Received: 7 February 2022 | Revised: 10 June 2022 | Accepted: 15 June 2022

DOI: 10.1002/rfe.1170

ORIGINAL ARTICLE

REVIEW of FINANCIAL ECONOMICS | THE UNIVERSITY of NEW ORLEANS | WILEY

Betting against sentiment? Seemingly unrelated anomalies and the low-risk effect

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Abstract

The negative CAPM alphas of high-beta and high-variance stocks are attributable to an unaccounted factor in the CAPM. We use eight seemingly unrelated anomalies to construct a composite factor in the spirit of the optimal orthogonal portfolio (FOP). Accounting for FOP re-establishes a positive relation between beta and average returns in time series regressions as well as cross-sectional and explains the negative alphas of high-beta and high-variance stocks. To analyze economic drivers behind FOP, we perform a horse race between leverage constraints, investor sentiment, and disagreement. Our results highlight investor sentiment as the most promising explanation for the low-risk effect.

KEYWORDS

CAPM, low-risk effect, optimal orthogonal portfolio

JEL CLASSIFICATION

G10; G12; G40

1 | INTRODUCTION

Empirical asset pricing provides rich and robust evidence that the relationship between average returns and the two most widely adopted risk measures in finance—market beta and volatility—points in the wrong direction (Baker et al., 2011). This so-called low-risk effect[1] presents a standing challenge to the capital asset pricing model (CAPM) which predicts a positive trade-off between market beta and returns, whereas diversifiable risk such as volatility should yield no significant risk premium at all.[2] While early evidence from Black (1972) traces back fifty years, the seminal papers of Ang et al. (2006) and Frazzini and Pedersen (2014) refueled the debate about the underlying mechanisms of the low-risk effect. Asness et al. (2018) boil down this debate to the two most promising explanations: Systematic risk induced by leverage constraints and idiosyncratic risk due to behavioral biases, for example, a preference for lottery-like returns.

Our paper seeks to resolve this debate and shows that the low-risk effect is both, behaviorally driven *and* attributable to a common systematic factor. To pin down this factor, we introduce the optimal orthogonal portfolio framework of MacKinlay (1995) and MacKinlay and Pastor (2000) who show that mispricing in the investor's factor model due to latent

We thank the editor, Tarun Mukherjee, an anonymous referee, Lena Dräger, Fabian Hollstein, Marcel Prokopzcuk, Jan Krupski, Lezgin Ay, and Michalis Stamatogiannis for valuable comments. We also thank the participants of the EEA Virtual 2020, the FMA Virtual Conference 2020, and the FMA Europe Virtual Conference 2021. We are grateful to Andrey Y. Chen, Kenneth R. French, Robert F. Stambaugh, Jeffrey Wurgler, and Tom Zimmermann for making valuable research data publicly available.

factors is embedded in the covariance matrix of factor model residuals. This way, we can distill the latent factor without conjectures about explicit proxies for it and impartially test theories of the low-risk effect.

Our empirical proxy for the optimal orthogonal portfolio—referred to as *FOP*—explains the negative CAPM alphas of high-beta and high-variance stocks, both in time series regressions as well as cross-sectional. Furthermore, controlling for the exposure to *FOP* re-establishes a significantly positive relation between beta and average returns. Having shown that *FOP* explains the flat and sometimes even negative slope of the empirical security market line (SML), we use *FOP* to challenge theoretical propositions for the low-risk anomaly, namely leverage constraints, investor sentiment, and disagreement. The Baker and Wurgler (2006) investor sentiment index is the most promising state variable behind *FOP* and our results suggest that high-risk stocks earn low returns because of sentiment-driven demand for high-beta and high-variance stocks.

We use the optimal orthogonal portfolio of MacKinlay (1995) to motivate the construction of the composite factor *FOP* from seemingly unrelated anomalies. *FOP* captures unaccounted factors in the CAPM, explains the anomalies in Fama and French (1993, 2015, 2016) with the exception of momentum and net issues, and spans the risk factor models of Fama and French (2018) as well as Stambaugh and Yuan (2017). By construction, *FOP* is uncorrelated to the market portfolio which allows us to extend the CAPM by *FOP* without affecting market beta estimates (β_{Mkt}). The extended CAPM predicts that high-beta and high-volatility stocks exhibit negative exposures to *FOP* which alleviates their negative CAPM alphas.

Once we extend the CAPM by *FOP*, the negative CAPM alphas of high-beta and high-variance (*Var*) stocks in univariate portfolio sorts become insignificant. Returns of β_{Mkt} decile portfolios increase in β_{Mkt} after controlling for the exposure to *FOP*. This result extends to 25 *Size*-β_{Mkt} and 25 *Size*-*Var* portfolios, albeit to a lesser extent. Accounting for the exposure to *FOP* re-establishes a positive trade-off between β_{Mkt} and average returns in Fama and MacBeth (1973) regressions and explains cross-sectional pricing errors of the CAPM. This finding is robust to model misspecification and errors-in-variables.

Turning to the economic explanations for the low-risk effect, we reevaluate the theoretical propositions of Frazzini and Pedersen (2014), Antoniou et al. (2016), and Hong and Sraer (2016), that is, leverage constraints, investor sentiment, and disagreement. More specifically, Frazzini and Pedersen (2014) argue that risky stocks substitute for leveraged investments, leading leverage-constrained investors to bid-up the prices of high-beta stocks. In contrast, Antoniou et al. (2016) and Hong and Sraer (2016) provide evidence that the high-beta stocks are subject to time-varying speculative demand, tantamount to high prices and low future returns. In Antoniou et al. (2016), this speculative demand is driven by sentiment-prone investors, whereas Hong and Sraer (2016) argue that high-beta stocks are sensitive to disagreement about common cash flows.

Despite different theoretical arguments, all three explanations share the common prediction that the slope of the SML is flatter when leverage constraints, sentiment, or disagreement are high due to the aforementioned demand for risky stocks. *FOP* fully resembles this time variation in the slope of the SML, and thus, facilitates an impartial horse race to discriminate between the three competing explanations. Any potential candidate for the low-risk effect should not only affect the slope of the SML, but also explain the time series dynamics of *FOP*. Investor sentiment is the only state variable which consistently satisfies both criteria and turns out to be the most promising candidate to explain the low-risk anomaly.

Our study contributes to three strands in the literature. First and most importantly, we shed further light on the mechanisms behind the low-risk effect. Picking up where Asness et al. (2018) left off the debate, we focus on the controversy between risk-based and behavioral explanations. Since *FOP* is a priori unrelated to the low-risk anomaly, our perspective on the explanation starts purely agnostic. In line with Asness et al. (2018), our results suggest that the low-risk effect is indeed systematic. The exposure to our composite factor *FOP* explains the underperformance of both high-beta and high-variance portfolios. *FOP* serves as a powerful tool to discriminate between so far observationally equivalent predictions of leverage constraints, disagreement and sentiment and our results make a strong case for a common sentiment-based, and thus, behavioral explanation.

In the current literature, papers explaining the low-risk effect with price pressure from demand for lottery-like stocks (e.g., Bali et al., 2017) focus on idiosyncratic risk. To this end, Liu et al. (2018) argue that volatility is the driver behind the anomaly and beta is guilty by correlation. Although high-variance stocks tend to have high market betas, returns significantly increase in market beta after controlling for *FOP*, but not in variance. Risky stocks are likely to be exposed to common but unaccounted factors attributable to investor sentiment—which goes beyond and above correlation between beta and volatility.

Second, there is closely related and growing evidence that investor sentiment affects the aggregate risk–return tradeoff (Antoniou et al., 2016; Shen et al., 2017; Yu & Yuan, 2011). Antoniou et al. (2016) and Shen et al. (2017) both investigate the spreads of market beta sorted portfolios and find that the slope of the SML decreases in sentiment and even becomes negative during periods of high sentiment. Our evidence offers a new perspective on this finding. The seemingly tilted SML is attributable to the negative exposure of risky stocks to the unaccounted factor *FOP* and the average return on *FOP* is higher during periods of high investor sentiment. During these periods, the component in the returns of high-beta and high-variance stocks that is attributable to *FOP* exceeds their expected returns from the CAPM, and the slope of the SML appears to be negative.

Third, empirical asset pricing recently went from a zoo of factors (Cochrane, 2011; Harvey et al., 2016) to a variety of factor models with substantial common ground (Hou et al., 2019). Clearly, the fact that *FOP* explains its constituting anomalies better than alternative factor models has little implication beyond the law of one price (Kozak et al., 2018). However, the explanatory power of *FOP* with respect to the seemingly unrelated low-beta and low-variance anomalies indicates that several characteristics may align with the exposure to a few common factors, as pointed out by Kelly et al. (2019). As Asness et al. (2018) argue, existing factors are correlated with one another or the market portfolio—for example, the Frazzini and Pedersen (2014) betting-against-beta (*BAB*) and the Bali et al. (2017) lottery demand factor (*FMAX*)—which impedes discriminating tests between the factors. *FOP* on the other hand captures only mispricing above and beyond market risk and leaves existing market beta estimates unchanged. Thus, the theoretically motivated factor *FOP* might help separating important from redundant factors without suffering from "guilt by association" (Liu et al., 2018).

2 | THE OPTIMAL ORTHOGONAL PORTFOLIO, SEEMINGLY UNRELATED ANOMALIES, AND THE LOW-RISK EFFECT

2.1 | Introducing the optimal orthogonal portfolio

Our explanation for the low-risk effect relies on unaccounted factors in the CAPM. We treat this factor as a latent variable and propose an empirical approach to the theoretical framework of MacKinlay (1995) and MacKinlay and Pastor (2000) who show that mispricing due to latent factors is embodied in the covariance matrix of factor model residuals. Starting from the CAPM, the excess return $r_{i,t}$ of asset $i \in \{1, \ldots, N\}$ in period t is

$$\begin{gathered} r_{i,t} = \alpha_i + \beta_{Mkt,i} r_{Mkt,t} + \epsilon_{i,t} \\ \mathbb{E}(\epsilon_t) = 0, \quad \mathbb{E}(\epsilon_t \epsilon_t{}') = \Sigma \quad \text{and} \quad \text{Cov}(\epsilon_t, r_{Mkt,t}) = 0 \end{gathered} \tag{1}$$

where $\beta_{Mkt,\,i}$ is the beta of asset i with respect to the market return $r_{Mkt,t}$, $\epsilon_{i,t}$ is the error in each time period, and α_i denotes mispricing. As long as an exact factor which proxies for additional state variable risk is missing in Equation (1), all deviations from the return generating process are embodied in a nonzero intercept α_i. In this case, MacKinlay and Pastor (2000) show that the covariance matrix Σ contains information about the missing factor driving α_i. This relationship can be developed using the optimal orthogonal portfolio (*OP*).[3] *OP* is optimal and orthogonal such that the inclusion of *OP* to the factor model in Equation (1) alleviates the mispricing α_i while preserving the coefficient estimate $\beta_{Mkt,i}$.

We denote the return on *OP* at time t by $r_{OP,t}$ which governs the asset return with sensitivity β_{OP}. Its first two moments are $\mathbb{E}(r_{OP,t}) = \mu_{OP}$ and $\text{var}(r_{OP,t}) = \sigma^2_{OP}$. Per definition, it holds $\text{Cov}(r_{Mkt,t}, r_{OP,t}) = 0$. Replacing α_i in Equation (1) with the return of the optimal orthogonal portfolio yields

$$\begin{gathered} r_{i,t} = \beta_{OP,i}\, r_{OP,t} + \beta_{Mkt,i} r_{Mkt,t} + \nu_{i,t} \\ \mathbb{E}(\nu_t) = 0, \quad \mathbb{E}(\boldsymbol{\nu_t \nu_t}') = \Phi, \quad \text{and} \quad \text{Cov}(\boldsymbol{\nu_t}, \boldsymbol{r_{Mkt,t}}) = \text{Cov}(\boldsymbol{\nu_t}, r_{OP}) = 0 \end{gathered} \tag{2}$$

Taking the unconditional expectations of Equations (1) and (2) leads to

$$\alpha_i = \beta_{OP,i} E(r_{OP}) = \beta_{OP,i} \mu_{OP}. \tag{3}$$

It follows that the variance of the residual in Equation (1) is positively linked to the mispricing vector α according to

$$\Sigma = \beta_{OP} \beta'_{OP} \sigma^2_{OP} + \Phi = \alpha\alpha' \frac{1}{s^2_{OP}} + \Phi, \tag{4}$$

where s_{OP} is the Sharpe ratio of *OP*. In absence of this link, near-arbitrage opportunities arise (MacKinlay & Pastor, 2000, p. 886). Additionally, MacKinlay and Pastor (2000) assume the covariance matrix Φ to be diagonal and proportional to the identity matrix *I*, that is, $\Phi = \sigma^2 I$. Under this so-called strong-form link, MacKinlay and Pastor (2000) propose an active portfolio which alleviates mispricing α in the observed factor model. They further show that the weight vector $\mathbf{w}$ of $N+1$ assets in the active portfolio is

$$\mathbf{w} = c\begin{bmatrix} \alpha \\ -\beta'\alpha \end{bmatrix}, \tag{5}$$

where *c* is a normalizing constant such that portfolio weights add up to one. Thus, the weights of the *N* assets in the active portfolio are proportional to the mispricing vector α (see MacKinlay & Pastor, 2000, p. 891). The weight $-\beta'\alpha$ in the $(N+1)^{th}$ asset, that is, the factor portfolio, guarantees that the active portfolio is orthogonal to the market factor.

2.2 | Tracking down the optimal orthogonal portfolio empirically

To construct an empirical counterpart to the optimal orthogonal portfolio, we employ the active portfolio in Equation (5). More specifically, we follow MacKinlay (1995) and use subsets $S \subset \{1, \dots, N\}$ of the *N* assets. The sample representation of the optimal orthogonal portfolio for a given subset *S* is then

$$FOP_S := \mathbf{w}_\mathbf{s}'[\mathbf{X}_\mathbf{S} \;\; Mkt], \tag{6}$$

where $\mathbf{w}_\mathbf{s}$ is the weight vector in Equation (5), $\mathbf{X}_S$ is a $T \times N$ matrix of returns for the *N* constituent assets, and *Mkt* is a $T \times 1$ vector with returns of the market portfolio, that is, the $(N+1)^{th}$ asset in the active portfolio. We estimate Equation (5) over the full sample period to reduce the measurement error. FOP_S thus represents an *ex-post* estimate for the optimal orthogonal portfolio with respect to the subset *S*.

A formal analysis of the theoretical framework above requires sample assets to construct *FOP* empirically. These sample assets should be informative about CAPM deviations and allow a precise estimation of the weight vector in Equation (5). Anomaly portfolios satisfy these requirements. Anomalies typically refer to patterns in stock returns which are not explained by the CAPM (Fama & French, 1996) and are thus particularly informative with respect to CAPM violations. Furthermore, the portfolios are homogeneous in the characteristics behind the CAPM deviation, thus reducing the measurement error of α.

We use decile portfolios of the following eight anomalies as base assets to construct *FOP*: Accruals (*Accr*), book-to-market (*BM*), investment (*Inv*), momentum (*Mom*), net share issues (*NetIss*), operating profitability (*Prof*), short-term reversal (*ShRev*), and size (*Size*). See Appendix A1 and Fama and French (1993, 2015, 2016) for further details and Section 3 for the motivation of this selection. We treat each set of anomaly decile portfolios as a subset and form eight FOP_S according to Equation (6). To further reduce dimensionality, we form a single composite factor for the optimal orthogonal portfolio, *FOP*, from the sample representations FOP_S. *FOP* is the linear combination of the sample FOP_S which maximizes the Sharpe ratio *s*. We estimate the maximum Sharpe ratio combination under the constraints that all weights are non-negative and add up to one. MacKinlay (1995) argues that for any given subset of S assets, it holds that $s^2_{FOP_S} \leq s^2_{FOP}$, so the maximum Sharpe ratio combination is expected to be a reasonable proxy for the optimal orthogonal portfolio. The linear combination with the highest Sharpe ratio is referred to as *FOP*.

2.3 | Seemingly unrelated anomalies and the low-risk effect

Now that we are equipped with an empirical measure for unaccounted factors in the CAPM, we can assemble the pieces in the novel context of the low-risk anomaly. In the first part of the paper, we take the source of the unaccounted factors as exogenous and focus on the asset pricing implications of the two-factor model in Equation (2). The second part is devoted to a search for the main drivers behind *FOP*. We facilitate the first part of the analysis in two testable predictions.

First, as an empirical counterpart of the optimal orthogonal portfolio, *FOP* is expected to embody all relevant asset pricing information for a given set of test assets (Asgharian, 2011). *FOP* should therefore not only explain the constituent anomalies, but also unrelated anomalies and ideally span multi factor models which rely on related anomalies, for example, the factor models in Fama and French (2015, 2018) and Stambaugh and Yuan (2017).

Second, turning to the low-risk effect, Equations (3) and (4) point out how unaccounted factors in the CAPM might explain the underperformance of risky stocks. For the negative CAPM alphas of high-beta stocks, the prediction of the two-factor model is straightforward. Since the expected return of *FOP* is positive by construction, Equation (3) implies negative β_{FOP} for high-beta stocks. To alleviate the low-beta anomaly, this exposure should account for the negative alphas of high-beta stocks and furthermore re-establish a positive trade-off between β_{Mkt} and average returns.

Implications for volatility as a risk measure are less obvious. To illustrate this, reconsider Equation (4) without further restrictions on the covariance matrix Φ. In this case, the diagonal elements in Φ vary across assets. Taking a closer look at diagonal element *i* of the matrix Σ yields

$$\sigma_{\epsilon,i}^2 = \beta_{OP,i}^2 \sigma_{OP}^2 + \sigma_{\nu,i}^2, \tag{7}$$

where $\beta_{OP,i}^2 \sigma_{OP}^2$ reflects systematic deviations from the return generating process due to the latent factor r_{OP} and $\sigma_{\nu,i}^2$ is *truly* non-systematic. Thus, if the investor's factor model is misspecified, that is, the latent factor *OP* is missing, the resulting measure for idiosyncratic risk depends on the asset's beta with respect to the latent factor (Chen et al., 2012). This component prevents the diversification of idiosyncratic risk to zero when forming a portfolio (MacKinlay, 1995). Thus, the negative CAPM alphas of high-volatility stocks might compensate for unaccounted factors in the initial model. We focus on return variance rather than the more common residual variance to measure idiosyncratic risk because the latter measure is model dependent and usually measured from multifactor models such as the Fama and French (1993) three-factor model. Robustness checks in Section 6, however, illustrate that our results are robust to this choice and *FOP* performs equally well in explaining the most common idiosyncratic volatility proxy proposed by Ang et al. (2006).

3 | DATA AND DESCRIPTIVE STATISTICS

We obtain value-weighted monthly returns of twelve decile portfolios for the following anomalies: accruals (*Accr*), market beta (β_{Mkt}), book-to-market (*BM*), dividend yields (*DivY ld*), investments (*Inv*), long-term reversal (*LRev*), momentum (*Mom*), net share issues (*NetIss*), operating profitability (*Prof*), short-term reversal (*ShRev*), size (*Size*), and return variance (*Var*). The motivation for this selection is unpretentious. We consider all anomalies on Kenneth R. French's website which presumably constitute the core anomalies of the cross-section of returns. Our benchmark model is the CAPM, and we apply the following screening criteria: First, for the value anomaly, we focus on book-to-market equity sorts as the most prominent measure of this anomaly and eliminate sorts based on earnings/price and cash flow/price for collinearity reasons (see Fama & French, 1996, p. 82). Second and relatedly, we drop sorts which do not constitute an anomaly, because the CAPM is not rejected at the 10% level in Gibbons et al. (1989) (GRS) tests. This holds true for *DivY ld* (*p*-value = .1677) and *LRev* (*p*-value = .1358). Third, we use return variance instead of residual variance because the latter is model dependent. Furthermore, we obtain 25 portfolios sorted by size and market beta (*Size*-β_{Mkt}, 5 × 5) and size and return variance (*Size*-*Var*, 5 × 5). We provide a detailed description with respect to portfolio formation in Appendix A1 and refer to Fama and French (2015, 2016) for further information.

The aggregate market return is proxied by the market factor *Mkt* which is the value-weighted excess return of all stocks in the CRSP universe. Moreover, we employ risk factors based on Fama and French (1993, 2015, 2018). *SMB* and *HML* are the Small-minus-Big and the High-minus-Low factors of the Fama and French (1993) three-factor model (*FF*3). Fama and French (2015) extend this model by the profitability factor *RMW* (Robust-minus-Weak) and the investment factor *CMA* (Conservative-minus-Aggressive) to form the five-factor model *FF*5. Most recently, Fama and French (2018) add the Carhart (1997) momentum factor *UMD* (Up-minus-Down) to constitute the six-factor model *FF*6.[4] All of the above data is from Kenneth French's website.[5]

The risk factors of the Stambaugh and Yuan (2017) mispricing model *M*4 are from the website of Robert F. Stambaugh.[6] The *M*4 model comprises the market *Mkt*, a size factor SMB_{M4}, and the mispricing factors *PERF* and *MGMT* which are formed on anomaly portfolios. As a main difference to the traditional Fama and French (1993) methodology, Stambaugh and Yuan (2017) use 20/80 breakpoints to form high minus low portfolios.

Our robustness checks in Section 6 challenge the explanatory power of *FOP* for the low-risk effect by using alternative sort variables and proxies for risky stocks. Those variables comprise alternative beta sorts following Frazzini and Pedersen (2014) and Dimson (1979) and idiosyncratic volatility as proposed by Ang et al. (2006). Stock characteristics for these sorts are from open source asset pricing (see Chen & Zimmermann, 2021).[7] We explain the alternative sort variables as well as the sorting procedure in further detail in Appendix A2. Individual stock returns and market capitalizations for the alternative sorts are from CRSP.

Other economic data are from common sources: The University of Michigan Index of Consumer Sentiment is from the University of Michigan website and Baker and Wurgler (2006) (BW) sentiment data are from Jeffrey Wurgler's website.[8] The TED spread is from the Federal Reserve Bank of St. Louis. Margin debt of NYSE customers is from Datastream (series USCBDMGNA) and NYSE market capitalization is from CRSP. Note that the margin debt series is discontinued as of the end of 2017. Disagreement as the standard deviation of analysts' long-term EPS growth forecasts (series LTSD) is also from Datastream. In constructing aggregate disagreement, we follow Hong and Sraer (2016) and weight the standard deviation of individual stocks by the pre-ranking market beta. Betas are estimated over the previous five years with monthly return data from Datastream as well. We follow the screening procedures of Ince and Porter (2006) to prepare Datastream data for the beta estimation. We thank our fellow colleagues for the provision of the research data.

To have a first look at the data, Panel A of Table 1 presents summary statistics, that is, mean, standard deviation as well as the 25%, 50% (median), and 75% quantile of the respective variable. Panel B presents correlations. Anomaly returns are the long-short portfolio returns of the highest minus the lowest decile of the sorting variable under consideration and are reported in percent per month. We include the baseline anomalies as well as the alternative variables for market beta and volatility.

Average returns of the long-short portfolios vary from −0.51% for *IVol* to 1.17% in the case of *Mom*. For our main variables of interest, the average long-short return of 0.15% is slightly positive for β_{Mkt} and highly negative for *Var* (−0.47%). The alternative market beta sorts, β_{Mkt}^{Dimson} and β_{Mkt}^{FP}, produce slightly negative return spreads of −0.07% and −0.18%, respectively. The correlations in Panel B indicate a highly positive correlation coefficient between β_{Mkt} and *Var* (0.84), in line with the findings of Liu et al. (2018). Likewise, both the returns of all beta estimates (0.58, 0.67, and 0.85) and the two volatility measures, *Var* and *IVol*, are highly positively correlated (0.93). With respect to economic variables, we find that all behavioral variables are positively related. However, even the highest correlation between Consumer Confidence and BW Sentiment amounts to only 0.32. The TED spread and margin debt, on the contrary, exhibit a slightly negative correlation of −0.08. In general, the correlation across economic variables does not raise any concerns about collinearity issues for the joint regressions in Section 5.2.

4 | EXPLAINING THE LOW-RISK EFFECT

4.1 | Seemingly unrelated anomalies and the optimal orthogonal portfolio

The first prediction states that *FOP* embodies all relevant asset pricing information for the given set of test assets. To illustrate that this prediction does not hold for the benchmark model, we start with the CAPM. Panel A of Table 2 presents results of the Gibbons et al. (1989) GRS test for the null hypothesis that the CAPM alphas of the decile anomaly portfolios are jointly equal to zero. We present the test statistic as well as a *p*-value for each of the ten anomalies. The sample period is July 1963 to December 2021.

The GRS test rejects the null hypothesis for each and every anomaly at the 10% level. This also holds true for β_{Mkt} and *Var* portfolios, indicating the existence of the low-risk effect in our sample. For β_{Mkt}, the GRS test rejects the null hypothesis at the 5% level with a *p*-value of 0.0357. The GRS test statistic for *Var* portfolios is more than twice as high, tantamount to a rejection of the null hypothesis at any conventional level. Consequently, the CAPM fails to price all anomalies and we compute FOP_S for the full set of anomalies according to the procedure in Section 2.2.

Panel B presents descriptive statistics for each FOP_S. We provide average monthly excess returns in %, a *t*-statistic for the null that this excess return equals zero as well as monthly standard deviations and an annualized Sharpe ratio. Unless stated otherwise, *t*-statistics throughout this paper are computed from Newey and West (1987) standard errors with six lags.

With the exception of *Size* with an average return of roughly 4 basis points (bps) and a *t*-statistic of 1.14, the average returns are statistically significant. These anomaly returns provide significant information after accounting for market risk. Significant average returns vary from 10 bps for FOP_{BM} to 155 bps for FOP_{ShRev} with *t*-statistics of 2.72 and 3.64, respectively. $FOP_{\beta_{Mkt}}$ and FOP_{Var} earn average returns of 47 bps and 66 bps with *t*-statistics of 2.67 and 4.59.

We use the full set of sample FOP_S except β_{Mkt} and *Var* to form a single factor representation as the linear combination which maximizes the Sharpe ratio. Since the FOP_S are zero investment portfolios, we form *FOP* with long-only portfolio

TABLE 1 Summary statistics and correlations

	β_{Mkt}	*Var*	*Accr*	*BM*	*Inv*	*Mom*	*NetIss*	*Prof*	*ShRev*	*Size*	β_{Mkt}^{Dimson}	β_{Mkt}^{FP}	*IVol*	Margin Debt	TED	Cons. Confidence	BW sentiment	Dis-agreement
Panel A: Descriptive statistics																		
Mean	0.151	−0.467	−0.336	0.357	−0.369	1.174	−0.412	0.252	−0.338	−0.250	−0.066	−0.181	−0.507	0.002	0.578	86.075	0.000	3.633
Standard dev.	6.580	7.909	2.784	4.775	3.248	7.173	3.181	4.198	5.538	4.817	5.232	7.228	6.964	0.032	0.433	12.690	1.000	0.725
25% quantile	−3.677	−5.010	−1.878	−2.478	−2.147	−1.500	−2.390	−2.082	−2.685	−2.755	−2.369	−4.283	−4.040	−0.022	0.268	76.375	−0.431	3.026
Median	−0.035	−0.775	−0.275	0.065	−0.335	1.650	−0.255	0.370	−0.280	−0.050	−0.209	−0.287	−0.717	0.002	0.460	89.600	0.024	3.574
75% quantile	4.022	3.545	1.200	3.173	1.768	4.970	1.528	2.470	2.487	2.692	2.302	4.123	2.674	0.031	0.720	94.800	0.534	4.017
Panel B: Correlations																		
Var	0.844																	
Accr	0.190	0.118																
BM	0.128	0.154	−0.013															
Inv	0.248	0.206	0.063	−0.456														
Mom	−0.244	−0.259	−0.175	−0.272	−0.003													
NetIss	0.601	0.609	0.101	0.093	0.314	−0.136												
Prof	−0.586	−0.687	0.040	−0.306	−0.001	0.145	−0.520											
ShRev	−0.291	−0.285	−0.025	−0.153	−0.085	0.411	−0.167	0.171										
Size	−0.531	−0.590	−0.137	−0.432	0.156	0.133	−0.456	0.539	0.168									
β_{Mkt}^{Dimson}	0.580	0.653	0.053	0.081	0.232	−0.232	0.436	−0.421	−0.286	−0.234								
β_{Mkt}^{FP}	0.845	0.816	0.153	0.138	0.262	−0.318	0.480	−0.480	−0.311	−0.354	0.674							
IVol	0.792	0.927	0.130	0.167	0.202	−0.272	0.616	−0.671	−0.247	−0.629	0.587	0.719						
Margin Debt	0.071	0.035	0.034	0.061	−0.053	−0.016	0.013	−0.042	−0.048	−0.032	0.005	0.087	0.030					
TED	−0.108	−0.074	0.027	−0.086	0.068	0.087	0.002	0.054	0.109	0.169	−0.058	−0.068	−0.094	−0.080				
Cons. Confidence	−0.068	0.003	−0.085	0.060	−0.031	0.046	−0.093	0.034	−0.013	0.033	0.027	0.023	−0.022	0.134	0.018			
BW Sentiment	−0.149	−0.174	0.004	0.006	−0.077	0.015	−0.207	0.176	0.039	0.108	−0.109	−0.131	−0.190	0.096	0.029	0.315		
Disagreement	−0.037	0.004	−0.071	0.093	−0.163	−0.037	−0.153	0.007	−0.084	−0.121	−0.043	−0.012	0.003	0.058	−0.275	0.179	0.108	1.00

Note: Summary statistics (Panel A) and correlations (Panel B) for our main variables. We cover the following anomalies: Market beta (β_{Mkt}), return variance (*Var*), accruals (*Accr*), book-to-market (*BM*), investments (*Inv*), momentum (*Mom*), net share issues (*NetIss*), operating profitability (*Prof*), short-term reversal (*ShRev*), size (*Size*), Dimson (1979) market beta (β_{Mkt}^{Dimson}), Frazzini and Pedersen (2014) market beta (β_{Mkt}^{FP}), and idiosyncratic volatility (*IVol*). Anomaly returns are the long-short portfolio returns of the highest minus the lowest decile of the sorting variable under consideration and are reported in percent per month. Economic state variables are as follows: Margin debt of NYSE customers in relation to NYSE market capitalization (margin debt), the TED spread (TED), the Baker and Wurgler (2006) Investor Sentiment Index (BW Sentiment), the University of Michigan Consumer Confidence Index (Cons. Confidence), and aggregate disagreement disagreement. The sample period for all anomalies under consideration is from July 1963 to December 2021. In case of economic variables, the sample period is from 1967 to 2017 for margin debt, 1986 to 2021 for the TED spread, 1965 to 2018 for BW Sentiment, 1978 to 2021 for Consumer Confidence, and 1982 to 2021 for disagreement.

TABLE 2 CAPM deviations and the two-factor model

	β_{Mkt}	*Var*	*Accr*	*BM*	*Inv*	*Mom*	*NetIss*	*Prof*	*ShRev*	*Size*
Panel A: GRS test for null hypothesis that all CAPM alphas are zero										
GRS statistic	1.9535	4.1594	3.0513	1.8361	3.7354	5.2219	5.1419	2.6236	1.7415	1.7771
p-value	.0357	<.001	<.001	.0513	<.001	<.001	<.001	.0039	.0680	.0612
N	10	10	10	10	10	10	10	10	10	10
Panel B: Average return of anomaly FOP_s										
Excess return in %	0.4718	0.6586	0.3313	0.1045	0.2257	1.0245	0.3226	0.4999	1.5501	0.0446
	(2.67)	(4.59)	(5.00)	(2.72)	(4.98)	(4.79)	(6.76)	(4.00)	(3.64)	(1.14)
SD in %	4.6774	3.8017	1.7553	1.0163	1.1996	5.6695	1.2641	3.3111	11.2971	1.0362
Sharpe ratio (p.a.)	0.3494	0.6002	0.6538	0.3562	0.6517	0.6259	0.8839	0.5230	0.4753	0.1492
Panel C: Weight in FOP										
Weight in %	–	–	17.71	3.69	3.55	2.76	18.00	7.25	1.37	45.68
Panel D: GRS test for null hypothesis that all alphas from selected models are zero										
Mkt+FOP										
GRS statistic	1.3606	1.0553	0.7167	0.9332	1.0117	2.3458	1.9148	0.9604	0.3121	0.6777
p-value	.1944	.3950	.7092	.5017	.4317	.0100	.0403	.4769	.9782	.7458
*FF*6 model										
GRS statistic	1.6930	2.0255	3.2389	1.1913	2.2307	3.3743	4.4978	1.3328	0.9716	1.7129
p-value	.0784	.0285	<.001	.2932	.0147	<.001	<.001	.2086	.4669	.0740
*M*4 model										
GRS statistic	1.2399	2.3128	2.2790	0.4491	0.9010	2.4413	2.1487	0.9380	1.4169	1.8912
p-value	.2619	.0113	.0126	.9218	.5319	.0073	.0193	.4974	.1685	.0435

Note: This table compares the CAPM with the two-factor model. The two-factor model extends the CAPM by the empirical factor for the optimal orthogonal portfolio *FOP*. Panel A presents Gibbons et al. (1989) (GRS) test statistics and the corresponding *p*-values for the null hypothesis that all CAPM alphas of the decile anomalies are zero. Panel B presents monthly excess returns (in %), *t*-statistics for the null hypothesis that excess returns are zero, monthly standard deviations in % as well as annualized Sharpe ratios for FOP_S from the anomaly portfolios. *t*-statistics in parentheses are computed from Newey and West (1987) with six lags. Panel C presents the weights of the respective FOP_S representations in the final *FOP* factor, note that β_{Mkt} and *Var* are excluded from the construction. Panel D repeats the GRS test for selected multifactor models: The two-factor model *Mkt+FOP*, the Fama and French (2018) six-factor model *FF*6 and the Stambaugh and Yuan (2017) four-factor model *M*4. The following anomalies are covered: Market beta (β_{Mkt}), return variance (*Var*), accruals (*Accr*), book-to-market (*BM*), investments (*Inv*), momentum (*Mom*), net share issues (*NetIss*), operating profitability (*Prof*), short-term reversal (*ShRev*), and size (*Size*). The sample period is July 1963 to December 2021 except for the *M*4 model which ends in December 2016.

weights. The exclusion of β_{Mkt} and *Var* guarantees that *FOP* is a priori unrelated to the low-risk effect. Panel C presents the weights in *FOP* which maximize its Sharpe ratio. Somewhat interestingly, *Size* attains the largest fraction in *FOP* with a weight of roughly 46%, followed by *NetIss* and *Accr* with both roughly 18%. Other than that, weights in *FOP* are rather balanced. Despite the high individual Sharpe ratio, *Mom* attains a relatively low weight of roughly 3%. The average monthly excess return of *FOP* is 23 bps with a Newey and West (1987) *t*-statistic of 9.47. *FOP* exhibits an annualized Sharpe ratio of 1.50, which is, by construction, higher than each of its subsample counterparts in Panel B of Table 2.

In Panel D of Table 2, we present the ability of *FOP* to explain its constituent anomaly portfolios as well as the two low-risk anomaly portfolios. Again, we present GRS test statistics and *p*-values for the null that all anomaly alphas are zero. With *p*-values of .1944 and .3950, respectively, the null hypothesis cannot be rejected for β_{Mkt} and *Var* portfolios, even though both anomalies have no part in the construction of *FOP*. Except for *Mom* and *NetIss*, this finding also extends to the other anomalies. In both cases, the GRS test rejects the null hypothesis for the two-factor model at the 5% level, which is nevertheless a substantial improvement over the CAPM in Panel A.

To put the negative result in the cases of *Mom* and *NetIss* into perspective, we report GRS test statistics and *p*-values for the factor models *FF*6 and *M*4 as well. It is worth pointing out that both models specifically account for *Mom*. In case of the *FF*6 model, the null hypothesis is rejected at any level for both anomalies. The *M*4 model performs better, but the GRS test still rejects the null hypothesis at the 1% level for *MOM* (*p*-value = .0073) and the 5% level for *NetIss* (*p*-value = .0193). Furthermore, both *FF*6 and *M*4 perform worse in explaining the low-risk effect. For the *FF*6 model, the null hypothesis is rejected for both, β_{Mkt} (*p*-value = .0784) and *Var* (*p*-value = .0285). The *M*4 model explains alphas of the β_{Mkt} decile portfolios, but not those of *Var*, where the null hypothesis is rejected at the 5% level (p-value = .0113). Although this is not a fair comparison since *FOP* is constructed from an ex-post perspective, this result illustrates that *FOP* captures the anomalies under consideration very well.

To further emphasize the latter finding, Table 3 presents spanning regressions which are less sensitive to the choice of test assets (see Barillas & Shanken, 2017; Hou et al., 2019).[9] In Panel A of Table 3, we regress risk factors of the factor models *FF*6 and *M*4 on *FOP* to evaluate the factor's alphas. We follow Stambaugh and Yuan (2017) and focus on unique factors, that is, we do not include *Mkt*, to analyze whether *FOP* subsumes the asset pricing qualities of multifactor models. We report coefficient estimates as well as t-statistics from Newey and West (1987) standard errors in parentheses.

The first Column, with *Mkt* as the dependent variable, indicates that *FOP* and *Mkt* are unrelated in statistical terms. The coefficient on *FOP* is close to zero and insignificant (t-statistic ≈ 0). This finding is in line with the orthogonality condition of the optimal portfolio and illustrates that *FOP* and *Mkt* are uncorrelated. We find quite the opposite for the *FF*6 factors *SMB*, *HML*, *RMW*, *CMA*, and *UMD*. All factor alphas are statistically insignificant. The GRS test for the joint alphas of the factor models *FF*3 and *FF*6 in Panel B does not reject the null hypothesis that all alphas are jointly zero. The p-values are .7326 and .9820, respectively. *FOP* consistently spans the *FF*6 risk factors and represents a reasonable univariate representation of the multifactor models in Fama and French (1993) and Fama and French (2018). This finding extends to the *M*4 model, however, to a lesser extent. *FOP* spans the mispricing factors *PERF* and *MGMT*, but not the *M*4 counterpart of the *SMB* factor. The difference between *SMB* and SMB_{M4} is due to different breakpoints in the portfolio formation. Nevertheless, the GRS test in Panel B does not reject the null hypothesis at the 5% level (p-value = .0726). We conclude that *FOP* does not only explain its constituent anomaly portfolios, but also subsumes the largest part of the information in the multifactor models *FF*6 and *M*4. *FOP* is a reasonable single factor representation of the multifactor models and embodies all important information for the set of test assets.

4.2 | Explaining the low-risk effect in time series regressions

Having shown that *FOP* satisfies the theoretical properties of the optimal orthogonal portfolio, we can turn to the performance of *FOP* in the context of the low-risk anomaly. The second prediction postulates that the inclusion of *FOP* into the CAPM alleviates the negative alphas of high-beta and high-volatility stocks.

Table 4 revisits the single sorted β_{Mkt} and *Var* portfolios in further detail and presents unadjusted monthly excess returns, alphas of several risk factor combinations, as well as the exposure of each decile with respect to the two factors *Mkt* and *FOP*. Returns and alphas are presented in % per month with Newey and West (1987) adjusted t-statistics in parentheses. Panel A presents β_{Mkt} decile portfolios. The unadjusted excess returns and the CAPM alphas confirm the beta

TABLE 3 Spanning regressions and GRS test

	FF6 Factors						**M4 Factors**		
	Mkt	***SMB***	***HML***	***RMW***	***CMA***	***UMD***	SMB_{M4}	***PERF***	***MGMT***
Panel A: Spanning regressions									
α	0.5892	0.0842	−0.0504	−0.0315	−0.0261	−0.0317	0.2216	0.0076	0.1069
	(3.22)	(0.68)	(−0.44)	(−0.36)	(−0.34)	(−0.20)	(1.81)	(0.05)	(0.93)
FOP	−0.0000	0.6129	1.3758	1.3103	1.2532	2.7968	0.9398	2.7708	1.9639
	(−0.00)	(2.91)	(7.02)	(8.93)	(9.63)	(10.19)	(4.48)	(10.67)	(9.99)
N	702	702	702	702	702	702	642	642	642
Panel B: GRS test for joint alphas of unique factors									
	$\alpha_{SMB} = \alpha_{HML} = 0$		$\alpha_{SMB} = \alpha_{HML} = \alpha_{RMW} = \alpha_{CMA} = \alpha_{UMD} = 0$				$\alpha_{SMB_{M4}} = \alpha_{PERF} = \alpha_{MGMT} = 0$		
GRS	0.3113		0.1432				2.3365		
p-value	.7326		.9820				.0726		

Note: Spanning regressions and Gibbons et al. (1989) (GRS) test results for different asset pricing factors. Panel A presents spanning regressions for the market factor *Mkt* and the asset pricing factors of the Fama and French (2018) six-factor model *FF*6 and the Stambaugh and Yuan (2017) four-factor model *M*4. *Mkt* enters both of the factor models *FF*6 and *M*4. t-statistics in parentheses are computed from Newey and West (1987) adjusted standard errors with six lags. In Panel B, we perform the GRS test for the null hypothesis that the alphas of unique asset pricing factors are jointly zero. We present the GRS test statistic and the corresponding p-value. The sample period is July 1963 to December 2021 except for the *M*4 model which ends in December 2016.

anomaly. The relation between β_{Mkt} and average excess returns is flat with an insignificant difference between high and low β_{Mkt} stocks of roughly 15 bps (t-statistic = 0.61). Controlling for *Mkt* leaves an alpha of approximately −43 bps with a t-statistic of −2.28. In contrast to the predictions of the CAPM, high-β_{Mkt} stocks significantly underperform low-β_{Mkt} stocks after accounting for market risk exposure.

Extending the CAPM by *FOP* alleviates the anomaly. Once we control for the portfolio's exposure with respect to *FOP*, but leave out *Mkt*, alphas increase in β_{Mkt} and the difference portfolio exhibits an alpha of approximately 82 bps with a t-statistic of 3.09. Adding *FOP* to the market factor *Mkt* fully wipes out this unexplained return and the alpha of the difference portfolio (24 bps) becomes insignificant. In line with our second prediction, high-β_{Mkt} stocks have negative exposures to *FOP* and β_{FOP} decreases from low to high β_{Mkt} deciles. Stocks in the high-β_{Mkt} deciles have a significantly negative β_{FOP} of −1.86 (t-statistic = −8.01), while low-β_{Mkt} portfolios exhibit a β_{FOP} of 0.967 (t-statistic = 6.71).

Panel B of Table 4 repeats this analysis for *Var* decile portfolios. The underperformance of high-*Var* deciles is stronger compared with β_{Mkt}. Unadjusted returns decrease from low to high *Var*, but the return of the difference portfolio is insignificant. This lies in stark contrast to the CAPM regressions. Here, high-*Var* stocks earn significantly negative alphas of roughly −90 bps (t-statistic = −4.65). The negative alpha of the difference portfolio of −109 bps is highly significant with a t-statistic of −4.48, even when considering the standards of Harvey et al. (2016).

Including *FOP* alone reveals an interesting pattern. The average alphas slightly increase in the *Var* deciles, but the difference of roughly 50 bps is now insignificant (t-statistic = 1.59). Although the *Var* decile portfolios and the beta sorted portfolios have almost identical β_{Mkt}, the increasing return pattern of the beta portfolios—when controlling for *FOP*—does not extend to the *Var* deciles. Again, combining *Mkt* and *FOP* wipes out unexplained returns in the individual decile portfolios and reduces the alpha of the difference portfolio to −12 bps (t-statistic = −0.50). High-*Var* deciles also exhibit highly negative exposures to *FOP*. While the positive exposures in the lowest decile of 0.9330 are similar to the β_{Mkt}-sorted portfolios, the negative β_{FOP} exposure in the highest decile is more than twice as large as for the top-β_{Mkt} decile.[10]

Next, we extend the set of test assets to double-sorted portfolios. Table 5 presents time series regressions for 25 portfolios sorted by *Size* and β_{Mkt}. Panel A (B) reports unadjusted excess returns (CAPM alphas), with corresponding t-statistics in parentheses, which confirm the results of the univariate decile portfolios.

In Panel A, the relationship between β_{Mkt} and excess returns is flat in each of the *Size* quintiles, whereas CAPM alphas in Panel B decrease from low to high β_{Mkt} quintiles. The beta anomaly persists in double-sorted portfolios. The GRS test rejects the null hypothesis for the CAPM at conventional levels with a test statistic of 2.35 (p-value < .001).

Panel C presents results for the two-factor model which includes *Mkt* and *FOP*. We present alphas as well coefficient estimates for β_{Mkt} and β_{FOP}. The GRS test statistic for the two-factor model amounts to 1.31 with a p-value of .1456, thus not rejecting the null hypothesis that all alphas are jointly zero. In contrast to the CAPM estimates in Panel B, none of the alphas in Panel C is statistically significant. Again, β_{FOP} decreases in β_{Mkt} quintiles and stocks in the highest β_{Mkt} quintiles have negative β_{FOP} except for the smallest quintile. These quintiles largely correspond to the stocks which exhibit negative CAPM alphas in Panel B. β_{FOP} estimates furthermore monotonically decrease from Small to Big quintiles.

Table 6 repeats this analysis for *Size*-*Var* portfolios. In Panel A and B, unadjusted excess returns and CAPM alphas decrease from low to high-*Var* quintiles. The highest *Var* quintiles exhibit significantly negative CAPM alphas over all *Size* quintiles. The strength of this relationship decreases from Small to Big quintiles. Consequently, the GRS test rejects the null in case of the CAPM with a test statistic of 5.279 at all conventional levels (p-value < .001).

The extended CAPM again reduces the mispricing considerably and largely accounts for the negative alphas of the highest *Var* quintiles. The smallest quintile—referred to as the lethal combination (Fama & French, 2016)—is the only exception and alphas still significantly decrease from low to high-*Var* quintiles. Similar to Table 5, portfolios with negative CAPM alphas exhibit negative β_{FOP}. Although *FOP* improves the asset pricing abilities of the CAPM, the GRS test still rejects the null hypothesis with a test statistic of 3.348 (p-value < .001).[11]

The time series regressions make another strong case for the second prediction. Adding *FOP* to the CAPM explains the negative alphas of high-β_{Mkt} and high-*Var* stocks in decile portfolios. As predicted, high-risk portfolios exhibit highly negative exposures with respect to *FOP*. Both findings extend to 25 double-sorted *Size*-β_{Mkt} portfolios, but the negative alphas of small high-*Var* portfolios remain statistically significant. The low-risk effect is likely to arise from unaccounted factors in the CAPM.

TABLE 4 Explaining the returns of decile portfolios

	Low	2	3	4	5	6	7	8	9	High	High-Low
Panel A: β_{Mkt} decile portfolios											
Unadjusted	0.5729 (4.40)	0.5478 (3.84)	0.6211 (4.00)	0.7352 (4.24)	0.6233 (3.45)	0.6928 (3.54)	0.6242 (3.02)	0.7515 (3.30)	0.7320 (2.87)	0.7237 (2.43)	0.1509 (0.61)
CAPM	0.2131 (2.63)	0.1173 (1.60)	0.1227 (1.94)	0.1616 (2.75)	0.0224 (0.38)	0.0450 (0.68)	−0.0609 (−0.88)	0.0052 (0.06)	−0.0892 (−0.88)	−0.2149 (−1.62)	−0.4280 (−2.28)
FOP	0.3460 (2.46)	0.3235 (2.10)	0.4309 (2.56)	0.6344 (3.36)	0.5469 (2.78)	0.7248 (3.39)	0.5979 (2.65)	0.8883 (3.59)	0.9231 (3.33)	1.1610 (3.60)	0.8150 (3.09)
FOP+Mkt	−0.0138 (−0.16)	−0.1071 (−1.39)	−0.0675 (−1.02)	0.0609 (0.96)	−0.0541 (−0.86)	0.0769 (1.07)	−0.0872 (−1.15)	0.1420 (1.62)	0.1020 (0.93)	0.2223 (1.61)	0.2361 (1.22)
β_{Mkt}	0.6106 (34.84)	0.7307 (46.33)	0.8459 (62.28)	0.9734 (74.97)	1.0199 (78.78)	1.0995 (74.49)	1.1627 (75.02)	1.2666 (70.54)	1.3936 (62.28)	1.5931 (56.34)	0.9825 (24.71)
β_{FOP}	0.9668 (6.71)	0.9562 (7.37)	0.8105 (7.25)	0.4295 (4.02)	0.3259 (3.06)	−0.1362 (−1.12)	0.1121 (0.88)	−0.5830 (−3.95)	−0.8147 (−4.43)	−1.8631 (−8.01)	−2.8299 (−8.65)
Panel B: *Var* decile portfolios											
Unadjusted	0.5466 (4.31)	0.6731 (4.57)	0.6526 (4.00)	0.6256 (3.54)	0.6874 (3.66)	0.7883 (3.80)	0.8509 (3.71)	0.7872 (3.10)	0.6450 (2.30)	0.0795 (0.24)	−0.4671 (−1.56)
CAPM	0.1932 (2.49)	0.2026 (3.27)	0.1232 (1.97)	0.0459 (0.72)	0.0688 (1.05)	0.1014 (1.48)	0.0986 (1.20)	−0.0353 (−0.36)	−0.2452 (−2.01)	−0.8950 (−4.65)	−1.0882 (−4.48)
FOP	0.3277 (2.40)	0.4831 (3.02)	0.5103 (2.88)	0.5378 (2.79)	0.6275 (3.06)	0.7841 (3.47)	0.9190 (3.68)	0.9483 (3.43)	0.9524 (3.12)	0.8240 (2.28)	0.4963 (1.59)
FOP+Mkt	−0.0257 (−0.31)	0.0126 (0.19)	−0.0191 (−0.29)	−0.0419 (−0.61)	0.0088 (0.12)	0.0971 (1.30)	0.1667 (1.87)	0.1258 (1.19)	0.0621 (0.48)	−0.1505 (−0.76)	−0.1248 (−0.50)
β_{Mkt}	0.5998 (35.69)	0.7985 (59.94)	0.8985 (65.85)	0.9839 (69.43)	1.0500 (71.98)	1.1659 (76.08)	1.2768 (69.83)	1.3960 (64.67)	1.5109 (56.81)	1.6538 (40.92)	1.0540 (20.76)
β_{FOP}	0.9330 (6.75)	0.8098 (7.39)	0.6067 (5.40)	0.3744 (3.21)	0.2554 (2.13)	0.0182 (0.14)	−0.2902 (−1.93)	−0.6865 (−3.87)	−1.3097 (−5.99)	−3.1726 (−9.54)	−4.1056 (−9.83)

Note: Panel A (B) presents returns and factor model alphas of decile portfolios sorted by market beta β_{Mkt} (return variance *Var*) as well as a difference portfolio which is long in the highest and short in the lowest decile. Alphas and excess returns are multiplied with one hundred and the *t*-statistics in parentheses are computed from Newey and West (1987) adjusted standard errors with six lags. *Mkt* is the market factor and *FOP* is the empirical factor for the optimal orthogonal portfolio. The sample period is July 1963 to December 2021.

TABLE 5 Explaining the returns of 25 *Size*-β_{Mkt} portfolios

$\beta_{Mkt} \rightarrow$	Low	2	3	4	High	Low	2	3	4	High
	Coefficients					***t*-statistics**				
Panel A: Unadjusted excess returns										
Small	0.8021	0.9030	0.9394	1.0374	0.8276	(4.34)	(4.62)	(4.16)	(4.18)	(2.66)
2	0.7221	0.8933	0.9747	0.9395	0.7510	(4.39)	(4.82)	(4.64)	(3.99)	(2.52)
3	0.6842	0.8884	0.8577	0.8440	0.7868	(4.69)	(5.03)	(4.29)	(3.71)	(2.74)
4	0.6854	0.8252	0.7883	0.6709	0.8234	(4.65)	(4.77)	(3.99)	(3.03)	(2.90)
Large	0.5380	0.6073	0.5928	0.6349	0.5656	(3.99)	(3.78)	(3.18)	(2.95)	(2.09)
Panel B: CAPM α										
Small	0.3481	0.3656	0.3176	0.3205	−0.0476	(2.62)	(2.99)	(2.25)	(2.25)	(−0.25)
2	0.2611	0.3478	0.3361	0.2211	−0.1586	(2.62)	(3.42)	(3.17)	(1.88)	(−1.07)
3	0.2590	0.3331	0.2279	0.1274	−0.1142	(3.15)	(4.19)	(2.56)	(1.25)	(−0.88)
4	0.2538	0.2640	0.1424	−0.0500	−0.0739	(3.08)	(3.94)	(1.98)	(−0.60)	(−0.60)
Large	0.1496	0.0802	−0.0251	−0.0729	−0.2705	(1.92)	(1.38)	(−0.41)	(−0.97)	(−2.07)
Panel C: two-factor model coefficients										
α										
Small	0.0680	0.0462	0.0129	0.1042	−0.0973	(0.48)	(0.36)	(0.09)	(0.68)	(−0.47)
2	−0.0168	0.0492	0.0619	0.0028	−0.1137	(−0.16)	(0.46)	(0.55)	(0.02)	(−0.71)
3	0.0255	0.1061	0.0291	−0.0152	0.0340	(0.29)	(1.26)	(0.31)	(−0.14)	(0.24)
4	0.0216	0.0982	−0.0235	−0.0725	0.1680	(0.25)	(1.38)	(−0.31)	(−0.80)	(1.26)
Large	−0.0694	−0.0545	−0.0060	0.0159	0.1829	(−0.84)	(−0.88)	(−0.09)	(0.20)	(1.35)
β_{Mkt}										
Small	0.7705	0.9121	1.0554	1.2167	1.4853	(26.41)	(34.29)	(34.05)	(38.54)	(35.39)
2	0.7823	0.9258	1.0838	1.2192	1.5438	(36.21)	(42.19)	(47.10)	(47.15)	(46.73)
3	0.7216	0.9425	1.0688	1.2162	1.5291	(40.59)	(54.85)	(54.96)	(53.89)	(52.90)
4	0.7324	0.9525	1.0962	1.2234	1.5229	(41.14)	(65.24)	(69.70)	(66.12)	(55.78)
Large	0.6592	0.8947	1.0486	1.2012	1.4190	(39.07)	(70.46)	(76.72)	(72.08)	(51.18)
β_{FOP}										
Small	1.1937	1.3612	1.2982	0.9216	0.2119	(4.97)	(6.22)	(5.09)	(3.55)	(0.61)
2	1.1841	1.2722	1.1684	0.9305	−0.1915	(6.66)	(7.05)	(6.17)	(4.37)	(−0.70)
3	0.9951	0.9673	0.8469	0.6075	−0.6314	(6.80)	(6.84)	(5.29)	(3.27)	(−2.66)
4	0.9895	0.7067	0.7069	0.0958	−1.0310	(6.76)	(5.88)	(5.46)	(0.63)	(−4.59)
Large	0.9332	0.5738	−0.0814	−0.3781	−1.9322	(6.72)	(5.49)	(−0.72)	(−2.76)	(−8.47)

Note: Time series regressions of 25 *Size*-β_{Mkt} portfolios on different factor models. Intercepts are multiplied with one hundred to facilitate interpretation. The corresponding *t*-statistics are presented in parentheses. Panel A (B) presents unadjusted excess returns (CAPM alphas). Panel C presents results for the two-factor model which extends the CAPM by the empirical factor for the optimal orthogonal portfolio *FOP*. We present the model Intercept as well as coefficient estimates for the two factors, that is, β_{Mkt} and β_{FOP}. The sample period is July 1963 to December 2021.

TABLE 6 Explaining the returns of 25 *Size-Var* portfolios

Var →	Low	2	3	4	High	Low	2	3	4	High
	Coefficients					***t*-statistics**				
Panel A: Unadjusted excess returns										
Small	1.0295	1.1768	1.0899	0.8250	−0.0428	(6.48)	(5.41)	(4.35)	(2.84)	(−0.12)
2	0.9071	1.0612	1.0819	0.9669	0.3869	(5.83)	(5.31)	(4.77)	(3.70)	(1.16)
3	0.7831	0.8521	1.0036	0.8985	0.5052	(5.49)	(4.62)	(4.86)	(3.83)	(1.65)
4	0.7323	0.7792	0.8378	0.7931	0.5727	(5.16)	(4.60)	(4.33)	(3.64)	(1.99)
Large	0.5105	0.6460	0.6123	0.5486	0.5647	(3.90)	(4.25)	(3.63)	(2.87)	(2.27)
Panel B: CAPM α										
Small	0.5985	0.5522	0.3598	−0.0089	−0.9690	(5.89)	(4.37)	(2.55)	(−0.05)	(−4.03)
2	0.4588	0.4631	0.3912	0.1574	−0.5959	(5.12)	(4.38)	(3.44)	(1.27)	(−3.28)
3	0.3655	0.2788	0.3569	0.1567	−0.4380	(4.61)	(3.22)	(3.79)	(1.52)	(−2.97)
4	0.3259	0.2473	0.2144	0.0835	−0.3394	(3.94)	(3.22)	(2.78)	(1.02)	(−2.74)
Large	0.1357	0.1660	0.0571	−0.0877	−0.2377	(1.77)	(2.47)	(0.96)	(−1.43)	(−2.41)
Panel C: two-factor model coefficients										
α										
Small	0.3058	0.2570	0.2121	0.0310	−0.6154	(2.85)	(1.91)	(1.38)	(0.17)	(−2.37)
2	0.0946	0.1261	0.1379	0.0452	−0.2760	(1.04)	(1.14)	(1.13)	(0.34)	(−1.41)
3	0.0407	0.0399	0.1031	−0.0088	−0.0633	(0.51)	(0.44)	(1.03)	(−0.08)	(−0.40)
4	0.0364	0.0547	0.0263	0.0363	0.0747	(0.42)	(0.67)	(0.32)	(0.41)	(0.58)
Large	−0.0458	−0.0303	−0.0473	−0.0645	0.1293	(−0.56)	(−0.43)	(−0.74)	(−0.97)	(1.27)
β_{Mkt}										
Small	0.7315	1.0600	1.2390	1.4152	1.5719	(33.31)	(38.39)	(39.45)	(37.65)	(29.57)
2	0.7608	1.0150	1.1722	1.3740	1.6680	(40.69)	(44.84)	(47.04)	(49.94)	(41.60)
3	0.7088	0.9729	1.0975	1.2591	1.6008	(42.98)	(51.90)	(53.71)	(55.13)	(49.81)
4	0.6896	0.9027	1.0580	1.2042	1.5480	(39.28)	(54.04)	(62.80)	(65.92)	(58.60)
Large	0.6360	0.8147	0.9424	1.0800	1.3618	(38.02)	(56.21)	(71.55)	(78.99)	(65.32)
β_{FOP}										
Small	1.2474	1.2582	0.6294	−0.1700	−1.5067	(6.91)	(5.54)	(2.44)	(−0.55)	(−3.45)
2	1.5519	1.4360	1.0795	0.4781	−1.3632	(10.09)	(7.71)	(5.27)	(2.11)	(−4.13)
3	1.3840	1.0180	1.0818	0.7051	−1.5968	(10.20)	(6.60)	(6.44)	(3.75)	(−6.04)
4	1.2338	0.8208	0.8018	0.2013	−1.7648	(8.54)	(5.97)	(5.79)	(1.34)	(−8.12)
Large	0.7739	0.8367	0.4449	−0.0991	−1.5638	(5.62)	(7.02)	(4.11)	(−0.88)	(−9.12)

Note: Time series regressions of 25 *Size-Var* portfolios on different factor models. Intercepts are multiplied with one hundred to facilitate interpretation. The corresponding *t*-statistics are presented in parentheses. Panel A (B) presents unadjusted excess returns (CAPM alphas). Panel C presents results for the two-factor model which extends the CAPM by the empirical factor for the optimal orthogonal portfolio *FOP*. We present the model Intercept as well as coefficient estimates for the two factors, that is, β_{Mkt} and β_{FOP}. The sample period is July 1963 to December 2021.

4.3 | Cross-sectional evidence

We further use the 25 double-sorted *Size*-β_{Mkt} and *Size-Var* portfolios to evaluate the asset pricing performance of the two-factor model in two-pass cross-sectional regressions in the spirit of Fama and MacBeth (1973). In the first pass, we estimate factor betas of the CAPM as well as the two-factor model which extends the CAPM by *FOP*. Then, we use factor betas as predictors to explain cross-sectional variation of average returns in the second-pass regression:

$$\mu_p = \gamma_0 + \gamma_1 \beta_{Mkt,p} + \gamma_2 \beta_{FOP,p} + \epsilon_p, \tag{8}$$

where μ_p is the average excess return, $\beta_{Mkt,p}$ ($\beta_{FOP,p}$) is the exposure to the market excess return (*FOP*), and ϵ_p is the residual of portfolio p. γ_1 and γ_2 are the risk premium estimates for *Mkt* and *FOP*, respectively, and γ_0 is the pricing error of the model.

A large body of literature, for example, Kan et al. (2013), Gospodinov et al. (2014), and Giglio et al. (2021), highlights several shortcomings of the inference from the traditional two-pass methodology, which are particularly important in the context of our analysis. First, the construction of *FOP* heavily relies on estimated quantities, thus inducing an errors-in-variables (EIV) problem due to estimation error (see Shanken, 1992). Second, our research hypothesis of a latent factor in the CAPM is tantamount to the null hypothesis that the reference model is misspecified (see Kan et al., 2013). In this case, the conventional Fama and MacBeth (1973) standard errors are inconsistent and lead to false rejections of the null hypothesis, especially in the presence of weak factors in the model (see Gospodinov et al., 2014).[12] Gospodinov et al. (2014) show that the usage of the misspecification-robust standard errors proposed by Kan et al. (2013) (KRS) restores the validity of the statistical inference from cross-sectional regressions, even in the presence of weak factors.

Table 7 presents the second stage coefficients of the two-pass cross-sectional regressions in Equation (8). The risk premium estimates are stated as percentages and the cross-sectional R^2 follows Kandel and Stambaugh (1995). Furthermore, we follow Lewellen et al. (2010) to include the factor portfolios of the respective model among the left-hand side assets. Fama and MacBeth (1973) t-statistics are stated in parentheses. To address the issues of model misspecifications, we follow Gospodinov et al. (2014) and further add t-statistics based on KRS standard errors which are robust against EIV and model misspecification in brackets. Both standard errors are adjusted for autocorrelation with six lags (Newey & West, 1987).

Panel A of Table 7 presents the results for 25 *Size*-β_{Mkt} portfolios. Comparing the CAPM in Model (1) and the two-factor specification in Model (2) reveals two major differences. First, the risk premium for β_{Mkt} increases from roughly 9 to 57 bps after the inclusion of β_{FOP}, with KRS t-statistics of 0.36 and 2.23, respectively. This estimate is close to the full sample risk premium for *Mkt* of 59 bps and is statistically significant at the 5% level. Second, the pricing error of the CAPM reduces considerably after including *FOP*. In Model (1), the pricing error is roughly 68 bps and highly significant at conventional levels (KRS t-statistic = 3.41). Including *FOP* in Model (2) fully explains this pricing error and the intercept turns insignificant (KRS t-statistic = 0.42). Since we include excess returns on the left-hand side, a nonzero intercept indicates mispricing. Model (2) prices the test assets more efficiently and explains a larger fraction of cross-sectional variation than the CAPM.

TABLE 7 Fama and MacBeth (1973) regressions for double-sorted portfolios

Model	Intercept			β_{Mkt}			β_{FOP}			R^2 in %	N
Panel A: 25 *Size*-β_{Mkt} portfolios											
(1)	0.6759	(3.58)	[3.41]	0.0921	(0.36)	[0.35]				3.10	26
(2)	0.0596	(0.41)	[0.42]	0.5651	(2.30)	[2.23]	0.2077	(2.80)	[2.83]	83.19	27
Panel B: 25 *Size-Var* portfolios											
(3)	1.0995	(5.14)	[4.92]	−0.3073	(−1.11)	[−1.08]				11.85	26
(4)	−0.0876	(−0.69)	[−0.55]	0.6563	(2.67)	[2.26]	0.3383	(5.21)	[4.48]	74.33	27

Note: Panel A (B) presents second stage Fama and MacBeth (1973) estimates for 25 *Size*-β_{Mkt} (25 *Size-Var*) portfolios. β_{Mkt} (β_{FOP}) is the beta with respect to the market portfolio (*FOP*). All coefficients are multiplied with one hundred. The t-statistics in parentheses are computed from Newey and West (1987) standard errors with six lags, t-statistics in brackets are computed from standard errors of Kan et al. (2013) (KRS) and account for errors-in-variables, model misspecification, and autocorrelation with six lags. The cross-sectional R^2 follows Kandel and Stambaugh (1995). We follow Lewellen et al. (2010) and include the respective right-hand side factors among the test assets. The sample period is July 1963 to December 2021.

This result also extends to Panel B in which 25 *Size-Var* portfolios serve as base assets. Again, the risk premium for β_{Mkt} increases from −30 bps to 66 bps due to the consideration of β_{FOP}. The latter estimate is statistically significant at the 5% level (KRS *t*-statistic = 2.26) and is once again close to the full sample market risk premium. The pricing error of the two-factor model becomes insignificant and reduces from roughly 110 bps (KRS *t*-statistic = 4.92) to −9 bps (KRS *t*-statistic = −0.55). In both cases, Fama and MacBeth (1973) *t*-statistics and KRS *t*-statistics lead to similar conclusions.

To further emphasize the difference between the CAPM and the two-factor model, Figure 1 plots average realized returns against the expected returns from the respective models. Panel a (b) plots expected returns from the CAPM model (CAPM extended by *FOP*) in % per month against the average realized returns of the 25 *Size*-β_{Mkt} portfolios. Panels c and d repeat the same analysis for the 25 *Size-Var* portfolios. The size of markers indicates the *Size* quintile (small to big) and β_{Mkt}/*Var* quintiles increase in the marker's color (light to dark). The solid 45-degree line corresponds to a perfect relationship between expected returns and average realized returns.

In case of the CAPM (Panels a and c), the relation between realized and expected returns is flat for both sets of test assets. In Panel a, low-β_{Mkt} portfolios earn a higher realized return than expected from the model, whereas the opposite is true for high-β_{Mkt} stocks. This result extends to Panel c where high-*Var* portfolios are plotted well above the 45-degree line, indicating that expected returns from the CAPM are too low compared with realized returns. In both cases, the model leaves significant pricing errors.

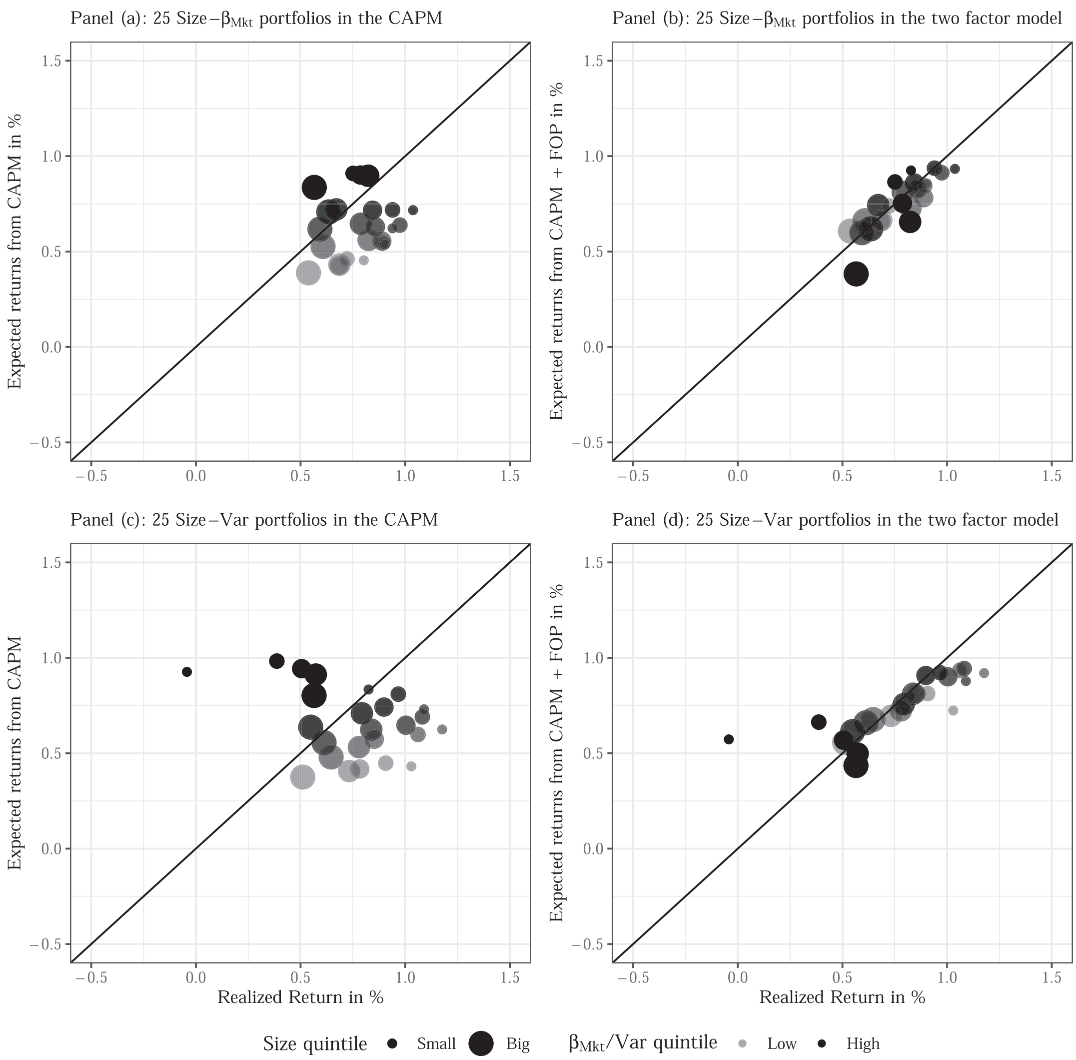

FIGURE 1 Expected returns of the CAPM versus the two-factor model. This figure plots expected versus realized returns of the CAPM (Panels a and c) and the CAPM extended by the mimicking factor for the optimal orthogonal portfolio *FOP* (Panel b and d). The test assets are 25 portfolios sorted by *Size* and β_{Mkt} (*Size* and *Var*) in Panel a and b (c and d). The 45-degree line indicates a perfect relationship between realized and expected returns. Marker size indicates the *Size* quintile (small to big), marker color indicates the β_{Mkt} quintile (light to dark) of the respective portfolio. The sample period is July 1963 to December 2021

Including β_{FOP} into the model improves both of the problems in the CAPM. Expected returns of 25 *Size*-β_{Mkt} and 25 *Size*-*Var* portfolios are now closer to the 45-degree line. As expected from the time series regressions, the two-factor model performs better in case of *Size*-β_{Mkt} portfolios since high-*Var* portfolios do not line up well with the 45-degree line in Panel c. The improvements over the CAPM, however, are easily visible.

Extending the CAPM with our composite factor *FOP* explains the underperformance of high-β_{Mkt} and high-*Var* portfolios. The negative exposure of risky stocks with respect to *FOP* explains their negative CAPM alphas. Furthermore, controlling fo *FOP* once more re-establishes a significant trade-off between β_{Mkt} and average returns.

5 | TESTING ECONOMIC THEORIES

5.1 | FOP and the slope of the security market line

The two-factor model solves the issues of the CAPM in pricing risky portfolios but remains agnostic with respect to the economic mechanisms behind *FOP*. The most prominent economic explanations—no matter whether they are based on leverage constraints, investor sentiment or disagreement—share the common prediction that the slope of the SML depends on the respective state variable. During periods of high leverage constraints, investor sentiment or disagreement, the SML takes on a flatter slope because high-β_{Mkt} stocks tend to be overpriced and earn lower future returns (see, for example, Frazzini & Pedersen, 2014; Hong & Sraer, 2016; Antoniou et al., 2016; Jylhä, 2018).

If *FOP* is related to existing theoretical explanations, we expect the same prediction for the SML. Since the CAPM suffers from the omission of the latent factor *FOP* in the first place, the exposure to β_{FOP} should fully account for variations in the slope of the SML. This powerful additional prediction is possible because the inclusion of *FOP* to the CAPM leaves estimates for β_{Mkt} unchanged. This section focuses on β_{Mkt}-sorted portfolios given the natural relation to the SML.

Figure 2 presents sample splits at the median of *FOP* for β_{Mkt} decile portfolios. We plot the monthly realized returns and expected returns from the CAPM against the post-formation β_{Mkt}. The dashed line is the theoretical SML as expected from the CAPM and the solid line plots the empirical relationship between realized returns and post-formation β_{Mkt}. Marker colors indicate decile portfolios from low to high (light to dark). For each sample split, we plot the return spread of the decile portfolios which is attributable to *FOP*, that is, β_{FOP} times the average return of *FOP* in the respective subsample period.

Panel a presents the full sample period from July 1963 to December 2021. In line with results in the previous literature, the empirical SML is flat. The difference between the theoretical and the empirical SML almost perfectly lines up with the *FOP* return spread in Panel b For example, the lowest β_{Mkt} decile earns an average return of 57 bps, while the expected return in the CAPM amounts to 36 bps. The β_{FOP} exposure times the average return on *FOP* is 23 bps, and thus, matches this difference. This finding, however, is no surprise considering the good performance of the two-factor model in the previous section.

Panel c presents the same estimates for the subperiod in which *FOP* is below the historical median and reveals the expected pattern. Now the empirical SML is steeper than its theoretical counterpart, in line with periods of low leverage constraints, disagreement, or sentiment as presented in Jylhä (2018), Hong and Sraer (2016), and Antoniou et al. (2016). Now that the realized returns exceed their expectations from the CAPM and the *FOP* return spread in Panel d lines up positively from low to high β_{Mkt} deciles. This switch is due to a negative average *FOP* of −15 bps in this subsample, whereas the β_{FOP} exposures of the β_{Mkt} decile portfolios hardly change and still decrease monotonically from low to high deciles.

Panel e plots the most interesting case: subperiods with *FOP* above the sample median. If *FOP* is consistent with the theoretical explanations above, the negative slope of the SML should be fully attributable to the β_{FOP} exposure. The slope of the empirical SML now turns negative, in line with previous studies. Again, the pricing error of the theoretical SML almost perfectly lines up with the *FOP* return spread. Interestingly, the spread is flat in the first three deciles and then decreases monotonically. The overall spread is stronger compared with Panel b which might reflect an arbitrage asymmetry as documented by Stambaugh et al. (2015).

Overall, the sample splits reveal familiar patterns with respect to the slope of the SML. The finding that this pattern is fully attributable to the exposure to *FOP*, however, provides another powerful implication to test theoretical propositions for the tilted SML. In order to constitute a consistent explanation for the low-risk effect, any potential state variable should induce a higher average return on *FOP* and significantly affect the sign of the β_{Mkt} decile return spread in the

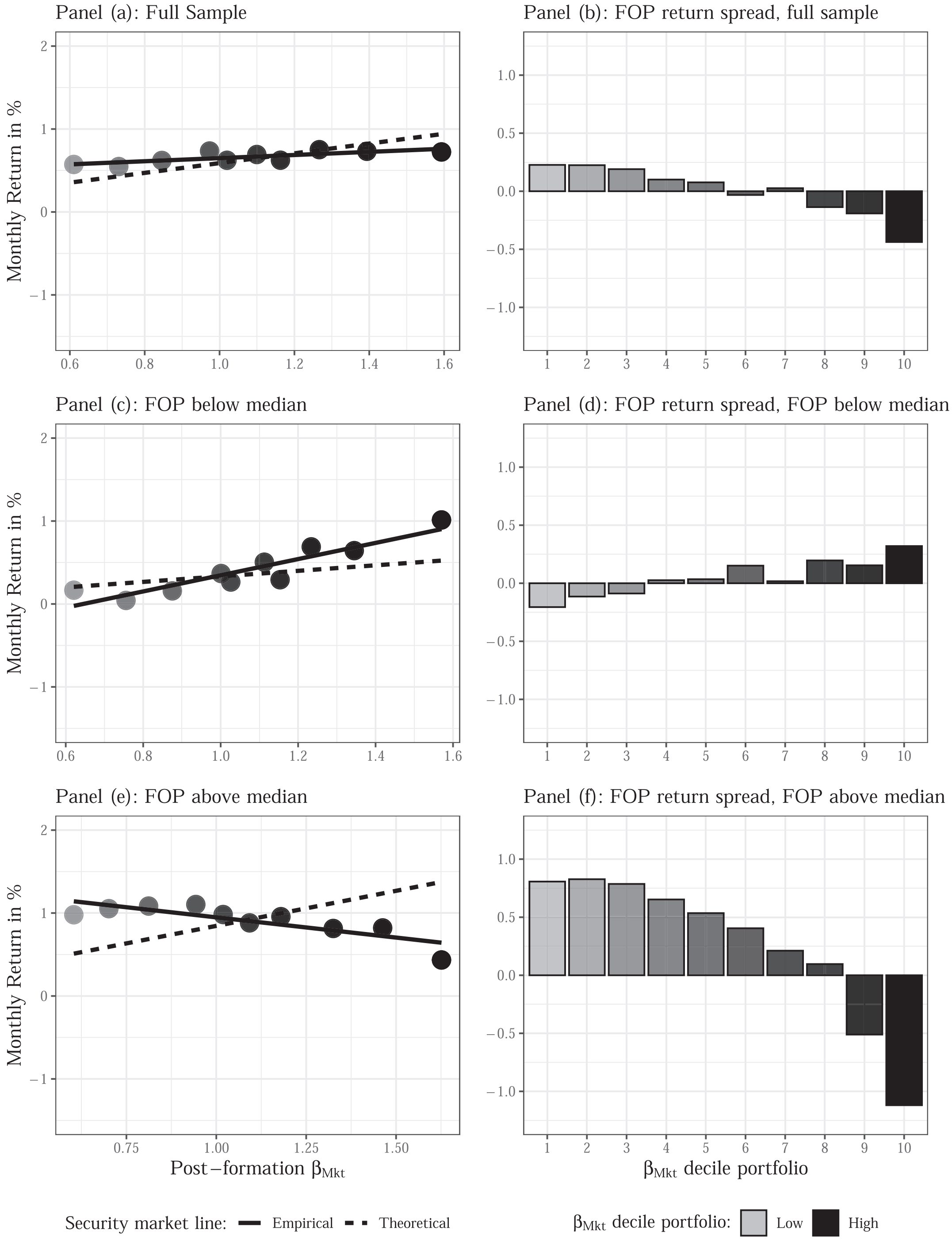

FIGURE 2 *FOP* and the slope of the SML. This figure plots the empirical (solid) versus the theoretical (dashed) slope of the Security Market Line (SML) for decile β_{Mkt} portfolios. Panel a and b consider the full sample, in Panel c and d (e and f), we present the slopes during months in which the empirical factor for the optimal orthogonal portfolio *FOP* is lower (higher) than the sample median. The sample period is July 1963 to December 2021

same direction as *FOP*. The factor *FOP* thus facilitates a horse race to discriminate between the otherwise observationally equivalent predictions of leverage constraints, disagreement, and sentiment.

5.2 | Leverage constraints versus behavioral explanations

We now turn to potential economic drivers behind *FOP*. Following Asness et al. (2018), we focus on leverage constraints and promising behavioral alternatives. Specifically, our analysis considers leverage constraints, investor sentiment, and

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TABLE 8 Leverage constraints versus behavioral explanations

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
	Levels						**First differences**					
TED	−0.0702 (−1.13)					−0.0311 (−0.47)	−0.0017 (−1.20)					−0.0010 (−0.59)
Margin Debt		23.6544 (0.31)				−86.5168 (−1.15)		4.6568 (1.38)				3.2524 (0.87)
BW Sentiment			0.0746 (2.16)			0.2915 (5.11)			0.0042 (2.49)			0.0054 (1.85)
Consumer Confidence				0.0046 (1.66)		0.0008 (0.25)				0.0001 (0.92)		0.0000 (0.35)
Disagreement					0.1303 (2.78)	0.1407 (3.51)					0.0022 (1.39)	0.0020 (1.07)
Intercept	0.2762 (4.89)	0.2436 (9.88)	0.2436 (10.10)	−0.1602 (−0.67)	−0.2446 (−1.53)	−0.3616 (−1.26)	0.0024 (6.25)	0.0024 (9.93)	0.0024 (9.69)	0.0023 (7.47)	0.0024 (6.88)	0.0026 (6.56)

Note: Time series regressions of *FOP* on predictor variables for constraints to arbitrage and investor sentiment. We include the following variables: The TED spread, margin debt of NYSE customers in relation to NYSE market capitalization, the Baker and Wurgler (2006) (BW) Investor Sentiment Index, the University of Michigan Consumer Confidence Index and aggregate disagreement. All coefficients are multiplied with one hundred. We include explanatory variables in terms of levels in Columns (1) to (6) and first differences in Columns (7) to (12). The sample period is 1986 to 2021 for the TED spread, 1967 to 2017 for margin debt, 1965 to 2018 for BW Sentiment, 1978 to 2021 for Consumer Confidence, and 1982 to 2021 for disagreement. The kitchen sink models in Columns (6) and (12) reduce the sample period from 1986 to 2017. *t*-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

disagreements as proposed by Frazzini and Pedersen (2014), Antoniou et al. (2016) and Hong and Sraer (2016).[13] We include the TED spread and margin debt of NYSE customers as two proxies for leverage constraints (Asness et al., 2018; Frazzini & Pedersen, 2014). Since the latter exhibits a time trend and is therefore nonstationary, we remove the trend in a linear regression. To facilitate the interpretation, we multiply margin debt by minus one such that a higher value of margin debt in our analysis reflects higher leverage constraints.[14] Our two proxies for sentiment are the BW Investor Sentiment Index and the University of Michigan Consumer Confidence Index. As in Hong and Sraer (2016), disagreement is the beta-weighted average of the standard deviation from analyst forecasts for the long-term EPS growth rate.

Table 8 presents time series regressions of *FOP* on proxies for leverage constraints, investor sentiment, and disagreement. We include explanatory variables in terms of levels in Columns (1) to (6) and first differences in Columns (7) to (12). The sample period is 1986 to 2021 in Columns (1) and (7), 1967 to 2017 in Columns (2) and (8), 1965 to 2018 in Columns (3) and (9), 1978 to 2021 in Columns (4) and (10), and 1982 to 2021 in Columns (5) and (11). The kitchen sink models in Columns (6) and (12) reduce the sample period to 1986 to 2017. All coefficients are multiplied with one hundred with *t*-statistics from Newey and West (1987) standard errors in parentheses.

Results in levels, that is, Columns (1), (2) and (6) provide little support for leverage-based explanations. The TED spread and margin debt exhibit insignificant coefficients, both in the univariate models in Columns (1) and (2) as well as the kitchen sink regression in Column (6). Both sentiment measures are significantly positive with coefficients of 0.0746 (*t*-statistic = 2.16) for BW Sentiment and 0.0046 (*t*-statistic = 1.66) in case of Consumer Confidence. Only the former sentiment proxy, however, survives when we control for all predictive variables in Column (6) with a highly significant coefficient of 0.2915 (*t*-statistic = 5.11). This also holds true for disagreement which is statistically significant at the 1% level in Columns (5) and (6) with coefficient estimates of 0.1303 and 0.1407, respectively.

In terms of first differences, results are mixed at best. Only BW Sentiment is statistically significant with a coefficient estimate of 0.0042 (*t*-statistic = 2.49) in the univariate model and 0.0054 (*t*-statistic = 1.85) in the kitchen sink regression. All other predictors turn insignificant if we use first differences instead of levels. The time series regressions highlight behavioral explanations, most importantly BW Sentiment and disagreement. Conversely, we find little support for leverage constraints.

As stated above, the two-factor model yields a second, even stronger prediction to identify economic state variables behind variations in the slope of the SML. In order to account for the effects in Figure 2, a regime switch from low to high states in the economic variable should induce a significantly positive change in *FOP* and negatively affect the sign of the return difference between high and low β_{Mkt} deciles. To test this prediction formally, we follow Stambaugh et al. (2012) and run the time series regression

$$r_t = \alpha_H d_{H,t} + \alpha_L d_{L,t} + \epsilon_{i,t}, \tag{9}$$

where $d_{H,t}$ ($d_{L,t}$) is an indicator variable which is equal to one if the respective predictor variable in the previous month is above (below) the sample median and zero otherwise. r_t is either the return on *FOP* or the difference between the highest and the lowest β_{Mkt} decile portfolio. In the latter regressions, we include *Mkt* to effectively measure the CAPM alpha of the β_{Mkt} decile spread.[15]

Table 9 presents the coefficient estimates as well as their respective difference $\alpha_H - \alpha_L$ which indicates whether the difference of the dependent variable in the two states is significantly different from zero. The dependent variables are the returns on *FOP* in Panel A and the return spread between the highest and the lowest β_{Mkt} decile in Panel B. Coefficients are stated as percentages and *t*-statistics in parentheses are computed from Newey and West (1987) robust standard errors.

Panel A, again, accentuates behaviorally motivated predictive variables over leverage constraint-based explanations. The difference in *FOP* between high and low leverage constraint regimes is insignificant, both in case of the TED spread and margin debt. Hence, leverage constraints are unlikely to explain variation in the state variable proxied by *FOP*. BW Sentiment, Consumer Confidence and disagreement all induce significantly higher average returns on *FOP*. For example, when the previous month's BW Sentiment is high, *FOP* is also higher on average and a high-*FOP* state—tantamount to a negative slope of the SML—is more likely.

The second condition refers to the sign of the β_{Mkt} decile spread during periods of high leverage constraints, sentiment, or disagreement. Panel B presents the coefficient estimates as well as their respective difference while controlling for *Mkt*. Once more, behavioral explanations attain more promising results. When sentiment is high—either measured by BW Sentiment or Consumer Confidence—the decile return spread on β_{Mkt} is significantly negative and insignificant otherwise. The difference estimates of −111 bps and −94 bps are statistically significant at the 5% and 10% levels with *t*-statistics of −2.75 and −2.16, respectively. Disagreement is not in line with the second prediction and does not

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TABLE 9 Testing economic theories

	Panel A: *FOP*			Panel B: β_{Mkt} decile spread		
	High	**Low**	**High-Low**	**High**	**Low**	**High-Low**
TFD	0.2264 (5.52)	0.2486 (6.15)	−0.0222 (−0.34)	−0.3754 (−1.09)	−0.7167 (−2.09)	0.3413 (0.69)
Margin Debt	0.2484 (8.50)	0.2389 (8.18)	0.0096 (0.21)	−0.4750 (−1.75)	−0.3929 (−1.46)	−0.0821 (−0.21)
BW Sentiment	0.2976 (10.08)	0.1896 (6.42)	0.1080 (2.19)	−0.9858 (−3.63)	0.1243 (0.46)	−1.1101 (−2.75)
Consumer Confidence	0.3083 (8.89)	0.1611 (4.66)	0.1472 (2.36)	−1.0594 (−3.54)	−0.1194 (−0.40)	−0.9400 (−2.16)
Disagreement	0.3056 (8.05)	0.1777 (4.69)	0.1279 (2.06)	−0.6987 (−2.12)	−0.6520 (−2.00)	−0.0467 (−0.10)

Note: Time series regressions with different indicator variables based on the median split of constraints to arbitrage and investor sentiment. The dependent variable is *FOP* in Panel A and the decile return spread of β_{Mkt} decile portfolios in Panel B. Regressions in Panel B include *Mkt* as an explanatory variable. We include the following variables: The TED spread, margin debt of NYSE customers in relation to NYSE market capitalization, the Baker and Wurgler (2006) (BW) Investor Sentiment Index, the University of Michigan Consumer Confidence Index and aggregate disagreement. All coefficients are multiplied with one hundred. The sample period is 1986 to 2021 for the TED spread, 1967 to 2017 for margin debt, 1965 to 2018 for BW Sentiment, 1978 to 2021 for Consumer Confidence, and 1982 to 2021 for disagreement. *t*-statistics calculated from Newey and West (1987) standard errors with six lags in parentheses.

significantly affect the β_{Mkt} decile spread. Again, all predictors for leverage constraints are insignificant. In summary, investor sentiment satisfies the predictions from Section 5.1 best and is a likely source to explain both parts of the low-risk anomaly.

6 | ROBUSTNESS CHECKS

Our main results in Table 4 employ the decile portfolios from Kenneth R. French, and thus, may depend on the specific definitions of β_{Mkt} and *Var* outlined in Fama and French (2016). In this section, we repeat our analysis with several alternative proxies for the riskiness of individual stocks to show that the explanatory power of *FOP* is not driven by our specific choice for the low-risk effect.

Liu et al. (2018) argue that the strength of the beta anomaly may depend on the estimation procedure of β_{Mkt}. Thus, we provide two alternative estimation techniques. The first procedure follows the Dimson (1979) sum of coefficients method which accounts for non-synchronous trading. The second estimation technique follows Frazzini and Pedersen (2014) and relies on independent estimation of correlations and volatilities. With respect to idiosyncratic risk, the total variance over the previous 60 days is arguably not the most adopted measure. Instead, the literature on the idiosyncratic volatility puzzle mostly takes into account the one-month standard deviation of residuals from the FF3 model, as proposed by Ang et al. (2006). We refer to this measure as *IVol*. Given these three characteristics, we form decile portfolios with NYSE breakpoints and compute value-weighted returns with monthly rebalancing. For a detailed description of the variable construction, we refer to Appendix A2. Table 10 repeats the baseline analysis with the three alternative sorts mentioned above. Other than that, the analysis is identical to Table 4.

Panels A and B show that *FOP* performs equally well when applying the alternative beta estimation procedures from Dimson (1979) and Frazzini and Pedersen (2014) to proxy for the beta anomaly. In both cases, high-beta stocks earn significantly negative CAPM alphas, which also manifests in highly significant alphas for the long-short portfolios of −44 bps in Panel A (t-statistic = −2.58) and − 84 bps in Panel B (t-statistic = −4.23). Both effects are stronger in statistical terms compared with the baseline analysis. Nevertheless, including *FOP* fully explains the negative CAPM alphas of high-beta stocks. This finding extends to *IVol* in Panel C. Again, the CAPM leaves highly negative alphas in the highest *IVol* deciles as well as the long-short portfolio, which are fully captured by the two-factor model. We conclude that our results are robust to the choice of proxies for systematic and idiosyncratic risk, that is, β_{Mkt} and volatility.

7 | CONCLUDING REMARKS

We employ seemingly unrelated anomaly portfolios to construct the composite factor *FOP* which approximates the optimal orthogonal portfolio of MacKinlay and Pastor (2000) and test the asset pricing implications of the extended CAPM. The exposure to *FOP* explains the negative alphas of high-beta and high-variance stocks and re-establishes a positive

TABLE 10 Robustness: Alternative portfolio sorts for the low-risk effect

	Low	2	3	4	5	6	7	8	9	High	High-Low
Panel A: Dimson (1979) β_{Mkt}^{Dimson}											
CAPM	−0.0526 (−0.59)	0.1496 (2.17)	0.0766 (1.20)	0.0669 (1.12)	0.1344 (2.38)	0.1457 (2.45)	0.0347 (0.59)	−0.0123 (−0.19)	−0.1025 (−1.30)	−0.4886 (−3.82)	−0.4360 (−2.58)
FOP + Mkt	−0.0839 (−0.86)	−0.0096 (−0.13)	−0.0136 (−0.20)	−0.0340 (−0.53)	0.0751 (1.23)	0.0397 (0.62)	−0.0060 (−0.10)	0.0343 (0.48)	0.0230 (0.27)	−0.0517 (−0.39)	0.0322 (0.18)
Panel B: Frazzini and Pedersen (2014) $\beta_{Mkt}^{F\&P}$											
CAPM	0.2809 (3.05)	0.2747 (3.63)	0.2261 (3.22)	0.2090 (2.80)	0.0879 (1.21)	0.0203 (0.27)	0.0274 (0.35)	−0.0615 (−0.69)	−0.1452 (−1.41)	−0.5628 (−3.80)	−0.8437 (−4.23)
FOP + Mkt	0.0259 (0.27)	0.0399 (0.50)	0.0074 (0.10)	0.0081 (0.10)	−0.0805 (−1.04)	−0.1116 (−1.40)	−0.0633 (−0.74)	−0.0963 (−1.00)	−0.1100 (−0.98)	−0.1122 (−0.72)	−0.1381 (−0.67)
Panel C: *IVol*											
CAPM	0.1624 (2.41)	0.1690 (3.17)	0.1674 (2.91)	0.0728 (1.26)	0.0134 (0.21)	0.0786 (1.22)	0.0171 (0.22)	−0.0105 (−0.11)	−0.3002 (−2.65)	−0.8328 (−4.65)	−0.9952 (−4.41)
FOP + Mkt	−0.0549 (−0.78)	0.0008 (0.01)	0.0259 (0.42)	−0.0096 (−0.15)	−0.0010 (−0.01)	0.0948 (1.35)	0.1500 (1.80)	0.1584 (1.60)	0.0479 (0.40)	−0.1600 (−0.87)	−0.1050 (−0.46)

Note: Factor model alphas of the CAPM and the two-factor model *Mkt + FOP* for decile portfolio sorts on various alternative risk measures. *Mkt* is the market factor and *FOP* is the empirical factor for the optimal orthogonal portfolio. Alphas are multiplied with one hundred and the *t*-statistics in parentheses are computed from Newey and West (1987) adjusted standard errors with six lags. Panels A and B repeat the baseline analysis with alternative estimation techniques for the market beta β_{Mkt} following Dimson (1979) in Panel A and Frazzini and Pedersen (2014) in Panel B. In Panel C, we follow Ang et al. (2006) and use idiosyncratic volatility (*IVol*) from the Fama and French (1993) three-factor model as a proxy for idiosyncratic risk. The sample period is August 1963 to December 2021.

trade-off between beta and returns. Our extended CAPM is theoretically motivated, computationally tractable, and allows a multidimensional approach to the identification of characteristics which provide independent information about average returns (Cochrane, 2011). Our evidence promotes sentiment as an explanation for the low-risk effect and is not supported by alternative predictors, for example, leverage constraints.

ACKNOWLEDGMENT
Open Access funding enabled and organized by Projekt DEAL.

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ENDNOTES

1 We follow Asness et al. (2018) and use the term “low-risk effect” to summarize the negative alphas of high-beta and high-volatility stocks. The literature mostly considers both phenomena separately.

2 If investors are unable to diversify properly, Merton (1987) predicts a positive risk premium for volatility-risk. The negative relationship, however, remains a puzzle.

3 MacKinlay (1995) defines the optimal orthogonal portfolio as “*the unique portfolio given* $\overline{N}$ *assets that can be combined with the factor portfolios to form the tangency portfolio and is orthogonal to the factor portfolios*” (MacKinlay, 1995, p. 8).

4 Technically, the six-factor model *FF*6 replaces the operating profitability factor *RMW* with a cash profitability factor RMW_C. However, this version of the factor is not publicly available, and we use the initial definition of *RMW* instead, but refer to the model as *FF*6.

5 https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

6 https://finance.wharton.upenn.edu/~stambaug/

7 See https://www.openassetpricing.com/data/ for the full data description.

8 See https://pages.stern.nyu.edu/~jwurgler/. We use the orthogonalized BW Investor Sentiment Index. Note that the BW Sentiment Index is available until 2018.

9 The objective of the spanning regressions is not a model comparison, but the indication to what extent *FOP* explains existing asset pricing factors to reduce dimensionality. For an extensive comparison of factor models we refer to Fama and French (2018), Ahmed et al. (2019) and Hou et al. (2019).

10 In line with the fact that β_{OP} enters the variance covariance matrix of returns in Equations (4) and (7) in squared terms, this exposure is fairly close to the squared exposure of high-β_{Mkt} deciles. This observation supports the hypothesis that both anomalies are driven by the same latent factor.

11 To put this into perspective, the *FF*6 model attains a GRS test statistic of 3.784 (p-value $< .001$) and the negative Var spread in the second *Size* quintile remains significant as well.

12 In a previous version, we estimated *FOP* from a mimicking factor regression, which adds an errors-in-weights problem as an additional layer of estimation error (see Jiang et al., 2014; Kleibergen & Zhan, 2018). We thank an anonymous referee for pointing out the issues highlighted above.

13 In unreported robustness checks, we account for the following alternatives, but find no significant evidence: The CBOE VIX (Ang et al., 2006; Barinov, 2018), average variance (Chen & Petkova, 2012), the CFNAI, Economic Policy Uncertainty of Baker et al. (2016), inflation (Cohen et al., 2005), the term spread, the earnings price ratio and the default yield spread (all as defined in Welch & Goyal, 2008).

14 We refer to Asness et al. (2018) for the discussion regarding the interpretation of margin debt as a measure of leverage constraints. Unreported robustness checks reveal that the detrended time series exhibits an even better predictive power for the Frazzini and Pedersen (2014) betting-against-beta factor *BAB*, a key result in Asness et al. (2018).

15 Including both, *Mkt* and *FOP* yields qualitatively identical results.

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How to cite this article: Dierkes, M., & Schroen, S. (2023). Betting against sentiment? Seemingly unrelated anomalies and the low-risk effect. *Review of Financial Economics*, *41*, 152–176. https://doi.org/10.1002/rfe.1170

APPENDIX A

A.1 | CORE ANOMALIES

Following Fama and French (1993, 2015, 2016), we consider the following setup of anomalies. All portfolios are formed on NYSE breakpoints at the end of June each year. Double-sorted portfolios are independent sorts with NYSE breakpoints as well.

Accruals (Accr): Sloan (1996) shows that companies with high accruals earn lower future returns. Accruals are the change in operating working capital per split-adjusted share divided by the book equity per share (Fama & French, 2016, p. 74).

Book-to-Market (BM): Fama and French (1993) show that average returns are related to the book-to-market ratio which is defined as the ratio of book equity to market equity.

Investments (Inv): Investments is the growth of total assets from the fiscal year $t-2$ to $t-1$ (Fama & French, 2015, p. 4).

Dividend Yield (DivYld): Fama and French (1988) show that the dividend/price ratio or dividend yield is informative about average returns and Fama and French (1993) use these sorts to challenge their three-factor model. The dividend yield used to form portfolios is the total dividends paid from July of $t-1$ to June of t per dollar of equity in June of t.

Investments (Inv): Investments is the growth of total assets from the fiscal year $t-2$ to $t-1$ (Fama & French, 2015, p. 4).

Long-term Reversal (LRev): Long-term reversal is the prior return over the prior 13 to 60 months.

Momentum (Mom): Momentum, as documented by Jegadeesh and Titman (1993), is the cumulative return over the prior 2 to 12 months (Fama & French, 2016, p. 75).

Net Share Issues (NetIss): Returns following share issues are lower, as documented by Loughran and Ritter (1995). We use decile portfolios formed on NetIss, defined as the change in the natural log of split-adjusted shares outstanding from fiscal year-end in $t-2$ to $t-1$ (Fama & French, 2016, p. 74).

Operating Profitability (Prof): Novy-Marx (2013) shows that profitable firms earn higher returns. Operating profitability is annual revenues minus cost of goods sold, interest expense, selling, general, and administrative expenses divided by book equity (Fama & French, 2015, p. 4).

Short-term Reversal (ShRev): Short-term Reversal is the return in the previous month.

Size (Size): Size is the market equity at the end of June.

Return Variance (Var): Ang et al. (2006) show that highly volatile stocks earn lower future returns. We consider portfolios on the variance of daily returns over the previous 60 days with a minimum of 20 days (Fama & French, 2016, p. 74).

Market Beta (β_{Mkt}): Market Beta is estimated over the previous 5 years of monthly returns with a minimum of 24 observations (Fama & French, 2016, p. 74).

A.2 | ALTERNATIVE SORT VARIABLES

Stock characteristics for the alternative sorts are from open-source asset pricing as documented in Chen and Zimmermann (2021) and all sorts are based on NYSE breakpoints with monthly rebalancing.

Dimson (1979) Beta (β_{Mkt}^{Dimson}): Chen and Zimmermann (2021) implement the Dimson (1979) sum of coefficients method by regressing the daily stock return of a firm on the same-day, one-day ahead and one-day lagged return of the market portfolio in one-month rolling windows. β_{Mkt}^{Dimson} is the sum of the individual coefficients. At least 15 valid daily returns are required.

Frazzini and Pedersen (2014) Beta ($\beta_{Mkt}^{F\&P}$): Frazzini and Pedersen (2014) estimate beta of stock i as

$$\beta_i = \rho \frac{\sigma_i}{\sigma_m}$$

where ρ is the correlation between returns of stock i and the market portfolio and σ_i (σ_m) is the volatility of stock i (the market portfolio). Volatilities are estimated from a one-year rolling window with daily log returns, correlations are estimated from five-year windows of overlapping three-day log returns to account for non-synchronous trading (see Frazzini & Pedersen, 2014, p. 8). At least six months (three years) of data are required to estimate volatilities (correlations).

Idiosyncratic Volatility (IVol): Idiosyncratic volatility is the standard deviation of Fama and French (1993) three-factor model residuals over the previous month, estimated from within-month regressions with at least 15 daily returns. High *IVol* stocks earn low returns and negative alphas (Ang et al., 2006).