Statistical analysis of high-speed jet flows

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Abstract

The spatiotemporal dynamics of pressure fluctuations of a turbulent jet flow is examined from the viewpoints of symbolic permutations theory and Kolmogorov-Smirnov statistics. The methods are applied to unveil hidden structures in the near-field of the two jets corresponding to the NASA SHJAR SP3 and SP7 experiments. Large Eddy Simulations (LES) are performed using the high-resolution Compact Accurately Boundary-Adjusting high-REsolution Technique (CABARET) accelerated on Graphics Processing Units (GPUs). It is demonstrated that the decomposition of the LES pressure solutions into symbolic patterns of simpler temporal structure reveals the existence of some orderly structures in the jet flows. To separate the non-linear dynamics of the revealed structures from the linear part, the results based on the pressure signals obtained from LES are compared with the surrogate dataset constructed from the original data.

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I. Introduction

High-speed jet flows are an example of frequently occurring free-shear flows in industrial and aviation applications. Despite decades of research, they continue to attract attention especially in the search for organised structures hidden under the random turbulence from the viewpoint of control of such flows. In early works [1] experimental techniques were used to visualise orderly structure in jet turbulence. Crighton and Gaster [2] suggested that the coherent structures in the jet can be described as linear instability waves with respect to the effective mean flow by analogy with laminar jets. More recently, the importance of large-scale coherent structures was emphasised in a series of works devoted to the wave-packet theory [3]. These structures can be extracted from the flow using techniques such as Spectral Proper Orthogonal Decomposition (SPOD) [4,5] by utilising a database of Large Eddy Simulations (LES). To characterise the jet development, three amplification mechanisms of hydrodynamic fluctuations in a turbulent jet, namely lift-up, Kelvin- Helmholtz, and Orr were identified via global resolvent analysis and SPOD [6]. The reduced-order footprints of these dynamic structures, such as the first few most energetic SPOD modes [6], are amenable to data-driven low-order modelling [7].

However, the analysis of dominant spatiotemporal dynamics of high-speed jets based on the conventional Fourier or Wavelet analysis using LES data have limitations due to the relative shortness of the computed flow signal. At the same time, the methods based on the coarse graining/symbolisation of noisy signal data can be applied to the jet time series to distinguish correlated (non-random) from uncorrelated (random) parts of the turbulence flow fields.

As a popular data analysis method, Permutation Entropy (PE) analysis, has been discussed in a number of works including [8-10]. The idea behind this approach lies in counting the occurrence of symbolic ordinal patterns characterizing the discretized signal. Following the original methodology developed by Brandt and Pompe [8], it becomes a useful technique to examine the determinism as a measure of complexity in time series across several scientific disciplines. For example, using the complexity-entropy measure developed by Rosso et al. [9], Kobayashi et al. [10] showed that jet velocity exhibits a stochastic behaviour in the near nozzle region, but contains strong deterministic component in the far-field where it becomes dominated by the chaotic nature. Although this approach gave an indication that low-dimensional dynamics may govern the time evolution of a turbulent jet in the far-field, it did not suggest any specific structure of dynamical systems (models) that can be responsible for the detected determinism. The present work can be viewed as an extension of the Kobayashi et al. [10] approach for extraction of hidden patterns in jet turbulence, which may lead to a model of the low-dimensional jet dynamics to be developed at the next step.

II.Conventional Statistical Analyses

The jet cases considered in the present study correspond to the conditions of the NASA Small Hot Jet Acoustic Rig (SHJAR) experiment. SHJAR jet is isothermal single-stream static jet, which issues from profiled axi-symmetric convergent nozzles of a small area ratio. Two Set Points (SP) 3 and 7 with acoustic Mach number Ma = 0.5 and 0.9, respectively, are considered [11,12]. The Large Eddy Simulation solutions of the SHJAR jets were obtained using the high-resolution CABARET method accelerated on GPU cards. The CABARET method and its implementation details can be found in [13-16].

Snapshots of unsteady pressure field in a cylindrical volume around the jet (in streamwise x / D_j, radial r / D_j and azimuthal θ coordinates) are extracted from the LES solution. There were 5000 time samples extracted in total. One such snapshot of SP7 jet at one azimuthal angle is shown in Figure 1.

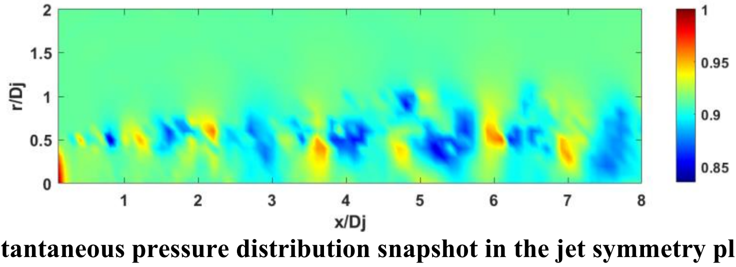
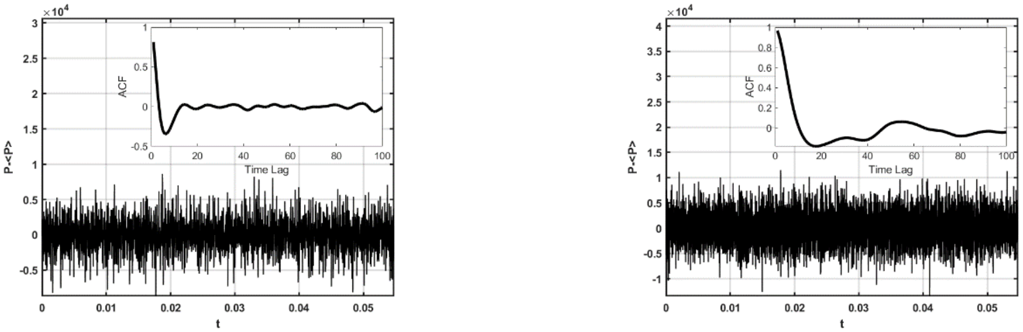


Figure 1: Instantaneous pressure distribution snapshot in the jet symmetry plane of SP7 jet.

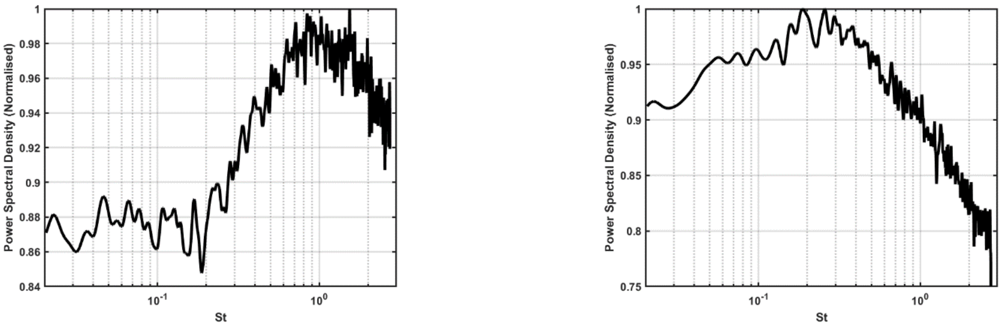
To illustrate the difference in the evolving jet between an upstream point in the early shear layers and in the end of the potential core region, Figure 2 shows the pressure time signals $p(t)$ and the autocorrelation function (ACF) at two spatial locations on the jet lipline at one azimuthal angle. As the jet flow is spreading downstream, the temporal scale grows larger compared to the upstream location, as expected in accordance with the jet flow physics.



(a) (b)

Figure 2: Pressure time series and autocorrelation function
$p(x/D_j = 6, r/D_j = 0.5, \theta = 1)$ **- (a);** $p(x/D_j = 6, r/D_j = 0.5, \theta = 1)$ **) - (b).**

The same trend is also observed in the pressure Fourier spectrum plots shown in Figure 3, where the peak frequency is reduced for the downstream probe locations compared to the upstream point.



(a) (b)

Figure 3: Power spectral density $\hat{p}(x/D_j = 1, r/D_j = 0.5, \theta = 1)$ **) - (a);** $\hat{p}(x/D_j = 6, r/D_j = 0.5, \theta = 1)$ **- (b).**

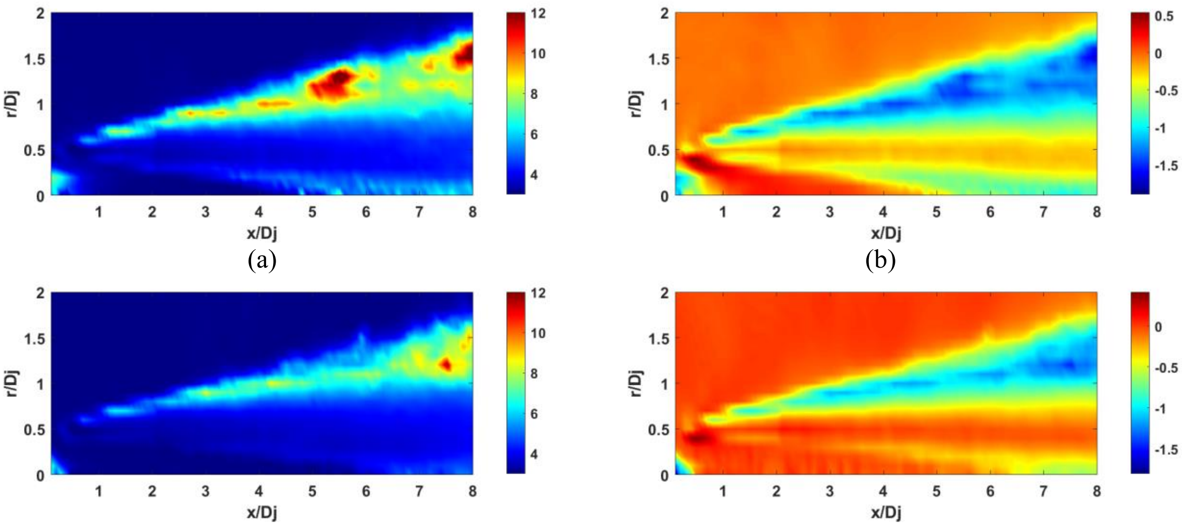
The frequency is plotted as the Strouhal number based on the nozzle exit diameter and jet velocity St=fD_j / U_j.

Next, third- and fourth- order statistical moments of the pressure fluctuations are considered,

$$s(p) = \frac{E(p-\mu)^3}{\sigma^3}, \qquad k(p) = \frac{E(p-\mu)^4}{\sigma^4} \tag{1}$$

where μ is the mean, σ is the standard deviation, and E represents the expected value of the pressure time series.

These moments are computed for each point of the cylindrical volume and averaged in the azimuthal direction to obtain a distribution in the jet symmetry plane. The results are shown in Figure 4. Clearly, the probability distribution of the fluctuating pressure is not Gaussian in the outer shear layer region. For example, high values of kurtosis (Fig.4a,c) indicate that the distribution becomes heavily tailed and negative values of skewness (Fig.4b,d) shows that pressure distributions become asymmetric around the mean. The non-Gaussian behaviour appears to be more prominent in the lower speed jet, Ma = 0.5 compared to the Ma = 0.9 case, which corresponds to thinner shear layers.



(a) (b)

(c) (d)

Figure 4: Distribution of kurtosis (a,c) and skewness (b,d) applied to pressure time series of SP3 (top row) and SP7 (bottom row) jets.

To further investigate the statistics of the pressure time series data under investigation, the Kolmogorov-Smirnov (K-S) test is considered [17,18]. The K-S test is a non-parametric test, which uses the cumulative distribution to decide about the specific distribution of the data. In other words, K-S test compares the empirical cumulative distribution function $F(x)$ (CDF) of data to a hypothesised CDF $G(x)$ which correspond to normal distribution. If the empirical CDF is not close to the hypothesized CDF in terms of the supremum norm (Eq.1), it means that the data contain complex components imposing certain structures in the probability space. The key parameter D_n specifying maximum deviation between calculated and theoretical distributions is defined as

$$D_n = \sup |F(x) - G(x)|, \tag{1}$$

Maximum deviation parameter $(D_n\sqrt{n})$ (where n is the number of samples) is then projected on K-S curve which is defined analytically as:

$$K(\chi) = \sum_{m=-\infty}^{\infty} (-1)^m e^{-2m^2\chi^2} \tag{2}$$

For verification purposes, we first consider a test case. 10,000 random time series were generated, for which the K-S test was applied. The resulting deviation parameter is projected on the K-S curve as show in Fig.5. Notably, the results are distributed within $0.2 \le D_n \le 1.8$, which confirms that the probabilities of the random time series correspond to Gaussian distribution, as expected.

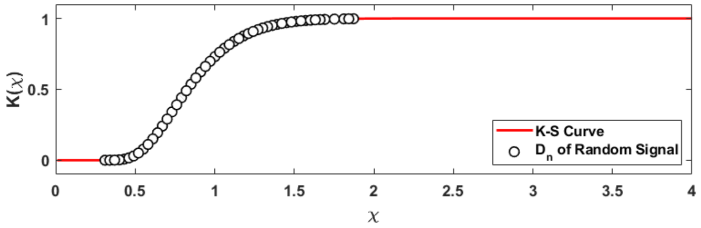
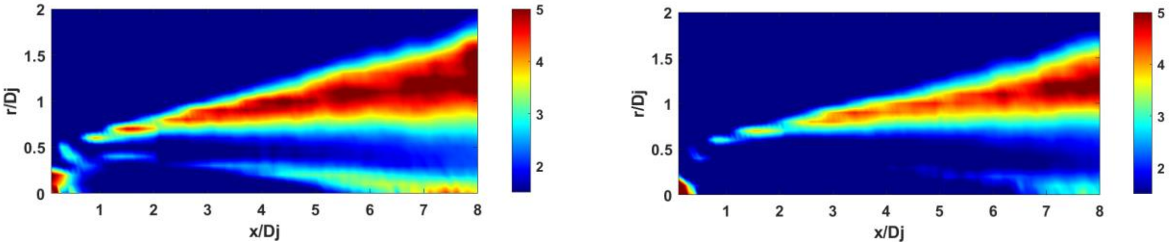


Figure 5: Kolmogorov-Smirnov test applied to random signal.

Having verified the K-S implementation on the random distribution test, the deviation parameter D_n has been estimated for all pressure time series from the jet calculation. The resulting values of maximum deviation parameter are averaged around the azimuthal coordinate and the resulting two-dimensional plots are shown in Fig.6 for both jets.

The obtained distributions confirm that non-Gaussian behaviour of the jet pressure signal occurs in the jet spreading region, in accordance with the previously performed analysis of the high-order moments (Fig.4).

However, apart from the non-Gaussian patterns in outer shear layers of the spreading jets, not much new information about the turbulent jet structures has been obtained by applying the conventional analysis.



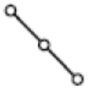
(a) (b)

Figure 6: The distribution of maximum deviation parameter of Kolmogorov-Smirnov test applied to pressure time series of SP3 (a) and SP7 (b) jets.

III.Permutation Entropy Analyses

Bandt and Pompe [8] proposed a robust symbolization algorithm to analyze nonlinear time series. The idea of which lies in the application of symbolic encoding to transform time series into a set of discrete states or patterns. Following [8], we recall that Permutation Entropy (PE) method has two model parameters, which are the number of symbols that form the pattern (also known as embedding space dimension d) and time delay τ . Briefly, if one considers embedding dimension $d=3$, three values $x_t, x_{t+\tau}, x_{t+2\tau}$ represent a pattern $\pi=312$, for instance, if $x_{t+2\tau} < x_t < x_{t+\tau}$. All possible combinations of $d!=6$ and their graphical representations are presented in Table 1.

Table 1. All possible patterns for embedding dimension of length three (patterns of circular sequence)

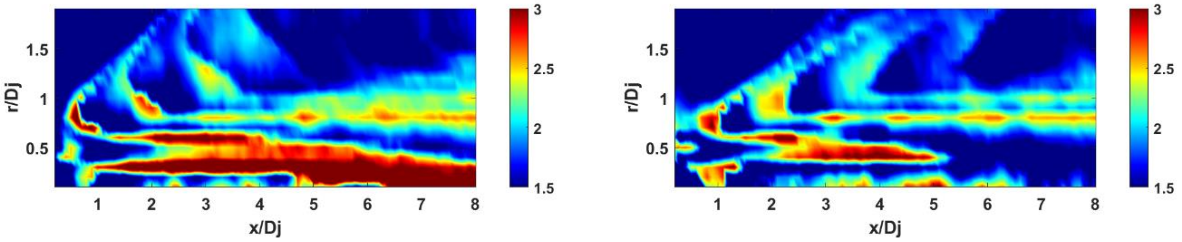
					
$x_1 < x_2 < x_3$	$x_1 < x_3 < x_2$	$x_2 < x_1 < x_3$	$x_3 < x_1 < x_2$	$x_2 < x_3 < x_1$	$x_3 < x_2 < x_1$
$\pi=123$	$\pi=132$	$\pi=213$	$\pi=312$	$\pi=231$	$\pi=321$

In the present study the embedding dimension equal to $d=5$ is considered, hence, one can construct the sequence of $d!=120$ possible permutations $\pi_i \geq 0, i=1,...,d!$. The embedding time delay choice between the adjacent real data points of considered time series, can be estimated as the decay time to reach the value of $1/2$ in the autocorrelation function. Hence, the number of patterns will be different with respect to the choice of time delay parameter. Results of the analysis of the pressure fluctuations of the turbulent jet using the above permutation-based symbolization technique and the statistical hypothesis testing using K-S are presented below.

Fig.7 shows results of the K-S procedure applied to the symbolized signal at each point of the cylindrical jet volume and averaged in the azimuthal direction. The red colour corresponds to non-random regions of the distribution (orderly structures driven by some deterministic dynamics). The blue colour corresponds to the random behaviour.

Remarkably, in comparison with the standard statistical tests, the PE method reveals some non-trivial structures. Outside of the jet flow, $r/D_j > 0.5$, multiple curved branches appear, which look similar to pressure wave interference patterns.

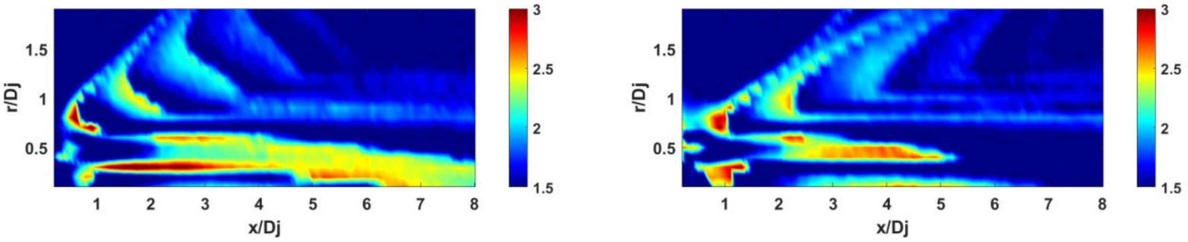
In particular, let us suppose that the stream-wise coordinate, x/D_j scales with the frequency of noise sources inside the jet (the higher the frequency, the more its sound radiated at oblique angles is reflected by the jet meanflow, which effect becomes weaker in the spreading jet shear layers) and the radial coordinate, r/D_j scales with the phase of the propagating acoustic waves through the jet. Then the multiple curved branches outside of the jet core in Fig.7 are qualitatively similar to the subtle pressure patterns in the wavenumber- frequency space, which emerge in accordance with the linear dispersion relation of the round jet flow analysed by Towne et al. [4]. For example, consider the outer boundary of these curved patterns, which would correspond to the dispersion relation branch of the upstream travelling acoustic waves. Fig.7 shows that for the faster jet at Ma = 0.9 compared to the Ma = 0.5 jet the outer branch is stretched closer to the jet axis, or the frequency axis in the wavenumber-frequency space, exactly as it should do in accordance with the jet dispersion relation.



(a) (b)

Figure 7: The distribution of Kolmogorov-Smirnov deviation parameter of symbolized pressure fields of SP3 (a) and SP7 (b) jets.

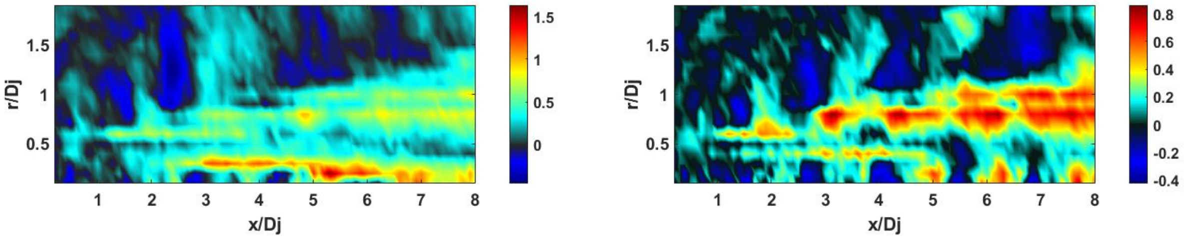
To further understand the nature of the emerged patterns, which could possibly be linked to either linear or nonlinear dynamics, several surrogate datasets were generated by shuffling the original pressure solutions of the LES dataset using a random phase. Then the same analysis is performed based on the PE method followed by the K-S test. The results of the surrogate models are presented in Fig. 8, which looks remarkably similar to Figure 7. The similarity of the multiple curved branches above the jet between the datum and the surrogate datasets suggests that the revealed subtle structures of the pressure field may indeed be related to the linear wave propagation effects as discussed above. At the same time, there are differences between the two datasets downstream of the potential jet core, $x/D_j > 6$, where the jet flow is highly nonlinear.



(a) (b)

Figure 8: The distribution of Kolmogorov-Smirnov deviation parameter of symbolized surrogate fields of SP3 (a) and SP7 (b) jets.

To further analyse the differences between the two datasets, we subtract the surrogate solutions from the original jet solutions at each spatial location in the jet symmetry plane, $(x/D_j, r/D_j)$ of each jet. Figure 9 demonstrates the result of the above procedure, which reveals cylindrical structures growing downstream of the jet core. One interesting question, which can be investigated in the future is the relation of these structures to modal and non-modal instability mechanisms in the turbulent jet flow.



(a) (b)

Figure 9: The difference between the distribution of Kolmogorov-Smirnov deviation parameter of symbolized pressure and surrogate fields of SP3 (a) and SP7 (b) jets.

IV.Conclusions

Pressure space-time distributions of high-speed jet flows are analysed using the symbolic permutations theory and Kolmogorov-Smirnov statistics in an attempt to uncover the hidden dynamics of jet turbulence. The jet cases correspond to the isothermal jets of the NASA SHJAR experiment, where the acoustic Mach number varies from 0.5 to 0.9. It is demonstrated that the decomposition of the pressure time series into simple symbolic patterns reveals the existence of orderly structures. Similar structures occur for both the Ma = 0.5 and Ma = 0.9 jet cases. The shape of these patterns becomes more stretched towards the jet flow for the faster jet. Furthermore, the structures are similar to the wavenumber-frequency branches of the linear dispersion relation of the round jet flow, if one assumes a distribution of the effective noise source inside the jet, which favors high frequencies further downstream of the nozzle exit, where the oblique sound waves are less affected by the reflection effect of the spreading jet shear layers. To further separate the linear effects from the nonlinear ones, surrogate datasets for the same two jets are generated from the pressure signals of the original LES data using the phase randomisation method. The results are analysed using the same methodology of the symbolized permutations and K-S test as the original LES dataset. Notably, both the original LES solutions and the surrogate models share the same curved branch patterns outside of the jet flow, which confirms that these patterns are related to linear pressure wave propagation effects. To reveal a part of the jet flow, which is dominated by the non-linear effects, the difference between the distribution of K-S parameter of the original symbolised pressure fields and that of the surrogate datasets is further calculated. This difference shows the existence of cylindrical structures downstream of the end of the potential core. Further investigation of these nonlinear structures will be a subject of our future work.

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