure 5), this may facilitate counting and it is also a signal of the integration of the number line concept. Children count from one extremity to another to ensure that they counted all the unit markers and therefore avoid accidental repetition. Thus, they rearrange the physical environment to offload cognition as the PDL theory explains [44].

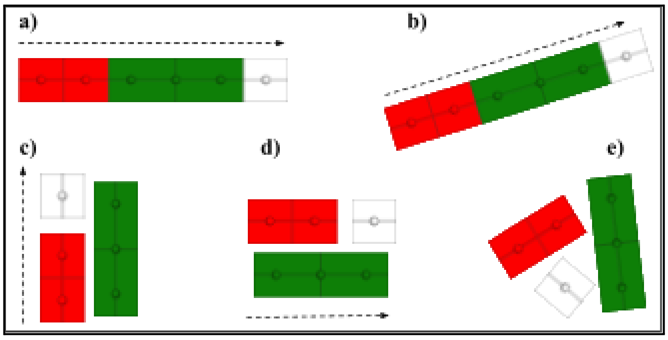

Figure 5. Observed composition arrangements. a) Horizontal line, b) Diagonal line, c) Vertical 2d, d) Horizontal 2d, e) No order

For the **blind children**, on the one hand, we observed that they relied on *fast vibration* and *sound and vibration* in a similar proportion during both activities. On the other hand, when digital feedback was available during the composition activity, they relied on the size estimation of blocks more than dot counting, a phenomenon that was not clearly observed during the find a block activity. These results suggest that, unlike children with low vision, for **blind children digital feedback is still useful in the context of the composition task**. Actually, they replaced in a higher proportion the size estimation action than the dot counting action. Thus, **dot counting might be an action more useful for the composition task than size estimation**, which makes sense since it allows to count the result of a composition, and even more when blocks are joined/aligned as it was the case for blind children since they used the number line working area.

Broken Blocks
Overall, we observed that **children were highly efficient in assessing if the digital mapping matched with the physical block**, and when asked to report why, to reveal their actual perception.

We also observed that for children it was easier to understand the PARALLEL *unit feedback* than the VIBRATION mode (see Appendix A - Observations and Results - Study 2 - Results). It seems that the slower and parsimonious strategy was more successful to convey the number representation without ambiguity.

One of the blind participants was not able to perceive and count sounds because he was always counting the physical unit markers, in this case, the experimenter held his hand gently constraining movement in order to avoid physical dot counting and asked him to count the sounds. Then, he was able to perceive the digital feedback correctly. This exposes the **predominance of the physical exploration over the digital feedback**, at least, during the first approaches with the system.

7. Final Design and Study 3

7.1.Final Design

We detail the final version of the design rationale of LETSMath in terms of the TLDF [5] categories as before.

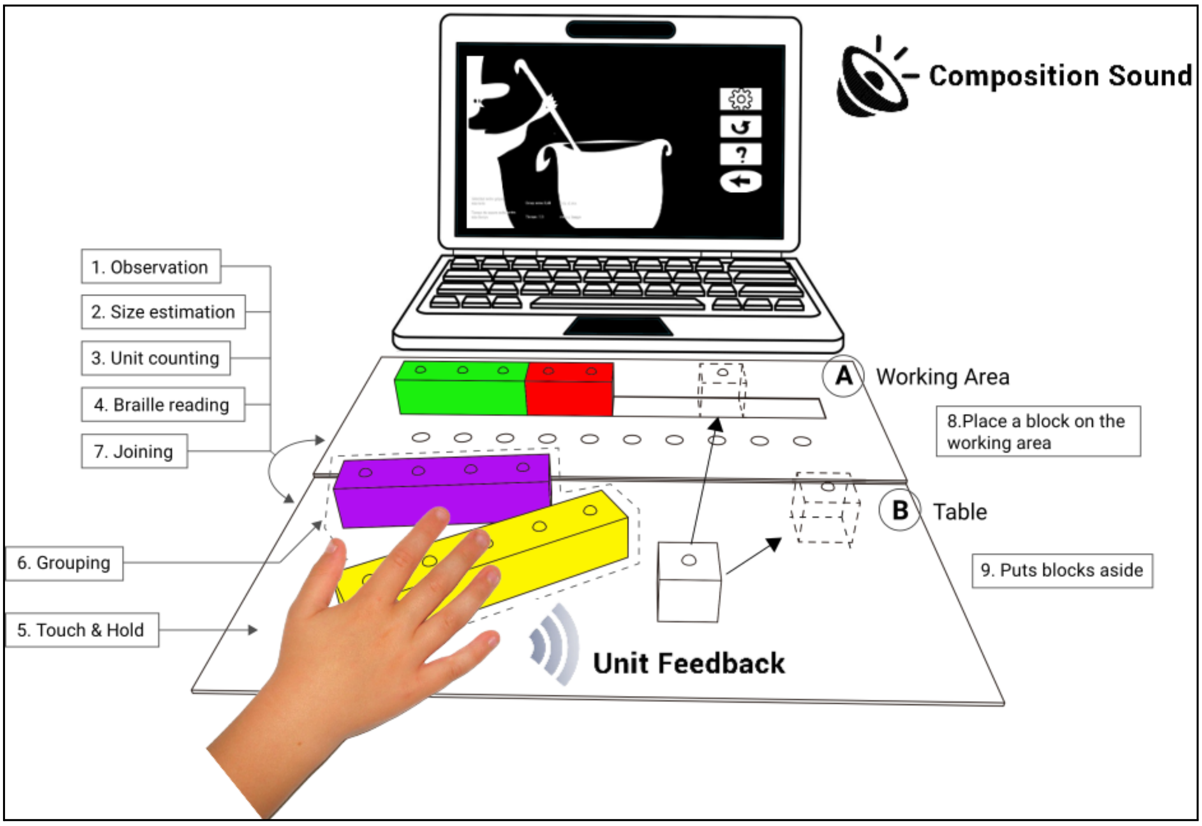

Figure 6. LETSMath prototype environment

Learning Activity

We implemented the learning activity through an adaptation of a narrative computer game (DR3), Logarin [50]. The game is based on an audio interface complemented with a high contrast GUI where a representative drawing of each level is displayed. The GUI is suitable for children with both full and low vision, but it is not essential to solve the mathematical challenges during the game, so it can be played by blind children (DR2). The game narrative (DR3) is about a 'recently graduated wizard' named Logarin who needs help to make different spells. Throughout the levels, Logarin has to perform different actions (stir, knock on a door, add ingredients, etc) N times in order to prepare the spells. The child has to compose the number N by combining blocks. Once they solve the problem, the wizard finishes the spell and advances to the next level. The activity of additive composition fosters mastery of number combinations, an important milestone in cardinality acquisition [61].

Physical Objects

Blocks: The length of a block is proportional to the number it represents, which is also identifiable by a different color (see Figure 7). The actual dimensions of block 1, which represents one unit, are 5 x 5 x 5 cm, and those of block 2, which represents two units, are 10 x 5 x 5 cm and so on. In order to adapt the Cuisenaire rods for CVIs, we included some extra physical features (see Figure 7), but they are still appropriate for children with full vision too (DR2). Each block has a physical representation of the number: equally spaced circular spots as a 3D relief as *unit markers* - for active touch and count (see 2.1), (DR4), as epistemic actions that offload cognition (See 2.3.2) [42] -, and the braille sign of the number - which is a symbolic representation of the quantities. We also included magnets inside the left and right sides of the blocks in such a way that they always attract one another. This encourages the joining action, creating (composing) a longer block and mimicking the number line representation, as a **physical representation** (See 2.3.1) that supports an **epistemic action** (See 2.3.2). Moreover, magnets have a novelty effect and increase children's enjoyment and engagement [42]. A guiding line connects the unit markers to support haptic exploration. Compared with usual Cuisenaire rods the size of the blocks is bigger to facilitate their appropriation by children with low vision.

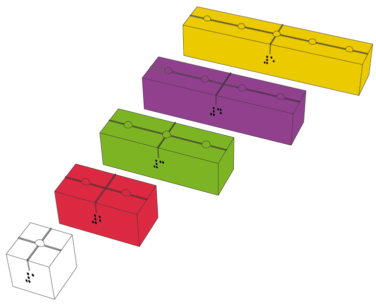

Figure 7. Physical blocks ranged in length with unit markers, braille signs and guiding lines to support exploration and identification

Number line working Area (stable input space): The working area is a board, as a physical representation (See 2.3.1) of the number line (see Figure 8) with a guide to place and fit the blocks, so that children can create their compositions. It includes braille signs of the numbers and magnets detected by the blocks when they are placed on it and sends a notification to the laptop. Thus, the system only evaluates the compositions performed on the working area. The board measures 61 x 16 x 1 cm, thus it can be placed, stored and easily transported between classrooms (DR1) without any special requirement. The shape of the number line working area physically constrains children to submit the solution using the number line representation.

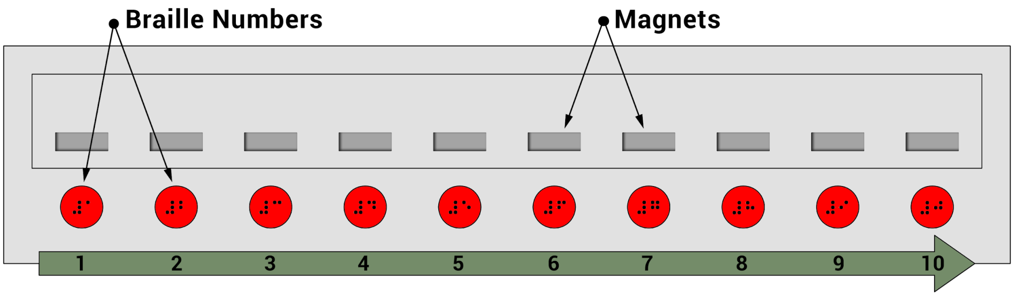

Figure 8. Working area.

The system is *suitable for active touch strategies* (DR4) which are developed by blind children when counting through tactile patterns [35] (we recall the typical strategy, preliminary scanning, count organizing and partitioning). In order to help children in their first approach to the system, the game asks them to clear the working area during the tutorial and the first two levels, i.e., they are forced to follow the composition strategy a) or b) as discussed later. For levels 3, 4 and 5 they are free to work without clearing the working area, so they can follow the strategy c).

Digital Objects and informational relations

There are two different kinds of digital objects. First, Logarin as the main character of the storytelling. And second, the digital number representations.

Logarin, whose voice guides the user through the game, asking for help, giving hints and congratulating the child when the answer is correct. Some high contrast pictures are displayed on the screen (see Figure 6).

Digital number representations
Composition sound: A thematic sound represents the target number to be composed (N) depending on the context of the story, e.g. the sound of drops when Logarin is preparing a spell or of knocks to open a magic door. The children have to pay attention to how many times the sound is repeated (N) and compose a block of the size N on the working area.
The *composition sound* provides rhythmic cycles of consecutive sounds (see Figure 9-a). This is followed by a hint, provided by Logarin, that depends on the current solution on the working area, for instance, if N = 4 and only block number 2 is placed on the working area, the hint might say “come on, try adding more blocks to the solution”. This cycle, thematic sounds followed by a hint, is looped until the correct solution is composed on the working area. When this happens, a special sound (called *block sound*) is played along with the thematic sound instruction just to indicate that the solution is correct (see Figure 9-b). After this, Logarin congratulates the child and the next level starts.

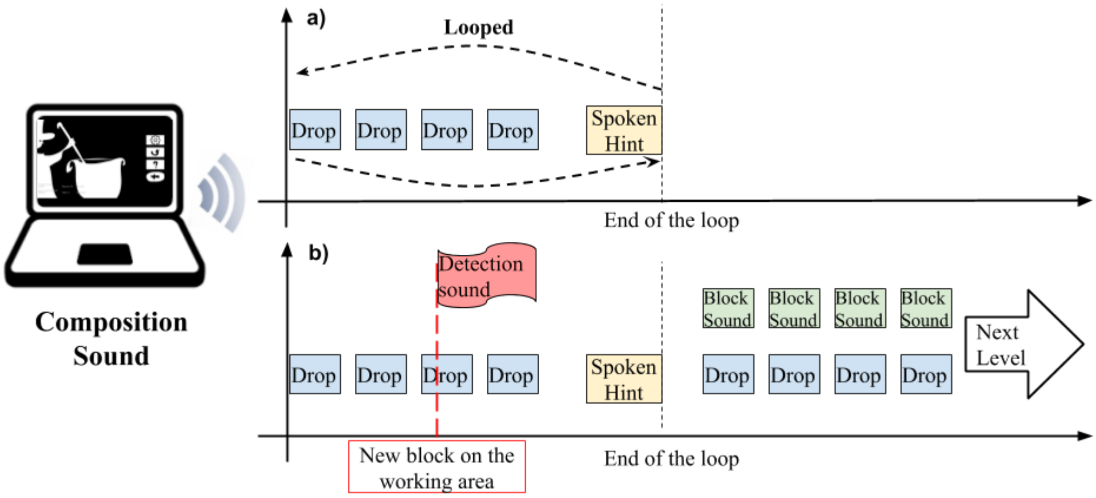

Figure 9. Composition sound. a) Looped composition sound b) After composing the correct solution

Unit feedback: While a block is being touched it plays the *unit feedback* composed by sounds and vibrations. This feedback represents the units (cardinality) of the block, i.e., a pattern of sound and vibrations is repeated matching the units of the block. The final version of the system combines, both the fast vibration and the slow beep feedback in sequence; the *sequential unit feedback* (SEQUENCE, see Figure 10). Actually, the vibration also generates an intrinsic collateral sound, i.e, vibration might be perceived by both the haptic and auditory channels. Thus, there are two patterns, whose key difference is the stimulus frequency, high and low.

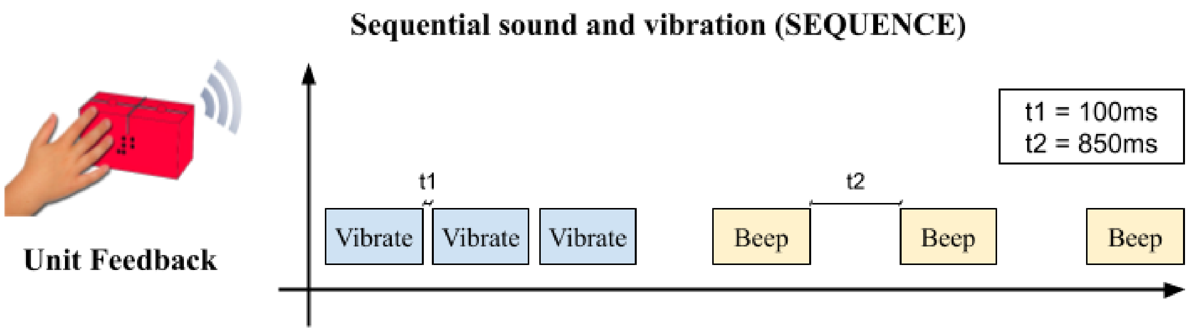

Figure 10. SEQUENTIAL *Unit Feedback*

Vibrations are presented with 100 ms inter-stimulus intervals while beeps have an inter-stimulus interval of 850 ms. Such a slower beeps pattern is easier to count, but there is a risk of children getting stuck in counting strategies, preventing them to acquire abstract number conceptualization [35]. High-frequency patterns (lower inter-stimulus intervals), may allow a kind of simultaneous perception of the whole and the parts in a similar way as visual perception does [35]. The automatic detection of small quantities through the auditory channel (auditory subitizing [29]) is possible when inter-stimulus intervals are around < 100 ms [65,66]. Furthermore, sounds played at less than 200 ms start being perceived as a continuum [57].

Actions

In this section, we summarize the actions performed with the blocks, describe how the different system design features encourage children to discover and perform such actions, and discuss their role in the problem-solving process. Some of them do have an impact on digital objects, others might be performed during the development of the solution as epistemic actions (see Figure 6 and Table 2 for a summary).

	Action	Role	Design affordance
Visual	Observation (size estimation/comparison/unit counting)	Mainly used for block recognition. Could be part of the composition if children perform mental computation	● Big size ● Blocks ranged in length ● 3D unit markers ● Colors
Physical	Size estimation/comparison	Mainly used for block recognition	● Big size, two hands grasping ● Blocks ranged in length
	Unit counting (part-whole identification)	Used for recognition and composition	● 3D unit markers ● 3D guiding lines
	Touch & Hold (digital feedback perception)	Mainly used for block recognition (feedback is individual)	● Sensors ● Unit feedback (sound/vibration)
	Grouping	Epistemic action used for composition	● Freedom to move blocks and rearrange the environment
	Joining	Epistemic action used for composition	● Magnets at the extremities of the blocks
	Place a block on the working area	Pragmatic action, used for composition	● Physical blocks fit on the working area
	Put blocks aside	Epistemic action, used during composition or recognition	● Freedom to move blocks and rearrange the environment

Table 2. Actions performed with the blocks and their respective design affordances.

Touching and Holding the blocks provide the sound and haptic *unit feedback*. It supports children to recognize the number of a block without a full physical scanning.
Unit counting Physical unit markers expose the whole-part relationship to facilitate *active touch* (see 2.1). Unit counting can be interpreted within a single block, or with blocks joined.
The ***size estimation*** action is mainly performed as a quick and gross alternative to recognize blocks when children search for a specific block.

Grouping relates to the conceptual metaphor that bringing objects together somehow adds, composes and creates a new object [42]. This grouping action supports the PDL [44] since children are adapting and reinterpreting the environment. Children also offload cognition by taking actions on objects [5] and by creating an external representation of groups [39]. ***Joining*** blocks implies creating an aligned group of blocks, making use of the magnets and imitating the number line representation. *Joining* exposes the whole-part relation between blocks and unit markers. *Grouping* and *joining* are epistemic actions, i.e., make the problem easier to solve but are not necessarily part of the solution [31].

Placing blocks on the working area is the only pragmatic action involved in the development of the solution. ***Putting blocks aside***, to exclude them from the solution and reduce the interference when composing, is its opposite, and it has been previously observed with sighted children [42].
Observation Blocks have different colors that facilitate the recognition of each block. In addition, blocks are big enough to allow children with low-vision, or eventually sighted children, to estimate their size, compare them to other ones and even count the unit markers depending on their degree of vision.

All the actions except *observation* fulfill the *suitable for active touch strategies* (DR4).

Informational relations

Touching and holding is the action that triggers the *unit feedback*. Placing blocks on the working area or placing blocks aside modifies the composition which will be evaluated as the solution.

The main informational relation is, first, the unit feedback (digital object), designed for children to recognize the number (through touch and hold action) represented in each block (physical object). It provides alternative external representations of the number and uses multiple representations through different sensory channels that might help to gain abstraction [2]. Such feedback contributes to digital enhancement for number recognition (DR6). Second is the relation of the blocks (physical object) when they are placed (action) on the number line (physical object), and the system that evaluates their composition and provides feedback (digital object). In case that the solution is correct, it also plays the block sound and then congratulates the child (digital object).

7.2. User Study 3

The aim of the third user study was to test the complete system LETSMath, including the narrative game Logarin and the number line working area as the input space of the interactive system. This way we validate the concrete model as part of a system aimed to train mathematics autonomously (complementing RQ1) and also test the number line working area and its impact on the strategies and performance (RQ3). Additionally, as observations made on study 2 already suggested that children understand both unit feedback (PARALLEL and VIBRATION), we incorporated and tested the SEQUENCE unit feedback as a combination of both alternatives.

Procedure
The test included 1) Warm-up, 2) Logarin gameplay and 3) Broken blocks activity. During the warming up stage, the experimenters explained the goal of the game, the relationship between the size and the unit markers, as well as the unit feedback mapping. After warming up, the Logarin game started with a tutorial. The game included five levels in which children were challenged to compose three (random) numbers from 1 to 7 except in level one that it was from 1 to 5 in order to make it easier. Lastly, participants played the Broken Blocks activity. Finally, if the child did not look tired, we asked how they recognized the blocks and if they would have liked to continue playing.

Results
In this section we present the main findings of study 3. More information and quantitative data can be found in Appendix A - Observations and Results - Study 3 - results.

Logarin Gameplay

According to the structured observations (See Appendix A - Observations and Results - Study 3), **all the children were able to understand the introductory tutorial, to successfully use the prototype, and play the five levels of the game** understanding the rules. Importantly, we observed that **children made fewer mistakes as the game advanced**.

The representation of the target number (N) through the *composition sound* was clear in most of the cases. Only one participant had problems interpreting the *composition sound,* having to wait until the game gave him an explicit hint like "You have to compose the number N".

We observed that some participants confused the concept of units with blocks. One of the reasons might be the oral hints provided by the game like: "try with more/less blocks" which is not always accurate, it would be better to say "try with smaller/bigger blocks" instead, this issue was also stressed by one of the teachers.

In some cases, children did not realize that the blocks were detected by the system when they put them on the working area. Other times, the interaction pace was not the expected, they did not understand that to get the system evaluation they had to wait until a new loop starts (see Figure 9-b). As a consequence, **idle time caused uncertainty** and sometimes they got impatient and removed the blocks from the working area before the system had evaluated the composition.

On the whole, based on the structured observations, we conclude that children understood the concrete model (RQ1) applied in the context of a narrative game. However, no one asked to continue playing and when they were explicitly asked if they wanted to continue playing, half of the participants said no, suggesting a low engagement with the game.

Regarding the incorporation of the number line working area and its impact on children's strategies (RQ3), we observed **a preference to compose the solution directly on the number line** (see Appendix A - Observations and Results - Study 3). At the same time, **we also observed that children make less mistakes when they clear the number line before composing the solution**. Working directly on the number line without clearing it produces the highest error rate.

Broken Blocks activity

Regarding RQ1, we might say that through this second instance of the broken blocks activity we observed that **children understood the informational relations**, i.e., the mapping between physical blocks, touch action and digital feedback in the *SEQUENCE* mode (see Table 3).

	Right	Wrong	Missed observations
Answer	88.2%	11.8%	-
Justification	63.5%	10.6%	25.9%

Table 3. Total right and wrong answers and right and wrong justifications among all participants of the Broken Blocks activity with SEQUENCE mode

In addition, this test validates the usefulness of the *SEQUENCE* unit feedback as a number representation. One participant relied only on vibration when perceiving the digital feedback during the Broken Blocks activity achieving a good performance, showing that fast vibration can be understood. However, another participant did not wait until the sounds were finished and this led to confusion and provoked errors. A possible solution might be to always play the full unit cycle feedback even when the child already raised the hand.

8. Results and Discussion

With the design-based research approach of LETSMath, we had a double objective: propose a novel, inclusive, portable and multimodal tool supporting basic mathematics learning for CVIs, including those with PDD, and gain insights in the design of this type of tools. The aspects of inclusiveness, portability and novelty have been discussed elsewhere. In this section we aim at a more integrated view of the interplay of the design features in this novel tool, and the CVIs interaction in this challenging mathematics learning context, discussing more in detail the results on the research questions. In Table 4 we summarize the main results linked with the respectives design requirements and research questions. Overall limitations and future work are discussed in Section 10.

Results	**DR / Studies / TLDF category**
RQ1: Do CVIs **understand** the proposed **concrete model**?	
All CVIs understood the **mapping between the physical design and the number representation.**	DR1 DR2 / 1-2 / Physical objects
Active touch strategies, such as spatial arranging and haptic exploration **were used to recognize the blocks.** All CVIs exploited shape, size and unit markers in order to estimate size and count. Children aligned blocks horizontally, **instantiating the number line conceptual metaphor** and taking advantage of this arrangement to partition and count, even when not constrained to use the number line during composition.	DR4 / 1-2 / Actions
Design Implication: In the context of a composition task, conceptual metaphors combined with spatial organization and restrictions, should be incorporated in order to encourage active touch strategies for CVIs.	
Despite the visual impairment severity, **Unit feedback comprehension** in its three modalities (*PARALLEL*, *VIBRATION* and *SEQUENCE*) has been validated through the Broken Blocks activity. ***Fast vibration*** is a more abstract (and challenging) representation, towards Leuders' direction [35], providing vibrotactile material closer for simultaneous perception rather than sequential counting, bringing CVIs closer to auditory subitizing [29]. Children understood the ***PARALLEL*** mode better, probably because it was slower and easier to count.	DR6 DR2 / 2-3 / Informational relations; Digital objects
Design Implication: starting by *PARALLEL* and then incorporating *VIBRATION* would scaffold the *unit feedback* comprehension. Combining both representations (*SEQUENCE*) children have two instances to perceive the number at different abstraction levels.	

RQ2: is the **digital feedback incorporated** during: **a)** the block recognition process, **b)** the composition task?	
(RQ2 a) Beyond recognizing blocks combining unit feedback, with spatial memory, vision (when it was possible) and active touch strategies, **unit feedback was mainly used to confirm the blocks**, particularly in the find the block activity.	DR6 / 2 / Actions; Informational relations
Design Implication: Materials should be **designed to progressively scaffold the transition from passive haptic feedback to digital feedback.**	
(RQ2-b) The participants performed dot counting while composing numbers, in line with the horizontal block arrangement. Our design additionally facilitates this action by the **inclusion of magnets** that **suggest joining blocks**: **affordances shape users' strategies** [42,51]. **Low vision children barely used unit feedback for the composition** task. This feedback relates to individual blocks and not to combined blocks, which is not useful to count the composition (and children who are learning to add need to count to check the result of the operation [61]). **Blind children relied on the same unit feedback pattern for both find a block and composition** tasks. For them it seems harder to disentangle the recognition process from the composition task. Composition is a demanding task, but they previously need to find the blocks. In the retrieval, they might also count the unit markers, perceive the digital feedback and compose in a more homogenous way throughout the different tasks.	DR6 DR4 / 2 / Actions; Informational relations; Digital objects
Design Implication: Physical affordances that encourage the development of domain specific problem-solving strategies should be envisaged.	
RQ3: Impact of the **input space** in slowing down the **interaction pace** and supporting **reflective** strategies for solving composition tasks	
In general, children compose the solution directly on the wooden number line, but they make more mistakes. The spatial restriction *number line - table* splits the working space in two and **slows down the interaction pace** as previously suggested [5,39,42,55]. The extra time of clearing the number line and rearranging blocks might also **discourage the trial and error strategy** and force momentary withdrawal causing **reflection** [2]. However, **composing directly** on the working area could increase the cognitive load as it demands to reformulate the problem as a difference. This makes the problem **harder for children**, probably leading to more composition errors. CVI's composition performance improves when they take an extra time to clear the number line and start the composition from scratch.	DR4 / 3 / Physical objects; Actions

***Design Implication*: A learning system should use interaction rules and physical constraints to slow down the interaction pace.**	
Delaying the system feedback contributes to slow down the interaction pace. However, **waiting states must be clearly communicated in order to avoid uncertainty generated by idle time**.	DR3 / 3 / Informational relations; Learning activity
***Design Implication*: Storytelling might help to fill out idle time and communicate waiting states to CVIs.**	

Table 4. Main results summary

9.Conclusions

Nowadays, CVIs learn mathematics in school mainly using traditional manipulatives (building blocks and tiles) or in some cases especially adapted materials. This current approach presents three main drawbacks. Firstly, while active haptic exploration might be supported by these materials, the auditory channel is not, leaving out multisensorial integration that would benefit abstract conceptualization of numbers. Secondly, in general, all these materials demand active supervision and guidance from educators. Finally, none of these materials permits the number identification through one gesture, even braille codes demand an active exploration and physical scanning. Thus, there is a lack of educational materials that provide CVIs similar opportunities to sighted children to learn mathematics exploiting multi-sensorial stimulation.

Building on cognitive theories and design requirements identified from previous related work, we have proposed a concrete model that exploits auditory and haptic feedback for mathematics learning especially, but not exclusively, for CVIs. In addition, some participants had PDD which is a quite general label and could involve many different cognitive disabilities at different levels. We approached this study aiming at an inclusive design for all (sighted, blind, low vision, PDD), not considering these conditions as diseases, but focusing on achieving a suitable design for inclusive education [67].

We validated the model through three user studies in which 19 CVIs were involved as well as some educators from their institutions. This process allowed us to gain insights, validate interaction techniques and contribute to design knowledge.

We observed that almost all the children understood the proposed concrete model as well as the narrative game. They were able to train mathematical skills with LETSMath showing a tendency to appropriate the features of the system during a single session. Providing guidance and hints facilitated this. So far, however, we still cannot claim a learning effect as a consequence of the system.

The inclusion of this kind of tangible systems enabling the exploitation of auditory and haptic channels in schools is a step forward to the equality of opportunities for CVIs.

10.Limitations and Future work

The system was tested through three user studies in different contexts, that was useful for the iterative design process, to inform the design decisions based on the feedback from users. However, all these were "one session" user studies where generally users have little time to get used to the system. The novelty effect was also present and first experiences may be cognitive demanding since everything is new, and interaction rules have to be incorporated at the same time as math problems are solved. In addition, in the first two user studies some participants presented cognitive disabilities, but not in the third one, thus we do not generalize the design and results for children with cognitive disabilities.

We thus have a prototype that should be tested in a longer-term user study where children use the system in classrooms for a longer period of time. To this aim, the system could be incorporated in the curriculum and contextualized as a classroom activity. This would also increase the ecological validity but maintain the experimental approach, which is a challenge when it comes to educational materials [38]. This would allow us to assess the learning gain and the efficiency of the system, for instance, using pre and post-tests [51].

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Appendix A: User studies detailed information

In this appendix we proportionate further information regarding the three user studies.

Methods, activities and prototypes status

Table A1 summarizes the participants, session average duration and how data was collected for each user study. Complementary, as the prototype evolved through an iterative process, in Table A2 we describe the current implementation of the system at the time of each user study, including what unit feedback modes were tested, how was the working area and which learning activities were evaluated.

Study	**Participants**		**Average Duration (minutes)**	**Data Collection**
	Low vision - Blind - PDD	**Age**		
1	N=6 (2 with low vision, 4 blind). All PDD	7-12	18	Conducted by five researchers who acted as participant observers and performed post video analysis
2	N=6 (2 with low vision, 4 blind). 3 PDD	7-12	20	Conducted by three researchers. They acted as participant observers and performed post video analysis
3	N=8 (4 with low vision, 4 blind). No PDD	6-8	18	Conducted by two researchers and one educator. They acted as participant observers and took structured observations

Table A1. User studies sessions' information

Study	**Unit Feedback**	**Learning Activities**	**Working Area**
1	OFF and PARALLEL (looped)	Exploration, Composition, Order	Rectangular wooden working area (no detection capabilities, wizard-of-oz)
2	OFF, VIBRATION and PARALLEL (not looped, counterbalanced)	Find a Block, Composition, Broken Blocks	Regular desk/table
3	SEQUENCE (not looped)	Logarin, Broken Blocks	Wooden Number line with magnets

Table A2. Description of the system and activities of each study

Observations and Results

In this section we detail how data was collected and how observations were discussed and interpreted by the researchers.

Study 1

Observations guidelines and experiment script

The first study was highly explorative and the purpose was to gain design insights and test the comprehension of the prototype. Figure A1 shows the experiment script including the questions that experimenters asked to the participants during the session.

Script user study prototype 1

1- Introduction

The cubes are introduced to the child.

Questions:

1. Are all the cubes identical?
2. Do they include braille notations?
3. Could you order them?
4. Could you hand me the smaller one?
5. Could you hand me the bigger one?

2- Game.

The child is asked to play the challenge audio game, in which the child is asked to compose a number by hearing a series of beats. The cubes must be placed on the platform.

Activities:

1. The working area is introduced to the child
2. The child is encouraged to put blocks on the working area in order to get feedback from the tablet (Exploration game)
3. The child plays the composition game guided by the tablet

Figure A1. Experimental script for the user study 1

Figure A2 shows the script of the interview that took place after the session with the blocks. The interview was a roleplay simulating a radio show.

Roleplay - Radio interview

Interview

Introduction: Experimenter gives the microphone to the child and does the role-playing pretending to be in a radio show.

Questions:

1. What were you supposed to do with the cubes?
2. How did the cubes work?
3. How would you explain to another child what the game is about?
4. What did you like more?
5. Do you want to play again?

Figure A2. Experimental script for the user study 1

The videos of the session with the blocks were analysed following the observation guidelines of the figure A3.

Observations guidelines

- Does the participant use the unit dots to count?
- Does the participant compare the size of blocks in a "Cuisenaire rods" fashion?
- Does the participant use the Braille sings? yes/no -> why? is there any problem with the braille pattern implementation?
- How does the participant explore the blocks? What is his/her reaction to the unit feedback?
- Does the participant understand the activities?

Figure A3. Observations guidelines for study 1

Results

In addition to the results presented in Section 6.1, we also observed that all participants had problems identifying the **braille** signs, which are made of 3D points. Due to technical 3D printing limitations we decided to increase the braille sign size, but this led children to misunderstand it and they often described these braille signs as "small points" or even confused them with the unit markers. In addition, one of the educators also stressed the importance of using the standard braille sign size [69] because each sign might fit under their fingertips. She was the same educator who also suggested adding the guiding lines between unit markers to lead the exploration to the next point, so in both cases the expert opinion matched with the field observations.

We also observed that some participants put the blocks closer to their ears because the sound of the unit feedback was a bit low in this first prototype. As a design implication we considered detecting this gesture and increasing the volume of the sounds. However, we did not incorporate this feature in the next prototype because of a matter of time.

Another interesting observation was that some participants tried to touch two blocks and count the sounds together. Thus, we decided to synchronize the blocks and make them play unit feedback together when they were attached with the magnets. However, the system was already complex enough and we decided not to incorporate this functionality in the nexts user tests.

Study 2

Structured observations sheets

Figures A4 and A5 show the structured observations sheet used by the two researchers that analysed the videos. All the data was filled during the video analysis done after the sessions. Regarding the composition arrangement, initially in the observation sheet there were three options: Horizontal line, vertical line, no order. However, during the video analysis three extra categories emerged: Horizontal 2D, Vertical 2D and Diagonal Line.

Participant Data: **User No.** __ **Duration:** __ **Sex:** M F **Age:** __
Scholar year: __ **Visual Condition:** Blind Low vision

Condition A - No unit feedback

A. Find a block

Trial 1, N= 3

Block	Dot counting	Braille	Size Estimation	Sound counting	Vibration	Other

Idem for Trial 2 (N=1) and Trial 3 (N=2)

B. Composition game

Trial 1, N=5

Block	Dot counting	Braille	Size Estimation	Sound counting	Vibration	Other

Composition strategy:

a. Horizontal line
b. Vertical line
c. No order
d. Other : ______________________

Idem for Trial 2 (N=4) and Trial 3 (N=7)

Condition B: Vibration unit feedback

A. Find a block

Trial 1, N= 2

Block	Dot counting	Braille	Size Estimation	Sound counting	Vibration	Other

Idem for Trial 2 (N=1) and Trial 3 (N=3)

B. Composition game

Trial 1, N=6

Block	Dot counting	Braille	Size Estimation	Sound counting	Vibration	Other

Composition strategy:

a. Horizontal line
b. Vertical line
c. No order
d. Other : ______________________

Idem for Trial 2 (N=7) and Trial 3 (N=5)

C. Broken Blocks

Broken Blocks GAME

Trial 1:

Answers: (mark with a circle and write justification)

- Block 1 : Broken / Works ---> Why?
- Block 2 : Broken / Works ---> Why?
- Block 3 : Broken / Works ---> Why?

Observations:

Trial 2:

Answers: (mark with a circle and write justification)

- Block 1 : Broken / Works ---> Why?
- Block 2 : Broken / Works ---> Why?
- Block 3 : Broken / Works ---> Why?

Observations:

Trial 3:

Answers: (mark with a circle and write justification)

- Block 1 : Broken / Works ---> Why?
- Block 2 : Broken / Works ---> Why?
- Block 3 : Broken / Works ---> Why?

Observations:

Trial 4:

Answers: (mark with a circle and write justification)

- Block 1 : Broken / Works ---> Why?
- Block 2 : Broken / Works ---> Why?
- Block 3 : Broken / Works ---> Why?

Observations:

Condition C: Parallel unit feedback

Same structure than in condition B with the following trials:

A. Find a block: Trial 1, N=1; Trial 2, N=3; Trial 3, N=2
B. Composition game: Trial 1, N=7; Trial 2, N=5; Trial 3, N=4
C. Broken blocks

Figure A4. Observations sheet for study 2

Final observations (sheet for researchers)

User No.

1. Understand the task? ☐ Yes ☐ No ☐ Partially. Obs:

2. Relates the asked **number** with the **blocks** he/she places? ☐ Yes ☐ No ☐ Partially. Obs:

3. Use a trial/error random **strategy**? ☐ Yes No

Figure A5. Final observations sheet for study 2

Results

In this section we present the quantitative analysis of the observations done during the second study. This data supports the results described qualitatively in Section 6.1 - Prototype and User study 2.

Regarding the quantitative data analysis, it is worth mentioning that one of the blind participants could count the unit markers using the physical blocks as well as the sounds and vibrations, but when he was asked to compose specific numbers he presented some problems and many times could not make it. The same situation happened when playing at the broken blocks game, it seemed that he did not understand the task. At the same time, there was a special event at the school and it was quite hard for him to focus on the tasks since we could hear that his classmates were playing outside the room where the study was taking place. Finally, considering all these factors we decided to not insist anymore since the session was lasting too long and he was already tired. Thus, he did not perform under the fast vibration condition. As a consequence, he was not included in the quantitative analysis for the broken blocks game of the sound and vibration condition neither in the activities under fast vibration mode.

Find a Block

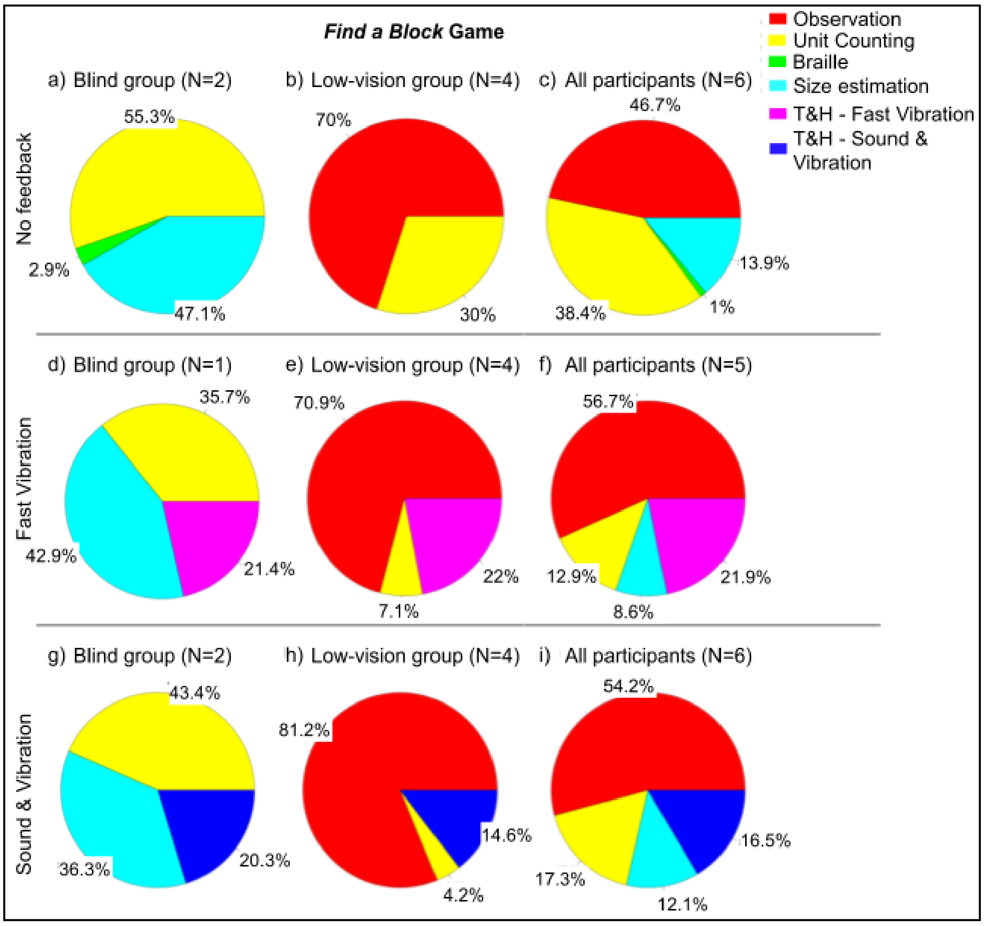

Figure A6. Actions performed during the Find Block game through the three conditions: No feedback, Fast Vibration and Sound and Vibration. Data grouped by visual condition group blind (a,d,g) low vision (b,e,h) as well as all together (c,f,i).

Composition Game

Figure A7 shows the composition arrangement observed for low vision children during the composition game. Blind children were not included in this quantitative analysis because they used the number line working area to compose the solutions, i.e., in these cases the arrangement was always "Horizontal Line".

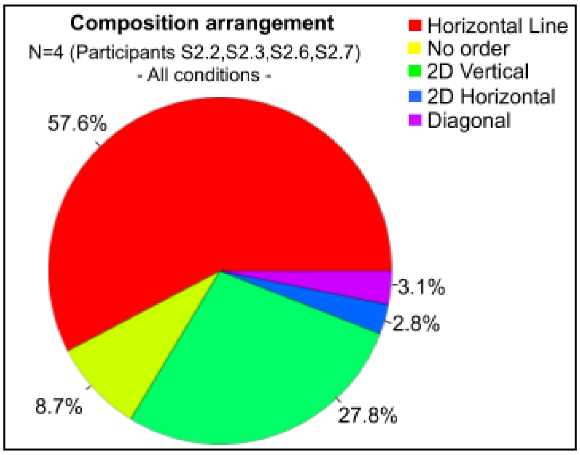

Figure A7. Blocks arrangement observed during the composition activity for low vision group

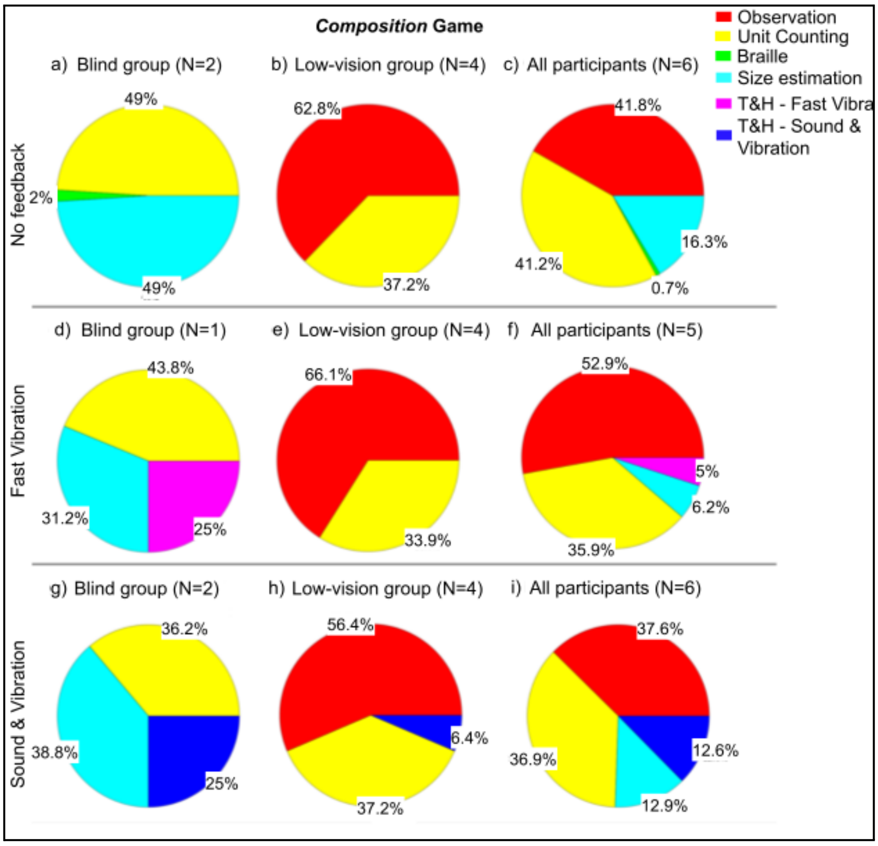

Figure A8. Actions performed during the Composition activity through the three conditions: No feedback, Fast Vibration and Sound and Vibration. Data grouped by visual condition group: blind (a,d, g) low vision (b,e,h) as well as all together (c,f,i).

Broken Blocks

We observed that children were highly efficient assessing if the digital mapping matched with the physical block (84%), and when asked to report why (justification), to reveal their actual perception, their responses were at a similar level (75%); a little decrease reflected that sometimes children had trouble explaining why they chose an answer (see Figure A9). Interestingly, children were not equally accurate in all conditions.

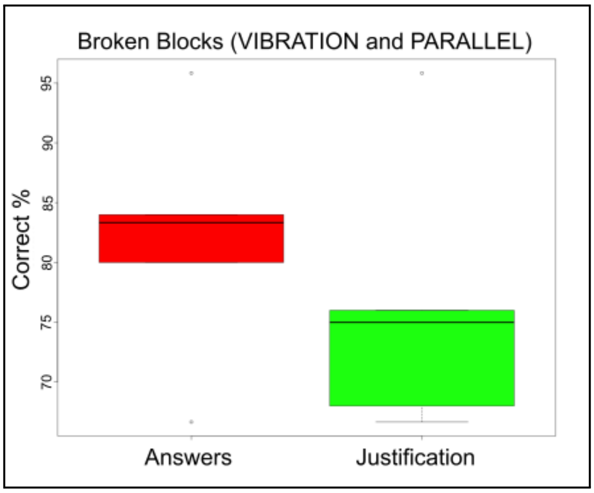

Figure A9. Broken blocks activity results, VIBRATION and PARALLEL unit feedback conditions

At the PARALLEL unit feedback condition, responses achieved 100% accuracy (correctly justified in 91% of the cases, see Figure A10). Whereas under the VIBRATION unit feedback condition a 65% of accuracy was reached (justification 59%, see Figure A11). It seems that the slower and parsimonious strategy was more successful to convey the number representation accurately.

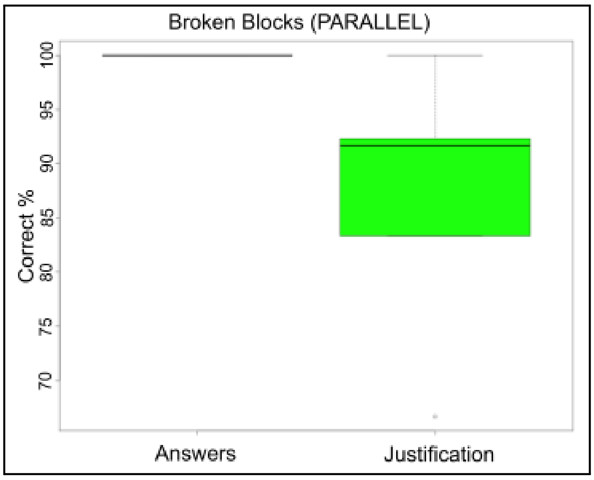

Figure A10. Broken blocks activity results, PARALLEL unit feedback condition

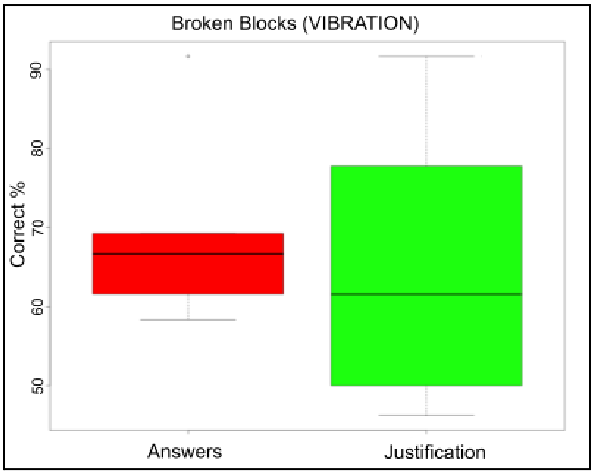

Figure A11. Broken blocks activity results, VIBRATION unit feedback condition

General observations

Figure A12 shows the quantitative results of the general observations. There was not any participant who could not understand the composition task at all or who did not relate the blocks with the challenge number to be composed (N).

Regarding their strategies to compose the numbers, in one case we clearly observed a **trial and error** strategy and in other two cases it was confusing, so we classified these cases as "partially" (see Figure A12). However, this does not necessarily mean that they could not count the composition. Many times the participants apparently joined or grouped the blocks randomly but then computed the composition by counting the unit markers. Therefore, showing signals of having understood the task and the value of the blocks and unit markers.

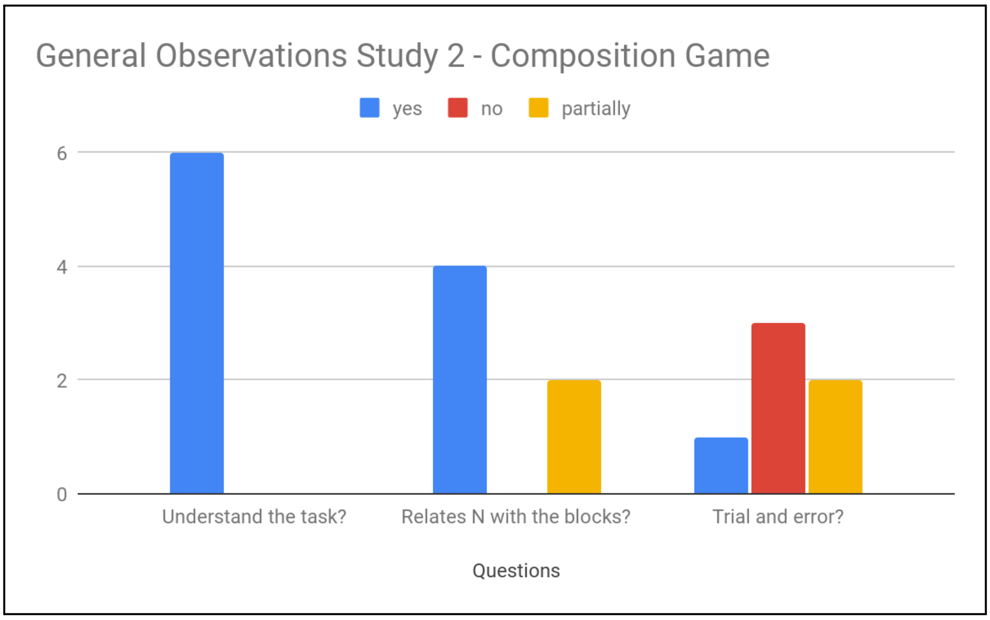

Figure A12. General observations results from study 2

Study 3

Structured observations sheets

During the third user study it was not possible to video record the sessions. Two researchers and one educator completed structured observation sheets in real time. Figure A13 shows the original observations sheet designed for this experiment. This sheet was completed by one of the researchers and later discussed with the second researcher who was guiding the participants during the execution of the study. The actions' table in this opportunity was quite challenging to fill up in real time, so we decided to discard such information during the analysis. Then, from the sheet in Figure A13 we only analysed the amount of error per trial, the composition strategy and the Broken blocks answers. Regarding the final observations (Figure A14), they were taken by both researchers and the educator, and later discussed until agreement was reached.

Observations Sheet (LETSMath evaluation)

Participant Data:

User No. __ **Duration:** __ **Sex:** M F **Age:** __ **Scholar year:** __ **Visual Condition:** Blind Low vision

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- Level 1 : Knocks level (3 trials)

Trial 1

Block	Dot counting	Braille	Size Estimation	Sound counting	Vibration	Other

Errors ☐☐☐☐☐☐☐ ☐☐☐☐☐☐☐

Observations

Composition Strategy:

a. Clear the w. area, compose the number on the table and then place blocks on the w. area
b. Clear the w. area and compose the numbers directly on the w. area block by block
c. Doesn't clear the w. area and perform the whole composition directly on it

Trial 2

Block	Dot counting	Braille	Size Estimation	Sound counting	Vibration	Other

Errors ☐☐☐☐☐☐☐ ☐☐☐☐☐☐☐

Observations

Composition Strategy:

a. Clear the w. area, compose the number on the table and then place blocks on the w. area
b. Clear the w. area and compose the numbers directly on the w. area block by block
c. Doesn't clear the w. area and perform the whole composition directly on it

Trial 3

Block	Dot counting	Braille	Size Estimation	Sound counting	Vibration	Other

Errors ☐☐☐☐☐☐☐ ☐☐☐☐☐☐☐

Observations

Composition Strategy:

a. Clear the w. area, compose the number on the table and then place blocks on the w. area
b. Clear the w. area and compose the numbers directly on the w. area block by block
c. Doesn't clear the w. area and perform the whole composition directly on it

General level observations: ____

- **Same observations structure for levels 2, 3, 4 and 5** (3 trials in each one)
- **Broken Blocks Game:** Same obervations structure than in study 2 (omitted)

Figure A13. Structured observations for study 3

Final observations (sheet for researchers and educators)

User No.

1. Understand the **tutorial**? ☐ Si ☐ No ☐ Partially. Obs:

2. Understand the **task**? ☐ Si ☐ No ☐ Partially. Obs:

3. ¿Identifies the **number** to be composed? ☐ Si ☐ No ☐ Ask for help.

4. Understand the game **rules**? ☐ Si ☐ No

5. Relates the asked **number** with the **blocks** he/she places? ☐ Si ☐ No ☐ Partially. Obs:

6. Use a trial/error random **strategy**? ☐ Si ☐ No

7. Understand that the blocks are detected by the system? ☐ Si ☐ No ☐ Needs help.

Extras:

1. Asked to repeat the levels? ☐ Si ☐ No

2. Wants to keep playing? ☐ Si ☐ No

Observations:

Figure A14. Final observations for Study 3

Results

Composition Strategy

We observed a preference for composing the solution directly on the number line (see Table A3). When children were forced to clear the number line working area, they showed a preference to compose numbers directly on the number line (b) rather than composing on the table and then moving the whole solution to the number line (a). When they were free to work without clearing the number line, 33.51% still prefered to clean it but they composed the numbers directly on the working area (b) and 30.45% directly composed the numbers on the number line without clearing it first (c). Only 25.42% performed the compositions on the table and then moved to solution to the number line.

Clear number line instruction	Composition strategy			
	a)	b)	c)	Missed observations
Must clear working area	39.58%	54.17%	-	6.25%
Free to work without clearing	25.42%	33.51%	30.45%	10.63%

Table A3. Percentage of Observed composition strategies (see Study 3, Methods, Data collection & analysis)

Regarding the impact of these strategies on the performance, we observed that when children compose directly on the number line (b and c) they make more mistakes (see Figure A15).

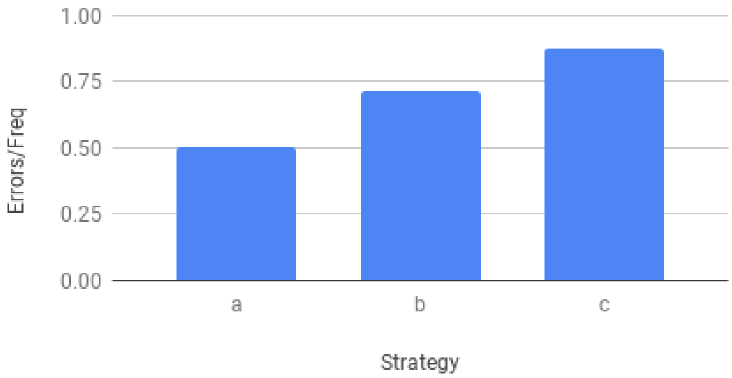

Figure A15. Error rate per composition strategy (Study 3)

Final Observations

Figure A15 shows the quantitative results of the final observations sheet presented in Figure A14. Such results were already discussed qualitatively in section 7.2 - Results.

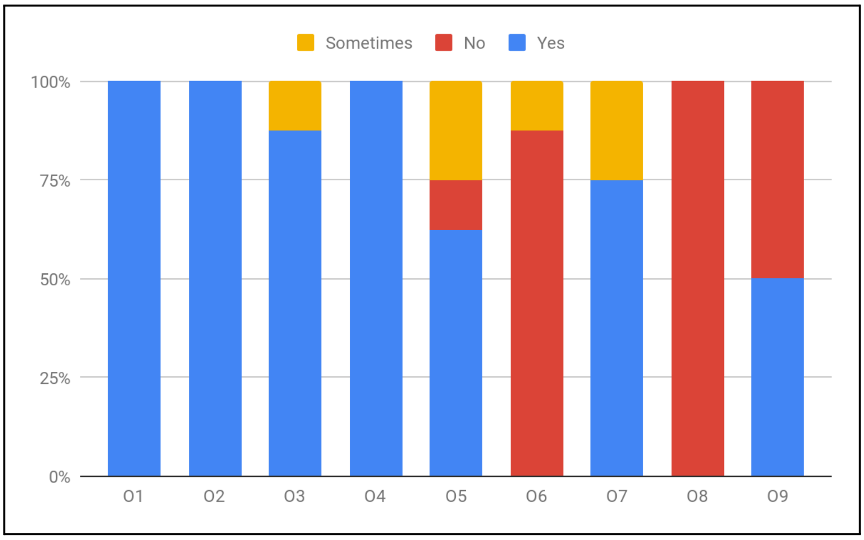

Figure A15. Study 3 Observations: O1 Understand the tutorial, O2 Understand the task, O3 Identify/recognize the proposed number to be composed, O4 Understand the game rules, O5 Match the requested number with the blocks placed on the working area, O6 Try blocks randomly, O7 Understand when the blocks are detected by the system, O8 Ask to play one more level, O9 Would you like to continue playing?