Chinese Journal of Polar Science, Vol. 10, No. 2, 111 - 116, December 1999

A theory for the formation of weak double layer

Hong Minghua (洪明华)[1], Wang Xianmin (王宪民)[2] and Tang Keyun (汤克云)[1]

1 *Institute of Geology and Geophysics, Chinese Academy of Sciences, Beijing 100101, China*

2 *Institute of Space Medico-Engineering, Beijing 100094, China*

Received September 3, 1999

Abstract Solitary electrostatic waves with ion acoustic speed were frequently observed by satellites in the auroral acceleration region. In this paper, the nonlinear ion acoustic waves are studied in the plasma which is composed of warm electrons with the Boltzman distribution and cold ions of equal density. The characteristics of solitary-like structure in the ion acoustic frequency range are derived with the methods of reductive perturbation and phase plane analysis. The results show that nonlinear ion acoustic waves may develop to a symmetric solitary structure which is compressive and no net potential drop when dissipation does not exist, and in the case with dissipation it may evolve to compressive solitary-like structure with asymmetric shape, produce net potential drop and form weak double layer. The above theoretical results are consistent with observations.

Key words solitary electrostatic wave, ion acoustic wave, weak double layer, auroral region.

1 Introduction

It is widely accepted that potential differences between the magnetosphere and ionosphere along the auroral field lines exist and are of importance in the magnetosphere and ionosphere interaction. Various different mechanisms, such as anomalous resistivity, magnetic mirror effect, and double layers, have been proposed to be the origin of the parallel electric field (Dovner *et al.* 1994; Shawhan *et al.* 1978; Burke and Heinemann 1983). However, there is still no general agreement regarding how the parallel potentials are maintained, and how the auroral plasma responds to these mechanisms.

Solitary electrostatic waves were frequently observed by both the S3-3 and Viking satellites in the auroral acceleration region within $3R_E$ geocentric distance. Recently similar structures have also been detected in the magnetotail out to more than $100R_E$ by the Geotail satellite. There are also indications of solitary electrostatic waves in the upper ionosphere from the Freja satellite (Eriksson *et al.* 1997). The satellite measurements have shown that the solitary waves are in the form of the small-scale holes in plasma density and electrostatic potential, propagating upward at the velocity of the order of the ion acoustic speed along the geomagnetic field lines. These solitary structure sometimes exhibit a net potential drop and constitute "weak double layer" (WDLs). The parallel electric field pointing downwards and upwards occur with probabilities of 0.42 and 0.58, respectively (Mälkki *et al.* 1993). Because they are usually observed in weak auroral acceleration region, it has been suggested that a large number of WDLs summed up along a geomagnetic flux tube would build up a quasistatic potential drop. Up to now no adequate theory of the formation and evolution of these WDLs has been presented.

The purpose of this paper is to investigate the formation mechanism of the solitary waves in the ion acoustic frequency range and their possible contribution to the parallel potential drop. We will show that the WDLs can appear when the ion acoustic solitary waves evolve to double layers in the presence of wave-particle interactions. In section 2 the wave equation is derived with reductive perturbation method. In section 3 we analyse the characteristics of the wave equation and its solutions. In section 4 we discuss the properties of solutions and the formation of WDLs.

2 Wave equations

We consider a collisionless plasma which is composed of warm electrons with the Boltzman distribution and cold ions of equal density, $n_e = n_i = n_0$, and neglect the electron inertial and ion temperature ($m_e/m_i \to 0, T_i/T_e \to 0$). In such a plasma, Landau damping due to ions can be omitted, while the effect of Landau damping due to electrons may be of the order of $(m_e/m_i)^{1/2}$. In order to clarify this, one must appeal to kinetic approach. For simplicity, here we introduce the anomalous damping rate ν characterizing the weak dissipative effect. In the presence of an external magnetic field in x-direction $B_0 = B_0 e_x$, the normalized ion dynamic equations are:

$$\frac{\partial n}{\partial t} + \nabla(nV) = 0 \tag{1}$$

$$\frac{\partial V}{\partial t} + (V \cdot \nabla)V + \nabla\phi + \Omega_c e_x \times V - \nu\nabla^2 V = 0 \tag{2}$$

$$\nabla^2\phi = \mathrm{e}^{\phi} - n \tag{3}$$

where ϕ is the electric potential defined by $E = -\nabla\phi$, and normalized by $(\mathrm{k_B}T_e)/\mathrm{e}$, n and V denote the ion density and velocity in units of n_0 and $(\mathrm{k_B}T_e/m_i)^{1/2}$, respectively. Ω_c is ion gyrofrequency, ν is the anomalous damping rate. All lengths have been normalized by the Debye length $(\mathrm{k_B}T_e/4\pi n_0 \mathrm{e}^2)^{1/2}$.

The reductive perturbation method is employed with the assumption that all variables are independent of z. First, we make use of small parameter expansion:

$$\begin{aligned} n &= 1 + \varepsilon n_1 + \cdots, \\ V_x &= \varepsilon V_{x1} + \varepsilon^2 V_{x2} + \cdots, \\ V_y &= \varepsilon^2 V_{y1} + \varepsilon^3 V_{y2} + \cdots, \\ V_z &= \varepsilon^{3/2} V_{z1} + \varepsilon^{5/2} V_{z2} + \cdots, \\ \phi &= \varepsilon\phi_1 + \varepsilon^2\phi_2 + \cdots. \end{aligned} \tag{4}$$

We look for solutions of Eqs. (1)-(3) that depend on x and t through the progressing wave transformation:

$$\begin{aligned} \xi &= \varepsilon^{1/2}(x - Mt) \\ \sigma &= \varepsilon^{1/2} y \\ \tau &= \varepsilon^{3/2} t \end{aligned} \tag{5}$$

and $\partial/\partial x = \varepsilon^{1/2}\partial/\partial\xi$, $\partial/\partial t = \varepsilon^{3/2}\partial/\partial\tau + \varepsilon^{1/2}M\partial/\partial\xi$, $\partial/\partial y = \varepsilon^{1/2}\partial/\partial\sigma$. Where $M = V_0/C_s$ is Mach number, V_0 is ion flow velocity, C_s is ion acoustic speed.

Substituting (4) and (5) into (1) - (3), and maintaining the first and second order terms in Eqs. (1) - (3), we find

$$\frac{\partial\phi}{\partial\tau} + \phi\frac{\partial\phi}{\partial\xi} - \frac{1}{2}\nu\left(\frac{\partial^2\phi}{\partial\xi^2} + \frac{\partial^2\phi}{\partial\sigma^2}\right) + \frac{1}{2}M^3\frac{\partial^3\phi}{\partial\xi^3} + \frac{1}{2}M^3(1+\Omega_c^{-2})\frac{\partial^3\phi}{\partial\xi\partial\sigma^2} = 0 \quad (6)$$

where $\phi = \phi_1$. Here after we will omit the subscript. Eq. (6) is just the nonlinear ion acoustic wave equation which describe the evolution of an arbitrary initial disturbance moving at roughly the ion acoustic speed. The origins of various terms in Eq. (6) are clear: the nonlinear term arises from convection, the dispersive term $\sim \partial^3\phi/\partial\xi^3$ is caused by the deviation from charge neutrality in Eq. (3), and the dissipative term $\sim \partial^2\phi/\partial\xi^2$ arises from wave-particle interaction. In the next section, we look for different types of solutions for ion acoustic waves and analyze their properties based on Eq. (6).

3 Analytic results

We are mostly interested in the behavior of waves propagating along field line. To focus on these waves we neglect the motion perpendicular to the magnetic field. Then Eq. (6) can be reduced to

$$\frac{\partial\phi}{\partial\tau} + \phi\frac{\partial\phi}{\partial\xi} - \alpha\frac{\partial^2\phi}{\partial\xi^2} + \beta\frac{\partial^3\phi}{\partial\xi^3} = 0 \quad (7)$$

where $\alpha = \nu/2$, $\beta = M^3/2$. Eq. (7) is the B-KdV (Burgers-Korteweg-de Vries) equation.

If there is no dissipation, $\alpha = 0$, then Eq. (7) reduces to the well-known KdV equation which has a steady progressive wave solution (Infeld 1990)

$$\phi = 3(M-1)\text{sech}^2\left[\frac{1}{\sqrt{2}}(M-1)^{\frac{1}{2}}(x-Mt)\right] \quad (8)$$

Fig. 1 show how ϕ vary with $\xi = M - xt$.

In the case with dissipation, Eq. (7) also has the steady progressive wave solutions with different shapes. We analyse this with a phase plane. Let

$$\phi = \phi(\xi), \qquad \xi = x - Mt. \quad (9)$$

From Eq. (7) and (9), we have

$$(\phi - M)\frac{\partial\phi}{\partial\xi} - \alpha\frac{\partial^2\phi}{\partial\xi^2} + \beta\frac{\partial^3\phi}{\partial\xi^3} = 0 \quad (10)$$

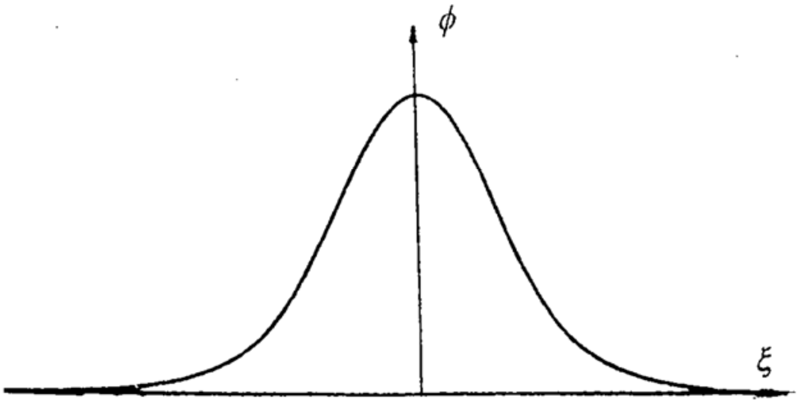


Fig. 1. Solution of solitary wave.

Integrating Eq. (10) to ξ, we obtain

$$-\alpha\frac{\partial\phi}{\partial\xi}+\beta\frac{\partial^2\phi}{\partial^2\xi}+\frac{1}{2}\phi^2-M\phi=A \tag{11}$$

in which A is the constant of integration. We now introduce the equivalent counterpart Eqs. of (11) as following

$$\begin{aligned}&\frac{\partial\phi}{\partial\xi}=\psi\\&\frac{\partial\psi}{\partial\xi}=\frac{\alpha}{\beta}\psi-\frac{1}{2\beta}(\phi^2-2M\phi-2A)\end{aligned} \tag{12}$$

In the phase plane, Eq. (12) have two steady states: $P(\phi_1, 0)$ and $Q(\phi_2, 0)$. where $\phi_1=M+(M^2+2A)^{1/2}$, $\phi_2=M-(M^2+2A)^{1/2}$ $(M^2+2A>0)$.

Eq. (12) can be analyzed in the following general cases:

(1) The case of $\alpha>0$, $\beta>0$. When $\alpha^2<4\beta(M^2+2A)^{1/2}$, the point P is an unstable focus, and Q is a saddle point. In the neighborhood of saddle point, the solution is corresponding to a solitary wave, while in the neighborhood of focus point, the solution represents a decaying oscillatory wave. saddle-facus trajectory correspond to a solitary-like solution (Liu and Liu 1991)

$$\phi=\begin{cases}\phi_1+\frac{\phi_1-\phi_2}{2}e^{\frac{\alpha}{2\beta}\xi}\cos\sqrt{\frac{\phi_1-\phi_2}{2\beta}-\frac{\alpha}{2\beta}}\xi & (-\infty,0)\\ \phi_2+\frac{3(\phi_1-\phi_2)}{2}\operatorname{sech}^2\sqrt{\frac{\phi_1-\phi_2}{8\beta}}\xi & [0,+\infty)\end{cases} \tag{13}$$

Fig. 2a plots ϕ vary with ξ in Eq. (13).

(2) The case of $\alpha>0$, $\beta<0$. When $\alpha^2<-4\beta(M^2+2A)^{1/2}$, the point P is a saddle point, and Q is a steady focus point. Similar to the case 1, there is a solution which takes the form of

$$\phi=\begin{cases}\phi_1+\frac{3(\phi_1-\phi_2)}{2}\operatorname{sech}^2\sqrt{\frac{\phi_1-\phi_2}{-8\beta}}\xi & (-\infty,0]\\ \phi_2-\frac{\phi_1-\phi_2}{2}e^{\frac{\alpha}{2\beta}\xi}\cos\sqrt{\frac{\phi_1-\phi_2}{-2\beta}-\frac{\alpha}{2\beta}}\xi & [0,+\infty)\end{cases} \tag{14}$$

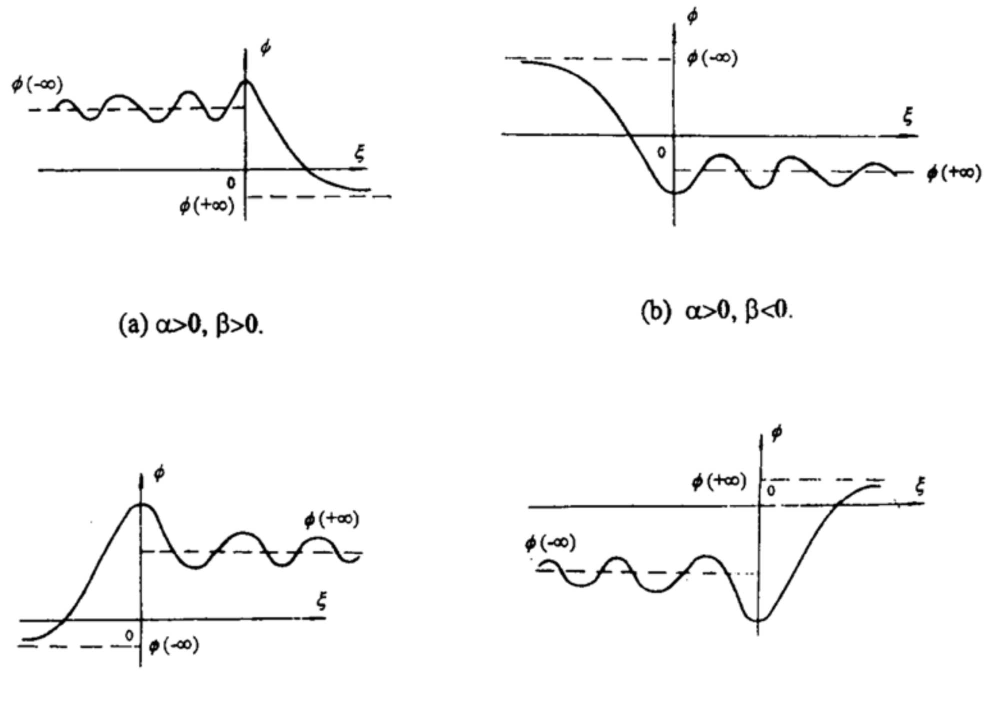
Fig. 2b shows ϕ vary as a function of ξ in Eq. (14).

With similar analysis in the cases $\alpha>0$, $\beta>0$ and $\alpha>0$, $\beta<0$, it is easy to see that Eq. (7) have solutions with wave shapes plotting in Fig. 2c and Fig. 2d, respectively.

4 Discussions and conclusions

In section 3, we analyzed the steady progressive solutions of Eq. (7) in a frame of reference moving at the ion acoustic speed. Different type solutions correspond to different conditions in which three effects (nonlinear, dispersion and dissipation) reach balances and the waves develop to different stable structures. It is clear that the structures of progressive waves are mostly dependent on the dispersive and dissipative effects.

In the case without dissipation, the solution is a symmetric solitary wave (Fig. 1). It is a compressive structure ($\delta n>0$) exhibiting potential barrier due to the dispersive coefficient $\beta=M^3/2>0$ in present model. This is not in agreement with the satellite



(a) $\alpha>0$, $\beta>0$.

(b) $\alpha>0$, $\beta<0$.

(c) $\alpha<0$, $\beta>0$.

(d) $\alpha<$, $\beta<0$.

Fig. 2. The Solitary-like solutions of Eq. (7).

observations which indicate that most of solitary structures are associated with density depletion corresponding to a potential well. This means that the plasma parameters neglected in the present model (such as ion temperature, electron temperature deviated from the Boltzman distribution) must be important for generating the density depleted structures and potential wells. Further more, the symmetric solitary wave in Fig. 1 cannot produce the potential differences (WDLs). This is probably due to omitting the dissipation. It may also be the reason why some other works (Gray *et al.* 1991) showed the density depleted structure, while did not see the asymmetry of the WDLs.

In the case of $\alpha>0$ (positive dissipation) regardless of $\beta>0$ or $\beta<0$, the wave structures exhibit potential drop in the direction of wave propagation (Fig. 2a and b). This suggests that the dissipative effect makes wave energy decrease. The wave shape is like a superposition of solitary and shock (hereafter we refer it to as solitary-like structure). If $\beta>0$, the solitary-like wave appears to be compressive; whereas if $\beta<0$, it exhibits depletive. The case of $\beta<0$ characterizes the density depleted structures and potential well.

On the other hand, in the case of $\alpha<0$, regardless of $\beta>0$ or $\beta<0$, the solitary-like structures exhibit potential increase, the direction of electric field is opposite to the wave propagation. This suggests that waves get energy from some free energy sources which have not been identified yet. Various possible energy sources have been proposed which include magnetospheric electron injection, ion-beam, field-aligned current, etc.. However, till now their relations with solitary waves have not been identified by observations. For example, Mälkki and Lundin (1994) show that there are well correlations between upward flowing ion beams and solitary structures, but which of them being the consequence is still an open question.

Within the context of present model (warm electrons possess the Boltzman distribution and ions are cold) there is only weak dissipation due to electron temperature. Therefore $\alpha > 0$ and $\beta > 0$, the wave have structure plotted in Fig. 2. It is seen in Fig. 2 that this type of structure can produce a parallel electric field along the wave propagation direction. This is consistent with the observations that solitary-like structures move upwards with a parallel electric field pointing upward (Dovner *et al.* 1994).

Based on the above discussions, we can get the following conclusions:

(1) For $\alpha = 0$, the nonlinear ion acoustic wave may develop into a symmetric solitary structure which is compressive with no net potential difference. On the other hand, when $\alpha \neq 0$, ion acoustic wave evolve to compressive solitary-like structure with a asymmetric shape, and produce a potential drop in the direction of wave propagation.

(2) The effects of wave-particle interaction must be important in the forming wave structures associated with the potential differences.

(3) The occurrence of the density depleted structures and potential well must be dependent on the background plasma parameters which affect the dispersive property of plasma. We are about to clarify this in further works.

In the end, based on all afore mentioned analyses, we suggest a possible scenario for the appearence of the WDLs: a symmetric structures may first be excited by local free energy sources, then during their subsequent propagation they evolve into WDLs due to wave-particle interaction. The WDLs may either transfer energy to the particles (potential drop) or absorb energy from particles (potential increase).

Acknowledgements This work is supported by the projects 49634160 and 49974033 of NNSFC. We are grateful to Prof. Pu Zuyin for help revising the manuscript in English and offering opinions.

References

Burke WJ, Heinemann M(1983): Origins and consequences of parallel electric field. In: Carovillano RL and Forbes JM, ed. Solar-terrestrial Physics. Dreidel, Hingham-Mass.

Dovner PO, Eriksson AI, Boström R, Holback B(1994): Freja multiprobe observations of the electrostatic solitary structures. Geophys. Res. Lett., 21(17): 1827 - 1830.

Eriksson AI, Mälkki A, Dovner PO, Boström R, Holmgren G, Holback B(1997): A statistical survey of auroral solitary waves and weak double layers: 2. Measurement accuracy and ambient plasma density. J. Geophys. Res., 102:11385 - 11398.

Gray PC, Hudson MK, Bergmann R, Roth I(1991): Decay of ion beam driven acoustic waves into ion holes. Geophys. Res. Lett., 18:1675 - 1678.

Infeld E(1990): Nonlinear Waves, Solitons and Chaos. Cambridge University Press.

Liu Shida, Liu Shigua(1991): KdV-Burgers model of the turbutence. Science in China (Series A), 9: 938 - 946.

Mälkki A, Eriksson AI, Dovner PO, Boström R, Holback B, Holmgren G, Koskinen HEJ(1993): A statistical survey of auroral solitary waves and weak double layers: 1. Occurrence and net voltage. J. Geophys. Res., 98:15521 - 15530.

Mälkki A, Lundin R(1994): Altitude distributions of upward flowing ion beams and solitary wave structures in the Viking data. Geophys. Res. Lett., 21:2243.

Shawhan SD, Fälthammar C-G, Block LP(1978): On the nature of large auroral zone electric fields at 1-R_E altitude. J. Geophys. Res., 83:1049 - 1056.