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Helium abundance in globular clusters: the R-method

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Summary. A new estimate of the helium abundance (Y) in globular cluster stars has been obtained through the application of the R-method (Iben, 1968) to a sample of galactic globular clusters.

A new calibration for the $(R-Y)$ relation has been derived. The influence on the calibration of some theoretical uncertainties and of phenomena like mass loss have been analysed.

The procedure, applied to a sample of 15 clusters whose data reach a level of completeness and photometric accuracy enough to give statistically significant results, led to a mean value for Y equal to 0.23 ± 0.02.

A constant Y with varying metal content seems consistent with the data.

The distribution of the number ratios of asymptotic and giant branch stars suggests the full development of the semi-convective zone for the stars of all the considered clusters.

Key words: globular clusters – helium abundance – R-method

1. Introduction

There are many different ways of estimating the mean helium abundance in globular clusters (Iben, 1974; Caputo, 1983 and references therein). Most of them agree on values of Y ranging in the interval 0.2–0.3 and, since the mean associated error is hardly less than 0.05, the usual conclusion is that the helium abundance of globular cluster stars is not inconsistent with the value ($Y=0.23$, Wagoner, 1973) given by the big bang cosmology.

However, apart from this generic deduction, the helium problem is still unsolved. In fact, a precise evaluation of the absolute helium content is far from being achieved. Moreover, Y-variations among galactic globular clusters have been deduced anticorrelated (Sandage, 1981) or correlated (Caputo and Cayrel, 1981) with the metal abundance depending on the method used for the determinations.

Given the conflicting results and the large impact of the problem on so many fields in astrophysics, a careful revision of the various methods and data seems absolutely necessary.

In this paper, we focus our attention on the R-method first suggested by Iben (1968). Since then, this procedure has been used on various samples of clusters by many authors (Iben and Rood, 1969; Iben et al., 1969; Demarque et al., 1972; Renzini, 1977; Caputo et al., 1978, 1980; Arimoto and Simoda, 1981) who reached however different conclusions.

The R-method consists in the comparison between theoretical lifetimes of model stars on the giant branch (RGB) and on the horizontal branch (HB) with the observed number ratios of RGB stars and HB stars: $R=N_{\mathrm{HB}}/N_{\mathrm{RGB}}=t_{\mathrm{HB}}/t_{\mathrm{RGB}}$. Iben and Rood (1969) showed that R depends almost exclusively on Y. The method, straightforward in principle, suffers however from the uncertainties in the theoretical calibration of the $(R-Y)$ relationship mainly due to the presence of semiconvection in HB stars, and should be applied only to complete and very populous samples.

In the following sections we derive a new calibration of R and apply the procedure to all the clusters for which complete and populous samples of stars have been observed and whose data have photometric quality good enough to allow separation between the different branches.

2. Theoretical relations

The procedure to get the theoretical $(R-Y)$ calibration has been extensively discussed by Renzini (1977) and need not be repeated here.

The basic relations we use have been obtained through analytic fits to the evolutionary tracks tabulated by Sweigart and Gross (1976, hereafter SG 76) for the HB and Sweigart and Gross (1978, hereafter SG 78) for the RGB.

a) Basic formulae

The HB tracks were computed by SG 76 following the so called "canonical semiconvection prescriptions" (see Renzini, 1977, for details). Since they end when the helium abundance in the core – Y_c – is 0.05, t_{HB} – the HB lifetime – has been increased by 5% to take into account the time spent during the phase of core helium exhaustion. Core helium exhaustion may last somewhat longer if the final helium burning does not proceed smoothly, as has been suggested by several calculations (Sweigart and Demarque, 1973; Gingold, 1976) which show a large and rapid growth of the convective core when Y_c is ~ a few percent. An increase in the length of the core helium exhaustion phase slightly larger (a few percent) than that here considered would lead to a lower estimated Y by less than 0.01. The fit we derived for t(HB) (in 10^6 yr) is:

$$\mathrm{Log}\, t_{\mathrm{HB}} = -0.22\, M_{\mathrm{HB}} - 2.58\, M_c - 0.21\, Y + 0.01\, \mathrm{Log}\, Z + 3.51 \quad (1)$$

Send offprint requests to: F. Fusi Pecci with an accuracy of about 4% in the range $M_c = 0.425–0.575 M_\odot$; $M_{\rm HB} = 0.45 - 1.05 M_\odot$ (being $M_{\rm HB}$ the total mass of the star in the HB); $Y = 0.2 - 0.3$; $\mathrm{Log} Z = -4$ to -2.

The core mass, M_c, is given by:

$$M_c = -0.023 M_{\rm RGB} - 0.221 Y - 0.009 \,\mathrm{Log} Z + 0.534 \qquad (2)$$

with an error of less than $0.003 M_\odot$ in the interval $M_{\rm RGB} = 0.7 - 1.3 M_\odot$ [being $M_{\rm RGB}$ the total mass of a star experiencing the RGB phase, see Eq. (7)]; $Y = 0.2 - 0.3$; $\mathrm{Log} Z = -4$ to -1.4.

The lifetime – $t_{\rm RGB}$ – spent on the RGB by stars brighter than the average luminosity of the RR Lyr variables – $L_{\rm RR}$ – can be computed making use of the following relations derived from the models:

$$\mathrm{Log}(t_{\rm RGB} - \tau) = \mathrm{A} \frac{\mathrm{Log} L}{\mathrm{Log} L_{\rm fl}} + \mathrm{B} \quad \text{for} \quad \frac{\mathrm{Log} L}{\mathrm{Log} L_{\rm fl}} = 0.0 - 0.8, \qquad (3a)$$

$$t_{\rm RGB} = t_{\rm RGB}(0.8) \left[\frac{1 - \frac{\mathrm{Log} L}{\mathrm{Log} L_{\rm fl}}}{0.2} \right]^\beta \quad \text{for} \quad \frac{\mathrm{Log} L}{\mathrm{Log} L_{\rm fl}} = 0.8 - 1.0, \qquad (3b)$$

where $L_{\rm fl}$ is the luminosity of the star at the helium flash given by:

$$\mathrm{Log} L_{\rm fl} = (0.75 Y - 0.30) M_{\rm RGB} + 0.09 \,\mathrm{Log} Z - 1.12 Y + 3.93 \qquad (4)$$

with an error of 0.03 mag in the range quoted for Eq. (2) and

$$A = (-5.05 - 0.53 \,\mathrm{Log} Z) 10^{-4} M^8_{\rm RGB} + 0.565 Y - 0.011 \,\mathrm{Log} Z - 3.032$$

$$B = 4.32\, 10^{-4} M^8_{\rm RGB} - 0.773 Y - 0.075 \,\mathrm{Log} Z + 3.243$$

$$\beta = -0.11 M^2_{\rm RGB} - 1.15 Y + 1.79$$

$$t_{\rm RGB}(0.8) = t_{\rm RGB} \quad \text{for} \quad \frac{\mathrm{Log} L}{\mathrm{Log} L_{\rm fl}} = 0.8 .$$

τ has been inserted to take into account the effects on evolutionary time-scales of the brief hesitation in the rate of evolution up the giant branch when the hydrogen-burning shell reaches the hydrogen discontinuity left by the inward penetration of the convective envelope (Rood, 1972; Sweigart, 1978). In many cases this hesitation is accompanied by an actual drop in the surface luminosity (SG 78). Moreover, due to this behaviour, there is an enrichment in the surface helium abundance by an amount ΔY_s. Interpolation on the tracks gives respectively:

$$\tau = t_{\rm DROP} \quad \text{if} \quad L \leqq L_{\rm DROP}$$

$$\tau = 0 \quad \text{if} \quad L > L_{\rm DROP}$$

with

$$\mathrm{Log} L_{\rm DROP} = (0.07 \,\mathrm{Log} Z + 0.67) M_{\rm RGB} + 1.97 Y - 0.49 \,\mathrm{Log} Z - 0.32, \qquad (5a)$$

$$\mathrm{Log} t_{\rm DROP} = (-3.064 Y + 0.179)(0.9 - M_{\rm RGB}) - 2.483 Y + 0.416 \,\mathrm{Log} Z + 2.308 \qquad (5b)$$

in the range $M_{\rm RGB} = 0.5 - 0.9 M_\odot$, ($t_{\rm DROP}$ in 10^6 yr), and

$$\Delta Y_s = 0.028 M_{\rm RGB} - 0.028 Y - 0.004 \,\mathrm{Log} Z + 0.009 \qquad (6)$$

in the range $M_{\rm RGB} = 0.5 - 1.3 M_\odot$.

Given the age of the cluster – t_9 (in 10^9 yr) –, the total mass of a star at the turn off point – $M_{\rm to}$ – and the mass of a red giant branch star – $M_{\rm RGB}$ – can be deduced from Ciardullo and Demarque (1977) through:

$$M_{\rm RGB} = M_{\rm to} + \Delta M, \qquad (7a)$$

$$\mathrm{Log} M_{\rm to} = (0.467 Z + 0.035 Y - 0.286) \,\mathrm{Log} t_9 + (0.116 - 0.05 Y) \,\mathrm{Log} Z + 0.016 \,\mathrm{Log}^2 Z - 0.920 Y + 0.609, \qquad (7b)$$

$$\Delta M = 0.011 \,\mathrm{Log} Z - 0.07 Y + 0.073 \qquad (t_9 > 13), \qquad (7c)$$

$$\Delta M = (0.01 Y - 0.002 \,\mathrm{Log} Z - 0.012) t_9 + 0.035 \,\mathrm{Log} Z - 0.18 Y + 0.222, \qquad (7d)$$

with an error of about 1% in the interval $t_9 = 1 - 15$; $Y = 0.2 - 0.3$; $\mathrm{Log} Z = -4$ to -1.4.

The whole procedure to derive the time along the RGB leads to an error of about 5.4% in the interval: $M_{\rm RGB} = 0.7 - 2.2 M_\odot$; $Y = 0.2 - 0.3$; $\mathrm{Log} Z = -4$ to -1.4. Thus, given t_9, Y and Z, $t_{\rm RGB}$ can be obtained by inserting $L_{\rm RR}$ into Eq. (3).

As far as $L_{\rm RR}$ is concerned, one has:

$$\mathrm{Log} L_{\rm RR} = \mathrm{Log} L_{3.85} + \Delta_{\rm ev}, \qquad (8)$$

where $L_{3.85}$ is the luminosity of the zero age horizontal branch (ZAHB) at $\mathrm{Log}(T_{\rm eff}) = 3.85$, and $\Delta_{\rm ev}$ is the mean luminosity excess of RR Lyr stars with respect to the ZAHB due to evolution. Furthermore:

$$\mathrm{Log} L_{3.85} = 3.04 M_c + 2.07 Y - 0.04 \,\mathrm{Log} Z - 0.48 \qquad (9)$$

with an error of about 0.02 mag in the same range as Eq. (1), and

$$\Delta_{\rm ev} = 0.5 Y - 0.05 .$$

Having defined $R = t_{\rm HB}/t_{\rm RGB}$, we have computed a grid of R-values with varying t_9, Z, Y using for the mass of the HB stars those pertinent to models with ZAHB effective temperature $\mathrm{Log}(T_{\rm eff}) = 3.85$. This mass – $M_{3.85}$ – is given by the following formula:

$$M_{3.85} = 1.89 M_c + 0.35 Y - 0.12 \,\mathrm{Log} Z - 0.72 . \qquad (10)$$

Finally, a numerical interpolation on the whole grid gives:

$$Y = 0.380 \,\mathrm{Log} R + 0.176 . \qquad (11)$$

Since from the computed grid it can be seen that the dependence of R on t_9 is negligible and that on Z is very weak (at least, until $\mathrm{Log} Z < -2.5$), both terms have been taken into account through a weighted mean of the different cases and thus the explicit dependence has been neglected in the final formula. In addition two more terms should have been included. They are related to the average excess of core mass of HB and RR Lyr stars respectively in comparison with the value derived from Eq. (2). As widely dealt with by Renzini (1977), these two terms can be ignored (as a first order approximation) since they have opposite signs and very similar coefficients. Therefore, what actually matters is their difference which is generally small and function of the HB morphology. In fact, if there is a dispersion in M_c among HB stars (with bluer ZAHB models with increasing M_c), this difference is positive if RR Lyr stars are redder than the average color of HB stars and negative if they are bluer.

The last term to deal with in Eq. (8) is $\Delta_{\rm ev}$. As can be seen from the HB evolutionary tracks, their width in luminosity is a sensitive function of Y and very accurate photometric data may allow some deduction on Y from the measure of the thickness of the HB (Sandage, 1969; Harris, 1982; Buonanno et al., 1983a). However, since $\Delta_{\rm ev}$ depends strongly also on the HB morphology, its contribution may be variable from cluster to cluster and can hardly be inserted in a precise way in the general formula (8).

Before analysing the relevance on the $(R-Y)$ calibration of the uncertainties involved in the theoretical determination and of other phenomena not included in the previous treatment, we deduce a second parameter $R'=t_{\rm HB}/(t_{\rm RGB}+t_{\rm AGB})$ where $t_{\rm AGB}$ is the time spent by stars on the asymptotic giant branch (AGB). The knowledge of R' may be useful for the study of clusters for which the separation of RGB and AGB stars is impossible due to photometric problems and/or to the intrinsic morphology of the two branches. A rough indication for R' can already be derived by assuming for $t_{\rm AGB}$ a mean lifetime of 15 millions years (Gingold, 1974). In this case one has:

$$Y=0.457\,{\rm Log}R'+0.204\,. \qquad (12)$$

A precise calibration requires instead the use of a wide grid of AGB evolutionary tracks not available up to now.

b) $M_{3.85}$ *and the average mass of* HB *stars in a cluster*

In the computation made to derive Eqs. (11) and (12) we used $M_{\rm HB}=M_{3.85}$ following the procedure applied by previous authors. However, the average mass of a HB star in a given globular cluster might differ significantly from the ZAHB mass at ${\rm Log}\,T_{\rm eff}=3.85$ depending on the HB morphology. This effect would introduce a systematic trend in the estimated helium abundance between globular clusters with very blue HB's and those with very red HB's since one has $M_{3.85}>M_{\rm HB}$ and $M_{3.85}<M_{\rm HB}$ in the two cases, respectively. Given the rather weak dependence of $t_{\rm HB}$ on $M_{\rm HB}$ [see Eq. (1)] this effect is usually small but may be important when the difference in mass between the mean $M_{\rm HB}$ and $M_{3.85}$ becomes significant as it happens for instance with high values of metal abundance. In the extreme cases, due to the dependence of $M_{\rm HB}$ on Z, the quoted difference may reach a value up to $0.2\,M_\odot$ which implies an uncertainty in the estimated Y of about 0.02.

c) Influence of mass loss

As previously stated, the computations which led to Eqs. (11) and (12) have been made neglecting phenomena – like mass loss – which take place in a significant way in the considered evolutionary phases. For instance, constant mass evolution would imply the existence of many very bright AGB stars, up to 2–3 mag above the red giant branch tip, in the C–M diagram of globular clusters contrary to all observational evidence (Renzini, 1977). Therefore, it may be worthwhile checking the importance of mass loss in the calibration of the $(R-Y)$ relation.

The need for a substantial mass loss ($0.2\,M_\odot$) for Population II stars was already suggested by Castellani and Renzini (1968) and Iben and Rood (1970) in their early studies on HB morphologies. It is now accepted that mass loss is efficient along both RGB and AGB evolutionary phases (Renzini, 1977) but both mechanism(s) responsible for, and the dependence on the basic parameters of the star are far from being understood.

In order to estimate the influence of mass loss on R and R' determinations we have thus computed some grids of R and R'-values taking into account mass loss through the procedure developed by Fusi Pecci and Renzini (1976) and Renzini (1977). Using Reimers' (1975) empirical formula for the mass loss rate, introducing the parameter η as an uncertainty factor and imposing $M_{\rm HB}=M_{\rm RGB}-\delta M$, where δM is the total amount of mass lost during the RGB phase, we have:

$$Y=(0.37-\eta/33)\,{\rm Log}R+(0.186-\eta/55)\,, \qquad (13)$$

$$Y=(0.43-\eta/50)\,{\rm Log}R'+(0.210-\eta/100)\,. \qquad (14)$$

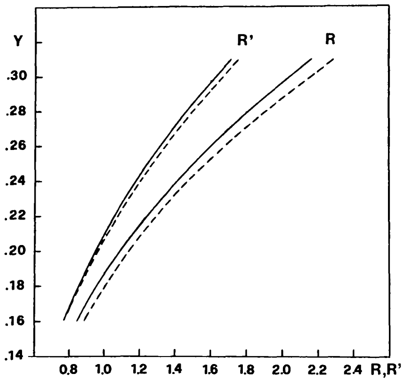

Fig. 1. Plot of the $(R-Y)$ calibration presented in Eq. (13) for two values of η. The solid curve refers to $\eta=0$, the dashed one to $\eta=1/3$

Figure 1 presents the theoretical $(R-Y)$ calibration given in Eq. (13). As can be seen from the two last equations if a value of η close to 1/3 is used (Renzini, 1981a), the error in Y made neglecting the presence of mass loss is of the order of 0.01.

d) Influence of the treatment of neutrino energy losses

A further aspect which can be taken into account using SG 78 tables is the treatment of neutrino energy losses via direct electron-neutrino interactions. Using F_ν – the numerical factor introduced by SG 78 – we estimate:

$${\rm Log}R(F_\nu)={\rm Log}R(1)-0.07(\sqrt{F_\nu}-1)\,. \qquad (15)$$

As in the case of mass loss the dependence is rather weak. If $F_\nu=1.48$ is used, as quoted by SG 78, the difference in the derived Y-values turns out to be of about 0.005.

e) Dependence on core mass, helium flash luminosity, and average RR Lyr *luminosity*

Uncertainties still remain also in the input physics which led to the determination of parameters like M_c, $L_{\rm fl}$, and $L_{\rm RR}$. Therefore, it may be useful to consider their possible influence on the $(R-Y)$ calibration. In the previous computations we used:

$$R=\frac{t_{\rm HB}\{M_c(M_{\rm RGB}\,;Y;Z);Y;Z;M_{\rm HB}\}}{t_{\rm RGB}\{L_{\rm RR}[M_c(M_{\rm RGB}\,;Y;Z)]\,;Y;Z;M_{\rm RGB}\,;L_{\rm fl}\}}$$
$$=R(Y)=R_{\rm obs}\,. \qquad (16)$$

This formula enables us to discuss various aspects separately remembering that any change in the input physics may have two distinct effects: the first, direct, and the second, indirect through the implicit dependencies. In this respect, the case of the helium flash luminosity is typical. In fact, a significant change (up to half a magnitude) in the flash luminosity has a negligible direct effect since it modifies by only a few million years $t_{\rm RGB}$ (SG 78). The flash luminosity strongly affects instead $M_{\rm HB}$ because of the importance of mass loss close to the red giant tip. On the other hand, as already discussed, the variation of $M_{\rm HB}$ induced by a change in $L_{\rm fl}$ has (usually) a small effect on the $(R-Y)$ calibration given the weak dependence of $t_{\rm HB}$ on $M_{\rm HB}$. Therefore, the direct dependencies on both $L_{\rm fl}$ and $M_{\rm HB}$ can be generally neglected and the discussion can be reduced to the analysis of the effects of changes in two main quantities: M_c and $L_{\rm RR}$.

The possibility that current "canonical" theoretical M_c values are not necessarily correct has been suggested by many authors in the attempt of explaining both the HB morphologies and the RR Lyr properties in globular clusters. In fact, phenomena like rotation and neutrino losses and the uncertainties in the electron-conduction coefficient in the partially relativistic regime (Renzini, 1983) may influence strongly the core mass leading presumably, if neglected, to an underestimate of M_c. As shown in formula (16), any change in M_c is inherently associated with changes in $t_{\rm HB}$ and, through $L_{\rm RR}$, in $t_{\rm RGB}$. From our computation we have $\partial\,{\rm Log}R/\partial M_c=-2.0$. Any change in the input physics which affects the core mass of the star during its evolution leads thus to a corresponding variation in the derived Y which can be obtained through the following procedure: from Eqs. (11) or (13)

(i) $Y=A\,{\rm Log}R+B$

the change in input physics leads to:

(ii) ${\rm Log}R^*={\rm Log}R+(\partial\,{\rm Log}R/\partial M_c)\delta M_c$

hence

(iii) $Y=A\,{\rm Log}R^*+B-A(\partial\,{\rm Log}R/\partial M_c)\delta M_c$.

Since the $(R-Y)$ calibration has to be used to derive Y starting from the observed R-value – $R_{\rm obs}$ – one has to substitute $R=R_{\rm obs}$ and $R^*=R_{\rm obs}$ and derive thus two different estimates for Y according to which formula is used:

$$Y_0=A\,{\rm Log}R+B$$

$$Y_1=A\,{\rm Log}R^*+B-A(\partial\,{\rm Log}R/\partial M_c)\delta M_c$$

and then:

$$\delta Y=Y_1-Y_0=-A(\partial\,{\rm Log}R/\partial M_c)\delta M_c=0.7\delta M_c\,.$$

A second aspect to deal with is represented by possible uncertainties in the input physics which affect $L_{\rm RR}$ without affecting the previous RGB phase. Renzini (1983), for instance, suggests that a revision of current opacity tables around 10^6 K may be necessary to explain observed period shifts among RR Lyr variables in cluster. This refinement of the input physics of the evolutionary models implies presumably a change in $L_{\rm RR}$ whose sensitivity to metal abundance should be increased. The effects of any variation in $L_{\rm RR}$ can thus be estimated starting from $\partial\,{\rm Log}R/\partial\,{\rm Log}L_{\rm RR}=0.84$ and through the procedure already applied, one has: $\delta Y=-0.29\,\delta\,{\rm Log}L_{\rm RR}$.

f) Influence of the treatment of semiconvection

As far as the dependence of the $(R-Y)$ calibration on the treatment of semiconvection is concerned, it is well known that the inclusion of semiconvection lengthens the HB lifetime by almost a factor of 2. Since this implies a strong reduction of the deduced value of Y, doubts on the validity of the R-method (Dupree et al., 1978) or on the full development of the semiconvection zone (Arimoto and Simoda, 1981) have been proposed. As suggested by Renzini (1977) and showed in the following section, most of the discrepancies found can be solved with a careful check of the sample of clusters whose data have been used to determine Y through the R-method. Moreover, a further insight on the problems related to semiconvection can be derived using a third parameter R_1, defined by Caputo et al. (1978) as the ratio of the number of AGB stars up to 2.5 mag above the luminosity of RR Lyr stars to the number of RGB stars above the same magnitude level. In fact, Caputo et al. (1978) obtained $R_1=0.8$ from calculations made in absence of semiconvection by Iben and Rood (1970) and Rood (1972), while $R_1=0.2$ is expected including semiconvection. Therefore, the estimate of R_1 from a sample of clusters with appropriate data can also clarify this aspect.

3. Observational data

The basic problem is thus the determination of R and R_1 from star counts for the widest number of clusters. Although C–M diagrams are available for the majority of galactic globular clusters, very few are made using complete and populous samples with the necessary photometric accuracy. Since a careful selection of the cluster to be considered in the sample is a fundamental step to draw significant results, it is important to fix the prerequisites which should be fulfilled by the data and/or should be taken into account in the determination of R.

First, the limit in magnitude of the plates used for the photometry should be deep enough to guarantee that no blue extension of the HB has been missed.

Second, all the stars above a fixed level of magnitude should be measured over a well defined area.

Third, contamination due to field stars has to be removed.

Fourth, one needs a criterion to separate AGB and RGB stars if the separation is not clear, and also the transition from the red HB to the AGB has to be clearly defined.

Fifth, the determination of $L_{\rm RR}$ requires a problematic extrapolation when the strip is not populated.

Sixth, a differential reddening correction must be applied between the instability strip position on the HB and RGB stars.

Seventh, all the RR Lyr variables included in the considered area should be inserted in the compilation.

Eighth, the radial behaviour of R can be examined and the mean value adopted unless a statistically significant radial variation is shown.

In this context, it is easy to realize how contradicting results can be deduced using the R-method. For instance, complete samples (in the sense of measuring all the stars down to a fixed magnitude level over a well defined large area) have never been made until the introduction into globular cluster photometry of reduction methods using PDS data analysis (see for instance, Buonanno et al., 1981). Moreover, many studies of cluster $C-M$ diagrams in the past were made to reach the RR Lyr magnitude level and large portions of the blue HB have been missed.

In order to obtain a sample of clusters as wide as possible without inserting bias estimates, we analysed all the available $C-M$ diagrams and selected only those fulfilling at a sufficient degree the quoted prerequisites. In particular, we looked carefully at the photometry of the HB's and at the degree of completeness of the samples over defined areas.

As can be seen from the references in Table 1, where the data for the considered clusters are given, besides the latest $C-M$

Table 1. Data for galactic globular clusters

Cluster	N_{HB+RR}	N_{RR}	N_{RGB}	N_{AGB}	R	R'	Y(R)	Y(R)	Y(R')	Y(R')	R1	References
104 47Tuc+	365	0	208	45	1.75±0.21	1.44±0.17	0.27±0.02	0.27	0.27±0.02	0.27	0.22±0.05	Lee 1977a
362	78	4	65	13	1.20 0.28	1.00 0.23	0.21 0.04	0.21	0.21 0.04	0.20	0.20 0.08	Harris 1982
1851	101	7	70	15	1.44 0.32	1.19 0.25	0.24 0.03	0.24	0.24 0.04	0.24	0.21 0.08	Stetson 1981
3201	175	60	121	19	1.45 0.24	1.25 0.20	0.24 0.03	0.24	0.25 0.03	0.25	0.16 0.05	Lee 1977c
4147	59	14	38	7	1.55 0.45	1.31 0.37	0.25 0.05	0.25	0.26 0.05	0.26	0.18 0.10	Sandage and Walker 1955
5272 M3	183	83	142	28	1.29 0.20	1.08 0.16	0.22 0.02	0.22	0.22 0.03	0.22	0.20 0.05	Sandage and Katem 1982
5904 M5	164	40	140	31	1.16 0.19	0.96 0.15	0.20 0.03	0.20	0.20 0.03	0.20	0.22 0.06	Buonanno et al.1981
6121 M4	148	38	113	20	1.31 0.23	1.11 0.19	0.22 0.03	0.22	0.23 0.03	0.23	0.18 0.06	Lee 1977b
6171 M107	45	8	29	6	1.55 0.52	1.29 0.41	0.25 0.05	0.25	0.25 0.06	0.25	0.21 0.12	Dickens and Rolland 1972
6218 M12	80	0	59	11	1.36 0.33	1.14 0.26	0.23 0.04	0.23	0.23 0.04	0.23	0.19 0.08	Racine 1971
6254 M10	70	0	48	11	1.46 0.39	1.19 0.30	0.24 0.04	0.24	0.24 0.05	0.24	0.23 0.10	Harris et al.1976
6341 M92	117	7	85	21	1.38 0.28	1.10 0.21	0.23 0.03	0.23	0.22 0.02	0.22	0.25 0.08	Buonanno et al.1983b
6752	97	2	64	13	1.52 0.34	1.26 0.27	0.25 0.03	0.25	0.25 0.04	0.25	0.20 0.08	Cannon and Lee 1981
6809 M55	209	7	158	45	1.32 0.20	1.03 0.14	0.22 0.02	0.22	0.21 0.02	0.21	0.28 0.06	Lee 1977d
7078 M15	152	33	107	22	1.42 0.25	1.18 0.20	0.23 0.03	0.23	0.24 0.03	0.24	0.21 0.07	Buonanno et al.1983a
							Eq.13	11	14	12		

+ see text, sect.4

diagrams derived with PDS analysis of the plates also clusters studied with iris-photometer reduction techniques have been considered. In these cases, as shown by Federici et al. (1983), the completeness of the samples cannot be guaranteed due to the impossibility of measuring highly crowded images.

However, to increase the population of the sample, we inserted clusters whose data were derived from regions where about 90% of the stars have been measured and no indication of bias in color-selection is present. Furthermore, we discarded clusters for which the HB and RGB populations were particularly poor, $N_{HB}<40$.

Using these criteria in the selection, the sample includes 15 clusters. In each case we derived the number of stars in the different branches (see Table 1) taking into account the quoted prescriptions.

(a) Contamination due to field stars has been removed using available data on cluster membership.

(b) Separation between the branches has been made independently by each of the authors and a mean value used. The red HB-AGB separation has been fixed in general about 1 mag brighter than the HB level, according to Gingold's evolutionary tracks (1974, 1976).

(c) As for the separation of the branches, the magnitude level of RR Lyr stars has been derived independently if not defined in the original papers, and a mean value has been used. At least in some cases (NGC 6752, for instance) the uncertainty in the definition of this magnitude level alone may slightly modify the resulting *Y*.

(d) Given the dependence of the red giant branch color on metal abundance ($\partial(B-V)o,g/\partial \operatorname{Log} Z \sim -0.2$) the differential bolometric correction between HB and RGB stars should vary with the metallicity of the considered cluster. However, inspection on the tables given by Buser and Kurucz (1978), Kurucz (1979), and Bell and Gustafsson (1978) for the ranges of metallicities and gravities typical for HB and RGB stars in clusters, indicates that the use of a mean value of 0.15 mag (as suggested by Iben, 1968, 1971) is compatible for all the clusters, taking into account the uncertainties in the parameters necessary to deduce a value for each single cluster and those involved in the computed models.

(e) If not given in the original papers, the numbers of RR Lyr variables included in the HB star counts in each considered region have been obtained from the Sawyer Hogg (1973) catalog.

(f) No trend of *R*-variation with respect to radial distance from the centre of each cluster revealed to be statistically significant, although large scatter between *R*-values computed from different annuli in the same cluster have been found (Lee, 1977a–d; Buonanno et al., 1981).

On the other hand, the significance of a value of R deduced from a particular region of the cluster can be tested by comparing roughly the number of HB stars counted in the sample with that forming the expected population of the HB in the whole cluster. The total number of HB stars in a cluster having integrated luminosity – L_{tot} – can be obtained through the use of Eq. (30) given by Renzini (1981b): $N_{HB}=1.7\,10^{11}\,L_{tot}t_{HB}$. Inserting $t_{HB}=10^8$, one has for instance $N_{HB}=1700$, 170, 17 for $L_{tot}=10^6$, 10^5, $10^4\,L_\odot$, respectively. Since the mean integrated luminosity of galactic globular clusters is of the order of $10^5\,L_\odot$, samples with N_{HB} less than 40–50 stars can hardly be significant.

Table 1 presents thus the data we derived for each cluster and the corresponding Y-values deduced through the use of Eqs. (11)–(14) with $\eta=1/3$ (Renzini, 1981a). The errors associated with the R-values have been computed from $[(N_{HB})^{-1/2}+(N_{RGB})^{-1/2}]N_{HB}/N_{RGB}$ (Iben, 1971), while those associated to Y represent the corresponding uncertainty from the same Eqs. (13) and (14).

The agreement between the values obtained with Eqs. (11)–(14) is excellent, whilst there are many differences with respect to the values presented by previous authors. These differences are due either to the use of new $C-M$ diagrams (see for instance M 15 and M 92) or to different criteria used in the counts (NGC 6171). This means that further revisions and checks of these results should be made whenever new appropriate $C-M$ diagrams become available.

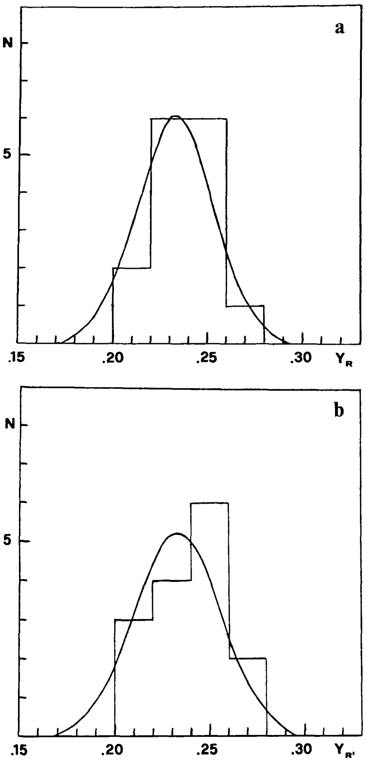

Fig. 2. a Histogram of the Y-values deduced inserting the R-values given in Table 1 in Eq. (13) of the text. The curve represent a Gaussian with a mean Y-value of 0.233 and $\sigma = 0.019$. **b** Histogram of the Y-values deduced inserting the R'-values given in Table 1 in Eq. (14) of the text. The curve represents a Gaussian with the same mean Y-value and $\sigma = 0.022$

4. Discussion

Many indications deducible from the data in Table 1 are worth noticing. First of all, the fact that the helium abundance obtained for the clusters in the sample turns out to be always $Y \geqq 0.2$. Therefore, contrary to previous analyses (Dupree et al., 1978), the R-method gives values compatible with the abundance expected from element synthesis during the big-bang.

Furthermore, the distribution of the Y-values allows the definition of a mean value with a good statistical significance. In fact, from the data in Table 1 (independently of the formula used), one has a mean value: $Y = 0.23 \pm 0.02$.

Figure 2 shows the histogram of the values. As can be seen in the figure, the distribution is well fitted by the curve and the scatter around the mean magnitude may be interpreted as due to observational uncertainties. If this is the case, this implies a constant helium abundance throughout the cluster system and a mean absolute value intermediate between 0.2 and 0.3 as often deduced by many studies. However, some more aspects should be considered.

First, the absolute value may also be affected by the theoretical uncertainties discussed in Sect. 2. This means that the non-formal total error to associate to the computed mean may raise to about ± 0.04 as a result of the sum of the contributions of observational plus theoretical uncertainties.

Second, the existence of possible trends with other basic parameters of the cluster, like metal abundance, requires further check.

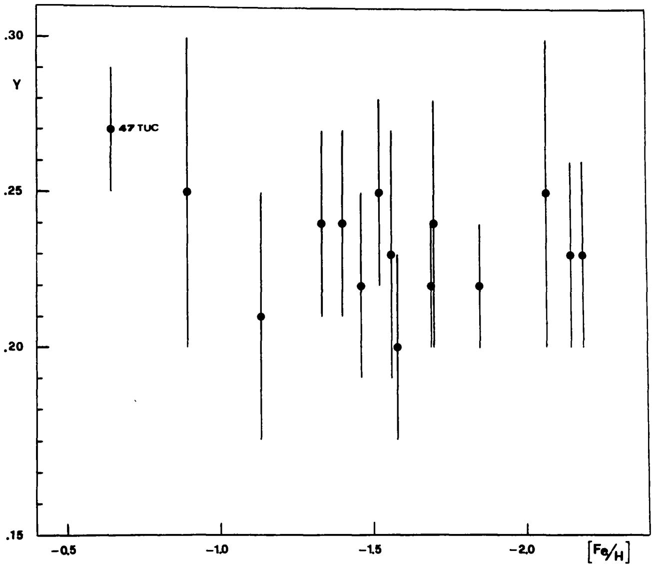

Fig. 3. Plot of the Y-values deduced from the R-values [from Eq. (13)] vs. [Fe/H] taken from Zinn (1980). See discussion in the text for 47 Tuc

Figure 3 shows a plot of the Y-values deduced using the $(R-Y)$ calibration of the present paper versus the metal abundance – [Fe/H] – expressed in Zinn's (1980) scale. A linear fit to the data gives:

$$Y = (0.027 \pm 0.020)\,[\mathrm{Fe/H}] + (0.275 \pm 0.031) \tag{17}$$

[error bars refer to a confidence level $(1-\alpha) = 95\%$].

This implies that a constant helium abundance with varying metal content seems compatible with the observations.

Different indications using the R-method were derived on this topic by Caputo et al. (1980). However, as already suspected by them, the variation they found of Y decreasing with decreasing overall metallicity of the cluster is due to the inclusion in the sample of metal poor clusters with very blue HB's for which the HB samples were far from being complete. In this respect, new photometry made in the extremely metal poor clusters M 15 and M 92 (Buonanno et al., 1983a, b) confirms the need for new observational surveys on clusters with blue HB's.

A possible dependence of the helium-abundance estimated through the R-method on the HB morphology must also be checked. As shown in Fig. 4, where the obtained Y-values versus the HB Dickens (1972) types are plotted, the present data do not display any trend going from class 1 (blue HB) to class 7 (red HB).

Since the majority of the clusters in the considered sample are intermediate metal-poor or metal poor and the population of moderately metal rich clusters ([Fe/H] > -1) is poorly represented, the previous correlation on the absence of any gradient of Y with metallicity, cannot be considered very strong. 47 Tuc, for instance, would have $Y = 0.27 \pm 0.02$ if the standard formula (13) is applied. This value is still within 2σ from the mean, nevertheless it might be interpreted as a weak indication in favour of a higher helium abundance in less metal poor clusters. There are however two possible reasons which may lead to a reduction of the Y-value obtained for 47 Tuc.

First, in Eqs. (11) or (13) used to compute Y, the Z-dependence has been averaged over the range $\mathrm{Log}\,Z = -4$ to -2.5. Since 47 Tuc is close to the metal rich extreme where the dependence on

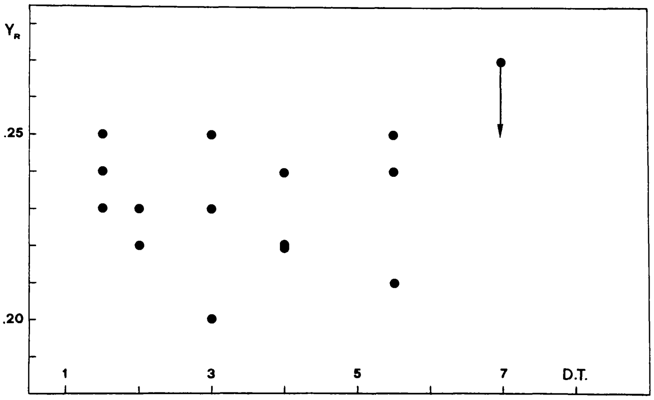

Fig. 4. Plot of Y-values [from Eq. (13)] vs. the HB Dickens type. The arrow indicates a lower limit for 47 Tuc still compatible with the data (see text for discussion)

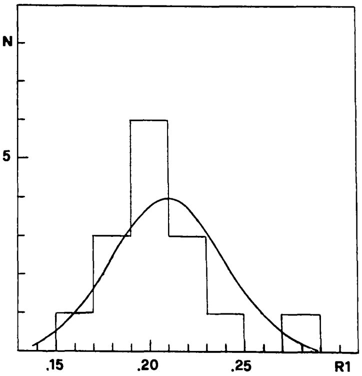

Fig. 5. Histogram of the R_1 values given in Table 1. The curve represents a Gaussian with mean R_1 equal to 0.210 and $\sigma = 0.029$

Z is higher, a precise computation made taking this term into account in an explicit way, would drop the deduced Y by about -0.02 (see Discussion at Sect. 2b).

Second, for values of Z close to that of 47 Tuc ($\mathrm{Log}\,Z = -2.34$, Zinn, 1980) and $Y = 0.25$, the peak present in the luminosity function of the RGB (due to the quoted fact that the hydrogen-burning shell encounters the discontinuity in the hydrogen abundance left by the maximum inward penetration of convection) falls at a luminosity comparable to L_{RR}. This means that due to observational errors a fraction of the stars in the magnitude interval of the peak can fall below the fixed luminosity threshold reducing the counted number of RGB stars. Given the high population of stars in 47 Tuc close to the considered limit, 10–30 stars may have been missed. Their inclusion would reduce R to about 1.6 and correspondingly Y to 0.25.

In summary, we are inclined to believe that a constant value of Y is suggested from the analysis of the clusters included in the sample. However, since an intrinsic scatter in the helium abundance in globular clusters of only 0.03–0.05 has been suggested in order to explain at least part of the problems involved in the analysis and classification of HB morphologies (the well known "second parameter problem", for instance), a firm conclusion cannot be drawn yet. In fact: (i) the present sample is probably still too poor, (ii) accurate data on R are still lacking for clusters like NGC 7006 and M 13 where the effects of the second parameter are thought to be more evident, and (iii) the mean standard deviation is too similar to the required intrinsic scatter to clarify the problem conclusively.

As far as the behaviour of R' is concerned, the data given in Table 1 and Fig. 2b, show a complete agreement between the Y-values derived using R and R'. This essentially means that the rough estimate of a mean AGB lifetime of 15 million years is consistent with the observations. Therefore, the use of $(R' - Y)$ calibration allows the deduction of an estimate of Y from star counts also for clusters with a $C-M$ diagram not suitable for a clear separation between AGB and RGB.

A last aspect worthy of remark is the analysis of the distribution of R_1. As discussed in Sect. 2, the estimate of R_1 can give some hints on presence and development of the semiconvective zone in HB stars. As can be seen from Table 1 and Fig. 5, where the histogram of the R_1-values is given, the mean value of R_1 is 0.21 ± 0.03. This estimate is in agreement with the expected value if full development of semiconvection is included in the models. Moreover, taking into account the poorness of the AGB samples and the mentioned difficulty in the separation between AGB and RGB, the small scatter around the mean R_1 may indicate that the development of the semiconvective zone is similar for the stars of all the examined clusters.

Possible evidences against this indication have been discussed by Arimoto and Simoda (1981). However, problems of completeness of the samples and very low number of considered clusters have probably affected their conclusions.

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