MARSIS Radar: TEC estimation analysis

Issue 1, Rev 0

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1. Introduction

In this document, we describe a study to evaluate the possibility to a further improvement of the Total Electron Content of Mars's Ionosphere using the outputs parameters produced by the Contrast Method algorithm used for the on-board and the on- ground processing of the MARSIS radar, in order to compensate the ionosphere distortions that affected the data collected by MARSIS in its subsurface operation mode.

2. The distortion introduced by the martian ionosphere on MARSIS signal

As it is well known the presence of the Martian ionosphere produces a variation of the refraction index respect to the vacuum.

Therefore, an electromagnetic wave of frequency f propagating in the ionosphere is characterized by the following refraction index

$$n(z) = \sqrt{1 - \frac{f_p^2(z)}{f^2 - jf\nu}} \cong \sqrt{1 - \frac{f_p^2(z)}{f^2}} \qquad (1)$$

where f_p is the plasma frequency, ν the electron-neutral collision frequency and z is the altitude above ground. Considering a typical MARSIS operation frequency (i.e. in the 1.3-5.5 MHz range), the imaginary term in the denominator of Eq. (1) can be neglected, because $\nu \sim 10 - 60$ kHz. The plasma frequency, in Hz, can be written as

$$f_p(z) = 8.98\sqrt{N_e(z)}, \qquad (2)$$

where N_e is the electron density in m^{-3}. The maximum value of f_p obviously corresponds to the maximum value of the electron density N_{emax}.

According to Eq. (1) all frequencies lower than f_p will be reflected, while if the radar signal has a wide band, the propagation speed is not constant through the band itself and a frequency dependent phase shift arises. In details, frequencies higher than f_p will be attenuated, delayed by an average delay (group delay) in signal travel time and dispersed depending on the electron density values encountered along the path.

The phase shift induced by the ionosphere in a radar signal of frequency f can be written as

$$\Delta\phi(f)=\frac{4\pi}{c}f\int_0^L[n(z)-1]dz=\frac{4\pi}{c}f\int_0^L\left[\sqrt{1-\left(\frac{f_p(z)}{f}\right)^2}-1\right]dz, \qquad (3)$$

where L is the ionosphere thickness and c is the speed of light in vacuum.

If f_0 is the central frequency of the radar signal band, we can perform a Taylor expansion of the integrand of Eq. (3) and then integrate each term of the expansion, so as to obtain

$$\Delta\phi(f)\cong a_0+a_1(f-f_0)+a_2(f-f_0)^2+a_3(f-f_0)^3+a_4(f-f_0)^4+..., \qquad (4)$$

where:

$$a_0 = \frac{4\pi}{c}\int_0^L \left(\sqrt{f_0^2 - f_p^2} - f_0\right) dz \quad [rad]$$

$$a_1 = \frac{4\pi}{c}\int_0^L \left(\frac{f_0}{\sqrt{f_0^2 - f_p^2}} - 1\right) dz \quad [rad / Hz]$$

$$a_2 = -\frac{4\pi}{c}\int_0^L \left(\frac{f_p^2}{2(f_0^2 - f_p^2)^{\frac{3}{2}}}\right) dz \qquad [rad / Hz^2] \qquad (5)$$

$$a_3 = \frac{4\pi}{c}\int_0^L \left(\frac{f_0 f_p^2}{2(f_0^2 - f_p^2)^{\frac{5}{2}}}\right) dz \quad [rad / Hz^3]$$

$$a_4 = \frac{4\pi}{c}\int_0^L \left(\frac{4 f_0^2 f_p^2 + f_p^4}{8\left(f_0^2 - f_p^2\right)^{\frac{7}{2}}}\right) dz \quad \left[rad / Hz^4\right]$$

3. TEC estimation methods

In Cartacci et al. 2013, we approximated the a_2 parameter of Eq. 5 as

$$a_2 \cong -\frac{4\pi}{c}\int_0^L \frac{1}{2}\frac{f_p^2}{f_0^3} dz = -\frac{2\pi}{c\, f_0^3}\ (8.98)^2 \int_0^L N_e\, dz \qquad (6)$$

The term a_2 is estimated through the Contrast Method (CM).

$$\boldsymbol{TEC} \cong -\frac{\boldsymbol{a_2 c\, f_0^3}}{\boldsymbol{2\pi(8.98)^2}} \qquad (7)$$

The TEC estimated in this way has a very good accuracy during the night side but the adopted approximation yields an overestimate of the TEC during the day side.

In Cartacci et al. 2017, we approximated the parameters a_1 and a_2 of Eq. 5 as

$$a_1 = \frac{4\pi}{c}\int\left(\frac{f_0}{\left(f_0^2 - f_p^2\right)^{\frac{1}{2}}} - 1\right)dz \approx \frac{4\pi}{c}\int\left(\frac{1}{2}\frac{f_p^2}{f_0^2} + \frac{3}{8}\frac{f_p^4}{f_0^4}\right)dz =$$

$$= \frac{2\pi}{c\,f_0^2}\,(8.98)^2\,TEC + \frac{3\pi}{2c\,f_0^4}\,(8.98)^4\int N_e^2(z)\,dz \qquad (8)$$

$$a_2 = -\frac{4\pi}{c}\int\left(\frac{f_p^2}{2\left(f_0^2 - f_p^2\right)^{\frac{3}{2}}}\right)dz \approx -\frac{2\pi}{c}\int\left(\frac{f_p^2}{f_0^3}\left(1 + \frac{3}{2}\frac{f_p^2}{f_0^2}\right)\right)dz =$$

$$= -\frac{2\pi}{c\,f_0^3}\,(8.98)^2 TEC - \frac{3\pi}{c\,f_0^5}\,(8.98)^4\int N_e^2(z)\,dz \qquad (9)$$

The term a_1 is estimated comparing the real and the simulated signal, while the term a_2 is estimated through the Contrast Method (CM).

$$\boldsymbol{TEC = \frac{(2a_1 + a_2 f_0)\,c\,f_0^2}{2\pi(8.98)^2}} \qquad \mathbf{(10)}$$

This approximation increases the TEC accuracy during the day side.

Starting from these results already published, we try to verify the possibility to improve again the TEC estimation adding one more term of the original Eq.5.

An alternative version of the CM estimates the term a_3 independently from the term a_2 (in the version implemented in the ground segment the term a_3 is estimated from the a_2 value). This solution allows to also consider the term a_3 in the algorithm to estimate the TEC.

Therefore, we approximate the parameters a_1, a_2 and a_3 of Eq. 3 as

$$a_1 = \frac{2\pi}{c\,f_0^2}\,(8.98)^2\,TEC + \frac{3\pi}{2c\,f_0^4}\,(8.98)^4\int_0^L N_e^2(z)dz + \frac{5\pi}{8c\,f_0^6}\,(8.98)^6\int_0^L N_e^3(z)\,dz$$

$$a_2 = -\frac{2\pi}{c\,f_0^3}\,(8.98)^2 TEC - \frac{3\pi}{c\,f_0^5}\,(8.98)^4\int_0^L N_e^2(z)\,dz - \frac{15\pi}{4c\,f_0^7}\,(8.98)^6\int_0^L N_e^3(z)dz \qquad (11)$$

$$a_3 = \frac{2\pi}{c\,f_0^4}\,(8.98)^2 TEC + \frac{5\pi}{c\,f_0^6}\,(8.98)^4 \int_0^L N_e^2(z)\,dz + \frac{35\pi}{4c\,f_0^8}\,(8.98)^6 \int_0^L N_e^3(z)\,dz$$

The term a_1 is estimated comparing the real and the simulated signal.

The terms a_2 and a_3 are estimated through the Contrast Method.

$$\boldsymbol{TEC = \frac{3a_1cf_0^2}{2\pi(8.98)^2} + \frac{11a_2cf_0^3}{8\pi(8.98)^2} + \frac{3a_3cf_0^4}{8\pi(8.98)^2}} \qquad (\mathbf{12})$$

Pushing further our analysis, we can try to use all the terms of eq. 5 to estimate the TEC.

In this case we obtain:

$$a_1 = \frac{2\pi}{c\,f_0^2}\,(8.98)^2 \left(TEC + \frac{3}{4}\,\frac{(8.98)^2}{f_0^2} \int_0^L N_e^2(z)dz + \frac{5}{8}\,\frac{(8.98)^4}{f_0^4} \int_0^L N_e^3(z)\,dz + \frac{35}{64}\,\frac{(8.98)^6}{f_0^6} \int_0^L N_e^4(z)\,dz \right)$$

$$a_2 = -\frac{2\pi}{c\,f_0^3}\,(8.98)^2 \left(TEC + \frac{3}{2}\,\frac{(8.98)^2}{f_0^2} \int_0^L N_e^2(z)dz + \frac{15}{8}\,\frac{(8.98)^4}{f_0^4} \int_0^L N_e^3(z)\,dz + \frac{35}{16}\,\frac{(8.98)^6}{f_0^6} \int_0^L N_e^4(z)\,dz \right)$$

$$a_3 = \frac{2\pi}{c\,f_0^4}\,(8.98)^2 \left(TEC + \frac{5}{2}\,\frac{(8.98)^2}{f_0^2} \int_0^L N_e^2(z)dz + \frac{35}{8}\,\frac{(8.98)^4}{f_0^4} \int_0^L N_e^3(z)\,dz + \frac{105}{16}\,\frac{(8.98)^6}{f_0^6} \int_0^L N_e^4(z)\,dz \right)$$

$$a_4 = -\frac{2\pi}{c\, f_0^5}\,(8.98)^2\left(TEC + \frac{15}{4}\,\frac{(8.98)^2}{f_0^2}\int_0^L N_e^2(z)dz + \frac{35}{4}\,\frac{(8.98)^4}{f_0^4}\int_0^L N_e^3(z)\,dz + \frac{63}{32}\,\frac{(8.98)^6}{f_0^6}\int_0^L N_e^4(z)\,dz\right)$$

$$\boldsymbol{TEC} = \frac{178}{61}\left(\frac{a_1 c f_0^2}{2\pi(8.98)^2}\right) + \frac{1247}{488}\left(\frac{a_2 c f_0^3}{2\pi(8.98)^2}\right) + \frac{291}{488}\left(\frac{a_3 c f_0^4}{2\pi(8.98)^2}\right) - \frac{5}{122}\left(\frac{a_4 c f_0^5}{2\pi(8.98)^2}\right) \qquad (13)$$

The possibility to use more terms allows to improve the approximation, increasing the accuracy in the TEC estimation during the day side.

During the night side the TEC will be estimated with the previous algorithm in order to reduce the processing time.

In fig. 1 there is a preliminary analysis of the obtained results.

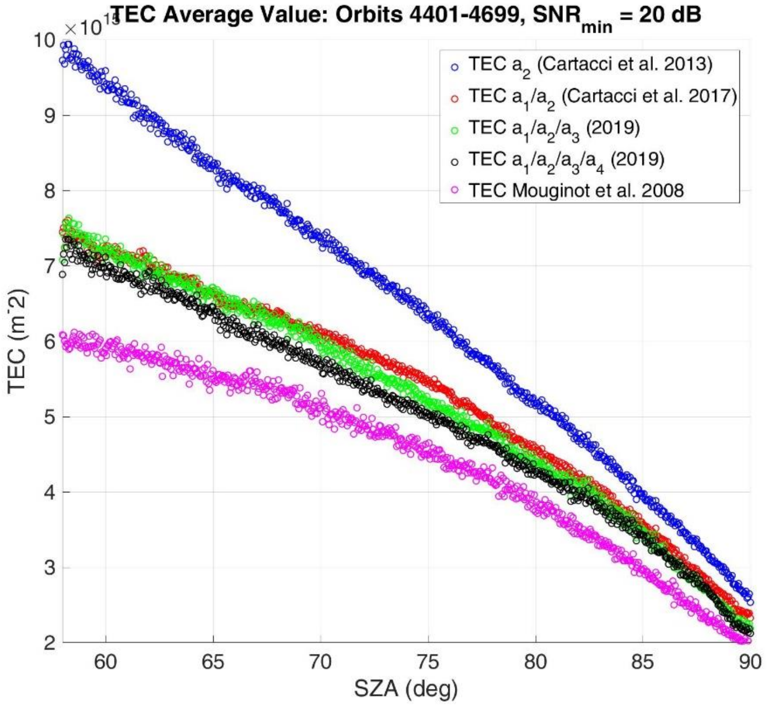

Fig.1

The figure shows the TEC values obtained averaging the values of almost 300 orbits collected with the same Solar Zenith Angle (SZA) with a range bin of 0.1° of SZA.

The only constraint is related to the SNR that must be greater than 20dB, in order to avoid that signals of poor performances can reduce the accuracy of this statistical analysis.

As it is clear from the figure, our study is focused on the differences that the various methods show during the day side while during the night side their accordance is good.

4. Discussion and summary

In this document we analyzed the performances of different methods to evaluate the Mars Ionosphere TEC.

The methods developed from eq. 5 seems to improve the TEC accuracy in particular respect to the older one obtained from Cartacci et al. 2013.

Anyway, there are some doubts regarding the opportunity to use all the parameters of eq. 5. Actually, there is the possibility that equations. 12 and 13 can decrease the estimation accuracy rather than increase it, due to the use of a consistent number of approximations in their development.

In order to solve our doubts, the next step of the analysis will consist in the integration of the NeMars Model of the ionosphere (Sánchez – Cano et al. 2013) in the algorithm, this could allow to verify the correctness of each TEC estimation.

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