Vol. 62 No 04 (244) 2022
DOI: https://doi.org/10.5564/pmas.v62i04.2679
PMAS
Proceedings of the Mongolian Academy of Sciences
ARTICLE

Corrections for regular identification of high energy positive particles in experimental data using lobachevsky space

Togoo Ravdandorj[1]*, Khishigbuyan Narankhuu[1], Shinebayar Janchiv[2]

[1] Department of Theoretical and High Energy Physics,Institute of Physics and Technology, Mongolian Academy of Sciences, Ulaanbaatar, Mongolia
[2] School of Educational Studies, Mongolian National University of Education, Ulaanbaatar, Mongolia

ARTICLE INFO: Received: 22 Nov, 2022; Accepted: 09 Jan, 2023

Abstract: In this work, high-energy positive charged particles are distinguished using the Lobachevsky space or Hyperbolic space, which is defined as the total rapidity multiplied by hyperbolic cosines of the transverse and longitudinal rapidity of the particles. Experimental data from eight different types of interactions detected in the bubble chambers accumulated in the high-energy sector were used in the calculations. The weights used to construct the proton and positive pion distributions for each of the interacting secondary particles have been eliminated, allowing such studies to be performed such as particle counting and clustering.These weights do not include calculated weights at azimuth angles, near the center of the star, or without momentum measurements. We now have the opportunity to study positive pions and protons. The percentage of confused particles increases with the beam energy.

After the reconstruction, we conducted a study of the temperature of the charged particles produced by the p + p interaction of 205 GeV, where Tsallis temperatures are close to Hagedorn T_1. On the other hand, Hagedor T_2 and T_0 temperatures are higher than Tsallis, which means that the unstable states exchange heat as they move to equilibrium.

Keywords: *Lobachevsky space; Hagedorn approach; Tsallis approach;*

INTRODUCTION

The interaction of collision’s reaction can be formulated in two ways:

The form noted in the most studies is I (projectile) + II (target)→ c + X, where c is the newly produced particles and the uninteracted fragments from the target and projectile nucleus. X is a particle that is not detected in the detector. Alternatively, 1 (projectile) + 2 (target)→ 3 + X, where 3 is the same as c from the previous reaction.

When studying the interaction process of elementary particles, there are many advantages in solving all problems by simultaneously studying the onward and transverse directions of the movement. For example, Lobachevsky's geometry [1, 2], or the so-called "total rapidity" quantity applied to the rapidity space, calculates the above-mentioned two directions simultaneously.

***corresponding author: togoo@mas.ac.mn** **https://orcid.org/0000-0002-7266-0564**

This geometry is represented by the following multiplication:

$$ch(\rho) = ch(\tau)ch(y) \quad (1)$$

Where y is the rapidity in the onward direction, which is defined according to the formula $y = 0.5 * \log\left[\frac{E+p_{||}}{E-p_{||}}\right]$, but transverse direction is defined as $\tau = m_T/m$. E, $p_{||}$, m_T and m are the constituents according to the total energy, the longitudinal momentum, the transverse mass and m is the rest mass of the particle.

In the work [2], we kept the expression ρ_{23} as is in terms of the second law of the collision reaction in order to distinguish the quantity ρ from the density notation. We have previously distinguished positive pion loss within protons at 10 GeV proton-carbon and 4.2 GeV per nucleon carbon-carbon interactions, and have collected experimental data using the above space using Monte Carlo methods based on well-described theoretical models [3, 4].

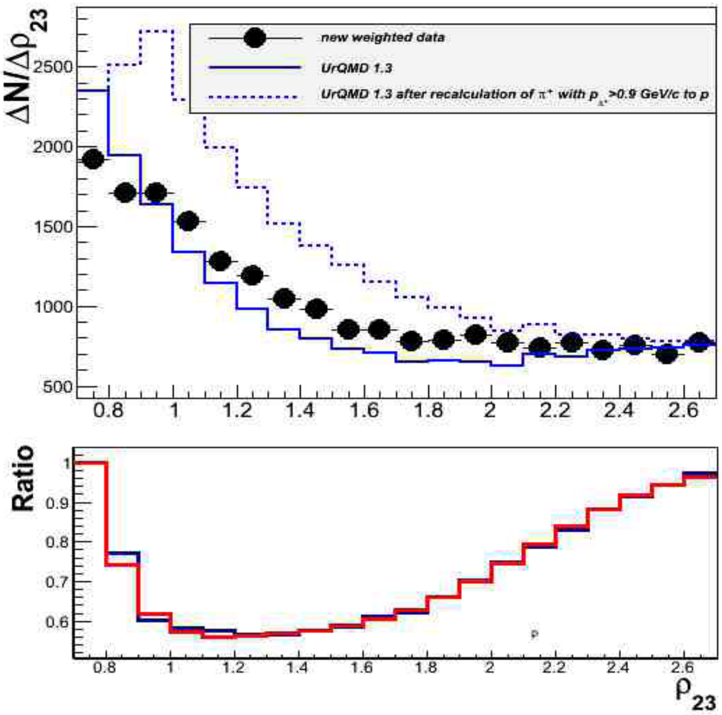

Figure 1. Total rapidity distribution of protons produced by relativistic interaction pC (Top) Here, the experimental and histograms in black points are the results of the UrQMD 1.3 model, while the dashed histograms are the results of experimental old conditions in the UrQMD 1.3 model. The bottom figure shows the distribution of "real" protons generated by the UrQMD model, or the histogram of protonation of π^+-mesons with a momentum greater than 0.9 GeV/s, compared to the distribution shown by the dashed histogram. Here, in red histogram, the correlation is approximated by the 7th order Legendre polynomial

The red histogram as in Figure 1 shows the probability of distinguishing a positive pion from a proton, and if the MC probability is above this curve for each value of ρ_{23}, it is assumed that the proton is transformed into a π^+-meson, and when its rest mass is changed, other physical parameters are also recalculated. Otherwise, the proton remains as a proton.

According to this principle, positive pions were separated from protons for other interactions, and protons from positive pions for the last three interactions.

The following table proves that by applying the Lobachevsky geometry eight types of data can be eliminated with maximum probability from not being able to distinguish high-energy positive particles. Theoretical models were used that thoroughly explained the experimental data in terms of many physical parameters. To date, there are many models that describe quite well experimental results.

DOI: https://doi.org/10.5564/pmas.v62i04.2679
PMAS
Proceedings of the Mongolian Academy of Sciences

Table 1. Proportion of corrected and separated positive particles with momentum greater than 1 GeV/c

Interaction	Beam energy, GeV	Correction interval of ρ_{23}	Percent of entangled particles
p + C	4.2	proton 0.8< ρ_{23}^{p}<2.1	π^+meson 19.7%
d + C	4.2 A	proton 0.8<ρ_{23}^{p}<1.9	π^+ meson 22.7%
He + C	4.2 A	proton 0.8<ρ_{23}^{p}<1.9	π^+ meson 17%
		π^+ meson 1.65< $\rho_{23}^{\pi^+}$<2.55	proton 2.9%
C + C	4.2 A	proton 0.8< ρ_{23}^{p}<1.7	π^+ meson 20%
		π^+ meson 0.5< $\rho_{23}^{\pi^+}$<0.8	proton 10%
p + C	10	proton 0.8< ρ_{23}^{p}<2.8	π^+ meson 25.3%
π^-+ p	40	π^+ meson 1.9< $\rho_{23}^{\pi^+}$<6.3	proton 75%
π^- + C	40	π^+ meson 1.8< $\rho_{23}^{\pi^+}$<6.4	proton 40%
p + p	205	π^+ meson 2.6< $\rho_{23}^{\pi^+}$<9.8	proton 67%

With this correction, we apply weights to each secondary particle, and the studies, such as counting particles with real numbers, creating clusters etc. can be carried out. Other research institutes have conducted extensive study in to negative pions recorded by our camera [5]. We are now in a position to investigate positive pions and protons.

To confirm this, we constructed the transverse momentum distributions of π^+ and π^- -mesons and protons generated by 205 GeV $p+p$ interaction, and we were able to determine the patterns of corresponding temperatures by the Hagedorn and Tsallis methods.

Hagedorn approach

The p_T spectrum of the above particles was approximated using the Hagedorn function. Assuming the Hagedorn thermodynamic model [6], and meeting the condition $m_T \geq T$, approximation can be made by the function of the following form:

$$\frac{1}{N}\frac{dN}{dpT} = F(p_T) = A\, p_T \sqrt{m_T T}\, \exp(-m_T/T) \quad (2)$$

where N is the total number of distributions and A is the approximation constant. We find that the previous expression is a Hagedorn function with one temperature. But this function can be the sum of two or more functions of the structure. If the sum of the two functions is the values of the two temperatures T_1 and T_2, then the p_T functions of our structure will have the following form:

$$\frac{dN}{NdpT} = F_1(p_T) + F_2(p_T) = A_1\, p_T \sqrt{m_T T_1}\, exp(-m_T/T_1) + A_2\, p_T \sqrt{m_T T_2}\, exp-(m_T/T_2) \quad (3)$$

Assuming that the first sum of this expression corresponds to a low value of p_T, let's mark it as a soft (Soft) process of interaction or $f_S(p_T)$ function, while the second sum represents a hard (Hard) process corresponding to a large value of p_T by $f_H(p_T)$ function. Consider how the general function $f_0(p_T)$ is defined by these two functions. On the other hand [7], the p_T spectrum of the experience covers a wide range, but can be considered as a combination of easy (soft or most of it reported) and difficult to decipher (hard or difficult to find reasons) processes. It is a two-component function:

$$f_0(p_T) = k_1 f_S(p_T) + k_2 f_H(p_T) \quad (4)$$

where k1(k2) - denotes the ratio of soft and hard processes, while many studies are conducted to determine the boundaries of the functions $f_S(p_T)$ and $f_H(p_T)$, we can also make our own contribution. The main condition for the sum of two and more functions above is $\int_0^{p_T^{max}} f_0\,(p_T)dp_T = 1$, where p_T^{max} is the maximum value of p_T. In expression (3), the soft part covers the range of low p_T and the hard part covers the whole range of p_T. In the low p_T range, soft and hard processes overlap.

According to Hagedorn model [8], our usual step is to use only an infinite and undefined number of different excited hadron fireballs to keep the sum of the two functions in thermodynamic equilibrium in classical and high-energy collisions. A necessary and very important conclusion and explanation of this mechanism is that the temperature is independent of the primary energy [9]. A simple proof was made earlier by Field and

Vol. 62 No 04 (244) 2022
DOI: https://doi.org/10.5564/pmas.v62i04.2679
PMAS
Proceedings of the Mongolian Academy of Sciences

Feynman a few years later [10]. A predictable problem now is that the transverse momentum (p_T) distribution of the ejected particles along the axis of relaxation will drive this temperature.

In some cases, the rate of resonance generation of pions in the range of very low p_T is considered, and resonances occur in the range of p_T 0.2~0.3 GeV/c. When the system has completely transitioned to the equilibrium state, its temperature is determined as follows:

$$T_0 = k_1 T_1 + k_2 T_2 \qquad (5)$$

Where T_0 is the equilibrium Hagedorn temperature.

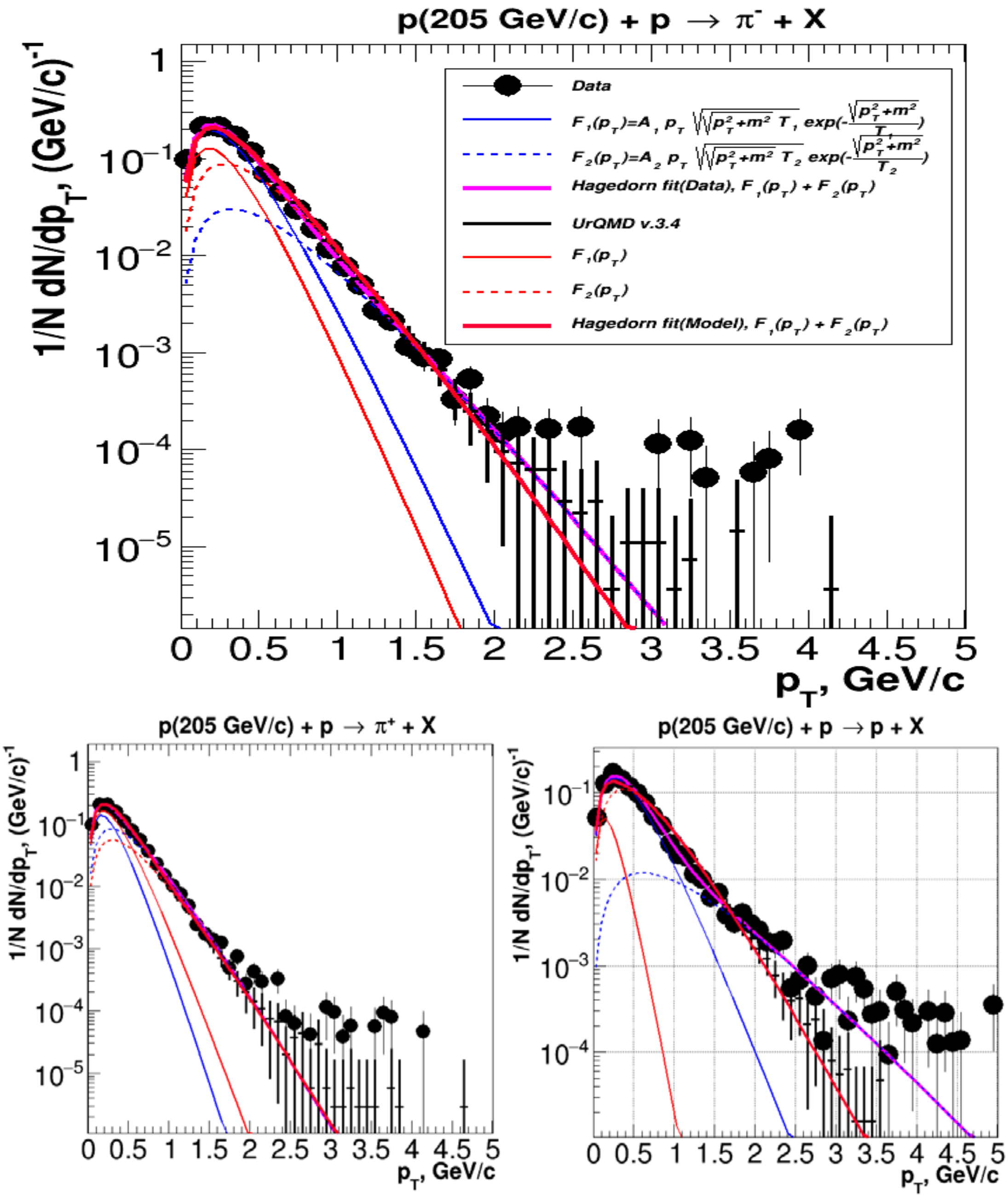

Figure 2. Transverse momentum distributions of charged pions and protons. Approximations by Tsallis functions

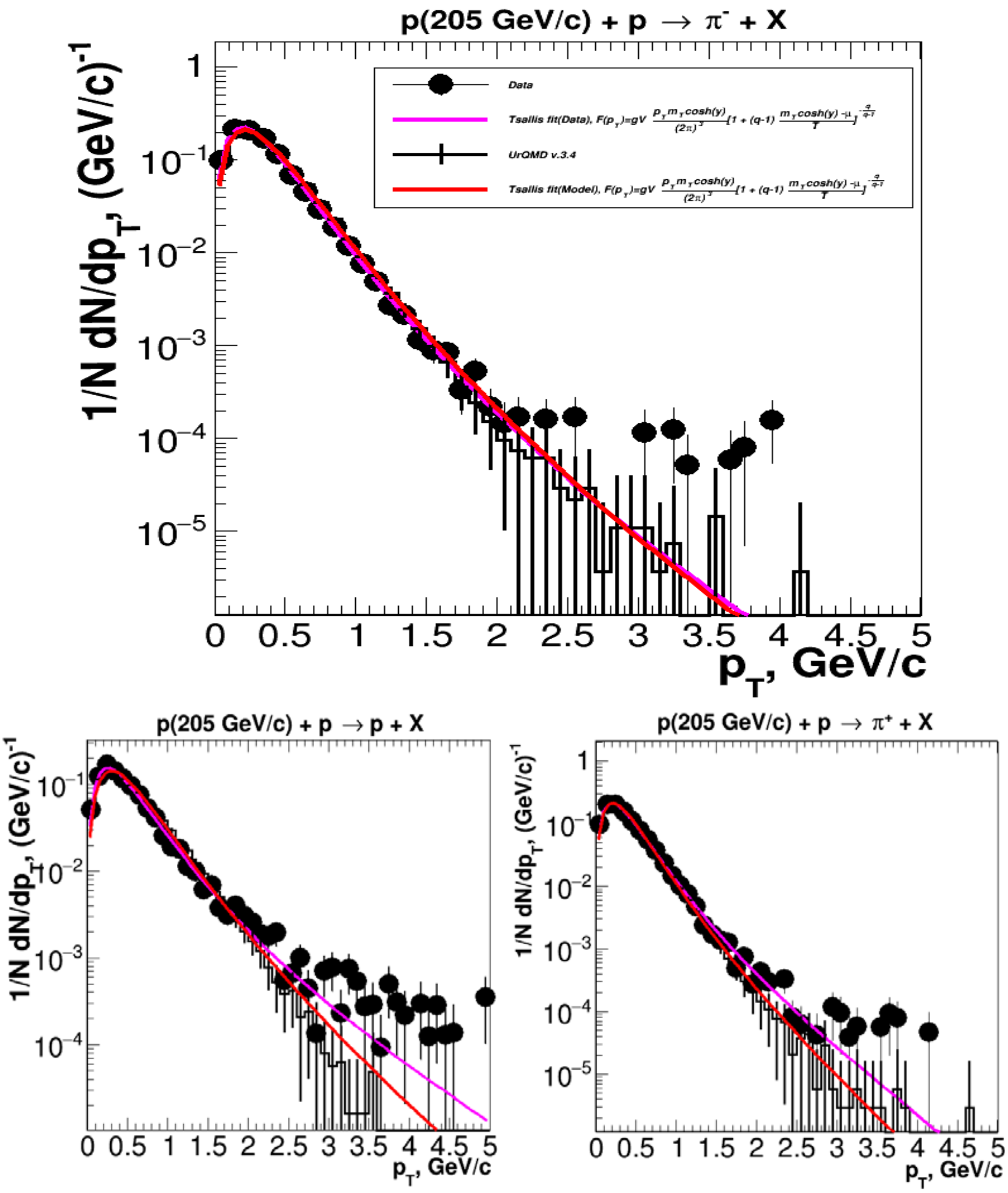

Figure 3. Transverse momentum distributions of charged pions and protons. Approximations by Hagedron functions

The following table reports the results of approximations with Hagedron and Tsallis functions derived from Figure 2,3.

Table 2. Parameters of approximations of Hagedron functions

	π^--мезон	π^--мезон	π^+-мезон	π^+-мезон	proton	proton
	Data	UrQMD	Data	UrQMD	Data	UrQMD
A_1 (GeV)-1	49±2	40±2	52±3	41±2	15±1	39±15
T_1 (MeV)	115±3	106±8	98±5	114±4	166±4	76±12
A_2 (GeV)-1	1.9±0.9	8.0±2.6	6.6±1.4	4.0±1.4	0.18±0.07	5.1±0.5
T_2 (MeV)	208±14	176±15	186±6	193±9	403±27	237±5
$\chi^2 / n.d.f$	24.1/28	4.2/30	41.8/29	3.2/34	48.1/42	13.9/31
k_1	0.79	0.48	0.48	0.64	0.82	0.14
k_2	0.21	0.52	0.52	0.36	0.15	0.86
T_0 (MeV)	135±4	144±8	143±4	141±4	198±5	214±9

Vol. 62 No 04 (244) 2022
PMAS
DOI: https://doi.org/10.5564/pmas.v62i04.2679
Proceedings of the Mongolian Academy of Sciences

Tsallis distribution

The Tsallis distribution was first proposed as a generalization of the usual exponential Boltzmann-Gibbs distribution about twenty-five years ago, and characterized by the parameters T (temperature), μ (chemical potential), g (particle spin reduction coefficient), and V (volume). An equation of this form exp(-E/T) is usually expressed in the general form of the Boltzmann-Gibbs exponential distribution:

$$\frac{d^3N}{dp^3} = \frac{gV}{(2\pi)^3}\left[1+(q-1)\frac{E-\mu}{T}\right]^{-\frac{q}{q-1}}$$

$$q \to 1 \;\; \to \;\; \frac{gV}{2\pi^3} exp\left(-\frac{E-\mu}{T}\right) \qquad (6)$$

Such an approach is known as non-extensive statistics in which the parameter q summarily describes all features causing a departure from the usual Boltzmann-Gibbs statistics[10]. In particular it was shown in [11] that $q-1 = Var\,(T)/\langle T\rangle^2$ and directly describes intrinsic fluctuations of temperature. However, the Tsallis distribution also emerges from a number of other more dynamic mechanisms, for example see [12] for more details and references. This approach has been shown to be very successful in describing multiparticle production processes of a different kind (see [11, 12] for recent reviews). If we show the Equation (5) in terms of transverse momentum and transverse mass, $m_T = \sqrt{m_0^2 + p_T^2}$, and rapidity y, Eq. (5) becomes as follows. Here, m_0 is the rest mass of a particle. If we include m_T and y variables in expression (5), the function of the particles structure can be expressed as below.

$$\frac{d^2N}{dp_T dy} = gV\frac{p_T\, m_T coshcosh\,(y)}{2\pi^2} \times \left[1+(q-1)\frac{m_T coshcosh\,(y)-\mu}{T}\right]^{-\frac{q}{q-1}} \qquad (7)$$

Approximation of the transverse momentum distribution of secondary charged particles resulting from p+p collisions with energy $\sqrt{s} = 19.7$ GeV was approximated by Tsallis distribution under three conditions. The particle spin reduction coefficient g in expression (5) is calculated as 1 for $\pi^{\pm}$ and 2 for protons. It can be seen in Figure 1 that the transverse momentum distribution is well explained by the Tsallis method in total rapidity intervals for $\pi^{\pm}$ -mesons and protons from experimental proton-proton collisions. Table 3 lists the values of volume (V), entropy index (q), temperature (T) and χ^2 per unit degree of freedom for total rapidity intervals.

Table 3. Parameters of Tsallis function approximation

	π^- -meson	π^--meson	π^+ -meson	π^+ -meson	proton	proton
	Data	UrQMD	Data	UrQMD	Data	UrQMD
gV (GeV)-1	1961±116	1429±72	1726±20	1700±20	1104±26	892±24
q	1.081±0.004	1.073±0.003	1.097±0.003	1.076±0.003	1.124±0.006	1.085±0.004
T (MeV)	101±1	108±1	104±1	107±1	130±2	152±2
χ2 / n.d.f	22.6/29	22.1/29	54.8/35	10.3/34	52.1/42	88.0/34

Tables 2 and 3 show that Tsallis temperatures (T) are approximate to Hagedorn temperatures (T_1). On the other hand, the Hagedorn temperatures T_2 and T_0 are greater than the Tsallis temperature (T) indicating that the unstable states are undergoing heat exchange as they transition to the equilibrium state.

CONCLUSIONS

We no longer use weights [13] as secondary particles, and studies such as count particles with real numbers, creating clusters, etc. are now performable.

We have now shown that the conditions for the study of positive pions and protons are in place. The fraction of entangled particles increases with initial energy.

Thallis temperatures (T) are close to Hagedorn temperatures (T_1). On the other hand, the Hagedorn temperatures T_2 and T_0 are greater than the Tsallis temperature (T), indicating that the unstable states undergo heat exchange as they move to the equilibrium state.

This research work was carried out within the framework of the 2019/20 project of the Ministry of Education and Culture.

Acknowledgement: I would like to express my gratitude to doctorant Ts. Banzrahch, who contacted Dr. Marcus Bleicher, one of the main authors of the UrQMD simulation, and made all types of the Fortran package of this specification usable at our institute.

REFERENCES

1. N. A. Chernikov (Dubna, JINR) "Lobachevsky geometry and relativistic nuclear physics" Dubna, 1997.
2. A. A. Baldin, E. G. Baldina, E. N Kladnitskaya, O. V. Rogachevsky, "Analysis of experimental data on relativistic nuclear collisions in Lobachevsky space", Phys.Part.Nucl.Lett. 1 (2004), 171-177, Pisma Fiz.Elem.Chast.Atom.Yadra 2004 (2004) 119, pp. 7-16.
3. R. Togoo, T. Tulgaa, A. Tursukh, O. V. Rogachevsky, M. Sovd and J. Shinebayar "A possibity to identify positive particles produced from the inelastic interactions detected by propane bubble chamber", Proc. of Mongolian Academy of Sciences,Vol. 56, No. 1(217),Ulaanbaatar, 2016, pp. 5-12.
4. R. Togoo, M. Sovd, T. Tulgaa, A. Tursukh, B. Khurelbaatar, O. V. Rogachevsky and J. Shinebayar "Coloumb interaction in C+C collisions at 4.2 A GeV/c momentum" Proc. Of the Institute of Physics and Technology, № 44, Ulaanbaatar, 2018.
5. Imran Khan and Kh. K. Olimov "Spectral Temperatures of Δ0(1232) Resonances Produced in p12C and d12C Collisions at 4.2 GeV/c per Nucleon" ISSN 1063-7788, Physics of Atomic Nuclei, 2013, Vol. 76, No. 7, pp. 883–887. https://doi.org/10.1134/S1063778813070089
6. R. Hagedorn and J. Rafelski, "Hot Hadronic Matter and Nuclear Collisions" Phys. Lett. B 97, 136 (1980). https://doi.org/10.1016/0370-2693(80)90566-3
7. Li-Li Li, Fu-Hu Liu "Kinetic Freeze-Out Properties from Transverse Momentum Spectra of Pions in High Energy Proton-Proton Collisions", MDPI Physics 2 (2020) 2, pp. 277-308, https://doi.org/10.3390/physics2020015
8. Hagedorn,R. "Multiplicities, p_T Distributions and the Expected Hadron $\to$ Quark - Gluon Phase Transition", Riv.Nuovo Cim. 6N10 (1983), pp. 1-50, https://doi.org/10.1007/BF02740917
9. Hagedorn, R., "Statistical thermodynamics of strong interactions at high-energies" Nuovo Cim.Suppl. 3 (1965), pp. 147-186.
10. Field, R. D. and Feynman, R. P., "A Parametrization of the Properties of Quark Jets" Phys. B 136, pp. 1-76 (1978). https://doi.org/10.1016/0550-3213(78)90015-9
11. G. Wilk and Z. Wlodarczyk, "Interpretation of the Nonextensivity Parameter q in Some Applications of Tsallis Statistics and Lévy Distributions" Phys. Rev. Lett. 84, 2000, https://doi.org/10.1103/PhysRevLett.84.2770
12. Grzegorz Wilk (Warsaw, Inst. Nucl. Studies), Zbigniew Wlodarczyk (Jan Kochanowski U.) "Consequences of temperature fluctuations in observables measured in high energy collisions" Published in: Eur.Phys.J.A 48 (2012), https://doi.org/10.1140/epja/i2012-12161-y
13. A. I. Bondarenko, R. A. Bondarenko, E. N. Kladnitskaya, "Study of nucleus-nucleus interactions with complete destruction of the target nucleus with momentum 4.2-GeV/c per nucleon" Dubna, JINR, R1-96-292, 1996.