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Evaluating Prospective Mathematics Teachers' Knowledge Regarding Teaching Strategies Through Teaching Practicum

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Abstract

The aim of this study is to evaluate prospective mathematics teachers' knowledge of teaching strategies on geometric objects by observing their instruction through teaching practicum. Because the observation is of prospective teachers' teaching-strategy knowledge in real life contexts, conducting the study in a classroom environment enables this study to get more valid and reliable results. On this basis, we followed an evaluative case study research-design based a on qualitative research paradigm. The participants were seven elementary prospective mathematics teachers studying at a state university in Turkey. In the data collection process, interviews and observations were used, and the data obtained were analyzed via both descriptive and content analyses techniques. The findings suggest that prospective teachers do not have a wide knowledge on the sort of strategies used in teaching geometric objects; they generally adopted traditional methods, and, due to the abstract nature of geometric objects, their instruction were ineffective at clarifying the dimensions of concept. Besides, it is seen that a real-life environment is beneficial in terms of achieving more realistic and in-depth results.

Keywords: Geometric objects; Learning and teaching; Teaching strategy knowledge; Teaching practicum

Introduction

Mathematics teachers need to be sufficiently prepared, skilled, and flexible in facing the changes of curriculum policies and 21st century teaching strategies (Masri et al., 2021). In their studies about addressing the issues regarding learning and teaching of mathematics in schools, the instructors emphasize the significance of teachers' technical expertise in mathematics in terms of promoting students' learning. However, this expertise is not the only factor for effective instruction (Ball et al., 2005; Loewenberg et al., 2008) because effective instruction includes many steps (Beach et al., 2020). Thus, beyond technical expertise, teachers need to have knowledge on how to design pedagogically driven instructions that are adapted to the level of students. Today, this knowledge is called as "Pedagogical Content Knowledge (PCK)" in the literature (Batur & Balcı 2013; Pitjeng-Mosabala & Rollnick, 2018; Stevens et al., 2009).

Shulman (1986) states that pedagogical content knowledge includes knowledge of students' characteristics, educational contexts, outcomes, aims, values, and their philosophical and historical foundations. An et al. (2004), inspired by Shulman's definition and in hopes of broadening its scope (1986), divided PCK into three categories: content, curriculum, and teaching. In the definition they put, content knowledge is composed of relevant mathematical content knowledge to the grade level as well as global mathematical knowledge. On the other hand, curriculum knowledge is related to the selection and use of appropriate curriculum materials and a thorough understanding of the objectives and keys, while knowledge on teaching covers the awareness of students' thinking and learning styles, and the skill to design instruction on this basis. The schema presented below (see Figure 1) clearly demonstrates these components of An et al.'s (2004) definitions.

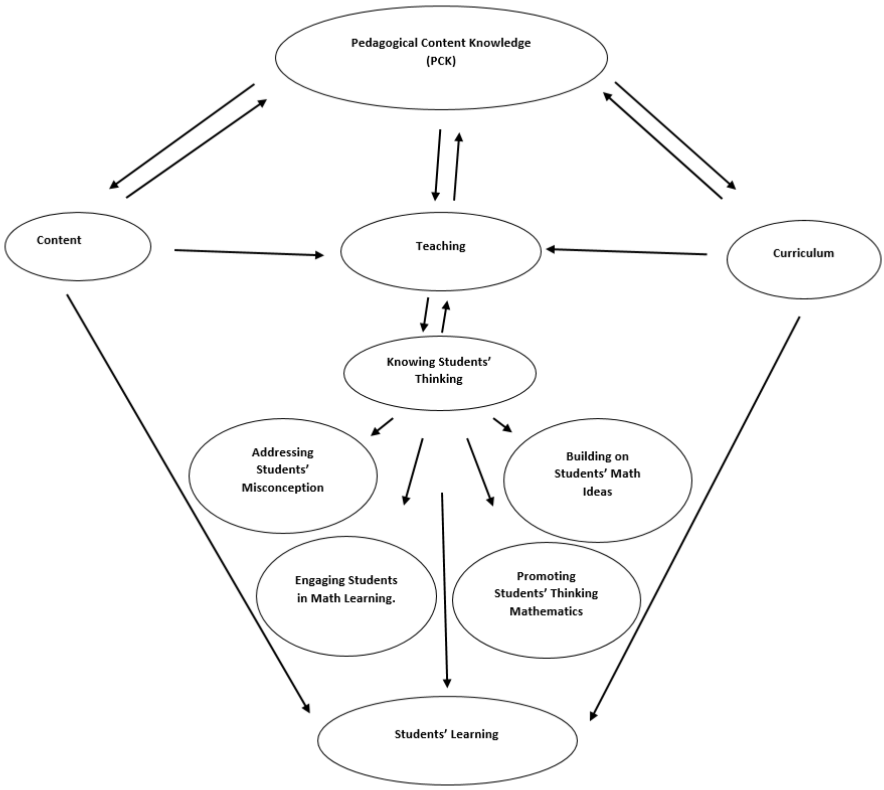


Figure 1. The Network of Pedagogical Content Knowledge

As it is seen in Figure 1, teaching knowledge interacts with curriculum knowledge and content knowledge, and it is basically referred to as the ability to know the students' thinking styles (An et al., 2004). Thus, the skills of identifying students' misconceptions, developing mathematical ideas, incorporating students in the mathematics learning process, and encouraging them to think mathematically are involved within the scope of teaching knowledge. Moving on these definitions, then, it seems that teaching knowledge is a type of knowledge based on a broad understanding of the interaction between the teacher and the students (Loughran et al., 2012).

In this regard, as classroom practice is important for facilitating the development of preservice teachers' teaching knowledge, teaching experience plays a vital role in promoting the development of instruction skills (Öztürk, 2021). Because it is important that a teacher can create an effective learning environment and adapt teaching to pupils (Zuljan, 2016). Without such opportunities, teaching knowledge development is at best, haphazard and at worst, barely apparent (Van Driel & Berry 2010). Therefore, it is essential to observe the prospective teachers' teaching strategy use in a real-life classroom environment in order to get valid results about their teaching strategy knowledge (Hersey, 2018). In the previous studies, it is seen that researchers mostly studied in environments that do not resemble a real-life classroom to evaluate prospective teachers' teaching strategy knowledge (Batur & Balcı, 2013; Creager, 2016; Didiş Kabar & Amaç, 2018; Koçak & Soylu, 2017; Kind & Chan, 2019; Kuennen & Beam, 2020), and the number of studies conducted in real life classroom settings, that is in primary or secondary school environments, is limited (Jackson et al., 2020; Speer et al., 2015; Wang, 2017). The current study differs from these studies from this aspect, for it was carried in real-life school contexts, which are officially operated by the Ministry of Education [MNE] and so it is significant in terms of providing valid and reliable results about the prospective teachers' teaching strategy knowledge. Therefore, this study focused on the evaluation of prospective mathematics' teachers teaching strategy knowledge by observing them in real-life contexts, specifically in teaching geometric objects. Within this framework study, we specifically aimed to evaluate prospective mathematics teachers' knowledge of teaching strategies on geometric objects and their basic elements, characteristics, expansions, surface areas, and volumes by observing their instruction through teaching practicum. The main reason for observing prospective teachers during the instruction of geometric objects is that this subject is one of the most cognitively challenging in mathematics (Horzum, 2017; Keşan & Akbulut, 2019; Kutluca & Baki, 2009; Masri et al., 2021). Geometric objects and the problem experienced in the comprehension of geometric objects is seemingly related to problems in instruction (Demirel et al., 2017; Flores-Bascuñana et al., 2019; Gökkurt, 2014; Hangül, 2010; Lee & Hollebrands, 2008; Toptaş, 2008). Accordingly, it is important to focus on prospective teachers' knowledge of teaching strategies. At this point, (Ministry of Education [MNE], 2018) in secondary school mathematics curriculum, the instruction of the geometric objects and shapes through examples from real life contexts without the use of mathematics jargon is emphasized. Besides, the curriculum emphasizes the need to progress from known to unknown concepts and to understand logical construction by establishing relationships among geometric objects and shapes through modeling. Namely, the aim of the instruction is, on the one hand to equip students with relevant knowledge and skills regarding geometry and, on the other hand, to enhance their of geometrical thinking (Baykul, 2014). Thus, it can be seen that knowledge of teaching strategies plays a significant role in delivering well-developed and outcome-focused instruction. Teachers with this knowledge possess instructional skills such as the ability to complete teaching activities during lessons, plan effectively, and evaluate lessons, which in turn positively affects the students' cognitive learning (Shulman, 1986). In other words, teaching strategy knowledge is directed towards the goal of improving students' learning. In addition, it was stated that there are 96 Education Faculties in Turkey (80 State Universities, 16 Foundation Universities) according to the fortifications of the Council of Higher Education (2023) and that 9% of undergraduate students learned in education faculties. Considering that the prospective teachers studying in education faculties are at a substantial level, it becomes more important to evaluate the teaching strategy knowledge, teaching methods and techniques.

It can be observed that previous research has not addressed whether prospective teachers have relevant instructional skills through observing them in secondary school classroom environments. Moreover, research has not fully considered the prospect of raising issues about the comprehension of geometric objects and the challenges students experience and the role of teaching strategy knowledge in this regard. It would seem, therefore, that further investigations are needed in order to evaluate prospective teachers' teaching strategy knowledge in the instruction of geometric objects by observing them in secondary school level classrooms, so the current study extends the observation of prospective teachers to real-life contexts, to secondary school platforms of (MNE), where there are sixth, seventh, and eight graders having mathematics classes about geometric objects and an effort is made to make valid judgements about prospective teachers' teaching strategy knowledge. This means that the results of this study can be used as a guide in planning activities involving teachers' teaching strategy knowledge in order to improve the quality of mathematics teachers and to ensure the effectiveness of teaching. Therefore, within the scope of this study, the evaluation of prospective teachers' knowledge of teaching strategies on geometric objects is important via the designing of new teaching methods and the development of teaching strategy knowledge in terms of presenting new ideas for teaching geometric objects at the cognitive level through.

Method

In this study, moving on the qualitative paradigm in which the research process is flexible and the data is examined in-depth and explicitly indicated at the end of the research (Kohlbacher 2006), the evaluative case study approach was adopted. Evaluative case studies involve description, explanation, and judgment (Merriam, 1998). Guba and Lincoln (1981) explain that case study is the best reporting form for evaluations (Merriam, 1998). Yin (2017) defines the case study method as the examination of a current phenomenon within its real-life framework without interfering with the circumstances. This is relevant for the current study because we addressed the issue of prospective teachers' teaching strategy knowledge in real-life context without any intervention in the process by just observing. Regarding the evaluative side, Merriam (1998) posits that the evaluation and inferences about the phenomenon or case of interest follow a detailed description and explanation of the relevant phenomenon or case. It is on this basis that we used many data collection tools such as interviews and observations in order to gain deeper insight into teaching strategy knowledge of prospective teachers and to make nuanced inferences.

Participants

The participants of the study are seven prospective mathematics teachers, who are seniors in secondary-school level mathematics teacher training department at a state university in Turkey. These prospective teachers were selected among 4^{th} graders in secondary-school level mathematics teacher training department who are having teaching practicum. Following the criterion sampling logic, these students were randomly involved in the study by considering whether they have all had classes about discipline specific and pedagogy specific knowledge as well as practice because only the students with the relevant knowledge would design and execute an instruction, which will demonstrate if they truly have sufficient teaching strategy knowledge. Before data collection, prospective teachers were informed about the study, and they accepted to participate voluntarily. Demographic information of prospective teachers is given in Table 1.

Table 1. Demographic Information of Prospective Teachers

Prospective Teachers	Gender	Affiliation	Grade Level
T_1	Male	Elementary Mathematics Teaching	4^{th} Grade

T_2	Female	Elementary Mathematics Teaching	4^{th} Grade
T_3	Female	Elementary Mathematics Teaching	4^{th} Grade
T_4	Female	Elementary Mathematics Teaching	4^{th} Grade
T_5	Female	Elementary Mathematics Teaching	4^{th} Grade
T_6	Female	Elementary Mathematics Teaching	4^{th} Grade
T_7	Male	Elementary Mathematics Teaching	4^{th} Grade

Regarding ethical concerns, during reporting the findings, the study used codes such as T_1, T_2, T_3, …, T_7 instead of their names.

Data Collection Tools

Observations and interviews as data collection tools were used sequentially and respectively to collect data because we would like to deepen our understanding about teaching strategy knowledge by digging the data gathered from observation through further interviews with participants.

For the first phase, we observed prospective teachers while instructing geometric objects during teaching practicum by using the observation sheet developed by Gökkurt (2014), the purpose being to examine prospective teachers' teaching strategy knowledge. Gökkurt (2014) explained that in the process of the development of the observation form, she considered the literature review, the definitions of PCK components and the notes during the observation process. It was expressed that expert opinion and pilot application were used for the validity and reliability of the observation form. The observation form created as a result consists of 21 items containing teaching skills (Appendix 1). However, there is a point we remarked while using this sheet; in this sheet, the items, which are believed relevant to the scope of the current study, were considered, and used during observation while the ones unfeasible to use for this study were eliminated as a result of the agreement with an expert in the field. The first item eliminated was preparedness for class because this study was conducted during teaching practicum, which stipulates prospective teachers to do preliminary preparation for instruction. As for the second item, using teaching strategy through research-examination, it was singled out, for the instruction duration was just for 80 minutes in the study, which coincides with only a two-hour class. Accordingly, the prospective teachers were observed on the basis of relevant learning outcomes about geometric objects, covered in the observation sheet, they were expected to teach.

As for the second phase of the study, following the observation, we did interviews with participants about the points, which remained enclosed and so further clarification was required in order to advance the credibility of the findings obtained from observation (Creswell 2015). Therefore, the questions in the interview were structured on the data provided through the observation sheet used. Before interviews, the participants were informed about the sort of questions they would be asked, how many minutes the interview would approximately take and what they would gain from this study. Also, the researchers got participants' permission in terms of the use of a tape recorder to avoid any missing data. Interviews were done in a silent setting to ensure clarity and openness of data collection and each interview took about between five and ten minutes.

Validity, Reliability and Ethics

The validity and reliability of a study substantially depends on research ethics. This is due to the fact that although the principles and guidelines to be followed have already been prepared, the application of ethical rules is closely related to the values and moral understanding of the individual researcher (Merriam, 2009/2015). In this study, many precautions were taken to increase reliability and validity within the framework of ethical reasons. In order to ensure the validity of the study, the data were processed, interpreted and reported in detail by the researcher impartially. In addition, interviews were conducted with the prospective teachers to confirm and interpret the data obtained from the observations. During the interviews, new questions were asked when needed, and the data were made detailed and understandable. In order to ensure the reliability of the study, the researcher first clarified her own role in the research process and thus, it was possible for other researchers doing similar research to assume a similar role and reach comparable results. Similarly, in order to ensure the reliability of the study, the researcher explained in detail the demographics of the participants who were the data source of the study, the research process, data collection and analysis methods, and how the results were combined and presented.

Role of the Researcher

The main of this study was to recognize that each prospective teacher has different teaching strategy knowledge and to discover these strategies. In order to accomplish this goal, the researchers followed a qualitative paradigm. Therefore, different data collection tools such as interviews and observations were used simultaneously, and the data obtained were aggregately handled during analysis. During the observations, as non-participant observers, in order to avoid that any participant would feel under pressure and to confirm they would conduct themselves routinely, the researchers reassured the anonymity of their names.

As for interviews, the researchers held an unbiased position by taking an objective listener role and, so avoided any judgmental discourse. Besides, the interview process was managed in a friendly atmosphere and each participant were allotted time in order to answer questions freely. Thus, thanks to this process management, prospective teachers had room for voicing their opinions.

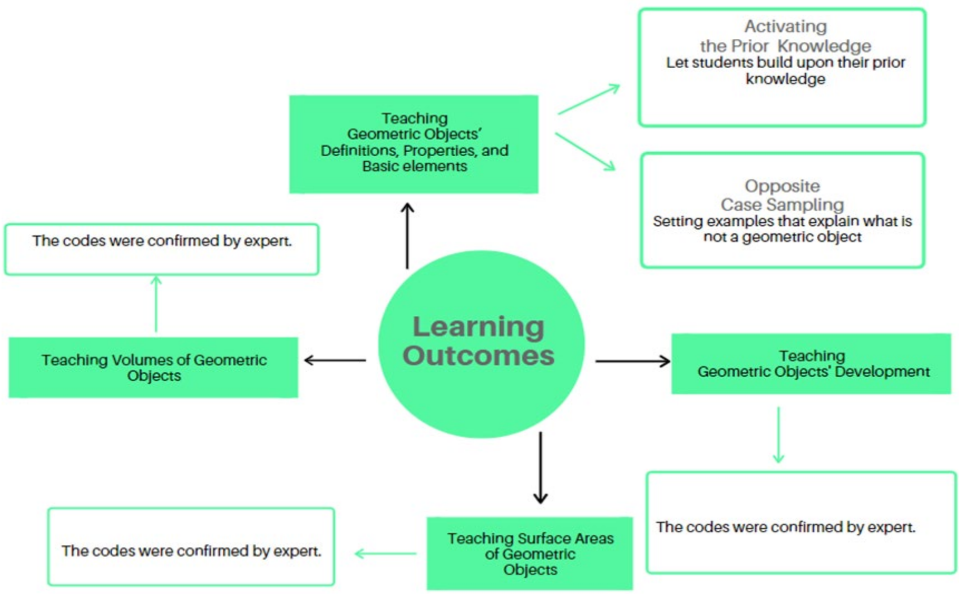
Interpretation of Obtained Data

The data obtained in this study were organized in accordance with the learning outcomes on geometric objects that the prospective teachers were required to teach. In this process, descriptive and content analyses were used sequentially moving on the existing and newly emerging cases.

Building on the nature of descriptive analysis, through which data are studied on the basis of already existed conceptual frameworks and themes (Bogdan & Biklen, 2007; Creswell & Poth, 2016), we examined the data obtained from a semi-structured observation sheet in which the codes relevant to prospective teachers' teaching strategies have been already embedded. In this regard, the data obtained were aligned with the codes in the sheet with a careful study on the thematic content of each data through identification and interpretation. During the descriptive analysis process, we realized there were some data that could not overlap with the codes in the observation grid; thus, we set out to use another data analysis technique, content analysis, to develop new codes which could openly account for these data (Miles et al., 2014). Here, we need to note that the data obtained from interviews, the verbatim records of which were rigorously transcribed, were also worked synchronically during analyses because these data would be explanatory for observations by looking for the rationale behind what prospective teachers did and why. After the labelling process was complete and the initial codes were fixed, we sent the data obtained from transcripts, the observation sheet, and codes to another expert for confirmation. Following consultation with the expert, we decided to add another two codes, and these codes were "activating the prior knowledge" and "opposite case exampling". Then, in order to ensure the reliability of the codes, the inter-rater reliability was checked through percentage agreement, and the overall agreement measured was 70% (McMillan & Schumacher, 2014).

Figure 2 demonstrates in which categories the initial codes were confirmed and used and in which category new code were added besides confirmed ones. The primary aim of this study was to recognize that each prospective teacher has different teaching strategy knowledge and to discover these strategies. In order to make this discovery, the researchers followed a qualitative paradigm.

Figure 2. The Process of Creating Categories and Codes



Findings and Comments

In this section, the findings from observations and interviews were presented as codes and categories, and the explanations about and further insights into these data were provided.

Table 2. The Themes, Categories, and Codes Regarding the Observation Data of Prospective Teachers' Lecturing Geometric Objects' Features and Development

Themes(Outcomes)	Categories (Strategies)	Codes (Techniques)	Prospective Teacher
Teaching Geometric Objects' Definitions, Properties, and Basic elements	Explicit Instruction	Concrete Materials	T_{7}
		Real Life Examples	T_{7}
		Explicit Instruction	$T_{1,7}$
	Implicit Instruction	Question-Answer Technique	$T_{4,5,6}$
		Demonstration Technique	$T_{3,5,6}$
		Information and Communication Technologies	$T_{3,4,5}$
		Activating the Prior Knowledge	$T_{4,5}$
		Concrete Materials	$T_{3,5,6,7}$
		Opposite Case Sampling	T_{2}
		Binding with Real Life Examples	$T_{2,3,4,5,6,7}$
Teaching Geometric Objects Development	Explicit Instruction	Explicit Instruction	$T_{1,7}$
	Implicit Instruction	Question-Answer Technique	T_{5}
		Demonstration Technique	$T_{3,4,6}$
		Information and Communication Technologies	$T_{3,4}$
		Concrete Materials	$T_{2,3,4,6}$

As seen in Table 2, for the explanation of characteristics, basic elements, and the development of geometric objects, five prospective teachers followed implicit instruction as a teaching strategy while two prospective teachers used explicit. Regarding the prospective teachers who adopted explicit instruction, they seemingly preferred to directly choose memorization techniques without any emphasis on the logic of subject. For example, one of the prospective teachers using this technique, T_1, was observed to mention properties, basic elements, and to show development of geometric objects through board drawings. A sample case about this observation in respect to T_1 is provided in Figure 3. Figure 3 clearly shows that T_1 took over the instruction through traditional teaching strategy while referring to definition, properties, and basic elements of right prisms; the prospective teacher drew geometric shapes.

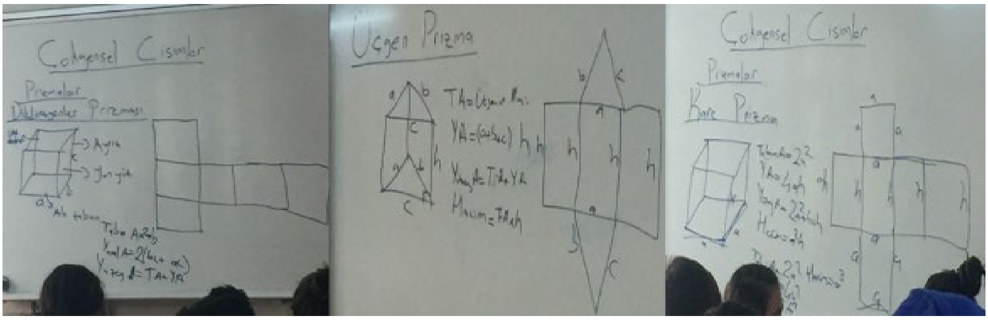


Figure 3.

T_1's examples of instructional explanations on the definition and properties of prisms and their development alongside and presented the formulas of lateral area, surface area, and volume by rote learning. Indeed, it is of importance to provide not only the definition but also the logic of concepts related to geometric objects, which would take 14 hours to explain the topic. Yet, it was seen that T_1 followed an accelerated instruction by lecturing all the dimensions within just two hours, which, as it was expected, caused information overload. Besides, the way the prospective teacher used the blackboard was sloppy; furthermore, it was observed that they could not draw the shapes as required. However, this teaching logic may be demanding especially for students with a spatial-visual intelligence tendency. Indeed, in this kind of instruction, prospective teachers may use tools such as rulers or dynamic geometry software such as GeoGebra for more accurate and intelligible drawings, which would make students more engaged in the topic and contribute to their comprehension of the shapes.

Thus, considering the prospective teacher's instruction performance based on these facts, it can be suggested that the prospective teacher's teaching strategy knowledge is too limited.

T_7 was observed designing their instruction using similar methodology to T_1, T_7, although they tried to provide real-life examples on geometric objects and used concrete materials during lecturing to carry out unsatisfying teaching; T_7 firstly mentioned properties, basic elements, and to show development of geometric objects through board drawings and then showed the concrete material as sample, which may indicate that this prospective teacher adopted a sort of instruction moving from abstract to concrete thinking without caring the basic principles of mathematical instruction.

On the other hand, as seen in Table 2, five prospective teachers tended to use the other instruction methodology, that is, implicit instruction, yet only two of these prospective teachers were seen to carry out their instruction by properly following the framework of implicit teaching methodology. A sample from the observation of one of these prospective teachers using implicit instruction, T_4, who discussed a cylinder in her lecture, is shown in Figure 4.

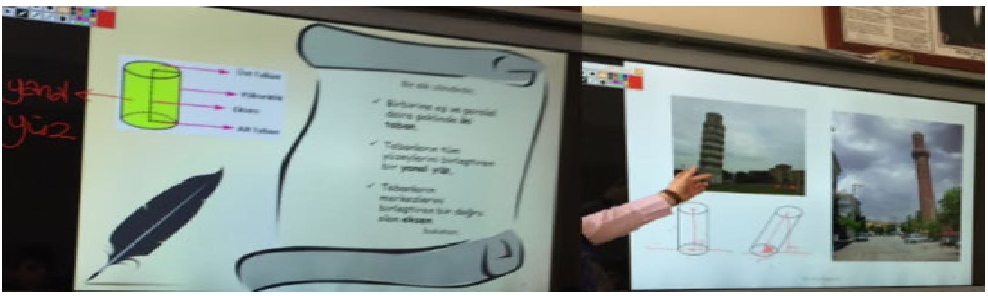


Figure 4. T_4's Examples of Instructional Explanations on the Definition and Properties of Cylinder

Figure 4 clearly shows that T_4, through information and communication technologies (ICT) and real-life examples, explained the definition and the logic of cylinder as a concept; moving onto opposite case sampling by showing the Tower of Pisa as real life example, the prospective teacher emphasized a vertical cylinder with non-circular cross, and, thus, T_4, demonstrated which sort of cases are, in fact, right circular cylinders and which are not. Besides, as it is seen in Figure 5, for expansion of cylinders, T_4 used both concrete materials she developed and ICT simultaneously.

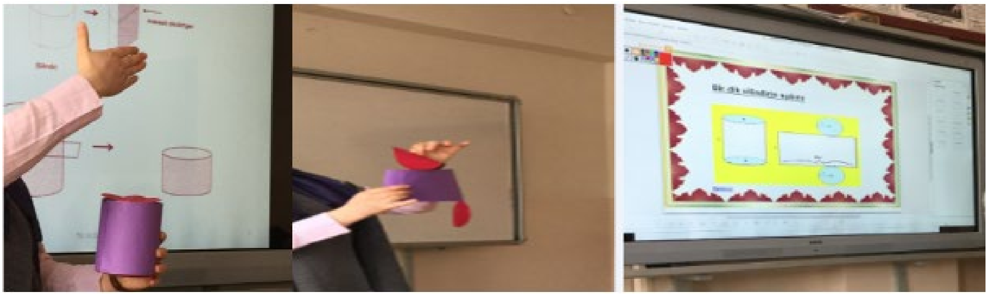


Figure 5. Concrete and ICT Materials Used by T_4 During Instruction of Cylinder Expansion

The materials T_4 developed for expansion of cylinders, as seen in Figure 5, are quite manageable and comprehensible; T_4, firstly, explained cylinders' expansion through concrete materials and then through semi-concrete materials by using ICT. By this instruction, it was observed that the 3D geometric objects, which are challenging abstractions for students to comprehend, could be concretized. Thus, the use of this technique by a prospective teacher may improve students' skills of envisaging the objects.; otherwise, students might have problems in envisaging the close and open shape of the same object. Besides, in order to develop students' envisaging skills, the same material could also have been developed in cooperation with students; thus, the prospective would have made the knowledge permanent through making the right circular cylinder by cooperating with students via the materials such as scissors, paper, and glue. In this regard, the findings seemingly suggest that the teaching methodology preferred by the prospective teacher was effective in making learning permanent and the informative explanations and teaching strategy knowledge the prospective teachers used in teaching definitions, properties, and the expansion of geometric objects were apparently satisfying.

On the other hand, when examining the instruction of the prospective teachers who could not use implicit instruction strategy effectively, it was observed that the students experienced difficulties envisaging 3D objects due to the prospective teachers' instruction style. The extract from the explanations of T_5 during instruction, one of the prospective teachers who could not use implicit instruction strategy, is given below as is:

"Who wants to draw the expansion of the prism (triangle prism) that you see on the board? (no response). Dear friends, try to envisage, we will expand the triangles in this way (pointing up and down with their hand), what is the lateral surface area?"

They made such explanations, but none of the students could draw the correct shape. At this point, in the teaching of 3D geometric objects, this sort of instruction style the prospective teacher followed through the expressions such as "*try to envisage, try to understand*" is generally evaluated as unapplicable because, to the readiness level of the students, it seems difficult for students to understand or envisage 3-D objects without concretization. Therefore, in mathematics, especially in the teaching of geometric objects, the importance of the concretization of objects and the use of concrete materials in this context is clear. In this regard, to choose the right material and to make teaching effective through the material is as significant as to use concrete material in instruction. Otherwise, students may have misconceptions or learning difficulties. The sample extract from the lectures of T_6, one of the prospective teachers, the explanations of whom might have caused misconceptions while using the instruction material, is as below:

(While she was holding a sheet of A4 paper and was folding it, stated that:)

"Dear friends, would this be a cylinder? Yes, it is, for the top and bottom parts, as you see, are two equal circles and there is a rectangle in-between, then this shape is a cylinder."

It is obvious that this instruction type of the prospective teacher, about which characteristics and elements should be looked for a shape to be cylinder, may result in misconstruction of the shape because, due to this lecturing, students, with a probable overgeneralization, may suppose that only the shapes having circular top and bottom parts and a rectangular in-between can be cylinders. Yet, the shape the prospective teacher mentioned is defined as a right circular cylinder; so, in terms of a general cylinder concept, the top and bottom parts do not have to be circle. Moreover, the top and bottom parts of the shape created by curling A4 paper were the circumference, not a circle. Thus, such instruction may cause misconceptions such that students may improperly think that the upright circular cylinder does not need to be closed, or the bottom and top may be open. Such lecturing may be due to the prospective teacher's lack of knowledge on cylinders. In this respect, the explanations of the prospective teachers, while giving real-life examples, are given below.

"The wheel is not a cylinder; it is a circle. Also, the wall clock is not a cylinder because it does not have a height like a pipe."

When these explanations of the prospective teacher upon the real-life examples of right circular cylinder, are considered, it is clearly seen that, for this prospective teacher, the wall clock does not indicate a cylinder and that they seemingly ignored the height of the shapes such as a wheel or a wall clock; therefore, they preferred wrong samples beyond real life. Thus, this suggests that the prospective teacher's field knowledge and accordingly their instructional explanations about the subject were insufficient. Additionally, it was observed that during the instruction about the basic elements of the right circular cylinder, the prospective teacher tended to follow traditional methods by drawing a closed right circular cylinder on the blackboard.

In this context, the prospective teacher obviously adopted a method based entirely on rote learning, which is far from permanent and meaningful learning. In fact, the prospective teacher could have brought concrete material to the class so that the students would observe and notice the basic elements of a cylinder. Alternatively, through dynamic software programs, the prospective teacher could have taught the basic elements and properties of the right circular cylinder on the basis of the open and closed forms of the cylinder, of which the students had difficulty envisaging. For, only such approaches would promote permanent and meaningful learning.

As was the case for T_6, T_3 could also not fully explain the characteristics that a shape should have to be cylindrical, and T_3's instruction was not effective as well. In this regard, the following extracts from the interview with T_3 clearly proves this:

R: Do you think it is a must that the upper and lower parts need to be disk for a shape to be cylindrical?

T_3: No, ma'am, it is not necessary, it might be a circle as well. In this case, it would be a cylinder with no upper or lower parts. Otherwise, if the bottom and upper parts are in other shapes, this is not a cylinder.

R: For example, if the bottom and upper parts are triangles, isn't that a cylinder?

T_3: No, ma'am, this will be triangular prism.

R: Is the prism a cylinder?

T_3: No, a cylinder is a prism. However, not every prism is a cylinder. For a shape to be cylindrical, the upper and lower parts must be circumference or circle.

Moving onto T_3's explanations, it is recognized that there are many problems related to the concept of a cylinder. First of all, T_3 stated that the upper and lower parts should be a circle in order for a shape to be a cylinder. However, for a shape to be a cylinder "*a closed, hollow object with bottom and upper parts, surrounded by a confined surface*" is required; then, it seems that the prospective teacher did not pay attention to the fact that the cylinder is a closed shape, and the explanation provided was wrong. Another point about which the explanation of T_3 was invalid is that not every prism is a cylinder, but the cylinder is a prism. It is known that "cylinders with a polygonal part are called prisms (for example, a cylinder with a rectangular part is called a rectangular prism." (Van de Walle et al., 2010/2013). Regarding this, it can be said that, due to the misconceptions of the prospective teacher, they associate the concept of the cylinder with only the right circular cylinder. Furthermore, it was observed that the prospective teacher made various mistakes and could not adopt an effective strategy while teaching this subject to the students. Within this framework, it might be concluded that, in terms of definitions, properties and basic elements of geometric objects, T_3 's knowledge of teaching is inadequate. Lastly, another concept that the prospective teachers do not understand and teach by adopting rote learning is the concept of edge. On the basis of the interview and observation data, it was observed that the prospective teachers used the concepts of side and edge interchangeably or did not use them in a meaningful way.

Table 3. The Themes, Categories, and Codes Regarding the Observation Data of Prospective Teachers' Lecturing Surface Areas and Volumes of Geometric Objects

Themes (Outcomes)	Categories (Strategies)	Codes (Techniques)	Prospective Teacher
Teaching the surface areas of geometric objects	Explicit Instruction	Concrete Materials	T_7
		Real Life Examples	$T_{2,6,7}$
		Explicit Teaching	$T_{1,2,6,7}$
	Implicit Instruction	Question-Answer Technique	$T_{4,5}$
		Demonstration Technique	$T_{3,5}$
		Information and Communication Technologies	$T_{3,4}$
		Recalling the Prior Knowledge	$T_{4,5}$
		Concrete Materials	$T_{3,5}$
		Binding with Real Life Examples	$T_{3,4,5,}$
		Explicit Teaching	$T_{1,2,6,7}$

	Explicit Instruction	Concrete Materials	T_2
Teaching the volumes of geometric objects		Question-Answer Technique	$T_{4,5}$
		Demonstration Technique	$T_{3,4}$
	Implicit Instruction	Information and Communication Technologies	$T_{3,4}$
		Concrete Materials	$T_{3,4,5}$

As for Table 3 summarizing the observation data of prospective teachers' lecturing the surface areas and volumes of geometric objects, it was seen that prospective teachers generally adopted explicit instruction when teaching surface areas and volumes of geometric objects. In this regard, the observation data showed that they mostly favored lecturing surface area and volume formulas directly to the students through rote learning, and so the content and scope of these concepts were seemingly overlooked. Accordingly, Figure 6 shows sample instruction of T_1 on surface areas and volumes of geometric objects.

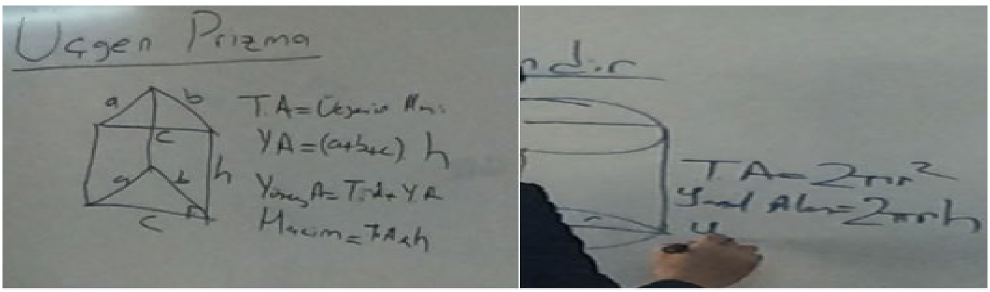


Figure 6. Sample Instruction by T_1 on Surface Areas and Volumes of Geometric Objects

Moving on from this instruction, it is seen that the T_1 wrote down the surface area and lateral area, floor space formulas and volume formulas of geometric objects directly on the blackboard. In this process, T_1 followed a math problem-solving technique by using the formula directly. However, such instruction can make students feel that there are too many meaningless formulas in geometry and lead them to the idea that geometry is an irrelevant course. Therefore, by explaining the content and scope of these concepts to students and establishing a relationship between the formulas, the course can become more enjoyable and meaningful.

The following Figure 7 shows another sample instruction by a different prospective teacher, T_2 who adopted a rote learning again in teaching the volume and surface area of the cylinder as T_1 did.

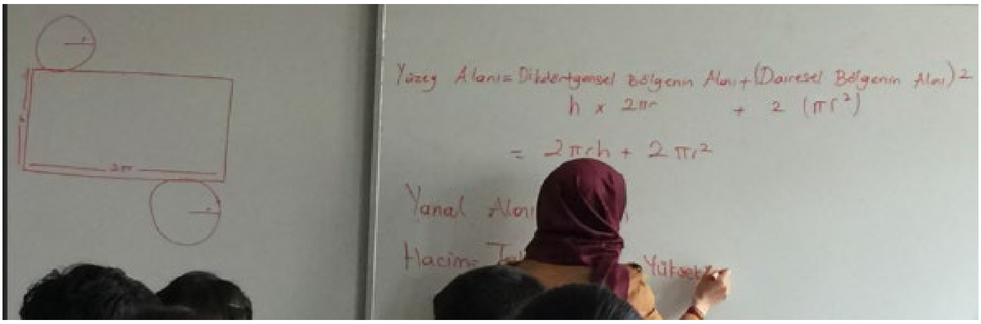


Figure 7. Sample Instruction by T_2 on Volumes of Geometric Objects

In this figure, it was seen that T_2, differently from T_1, taught the concepts of surface area and volume by benefiting from the expansion of the cylinder. However, T_2 wrote down the area formulas of circles in the expansion of right circular cylinder and the area formula of the rectangle on the blackboard; as a result, the expansions of the geometric object in this way had no effect on the meaningful teaching of the concept of surface area. Furthermore, T_2 herself solved the questions or made the students solve the questions by using formulas directly and thus they could not provide further explanation. In this regard, Figure 8 shows T_2 prospective teacher's observation data regarding the sample questions solution.

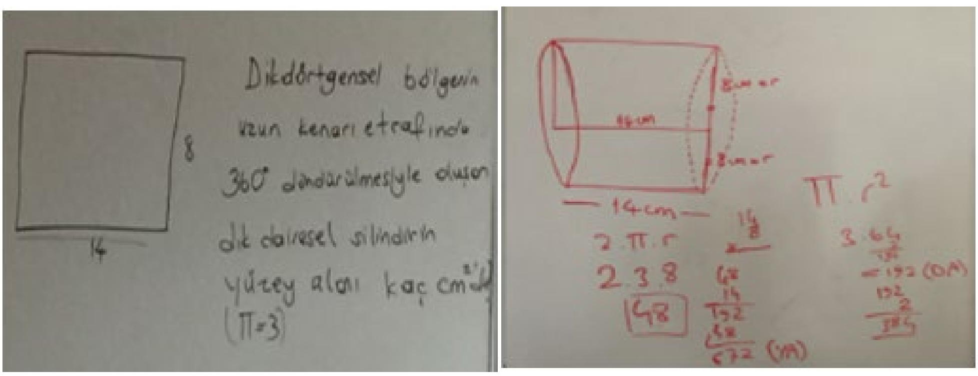


Figure 8. Sample Questions Solution by T_2 on Surface Areas of Geometric Objects

In this sample instruction, T_2 first drew a rectangle and then asked the students to convert this shape into a cylinder and then calculate the surface area of this cylinder. However, students could not do these mathematical operations. Accordingly, T_2 herself, by interfering in the process, transformed the shape into a cylinder and showed the radius and height on the shape. Subsequently, it was seen that the students wanted to go to the blackboard and a student got up to the blackboard and, by following the formula directly, found the proper result. In this regard, it can be stated that the instruction method T_2 followed in teaching surface areas and volumes of geometric objects, was ineffective due to the rote-learning technique through which students could only answer questions by directly following the formula T_2 provided. Also, although, at this stage, T_2 was expected to question the reasons why students could not convert a rectangle into a cylinder and to investigate where they made mistakes and at which point, they could not learn, he did not do this.

At this point, as seen in Figure 7 below, the other prospective teacher, T_3, followed a different technique from T_2, in which, rather than using formulas directly, they preferred to emphasize the characteristics of the cylinder.

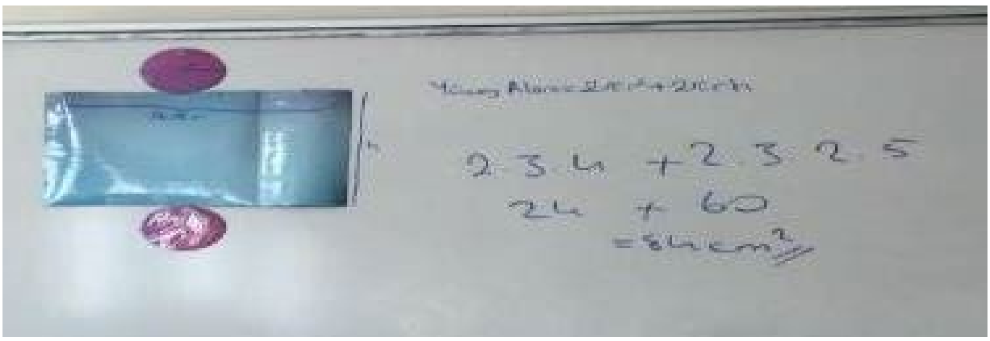


Figure 9. Sample Instruction of T_3 on Surface Areas of Geometric Objects

In this sample, it was seen that in problem-solving the prospective teacher used the expansion of the cylinder at least in the first examples and on this basis solved it by writing the formula of the surface area of the cylinder. In addition, T_3 provided clues for the students, the answers of whom were wrong, guided them, and performed one-on-one instruction where necessary. Furthermore, in teaching the concept of surface area of the cylinder, T_3 underlined the concept of the surface area instead of giving the formula directly and adopted an effective learning strategy and followed similar strategies while lecturing about the concept of volume of the cylinder. Moving on this case, it can be suggested that the T3's strategic knowledge about teaching the concepts of surface area and volume of the cylinder is sufficient. Similar to T_3's instruction, the prospective teacher T_4 also emphasized the meaning of the concepts in the teaching of the concepts of surface area and volume, which is shown in Figure 10.

Figure 10. T_4's Sample Instruction on the Surface Area of the Cylinder

At this point, it was observed that T_4 benefited from the material they brought to the classroom related to the concept of surface area. T_4 used this material both to show the closed and open forms of the cylinder and to explain the surface area. T_4's explanations about the surface area using the material shown in Figure 8 are given below.

Prospective Teacher(T): Friends, I've covered the tomato paste box with this cardboard, and now when I open it, I'll calculate how many cartons I used to cover this tomato paste box. (Repetition that the right circular cylinder consists of rectangles and circles) Do we know the long edge of the rectangle?

Students(S): No

T: Does the long edge of the rectangle cover the perimeter of the circle? (Curled the rectangle again.

S: Yes.

T: So what was the perimeter of the circle $2\pi r$ (they said with the students), then the long edge would be $2\pi r$, what was the height h (they said with the students). So what was the area of the circles πr^2 (they said together with the students), as there are two circles it will be $2\pi r^2$ So let's write down what the surface area equals to; It is $2. Base\ Area + \ Lateral\ Area = \ 2\pi r^2 + 2\pi rh$.

When the material the prospective teacher presented and the explanations during teaching made by them were considered, it was recognized that by emphasizing the logic of the surface area through covering the tomato paste box and by opening this shape and calculating the areas one by one with the students, they made the students realize what the concept of surface area is. On this basis, it is understood that they reached the formula with the students. Thus, the prospective teacher seemingly followed a meaningful method in instruction and accentuated the meaning of concepts. As for the question-solving activities of the T_4 followed, it was observed that they did not follow the same technique and, from the very first example, they led the students to use formulas directly, so, they did make calculations by looking at the formulas written on the board and they could not write the formula by themselves. However, if the prospective teacher had followed the method that they adopted in instruction, and also in solving questions, and explained by emphasizing the meaning of the concepts instead of using formulas directly, the students could have learnt the subject more permanently and so they could have formed the formulas related to the surface area without looking at the board; thus, they could have realized the formulas such as lateral area and floor area involved in the surface area by themselves. Yet, due to the method T_4 followed, the students stated they did not know the formula in the questions about the lateral area, and so they could not solve the question. As a result, while prioritizing meaningful learning in the teaching of the subject, T_4 seemingly adopted the use of formulas directly in solving questions.

Besides these, on the other hand, it was discovered that there were five prospective teachers with a lack of knowledge about the concept of volume, and so they could not teach effectively. In fact, the dialogue between the prospective teacher T_5 and the students, during lectures on the topic of volume, is one of the samples clearly proving this knowledge gap:

Prospective Teacher(T): Dear friends, what is volume, how do you explain it?

Students(S): Multiplication of width, length and height

T: Yes, it's true but I want you to put it in a clearer way. For example, (by showing the empty pet bottle to the students) do you think it has volume?

S: No

T: (By showing the plastic bottle filled with water in his hand) does this have a volume?

S: Yes.

T: So, to speak of a full volume, the shape must be completely filled and closed.

Moving on the explanations of T_4 above, the explanations about the meaning of the notion of volume with an emphasis on that the empty plastic bottle does not have volume and that the plastic bottle has to be completely filled with water and be a closed shape in order to have volume are wrong. What is more, for students, these explanations of T_4 might have caused confusion such as the fact that an object should be full of water to speak of its volume, or the empty objects have no volume. In addition, it was also realized that in solving questions about the surface area and volume, T_4 used the formula directly and did not emphasize the meanings of the concepts. Similar to T_5, an example of the question-solution of the prospective teacher T_6, who used formulas directly in solving questions and could not perform effective instruction, is given in Figure 11.

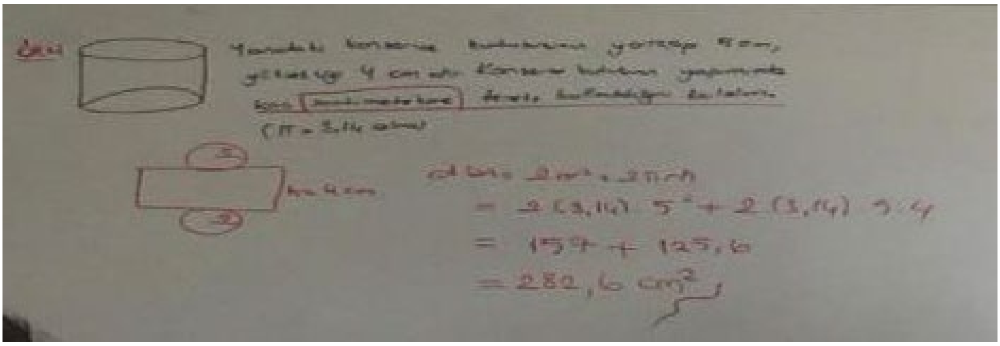


Figure 11. T_6's Sample Instruction on the Surface Area of Geometric Objects.

Moving on this figure, it was seen that T_6 found the surface area of the cylinder by writing the formula directly. Through this strategy T_6 adopted, even if no subject is taught for the students, via rote learning of the formulas, they can reach the correct result. In a system where only formulas are given without the need for further lecturing, it will be inevitable to educate students who do not understand mathematics and do not develop mathematical skills. Therefore, it is required for the prospective teachers to give up their tendency to make students memorize formulas and, instead, adopt effective mathematics teaching strategies.

Overall, the findings of the study regarding teaching strategy knowledge of prospective teachers suggest that their knowledge on the basic properties, expansions, surface areas and volumes of the geometric objects is insufficient. Indeed, it was seen that they have a critical knowledge gap in terms of geometric objects and their scope and so they experienced difficulty in teaching the relevant concepts. Besides, they were observed that, before instruction, they did not do any research on the subjects, for example regarding what sort of mistakes students would make on this topic, what misconceptions that they may fall into, etc.; therefore, the instruction they adopted was an unplanned one that may have led to misconceptions or learning difficulties.

Conclusion and Discussion

It was observed that the prospective teachers tended to use materials and computer technologies in teaching basic elements, properties, and expansions of geometry as well as to present real life samples. However, they could not reflect these techniques effectively in their instruction and continued to utilize a rote learning approach. In the new mathematics curriculum, the use of real-life samples regarding geometric objects is emphasized and widely recommended (MNE, 2018). Yet, in real-life classroom settings, the effectiveness of this method largely depends on the skill of the teachers. In this regard, in the current study, the prospective teachers' real-life samples regarding geometric objects are obviously insufficient. At this point Peterson et al. (1989) emphasize that the teaching knowledge of teachers plays a crucial role in their choice of the classroom activities and in affecting students' learning in the classroom. In this respect, any knowledge gap of the teachers about strategies and techniques to be used in the instruction will directly affect students' academic achievement in school. Indeed, the studies show a significant positive correlation between teacher's knowledge of the teaching strategies and student achievement, with qualified teaching knowledge serving as an important predictor for student's mathematics learning gains (Baumert et al. 2010; Carnoy & Arends, 2012). In the literature, there are also studies that point out the problems of teachers in terms of envisaging geometric objects as concepts (Karakuş, 2018), and, due to this, their inefficacy in choosing and implementing relevant teaching strategies (Gökkurt et al., 2016). This ineffectiveness is considered one of the reasons students struggle to cognitively grasp geometric objects and achieve desired learning outcomes (Gökkurt, 2014; Hangül, 2010; Lee & Hollebrands, 2008; Toptaş, 2008).

The other issue observed in this study is that the prospective teachers used only slides or videos, which for them are the tools of information technologies, and did not benefit from dynamic software such as Geogebra, Mathematica, Cabri etc. In this regard, Van de Walle et al., (2010/2013) emphasizes the importance of using information technologies, especially dynamic geometry software, for promoting students' geometric objects knowledge. In fact, the dynamic geometry softwares increase their self-confidence as they discover new relationships, features, and patterns on geometric structures, and thus meaningful learning could be established rather than memorization (Flores-Bascuñana et al. 2019; Tutak et al., 2015). At this point, the prospective teachers in the current study did not prefer to use dynamic geometry software, and rather followed traditional methods and mostly planned their instruction on the basis of solving math problems. Generally, problem-solving is a one of the commonly used methods in math teaching, but it requires to be done effectively. In this study, some prospective teachers completed lecturing in a short time and immediately passed on problem-solving, before the subject had not been fully understood, and just made problem-solving using the formulas directly. Moreover, it was realized that the prospective teachers were not able to manage the problem-solving activities successfully and some of their explanations might have resulted in misconceptions or learning difficulties. On this basis, these findings of the study are in line with the conclusions of other studies in the literature stating that teachers or prospective teachers could make explanations seemingly causing misconceptions or learning difficulties and this may be due to their own field knowledge, learning difficulties, or misconceptions (Ball et al. 2005; Koçak & Soylu, 2017) because a good teaching strategy knowledge requires good field knowledge first (Loewenberg Ball et al. 2008).

In conclusion, it was seen that, in teaching geometric objects, the teaching strategy knowledge the prospective teachers have is seemingly poor and they generally adopted traditional methods, as a result of which their instruction was not effective. Studies conducted in this context also show that the knowledge of many prospective teachers presented in the lesson plans is quite weak, their pedagogical content knowledge is at a low level and use use traditional methods (Dobrota & Benković, 2014; Krijan, Opić & Rijavec, 2017). There are necessity for development pedagogic content knowledge of propective teachers' if acquisition of knowledge, attitude and values required by teaching profession is intended. Because there are a significant and high-level positive relationship between teachers’ pedagogical competence perceptions and their attitudes about teaching profession (Adıgüzel, 2017).

Recommendations

Qualified teaching strategy knowledge, for the professional development of students can be improved through more systematic and practice-focused teacher training education in teaching practicums so that they can monitor their own development. It is believed that only in real-life classroom settings can prospective teachers engage directly with students, thereby developing a thorough and realistic understanding of teaching strategy. One of the methods for the professional development of prospective teachers is lesson study. According to the literature on lesson studies, teacher’ professional knowledge and self-confidence developed through three stages that include planning, observing both students’ cognitive learning and the teachers themselves during the implementation of the plan, and discussing the plan at the end of class (Akiba et al. 2019; Boran & Tarım, 2018; Dotger & Walsh, 2015; Fernandez & Yoshida, 2004). Therefore, it might be suggested to use methods such as lesson study in the development of the strategic knowledge of prospective teachers. In addition, training guides can be prepared that teachers can use in lessons. These guides may include samples of teaching materials and lesson plans. This can be possible with a project in which both teachers and researchers collaborate.

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Appendix 1. Instructional Strategies Knowledge Observation Form

The purpose of this measurement instruments is to determine which teaching methods, techniques and strategies are used by secondary school mathematics teachers on the subject of geometric objects and the behaviors they exhibit during the lesson.

Observation school : ………………	Observation start time : ………..
Observed teacher : ..……............	Observation finish time: …....….
Observer : …….…...........	Observation class : ……….....
Subject : …………….....	Number of students : ……..….
	Date of observation :………..

Instructional Strategies Knowledge	Number	Target Behavior	Y	P	N	Explanations and Comments
	1	They came prepared for class				
	2	They reminded of the students' preliminary information				
	3	They talked about the importance and justification of the subject.				
	4	They established a relationship between concepts and real-life environment while giving lecture.				
	5	The teaching method they used was appropriate for the course objectives, students' level, number of students and the physical conditions of the class.				
	6	While explaining the subject, they benefited from various course equipment				
	7	They used concrete materials or models while explaining the subject.				
	8	While teaching the lesson, it ensured the continuous participation of the student in the lesson.				
	9	They took into account the principles of teaching mathematics.				

	10	They benefited from various activities while explaining the subject to the students.					
	11	They pointed up different indicators/representations of concepts.					
	12	Using reinforcement.					
	13	They established relationships between concepts and other disciplines (science, social, etc.).					
	14	They used expository teaching technique.					
	15	They used discovery teaching technique.					
	16	They used research and investigation teaching technique.					
	17	They used direct instruction technique.					
	18	They used demonstration technique.					
	19	They used demonstration-perform technique.					
	20	They used question-answer Technique					
	21	Using other teaching methods and techniques (discussion, problem-based teaching, computer-assisted instruction, information technologies such as electronic tools and computers, etc.)					
The meaning of the abbreviations opposite the items in this observation form: (Y): Yes, (P): Partly, (N):No							