A CASE STUDY EXAMINING THE EXPERIENCES OF MIDDLE SCHOOL TEACHERS WHO USE MANIPULATIVES TO FOSTER THE SELF-EFFICACY OF STUDENTS WITH MATH DIFFICULTIES

by

Krishanta Katira Wildgoose-Butcher

Liberty University

A Dissertation Presented in Partial Fulfillment

Of the Requirements for the Degree

Doctor of Education

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2023

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APPROVED BY:

Gail Collins, EdD, Committee Chair

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Abstract

The purpose of this instrumental case study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Bandura's theory of self-efficacy was the theoretical framework for this study. The study was conducted at the Starlight School for Exceptional Children and at the Kingsway High School in The Bahamas. I recruited 15 participants for this study that included a combination of special and general education teachers who use manipulatives to provide math instruction to middle school students. To attain optimum results, I collected the data in the following order: individual interviews, focus groups, and classroom observations. Data analysis procedures were based on the guidelines outlined by Yin and Stake. The central research question that guided the study asked, How do middle school teachers describe the factors that influence the development of their students' self-efficacy while using manipulatives during mathematics instruction? The five major themes that emerged from the data analysis included the following: (a) consequences, (b) deliberate practice, (c) modeling, (d) targeted feedback, and (e) instructional changes. The interpretation of findings, along with relevant implications, limitations and delimitations of the study, and recommendations for future research are also discussed.

Keywords: mathematics difficulties, manipulatives, mathematics, instruction, self-efficacy, middle school teachers, assistive technology

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Dedication

I dedicate this dissertation to God, the author and finisher of my faith and to my family. God has truly proven His favor over my life and has been faithful from the beginning until the end of my doctoral journey. To my family, especially Renardo, Karen, Erskine, Adonai, Isabelle, Kestacia, and Kaviar: I am forever grateful for your unwavering support, words of encouragement, prayers, and love. You allowed me to balance being a wife, daughter, mother, and sister without complaining about the countless hours I had to sacrifice away from spending quality time with you. I love you all dearly, and I have found a fulfilling feeling of joy in knowing that I have made you proud!

Acknowledgments

Just saying thank you cannot suffice for the amount of encouragement I have received from my support system which includes my friends, fellow church members, principal, and colleagues. Your words of wisdom, advice, humor, stern motivation, and check-in calls or visits truly kept me going. A special thank you to my family, especially Andree and Rico for either ensuring that you were present to witness my accomplishments, or making sure that my devices were up to date with all of the latest and convenient programs I needed to complete my assignments.

I would like to acknowledge my past and current professors who have influenced the planning and writing of my dissertation, especially my dissertation chair, Dr. Gail Collins. I am very appreciative of all of her feedback, encouraging words, guidance, and patience. A special thank you is also extended to my committee member, Dr. Tony Ryff, who also shared wise and useful suggestions with me.

To my mentors Mrs. Barbara Barton, Ms. Juliette Brown, and Mrs. Mary Cooper (deceased), thank you for your guidance and inspiring me to pursue my doctorate degree in the field of special education.

I would also like to thank all of the participants who unselfishly took the time out to participate in my study. Your willingness, patience, and contributions were very valuable and were greatly appreciated.

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List of Abbreviations

Concrete Representational Abstract (CRA)

Individualized Education Program (IEP)

International Society for Technology in Education (ISTE)

Learning Difficulties (LD)

National Assessment of Educational Progress (NAEP)

Science, Technology, Engineering & Math Education (STEM)

CHAPTER ONE: INTRODUCTION

Overview

Schools throughout the world have made an enormous investment in technology to keep up with the educational demands of the 21st century. These large investments led to the continuous changes in teaching and learning that enhance student achievement and increase teachers' engagement and excitement regarding new technological strategies (Koonce, 2020). Despite the evidence that conveys that technology has a positive effect on student achievement, teachers' beliefs and experiences regarding their lack of expertise, training, workload, and time negatively impact their ability to provide effective technological instruction (Panisoara et al., 2020). In addition, Adov and Mäeots (2021) posited that teachers' attitudes toward the use of technology determine their willingness to use technology during instruction. This decrease in quality instruction when using technology negatively impacts students' self-efficacy and student achievement, especially for students who are challenged with grasping academic concepts. The purpose of this qualitative study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. This chapter provides historical, social, and theoretical contextual background for the problem. The problem statement is articulated followed by the purpose statement. Next, the empirical, practical, and theoretical significance of the research is introduced followed by the research questions. Relevant definitions applicable to the study are also provided for the audience.

Background

Studies conducted by contributors to the field have provided the most relevant literature regarding the historical, social, and theoretical concepts of the featured phenomenon. This section summarizes the most relevant literature and signifies how the problem has evolved by providing information regarding the evolution of the use and popularity of virtual manipulatives. Information regarding relevant social context that helps frame the study is also presented and explains how society, the community, and education systems are affected by the problem. This section concludes with a discussion of relevant theoretical concepts that have developed the phenomenon under examination and the principles underpinning the research.

Historical Context

Many students experience mathematics difficulty, especially as they begin middle school where the demand for mathematics proficiency increases drastically (Powell et al., 2021). Students experiencing math difficulties may have challenges with grasping foundational and more advanced math concepts (Myers et al., 2021). These ill-fated math difficulties may arise for students due to deficiencies in cognitive skills (Geary, 2000; Jordan & Hanich, 2000; Myers et al., 2021). The problem that motivates my study is that students who have mathematics difficulties often struggle with mastering the learning objectives outlined in mathematics curricula and demonstrate poor conceptual understanding of mathematical concepts (Doabler & Fien, 2013; Myers et al., 2021; Park et al., 2021; Satsangi, Hammer, & Evmenova, 2018; Vaughn & Bos, 2019).

According to Myers et al. (2021), recent math scores from the National Assessment of Educational Progress raise alarming concerns. Results suggested that many students lack proficiency in core math standards and experience math difficulties in learning and performance (National Center for Education Statistics, 2020). Researchers previously examined the nature of cognitive development related to mathematics difficulties experienced by young children (Geary, 1990, 1993; Griffin et al., 1994; Jordan et al., 2006). In addition, Bryant et al. (2008) noted that increasing attention is concentrated on the identification of young students with mathematics difficulties and Doabler and Fien (2013) revealed that becoming mathematically competent can be challenging for students with mathematics difficulties. These data communally suggested that students with mathematics difficulties have inadequate access to quality curriculum, instruction, or resources to meet their academic needs.

The relatively large body of research on students' mathematical cognition (Geary, 2000; Jordan et al., 2003) led to a rising interest in students with mathematics difficulties in recent years. According to Myers et al. (2021), math learning expands in complexity as students begin middle school, and many students with math difficulties continue to experience lower math achievement and performance than their peers. Mathematics performance is a vital issue that has been addressed since it is a determinant or predictable factor of future achievements such as high-school graduation, college acceptance, college completion, and employment (Myers et al., 2021; Powell et al., 2021). Myers et al. (2021) purported that attention be focused on the provision of effective instructional methods that may enhance the math achievement of students with math difficulties and may foster a trajectory of positive or desirable outcomes.

Research concerning the first computer-based manipulatives has promoted computer-based manipulatives as instructional and evaluative mathematical tools that teachers can implement (Berlin & White, 1986; Clements & Battista, 1989; Clements & Sarama, 2007; Moreno & Mayer, 1999). According to Shin et al. (2021), virtual manipulatives were first developed as Flash or Java programs and have been available to the public since the late 1990s. Researchers then focused greatly on how virtual manipulatives can be used to address the problem of learning difficulties (Satsangi, Hammer, & Hogan, 2018; Shin et al., 2017). Bouck and Flanagan (2009) discussed three main types of mathematics-based assistive technology for students who struggle with grasping math concepts (anchored instruction, computer-assisted instruction, and calculators). However, virtual manipulatives were missing from Bouck and Flanagan's (2009) studies as well as follow-up studies conducted by Kiru et al. (2018) and Bowman et al. (2019). Nonetheless, further studies supported the use of virtual manipulatives to enhance math instruction (Bouck et al., 2018; Gecu-Parmaksiz & Delialioglu, 2019; Moyer-Packenham & Suh, 2012; Park et al., 2021; Satsangi, Hammer, & Evmenova, 2018; Shin et al., 2017).

Social Context

Soykan and Kanbul (2018) reported that the skills expected of 21st century students include critical thinking and problem-solving, creativity and renewal, communication and collaboration, flexibility and adaptation, knowledge, media and technology literacy. In a society of evolving technology and educational advancements, it is paramount that education systems prepare students, teachers, and leaders with the necessary knowledge and skills needed to fully function in society. According to Lafay et al. (2019), many aspects of daily life involve mathematics, and thus, persons with mathematical deficiencies may be marginalized and their social and professional integration may be affected.

Education systems are affected by the problem because teachers and curriculum leaders are challenged with providing effective math instruction to meet the needs of students who experience mathematical difficulties. As previously mentioned, virtual manipulatives were missing from Bouck and Flanagan's (2009) studies; however, nearly 10 years later, Bouck et al. (2018) reported that virtual manipulatives have aided in the resolution of the problem of not sufficiently meeting the mathematical needs of students who struggle with mathematics. Society is also affected by the problem because Aud et al. (2011) revealed that a significant number of students who have mathematics difficulties may originate from economically and educationally disadvantaged backgrounds. This revelation highlights the societal issue of digital divide. Schrum and Sumerfield (2018) revealed that the increased prevalence of digital materials along with the increased necessity for digital fluency has amplified the need to address educational and digital equity. The authors also purported that digital equity is an issue of social justice because it is a newly constituted civil right that enforces the right to connect to needed technological resources anywhere and at any time.

This unfortunate divide may be addressed by considering the five elements of digital inclusion and the four opportunities of technology proposed by the International Society for Technology in Education (ISTE). The five elements of digital inclusion that should evolve as technology advances include (a) affordable, robust broadband internet service, (b) internet-enabled devices that meet the needs of the learner, (c) access to digital literacy training, (d) quality technical support, and (e) applications and online content designed to enable and encourage self-sufficiency, participation, and collaboration (National Digital Inclusion Alliance, n.d.). The four opportunities where technology can escalate access to learning opportunities are (a) access to learning resources, (b) access to expertise, (c) access to personalized learning, and (d) access to planning for higher education. Denied, limited, or no access to any of these mentioned elements or opportunities produces the current issues that surround educational and digital access. According to Pittman et al. (2021), basic access to information technology must be disseminated into educational systems of less fortunate societies to combat the negative effects of digital inequality. The authors emphasized the need for teachers and learners to be more prepared to use technology for the academic advancement of all.

Digital equity is a social justice issue because it disproportionately impacts specific student populations such as low-income students, students of color, and historically underserved students (Veletsianos, 2021). According to Xie et al. (2021), the usage of new technologies also enhances the existence of social stratification and inequality. An equity gap is created when learning goals for students are not clarified or when students are not provided with the knowledge and skills they need to be effective and empowered in and out of school (Schrum & Sumerfield, 2018). In summation, the persistent challenges associated with mathematics difficulty are a critical social problem; therefore, it is imperative to prevent or reduce the occurrences of mathematics difficulties (Fuchs et al., 2009).

Theoretical Context

According to Bandura (1997), self-efficacy controls one's thinking, behavior, and emotions; therefore, self-efficacy influences one's motivation, perseverance, and performance. It is then inferred that the more positive experiences one has had, the more self-efficacious they are, and the more effective they will become in learning (Carbonneau et al., 2018). Many students struggle with the educational demands presented in the mathematics curricula; therefore, researchers have conducted studies regarding pedagogical practices and classroom technologies to investigate, address, and combat the problem (Satsangi, Hammer, & Evmenova, 2018). Based on research findings regarding the implementation of virtual manipulatives, classroom teachers can integrate virtual manipulatives into their math instruction to teach new concepts and to provide students with ample opportunities to engage in interactive tasks as they navigate guided or independent practice (Satsangi, Hammer, & Hogan, 2018; Shin et al., 2017).

The theoretical lenses that other researchers have used to examine the problem include Bouck et al.'s (2018) presentation of a graduated instructional framework named the Concrete-

Representational-Abstract-Framework (CRA) as an instruction strategy that incorporates mathematics manipulatives to meet the needs of students who struggle with grasping mathematical concepts and solving mathematics problems. In addition, researchers have applied theories of cognition and the social construction of knowledge (Cobb, 1995; Vygotsky, 1978) that considered manipulatives to be cognitive and cultural tools that are conveyed in instructional processes. Research that investigates or addresses the problem has also explored Piaget's (1952) findings of clinical reviews, which recommend that students' learning of abstract mathematical ideas is supported with concrete manipulatives, and Bruner's (1960, 1986) proposal that students' understanding of mathematical concepts occurs in three stages (enactive, iconic, and symbolic), in which the first stage involves students interacting with objects such as manipulatives.

Grade level, teacher beliefs, and teacher experiences with the usage of concrete and virtual manipulatives are vital predictors of how effectively teachers use them with students during mathematics instruction (Moyer-Packenham et al., 2013). According to Wang and Tseng (2018), evidence in the literature indicates that concrete manipulatives enhance student learning, engagement, and motivation as the use of virtual manipulatives has also demonstrated positive learning outcomes for students. Nonetheless, C. Wang et al. (2022) noted that virtual manipulatives are progressively implemented in inquiry-based instruction as they emulate concrete manipulatives and have emerged and advanced with the development of information technologies. Since self-efficacy is developed through four main sources (Bandura, 1986) and is very interrelated to one's account of past experiences (Carbonneau et al., 2018), this study extends the body of existing literature by examining the experiences of teachers who use manipulatives to foster students' self-efficacy as their students' academic performance is enhanced in mathematics.

The theoretical lens that was used to examine the problem identified in this study is Badura's theory of self-efficacy. This theory is suitable for this study because Bandura (1997) hypothesized that individuals form their self-efficacy by selecting and interpreting information from four primary sources: mastery experience, vicarious experience, social persuasions, and physiological/affective states. By examining teachers' experiences, I was able to address the problem of learning difficulties (LD) in math through the lens of the origins of their students' self-efficacy as I also added to the body of existing literature regarding the phenomenon.

Problem Statement

The problem is that many students, especially those who have LD, often struggle with mastering the learning objectives outlined in mathematics curricula and may demonstrate poor conceptual understanding of mathematical concepts (Park et al., 2021; Satsangi, Hammer, & Evmenova, 2018; Vaughn & Bos, 2019). According to Park et al. (2021), students with mathematical difficulties demonstrate notable improvement after receiving instruction that utilizes manipulatives. In addition, Satsangi, Hammer, and Hogan (2018) suggested that to address these mathematical struggles, teachers must implement appropriate assistive technology to support their classroom instruction. In an effort to negate this problem, researchers have also studied innovative solutions such as concrete and virtual manipulatives to establish evidence-based instructional strategies that could support students who struggle with mastering math concepts (Bone et al., 2021; Bouck, Park, & Stenzel, 2020; Satsangi et al., 2016).

Researchers revealed promising findings regarding the use of manipulates during math instruction and suggested that manipulatives enhanced achievement for students experiencing mathematics difficulties (Bouck, Park, & Stenzel, 2020; Park et al., 2021). Since student achievement is proven to be improved by using manipulatives, there is a need to examine the experiences of teachers who have implemented manipulatives in their math instruction. Teachers who report their experiences of using instructional aides can either provide positive or negative insights regarding student achievement (Koonce, 2020). Several studies have explored and compared concrete and virtual manipulatives to improve teacher instruction and student achievement; however, a research gap is found in the lack of reported studies that have focused on teachers' experiences of using manipulatives while rendering math instruction. This study was needed because it provides important contributions to the literature by revealing a comprehensive understanding of the authentic experiences of teachers who use manipulatives to foster the self-efficacy of students with mathematics difficulties.

Purpose Statement

The purpose of this instrumental case study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Self-efficacy is defined as the belief in one's own ability to effectively accomplish a goal or task (Bandura, 1997). This study provides insights beneficial to the field of education; more specifically, it enhances math instruction and student achievement. Bandura's (1977) theory of self-efficacy guided this study. This theory entails four distinct sources that influence an individual's ability to develop self-efficacy. The four sources are mastery experience, vicarious experience, social persuasions, and physiological or affective states.

Significance of the Study

According to Moyer-Packenham et al. (2013), the use of manipulatives has an extensive trajectory that led to their prevalence and use by teachers to enhance mathematics instruction.

The history involves various studies (Fuson & Briars, 1990; Moyer, 2001; Moyer & Jones, (2004;); however, research that focused specifically on virtual manipulatives began over 30 years ago as a result of emphasis placed on technology integration into mathematics instruction. Despite the lack of recognition and research concerning virtual manipulatives over the decades, they are increasingly becoming a popular tool used by teachers for mathematics instruction (Bouck & Park, 2018). This increase in attention in the recent years can be attributed to the recent advancement in technologies and the increased use of advanced technologies in K–12 instruction (Satsangi & Miller, 2017). Satsangi and Miller (2017) discussed the benefits of implementing various technological practices that provide effective instruction for students who have mathematical difficulties; nonetheless, even with the expanding prevalence of computer-based technologies in today’s classrooms, some areas of study, such as those focusing on teachers’ and students’ use of manipulatives, lack sufficient exploration (Satsangi et al., 2016). This section contains a description of the contributions that this proposed study makes to the knowledge base or discipline from a theoretical, empirical, and practical perspective.

Theoretical Significance

According to Koonce (2020), many teachers experience challenges and difficulties when leaving their comfort zones to use technology. These unfortunate experiences encompass lack of time, professional development, resources, energy, administrative support, incentives, and commitment to use technology every day. This study contributes to the theoretical underpinnings of the problem by utilizing Bandura’s self-efficacy theory that provides a comprehensive framework based on the premise that an individual’s self-efficacy beliefs can be developed from four main sources of influence. According to Bandura (1997), students with elevated levels of self-efficacy are more likely to perceive difficult tasks as challenges and are more willing to perform a variation of tasks or try innovative ways to accomplish them than students with lower self-efficacy.

This study adds to Bandura's self-efficacy theory by providing a comprehensive understanding of teachers' experiences as they navigate the process of using manipulatives to foster their students' self-efficacy. This study also extends the theory by providing knowledge of how to effectively increase students' self-efficacy levels by using innovative instructional methods. Further enquiry into teachers' experiences provides an in-depth understanding of the effect that the use of manipulatives may have on students' sense of self-efficacy through the lenses of four factors (mastery experience, vicarious experience, social persuasions, and physiological or affective states). Descriptive accounts of teachers' experiences in this study revealed common or diverse insights of how their use of manipulatives potentially fostered the self-efficacy of students with math difficulties.

Empirical Significance

Similar studies involving the use of manipulatives have shown that neither concrete nor virtual manipulatives result in meaningful learning unless they are effectively embedded in instructional practice (Carbonneau et al., 2013; Laski et al., 2015). Once teachers effectively implement manipulatives into their instruction, students who are challenged with mathematical difficulties garner great academic benefits (Park et al., 2021; Satsangi et al., 2016). Accordingly, the data collection methods for this study focused on teachers' experiences of using manipulatives. Undertaking a qualitative methodological approach added to the literature because I was able to seek a better understanding of the problem by entering the setting with an open mind, immersing myself into the depths of the situation as I interacted with participants (Leedy & Ormrod, 2005). As a qualitative researcher, I embraced Leedy and Ormrod's (2005) suggestion to become the research instrument as I operated under the assumption that reality is not confined by discrete, measurable variables. I also heeded the suggestion of Ary et al. (2010) and attained participants who could best shed light on the phenomenon being investigated and analyzed their experiences to better understand the situation. The findings of this case study yielded empirical data regarding teachers' experiences of using manipulatives to enhance learning through an increased sense of self-efficacy and subsequently may inform pertinent decision-makers regarding implications for practice and policy.

Practical Significance

Kouzes and Posner (2017) asserted that the most significant contribution leaders can make is to promote change and to devise the future long-term development of individuals and institutions to ensure adaptability, change, prosperity, and growth. The knowledge that was generated from this study may be significant to educational leaders, teachers, students, and curriculum developers. According to Park et al. (2021), mathematics is perceived by many students as a difficult subject to master but is essential for student success because it provides students with the fundamental knowledge and skills needed to access science, engineering, and technology, industries that are expected to provide future job opportunities. Park et al. (2021) also revealed that mathematical learning also fosters inquisitiveness, critical thinking skills, and problem-solving skills.

This study could benefit teachers who provide math instruction, students who struggle with grasping mathematical concepts, and mathematics curriculum developers because manipulatives have the ability to support students' attitude and learning in mathematics (Park et al., 2021). In addition, since a significant number of students who have mathematics difficulties may originate from economically and educationally disadvantaged backgrounds (Aud et al., 2011), both students and teachers who lack access to resources and digital equity may also benefit or use the findings from this research. According to the Nation's Report Card (2022), of the fourth-grade and eighth-grade students who learned remotely during the 2020–21 school year, low performers (students who perform below the 25th percentile) had less frequent access to (a) a desktop computer, laptop, or tablet; (b) a quiet place to work; and (c) an available teacher to assist them with math assignments as compared to higher performers (those who perform at or above the 75th percentile). According to Schrum and Sumerfield (2018), the digital divide has evolved from an issue of access to an issue of use, knowledge, skill, and attitude among stakeholders. The authors warned that the digital divide has the potential to widen if not carefully addressed by interested stakeholders such as researchers and teachers. Findings from this study may encourage stakeholders to advocate for students and teachers by communicating their needs for digital resourcing such as manipulatives to support learners both inside and outside of the learning environment.

The findings of this study may also inform the structure of more effective professional development opportunities designed to support teachers who utilize technology in the classroom. For teachers, the adoption of technological trends likely mean that intense and mandatory professional development opportunities must be inserted in teaching and learning schedules. As Lalor (2017) suggested, the ideal curriculum should be stimulated with quality professional development. Because approximately 10%–20% of school-aged students are reported to have significant mathematics difficulties (Geary et al., 2012), it is imperative that educational leaders, teachers, and curriculum developers identify and implement strategies and programs beneficial to educational advancements in mathematics. One of these identified strategies is the use virtual manipulatives.

Hansen et al. (2016) expressed the need for educational researchers, teacher trainers, and software designers to identify ways to not only involve students but to effectively involve teachers in the design process of virtual manipulatives as well. This study involved accounts of teachers' experiences of the use of manipulatives and may provide insight to educational researchers, teacher trainers, and manipulative designers as they design and develop manipulatives that can be used within mathematics instruction. Since teachers face great challenges because of their workload, designers of manipulatives should value the input teachers can contribute to the design process as their aim should be to (a) involve additional participants in the design process and (b) to attain a mutual benefit for all parties involved (Hansen et al., 2016).

Sometimes, studies are conducted to provide insights and implications for futures planning. The practical significance of this study may be helpful for curriculum leaders in the futures planning process. According to Parkay et al. (2019), the futures planning process involves collaborative efforts among curriculum leaders and other stakeholders who aim to address present trends and develop alternate scenarios after they have forecasted and projected the effects of one trend compared to another. Therefore, strategic futures planning should be carried out in all educational settings so that uncertainties are curbed and failures are prevented. To reap the desired benefits of futures planning, all curriculum leaders should be lifelong learners who remain proficient in and informed of current digital trends.

Kouzes and Posner (2017) encouraged educational leaders to look forward in times of rapid change in a world that is becoming more volatile, uncertain, complex, and ambiguous, where visions are even more critical for human survival and success than when times are calm, predictable, simple, and clear. The sudden and unprecedented effects of the COVID-19 crisis led to massive changes in the education arena. Numerous educational systems were forced to adapt to technology-mediated teaching and learning modalities (Adov & Mäeots, 2021; Barlovits et al., 2021; Rosillo & Montes, 2021). According to the International Association of Universities (2020), more than one billion students throughout the world were affected by the instructional interruptions caused by the pandemic. It was also reported that the COVID-19 crisis continues to negatively impact the educational system, especially the vulnerable segment of students. Rapanta et al. (2020) stated that the unpredictable changes posed by the pandemic may have created a number of challenges and demands such as teachers' inability to teach virtually. The authors claimed that these difficulties impede the effectiveness of teachers and learners and may hinder student progress. It was postulated that students suffered conceptual learning gaps during the lockdown, and it is projected that the unfortunate learning deficits will plague learners in the future (Cahapay, 2021).

According to Rosillo and Montes (2021), students often possess negative perceptions of mathematics that are mainly due to factors relating to their affective domains such as attitudes, beliefs, and emotions. The authors claimed that the online learning platform decreased student motivation and performance levels in mathematics. In addition, Sawchuk and Sparks (2020) revealed that the disruptions caused by the pandemic negatively affected students' math gains. The authors claimed that learning loss was more prevalent in mathematics than in other subjects because of various reasons: (a) math is formally learned in face-to-face classroom settings, (b) parents were not fully equipped to assist their children with grasping mathematical concepts, (c) many students' math anxiety levels were worsened due to the stress and trauma caused by the pandemic, (d) teachers lacked sufficient training regarding how to navigate new ed-tech platforms and tools, and (e) it was more challenging for teachers to provide effective math instructional practices via the available remote platforms. On the other hand, changes such as the implementation of virtual learning amid the COVID-19 crisis provided stakeholders with favorable opportunities to find solutions to the emerging problems. Educators around the world explored the role of instructional technology and found ways to facilitate the continuity of education (Adov & Mäeots, 2021; Barlovits et al., 2021). According to Cahapay (2021), the pandemic pressured school systems to engage in a widespread technological transformation as educators relied on integrating technology into their instructional practices. Educators resorted to using various modalities of remote education to convey important instructional content.

COVID-19 caused an abrupt and compulsory transformation to education systems throughout the world. This transformation led to the transition from the traditional face-to-face teaching method to unconventional virtual learning platforms. This transition was met with both positive and negative perceptions from math teachers. Issues such as digital inequality and lack of teacher training created unfortunate learning gaps that is predicted to affect students in the future. As stakeholders seek to address and remediate these issues, it is undeniable that the pandemic forced the advancement of instructional technology that will continue to benefit all educational sectors, including students who struggle with learning difficulties in math.

Research Questions

Previous research has shown that the implementation of manipulatives presents learning opportunities for students who struggle with grasping mathematical concepts by allowing them to explore and understand concepts visually and tactilely (Gecu-Parmaksiz & Delialioglu, 2019; Park et al., 2021). Moreover, Moyer-Packenham and Suh (2012) found that low-achieving students appeared to benefit the most from teachers' implementation of virtual manipulatives. This study examined the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Bandura's theory of self-efficacy was the theoretical lens that was used to examine the problem identified in this study. As noted earlier in this chapter, Bandura (1997) hypothesized that students form their self-efficacy by selecting and interpreting information from four primary sources: mastery experiences, vicarious experiences, social persuasions, and physiological/affective states.

Central Research Question

How do middle school teachers describe the factors that influence the development of their students' self-efficacy while using manipulatives during mathematics instruction?

Sub-Question One

How do middle school teachers describe their experiences of using manipulatives to facilitate the mastery experiences of students with math difficulties?

Sub-Question Two

How do middle school teachers describe their experiences of using manipulatives to enhance vicarious experiences for students with math difficulties?

Sub-Question Three

How do middle school teachers describe their experiences of providing social persuasions when using manipulatives with students with math difficulties?

Sub-Question Four

How do middle school teachers describe their experiences of addressing the physiological or affective states of students with math difficulties when using manipulatives?

Definitions

1. *Assistive Technology* – Anything that essentially benefits a student with a disability (Bouck, 2017).

2. *Concrete Manipulatives* – Physical objects that can be manipulated to aid students in understanding different mathematics (Bouck et al., 2018).
3. *Digital Equity* – “A condition in which all individuals and communities have the information technology capacity needed for full participation in our society, democracy, and economy” (Schrum & Sumerfield, 2018, p. 128).
4. *Manipulative* – “Any tangible object, tool, model, or mechanism that may be used to demonstrate a depth of understanding, while problem-solving, about a specified mathematical topic or topics” (Kelly, 2006, p. 184).
5. *Mastery Experiences* – The interpreted result of an individual’s own previous successes that signifies that “successes build a robust belief in one’s efficacy. Failures undermine it, especially in earlier phases of self-development” (Bandura, 1999, p. 181).
6. *Physiological/ Affective States* – Interpretation of one’s own physiological arousal as an indicator of personal competence. An individual read their “tension, anxiety and depression as signs of personal deficiency” (Bandura, 1999, p. 181).
7. *Self-Efficacy* – A person’s particular set of beliefs that determine how well one can execute a plan of action in prospective situations (Bandura, 1997).
8. *Social Persuasions* – Acts of receiving encouragement from other important individuals such as parents, teachers, and friends that increase an individual’s confidence. “If people are persuaded that they have what it takes to succeed, they exert more effort and are more perseverant than if they harbor self-doubts and dwell on personal deficiencies when problems arise” (Bandura, 1999, p. 181).

9. *Vicarious Experiences* – Acts of individuals witnessing others like themselves perform the same task. “If people see others like themselves succeed by sustained effort, they come to believe that they, too, have the capacity to succeed” (Bandura, 1999, p. 181).
10. *Virtual Manipulative* – “An interactive, web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer et al., 2002, p. 373).

Summary

The purpose of this instrumental case study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. This introductory chapter provided a historical, social, and theoretical contextual background for the study. Satsangi, Hammer, and Evmenova (2018) reported that many researchers have orchestrated studies of numerous instructional practices and technologies used by educators to combat the struggles faced by students who find it challenging to grasp mathematical concepts; however, research regarding the use of manipulatives is lacking exploration. Bandura’s self-efficacy theory guided this study to add to the body of literature regarding the usage of virtual manipulatives. A central research question was posed along with four sub-questions. Information outlining the theoretical, empirical, and practical significance of the study was presented as well as a list of relevant definitions regarding the study.

CHAPTER TWO: LITERATURE REVIEW

Overview

This chapter entails the overview, a theoretical framework section, a related literature section, and a summary. The theoretical framework section provides a clear sense of Bandura's (1997) self-efficacy theory and its relationship to the phenomenon that I identified in my study. The related literature follows the theoretical framework section and provides a synthesis of the existing knowledge on the topic and links the existing knowledge to the proposed study. Topics discussed in the related literature section include LD in mathematics, self-efficacy and achievement, remediated instruction, and manipulatives. A rationale of how this study fills the identified gap in the literature and advances understanding in the field is also discussed.

Theoretical Framework

A theoretical framework is important in a qualitative research study because it guides the study and assists researchers with the organization of their study. According to Maxwell (2012), a theoretical framework provides the audience with a clear sense of the researcher's theoretical approach to the phenomenon. The theoretical framework also gives the audience an opportunity to conceptualize the research findings within a greater context. Without a theoretical framework, researchers would lack focus and be unable to ground their study to previous and relevant work (Maxwell, 2012). In this section, the theory that guided this study is described along with its origination and major theorist. This description is followed by a discussion of how the theory has advanced or informed the literature on my topic. This section concludes with an explanation of how my study utilizes the theory and how it may potentially advance or extend the theory. The theoretical framework that efficiently guided this study is Bandura's self-efficacy theory, a component of Bandura's (1986) social cognitive theory.

Bandura’s Social Cognitive Theory

Bandura’s (1986) social cognitive theory postulates that learning occurs in a social context. Bandura (1989) claimed that human behavior is a cyclical and reciprocal interaction of the individual’s personal influences (self-efficacy, motivation, anxiety, and experience), observations of the actions of others called behavioral influences (cognitive strategies, metacognitive strategies, and feedback), and environmental influences (modeling, achievement, and input from others). Bandura’s (1989) social cognitive theory provides clarity as to how behavior can be predicted, comprehended, and changed, and it provides insight as to how internal and external factors influence individuals’ perceptions of their abilities. Bandura’s social cognitive theory provides an important cognitive perspective on motivation. It assumes that motivation is a goal-directed behavior initiated and sustained by an individual’s expectations regarding the expected outcomes of their actions and their self-efficacy for performing those actions (Schunk, 2012).

Bandura’s theory also assumes that facets of learning such as knowledge, rules, skills, strategies, beliefs, and attitudes are attained through the observation of others in a social environment (Schunk, 2012). Based on the theory, a person’s perception of progress increases self-efficacy and nurtures motivation. Thus, individuals learn to set goals and undergo vital self-regulation processes (self-observation, self-judgement, and self-reaction) to regulate their cognitions, emotions, behaviors, and environments in ways that facilitate the achievement of those goals (Schunk, 2012). The focus of this study was based on Bandura’s (1997) self-efficacy theory, a subset of Bandura’s (1986) social cognitive theory.

Bandura's Self-Efficacy Theory

Bandura (1986) defined self-efficacy as the beliefs in one's own capabilities to organize and perform the courses of action required to produce given attainments. Within his self-efficacy theory, Bandura theorized that individuals lack the incentive to act unless they believe that they can produce desired outcomes. In addition, Usher and Pajares (2008) stated that the self-efficacy beliefs of individuals develop as the result of emotional, cognitive, or motivational processes, behavioral indicators, or social environments. In educational settings, students' self-efficacy beliefs can be enhanced when students modify personal factors such as their emotions and thoughts, when their teachers use environmental factors such as effective classroom structures, and when students improve self-regulation (Usher & Pajares, 2008). This study sought to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. According to Bandura (1997), individuals form their self-efficacy beliefs by selecting and interpreting information from the previously identified four primary sources: mastery experience, vicarious experience, social persuasions, and physiological or affective states. As teachers use manipulatives during math instruction, they should consider if avenues are provided for students with mathematics difficulties to select and interpret information from these four primary sources of self-efficacy and for students to formulate their self-efficacy beliefs.

Bandura's (1997) self-efficacy theory provides a framework to illuminate the lived experiences of teachers who use manipulatives to foster students' self-efficacy during math instruction. Through this study the teacher participants were given the opportunity to share their experiences and provide a comprehensive understanding of the phenomenon based on the perspectives of teachers rather than on the learning outcomes of students. Bandura's self-efficacy theory may inform and advance the literature on this topic because it sheds light on teachers' shared experiences regarding their use of manipulatives through the lens of their students' sources of mastery experience, vicarious experience, social persuasions, and physiological or affective states.

Mastery Experiences

The most powerful source of self-efficacy information is mastery experience (Usher & Pajares, 2008). Mastery experiences entail an individual's previous performances, which are perceived as successes or failures that influence one's future performances on similar tasks, conveying the notion that past successes result in future successes (Webb-Williams, 2017). According to Bandura (1997), building self-efficacy from mastery experiences requires time as individuals undergo a cycle of successes and failures as they achieve mastery. A student's own previous performance is the most influential source of information concerning self-efficacy because it provides the most authentic and accurate evidence regarding one's ability to successfully complete a task (Bandura, 1997; Schunk, 2012). Bandura claimed that the development of self-efficacy is not based on the success or failure of the performance but on what the performance reveals about one's effort and capability.

Based on Bandura's theory, successful performances that are attained with minimal effort do not produce a consistent sense of self-efficacy because an individual is more likely to avoid challenging tasks that require significant effort. Moreover, when individuals fail to accomplish an easy task, their self-efficacy development is negatively impacted (Bandura, 1997). In addition, students' self-efficacy may decrease when they experience successive unsuccessful performances which may cause them to experience even more unsuccessful performances in the future. Individuals who are accustomed to attaining easy successes look forward to receiving continuous instant success and oftentimes become frustrated when faced with failures (Bandura, 2012; Shipherd, 2019). To increase self-efficacy, Bandura (1997) postulated that individuals should attempt to complete challenging tasks to achieve continual efficacious performances.

Vicarious Experiences

One's self-efficacy can also be derived indirectly from vicarious experiences through which persons observe the actions of others accomplishing a task. Vicarious experience entails the expertise, behaviors, and beliefs of others and not one's own level of mastery (Webb-Williams, 2017). Therefore, vicarious experiences enable people to change their own self-perceptions by observing, evaluating, comparing, and modeling others (Bandura, 1997; Usher & Pajares, 2008). Vicarious experiences influence self-efficacy by providing opportunities for individuals to observe others as they display perseverance to successfully complete a task (Bandura, 2012). Witnessing the successful performances of others enables the observers to develop a stronger sense of self-efficacy as they begin to believe that they too will perform successfully if they persevere. Shipherd (2019) stated that observation of the successes and failures of other individuals affects the observers' perceptions of their own capabilities to perform similar tasks. Vicarious experiences influence self-efficacy because persons usually measure their capabilities by comparing their performances to the performances of others. According to Bandura (1997), the more the observer believes that they are like the performer, the more likely the success or failure of the performance will impact the observer's self-efficacy. Observations can also influence the development of self-efficacy because observing failures may also empower observers to utilize the skills they possess to successfully perform in similar situations (Usher & Pajares, 2008).

Vicarious experiences can also influence self-efficacy through modeling. Modeling involves teaching others new skills, coping strategies, and problem-solving. According to Shipherd (2019), modeling is also a source of motivation. When a person observes another person successfully completing a task, his or her motivation to complete a similar task may increase. One can become a model for others when they possess a strong sense of self-efficacy and exhibit positive behaviors such as encouraging self-affirmations, confidence, and determination as difficulties are encountered (Bandura, 1997).

Social Persuasions

According to Bandura (1997), self-efficacy increases when a trusted figure affirms their confidence in someone's ability to complete a task. This verbal feedback or social persuasion must be realistic to ensure that the receiver's perceptions are not sabotaged or the trusted figure is not compromised (Bandura, 1997). Social persuasion can build or hamper the development of self-efficacy (Bandura, 1997). Persons build their self-efficacy beliefs from social persuasions given by teachers, parents, and peers who provide constructive feedback and appraisals regarding their performance (Webb-Williams, 2017). Of the four self-efficacy sources, social or verbal persuasion is the easiest influencer even though it is weaker than mastery experience and vicarious experience (Bandura, 1977). According to Shipherd (2019), social persuasion happens when someone such as a teacher or peer uses verbal encouragement or discouragement to influence an individual's self-efficacy. The verbal encouragement or discouragement can either develop or weaken the individual's self-efficacy (Bandura, 1997). Failure is often met with negative feedback that provides minimal suggestions of how to attain success; therefore, one must take this into consideration when providing feedback to persons experiencing failure.

Physiological or Affective States

The fourth source of self-efficacy is physiological or affective states. This source denotes that self-efficacy beliefs are informed by situational emotions such as excitement, stress, anxiety, mood, and exhaustion (Usher & Pajares, 2008; Webb-Williams, 2017). According to Shipherd (2019), physiological states involve feelings of pain and fatigue, whereas affective or emotional states involve feelings of arousal, anxiety, or pleasure. Physiological and affective states are weaker predictors of ability than mastery experiences or social persuasions; nonetheless, self-efficacy is constructed when the four sources of self-efficacy are processed cognitively (Bandura, 1997).

Related Literature

Mathematics competence is critical for student achievement; however, many students, especially those who have LD struggle with mastering the learning objectives outlined in mathematics curricula and demonstrate poor conceptual understanding of mathematical concepts (Ok et al., 2020; Park et al., 2021; Satsangi, Hammer, & Evmenova, 2018; Vaughn & Bos, 2019). According to the Nation's Report Card (2022), there was a significant decline in fourth-grade mathematics scores at all five selected percentiles, which was the largest score decline since the initial mathematics assessment in 1990. The report revealed that fourth-grade scores are consistently declining across the percentiles and across all regions of the United States and in 43 states/jurisdictions. In addition, the percentage of fourth graders performing below the National Assessment of Educational Progress (NAEP) basic level ranged from 8% to 90%. Moreover, Ok et al. (2020) revealed that this trend of low mathematics achievement is likely to progress at middle school level and continue throughout the high school level. The importance of academic achievement for students who experience mathematical difficulties has amplified over the years.

To meet the needs of this population, educators have turned to innovative and evidence-based techniques and technologies such as manipulatives to improve instruction in the classroom and to address the struggles faced by students in academic areas such as mathematics. This related literature review section discusses pertinent topics related to the phenomenon being studied. The topics include middle school mathematics, LD in mathematics, self-efficacy and achievement, the importance of student–teacher relationships, remediated instruction, evidence-based practices for mathematics instruction, and manipulatives.

Middle School Mathematics

According to Ketterlin-Geller and Chard (2011), mathematics requirements increase significantly at the onset of the beginning of middle school. Middle school mathematics performance and achievement serve as a foundational predictor of future mathematics achievement (Norton, 2019). Although students experiencing mathematics difficulties can make rapid growth in the elementary grades, many students reach a plateau in their mathematics growth by middle school (Powell et al., 2021; Wei et al., 2013). Middle school entails the school period between elementary and high school. Wilkie and Sullivan (2018) identified middle school as Grades 5–8 with students of the age range of 9 to 14 years old; Powell et al. (2021) and Klingbeil et al. (2019) identified middle school as Grades 6–8 with students of the age range of 11 to 14 years old; and Norton (2019) identified middle school as Grades 7–10 with students of the age range of 11 to 15 years old.

Despite the differences in grade and age ranges, Powell et al. (2021) stated that the focus of mathematics instruction in middle school primarily involves readiness for algebra and preparation for formal algebra coursework. The authors also noted that when students complete middle school, they are expected to be ready for the rigors of algebra coursework. Traditional math instruction involves well-designed lectures by teachers and the completion of worksheets and textbook problems by students. Even though these traditional instructional strategies do provide valuable learning opportunities, heavy dependence on them is not very effective in increasing students' understanding or motivation (Rittle-Johnson et al., 2021). Given the details regarding such a critical time in the lives of students who struggle in mathematics, the next section includes a discussion about LD in mathematics.

Learning Difficulties in Mathematics

Elastika et al. (2021) stated that mathematics is a language entailing careful, clear, and accurate notations that can assist people in comprehending and solving social, economic, and natural problems. The authors also claimed that mathematics is used in numerous fields, and when learned proficiently, students may develop critical, logical, systematic, careful, effective, and efficient cognitive thinking skills. Lei et al. (2020) also noted that mathematics literacy equips students with the necessary skills they need to analyze information and communicate, both in academic environments and in their daily lives. According to Zhang et al. (2020), mathematical difficulties have been described as mathematics learning disability and a persistent low achievement in math. This underachievement is determined by a student's expectancy level based on age, intelligence, and education (American Psychiatric Association, 1994).

According to Klingbeil et al. (2019), students who are identified with mathematics difficulties or students who are expected to experience future mathematics difficulties often attain scores that range below proficiency levels on their end-of-year statewide achievement tests. These authors stated that educators refer to a combination of useful data sources such as grades and test scores to identify and predict students at risk of mathematics difficulties in middle school. According to Ekstam et al. (2018), attaining basic mathematics competencies is a very important prerequisite for accomplishing day-to-day routine activities in the 21st century. At the completion of middle school, students should have already been equipped with the basics for mathematics success and are expected to be prepared for even more demanding work in their final high school years.

LD in math can be influenced by various internal and external factors (Elastika et al., 2021). According to Albelbisi and Yusop (2019), these internal factors may include learning motivation, problem solving, intelligence, and personality, whereas external factors may include learning methods. Watson and Gable (2012) stated that students experiencing mathematics difficulties may have a specific learning disability in mathematics. Additionally, students with mathematics difficulties usually fall within two categories as those who (a) have prescribed Individualized Education Program (IEP) goals in the mathematics domain or (b) have been diagnosed with dyscalculia (Myers et al., 2021; Rapin, 2016; Skagerlund & Träff, 2016).

Dyscalculia is a mathematics learning disability that is characterized by a cognitive deficiency in numerical processing (Butterworth, 2005; Lei et al., 2020; Mussolin et al., 2010), whereas a specific learning disability may be due to cognitive difficulties with working memory, attention, or spatial reasoning (Powell et al., 2021). Dyscalculia affects an estimated 6%–8% of school aged children (Shalev, 2007). According to Lewis et al. (2022), researchers who attempt to discover the characteristics of dyscalculia rely heavily upon the vague indications of low mathematics achievement. Mazzocco (2007) also noted that students who experience mathematics difficulties may not necessarily have a school-identified disability but may have simply attained below average scores on proficiency tests or academic assessments. This coincides with Powell et al.'s (2020) findings that many students have mathematics difficulty but are not officially diagnosed with a disability.

Students with mathematics difficulties often struggle with grasping mathematical concepts such as counting, comparison, operations, symbolic representations of numbers, number sense, fact retrieval, problem solving, performing multistep problems, and understanding and applying mathematics vocabulary (Butterworth et al., 2011; Forsyth & Powell, 2017; Geary, 2004; Geary et al., 2012; Koponen et al., 2018; Zhang et al., 2014, 2020). A main area of difficulty for students with mathematics difficulty is solving word problems (Jitendra & Star, 2012). Wei et al. (2013) found that even though students with mathematics difficulties in elementary grades find success, this success begins to diminish by middle school. At this time of transition, mathematics learning becomes more demanding and complex as early learning gaps result in future challenges and impede math achievement (Witzel, 2016).

Timing is crucial, and as Ketterlin-Geller and Chard (2011) stated, the development of proficient algebraic reasoning skills is a critical objective to meet, especially for students with mathematics difficulties. Students with mathematics difficulties continue to experience difficulties with foundational mathematics skills and pre-algebra skills in middle school (Jitendra et al., 2017; Shin & Bryant, 2017). Lee (2012) asserted that middle school mathematics achievement provides a foundation for future mathematics achievement since middle school mathematics performance scores predict high school graduation, college readiness, and college completion (Lee, 2012). Research found that failing a sixth-grade mathematics course serves as a more reliable predictor of whether a student would successfully complete high school, as opposed to language barriers, race or ethnicity, and economic status (Balfanz et al., 2007; Siegler et al., 2012). Given the critical information regarding LD in mathematics, the next section includes a discussion about self-efficacy and achievement in students who experience mathematics difficulties.

Self-Efficacy and Achievement

Empirical research has established that mathematics self-efficacy is positively associated with mathematics achievement (Pajares & Graham, 1999; Pietsch et al., 2003; T. Stevens et al., 2004). Studies have also shown the effect that mathematics self-efficacy and performance results have on each other (Pajares, 1996; Pampaka et al., 2011). Self-efficacy beliefs are one’s discernments about their perceived ability to perform future tasks (Bandura, 1997), such as students’ beliefs that they can attain satisfactory scores on an impending math assignment. According to McCabe (2003), self-efficacy greatly influences motivation, which is the degree to which an individual will participate in or use energy to complete a task. In this section, the ways in which self-efficacy can influence achievement are discussed.

Ben-Naim et al. (2017) related that Bandura’s theory of self-efficacy implied that students with low self-efficacy levels often (a) fail to set high goals, (b) fail to attempt completing complex tasks, and (c) fail to face challenges and setbacks that consequently affect their ability to accomplish personal achievements and increase their susceptibility to poorer state of well-being. Additionally, Vukman et al. (2018) reported that students with LD perceive lower levels of both social and academic success compared to students without LD. Students with mathematics difficulties experience low mathematics achievement as measured by performance criterion such as screening assessments (Nelson & Kiss, 2021). These students experience difficulties with counting, computation strategies, and performing multiple-step problem solving (Nelson et al., 2022). Bandura (1986,1997) stated that mathematics self-efficacy can be acquired through four main sources: mastery experience, vicarious experience, verbal persuasion, and emotional and physiological states. According to Street, Stylianides, and Malmberg (2022), task difficulty may affect one’s self-efficacy beliefs through appraisals as to the relevance of the self-efficacy source for them to potentially succeed in the future. Perceptions of one's mastery experiences is deemed to be the strongest of the four named sources of self-efficacy (Byars-Winston et al., 2017; Usher & Pajares, 2008).

Self-efficacy in mathematics pertains to engagement, persistence, and academic performance (Street, Malmberg, & Stylianides, 2022). Students' motivation and achievement can be impacted by students' self-efficacy levels because students' high self-confidence in their ability to perform certain tasks will drive them to attain and sustain planned endeavors. Vukman et al. (2018) revealed that students with LD need support systems that are designed to enhance social skills and self-efficacy since they are vulnerable to higher levels of anxiety and lower levels of perceived self-efficacy. Unfortunately, low perceived self-efficacy among students with learning LD also puts them in a state of distress during their school years that continues to be dominant during their adult life (Ben-Naim et al., 2017).

To curb these dooming trends, innovative avenues need to be explored that may increase students' self-efficacy levels to attain and sustain reputable student motivation and achievement. A major innovative avenue that can be explored involves thoroughly investigating and analyzing teachers experiences regarding their use of instructional resources that may foster student self-efficacy. Regarding specific factors that may increase students' perceived self-efficacy levels, Miesera and Gebhardt (2018) discussed factors such as policies, student participation, classroom practices, and teacher training and professional development; however, the body of literature also revealed other factors that increase students' self-efficacy: (a) collaboration and (b) student–teacher relationships and support programs. In essence, students' self-efficacy can also influence their own achievement and engagement. Vukman et al. (2018) reported that students with LD are vulnerable to lower levels of perceived self-efficacy. This low level of perceived self-efficacy among students with LD places them in a stressful condition that is prevalent during their school years as well as their adult life (Ben-Naim et al., 2017). Regarding Bandura's theory of self-efficacy, Ben-Naim et al. (2017) also claimed that students with perceived low self-efficacy lack the ability to achieve personal accomplishments because they seldom (a) set high goals, (b) complete difficult tasks, and (c) face challenges.

Bandura (1986) also claimed that self-efficacy beliefs inform outcome expectations. Nilsen (2017) addressed a barrier that affects the self-efficacy of students with LD when they are given access to the general education curriculum. The dooming barrier is the lack of cooperation and coordination among special education curriculum leaders and general education curriculum leaders. This barrier impacts the learning process as well as learning outcomes. The evident lack of coherency and continuity that students with LD experience when they are given access to the general education curriculum causes them to be at risk of having to learn and perform in different ways that may also lower the chance of their receiving continual learning support. Students with higher perceived self-efficacy will learn and perform at a higher standard than students with lower self-efficacy levels (Öqvist & Malmström, 2018). Accordingly, students' engagement and achievement are certainly influenced by self-efficacy levels because their level of self-confidence regarding their ability to perform specific tasks will motivate them to attain and sustain delineated goals.

Students' perceptions regarding their mathematics self-efficacy can impact mathematics achievement by influencing some behavioral and psychological processes (Bandura, 1986, 1997). Unfortunately, teachers' perceptions may not coincide with students' perceptions. Consequently, students' self-efficacy levels can remain dormant and negatively affect their achievement and motivational levels. Curriculum leaders and educators must take students' perceptions of the effectiveness of the general education curriculum into account. Connor and Cavendish (2020) found that a student with a diagnosed LD is likely to achieve higher self-efficacy levels when their teacher adopts a curriculum that embodied certain characteristics and pedagogical practices. Some of these desired elements include (a) flexibility, (b) diversified instruction, and (c) individualized feedback.

These elements are critical to this literature review because they provide evidenced-based implications for educational reform and advancement. Failure to relay and consider students' perceptions of their self-efficacy can hinder the reform and implementation of quality curriculums that cater to students with LD. The self-efficacy and achievement of students with mathematics difficulties are important factors to discuss because students with low mathematics self-efficacy are less likely to attempt mathematics tasks, exert less effort when completing tasks, fail to persevere when facing challenges, and subsequently fail to achieve math goals (Bandura, 1997; Pajares, 1996; Schunk, 1981). Given the importance of providing a curriculum that is appropriate for students who struggle in mathematics, the next section will include a discussion about the importance of establishing student–teacher relationships that enhance how students with mathematics difficulties learn.

Importance of Student–Teacher Relationships

Bandura's (1986) social cognitive theory suggests that learning occurs in a social context. In this social context, Bandura (1989) implied that human behavior involves a reciprocal interaction of the students' and teachers' personal influences such as self-efficacy, behavioral influences, and environmental influences. Behavioral influences involves feedback, and environmental influences entail input from others. Therefore, it is important that a healthy relationship filled with quality interactions is established between students and teachers.

According to Vaughn and Bos (2019), the learner and the teacher are the key players in the teaching–learning process model, and Schwab et al. (2018) noted that teachers are the key factors in facilitating reciprocal relationships with their students. These relationships will create favorable classroom climates. Student–teacher relationships are important because they foster self-determination to reach personal and academic goals (Friend & Cook, 2017).

In a healthy student–teacher relationship, the role of the student is to is to provide knowledge, attitudes, and experiences for the teacher to build upon during the learning process (Vaughn & Bos, 2019). These contributions made by the learner aid the teacher in determining ways to facilitate learning. According to Vaughn and Bos (2019), the role of the teacher is to provide knowledge, skills, beliefs, and attitudes regarding teaching, learning, and the world and to implement practices that promote student learning and engagement. The role of the teacher is to also keep abreast of new universal educational trends such as the use of manipulatives and to continuously seek and pursue avenues for professional development to ensure student success since teacher success is correlational to student success.

Moyer (2001) stated that teachers' content knowledge is a key factor that plays a critical role in the effective use and integration of concrete manipulatives during math instruction. According to Hansen et al. (2016), teacher professional development has undertaken significant changes within recent history, from portraying teachers as passive recipients of information imparted by experts in the 1950s to shifting to teachers taking a more active role in their professional learning by the 1980s–1990s. In this current day, this shift or change is still apparent as teachers' own expectations of themselves have increased, which causes them to seek professional development opportunities regarding new ideas and educational innovations that improve student outcomes (Collinson et al., 2009; Sancar et al., 2021). This shift was based on increased demands for higher educational standards for curriculum reform and on the increased realizations that curricular, pedagogical and assessment methods concerning student learning could provide powerful learning opportunities for teachers within their own contexts to encourage them to focus on students' conceptual development (Hansen et al., 2016).

According to Sancar et al. (2021), professional development in mathematics education seeks to enhance teachers' knowledge of mathematics and use of innovative instructional strategies and resources. Norton (2019) related that the level of teachers' knowledge of mathematics determines their ability to use instructional materials. Ekstam et al. (2018) noted that teachers' knowledge of mathematics positively affects the achievement levels of students with mathematics difficulties in middle school and in higher grades. Sztajn et al. (2011) stated that the goals of professional development in mathematics education are to (a) create a shared vision for mathematics learning and instruction, (b) foster sound mathematical understanding relevant to the instructional level, (c) generate a conception of how students learn mathematics, (d) garner in-depth pedagogical content knowledge, (e) comprehend the role of equity in mathematics education, and (f) develop a sense of self as a mathematics teacher.

Lack of professional development in the educational field, especially in the area of providing effective instruction, makes it difficult for some teachers to meet the individual and diverse needs of students with specific LD (Gottfried et al., 2019; Norton, 2019; Schwab et al., 2018). Thus, conducive learning climates are obstructed when teachers are unable to diversify instruction or fail to use effective instructional resources such as manipulatives since teachers' mathematical knowledge involves how to use a variety of manipulatives to represent mathematical concepts and procedures (Ekstam et al., 2018; Hill et al., 2005).

As students transition into middle school, they must adjust to a more demanding social and academic environment; therefore, it is critical that teachers develop reciprocal relationships with students who struggle to learn. According to Wilkie and Sullivan (2018), middle school students have been described as disengaged, having low self-efficacy, and as low achievers in mathematics classrooms, and it was found that it is more difficult for teachers to establish positive relationships with these students. These authors also studied middle school students' affective development in mathematics and found that the students' relationships with their teachers were based on key intrinsic factors such as attitude, beliefs, values, effort, self-efficacy, mindset, and goal orientation; extrinsic factors such as social relationships with teachers and peers; and environmental factors that influence their motivation. Therefore, special education teachers could use these factors to ensure that their students with math difficulties are willing to learn.

According to Kouzes and Posner (2017), unity is forged, not forced; therefore, factors that should be considered when establishing student–teacher relationships include communication barriers, limited resources, accountability, competence, cultural identity, and individuals' levels of commitment (Friend & Cook, 2017). To nurture good student–teacher relationships, support programs should also be considered. These support programs should be designed to enhance motivation and stimulate perceived self-efficacy for students. Vukman et al. (2018) suggested that schools should provide consistent counseling support tailored to reduce teachers' and students' frustration regarding instructional expectations and aiding the growth of self-efficacy and self-regulation strategies. Given the importance of establishing healthy student–teacher relationships that are appropriate for students who struggle in mathematics, the next section includes a discussion of how teachers' experiences may influence students' perceptions of self-efficacy and how certain factors may influence teachers' experiences.

Teachers' Experiences

According to Shipherd (2019), self-efficacy has a great influence on performance; therefore, it is critical for teachers to undergo rich experiences that will assist them in understanding how their students' self-efficacy beliefs are formed to provide effective remediated instruction. Some factors that may influence teachers' experiences are teacher characteristics, pedagogical skills, and motivation. To foster students' perceived self-efficacy levels, teachers can exhibit certain character traits and implement various practices that students perceive as constructive and effective. Connor and Cavendish (2020) proposed reliable suggestions for teacher characteristics and pedagogical skills that teachers can adopt to become more effective in enhancing students' self-efficacy and in nurturing student motivation and achievement. When talking about teachers' characteristics, the authors also stated that teachers can increase students' perceived self-efficacy by (a) being respectful, (b) finding out personal information about their students, and (c) being empathetic, patient, and humorous. The authors also talked about the importance of teachers having the following pedagogical skills: (a) fostering motivation though engagement, (b) individualizing instruction, (c) utilizing multimodal learning styles and differentiated instruction, (d) granting addition time for one-on-one teacher access, (e) practicing flexibility when pacing lessons, and (f) instituting proximity control for redirecting students' focus. Öqvist and Malmströ (2018) also claimed that teachers who are guiding, supportive, dedicated, and engaging are able to cultivate students' self-efficacy and motivation.

Regarding pedagogical practices, students identified ineffective teachers as individuals who are (a) not motivating and not engaging, (b) unable to use multi-modalities or diversify instruction, (c) disallowing of monitoring opportunities and inconsiderate of the pacing of lessons, (d) unable to give clear explanations, and (e) unable to individualize feedback to struggling learners.

Regarding teacher motivation, Kanwal (2015) stated that teachers become unmotivated because of inadequate salary, lack of status recognition of the job title, poor working conditions, limited career advancement, and lack of consequences for teacher accountability. Schiefele (2017) have established three dimensions of teacher motivation: (a) teacher interest in subject matter, (b) teacher interest in teaching methods, and (c) teacher interest in motivating students to learn, especially those with learning and behavioral problems. When teachers lose interest in these areas, they become unmotivated and students' perceived self-efficacy and learning are negatively affected.

Han and Yin (2016) have also discussed factors that cause teacher demotivation such as lack of school funding and resources, teacher burnout and stress, insufficient motivators or incentives, lack of experience to deal with students with LD and behavioral problems, teachers' inability to retain students' attention, and poor leadership skills of administration. The negative effects of unmotivated teachers greatly impact the motivation of learning of students, especially those with LD. According to Han and Yin (2016), one of the prominent needs to address the issue of teacher demotivation derives from reported teacher shortages by a number of western countries, Australia, and other European countries. Unmotivated teachers fail to provide effective remediated instruction in a learning environment that is conducive to nurturing students' learning (Schiefele, 2017). Increasing teacher motivation will subsequently assist in correcting this issue as teachers would be more inclined to embrace the use of more innovative resources and technologies such as mathematics manipulatives. Given the importance of garnering rich experiences when providing instruction to students who struggle in mathematics, the next section includes a discussion of the ways in which teachers have attempted to remediate mathematics instruction to enhance how students with mathematics difficulties learn.

Remediated Instruction

To proficiently identify and academically support at-risk students who may need remediated instruction in mathematics, some researchers have proposed that educators provide screenings in middle school that use a combination of historical and current collected data (Geary et al., 2012; Klingbeil et al., 2019; Shin et al., 2017). The No Child Left Behind Act of 2001 stipulated that student achievement be increased based on the use of evidenced-based research practices. Since this mandate was made, there has been an increase in research regarding how to improve mathematics instruction to meet the needs of students with mathematics difficulties (Powell et al., 2021). Researchers have provided syntheses in which teachers have attempted to modify instruction to provide effective strategies to students who struggle with mathematics difficulties (Myers et al., 2021; Powell et al., 2021). According to the Nation’s Report Card (2022), data from the (NAEP revealed that some students were failing to meet specified benchmarks for mathematical proficiency. The report also revealed that 38% of eighth-grade students performed below NAEP Basic which was larger by 7 percentage points compared to 2019 and larger than all preceding assessments dating back to 2003. It was also revealed that average scores declined in all five math content areas (number properties and operations; measurement; geometry; data analysis, statistics, and probability; and algebra) at the fourth- and eighth-grade levels as compared to 2019 scores. A total of 11 states who accommodate students with math disabilities attained higher scores in 2022 than in 2019 while the remining 40 states received lower scores. Since there was such a high percentage of states receiving lower math scores, it may be speculated that this decline is related to the COVID-19 challenges that teachers experienced as they tried to deliver math instruction in virtual settings.

According to Powell et al. (2021), middle school is an opportune setting for teachers to provide mathematics intervention to students who experience mathematics difficulties. To reach this group of students, the authors suggested that intervention efforts may involve reteaching math content from elementary grades. This remediated instruction should be targeted and systematic since students experiencing mathematics difficulties are more likely to experience these difficulties throughout middle school and high school (O'Shea et al., 2017; Wagner et al., 2006). Webel et al. (2015) reported that teachers tend to value curriculum-based materials that they perceive as engaging, diverse, and manageable. Manageability ensures that activities are not too challenging for students and that materials support the varying learning or readability levels of students.

Curriculum-based technology must emulate evidence-based practices to transfer credibility. Student success depends on mathematics curriculum decision-makers' ability to identify, locate, and use potentially effective materials (Webel et al., 2015). This implies that curriculum decision-makers should be competent and qualified. This competence can be gained through aligning professional development sessions to pedagogical approaches related to curriculum development (Nuttall et al., 2019). Beneficial student outcomes of incorporating evidence-based materials in the curriculum provide insight regarding students' learning needs and give students access to effective instructional resources (Gordon et al., 2019). A critical component of the process of preparing to teach mathematics is to evaluate the structure, cohesion, and audience appropriateness of instructional materials such as manipulatives for usability and friendliness; in so doing, teachers will be able to determine whether an instructional tool needs to be modified, augmented, or adapted (Vaughn & Bos, 2019). Mathematics necessitates critical thinking skills; reasoning skills; and the understanding of principles, thoughts, ideas, and patterns in our environment. Research indicated that most interventions used by teachers to teach math to students who struggle with mathematics difficulties garnered positive outcomes (Chodura et al., 2015; E. A. Stevens et al., 2018). It was also found that special education teachers' experience in mathematics instruction also influences students' mathematical achievement (Bolyard & Moyer-Packenham, 2008; Ekstam et al., 2018). Given the importance of remediating mathematics instruction to enhance how students with mathematics difficulties learn, the next section includes a discussion of evidenced-based practices that have proven to be successful in the educational advancement of students who experience mathematics difficulties.

Evidenced-Based Practices for Mathematics Instruction

Stevenson and Reed (2017) proposed that core instruction is not sufficient to meet the needs of students experiencing mathematics difficulties. They suggested that these students need more intense supplemental intervention. According to Zhang et al. (2020), teachers often resort to evidenced-based practices to tackle the challenges faced by students with mathematics LD. In addition, Kellems et al. (2020) noted that evidenced-based practices are continually developed and implemented to prepare students with learning disabilities to live productive lives. Horner et al. (2005) referred to evidence-based practices as educational approaches that are validated by research and has garnered sufficient empirical support. The most prevalent evidence-based practices used to attain positive mathematics outcomes for middle school students experiencing mathematics difficulties include explicit instruction, problem-solving, teaching mathematics vocabulary, use of mnemonics, use of graphic organizers, and use of multiple representations. Each of these interventions, as discussed further in the following paragraph, have been proven in research and practice to assist students with learning disabilities in mathematics.

Explicit instruction involves modeling, guided and independent practice, timely feedback, and clear instructions (Archer & Hughes, 2011; Flores & Kaylor, 2007; Hunt & Vasquez, 2014; Witzel, 2005). Problem-solving instruction involves teaching students how to solve problems in real-world contexts (Bottge et al., 2001) and teaching students the skills needed to solve word problems (Brawand et al., 2020). Teaching mathematics vocabulary involves teaching students relevant vocabulary words and how to use the language of mathematics (Bryant et al., 2020; Butler et al., 2003; Shin & Bryant, 2017). Use of mnemonics was explored (Brawand et al., 2020; Cade & Gunter, 2002; Cuenca-Carlino et al., 2016;) and the use of graphic organizers (Butler et al., 2003; Jitendra et al., 2016; Shin & Bryant, 2017). The use of multiple representations involves physical manipulatives, pictorial or virtual drawings, three-dimensional concrete manipulatives, and virtual manipulatives (Barrett & Fish, 2011; Bottge et al., 2010; Bouck & Park, 2020; Bouck et al., 2019; Bouck, Park, & Stenzel, 2020). Peltier et al. (2020) acknowledged the use of multiple representations as an effective element of numerous mathematical interventions. Given the importance of using evidence-based practices such as manipulatives to deliver mathematics instruction, the next section includes a discussion about manipulatives and how they may enhance the learning of students who experience mathematics difficulties.

Manipulatives

According to Carbonneau et al. (2018), mathematics manipulatives are one of the most common strategies used to deliver mathematics instruction. Manipulatives are evidence-based tools used during mathematics instruction to support students' conceptual comprehension of mathematical concepts and are considered beneficial for students with disabilities (Maccini & Gagnon, 2000; McNeil & Jarvin, 2007). Manipulatives increase student attention and engagement during mathematical tasks (Belenky & Nokes, 2009; Jimenez & Stanger, 2017; Moyer, 2001). The use of manipulatives makes abstract concepts more concrete (Goodman et al., 2016) and may result in improved mathematical performance (Carbonneau et al., 2013). According to Vessonen et al. (2021), teachers' beliefs regarding the use of manipulatives significantly influence how effectively they use manipulatives during math instruction. Powell et al. (2021) suggested that teachers who use multiple representations, such as manipulatives, to help middle school students who are experiencing mathematical difficulties may help these students understand abstract mathematical concepts. These authors also noted that manipulatives have the potential to expand students' imaginations, to prohibit them from assuming passive roles, and to encourage them to assume active roles that require them to explore their respective ingenuities.

Vaughn and Bos (2019) suggested that mathematics instruction should be student-centered rather than teacher-centered. Instruction regarding the use of manipulatives should incorporate constructivist's practices. As teachers use manipulatives, they should facilitate learning, permit the students to link new concepts to previously learned information, and foster the idea of learning by doing. Gordon et al. (2019) also implied that a constructivist-based instructional tool entails thought-provoking learning tasks that motivate students by developing their self-efficacy, self-regulation, and control over their learning.

Gecu-Parmaksiz and Delialioglu (2019) revealed that both concrete and virtual manipulatives are supportive instructional tools used to enhance instruction. Researchers have also claimed that concrete manipulatives are equally as effective as virtual manipulatives when used during math instruction for students with math difficulties (Bouck et al., 2017,2018; Roberts et al., 2020; Vessonen et al., 2021). Shurr et al. (2021) found that both concrete manipulatives and virtual manipulatives were highly beneficial instructional tools and were both significantly effective in increasing independence and accuracy when acquiring mathematical skills. Nonetheless, manipulatives can be a financial stress for teachers as their use is based on their access to resources and digital devices (Satsangi & Miller, 2017). In addition to financial burdens, teachers may hesitate to use manipulatives due to factors such as poor teacher collaboration or limited professional development opportunities.

Concrete Manipulatives

Concrete manipulatives are physical objects one can manipulate to help build conceptual understanding and solve mathematical problems (Bouck & Flanagan, 2009). According to the National Center on Intensive Intervention (2016), concrete manipulatives have been identified as an evidenced-based practice used to enhance the learning of students with disabilities. Common concrete manipulatives include base 10 blocks, fraction pieces, geoboards, geometry tiles, chips, and algebra tiles (Bouck et al., 2018; Carbonneau et al., 2013; Disbudak & Akyuz, 2019). Students may acquire experiential learning through physical interaction with objects because concrete manipulatives involve sensory factors such as touch and sight (Carbonneau et al., 2013; Disbudak & Akyuz, 2019). The use of concrete manipulatives has been used to enhance learning

(Carbonneau et al., 2013; Hidayah et al., 2021) and has been used in science, technology, engineering, and mathematics (STEM) disciplines (Justo et al., 2022). According to Roberts et al. (2020), concrete manipulatives cannot illustrate mathematical relationships in isolation; rather, they enable students to develop a comprehensive understanding of mathematical knowledge via the exploration of mathematical relationships that concrete manipulatives help model.

Disbudak and Akyuz (2019) expressed the importance of emphasizing the purpose of using concrete manipulatives to learners before they are taught mathematical concepts so that they can make sense of the manipulatives when they are using them. The authors stated that teachers' intentions and students' perceptions regarding the use of concrete manipulatives must coincide with each other, or the students may treat the manipulatives like toys. According to Roberts et al. (2020), the use of concrete manipulatives can foster learning environments that are conducive to student mathematical learning and engagement. These physical manipulatives are beneficial to students with and without learning disabilities, and they promote active learning, engagement, and increased student motivation (Satterthwait, 2010). According to Carbonneau et al. (2013), concrete manipulatives provide students with opportunities to construct better understandings regarding how abstract concepts relate to real-life situations, which can be a challenging task for students with learning disabilities.

Virtual Manipulatives

Although studies have demonstrated that the use of virtual manipulatives increases the mathematical accuracy of students with mathematics difficulties in concepts ranging from operational skills to algebra, virtual manipulatives have a limited research base in mathematics as compared to concrete manipulatives (Bouck et al., 2014; Satsangi et al., 2016; Shin & Bryant, 2017; Shin et al., 2021). As a matter of fact, a great portion of the research published in the last 5 years regarding multiple representations focused on virtual manipulatives (Bouck & Park, 2020; Bouck et al., 2019). Nonetheless, Bouck, Mathews, and Peltier (2020) admonished that virtual manipulatives should not replace concrete manipulatives but should complement concrete manipulatives. Virtual manipulatives are two- and three-dimensional objects that are represented via computer-based applications that allows users to rotate, flip, or enlarge images on the screen (Moyer-Packenham & Suh, 2012). In essence, virtual manipulatives are interactive, web-based visual representations of an object that affords opportunities for constructing mathematical concepts (Moyer et al., 2002).

The use of virtual manipulatives is rising in classroom settings (Keldgord & Ching, 2022). According to Satsangi, Hammer, and Hogan (2018), virtual manipulatives gained attention in research and practice over the past two decades, as digital access became prominent in K–12 educational settings. The use of virtual manipulatives provides teaching and learning opportunities of a wide range of mathematical concepts including operational skills, geometric properties, and properties of lines (Satsangi, Hammer, & Hogan, 2018). According to Reiten (2020), teachers resort to using virtual manipulatives during instruction because they provide opportunities for immediate feedback, differentiated instruction, and visual learning. Studies have shown that the use of virtual manipulatives have revealed promising results for elementary and secondary students with mathematics difficulties and disabilities (Moyer-Packenham & Suh, 2012; Park et al., 2021; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018).

Research has also shown that the use of virtual manipulatives is effective in supporting mathematics instruction to students with disabilities (Bouck & Park, 2018; Satsangi, Hammer, &

Hogan, 2018; Satsangi & Miller, 2017; Shin et al., 2017, 2021). In addition, Satsangi, Hammer, and Evmenova (2018) noted that the increase of digital access in schools has provided greater opportunities for educators to utilize promising tools such as virtual manipulatives. Despite these promising findings and the expanding prevalence of computer-based technologies in today's classrooms, Ok et al. (2020) revealed that the percentage of students experiencing mathematical difficulties is increasing and the mathematics achievement of students with mathematics difficulties has garnered little improvement. Moreover, Herold (2015) postulated that teachers have been excruciatingly slow to transform the way they teach despite the immense influx of innovative and effective technology into their classrooms. This slow improvement in student academic performance and slow teacher willingness to use more innovative technologies have established a gap in the literature. Therefore, more studies are needed to examine middle school teachers' experiences to identify or expand on potential causes of this lag in progress or usage of innovative resources.

Virtual manipulatives may help students develop their self-efficacy in mathematics by providing teachers with an innovative opportunity to draw upon students' sources of self-efficacy to overcome their learning disabilities. According to Bandura (2001), subjective belief in one's own abilities to complete a task is a major factor that determines success. Teachers can strive to foster a student's self-efficacy through their use of virtual manipulatives since self-efficacy is highly correlated with academic achievement (Pajares & Schunk, 2001).

As technology continues to evolve, more technologies are becoming available in educational settings. According to Moyer-Packenham and Suh (2012), numerous virtual manipulative programs and apps provide built-in supports including visual and textual prompting, which aid students in navigating through problem-solving processes and potentially increase their independence.

Resource Divide and Digital Equity

In this section, I discuss how digital inequality can deter the use of virtual manipulatives. Reiten (2018) suggested that virtual manipulatives appeal to teachers because they promote student understanding and engagement and can encourage students' development of conceptual understanding. Unfortunately, the literature merely focuses on the issue of digital inequity within school systems that may negatively affect teachers' experiences when transitioning to using more innovative instructional tools such as virtual manipulatives. Schrum and Sumerfield (2018) revealed that the increased prevalence of digital materials along with the increased necessity for digital fluency has amplified the need to address educational and digital equity.

Since the onset of COVID-19, a global pandemic that continues to affect our educational, economic, and health systems, the urgency to address the issue of digital equity has been expedited. According to Buchholz et al. (2020), the COVID-19 pandemic exposed digital inequities that were less obvious during the era of face-to-face learning. Technology has influenced instructional decisions such as enhanced learning opportunities for students and teachers. Ali et al. (2020) reported that education is observed as a vital process for the future of an individual and society and innovative trends foster the development of effective and engaging learning atmospheres. Limited access to educational and digital resources can impede this desired development.

The digital, pedagogical, and resource divide exists because of the inability of educational systems to fully enforce and execute digital inclusion. According to Schrum and Sumerfield (2018), digital inclusion refers to the activities needed to ensure that all individuals and communities (including the most disadvantaged) have access to and use of information and communication technologies (ICTs). This unwarranted divide might best be addressed by considering the five elements of digital inclusion and the four opportunities of technology proposed by the International Society for Technology in Education (Schrum & Sumerfield, 2018). Denied, limited, or no access to any mentioned elements or opportunities for digital inclusion produces the current issues that surround educational and digital access and may contribute to teachers' hesitance in embracing the use of more innovative resources and instructional technologies such as manipulatives.

Summary

This literature review outlined the need for future research that will provide additional insight into teachers' experiences regarding their use of both concrete and virtual manipulatives to foster the self-efficacy of students with mathematics difficulties. The theoretical framework that efficiently guided this study is Bandura's self-efficacy theory. This study provided pertinent information that can increase awareness of the importance of teacher perceptions and, thus, enhance overall teacher and student motivation and achievement. Currently, the literature exposes that many students, especially students who have LD, often struggle with mastering the learning objectives outlined in mathematics curricula and demonstrate poor conceptual understanding of mathematical concepts (Ok et al., 2020; Park et al., 2021; Satsangi, Hammer, & Evmenova, 2018; Vaughn & Bos, 2019). The literature also revealed that the use of virtual manipulatives has acquired promising results for elementary and secondary students with mathematics difficulties and disabilities (Moyer-Packenham & Suh, 2012; Park et al., 2021; Satsangi et al., 2016; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). Despite these promising findings, Ok et al. (2020) revealed that the percentage of students experiencing mathematical difficulties is not declining and Herold (2015) postulated that teachers have been excruciatingly slow to transform the way they teach despite the immense influx of innovative and effective technology into their classrooms. Results of this study may narrow the gap in the literature and provide practical solutions to the problem in the educational field to improve or inform professional practices.

CHAPTER THREE: METHODS

Overview

The purpose of this instrumental case study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. To conduct a thorough examination, I utilized a qualitative case study approach. This chapter provides a comprehensive description of the research design, procedures, and data analysis utilized in the study. The contents and organization of the chapter begin with an explanation of the research design and the listing of the research questions. The research questions are followed by details discussing the setting, participants, the researcher's positionality, procedures, data collection plan, and trustworthiness respectively. The data collection sources include individual interviews, focus group interviews, and classroom observations. The participants included 15 middle school teachers who provide math instruction at schools in a school district in The Bahamas.

Research Design

According to Yin (2009), a case study design increases the chances that holistic and meaningful characteristics of real-life events are retained as it is "an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident" (p. 18). This section entails a discussion of the research method, the research design, and the research approach. The research method of this study is qualitative, and the research design of this study is a case study. The research approach of this study is an instrumental case study. The discussion articulates a rationale for why a qualitative instrumental case study is an appropriate selection for this study.

Research Method

To conduct a thorough examination of this case study, I utilized a qualitative method. Qualitative researchers provide detailed accounts of human experiences in real-life conditions (Merriam, 1998; Yin, 2014). Creswell and Poth (2018) illustrated the qualitative approach as an inquiry process that enables the researcher to be a key instrument who conducts the research in a natural setting, uses multiple methods to dissect participants' multiple perspectives and meanings, comprehends contextual features, establishes patterns, builds themes from participants' voices and researcher reflectivity, and provides a holistic account. A qualitative approach is appropriate for this study because I analyzed teachers' experiences and contextualized the findings. Gall et al. (2007) claimed that a qualitative design would allow the researcher to bring a case to life through a process of thick description. Based on the assertions of Creswell and Poth (2018), I concluded that the use of the qualitative design is ideal for this study because I could gain a detailed understanding of the issue, empowering the participants to share their experiences as their voices were heard and the researcher–participant power relationship was minimalized. According to Stake (2010), qualitative inquiry entails personal experiences that support the refining of a theory. I interviewed and observed 15 middle school teachers about their personal experiences using the self-efficacy framework.

Research Design

Creswell and Poth (2018) claimed that researchers must determine their style of qualitative inquiry so that they may present it as a highly developed study. According to Merriam (1998), a case study design is utilized to attain an in-depth understanding of the phenomenon as interpreted by the participants and is focused on process, context, and discovery. I expected that a case study design would lead to the discovery of contextual characteristics that provide enlightenment regarding how the self-efficacy of students with mathematics difficulties can be fostered through teachers' use of manipulatives. A case study design was an appropriate design for my research study because it aligns with Yin (2014, 2017), who posited that the main research questions are detailed how or why questions, and the research topic includes the exploration of a real-life case over time through in-depth and diverse data collection to document and report case description and themes. A case study was selected for this study because I intended to thoroughly analyze the different perspectives of 15 middle school teachers and I had minimum or no control over behavioral proceedings (Yin, 2014). In addition, Yin (2018) characterized case study research as inquiry that relies on multiple sources of evidence. A case study design was ideal for this current study because three data collection methods were implemented to provide multiple sources of evidence: individual interviews, focus group interviews, and observations.

Research Approach

The type of approach for this study was instrumental case study. This design type was suitable for my study because it allowed me to examine the experiences of middle school teachers to best understand how they navigate the usage of manipulatives to foster student self-efficacy. An instrumental case study examines a particular case to provide insight into an issue (Creswell, 2013). According to Yin (2018), instrumental case studies provide the researcher with a comprehensive understanding of a phenomenon through a specific case. In addition, an in-depth analysis of multiple accounts helped me to determine if participants' experiences were similar or different to gain a better understanding of a phenomenon (Stake, 1995).

Research Questions

This study examined the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Bandura's theory of self-efficacy was the theoretical lens used to examine the problem identified in this study. Bandura (1997) hypothesized that students form their self-efficacy by selecting and interpreting information from four primary sources: mastery experience, vicarious experience, social persuasions, and physiological/affective states. For this study, the following research questions were posed:

Central Research Question

How do middle school teachers describe the factors that influence the development of their students' self-efficacy while using manipulatives?

Sub-Question One

How do middle school teachers describe their experiences of using manipulatives to facilitate the mastery experiences of students with math difficulties?

Sub-Question Two

How do middle school teachers describe their experiences of using manipulatives to enhance vicarious experiences for students with math difficulties?

Sub-Question Three

How do middle school teachers describe their experiences of providing social persuasions when using manipulatives with students with math difficulties?

Sub-Question Four

How do middle school teachers describe their experiences of addressing the physiological or affective states of students with math difficulties when using manipulatives?

Setting and Participants

This section describes the setting of the study and the criteria for the participants. It was my aim to provide a vivid description of the setting for the audience in sufficient detail so that they may be able to visualize the setting. The criteria for participation in the study are also articulated.

Setting

The study was conducted at the Starlight School for Exceptional Children (pseudonym) and at the Kingsway High School (pseudonym). These sites are located in The Bahamas. Starlight School for Exceptional Children enrollment is currently 144 students (78% males, 22% females) of which approximately 99% are reported to be Black and 1% White. The school caters to students with diverse learning disabilities, specifically 35% Learning Disabilities, 15% Intellectual Disabilities, 14% Autism, 12% Behavioral & Emotional Disorder, 10% Down Syndrome, 7% Developmental Delay, 4% Attention Deficit Hyperactivity Disorder, and 3% Blind (The Bahamas Education Management Information System, n.d.). The school is a public school that caters to students who require special education services and also partners with general education settings that provide services for special needs students in inclusive classrooms and self-contained classrooms (The Bahamas Education Management Information System, n.d.). This is a public all-aged special education school (ages 5–22; preschool, elementary, middle school, and secondary school) located in an urban area. The majority of the population are economically challenged as 98% of students qualify for Social Service assistance. Instructional mathematics sessions last for 40 minutes and the school employs 45 teaching staff members consisting of trained teachers and teachers’ aides (The Bahamas Education Management Information System, n.d.). The leadership structure of the school is composed of a principal, senior assistant, and department heads that supervise each department of the school (primary and secondary school).

The Kingsway High School enrollment is currently 1,212 students (63% males, 37% females) of which approximately 98% are reported to be Black and 2% White. The school is a public school that mainly caters to general education students but also provides inclusive and self-contained classes for students who struggle with mathematics and reading (The Bahamas Education Management Information System, n.d.). The Kingsway High School is also located in an urban area, and the majority of the population are economically challenged as 62% of students qualify for Social Service assistance (The Bahamas Education Management Information System, n.d.). Instructional mmathematics sessions last for 45 minutes, and the school employs 86 teaching staff members (The Bahamas Education Management Information System, n.d.). The leadership structure of the school is composed of a principal, two vice principals, two senior masters, two senior mistresses, and 17 subject coordinators that supervise teachers of each subject taught in the school.

Both of these sites were selected because teachers currently use both concrete and virtual manipulatives at these schools and significantly increased their use of virtual manipulatives since September 2019 due to the aftermath of Hurricane Dorian and the onset of the COVID-19 pandemic (The Bahamas Education Management Information System, n.d.). Teachers were obligated to identify and efficiently use effective virtual platforms and concrete resources. These schools were ideal to acquire the authentic experiences of participants who have used manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties.

Participants

According to Yin (2014), participants should be purposefully selected. Gall et al. (2007) stated that determining the number of cases in qualitative research is a matter of judgement. In this section, information regarding the sample pool, the sample size, and the type of sample is discussed. The sample pool consists of all middle school teachers who use manipulatives during math instruction and are employed at schools in The Bahamas. The total number of possible participants was 3,097 (The Bahamas Education Management Information System, n.d.). According to Stake (1995), it is of vital importance that participants in a case study possess shared experiences to acquire relatable themes. Participants in this study were teachers who provide math instruction to middle school students. I recruited 15 participants for this study because Stake (1995) suggested that instrumental case studies should be comprised of a smaller sample size.

The participants for this study were selected through the combination/mixed purpose sampling method. Purposeful sampling provides a comprehensive understanding of the case such as supplementary sources of evidence that can be considered by the researcher (Yin, 2014). The sampling method that I used was the criterion sampling method. The criterion sampling method is utilized to identify cases that meets the prescribed criterion to foster quality assurance (Creswell & Poth, 2018; Gall et al., 2007). Gall et al. (2007) also revealed that criterion sampling is useful for educational studies and that a study of cases that meets the established criteria would most likely yield rich information regarding the phenomenon being studied.

The teacher participants were recruited from a mathematics classroom setting. I identified a gatekeeper who forwarded recruitment emails to determine participant selection. My identified gatekeeper was able to solicit sufficient responses. All participants met the following inclusion criteria: (a) is a trained teacher possessing a bachelor's degree or higher in education, (b) has administered math instruction using manipulatives to middle school students within the past 3 years, (c) has 3 or more years of teaching experience, and (d) ranges between the ages of 25–55 years.

Researcher Positionality

This section entails my motivation for conducting this study. My interpretive framework is articulated followed by a discussion of the three philosophical assumptions that guided the study: namely ontological, epistemological, and axiological assumptions. This section concludes with an explanation of the researcher's role.

My motivation for conducting this study stems from my experiences of assisting students who struggle with grasping mathematical concepts. As a special education teacher, I have the privilege of providing instruction for students with special needs as well as general education students. I am constantly searching for effective strategies to enhance my students' learning, and I am always eager to share my experiences of implementing these strategies with my colleagues so that their students can also reap the benefits of the strategies as well. In The Bahamas, all educators are familiar with the use of concrete manipulatives; however, due to lack of finances, training, and resources, all schools do not have access to reliable internet and sufficient technological devices to adopt the use of virtual manipulatives. As teachers seek to enhance mathematical education, digital equity and universal access to quality learning environments should also be kept in the balance.

I was also motivated to conduct this study because I am also an advocate for science, technology, engineering, and math (STEM) education. According to Schrum and Sumerfield (2018), STEM is a curriculum based on the idea of educating students in four disciplines—

science, technology, engineering, and mathematics—while using "an interdisciplinary, hands-on approach that relates to real-world applications" (p. 8). Schrum and Sumerfield (2018) also noted that the model of STEM education is designed to develop learners' conceptual knowledge of the interconnected nature of science and mathematics as they advance their understanding of engineering and technology. Regarding ethical considerations and best practices, English (2016) noted that that educators, policy developers, and other stakeholders are emphasizing the urgency for improving STEM skills to meet current and future social and economic challenges. In addition, Struyf et al. (2019) noted that educational researchers and policymakers are prioritizing the need to ensure students' engagement and achievement in the four STEM disciplines globally as well as prepare students for the STEM-influenced and dominated labor market. In the government school systems in The Bahamas, the lack of STEM education deprives students of opportunities that encourage them to take active roles in their own learning and sometimes forces them to be passive receivers of information. I hope that the results of this study provide insights that will encourage educational leaders to invest in reliable resources to make an informed decision as to whether the implementation of manipulatives is desired by teachers and beneficial to students.

Interpretive Framework

The interpretive framework or research paradigm that guided this study is pragmatism. According to Creswell and Poth (2018), pragmatism emphasizes that the significant feature of research is the problem being studied and the questions asked about the problem. The case study approach originated from the domains of anthropology and sociology and also has historical popularity in the social sciences such as psychology, medicine, law, and political science. Correspondingly, pragmatists believe that research transpires in social, historical, political, and other contexts (Creswell & Poth, 2018). Pragmatism enables a researcher to direct their focus towards the outcomes of the research such as the actions, situations, and consequences of the investigation instead of focusing on antecedent conditions (Creswell & Poth, 2018). My goal was to contextualize the findings of the research based on teachers' experiences rather than generalizing the findings based on known variables and established guidelines. According to Gutek (2011), pragmatics view the world as pluralistic, tentative, open and changing; and believe that human interactions with the environment and relationships are flexible, malleable, and constantly needing reappraisal and readjustment. The transition process in educational domains is flexible, changing, and forever in need of reappraisal and readjustment to bring about change that would foster academic achievement. Pragmatism is an appropriate interpretive framework for this study because pragmatists view philosophies as instruments or hypotheses arrived at by humans, to be developed, acted on, and examined in the reality of experience (Gutek, 2011). The purpose of this study was to examine the experiences of the study participants.

Creswell and Poth (2018) also stated that researchers who use pragmatism will (a) utilize multiple methods of data collection that best answer the proposed research questions; (b) be attentive to the practical implications of the research; and (c) value research that is conducted to best address the research problem. I utilized multiple methods of data collection such as individual interviews and focus groups to ensure that the research questions were satisfactorily answered. I also ascertained and shared practical implications of the research as I meticulously addressed the research problem.

Philosophical Assumptions

Researchers make philosophical assumptions when they commence qualitative studies (Creswell & Poth, 2018). Philosophical assumptions are based on the researcher's values and belief systems and help the audience comprehend the approach to the study from the researcher's point of view. In this section, three philosophical assumptions are addressed: (a) ontology, (b) epistemology, and (c) axiology.

Ontological Assumption

According to Creswell and Poth (2018), ontological assumptions entail the nature of reality and its characteristics. Guba and Lincoln (1988) asserted that something is real in qualitative research when it is constructed in the minds of the participants who are involved in or have experienced the phenomenon. The ontological assumption involves accepting the idea of multiple realties. As a qualitative researcher, I believe that reality is socially constructed and built upon persons' experiences. Therefore, I must value and use the responses and views of all participants to provide representations of diverse perspectives and to establish credible themes. Creswell and Poth (2018) supported this action by stating that qualitative researchers must intentionally embrace and report multiple realities. To provide evidence of multiple realities, I incorporated various forms of evidence in established themes using the authentic words of the 15 participants and presenting their diverse perspectives in the research findings.

Epistemological Assumption

The main philosophical assumption that led to my choice of research is the epistemological assumption. According to Creswell and Poth (2018), knowledge is known through the subjective experiences of the participants and the longer the researcher remains in the field or becomes familiar with the participants, the more they would "know what they know" from direct evidence. I became familiar with my participants by seeking their truths and perspectives through interviews and focus groups. Accordingly, Creswell and Poth (2018) stated that epistemological assumptions involve attaining subjective responses from the participants as the researcher collaborates and spend time with the participants in the field. Epistemological assumptions were beneficial to this educational study because they examined instructional processes and provided implications of how students learn and how educators should teach. According to Gutek (2011), if an educator's teaching methods follow the way students learn, then there is "a better chance that they would be effective" (p. 40). Based on the findings of this study, educators would be able to reflect upon their experiences of using manipulatives and to assess if manipulatives are compatible with the students' academic achievement and, therefore, would be able to determine effectiveness or ineffectiveness of the use of the strategy.

Axiological Assumption

The axiological assumption entails the role of the researcher's values in the study. According to Creswell and Poth (2018), the researcher discloses the value-filled nature of the study and gives an account of their values and biases "as well as the value-laden nature of information gathered from the field" (p. 21). Second Peter 1:10 (*New International Version*, 1978/1992) reads, "Therefore, my brothers and sisters, make every effort to confirm your calling and election. For if you do these things, you will never stumble." As a Christian, I believe that I find aspects of education rewarding because I have pursued a profession that is my calling. My calling is to impart knowledge and Christian values to children; therefore, I believe that every one of my students possesses value even if they have a disability. The words of Psalm 139:13–14 (*New International Version*, 1978/1992) state, "For you created my inmost being, you knit me together in my mother's womb. I praise you because I am fearfully and wonderfully made; your works are wonderful; I know that full well." As a special education teacher, I view each one of my students as ones who can gain a positive learner identity.

Regarding my social position, I am a citizen of The Bahamas who has personal experiences in providing mathematics instruction in the general and special education field; I also have experience teaching face-to-face and online math instruction with manipulatives. Berger (2015) stated that the researcher must also describe aspects such as experiences, beliefs, gender, age, race, and immigration status as well to convey values or positionality regarding the context and setting of the research to the audience. Being aware of values and biases aided bracketing and solidified the study's trustworthiness. I am a special education teacher who has been employed at a special education school for 9 years. I have also previously taught in the general education setting for 2 years.

Researcher's Role

Titus 2:7–8 (*New International Version*, 1978/1992) reads, "In everything set them an example by doing what is good. In your teaching show integrity, seriousness, and soundness of speech that cannot be condemned." Therefore, I believe that the main role of the researcher is to set a good example by upholding godly principles to attain favorable outcomes in these challenging times of adversity and uncertainty that the COVID-19 pandemic and other unprecedented devastations have created.

According to Gall et al. (2007), the role of a qualitative researcher is to be the primary "measuring instrument" (p. 458). My role as the human instrument in this study was to create a viable research proposal, attain the relevant institutional review and approval, and obtain permissions from the schools in which I conducted my study. My role in the setting was to conduct interviews, focus groups, and classroom observations. I had no prior social relationships with the participants of this study, and I did not have any authority over the participants. During data collection my role was to become personally connected to the phenomenon being examined.

I worked collaboratively with the research participants and used empathy and other psychological processes to gain a comprehensive understanding of the phenomenon as it is experienced by the participants (Gall et al., 2007). I recorded and honored differing views by not plagiarizing or falsifying information. I achieved this through honesty, avoiding deception, and accepting responsibility for my own work (Yin, 2014). I also avoided biasness during data collection and data analysis procedures.

According to Lämsä et al. (2018), students with disabilities must be motivated and engaged to practice and acquire skills that were proven difficult for them to learn. Other than my personal experiences, my educational and personal philosophies also impacted how I viewed the data and research surroundings. Like Vaughn and Bos (2019), I believe that learning is not simply the accumulation of knowledge and skills but is also the development and transformation of ideas based on observations and experiences. In my view, the purpose of education is to improve students' social, academic, and intellectual development to produce high-performing, motivated students who are equipped with proficient skills to become contributive citizens of society who are driven by social, moral, and cultural responsibility.

Procedures

In this section, I descriptively outline the steps that I used to conduct the study. This description includes the appropriate site permissions, Institutional Review Board (IRB) approval from Liberty University to conduct the study, and the procedures for recruiting participants.

Permissions

Firstly, I received district permission and placed my district approval letter temporarily in Appendix A. This letter was replaced with my IRB approval letter in my final dissertation to preserve the confidentiality of the school district. I did not attempt to contact the schools' principals or recruit any teachers to participate in this study until I had acquired IRB approval. After acquiring IRB approval, I conducted a pilot study. According to Yin (2003), a pilot study allows the researcher to refine their data collection plans and procedures. This pilot study enlisted three individuals who were not selected as participants in the study but matched the criteria for the study participants. I completed all three aspects of the planned data collection with them for the purpose of practicing these methods and ensuring that the planned data collection would be able to answer the research questions. I did not formally analyze this data but completed adequate analysis to ensure that the data acquired would answer the research questions. I used three data collection methods (individual interviews, focus group interviews, and classroom observations). As I conducted minor analysis of the collected pilot study data, I determined that I needed to modify one of the interview questions, insert an additional focus group question, and prepare some follow-up questions. After I completed the pilot study, I initiated my recruitment plan.

Recruitment Plan

To begin the recruitment process, I identified a gatekeeper who forwarded a recruitment email (see Appendix B) to prospective participants. A screening survey (see Appendix C) was hyperlinked within the recruitment letter. I reviewed the screening surveys of those individuals who showed an interest in participating in this study. I then emailed those individuals who met the study's participant criteria to let them know that they were selected as participants (see Appendix D). In this email, I included a hyperlink for the Consent Form (see Appendix E). The email also entailed the directions regarding how to sign the consent form and a statement instructing them to return the signed document to me via email. An email was also sent to those individuals who completed the screening survey but were not selected (see Appendix D). The consent form gave a clear indication that participation in the study was voluntary and posed no risks (Creswell & Poth, 2018). Data were collected via individual interviews, focus groups, and observations. A parent permission form (see Appendix F) was also distributed to the parents of each student in each of the participants' classes as requested by my district.

Data Collection Plan

A critical aspect of qualitative inquiry is rigorous and varied data collection techniques. Leedy and Ormrod (2005) stated that qualitative data collection sources are restricted only by the researcher's open-mindedness and creativity. According to Vaughn and Bos (2019), data collection in a qualitative study is time consuming, and the researcher should record any potentially useful data thoroughly, accurately, and methodically, using any suitable means. Yin (2017) presented six types of data collection sources for conducting case studies: documentation, archival records, interviews, direct observations, participant-observation, and physical artifacts. I gathered data from individual interviews, focus groups, and classroom observations. According to Yin (2017), using three or more data collection sources promotes data triangulation, which enhances research trustworthiness. To attain optimum results, I chose to collect the data in the following order: individual interviews, focus groups, and observations. This sequence allowed me to conduct preliminary data analysis that would enhance subsequent data collection methods.

Individual Interviews

Fowler (2001) noted that interviews can potentially influence the quality of the data collected, and Yin (2014) insinuated that the interview is the most valuable instrument in the data collection process. Seidman (2006) iterated that a good interviewer must remain attentive to eradicate any personal biases that could threaten the accuracy of the data collected. As a researcher, I must be able to refrain from verbally imposing my own perspectives even though they may be constructing their own perceptions mentally. I must also possess superb listening skills to conduct interviews. Listening skills are crucial to successful interviewing, for both the interviewer and the interviewee. Seidman (2006) noted that researchers need to listen to what the interviewee is saying and to listen while remaining cognizant of process and nonverbal cues. Brinkmann and Kvale (2015) described the qualitative research interview as an effort to comprehend the world from the participants' point of view, "to unfold the meaning of their experience, [and] to uncover their lived world" (p. 163). As I conducted the interviews, I adopted some key strategies that were suggested by Ary et al. (2010): (a) refrain from enforcing personal agenda or bias; (b) refrain from using specific terms or over-cueing the participants; (c) identify inconsistencies between the participant's verbal and nonverbal behaviors; (d) direct the participants' answers to actual and in-depth accounts instead of accepting generalizations. Seidman (2006) also prescribed some other strategies that I adopted: (a) explore laughter; (b) refrain from interrupting the participants' responses, (c) avoid sharing personal experiences, and (d) permit silence. In addition, Yin (2018) noted that interviews allow the researcher to gather detailed information regarding the participants' personal perspectives through detailed questions that lead to the achievement of higher response rates. The semi-structured questions for the individual interviews are as follows:

***Individual Interview Questions* (see Appendix G)**

1. Please share your reasoning for becoming a teacher.
2. What math resources have been the most beneficial to your students' achievement throughout your teaching career?
3. Describe your experiences as a math teacher. CRQ

In the rest of this interview, I would like to know how you used both concrete and virtual manipulatives to answer the questions given. I will simply say manipulatives in each question, but please remember to broaden your answer to talk about your use of either or both concrete or virtual manipulatives.

4. Describe how you make your students feel confident about their abilities in math. CRQ
5. How do you think your students' self-efficacy impacts your delivery of math instruction? CRQ
6. Describe your experiences of using manipulatives to provide opportunities for your students to practice new skills. SQ1
7. Describe your use of manipulatives to aid students in overcoming math difficulties. SQ1
8. Describe your use of manipulatives to teach students that success requires perseverance/sustained effort? SQ1
9. Describe your experiences of having students observe social models to successfully complete a task. SQ2
10. Describe your use of manipulatives to increase students' beliefs that they possess the capabilities needed to master similar activities to succeed. SQ2
11. Describe the positive feedback that you have given to students with math difficulties when using concrete and/or virtual manipulatives to build their self-efficacy. SQ3
12. Describe the negative feedback or discouraging aspects experienced by students with math difficulties when using manipulatives. SQ3
13. Describe your experiences of accommodating students' physiological or affective states when using concrete and/or virtual manipulatives. SQ4

14. How are students’ positive moods reduced or increased when using manipulatives to build their self-efficacy. SQ4
15. What else would you like to add to our discussion regarding your experiences of using manipulatives to enhance the self-efficacy of students with math difficulties?

Shipherd (2019) stated that only a few studies have explored information regarding how the sources of self-efficacy has changed over time despite the research findings which proved that self-efficacy is a main predictor of performance in both physical and cognitive tasks. These interview questions were designed to elicit teachers’ experiences of using manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. The purpose of the first five interview questions was to establish rapport for a good interview so that the participants were comfortable with sharing their experiences and providing more comprehensive responses to the succeeding questions (Marshall & Rossman, 2015). According to Patton (2015), asking open-ended questions garners the participant’s own categorical worldview. These questions were designed to elicit information regarding the participants’ personal beliefs, experiences, motivation, and attitudes toward teaching math. According to Hammad et al. (2022), teachers’ attitudes toward providing math instruction and supporting their students’ learning is recognized as a factor that can either influence students’ academic development positively or negatively. Thus, teachers’ positive attitude towards math instruction can foster students’ self-efficacy (Fan & Wolters, 2014). In addition, Frenzel et al. (2018) found that teachers who enjoyed teaching math transferred this enjoyment to their students, and Villavicencio and Bernardo (2016) found that when students experience enjoyment during math instruction, they develop a positive sense of mathematics self-efficacy.

According to Bandura (1997), individuals develop their self-efficacy by selecting and interpreting information from four primary sources (mastery experience, vicarious experience, social persuasions, and physiological or affective states). Of these four sources, mastery experience was found to be the most powerful source (Bandura, 1997; Shipherd, 2019). Interview Questions 6, 7, and 8 sought to identify teachers' description of their experiences of using manipulatives to facilitate the mastery experiences of students with math difficulties. These questions were designed to reflect the main source of creating a strong sense of efficacy outlined in Bandura's theory. Interview Questions 9 and 10 sought to discover the participants' experiences of using manipulatives to enhance vicarious experiences for students with math difficulties. These questions were designed for teachers to discuss how they provide opportunities for students to observe the actions of others to build their self-efficacy. Bandura (1997) hypothesized that modeling plays a significant role in the development of students' self-efficacy. Interview Questions 11 and 12 focused on soliciting participants' experiences of providing social persuasions when using virtual manipulatives with students with math difficulties. These questions were designed to determine how teachers provide opportunities for students to form self-efficacy from the social persuasion that they receive from others.

Bandura (1997) implied that students may depend on teachers and peers to provide constructive feedback, appraisals, or coaching regarding their academic performances. Interview Questions 13 and 14 examined teachers' experiences of addressing the physiological or affective states of students with math difficulties when using manipulatives. According to Bandura (1997), physiological or affective states inform self-efficacy. Research has also shown that the way in which teachers provide math instruction determines the physiological or affective states of their students in math-related situations (Goldin, 2014; Jackson & Leffingwell, 1999). These questions were designed for teachers to express how states such as anxiety, mood, and fatigue affect students' self-efficacy. Interview Question 15 was designed to give participants an opportunity to have the final say (Patton, 2015). Experts in the field reviewed and approved my interview questions before they were administered to the participants. Individual interviews were conducted in a face-to-face setting.

Individual Interview Data Analysis Plan

After each semi-structured interview, I personally transcribed the audio recordings to facilitate the data analysis phase of the research. I transcribed the data verbatim to avoid potential bias or interpretation that may transpire with summarizing as suggested by Yin (2018). During transcription, I included notes that documented nonverbal information that enhanced meaning, and I refrained from making grammatical corrections to words and phrases to avoid changing the intended meaning of responses. This process protects researchers from missing pertinent information, and it enhances depth and comprehensiveness of the analysis. I completed member checking by sending a transcript of each participants' interview (without my notes) to them to check for accuracy. After receiving the member checked transcript from each participant, I embedded my observational comments in the transcribed text. I read the transcriptions multiple times to gain an in-depth understanding of the participants' experience. Maxwell (2005) affirmed that qualitative researchers begin data analysis immediately after finishing the first interview and continue to analyze the data as long as they are working on the research. I began to derive preliminary codes as I examined the interview responses.

Focus Groups

Prior to the focus group session, participants were informed of the agreed upon meeting time. The focus group interviews were conducted for approximately 40–50 minutes. According to Yin (2018), focus group interviews are used to discuss, validate, and clarify information obtained from the individual interviews. Focus groups allow the participants to assemble and discuss the topic freely as they respond to the researcher and other participants (Creswell, 2013). Focus groups assist the researcher by revealing patterns within the study as the participants provide insightful descriptions of their experiences of the phenomenon (Patton, 2002). Interactions in focus groups enables the researcher to identify how the participants incorporate the views of the other participants to structure their own understandings (Ary et al., 2010). I arranged two focus groups consisting of an approximately equal number of participants (eight in one group; seven in the other group). Focus groups provide an opportunity for me to interact with multiple participants at the same time while encouraging dialogue amongst participants about the area being researched. Focus groups are especially useful for exploring complex, multi-layered concepts from the perspectives of the participants. Focus groups are an excellent means to create triangulation using varied sources of evidence in a study. Focus group questions should avoid re-asking questions already asked during individual interviews. Additionally, when using a focus group as a source of triangulation for individual interviews, I modified the focus group protocol after the study was underway to follow up most effectively on initial data findings of individual interviews. I scheduled focus group sessions based on convenience and teacher availability. I conducted focus groups in a face-to-face setting.

The sessions were regulated to ensure that all participants were actively involved and that no one was dominating the sessions (Creswell, 2013). The semi-structured questions for the focus group interviews were as follows:

Focus Group Questions (see Appendix H)

1. Please introduce yourselves to the group by stating your name and school where you teach.
2. Describe challenges you faced as a math teacher.
3. Describe your experiences regarding the advancement of technology in math instruction. CRQ

Just like in your individual interview, all of these questions will simply say manipulatives but please remember to broaden your answers to talk about your use of either or both concrete or virtual manipulatives to answer the questions given.

4. How do you provide opportunities for your students to practice new skills when using manipulatives? SQ1
5. What experiences have you encountered that may have prohibited you from using manipulatives during instruction with students with math difficulties? SQ1
6. Describe how you have used manipulatives to aid students in overcoming math difficulties. SQ1
7. Describe your students' reactions to being assigned social models to successfully complete a task. SQ2
8. Describe how positive and negative feedback given to students with math difficulties when using concrete and/or virtual manipulatives impacts their self-efficacy. SQ3
9. Describe ways teachers can use to accommodate their students' physiological or affective states when using concrete and/or virtual manipulatives. SQ4

10. What else would you like to add to our discussion regarding your experiences of using concrete and/or virtual manipulatives to enhance the self-efficacy of students with math difficulties?

The purpose of the first three focus group questions was to establish rapport as in the individual interviews. These questions allowed teachers to share their triumphs and challenges experienced when using manipulatives during math instruction. According to Kitsantas et al. (2011), teachers' accounts of their experiences significantly affect their students' self-efficacy, thus conveying that fostering the self-efficacy of students with math difficulties improves their mathematical achievements. This coincided with claims which indicated that self-efficacy is a credible predictor of academic achievement (Ben-Naim et al., 2017; Vukman, et al., 2018). Questions 4, 5, and 6 sought to discover the participants' views regarding their experiences with mastery experiences. Question 7 sought to discover the participants' experiences regarding vicarious experiences. These questions were formulated because they support Bandura's (1997) notion that self-efficacy could be fostered through observing the success of others who are more knowledgeable than the observer. Question 8 sought to examine teachers' experiences regarding social persuasions. I asked Question 8 to focus on soliciting participants' experiences regarding teachers' experiences of addressing the physiological or affective states of students with math difficulties when using manipulatives. Research has shown that students with math difficulties often exhibit negative feelings such as helplessness, anxiety, or shame (Goldin, 2014; Park et al., 2021; Satsangi, Hammer, & Evmenova, 2018). Goldin (2014) found that when students completed difficult mathematical tasks, they exhibited positive feelings such as pride or enjoyment. Question 9 allowed focus groups to discuss if their use of manipulatives elicited positive feelings in students with math difficulties. Shipherd (2019) also noted that it is crucial to comprehend how beliefs of self-efficacy are formed since self-efficacy has such a great influence on students' behavior. Once again, Question 10 was designed to give participants an opportunity to have the final say (Patton, 2015).

Focus Group Data Analysis Plan

I followed the same procedure for transcriptions as noted under the interview data analysis section. I completed member checking by sending a transcript of each groups' interview to each participant of the group to check for accuracy. After receiving the member checked transcript for each participant, I continued to derive preliminary codes.

Classroom Observations

Leedy and Ormrod (2005) claimed that observations in qualitative research promote flexibility and are purposefully free flowing and unstructured. This approach is beneficial because researchers can acquire unplanned data sources if they arise (Leedy & Ormrod, 2005). During the classroom observations I (a) remained as quiet and inconspicuous as I could and (b) refrained from confusing my actual observations with my personal preliminary interpretations of them (Leedy & Ormrod, 2005). I assumed the participant-observer role where I observed and closely interacted with the participants without engaging in their instructional activities. This role allows researchers to maintain their researcher identities while actively participating in the setting but not in the core roles of the participants (Gall et al., 2007). In addition, I observed the behaviors and interactions of the participants with their students as they utilized manipulatives not only to extend my time in the field to familiarize myself with the participants but also to identify accidental data sources if they arose.

The schedule of the observations was prearranged based on the math teachers' times of convenience. Participants were observed for one teaching session where they used manipulatives. Each observation session was no shorter than 15 minutes and no longer than 45 minutes. Participants were observed in their classrooms as they provided math instruction to their students. Using the observation protocol (see Appendix I), I took detailed descriptive field notes on a notepad recording everything that I saw such as interactions and facial expressions. At the end of each observational session, I composed reflective notes on the observation protocol.

Observations Data Analysis Plan

The data analysis plan for the observations involved taking written field notes on an observation protocol document and then analyzing the notes regarding the study's setting and the behaviors and interactions of the participants (Ary et al., 2010). Once again, I looked for preliminary codes based on the observation data set.

Data Synthesis

This section explains how I synthesized all of the collected data into a coherent singular body of evidence that identified themes and offered answers to the research questions. The data analysis plan that I used to derive themes and subthemes followed the following sequence: (a) organize the transcribed details of the case in logical order, (b) identify additional categories to cluster the transcribed data into meaningful groups, (c) examine the transcribed data that may seem insignificant to determine if they are related to the case, and (d) scrutinize the transcribed data and relevant interpretations to discover additional emerging themes (Stake, 1995). Data from all data collection methods were collected and analyzed before the themes were derived. I also considered Braun and Clarke's (2022) suggestions that themes should be presented in an order that best tells a coherent and comprehensive story of the observed data. Braun and Clarke (2022) also recommended that researchers discuss two to six themes that are rich, complex, multifaceted, and not overlapping.

According to Yin (2018), qualitative data analysis entails "organizing the data, conducting a preliminary read-through of the database, coding and organizing themes, representing the data and forming an interpretation of them" (p. 165). When I triangulated the three sets of data (individual interviews, focus groups, and classroom observations), I organized all data sets into digital files and created a file naming system that ensured that the data were easily accessible (Creswell & Poth, 2018). I continued the synthesis process by reading and rereading the transcripts in their entirety to immerse myself in the detail as I worked towards identifying significant patterns and determining what was important. I read and reread notes and transcripts and listened repeatedly to the audiotapes. I then created a complete list of data sources, manually sorted the data by using color-coding, and then organized the files. To provide a thick description by conveying meanings, I made conceptualizations based on the connections established among the categories and patterns and then presented a detailed view of the case using a theme development table and in-depth descriptions of each theme in Chapter Four.

Trustworthiness

According to Lincoln and Guba (1985), the standard of rigor in qualitative research includes credibility, transferability, dependability, and confirmability. The authors stated that trustworthiness determines whether one's research findings are worthy of attention or accurately conveys the responses of the participants. Each standard of trustworthiness is discussed below.

Credibility

Credibility relates to the truthfulness of the study's findings (Lincoln & Guba, 1985). Credibility determines how confident the readers can be in the researcher's observations, interpretations, and conclusions (Ary et al., 2010). Techniques that can be used to establish credibility include the following: (a) prolonged engagement, (b) persistent observation,

(c) triangulation, (d) peer debriefing, (e) negative case analysis, (f) referential adequacy, and (g) member-checks. Three methods that I used to ensure the credibility of this study were prolonged engagement, triangulation, and member-checking. Firstly, I conducted extensive fieldwork by conducting investigations over a prolonged period to ensure that my findings were credible. This process increased the credibility of the study because it allowed me to gain the trust of the participants and, thus, obtain more detailed and honest responses (Ary et al., 2010). Regarding data triangulation, Lincoln and Guba (1985) stated that triangulation consists of using multiple data sources in research to yield understanding and develop a comprehensive finding. These authors insisted that using one single method is inadequate to clarify a phenomenon or facilitate in-depth understanding. I collected data using various procedures (individual interviews, focus group, and observations) to support observations and conclusions. According to Lincoln and Guba (1985), implementing various methods of data collection safeguards the credibility of the study. Triangulation increases the credibility of the study because the convergence of a major theme or pattern in the data from several sources gives credibility to the findings. According to Ary et al. (2010), triangulation not only involves confirming data by using multiple data collection procedures but by also conducting multiple observations. For instance, Gall et al. (2007) noted that observations promote the corroboration of evidence to enhance the validity of qualitative research findings and to eliminate inaccurate interpretations.

In reference to member checks, I asked participants to review and critique the collected data for accuracy. This process increases the credibility of the study because it allows the researcher to clarify miscommunication, detect inaccuracies, and gain further insight from the data (Ary et al., 2010).

Transferability

Transferability is the degree to which the findings of a qualitative study can be applicable or generalized to other contexts (Lincoln & Guba, 1985). I provided thick descriptions as an approach to enhance transferability in my study. I provided accurate, detailed, and complete descriptions of the context and participants of the study. This process is required to guarantee transferability of findings (Creswell & Poth, 2018).

In addition to this, another way to ensure transferability is to create an audit trail (see Appendix J) to record the major steps taken to complete this study so that another researcher can replicate this study. According to Lincoln and Guba (1985), an audit trail is a transparent account of the steps taken from the beginning of the research until the findings of the study are reported. Audit trails nurture transferability as they are accounts that are recorded concerning what was done throughout the investigation. Ary et al. (2010) noted that the audit trail is the most valuable strategy that enables other researchers to achieve or not achieve the same conclusions given identical data and context. According to Lincoln and Guba (1985) transferability proves that the research findings are consistent and duplicable via thorough descriptions of the procedures undertaken for the research. The audit trail is comprised of a dated list of all tasks that I completed to conduct the research study. The list of tasks started with acquiring the permission letter to conduct research and included other tasks such as attaining IRB approval and completing the pilot study.

Dependability

According to Lincoln and Guba (1985), dependability refers to the extent to which the finding can be tracked or explained. Dependability involves variability and consistency and can be accomplished through strategies such as peer debriefing and the intrarater method.

Dependability involves showing that the findings are consistent and could be repeated (Lincoln & Guba, 1985). One way to ensure dependability is peer debriefing. I selected two of my peers who were encouraged to engage in discussions to determine consensus or if my interpretation of the data were reasonable. Lincoln and Guba (1985) defined this method as a "process of exposing oneself to a disinterested peer in a manner paralleling an analytic session and to explore aspects of the inquiry that might otherwise remain only implicit within the inquirer's mind" (p. 308). The peer review method increases the dependability of the study because investigator triangulation makes it "less likely that outside reviewers of the research will question the data" (Ary et al., 2010, p. 499). I used knowledgeable persons to review my findings who have experience in qualitative research methods and were familiar with the problem addressed in this study.

To apply the intrarater method, I manually coded the data, left the analysis for a period of time, then recoded the data to compare both sets of coded materials (Ary et al., 2010). This process increases the dependability of the study because comparison of results will determine whether both coded results obtained similar labels.

Confirmability

According to Lincoln and Guba (1985), confirmability entails the degree of neutrality and the extent to which the results of the study are molded by the participants' bias, motivation, or interest and not those of the researcher (Lincoln & Guba, 1985). Ary et al. (2010) stated that confirmability ensures that the research is free of bias in the procedures and results interpretation. A technique that is used to establish confirmability is reflexivity. I used a researcher's reflexive journal (see Appendix K) to record my biases, both before starting this study and throughout the research process. This process increases the trustworthiness of the study because it identifies and controls the researcher's own biases (Ary et al., 2010).

Ethical Considerations

According to Braun and Clarke (2022), ethical considerations are crucial requirements of all research designs and practices and relate to the procedures researchers follow when dealing with participants. They also apply to the sociopolitical domains of research that determine the power relationship between the researcher and the participants and the researcher's values. In addition, Creswell and Poth (2018) noted that ethical issues should be expected and planned for when designing qualitative studies since they can appear in several stages of the research process. I ensured that I requested permission to access the site and participants (Creswell & Poth, 2018). To ensure that my research was guided by ethical standards, I sought Liberty University's IRB approval to conduct this research. I obtained informed consent from participants and communicated the voluntary nature of the study to the participants. I informed the participants of the purpose of the study, attained voluntary consent forms, and respected diversity (Creswell & Poth, 2018). I remained professional to avoid becoming ethically vulnerable by monitoring my relationship to the participants. To ensure reciprocation, I ensured that each of the participants was awarded a $25 grocery voucher as an incentive in return for their time and cooperation. Data collected as part of this study may be shared for use in future research studies or with other researchers. If data collected from the participants are shared, any information that could identify any of the participants will be removed before the data are shared. Additionally, pseudonyms were used for all participants, schools, and school districts to preserve the confidentiality of the participants and site. To avoid deception, I cross-referenced participants' individual interview responses with responses from the focus group session. To protect the data, I created back up versions or copies of the data on multiple devices that require passwords, and I informed the participants that the data would be erased after 3 years. When I reported the data, I reported honestly and used appropriate language (Creswell & Poth, 2018). I also informed the participants that the study posed no known risks, and that the completion of the study will be beneficial to the special education field, especially to the advancement of effective practices for math instruction.

Summary

This chapter details the qualitative research methods used in this instrumental case study that examined the experiences of middle school teachers who use manipulatives during math instruction. Four research questions were outlined, followed by information outlining the setting and participants of the study. The researcher's positionality section followed, discussing my motivation for conducting this study. My interpretive framework was then articulated, followed by a discussion of the three philosophical assumptions that guided the study. This section concluded with an explanation of the researcher's role and was followed by the procedures section which detailed my plans for permissions and recruitment. Next, the data collection plan section discussed the three approaches that I used which include individual interviews, a focus group, and classroom observations. This section culminated with the data synthesis and was trailed by the trustworthiness section. The trustworthiness section detailed how I established credibility, transferability, dependability, and confirmability in my study. Chapter Three concluded with a discussion of the ethical considerations that are inherent to this specific case study.

CHAPTER FOUR: FINDINGS

Overview

The purpose of this instrumental case study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Participants included 15 middle school teachers who deliver math instruction to students with math difficulties. Chapter Four begins with a description of each participant, followed by the results of the data analysis. Data were collected from the participants using individual interviews, focus groups, and classroom observations. The results section presents and provides explanations of the emerging major themes and subthemes that were identified from the participants' experiences. From the collected data, the following major themes were developed: (a) consequences, (b) deliberate practice, (c) modeling, (d) targeted feedback, and (e) instructional changes. The organization of the results is based on the research questions that guided the study. The chapter concludes with a summary.

Participants

This section provides a profile of the participants. The description of the teachers includes their years of teaching experience, highest level of degree obtained, age, content area(s) taught, setting, and type(s) of manipulative used during math instruction. All of the participants met the study's inclusion criteria and possessed a comprehensive understanding of the phenomenon being studied.

Upon completion of my pilot study, I contacted my identified gatekeeper who forwarded the recruitment email with a hyperlinked screening survey to prospective participants employed at the sites. Based on the screening survey responses, 15 individuals who met the study's participant criteria were selected as participants. The inclusion criteria for the study required that each participant: (a) was a trained teacher possessing a bachelor’s degree or higher in education, (b) had administered math instruction using manipulatives to middle school students within the past 3 years, (c) had 3 or more years of teaching experience, and (d) ranged between the ages of 25–55 years. My rationale for this criterion was to accumulate a rich and authentic collection of varied but practical experiences. All of the participants were passionate about education and were eager to candidly share their experiences. Demographics for the teacher participants are presented in Table 1. Following the table, I included participant profiles reflecting their status and/or personalities at the time of this study. This information was gleaned from my observations and interactions with them during this study and not necessarily from what they told me.

Table 1

Teacher Participants

Teacher Participant	Years Taught	Highest Degree	Content Area	Setting	Concrete/Virtual Manipulative(s)
Avah	36	PhD	All Content Areas	Special Ed.	Both
Abigail	14	Master’s	All Content Areas	Special Ed.	Both
Aubree	6	Bachelor’s	All Content Areas	Special Ed.	Both
Tanya	7	Bachelor’s	All Content Areas	Special Ed.	Both
Emily	14	Bachelor’s	All Content Areas	Special Ed.	Both
Faith	7	Bachelor’s	All Content Areas	Special Ed.	Both
Rachel	4	Bachelor’s	All Content Areas	Special Ed.	Both
Patrice	20	Master’s	Math	Inclusive	Both
Nicole	5	Bachelor’s	Math	Inclusive	Both
Esther	9	Bachelor’s	Math	Inclusive	Both
Latesha	8	Bachelor’s	All Content Areas	Special Ed.	Both
Lynette	5	Bachelor’s	All Content Areas	Special Ed.	Both
Bianca	20	Master’s	Math	Inclusive	Both
Alli	4	Bachelor’s	All Content Areas	Special Ed.	Both
Gia	4	Bachelor’s	All Content Areas	Special Ed.	Both

Avah

Avah is an adventurous, vibrant, and loving educator who has been teaching for 36 years. She currently teaches in a special education classroom setting and has a PhD in educational leadership. Avah boasts that she had some amazing teachers throughout her primary and high school years who always found "innovative ways to make lessons fun and engaging." In retrospect, they went above and beyond to bring their classes to life and made her learning experiences as real as they possibly could. Avah strives to bring this same energy and drive to her classroom so that her students can have that same meaningful learning experience. She looks forward to those "ah-ha!" moments when her students grasp a new math concept. Avah believes that math instruction can only be successful when teachers use resources that are appropriate and effective. In her spare time, Avah enjoys reading and traveling.

Abigail

Abigail is currently employed as a special education teacher. She has taught in special education settings for 11 years but previously worked in a general education setting for 3 years. Abigail is currently pursuing her master's degree in curriculum and instruction. She expressed that "teaching is in her blood" as many of her relatives are also educators. She followed her passion for education when her mentor introduced her to a teacher cadet program when she was only in the 10th grade. Abigail said that she was "drawn to the special education field" because she lives with a relative who is diagnosed with a disability. Abigail is creative, inquisitive, and resourceful. She claims that she is committed to finding ways to meet the needs of students with academic challenges. She excitedly proclaimed, "Their challenges become my challenges… and oh I love a challenge!" Abigail claimed that she has a love-hate relationship with math. She loves the subject because it is fun to teach, but she hates it because her childhood traumas of struggling in math are sometimes activated when she witnesses her students also struggle with grasping new concepts. Abigail enjoys dancing and baking in her spare time.

Aubree

Aubree is an energetic, intelligent, and ambitious teacher who enjoys incorporating music and humor into her lessons. She has been teaching students with academic difficulties for 6 years in the public school system and has a bachelor's degree in education. Aubree became a teacher because she wanted to make a difference in the lives of children, and she wanted to make a positive impact within the education system. She believes that math is a very difficult subject for students to master, and claims that her ability to break this barrier has been "one of her greatest experiences in teaching math." She enjoys embedding videos, pictures, and manipulatives such as clocks in her lessons. She also loves trying new things to help her students overcome difficulties. Aubree has a bubbly personality and high confidence and she constantly challenges her students to match her energy. Her hobbies include making crafts and gardening.

Tanya

Tanya is a special education teacher who has been teaching for 7 years. She has a bachelor's degree in education and is the coordinator for the entire primary department of a special education, all-aged school. Tanya is kind, fun, and firm and has a passion for gaining new knowledge. She loves to read and share what she has learned. Tanya is also a hands-on teacher who is very practical. She claimed that she loves English and only learned to love math in college. She realized that math, like English, also develops articulation skills and helps students present logical explanations for their ideas. In her spare time, Tanya enjoys sewing and singing.

Emily

Emily is a caring, reserved, and funny educator who has been teaching for 14 years. She is the coordinator for the entire secondary department of a special education, all-aged school. Emily has a bachelor's degree in education and has prior experience teaching in a general education setting as a technical teacher in a secondary school. She says that she became a teacher "just by chance" and now has developed a passion for the field. Emily stated that math is a challenging subject to learn but is fun and interesting to teach as long as you are equipped with the right resource: "Whether they are physical or virtual, they have to be efficient and readily available." Emily enjoys traveling and watching a good movie in her spare time.

Faith

Faith is a fun, spontaneous, and passionate teacher who has been teaching for 7 years. She was a Sunday school teacher from a very young age, and she pursued a teaching career because she believed that it was a noble profession and a form of ministry. She claims that teaching "gave value to her life." Faith currently provides instruction to a class of all boys who are challenged with academic difficulties, and she enjoys singing, dancing, and sleeping. Faith shared that raps, virtual manipulatives, chants, and poetry are her favorite math aides, but she admitted that math is a subject area that many students struggle with. She claimed that she always has to "put in extra work" to increase her students' will to do math. She lets her students know that she also has strengths and weaknesses when it comes to her ability to effectively teach math.

Rachel

Rachel is a modest, compassionate, kind, and reliable educator who has been teaching for 4 years. Rachel became a teacher because she wanted to have a great influence on the future generation. She believes that she can make her students believe in themselves no matter what challenges they may face in life. She recalled that when she was in school, “teachers always taught one way as though everyone learnt the same way,” but she believes that every child has different learning styles. Rachel believes that she can be the change by being the best teacher she can possibly be and by giving every child a fighting chance. Rachel also believes that math is very rewarding. She claimed that teachers must have an abundance of patience when teaching math as the “mastery of objectives may not occur how and when you want them to.” In her spare time, Rachel enjoys working out and traveling to exotic destinations.

Patrice

Patrice is an optimistic, dedicated, and helpful educator who has a passion for the profession. She has been teaching for 20 years and has a bachelor’s degree in education. Patrice became a teacher to assist and support students in their development holistically. Patrice claims that teaching math is fun and exciting. She brags that her math lessons are very creative as she utilizes “numerous hands-on activities to introduce and teach new math concepts.” Patrice is also certified in speech therapy and enjoys traveling and dancing. She claimed that math is a subject that allows flexibility. She enjoys finding different ways to teach math skills. In her view, “There are visual ways, written ways, and ways to verbally communicate how to solve math problems.”

Nicole

Nicole is a quiet, friendly, and patient math teacher who teaches in the junior department of a secondary school. She has been teaching for 5 years and has a bachelor’s degree in education. Nicole loves children because they are “innocent, inquisitive and special in their own way.” She became a teacher because in her view, teaching is one of the noblest professions. She believes that teaching is the reason why a functional society and other honorable jobs exist. As a teacher, Nicole sows the seeds of knowledge and power that are needed to be successful in her students. She stated that teachers “give students support by helping them with materials or personal issues.” In her view, teachers help the world move on and forward. She is certified in teaching primary school math but found herself teaching middle school grades. She is eager to transfer into a primary school, but in the meantime, she is using her expertise to teach seventh- and eighth-grade leaners who are struggling in math. Nicole’s hobbies include camping and hiking.

Esther

Esther is a creative, empathetic, and wise teacher who has been teaching for 9 years. She holds a bachelor’s degree in education. She became a teacher because she wanted a career that focused on helping children. She takes utmost pride in making her lessons come to life as she makes learning fun and has experienced teaching in both special education and general education settings. Esther shared that teaching math requires a lot of planning, resources, and strategies. She iterated, “You have to have them at your fingertips and be ready to use them interchangeably at any given moment as the success of your lesson depends on it.” Esther enjoys traveling, cooking, and, of course, eating!

Latesha

Latesha is an easy-going, personable, and polite teacher. She wanted to become a teacher to assist struggling students who are just “pushed through the system,” who are deprived of teachers like her who are driven to take the time out to help struggling learners. She is committed to helping these students by pushing them to their full potential instead of pushing them through the system. As an individual who struggled with math herself, Latesha believes that students can benefit from the use of manipulatives because they provide hands-on experiences and without them, students “will struggle more because they don’t have any materials to guide or engage them.” Latesha enjoys watching movies and spending quality time with her family.

Lynette

Lynette is a special education teacher who is fun-loving, empathetic, and kind. She holds a bachelor’s degree in special education and is always seeking avenues for professional development. Lynette became a teacher because she always wanted to join a profession where she “directly had a positive impact on the lives of others.” She also has a few years of experience working as a teacher’s aide in a special education setting. Lynette described math as a very tough subject, especially for students in the special education sector. She shared, “Teaching math itself isn’t the difficult part, but getting the children to retain what they learn is.” Nonetheless, Lynette has found resources such as YouTube videos, ABC Mouse, manipulatives, and PBS kids to be the most beneficial to her students. In her spare time, Lynette enjoys traveling and dancing.

Bianca

Bianca is a compassionate, courageous, and pleasant teacher who holds a master’s degree in education. She has been teaching for 21 years. She became a teacher because she loved working with children, and she was sure that she would be able to “help them in becoming educated and positive adults.” Bianca has mixed feelings towards math as she has lived both positive and negative experiences. She has experienced successes with using resources such as virtual manipulatives, counters, and instructional videos; however, some factors such as lack of resources and lack of training in the use of some virtual manipulatives have hindered her students’ progress. Bianca lectures a local university and also uses her bachelor’s degree in Christian counseling to assist those in need. Her hobbies include writing and surfing the internet.

Alli

Alli is a special education teacher who has been teaching for 4 years. She is quiet, creative, and funny. Alli shared that she knew from a young age that she wanted to help students in any way that she possibly could have. She knew that she had to “be in a career that involved helping children.” When she began her pursuit of her studies in the special education field, she admitted that she was not familiar with the field; however, as soon as she gained knowledge about what special education was about, she knew that this was the field that she wanted to pursue. Alli describes the math field as one that is challenging but rewarding. She shared that some math manipulatives that have been most beneficial to her students’ success are play money and clocks. She always introduces a new concept by using manipulatives even if certain students do not need them to show them “how much easier their work can be.” She tries to portray manipulatives as tools people would use not just because they are struggling in math, but she would “also show them in a positive light as tools that can assist even the brightest of the brightest.” Alli enjoys performing spoken words and taking walks on the beach.

Gia

Gia is a kind, blunt, and loving special education teacher. She holds a bachelor’s degree in education and has been teaching for 4 years. She became a teacher because she realized from a young age that she loved helping her classmates when they were not able to understand the given assignments. She has a passion for working with children. Due to a very horrible encounter with a high school teacher, Gia vowed that she had to become a teacher so that her students would have the opportunity to “at least have one teacher who truly cared about them becoming successful… not just academically, but successfully as an all-around student.” Gia shared that she uses a variety of things to teach math including white boards, file folders, and online games.

She incorporates things that her students like in her math lessons to ensure that she keeps her students' attention and increases their engagement levels. Gia claimed that she makes her students feel as if they will make meaningful contributions to their communities despite whatever challenges they may have faced or will face. Gia enjoys taking long walks on the beach and traveling.

Results

The results of the data analysis process are presented in this section. The data were collected using teachers' face-to-face individual interviews followed by face-to-face focus groups and classroom observations. The findings are presented and supported with in vivo quotes derived from the participants' responses. All of the participants' quotes given in this manuscript, including dialect and grammatical errors in speech and/or writing, are presented verbatim to accurately depict their voices. The central research question explored asked, How do middle school teachers describe the factors that influence the development of their students' self-efficacy while using manipulatives during mathematics instruction? The four sub-questions sought to examine how middle school teachers describe their experiences of using manipulatives to facilitate the mastery experiences, vicarious experiences, social persuasions, and physiological/affective states of students with math difficulties. I analyzed the collected data using codes that developed into themes.

Theme Development

The following section presents the theme development and research questions responses. The theme development is organized in Table 2. An additional table was generated to provide a visual description of the number of participants who identified with each theme and subtheme that emerged from the collected data (see Appendix L). Following the theme development table, a narrative discussion of each theme and the respective subthemes are discussed using in vivo quotes from the participants.

Table 2

Theme Development

Key Words/Phrases	Subthemes
Major Theme 1: Consequences	
progress, better grades, higher scores, student accomplishments, better performance, mastered skills, meet IEP goals, reach milestones, I can do it by myself	Student Achievement
difficult, complicated, struggling, fear, complex, discouraging, hard, demanding, confusing, get tense	Intimidation
increased motivation, excitement, participation, involvement	Student Engagement
indirectly, unintentional, accidental, motor skills development, sensory learning	Incidental Learning
lack of resources, not enough, accessibility, insufficient devices, unreliable internet, digital illiteracy, no training, affordability, costly, expensive, make your own, expensive, end up using ineffective resources, not readily available, sharing resources	Digital Inequality
overreliance, lazy, dependent, hinder independent success	Dependence
Major Theme 2: Deliberate Practice	
do it over, do it again, practice makes perfect, repeat the steps, long-term memory	Repetition
build on prior knowledge, CRA model, discovery learning, continual learning, real world application, lifelong use, start from the ground up, each child is a blank slate, step by step	Constructivism
Major Theme 3: Modeling	
cooperative, stronger learner, set example, student-led lessons, group work, peer tutor, mimicry, demonstrating, gradual release, provide examples, guided practice, responsible for own learning, guide students until independent, “I do, We do, You do” approach, role play	Peer Teaching
play games, competitions, learn trough play, compete, challenges, compete, races	Gamification

Major Theme 4: Targeted Feedback	
Pep talks, affirmations, validations, you can do it, high five, approval, reassure, positive feedback, praises, chants, motivate, boost confidence, cheer on, congratulations, reassure, commend, good job, way to go, recognition, lots of celebrating	Verbal Encouragement
Prizes, pizza, stickers, rewards, incentives, treats, privileges	Tangible Rewards
Proximity, sit by my desk, pat on the shoulder, spend individual time with	One-on-One Instruction
Major Theme 5: Instructional Changes	
diverse needs, student choice, different learning styles, flexible groupings, student interests, safe learning environments, accommodating, facilitating, student-centered learning, IEP goals, change teaching approaches, kinesthetic learners	Differentiated Instruction
self-checks, self-evaluations, self-questioning, soul searching, professional development, better myself, upgrade, polish my skills, better equipped, professional growth, professional advancement	Teacher Self-Reflection

Note. CRA = Concrete Representational Abstract

Consequences

The most prevalent theme that emerged from the participants' responses regarding their experiences of using manipulatives to foster the self-efficacy of students with math difficulties was consequences. This theme was derived from the responses of all 15 of the participants. During her interview, Bianca stated that "there are always gains and losses when it comes to using any educational resource used to meet the needs of children with math difficulties." In a focus group session, Gia's comment also coincided with this statement when she said, "When you deal with students with math difficulties, you have to be able to stay focused on even the small victories because there will always be deficits … but we must learn how to aggressively tackle these barriers." It was clear from the teachers' comments that they were fully aware of the benefits and challenges related to using manipulatives to foster the self-efficacy of students with math difficulties.

It was also clear that teachers were passionately working towards reaping the benefits of using manipulatives to foster their students' self-efficacy; however, it was also evident that in their pursuit of success, they encountered certain undesirable facets that they needed to navigate. When speaking about her experiences with consequences, Emily shared in her interview that "pros and cons will always exist … it's up to the teacher to make sure that the pros always outweigh the cons, especially when dealing with students with math difficulties." Alli stated, "Manipulatives can be harmful or positive, it all depends on how it's being used in the classroom." Some teachers also voiced their concerns regarding the importance of being proactive when it comes to circumnavigating challenges. Avah reported, "Challenges related to the use of manipulatives happen if planning was not effective … and that's possible. The resources chosen may not be appropriate, because of its size or difficulty level." This major theme can be explained further through six subthemes: (a) student achievement, (b) intimidation, (c) student engagement, (d) incidental learning, (e) digital equity, and (f) dependence.

Student Achievement. Student achievement was the first subtheme to emerge and was reported by all 15 of the participants. When teachers use manipulatives to foster the self-efficacy of students with math difficulties, students attain better performance scores. This achievement is demonstrated through assessments, analyses of work samples, posttests, and progress charts. In her interview, Patrice said, "I see progress as moving from the basic level and continuing to challenge yourself to reach a higher skill level." Also sharing her experiences of student achievement, in her interview, Latesha said, "Using manipulatives surely boosted my students' confidence and this pushed them to want to learn more and surpass the academic expectations outlined in their Individualized Education Plans." Avah also supported the increase in student achievement by sharing how her students "monitor their own academic growth on a progress chart and are more inclined to use manipulatives to ensure that there are less errors on their math assignments." During a classroom observation, Rachel said, "Now try do these problems again, but use your counter this time. You will do a better job." After the students got a higher score, she said, "I told you so: counters get the job done."

Intimidation. The second subtheme to emerge was intimidation. This subtheme was also reported by all 15 of the participants. All of the teachers reported that many of their students find mathematics to be intimidating, and they expressed how the learning and teaching process in a math classroom can be negatively affected student when students are intimidated by math. There was a consensus amongst the teacher participants that math is a difficult subject for all students to master, as they themselves struggled with math as children. In her interview, Emily shared, "In school I dreaded math … in class I would become so tense because I knew that I really wasn't good in math." In her interview, Latesha also claimed, "I struggled through primary school in math but no one took the time out to help me." Teachers also expressed their past disdain for math. In a focus group session, Tanya shared, "I didn't originally love math until college." Abigail also shared her sentiments and stated, "I also hated math. If you take a look at my college transcript, you would see that my lowest grade was in math." Teachers agreed that their former negative feelings towards math are comparable to the feelings of their current students.

The intimidation that some of the teachers experienced in their childhood years also affected their ability to deliver math instruction. In her interview, Nicole shared her experience:

> As a trained teacher in lower middle school math, teaching math in upper middle school is very challenging. In order to teach my students, I would have to go and study the math concept myself, then select the easiest method of grasping the skill. Thankfully, I can rely on the upper middle school teachers in my department to assist me with strategies when necessary.

Bianca corroborated the notion that teaching math can be difficult for teachers: “Math was never my strong skill when I was in school, which is one reason what makes teaching math difficult.” In an interview, Esther also stated that “teaching math was challenging at first, but now I have the hang of it.”

Student Engagement. Regarding student engagement, 11 teachers agreed that when they use manipulatives to increase their students’ self-efficacy, their students’ engagement levels increase. During her interview, Aubree emphasized the need to ensure that students are engaged in math lessons: “Any lesson becomes unproductive if your students are not participating fully, and by fully, I mean giving their undivided attention, using the manipulatives appropriately, and being accountable for their own learning.” Faith explained, “When I use manipulatives, I notice that my students become more invested in their learning, their excitement levels increase, and I don’t have to beg for their attention.” To ensure that her students remain engaged, Rachel detailed how she uses “colorful, fun, and well-designed manipulatives to make sure that my students don’t call my lesson boring or uninteresting.”

Occasionally, it may also be possible that students’ engagement levels affect the teacher’s delivery of instruction. As Alli explained, “If I notice that my students shut down or have no interest in completing their work, this affects me and can hinder me from wanting to teach because I feel as if they aren’t getting it.” Keeping students engaged in a lesson requires teachers to establish a relationship between mathematical concepts and their students' life experiences and interests. As Bianca stated, "I instill in my students that math and manipulatives are needed in our everyday life. I teach them the connection of math to the real world." Abigail also iterated,

> My students' self-efficacy level determines their level of engagement in the lesson. If their self-efficacy seems low, my motivation and excitement is decreased, causing my delivery to be ineffective so therefore my math instruction is interrupted. If their efficacy is high, my delivery is engaging, impactful, and successful.

During a classroom observation, Latesha said, "Now to make sure you guys are paying attention, I brought some of your favorite counters to use."

Incidental Learning. According to the teachers' responses, nine participants agreed that when they use manipulatives to foster their students' self-efficacy in math, they have observed that the students also garnered heightened self-efficacy in other areas such as motor skills development and sensory learning. Tanya shared, "Using rocks and bottle tops have not gone out of style, you can even paint the rocks or other objects to change their texture to incorporate a sensory approach to learning." Lynette and Aubree discussed how the use of manipulatives improved the fine motor skills of their students who are diagnosed with physical disabilities. Lynette explained, "They have to grasp the manipulatives or devices with their hands; it's similar to hand exercises. Doing this on a constant basis forces them to use and strengthen their weaker hands."

Digital Inequality. For this subtheme, many of the participants highlighted the importance of digital inequality in their plight to foster their students' self-efficacy through the use of manipulatives. The digital divide may hinder students from being able to fully participate in rich and meaningful math lessons. As Esther explained, "Notwithstanding all of the challenges that we face as teachers, a great barrier to overcome regarding manipulatives is the lack of funding, which magnifies digital inequities." Patrice concurred with this statement by saying, "Some manipulatives are so expensive, and even if we purchase or subscribe to virtual platforms, the internet is so unreliable." Faith replied to this comment in a focus group session; she advised, "Create your own manipulatives to enhance your lesson, get things that are inexpensive but yet effective." Tanya then supported her statement and said, "Make your manipulatives from scratch, bring your lessons to life!" During a classroom observation in Aubree's class, Aubree said, "I couldn't afford enough fancy manipulatives, so I just made some cheap ones from scratch." Instead of having counters that were manufactured in stores, the students were using water bottle covers as counters.

As educators seek to enhance math education, digital equity must be considered and addressed to provide equitable access to the most current and effective manipulatives. As Abigail stated, "Manipulates have undeniable potential; however, some of us or our students may never experience this greatness in its entirety due to the equity gap that seems to widen day by day." During a classroom observation in Abigail's class, I noticed that some of the students were sharing devices. Some of the students had to wait until the others completed a task because of the lack of devices that were compatible with the virtual manipulatives software.

Dependence. A negative consequence that several of the participants noted was dependence. Sometimes students become over-reliant on assistive technology or resources and are left feeling helpless when they cannot use manipulatives. Faith explained, "We have to know when to take the manipulatives away from them and put them back on the shelf. We cannot allow students to feel as if it's the manipulative that is in control, we have to teach them that they are the ones who control the manipulative." Alli also shared her view regarding this subtheme during her interview: “Sometimes I notice that my students would tend to rely on the manipulatives to get the answers, that means that they are not even trying to independently solve any problems or try different methods.”

Deliberate Practice

Practice is key in learning mathematical concepts. Based on data collected from the three data sources, the second most effective approach that teachers used to influence the development of their students’ self-efficacy while using manipulatives during mathematics instruction was deliberate practice. Students need to engage in ample opportunities to practice new skills in order to overcome math difficulties. This repetitive practice teaches students that success demands persistence and sustained effort. As Nicole explained during her interview, “Math is a skill that can’t only be studied into remembrance, but only through studying and practicing will a skill be mastered.” Abigail also shared this same sentiment by saying, “Teaching and learning how to use manipulates requires time, patience, and continuity.” When speaking about her experiences of having her students engage in deliberate practice, Patrice stated:

> Using manipulatives provide students with the visual foundation for understanding and practicing a new skill. The concept can be broken down into small achievable steps using manipulatives every day. This cultivates the belief that success requires practice, consistency, and diligence. It also teaches how to persevere, and it builds students’ confidence.

Deliberate practice enables students to direct sustained effort into enhancing their math performance. As Lynette shared, “If students begin the lesson doubting that they can actually do the task, they are less likely to perform well and tend to give up quickly. Practice teaches them perseverance.” During a classroom observation, Nicole referred to a poster in her classroom that featured a quote by Aristotle which read: “If we are what we repeatedly do, excellence then is not an act but a habit.”

Repetition. Repetition was the first subtheme to emerge from deliberate practices and was reported by all 15 of the participants. Repetition is a key strategy in building student memory. During her interview, Faith revealed, “My students and I make chants and adapt poems to recite when we are using manipulatives so that we can remember the next step that is needed. When the students repeat the lines, you can hear the confidence in their voices.” In her interview, Tanya shared, “After we have done something a few times, and completed numerous examples, the students can now work independently. With virtual manipulatives, they can even go back and check their answers and see where to make corrections.” Lynette also shared that reviewing previous concepts before introducing new concepts and utilizing other repetitive methods have proven to be beneficial for her students. In her view, “Any form of repetition will accustom the students to practice using the manipulatives in an appropriate fashion.”

Constructivism. Constructivism approaches were present in most of the participants’ responses. The teachers agreed that components of constructivism such as building upon prior knowledge, the Concrete Representational Abstract (CRA) model, discovery learning, scaffolding, and continual learning refines deliberate practice. Regarding building upon prior knowledge, Emily claimed, “I always tell my students, math is simply building on one concept to another concept, to another concept … all of them are connected to each other in some form or fashion.” Patrice also reported in a focus group that she reminds her students to “self-check the steps on an anchor chart” that displays prior learning and instructs them to use the manipulative again when they encounter math difficulties.

Esther and Faith referred to their students as “blank slates.” According to Abigail, “New skills should be introduced with an evaluation of prior knowledge.” Avah explained, “A new skill can sometimes be an abstract thought, so we must build upon previous knowledge because math concepts such as area and volume usually follow a certain hierarchy.” The CRA approach involves the use of concrete and visual aids to expand students’ understanding of mathematical concepts, and some of the responses denoted the importance of this strategy. Gia explained, “Manipulatives make the new skill seem less challenging and gives the illusion that students are eased into the new skill through play such as the CRA model.”

Teachers should scaffold instruction to provide support for students while they use manipulatives to enhance learning and enable the acquirement of skills. Avah reported, “The manipulatives help to reduce the cognitive pressures and provide an excellent scaffold to aid understanding.” Aubree also agreed during a focus group discussion that the use of manipulatives “is a good way to scaffold instruction until mastery.” Patrice also supported this view: “I began teaching a skill at the basic level and continued to challenge students to a higher skill level.”

Teachers also shared their beliefs regarding how they promote discovery learning. Avah explained, “Manipulatives is student-centered and the child is discovering on their own.” Aubree insisted, “Teachers must find out or understand what it is that their students need to discover by themselves without telling them what to do all the time.”

In reference to continual learning, data analysis revealed that many teachers attributed a degree of math difficulties in older students to teachers’ unwillingness to use manipulatives with students in the upper middle school grades. There was a consensus among the participants that lifelong learning is key. In her interview, Avah said, “We tend to think that students at a particular level or age do not need physical or virtual manipulatives. I think that's a myth." Abigail's comment solidified this view when she said, "Manipulatives are oftentimes viewed as toys or tools only for preschoolers and lower elementary kids. They disappear in higher grades. If math difficulties follow students into adulthood, so should manipulatives that are proven to help them overcome said difficulties." Lynette said, "Continual learning is extremely important, especially with the special needs kids that I have. It takes them a really long time and continual effort to master concepts." During a focus group discussion, Alli stated, "There's so much to be discussed and considered when talking about or teaching manipulatives to older students. They are more than just a tool being used for play but for surviving in the real world."

Modeling

Modeling emerged as another major theme from the teachers' responses that aids in the development of students' self-efficacy. Teachers agreed that modeling through narrated demonstrations while using manipulatives enhances learning and fosters self-efficacy. Faith claimed that "modeling equips students with strategies, which is power," and Tanya reported, "I allow the students to role-play teaching with the manipulatives like they are me using the mimicry method. This surely builds their confidence." When speaking about her experiences of modeling, Alli stated, "I would show my students step by step how to use a manipulative to solve a problem." Gia shared, "To make my students feel confident about their math abilities, I always try to incorporate things I know they like in the math problem. I always model what I expect them to do first." Abigail also shared an analogy during a focus group discussion:

> Manipulatives give my students confidence and courage, like a weapon in battle. If you are not armed, you are vulnerable and susceptible to defeat. Manipulatives seem to empower the students … give them a new sense of hope. It arms them with confidence

and they believe that they are capable of achieving what their social models have achieved. Goliath had the best resources but David only needed the right resources.

Peer Teaching. Teachers agreed that peer teaching fosters students' self-efficacy. Patrice disclosed that peer teaching is a successful strategy that she has employed with her students. She reported, "In the lesson's introduction, students can model the steps of the concept with their peers. Sometimes students can teach the lesson to each other or the whole class." Avah boasted, "I allow students to model the concepts to each other and I see where the students are able to learn from their more abled peers." Aubree also shared, "Peer teaching makes students excited to teach what they have learned. I have created an environment where the students feel more comfortable with their peers and are less intimidated when they make mistakes." During Faith's class observation, she had a few students go by the board to complete a problem using manipulatives as they taught the concepts and steps related to solving the problem.

During a focus group discussion, Tanya shared, "I allow the students to teach the group or class. I alternate the strong and weak student." Faith claimed that peer teaching can "help students trust the sound of their own voice," and Bianca shared that she uses role-playing when using manipulatives "to help the students teach like me or like their more abled peers." Esther chimed in and said that "students are mini facilitators and can mirror desired behaviors when showcasing how to use manipulatives."

Faith and Esther also agreed with each other by expressing that they both love using the "I do, we do, you do" strategy as well as the think-pair-share approach. The "I do, we do, you do" instructional model is based on the notion that explicit instruction is systematic and that it is crucial that students be supported before they are expected to independently complete tasks. According to Deagon (2021), teachers may use this strategy to scaffold a learning activity. In the

"I do" phase, the teacher models the activity. In the "we do" phase, a structured learning task is completed as a group. In the "you do" phase, students complete the activity without receiving initial support but receive subsequent peer and teacher feedback. The think-pair-share approach is a collaborative discussion strategy. This strategy allows students to work together to solve problems while feeling comfortable in sharing their ideas. This model has been proven to provide better mathematics learning outcomes for students (Kusuma et al., 2020).

Gamification. Gamification is the art of turning difficult tasks into fun experiences. When using manipulatives, teachers can motivate students to complete tasks by implementing games as they also foster their students' self-efficacy. In her interview, Patrice shared, "Using manipulatives has taught me that learning through play kept students engaged with a variety of interactive experiences they can share with their peers." Rachel stated, "I play games instead of doing book work all the time … we switch it up and make learning a fun experience by playing games. During the game they are learning and having fun at the same time." During her interview, Abigail said, "I turn everything into a game; they think they are playing but it's actually me, the teacher, who is winning in the end." Faith also expressed her view, "Concrete and virtual manipulatives allow you to turn math tasks into competitions. My students love competing! It makes them push each other to do better." This claim was substantiated during her classroom observation session. In support of the importance of gamification to build students' self-efficacy, Gia stated, "When students become overwhelmed with the manipulatives, I let them take a break and play a game where they can win a prize if they solve all the problems. This usually motivates them to complete tasks quickly." Bianca shared her experiences by saying, "When it's time for games, the once quiet students would come alive. Their involvement and interest in the lesson is increased, even if they did not solve the problem correctly … I guess confidence is what matters the most." During Bianca's classroom observation, she had the students compete during a cooperative learning activity. The students had to model how to solve a problem with base 10 blocks and pass it on to the next team member who had to do the same. The team to have each member solve the problem correctly was the winner.

Targeted Feedback

Teachers revealed that providing targeted feedback can also be used to influence the development of students' self-efficacy while using manipulatives during mathematics instruction. Both teachers and students benefit from targeted feedback that specifically addresses areas that need to be addressed or corrected immediately. Feedback births confidence and motivation. When speaking about her experiences of providing immediate feedback, Avah noted, "Rather than just marking problems wrong, my students and I go through the process of using the manipulatives to see what aspect is challenging, then they complete activities to strengthen that specific area."

Interestingly, a candid reply regarding targeted feedback was brought up by Nicole, who said:

> A negative encounter I had from a student experiencing math difficulties was when he was using the manipulatives, he would try to be a class clown. The reason he would act like a clown is because he didn't understand or wasn't paying attention as to how to use the manipulative to complete a task. Instead of asking for assistance, he preferred to act out instead. It's difficult to give positive feedback in these cases.

Verbal Encouragements. Students need constant verbal encouragement, especially those experiencing learning difficulties. Verbal encouragement builds their confidence levels and motivates them to engage in lessons. As Rachel said in her interview, "When your students hear 'good job,' or 'I know you could do it,' they are proud of themselves and get excited about what they have accomplished. The students start smiling or jumping with excitement." In a classroom observation, Latesha had charts displayed in her classroom that promote confidence building skills. One of the posters stated, "Shoot for the moon. Even if you miss, you'll land amongst the stars." Emily and Faith both claimed that they always use pep talks to encourage their students, and Faith said: "I tell them to not doubt yourself, be confident." In a focus group session, Nicole reported, "The majority of the classes I teach are struggling students. To make them feel confident, I would let them know that it's ok to make mistakes and that no one is perfect. This eliminate feelings of being minimalized."

During Avah's classroom observation, one of the students was using a measurement tool incorrectly, but she allowed him to continue using it. He was constantly looking at her for feedback, but she did not give any. When she finally asked, "Do you need any help?" he replied, "I thought you'd never asked." In her interview, Gia reported, "If my students fail to solve a problem, their peers and I still congratulate them for trying. If they are successful, they love when I allow them to do a little dance then get a high five." Alli also shared a point, "I will constantly reassure them that it's perfectly fine to make mistakes, but when making the mistake, it's also more important to learn from that mistake." In Tanya's words, "I love giving praise … 'Good job! Look at you! Give me high five!'"

Tangible Rewards. Tangible rewards are appealing incentives that grab the attention of students and motivate them to complete tasks. Teachers can boost students' confidence levels by giving them tangible rewards to congratulate and recognize their minor achievements or grand accomplishments. In her explanation of the theme, Lynette said, "I give treats, stickers, and other types of rewards to my students." Tanya reported, "At the end of the month, we have a pizza party. There is lots of celebrating their efforts." This claim was observed during a classroom observation. In her interview, Esther responded, "Stickers would suffice for younger kids, but the older ones want expensive things, like lunch from a fast-food restaurant." I observed the use of tangible rewards in some of my classroom observations. For example, during a classroom observation in Lynette's class, students were given stickers. In Abigail's class students were given play money that they could save to redeem at a later date for a prize out of a prize box that was in the corner of the class.

One-on-One Instruction. One-on-one instruction provides opportunities for teachers to administer targeted feedback to their students in a more private and less embarrassing forum. This personal connection between instructor and learner nurtures trusting relationships and also builds confidence. As Aubree shared, "I work with the struggling student one-on-one at my desk, then I gradually release him or her back into the general population where they can participate more confidently." Nicole stated, "When a student is struggling, I would try to talk to them one-on-one in private to see if there is an underlying issue as to why they're struggling, then I help them resolve the problem through mentorship." During a classroom observation in Nicole's class, Nicole called students to her desk to assist them individually. She made them recite the steps they needed to identify where they were going wrong. When they were able to recite the steps correctly, she allowed them to return to their seats.

Instructional Changes

The final theme that was identified was instructional changes. To assist students struggling with grasping mathematical concepts, teachers may modify aspects of their instructional approach and learning environments to enhance the development of their students' self-efficacy while using manipulatives. They must be able to determine and alternate which approach or strategies are the most appropriate and effective. When speaking about her experiences of instructional changes, Emily said, "There is no one manipulative that is the best, it all depends on the learner and their ability." Patrice shared, "When I considered my students' affective states when using manipulatives, sometimes I had to change the type of manipulatives to utilize, take the lesson outside, or even have an invited guest." Correspondingly, Avah asserted, "Manipulatives can be used as a unique strategy to teach students that they must or can always find or adapt to a different method or technique to aid their understanding rather than quitting." In her interview, Abigail stated, "When students are feeling frustrated or anxious, I ask them to explain how they feel, I use calming strategies, I allow them to leave the problem and come back, I even change the manipulatives that I'm using." In Faith's words during a focus group discussion, "Switch it up, make changes with how you use manipulatives if you have to." Tanya agreed and said, "Do whatever it takes to reach your students, be the change." Nicole shared, "Teaching during COVID-19 time was a bit challenging because we had to change the typical way we taught and come up with creative ways to teach students virtually. This pushed me to utilize virtual manipulatives more." Esther admonished other participants in her focus group session to "look at it from a different way, use different methods … use different manipulatives."

Differentiated Instruction. To foster self-efficacy, teachers must gain an understanding of how each of their struggling students learns best. After deciphering how they learn, teachers must then tailor their instruction to meet these students' individual needs. Alli explained by saying:

> It is important for teachers to know exactly what manipulatives to use for the different types of students in their classroom. Not because one manipulative worked for one

student indicates that it will work for all, so don't force it, but simply find what suits that specific child.

In her interview, Lynette shared that there is no "one size fits all" approach, and "with the varying levels of cognitive and physiological development in my classroom, I need to research better strategies to enhance self-efficacy." Rachel stated, "I believe that every child has different learning styles; so therefore, teachers must use strategies to enhance every child's learning ability so that no child is left behind."

Differentiating instruction also involves student-centered learning and promoting student choice. In Tanya's words, "I encourage my students to choose which manipulative that they would like to use to solve a problem. We are now exploring STEM math tools to enhance their independent skills." Faith shared her view in her interview by saying, "The key is that we all want to reach the same goal but let students chose different manipulatives to solve the same problem." During a classroom observation in Esther's class, I observed that there were three ability groups. The first group was instructed to solve 10 problems using manipulatives independently. The second group had to solve five problems using manipulatives, and they received help if needed. The third group was instructed to only solve three problems using manipulatives, but they were given ongoing assistance. The students were able to choose the type of manipulative they wanted to use. Some chose popsicle sticks, but the majority chose to use colorful mini teddy bears. Building a safe learning environment also nurtures differentiated learning. Aubree suggested, "Math can be intimidating for so many of our students, so provide stimulating learning experiences and environments that can help to reduce their fear and anxiety." In Gia's words, "I had to incorporate a lot of positive self-image building initiatives.

Teach the students how to build each other up. I tried to build their self-worth and a positive environment was key." In her interview, Avah shared:

> If I want my students to feel better about themselves and their math ability, I must create the environment that will foster and build their confidence that will give them a desire to even participate in the lesson, and come out feeling like "I can do this! I have achieved this! I am making progress!" So, their belief plays a key role in my delivery, as a matter of fact, it informs the resources and techniques I would use to teach a particular concept.

Teacher Self-Reflection. The teacher participants agreed that an honest and robust self-reflection allows them to become self-aware and to evaluate their own instructional processes or methods of delivering instruction. They believed that through teacher self-reflection, they would be able to determine the root of their successes and failures, then strategically make the most appropriate decisions and modifications to their instructional approach to better meet the needs of their struggling students and seek professional development opportunities. Avah asserted, "I ask myself, what is it that may be the problem? Is it the manipulatives? Is it my strategy or language? Something in the equation is not working so I have to evaluate what need to change."

Lynette shared that she sometimes had to delay, pause, or cancel lessons to accommodate her students' moods. Based on her self-reflections, she claimed that she observed that her students' positive moods "are usually increased when they are using manipulatives that they like." During a focus group discussion, Bianca replied to Lynette's comment and said, "This has a domino effect on the lesson because the students are more willing to complete tasks when they are stimulated." She claimed that students' positive moods are reduced and they begin to shut down when they use a manipulative that does not assist them in attaining desired results. Tanya expressed her view by saying, "My students' confidence is a form of self-reflection because it lets me know that it's time to move on to the next step, and encourages me to be more creative with my instructional methods." In her interview, Latesha also made the following suggestion: "If your students have low self-efficacy, ask yourself: Should I use different strategies or types of manipulatives? Is it time to upgrade myself with a professional development course? Try different ways to reach the child."

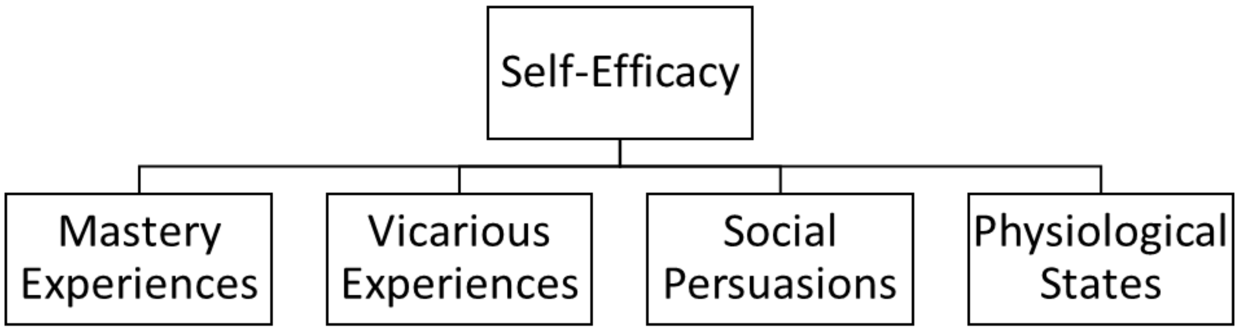
This theme development section entailing a narrative discussion of each theme and the respective subthemes is followed by a discussion of the research questions responses using in vivo quotes from the participants. The following section provides a snapshot of the responses to the research questions of this case study. The responses were based on the major themes and subthemes that emerged from the data collected from the teacher participants.

Research Question Responses

This study was guided by a central research question and four sub-questions. This section entails a synopsis of the participants' responses to the questions and presents the answers using in vivo quotations. The central research question asked, How do middle school teachers describe the factors that influence the development of their students' self-efficacy while using manipulatives during mathematics instruction? The explanations reflect the major themes and subthemes that emerged from the data collected from all teacher participants. Figure 1 below depicts a model of Bandura's four sources of self-efficacy that was used to guide this study. A table was generated to provide a visual description of the number of participants who identified with each theme and subtheme that emerged from the collected data (see Appendix L).

Figure 1

Bandura's Sources of Self-Efficacy



Central Research Question

How do middle school teachers describe the factors that influence the development of their students' self-efficacy while using manipulatives during mathematics instruction?

There was a consensus among the teacher participants that the factors that influence the development of their students' self-efficacy while using manipulatives during mathematics instruction may be difficult to achieve but are rewarding. Five major themes emerged from the collected data: (a) consequences, (b) deliberate practice, (c) modeling, (d) targeted feedback, and (e) instructional changes. For Abigail, her experiences have been "bitter sweet" as she has "experienced high and lows when fostering students' self-efficacy." Latesha added her view to Abigail's statement in a focus group discussion:

> For the most part, I think teachers have great experiences because we are trained to be optimistic and be problem solvers, but then there are those times when issues such as no access to effective supplies or resources hits you hard in the face and the reality is that even proper training can't really prepare you for those moments.

In her interview, Gia said,

> Some of our experiences are good and some are bad, but I guess when you have more good ones than bad ones, that means you have found proactive ways to combat the challenges or have a real good ability to adapt to undesired situations.

Sub-Question One

How do middle school teachers describe their experiences of using manipulatives to facilitate the mastery experiences of students with math difficulties?

Mastery experiences involve increasing self-efficacy through having direct experiences of achieving success by overcoming challenges through sustained effort and perseverance (Bandura, 1999). The teachers’ perspectives conveyed that the themes of consequences and deliberate practice described their experiences of using manipulatives to facilitate the mastery experiences of students with math difficulties. The participants’ responses concurred with Bandura’s (1997) notion that mastery experiences are the greatest source of self-efficacy as many of their responses leaned towards this domain. Teachers expressed that their positive experiences regarding providing mastery experiences for their students involved (a) being cognizant of both good and bad consequences of using manipulatives and being able to balance them to decrease the occurrences of failures, and (b) providing opportunities to students that engage students in activities that promote deliberate practice. These same themes also indicated that even though getting students to develop self-efficacy in math through manipulatives can be challenging, teachers can still successfully facilitate the students’ mastery experiences through integrating aspects of the constructivism approach and repetition strategies into their lessons as they find ways to eliminate digital inequality, feelings of intimidation, and over-dependency. The result will be increased student achievement and student engagement.

As Tanya stated, "Building on what students know, and constructing knowledge to increase their independence from the basics to more complex skills is key." During a classroom observation of a math lesson on comparing numbers, Faith modeled and verbally relayed the steps needed to solve a problem to her students. As the students used manipulatives to solve the problems, she urged them to continually repeat the steps needed to solve a math problem until they felt confident enough to independently complete subsequent problems. She said: "Now do what I did and say what I said over and over until you get it right."

Sub-Question Two

How do middle school teachers describe their experiences of using manipulatives to enhance vicarious experiences for students with math difficulties?

The teachers' perspectives conveyed that the theme characterized as modeling described their experiences of using manipulatives to enhance vicarious experiences for students with math difficulties. Vicarious experiences enable students to see people similar to themselves succeed through sustained effort, which raises the student's beliefs that they too possess the capabilities to master the activities needed for success. Teachers reported that providing vicarious experiences resulted in increased student self-efficacy; however, it required patience. Aubree described the process of providing vicarious experiences as "strategic, time consuming, and sometimes tiresome ... but worth it." Teachers agreed that students' self-efficacy may be increased when teachers model desired behaviors to their students by embedding peer teaching and gamification procedures within their math instruction when using manipulatives. Through gamification, the struggling students saw the success of others in the game and knew that they too could succeed. In reference to the emerging themes, Aubree stated: "My students love role-playing and acting like they are teachers. Their peers love it and learn from it and it makes my job way easier." As she shared her remarks on gamification in an interview, Esther said:

> Games are the epic moments of my math lessons. The ones who are losing are the ones learning from the ones who are winning the games, and the ones winning the games are the ones teaching the ones who are losing or struggling. This is modeling in its finest fashion.

Rachel also stated, "When the students observe other peers completing tasks, they are motivated to try it for themselves." These activities provide vicarious experiences because they allowed the struggling students to witness their peers successfully complete tasks. Witnessing this success made the students believe that they too could succeed.

Sub-Question Three

How do middle school teachers describe their experiences of providing social persuasions when using manipulatives with students with math difficulties?

The teachers' perspectives conveyed that the theme of targeted feedback described their experiences of providing social persuasions to students with math difficulties when using manipulatives with students with math difficulties. Experiences that teachers found to promote self-efficacy through social persuasions included giving targeted feedback to their students through verbal encouragements, awarding students tangible rewards, and providing one-on-one sessions with students in need of further academic or social support when using manipulatives in math. In her interview, Lynette said, "Boosting my students' confidence levels happen when I give them pep talks and let them repeat daily affirmations to remind them that they can do it." In support of this remark, Alli stated,

> Immediate feedback ... whatever it may be ... the pep talks, a toy, 30 minutes of playtime on online games ... a simple sticker ... even verbal praises, all these things increased my students' self-efficacy levels. We can temporarily use incentives to our advantage even though the long-term benefit is for the kids and not us.

The teacher participants were very excited to share their experiences of encouraging their students and exhibited more joy in their voices when answering questions related to this research question than the others. Emily spritely said, "I use lots of encouraging words and self-affirmations … the kids are always motivated." With a warm smile on her face, Lynette shared her sentiments during her interview:

> I make my students feel comfortable about their abilities by giving them positive reinforcements. I give verbal praises before, during, and after their assignments are completed. Even if the problem is not done correctly, I encourage them to try again.

Sub-Question Four

How do middle school teachers describe their experiences of addressing the physiological or affective states of students with math difficulties when using manipulatives?

The teachers' perspectives conveyed that the theme identified as instructional changes described their experiences of addressing the physiological or affective states of students with math difficulties when using manipulatives. Based on teachers' responses, the physiological/affective states of students were also proven to be important to the development of their students' self-efficacy when using math manipulatives. The teachers addressed their students' physiological states by allowing their students to take breaks when they are feeling overwhelmed or experiencing fatigue. As Abigail reported, "Even I get tired sometimes, so imagine the students. I give the students little breaks throughout the lesson to catch themselves."

To heighten the development of self-efficacy, the participants agreed that teachers must also consider students' affective states and initiate transformative instructional changes to meet the diverse needs of their struggling students. As Emily stated, "If a student is frustrated, I need to slow down my teaching pace and reteach the topic until the student is able to understand and apply what is being taught." Teachers can accommodate students' affective states by making instructional changes in delivery, resources, and classroom environment. Instructional changes encompass the following: (a) differentiating instruction to accommodate the affective states and learning needs of each child, and (b) completing teacher self-reflections to identify the affective states and teaching needs of the teacher.

In a focus group session, Bianca spoke about the importance of teacher self-reflections:

> A few years ago, I thought that I was up to date with the latest math resources, but I realized this was not so when hurricane Dorian and COVID-19 came around. That's when I learned about virtual manipulatives. We were forced to use online platforms and virtual resources that I had never even heard of.

In response, Abigail added:

> All of us could relate to this ... our eyes were opened to so many emerging technologies ... I felt robbed of knowledge. That's why we need to evaluate ourselves more, make changes, and seek more professional development opportunities. If not, we will get left behind ... along with our students who are depending on us to provide them with the best learning environment possible.

Emily's comments from her interview also coincided with this theme when she said:

> Change is inevitable, whether we like it or not, it's up to us now to keep abreast of these changes, be ready to adapt and make the necessary accommodations for our students ... you know ... find ways to ease our students' math anxiety, frustration, and tension levels.

Summary

This chapter provided individual descriptions of the teacher participants involved in this study and a description of the results of the data analysis. Data were collected from the participants using individual interviews, focus groups, and classroom observations. Five major themes and 15 subthemes were derived from the data. The five major themes that emerged included (a) consequences, (b) deliberate practice, (c) modeling, (d) targeted feedback, and (e) instructional changes. Explanations were presented to explain how each theme related to the central research question and the four sub-questions. The themes of consequences and deliberate practice illustrated how the teachers described their use of manipulatives to foster mastery experiences for their students in response to Sub-Question One. The theme identified as modeling depicted how the teachers described their use of manipulatives to foster their students' vicarious experiences in response to Sub-Question Two. In response to Sub-Question Three, the theme identified as targeted feedback captured how teachers fostered their students' self-efficacy through social persuasions. The theme identified as instructional changes illustrated how the teachers described their use of manipulatives while accommodating their students physiological/affective states to foster their self-efficacy in response to Sub-Question Four. Throughout this chapter, the participants' experiences were portrayed in descriptive details that were supported by in vivo quotes.

CHAPTER FIVE: CONCLUSION

Overview

The purpose of this instrumental case study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Chapter Five provides a discussion of the study's findings in light of the developed themes. This chapter is comprised of six distinct subsections including (a) summary of findings, (b) interpretation of findings, (c) implications for policy or practice, (d) theoretical and empirical implications, (e) limitations and delimitations, and (f) recommendations for future research. Chapter Five concludes with a summary.

Discussion

This instrumental case study examined the experiences of fifteen middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Major themes and subthemes evolved from the data gathered through individual interviews, focus groups, and classroom observations. Bandura's (1997) theory of self-efficacy served as the theoretical framework of this study. In the following sections, a summary and interpretations of the findings are discussed, followed by a discussion regarding the connection between the theoretical and empirical literature previously discussed in Chapter Two. Following this, implications for policy and practice are presented. Identified limitations, delimitations, and recommendations for future research are also discussed.

Summary of Findings

The theoretical lens that was used to guide the study was Bandura's (1997) theory of self-efficacy. According to Bandura (1997), students form their self-efficacy beliefs by selecting and interpreting information from the identified four primary sources: mastery experience, vicarious experience, social persuasion, and physiological or affective states. A case study research design was used to provide insights regarding how the self-efficacy of students with mathematics difficulties can be fostered through the teachers' use of manipulatives. This research design was utilized to present an in-depth understanding of the phenomenon as interpreted by the teacher participants (Merriam, 1998). To answer the research questions, 15 middle school teachers who met the inclusion criteria were participants in this study. Data were gathered using three different methods: individual interviews, focus groups, and classroom observations. Data were analyzed using Stake's (1995) methods of data analysis. Based on the data analysis, five major themes emerged and were addressed. The five themes that emerged were consequences, deliberate practice, modeling, targeted feedback, and instructional changes. The central research question asked, How do middle school teachers describe the factors that influence the development of their students' self-efficacy while using manipulatives during mathematics instruction? There was a consensus among the teacher participants that fostering students' self-efficacy was challenging but rewarding.

The first sub-question asked, How do middle school teachers describe their experiences of using manipulatives to facilitate the mastery experiences of students with math difficulties? The participants' experiences were characterized as consequences and deliberate practice. Based on the data, the teachers experienced both positive and negative encounters when using manipulatives to foster the student's self-efficacy. As Aubree explained, "Nothing is more powerful than experiencing repeated success, whereas when you experience failure, that will undermine your self-efficacy belief." In regard to deliberate practice, teachers experienced increased student achievements and engagement when they implemented constructivism and repetition teaching approaches.

The second sub-question asked, How do middle school teachers describe their experiences of using manipulatives to enhance vicarious experiences for students with math difficulties? The participants shared that their students' self-efficacy was increased when they implemented modeling strategies including peer teaching, gamification, and one-on-one instructional sessions to enhance their students' vicarious experiences. Bianca explained,

> We all observed people around us. I observe my role models, and so I want my students to observe me. I want them to believe that they too can possess the capabilities they need to master the activities needed to have success in math.

The third sub-question asked, How do middle school teachers describe their experiences of providing social persuasions when using manipulatives with students with math difficulties? The participants shared that they experienced increases in student's self-efficacy when they provided targeted feedback to the students. The teachers' targeted feedback provided social persuasions by giving verbal encouragement, tangible rewards, and one-on-one instructional time. During all of the classroom observations, I observed teachers giving their students verbal praise, tangible rewards, and one-on-one sessions. These forms of feedback made it more likely for the students to put in the sustained effort they needed to succeed even when they came across challenging tasks.

The fourth sub-question asked, How do middle school teachers describe their experiences of addressing the physiological or affective states of students with math difficulties when using manipulatives? Participants shared their experiences and suggested that negative states such as anxiety result in poor performance whereas positive emotions can boost students' confidence. From my classroom observations it was clear that the state the students were in influenced how they judged their self-efficacy. According to Lynette, "Feeling such as anxiety, stress, or frustration dampen my student's confidence in their abilities to complete math task whether they were using manipulatives or not."

Interpretation of Findings

The findings of the study indicated that there are four key factors that teachers can use during math instruction when fostering the self-efficacy of students who struggle with math difficulties. I conducted classroom observations of each of the 15 participants as they used manipulatives to teach a math lesson. Information collected from these observations was triangulated with data collected from individual interviews and focus groups. I found that for teachers to foster the self-efficacy of their students with math difficulties, they must keep in mind that (a) teachers' attitudes affect students' self-efficacy, (b) a balance must be maintained among the four sources of self-efficacy, (c) a supportive and safe learning environment increases students' self-efficacy, and (d) teachers' experiences may inform opportunities for instructional changes. The findings are interpreted based on the results from individually interviewing 15 teachers who also participated in focus group sessions and were observed using math manipulatives during classroom math instructional sessions.

Teachers Attitudes Affect Student Self-Efficacy

I discovered that teachers' attitudes toward math and instructional approach may influence students' self-efficacy. As I interpreted the data, I concluded that the teacher participants had past negative encounters regarding their experiences with math, which caused them to have low self-efficacy during their school years. The participants noted that their childhood teachers had poor attitude towards math and did not take the time out to provide effective math instruction. The teacher participants claimed that they were not exposed to all of the sources of self-efficacy as students, and since they had low self-efficacy, their former teachers' delivery of instruction was also negatively affected. On the other hand, the participants in this study said that now that they enjoy teaching math and have great attitudes toward it, this is transferred to their students who have increased self-efficacy, and in turn their instructional methods remain effective. In addition, I observed that teachers who experienced past low self-efficacy or had poor attitudes toward math did not let negative experiences affect their current instructional delivery. Instead of continuing to provide instruction that is ineffective, they found ways such as the techniques that emerged in this study to foster the self-efficacy of their students.

A Balance Must Be Maintained

As I interpreted the data, I concluded that there should be a balance between deliberate practices alongside modeling targeted feedback and instructional changes. These methods should not be used in isolation but should all be used interchangeably whilst implemented in a mass classroom setting to foster the self-efficacy of students who struggle with math. All of the teachers who participated reported an increase in student achievement and student engagement levels when using all of these techniques in their classrooms. A correct balance between these methods would ensure that students are exposed to a variety of teaching methods that can cater to the individuality and diverse needs of the students. Throughout the data collection process, the data conveyed that teachers were willing to use different methods to reach their students. Instead of focusing only on one of the sources of self-efficacy, they focused on the collective four. They did this by providing students access to methods that enabled mastery experiences, vicarious experiences, social persuasions, and physiological or affective states. Thus, creating a balance between the sources of self-efficacy will ensure that students are not deprived in any of the areas.

Therefore, the more sources they are privileged to access, the higher their level of self-efficacy will be.

A Supportive and Safe Learning Environment Must Be Created

As I interpreted the data, I concluded that teachers were very committed to providing their students with a safe and supportive learning environment. Many of the teachers conveyed the idea that their classrooms were a safe space where it was acceptable for students to make mistakes. Their use of one-on-one instructions along with verbal encouragements, tangible rewards, and differentiated instruction allowed students to have a choice of the type of manipulatives they were more comfortable with using. Their willingness to make instructional changes gave evidence that they are willing to create supportive and safe learning environments for their struggling students. If students do not feel safe or supported in their classroom settings, their self-efficacy levels will be impacted in a negative way. This, in effect, will cause them to not feel safe enough to make mistakes, causing them to feel as if they might not receive the appropriate or needed support they require to succeed. A supportive and safe learning environment that fosters self-efficacy creates a haven where students can feel as though their math difficulties can be overcome or managed.

Teachers' Experiences May Inform Opportunities

Due to aspects of the ever-changing global society, the expansion of instructional research and findings, and the constant development of digital trends and technology, instructional changes are inevitable and should be approached with strategic action steps and unwavering involvement and support from all stakeholders. As I interpreted the data, I concluded that teachers' experiences of using manipulatives provide opportunities for instructional changes. Therefore, the use of manipulatives while teaching math can inform instructional changes. These changes can be implemented in the curriculum and be the focus of policies or decisions regarding distribution of resources. Teachers' experiences can also lead to the identification of areas for professional development that may enhance specific educational systems. The use of manipulatives during math instruction can also promote avenues for incidental learning or the development of other skills. In my view, stakeholders should be challenged to embrace innovative ideas such as integrating fine motor skills development into math curriculums, making it mandatory for teachers to obtain professional development training in the use of manipulatives and the procedures of differentiated instruction. Teachers must reiterate that mistakes are inevitable, but the key is to learn from failures.

From my experience, the word change is often greeted with negative perceptions due to its association with fear or anxiety in the field of education. This causes educators to refrain from leaving their comfort zones and exploring new innovations. For example, many educators evade the use of current technology despite findings which revealed that teachers are "active agents for change in implementing and designing technological innovations" (Koonce, 2020, p. 295) and that the correct use of evidence-based technology positively impacts student achievement. Gordon et al. (2019) also reported that educational technology enhances student learning as long as educators seek ongoing professional training. 1 Thessalonians 5:21 (*New International Version*, 1978/1992) encourages us to test everything and to hold fast to what is good. How can teachers embrace instructional changes if they are unwilling to test their effectiveness?

Theoretical and Empirical Implications

The purpose of this section is to discuss the theoretical on empirical implications of this study. This study confirmed that mastery experiences are the greatest source of self-efficacy as identified by Bandura (1997). The findings of this instrumental case study suggest implications for implementing more strategic and focused support for students who struggle with math difficulties. The relationships between the findings in this study and the theoretical and empirical literature are discussed below.

Theoretical Implications

Bandura's (1997) theory of self-efficacy stated that the four elements that should be present in a teaching environment to aid in the development of self-efficacy are mastery experiences, vicarious experiences, social persuasions, and physiological/affective states. The results of this study indicated that the participants all had positive experiences regarding the importance of providing mastery experiences to their students to increase self-efficacy. In addition to increase self-efficacy, the teachers expressed the importance of implementing avenues for deliberate practices to enhance the students' personal experience. This appeared as one of the most effective ways to increase their students' self-efficacy beliefs. This constant reference to the occurrences of deliberate practices denotes that the use of manipulatives will transition students from being passive learners in the classroom to assuming more active roles in their learning processes. This student-centered approach resulted in higher student achievement and engagement.

This study added to Bandura's (1997) self-efficacy theory by providing a comprehensive understanding of teachers' experiences as they navigated the process of using manipulatives to foster their students' self-efficacy. This study also extends the theory by providing knowledge of how to effectively increase students' self-efficacy levels by using innovative instructional methods. Further enquiry on teachers' experiences provided an in-depth understanding of the effect that the use of manipulatives may have on students' sense of self-efficacy through the lenses of four factors that are necessary for self-efficacy to occur (mastery experience, vicarious experience, social persuasions, and physiological or affective states). Descriptive accounts of teachers' experiences revealed common or diverse insights of how their use of manipulatives potentially fostered the self-efficacy of students with math difficulties.

Empirical Implications

The participants all experienced the need to make instructional changes to best meet the diverse needs of their students. The teachers all reported that they implemented some sort of modifications or adaptations to their instructional delivery or to the learning environments to accommodate their students' weaknesses. These accommodations included creating supportive and safe learning environments, using appropriate math resources, making decisions based on robust teacher self-reflections, and seeking professional development opportunities.

My study corroborates previous studies by Carbonneau et al. (2013) and Laski et al. (2015) by confirming that involving the use of manipulatives has shown that neither concrete nor virtual manipulatives result in meaningful learning unless they are effectively embedded in instructional practice. The results of my study also coincide with previous research conducted by Park et al. (2021) and Satsangi et al. (2016) that revealed that once teachers effectively implement manipulatives into their instruction, students who are challenged with mathematical difficulties garner greater benefits academically. As the research instrument, I was able to attain participants who could best shed light on the phenomenon being investigated and analyzed their experiences to better understand the situation. The findings of this case study yielded empirical data regarding teachers' experiences of using manipulatives to enhance learning through an increased sense of self-efficacy and subsequently inform pertinent decision-making and implications for practice and policy.

Implications for Policy or Practice

The purpose of this instrumental case study was to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Previous studies emphasized the importance a fostering self-efficacy in students who struggle in math (Park et al., 2021; Satsangi, Hammer, & Evmenova, 2018; Satsangi, Hammer, & Hogan, 2018). The findings of this study add to the existing literature discussed in Chapter Two. Previous studies addressed the importance of using manipulatives to increase academic achievement (Belenky & Nokes, 2009; Carbonneau et al., 2013; Goodman et al., 2016; Hidayah et al., 2021; Jimenez & Stanger, 2017; Maccini & Gagnon, 2000; McNeil & Jarvin, 2007; Moyer, 2001; Roberts et al., 2020; Vessonen et al., 2021); however, these studies did not emphasize the importance of fostering students' self-efficacy and teachers' experiences regarding their knowledge of and use of the four sources of self-efficacy. This study highlighted the teachers' experiences as it relates to fostering students' self-efficacy through navigating consequences, deliberate practice, modeling, targeted feedback, and instructional changes.

Implications for Policy

According to the findings of this study, I concluded that education systems will see an increase in student math achievement and student engagement if policymakers consider and include the contributions of math teachers in their policymaking decisions. These considerations can be addressed as policymakers plan and implement policies that inform or govern instructional and learning processes. Policymakers need to be enlightened about teachers' experiences to create more effective policies so that teachers can implement strategic and beneficial instructional approaches within their math classrooms. Teachers who provide instruction to students with math difficulties should be properly trained and adequately equipped with the appropriate math resources that include both concrete and virtual manipulatives to meet the diverse needs of their students to increase their self-efficacy. Policies that are based on teachers' experiences should be further encouraged and supported by the relevant policy makers. Teachers' experiences regarding student performance should be the driving force behind the drafting of educational policies and life-changing decisions. The failure to consult teachers who experience the phenomenon in the policymaking process may be the reason for ineffective instructional mandates that result in poor teacher attitudes, low student self-efficacy, high levels of teacher and student intimidation, and decreased student achievement and engagement.

As it relates to school-level policymaking, math curricula should be reviewed and selected by teachers currently employed in the respective schools where the curricula are implemented. According to Vaughn and Bos (2019), a significant feature of a comprehensive math curriculum is to teach relevant concepts and specific vocabulary to ensure that students grasp the language of mathematics. The authors expressed that students with math difficulties are less likely to comprehend mathematical language and can be deprived of optimum learning experiences that enhance mathematical proficiency. Since traditional math curricula impede the learning progress of students with math difficulties by providing difficult readability levels, unorganized formats, and inadequate opportunities to practice math concepts and math application skills, school districts should employ policies that are based on teachers' diverse experiences to ensure that robust math instruction includes the resources and training for effective use of manipulatives. In doing this, policymakers would be able to improve and support student learning in a safe and stimulating environment that will encourage stakeholders to exude engagement, competence, and accountability.

Some policies that I believe districts should employ include the following: (a) assign teachers with responsibilities after they are familiarized with the goals, objectives, and purpose of the proposed policies; and (b) provide additional training for teachers regarding how to provide rigorous and relevant, well-planned math instruction that involves the use of manipulatives. Policymakers should also organize collegial planning to (a) improve math instructional delivery methods, (b) use the analysis of the gathered data regarding teachers' experiences to plan strategically, and (c) determine how the proposed policies will best meet students' needs.

Implications for Practice

Teachers should attempt to implement a variety of strategies and make necessary instructional changes to foster their students' self-efficacy. These strategies and instructional changes may result in improved student achievement and increased student engagement for the middle school student who is struggling in math. As Hansen et al. (2016) suggested, educational stakeholders need to identify ways to effectively involve teachers in the design process of manipulatives as well. Therefore, teachers' experiences must be considered as a form of consultation when planning and implementing instructional changes.

A supportive and safe learning environment characterized with modeling and targeted feedback may help the students who make mistakes to feel comfortable overcoming failures through completing academic tasks that involve deliberate practice. Many teachers have experienced both positive and negative encounters while learning to use manipulatives. Nonetheless, there is a need to have a discussion among educational stakeholders on how to decrease and eventually eradicate these negative encounters such as digital inequalities, lack of professional development opportunities, and lack of resources. Once these areas are adequately addressed, teachers' experiences may be of a more positive nature.

Limitations and Delimitations

This instrumental case study was designed to fill the gap in the literature by examining the experiences of middle school teachers who use manipulatives during instruction to foster the self-efficacy of students who struggle with math. To ensure that the essence and credibility of the teachers' experiences were captured, I incorporated certain delimitations. I utilized a case study design to garner concrete and in-depth knowledge of the context being examined in the study (Creswell & Poth, 2018). Additionally, I ensured that all teacher participants had first-hand experiences with the phenomenon by limiting the inclusion criteria to middle school teachers who teach students with math difficulties and have had experiences using manipulatives during math instruction. This criterion ensured that teachers were knowledgeable of the use of manipulatives during classroom observations and were able to satisfactorily answer individual interview and focus group questions.

This study also had limitations. First, the geographic location of The Bahamas was a limitation. The Bahamas is an archipelago, meaning that school districts are located on different islands, which limits the generalizability of the study. A second limitation was the geographical location of the participants, which also limited the generalizability of this study. The participants were employed in the same school district. A third limitation was that all of the participants were females. There is a shortage of male teachers in their respective school district. Because of this, the study results may not be entirely generalizable as no male perspectives were included.

Recommendations for Future Research

In consideration of the study findings, limitations, and the delimitations placed on the study, several recommendations are made for future research. Firstly, the study can be replicated in school districts that have different demographics such as private school sectors. The study can also be conducted in different school districts on different islands in The Bahamas, in both rural and urban areas as well as in mainland schools in the United States. Secondly, this study should be conducted among male participants to determine if their experiences are similar to the experiences of the female math teachers. For future research, I would recommend that studies be conducted to see the extent to which the practices mentioned in teachers' reports are effective in fostering student self-efficacy such as a quantitative study that measures math achievement on an end-of-year testing assessment.

The participants in this study gave several positive and negative aspects of their use of manipulatives, but only two of them mentioned professional collaboration or administrative support. Therefore, further research should be explored to determine if professional collaboration affects teachers' ability to foster the self-efficacy of students struggling with math difficulties. Further research should also be explored to determine whether the level of administrative support affects teachers' use of manipulatives. Additionally, only one participant mentioned student behavior as a negative factor that may hinder their use of manipulatives. A future study should also be conducted to see if student behavioral issues impact the use of manipulatives by teachers during math instruction. In addition, other case studies should be conducted in the different districts in The Bahamas to create a more in-depth description of the phenomenon to increase generalizability.

Conclusion

This study examined the experiences of middle school teachers who used manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Bandura's theory of self-efficacy was the theoretical framework that guided this study. The study was conducted in a school district in The Bahamas. Data were collected from individual interviews, focus groups, and classroom observations of 15 participants. A central research question and four sub-questions were explored that led to the development of major themes and subthemes for this study, adding to previous literature regarding self-efficacy and mathematics. The five major themes that emerged from the data analysis included the following: (a) consequences, (b) deliberate practice, (c) modeling, (d) targeted feedback, and (e) instructional changes. The findings indicated that teachers' experiences of providing mastery experiences for their students involved navigating consequences such as student achievement, intimidation, student engagement, incidental learning, digital inequality, and dependence. Teachers' experiences of providing mastery experiences also involved engaging students in learning processes that nurture deliberate practices such as repetition and constructivism. In their plight to provide vicarious experiences for their students, teachers engaged students in modeling practices such as peer teaching and gamification. Teachers reported providing targeted feedback through verbal encouragements, tangible rewards, and one-on-one instruction to increase their students' self-efficacy through social persuasions. Students self-efficacy was also increased due to instructional changes teachers initiated when they observed that their students' physiological/affective states warranted such changes. These instructional changes related to differentiated instruction and teacher self-reflections.

Teachers' experiences conveyed that teachers' attitudes affect students' self-efficacy as teachers who display negative attitudes and use ineffective teaching practices fail to foster their students' self-efficacy. Teachers' experiences also conveyed that a balance must be maintained among the four sources of self-efficacy to provide sufficient avenues for students to develop self-efficacy. The findings of this study also conveyed that a supportive and safe learning environment increases students' self-efficacy and that teachers' experiences may inform opportunities for instructional changes.

Despite the challenges of teaching students who struggle with math difficulties, teachers effectively used strategies to foster the self-efficacy of their students as they skillfully balanced the four influential sources of self-efficacy to increase their students' achievement and engagement levels. Results of this study provide implications for policy and practice to enhance teachers' delivery of instruction and the learning of students with math difficulties who possess the potential of becoming contributing members to an ever-changing global society.

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Appendices

Appendix A: IRB Approval Letter

LIBERTY UNIVERSITY.
INSTITUTIONAL REVIEW BOARD

March 2, 2023

Krishanta Butcher
Gail Collins

Re: IRB Exemption - IRB-FY22-23-978 A Case Study Examining the Experiences of Middle-School Teachers Who Use Manipulatives to Foster The Self-Efficacy of Students with Math Difficulties

Dear Krishanta Butcher, Gail Collins,

The Liberty University Institutional Review Board (IRB) has reviewed your application in accordance with the Office for Human Research Protections (OHRP) and Food and Drug Administration (FDA) regulations and finds your study to be exempt from further IRB review. This means you may begin your research with the data safeguarding methods mentioned in your approved application, and no further IRB oversight is required.

Your study falls under the following exemption category, which identifies specific situations in which human participants research is exempt from the policy set forth in 45 CFR 46:104(d):

Category 2.(iii). Research that only includes interactions involving educational tests (cognitive, diagnostic, aptitude, achievement), survey procedures, interview procedures, or observation of public behavior (including visual or auditory recording) if at least one of the following criteria is met:
The information obtained is recorded by the investigator in such a manner that the identity of the human subjects can readily be ascertained, directly or through identifiers linked to the subjects, and an IRB conducts a limited IRB review to make the determination required by §46.111(a)(7).

Your stamped consent form(s) and final versions of your study documents can be found under the Attachments tab within the Submission Details section of your study on Cayuse IRB. Your stamped consent form(s) should be copied and used to gain the consent of your research participants. If you plan to provide your consent information electronically, the contents of the attached consent document(s) should be made available without alteration.

Please note that this exemption only applies to your current research application, and any modifications to your protocol must be reported to the Liberty University IRB for verification of continued exemption status. You may report these changes by completing a modification submission through your Cayuse IRB account.

If you have any questions about this exemption or need assistance in determining whether possible modifications to your protocol would change your exemption status, please email us at irb@liberty.edu.

Sincerely,
G. Michele Baker, MA, CIP
Administrative Chair of Institutional Research
Research Ethics Office

Appendix B: Recruitment Letter

Dear Teachers,

My name is Krishanta Butcher, and I am a student of Liberty University currently enrolled in the Doctor of Education in Curriculum and Instruction-Special Education program. As a graduate student, I will be conducting a study as part of the requirements of this degree. The purpose of my research is to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. I am writing to invite eligible participants to participate in this study.

All potential participants should meet the following inclusion criteria: (a) is a trained teacher possessing a bachelor's degree or higher in education (b) have administered Math instruction using virtual manipulatives to middle-school students within the past 3 years, (c) have 3 years or more of teaching experience, and (d) range between the ages of 25-55 years. Participants, if willing, will be asked to take part in an audio-recorded interview (45-60 minutes), participate in a focus group (45-60 minutes), and be observed in your classroom by the researcher (15-45 minutes). Names and other identifying information will be requested as part of this study, but the information will remain confidential.

If you meet these requirements and desire to participate, please click the link below to complete the screening survey. You will be contacted if you are selected and be intreated to sign and return a consent form.

A consent document will be sent to you via email if you are eligible. The consent document contains additional information about my research. If you are deemed eligible and choose to participate, you will need to sign the consent document and return it to me at the time of the interview.

Participants will be compensated for participating in this study. All participants will be given a $25 grocery voucher by the researcher after all the participant tasks have been completed. To be eligible for compensation, the participant will have to complete an individual interview, participation in one focus group conversation, be observed for one math lesson, and review transcripts. Failure to complete all procedures will result in the forfeit of the voucher.

Please feel free to contact me with any questions or concerns at [redacted]

Yours faithfully,
Krishanta Butcher
Liberty University Student

Appendix C: Screening Survey

A Case Study Examining the Experiences of Middle-School Teachers Who Use Manipulatives to Foster the Self-Efficacy of Students with Math Difficulties

* Required

1. Full Name *

2. Cell Phone Contact *

3. Email *

4. School District *

5. School *

6. Grade Levels Taught in the past 3 Years *

7. Job Title *

8. Highest Level of Education & Degree Obtained*

9. Years of Teaching Experience *

10. Sex *

Mark only one oval.

Male ⬭

Female ⬭

11. Have you used concrete and/or virtual manipulatives during math instruction to middle school students within the past 3 years? *

Mark only one oval.

Yes ⬭

No ⬭

12. Have you used concrete and/or virtual manipulatives to enhance your students' confidence levels? *

Mark only one oval.

Yes ⬭

No ⬭

Appendix D: Acceptance/Rejection Email

Email for those accepted as participants:

Dear Sir/Madam,

You have been selected to participate in a research study that seeks to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties. Please review and sign the consent form attached to this email and return it to [redacted] within 5 days if you still wish to participate in this study. After I receive your signed consent form, you will be given more information. I want to thank you in advance for helping me to complete this research.

Thank you in advance!
Krishanta Butcher

Email to participants who completed the survey but were not selected for the study:

Dear Sir/Madam,

Thank you for taking the time out of your busy schedule to complete the screening survey regarding my research that seeks to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties.

Your assistance was greatly appreciated!

Best regards,

Appendix E: Consent Form

Title of the Project: A Case Study Examining the Experiences of Middle-School Teachers Who Use Manipulatives to Foster the Self-Efficacy of Students with Math Difficulties

Principal Investigator: Krishanta Butcher, Liberty University

Invitation to be Part of a Research Study

You are invited to participate in a research study. Participants must fit the following eligibility criteria: (a) is a trained teacher possessing a bachelor's degree or higher in education (b) have administered math instruction using manipulatives with middle-school students within the past 3 years, (c) have 3 years or more of teaching experience, and (d) range between the ages of 25-55 years. Taking part in this research project is voluntary.

Please take time to read this entire form and ask questions before deciding whether you would like to take part in this research project.

What is the study about and why are we doing it?

The purpose of this study is to examine the experiences of middle school teachers who use manipulatives during math instruction to foster the self-efficacy of students with mathematics difficulties.

What will participants be asked to do in this study?

If you agree to participate in this study, I will ask you to do the following:

1. Participate in a face-to-face interview lasting between 45-60 minutes. This interview will be audio-recorded.
2. Participate in a face-to-face focus group interview lasting between 45-60 minutes. This focus group will be audio-recorded.
3. Agree to be observed during a math class in which you are using concrete and/or virtual manipulatives as you teach. The observation session will be no shorter than 15 minutes and no longer than 45 minutes.
4. Review the transcript of your interview and your parts of the focus group to check for accuracy. This should take approximately 15 minutes.

How could participants or others benefit from this study?

Participants should not expect to receive a direct benefit from taking part in this study.

Benefits to society include discovering insightful knowledge and proactive measures that are needed to enhance mathematical instruction for students; especially those who struggle with grasping mathematical concepts.

What risks might participants experience from being in this study?

The risks involved in this study are minimal, which means they are equal to the risks you would encounter in everyday life.

How will personal information be protected?

The records of this study will be kept private. Published reports will not include any information that will make it possible to identify a subject. Research records will be stored securely, and only the researcher will have access to the records.

- Participant responses will be kept confidential through the use of pseudonyms.
- Interviews will be conducted in a location where others cannot hear the conversation.
- Confidentiality cannot be guaranteed in focus group settings. While discouraged, other members of the focus group may share what was discussed with persons outside of the group.
- The raw and transcribed data will be stored on a password-locked computer and may be used in future presentations. After three years, all electronic records will be deleted.
- Recordings will be stored on a password locked computer until participants have reviewed and confirmed the accuracy of the transcripts and then deleted. Only the researcher will have access to these recordings.

How will participants be compensated for being part of the study?

Participants will be compensated with a grocery voucher valued at $25 for participating in this study. The vouchers can be redeemed at a local grocery store located within the school district.

Is study participation voluntary?

Participation in this study is voluntary. Your decision whether or not you want to participate will not affect any related current or future relations with Liberty University or the [redacted]. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting those relationships.

What should be done if a participant wishes to withdraw from the study?

If you choose to withdraw from the study, please contact the researcher at the email address or phone number included in the next paragraph. Should you choose to withdraw, data collected from you, apart from focus group data, will be destroyed immediately and will not be included in this study. Focus group data will not be destroyed, but your contributions to the focus group will not be included in the study if you choose to withdraw.

Whom do you contact if you have questions or concerns about the study?

The researcher conducting this study is Krishanta Butcher. You may ask any questions you have now. If you have questions later, **you are encouraged** to contact her at [redacted] or kkbutcher@liberty.edu. You may also contact the researcher's faculty sponsor, Dr. Gail Collins, via email at glcollins2@liberty.edu.

Whom do you contact if you have questions about rights as a research participant?

If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher, **you are encouraged** to contact the Institutional Review Board, 1971 University Blvd., Green Hall Ste. 2845, Lynchburg, VA 24515 or email at irb@liberty.edu.

Disclaimer: The Institutional Review Board (IRB) is tasked with ensuring that human subjects research will be conducted in an ethical manner as defined and required by federal *regulations. The topics covered and viewpoints expressed or alluded to by student and faculty researchers are those of the researchers and do not necessarily reflect the official policies or positions of Liberty University.*

Your Consent/Opt-Out

By signing this document, you are agreeing to participate in this study. Make sure you understand what the study is about before you sign. You will be given a copy of this document for your records. The researcher will keep a copy with the study records. If you have any questions about the study after you sign this document, you can contact the study team using the information provided above.

Statement of Consent:

- ☐ I have read the above information. I have asked questions and have received answers. I consent to participate in the study.

- ☐ The researcher has my permission to audio-record/video-record me as part of my participation in this study.

Printed Participant Name: ______________________________

Signature: ______________________________ Date: ____________________

Signature of Investigator: ________________________ Date: ____________________

Appendix F: Parent Permission Form

Dear parent,

My name is Krishanta Butcher, and I am a student at Liberty University currently enrolled in the program: Doctor of Education in Curriculum and Instruction-Special Education. I am now enrolled in my dissertation courses and must conduct a research study as part of the requirements of the doctoral program. One of the data collection methods that I am using is classroom observations. In this regard, I hereby request that you sign this permission form to allow me to conduct classroom observations during math instructional time in the classroom that your child is in. The observations will be range between the time frame of 15 to 45 minutes. I will be a non-participant observer and will not be talking to your child or using any type of video or audio recording equipment that will capture photos of your child. I will not refer to your child in any way in the report. Your child will not be a participant in my study. My purpose in conducting these classroom observations is to observe how the teacher is using manipulatives to teach math.

Please feel free to contact me with any questions or concerns at [redacted] or via email: kkbutcher@liberty.edu

Thanking you in advance for your support.

Sincerely,
Krishanta Butcher
Liberty University Student

Statement of Permission:

- ☐ I have read the above information. I give permission for my child to be in the classroom during math instruction while Mrs. Butcher completes a classroom observation.

- ☐ I have read the above information. I DO NOT give permission for my child to be in the classroom during math instruction while Mrs. Butcher completes a classroom observation.

Printed Student Name: ____________________________

Parent Signature: ____________________________ Date: ___________________

Appendix G: Individual Interview Questions

1. Please share your reasoning for becoming a teacher.
2. What math resources have been the most beneficial to your students' achievement throughout your teaching career?
3. Describe your experiences as a math teacher. CRQ

In the rest of this interview, I would like to know how you used both concrete and virtual manipulatives to answer the questions given. I will simply say manipulatives in each question, but please remember to broaden your answer to talk about your use of either or both concrete or virtual manipulatives.

4. Describe how you make your students feel confident about their abilities in math. CRQ
5. How do you think your students' self-efficacy impacts your delivery of math instruction? CRQ
6. Describe your experiences of using manipulatives to provide opportunities for your students to practice new skills. SQ1
7. Describe your use of manipulatives to aid students in overcoming math difficulties. SQ1
8. Describe your use of manipulatives to teach students that success requires perseverance/sustained effort? SQ1
9. Describe your experiences of having students observe social models to successfully complete a task. SQ2
10. Describe your use of manipulatives to increase students' beliefs that they possess the capabilities needed to master similar activities to succeed. SQ2
11. Describe the positive feedback that you have given to students with math difficulties when using concrete and/or virtual manipulatives to build their self-efficacy. SQ3

12. Describe the negative feedback or discouraging aspects experienced by students with math difficulties when using manipulatives. SQ3
13. Describe your experiences of accommodating students' physiological or affective states when using concrete and/or virtual manipulatives. SQ4
14. How are students' positive moods reduced or increased when using manipulatives to build their self-efficacy. SQ4
15. What else would you like to add to our discussion regarding your experiences of using manipulatives to enhance the self-efficacy of students with math difficulties?

Appendix H: Focus Group Questions

1. Please introduce yourselves to the group by stating your name and school where you teach.
2. Describe challenges you faced as a math teacher.
3. Describe your experiences regarding the advancement of technology in math instruction. CRQ

Just like in your individual interview, all of these questions will simply say manipulatives but please remember to broaden your answers to talk about your use of either or both concrete or virtual manipulatives to answer the questions given.

4. How do you provide opportunities for your students to practice new skills when using manipulatives? SQ1
5. What experiences have you encountered that may have prohibited you from using manipulatives during instruction with students with math difficulties? SQ1
6. Describe how you have used manipulatives to aid students in overcoming math difficulties. SQ1
7. Describe your students' reactions to being assigned social models to successfully complete a task. SQ2
8. Describe how positive and negative feedback given to students with math difficulties when using concrete and/or virtual manipulatives impact their self-efficacy. SQ3
9. Describe ways teachers can use to accommodate their students' physiological or affective states when using concrete and/or virtual manipulatives. SQ4

10. What else would you like to add to our discussion regarding your experiences of using concrete and/or virtual manipulatives to enhance the self-efficacy of students with math difficulties?

Appendix I: Observation Protocol

Date: March 21st, 2023

Time Started: 1:15 PM **Time Ended:** 2:00 PM

Participant: Faith

Grade Level: 5

Observations	Preliminary Interpretations	Reflections
The teacher began the math lesson with a chant.	The teacher has to introduce the lesson with an interesting introduction in order to gain the students' attention.	The teacher tries to get the students excited about the lesson before introducing the new concept.
The teacher reviewed the previous math lesson.	The teacher is building on prior knowledge.	The students recalled a lot of information from the previous lesson, so they are ready to move on to a new concept. This appears to relate to the source of mastery experiences.
The teacher introduced the new lesson that was based on comparing numbers. She used manipulatives such as number cards and cups to introduce the concept. She modeled how to identify the largest number and used a fake/play alligator mouth to demonstrate which number goes into which cup.	As she taught the lesson, she explained each step and asked the students follow up questions to ensure that they were paying attention and grasping the concept.	The step-by-step teaching approach that the teacher used simplified the lesson. This appears to relate to the source of vicarious experiences.
She worked with them one-on-one, as the other students worked together in pairs solving other problems.	The teacher uses pair modeling strategies and work with struggling students one-on-one.	Modeling is a very effective strategy to make students confident and more knowledgeable about the process needed to solve problems. This appears to relate to the source of vicarious experiences.
Throughout the lesson, the teacher is constantly giving verbal praise to the students who are struggling with grasping the concepts. They	The students feed off of her feedback in order to know if they are doing well, or if they need further assistance.	The teacher practices giving feedback in a timely manner and I can tell that the students are accustomed to this. They are often looking to her

are also given verbal encouragements.		for affirmations. Constantly ask her questions, such as, “Am I doing it right?” This appears to relate to the source of social persuasions.
The teacher allowed the students to choose the type of manipulatives they wanted to use to solve the problems. The students were able to choose which manipulative they wanted to use and if the manipulative was ineffective I noticed that they went back to the math center and were able to choose a more appropriate manipulative.	The students love being in charge of their own learning process. They love being able to choose which manipulative they want to use and they enjoy going through the steps independently.	Even though the students seemed very confident in their ability to complete the task, they usually look towards their parents or their teacher for confirmation. Especially when completing independent assignments. This appears to relate to the source of physiological/ affective states.
As the lesson came to a close, the teacher asked follow-up questions to see if the students had any questions for her. She allowed two of the students to say what they would have done to make the lesson more interesting or what she could have done to make sure that everyone grasped the concept faster.	The teacher also looks for feedback from the students to improve her lessons. The majority of the students said that the lesson could not have gone any better and some said that she should have allowed them to play a little longer.	Even though the students did not give answers that she probably will be introducing in any of her future lessons, I found it commendable that she gave the students a voice and opportunity to make contributions in the evaluation process. This appears to relate to the source of physiological/ affective states.
The students had to share manipulatives when completing the assignments.	There is a lack of resources or sufficient manipulatives.	Sharing manipulatives has a negative impact on time management and interrupts with the smooth flow of the lesson.

Date: March 20th, 2023

Time Started: 9:15 AM **Time Ended:** 10:00 AM

Participant: Abigail

Grade Level: 5

Observations	Preliminary Interpretations	Reflections
The teacher introduced the lesson by playing a video about time. She instructed the students to pay attention to the video and to try and remember what they hear and see.	Videos increase students' engagement.	Videos that feature adults and children using manipulatives is also a way to introduce modeling.
The teacher then used a clock manipulative to demonstrate how to tell time to the hour. She used large flashcards that featured the steps the students needed to take to solve the problems themselves. After she used each card, she placed them in order and in an area where the students can refer to them when needed.	Step-by-step learning is a form of constructivism. As the students were able to build upon what they knew, I realized that they were becoming more involved in the lesson.	Repetition is a key strategy that aids memory. This appears to relate to the source of vicarious experiences.
The teacher gave one-on-one assistance to students who had a hard time grasping how to manipulate the clocks. As she provided this one-on-one instruction to some of the students, she allowed the students who grasped the concept well to work with weaker students. As the stronger students worked with the other students, they took on a teaching persona and were able to teach the other students how to manipulate the hands of the clock correctly to solve problems.	Peer teaching is an effective strategy to use when more than two or three students are struggling to grasp one concept.	Peer teaching is a very convenient tool. Sometimes the teacher cannot tend to or help a large number of students during a lesson. Empowering other students to have the confidence to teach their weaker peers will assist with time management and foster a supportive learning environment. This appears to relate to the source of vicarious experiences.
Throughout the lesson the teacher had the students play lots of games. She grouped the students based on their ability levels during some of the games, but for most of the games she paired a weaker student with a stronger student.	The students were very excited to play the games and seemed to grasp the concept more because they were	Games and free student engagement build student confidence levels. This appears to relate to the source of vicarious experiences.

The students seem very engaged in the lesson and were more eager to use the manipulatives so that they could win the games.	playing. When the games were over, they were in a more subtle mood, and requested more assistance to complete tasks.	
Throughout the lesson the teacher was giving verbal encouragements; however, the students seemed to enjoy their tangible rewards that they received at the end of the lesson. The tangible rewards were stickers and small containers of slime.	Rewarding students for their hard work on accomplishments boosted their confidence.	Even though students seemed to enjoy tangible rewards, they can also be costly. This appears to relate to the source of social persuasions.
The teacher was mainly using concrete manipulatives during the lesson; however, a few of the students were asking for their devices to use virtual manipulatives and she promised them that they would be able to use the devices at the end of the lesson, after they had mastered use of the concrete clock.	Sometimes virtual manipulatives or devices are used as tangible rewards.	Teachers and students must learn how to balance the use of concrete and virtual manipulatives, to ensure that one is not overused or ineffective for certain students. This appears to relate to the source of social persuasions.
The teacher had three motivations charts displayed in the classroom.	Motivational charts provide constant reminders to strive for success and increase a student’s confidence.	As a student, I always enjoyed seeing motivational posters in my classroom. So when I became a teacher, I ensured that I had at least two or three motivational banners or charts in my classroom. This appears to relate to the source of social persuasions.

Appendix J: Audit Trail

Date:	**Task:**
12/21/2022	Acquired the District Permission Letter
03/02/2023	Obtained IRB approval
03/05/2023	Completed the pilot study
03/05/2023	Gatekeeper distributed recruitment letters
03/07/2023	Distributed accepted and rejection letters
03/13/2023	Completed face-to-face individual interviews
03/15/2023	Completed interview transcriptions and member checking
03/16/2023	Completed Focus Group Sessions 1 and 2
03/17/2023	Completed focus group transcriptions, member checking, and began classroom observations
03/22/2023	Completed classroom observations
03/23/2023	Compiled data and began data analysis

Appendix K: Researcher's Reflective Journal

Date:	Comments:
12/13/2022	Since the onset of the COVID-19 pandemic, even more teachers in The Bahamas are more aware of and open to the use of virtual learning tools such as virtual manipulatives. I believe that manipulatives are only effective when they are used by teachers who are trained to use them and have positive experiences with them.
12/21/2023	I have just received District approval. According to the letter I have to complete all of my data collection by April 10th, 2023. This date seems unreasonable; however, I will not be issued another letter. This state is unreasonable because I have to also await IRB approval, which takes weeks. I am hoping that I get everything done in time.
03/02/2023	I have just acquired IRB approval. I will now begin recruitment for my pilot study.
03/05/2023	Today I have completed my pilot study. There are minor changes that I have to make to my individual interview questions. I am considering using the same members as participants for my study. As I conducted the interviews, I had to remind myself to suppress my own experiences as I conversed with the individuals. I hope this won't be a challenge in the future.
03/05/2023	I have identified a very reliable Gatekeeper. She is fully aware of my urgency to complete my data collection process in order to meet the deadlines outlined in my district approval letter. Today she has sent out recruitment emails to the principals of the two sites who will forward the recruitment emails to all staff members at their respective schools.
03/07/2023	Today I was able to distribute my acceptance emails and my rejection emails. It is such a relief that I found enough participants willing to complete the study. I've noticed in my class discussion board forums, finding participants is always a challenge. I am grateful for my participants' willingness to render their time and effort. I hope that they complete all tasks outlined in the consent form.
03/13/2023	Today I completed all of my face-to-face interviews. All of the participants were able to give valuable input. On numerous occasions I had to stop myself from making interjections. As a teacher, I had to set my own experiences aside.
03/15/2023	Today I completed the transcription of the individual interviews and sent them out for member checking. The majority of the participants sent them back within the hour. Two of the participants requested another day to complete member checking.
03/16/2023	Today I completed the first focus group session. The participants came to enjoy this more than the individual interview questions. It was a relief that all of the participants were willing to express their views and share their experiences with hardly any prompts given. They seem to be comfortable in their setting and were very candid with their answers. Today I also completed the second focus group interview session. The participants in this session were also open to sharing their experiences. It appeared as if they

	were waiting on an opportunity to be able to express their feelings regarding that instructional approaches and the struggles that their students faced. Again, I had to avoid sharing my own experiences, especially when the discussions were getting exciting.
03/17/2023	Today I completed the focus group transcriptions and sent them out to be member checked by the participants. Today I was also able to begin my classroom observations at one of the sites.
03/22/2023	Today I completed the remainder of the classroom observations. The experience was tedious and time consuming; however, I enjoyed every observation and built a relationship with each of the participants. During the observations, I had to keenly watch the participants' social cues. I also had to ensure that my facial expressions and body language appeared neutral.
03/23/2023	Today I compiled the data that I collected and began the data analysis process. As I go through this process, I have to deny the urge to prematurely fit codes into categories/themes based on my personal biases but rather on my participants' experiences.

Appendix L: Emerging Themes and Response Occurrences

MT	***Consequences***						***Deliberate Practice***		***Modeling***		***Targeted Feedback***			***Instructional Changes***	
RQ	*SQ1*						*SQ1*		*SQ2*		*SQ3*			*SQ4*	
Subthemes:	Student Achievement	Intimidation	Student Engagement	Incidental Learning	Digital Inequality	Dependence	Repetition	Constructivism	Peer Teaching	Gamification	Verbal Encouragements	Tangible Rewards	One-on-One Instruction	Differentiated Instruction	Teacher Self-Evaluation
TP															
Avah	x	x	x	x	x	x	x	x	x		x	x	x	x	x
Abigail	x	x				x	x	x	x	x	x	x			x
Aubree	x	x	x	x		x	x	x	x	x	x	x	x	x	
Tanya	x	x	x	x			x	x	x		x	x		x	x
Emily	x	x		x	x	x	x	x	x		x			x	
Faith	x	x	x				x	x	x	x	x	x	x	x	
Rachel	x	x			x		x		x	x	x	x		x	
Patrice	x	x	x	x	x	x	x	x	x	x	x	x			
Nicole	x	x	x		x	x	x		x	x	x	x	x	x	x
Esther	x	x	x		x		x	x	x	x	x	x			x
Latesha	x	x	x	x			x	x	x	x	x				x
Lynette	x	x	x	x		x	x	x	x	x	x	x	x	x	x
Bianca	x	x	x				x	x	x	x	x	x			
Alli	x	x		x		x	x	x	x	x	x	x		x	
Gia	x	x	x	x			x	x	x	x	x	x		x	
	15	15	11	9	6	8	15	13	15	12	15	13	5	10	7
Total:	*64*						*28*		*27*		*33*			*17*	
Key:	RQ = Research Question, SQ = Sub-Question, TP = Teacher Participant, MT = Major Theme														