A framework for practically effective creation of postprocessors for 5-axis CNC machines with all possible configurations and working mechanisms

Introduction

The CNC machining is based on the subtractive manufacturing principle, in which the parts are manufactured by successively machining the material away from a solid block of material. CNC machines are numerically controlled via the CNC programs. With the rapid advancement in design and manufacturing technologies, there are different options for multi-axis CNC machining, from 3-axis CNC machining to 5-axis CNC machining, or even 9-axis CNC machining which has the hybrid manufacturing capability, with the combination of additive manufacturing and subtractive manufacturing in a single system,[1] or the combination of the 4-axis lathe and the 5-axis milling machine such as the multi-axis CNC machining system by Tornos Swiss Deco[2]; in which the 3-axis and 5-axis CNC machines are commonly used. In 5-axis CNC machining, the tool moves in different directions, including three linear axes *X*, *Y*, and *Z*, and two rotary axes *A*, *B* and/or *C* with different combinations such as *A* and *B*, *B* and *C*, or *A* and *C*. The 5-axis CNC machining provides the excellent capability of machining a part from all directions in a single operation and machining setup, to eliminate the need of manually repositioning the workpiece between the operations such as done in the 3-axis CNC machining; and therefore, the productivity and accuracy are enhanced, while reducing the manufacturing time in 5-axis CNC machining. In this way, the 5-axis CNC machining plays an important role in manufacturing, especially in precision machining, and making parts and products of complex shapes such as turbine or propeller blades.

The strong points of 5-axis CNC machining are well-recognized and documented, especially when compared to 3-axis CNC machining. However, besides the high cost of investments, there are always challenges and difficulties when working with 5-axis CNC machines in industrial practices. The most challenging task is to optimally generate the CNC programs for different 5-axis CNC machines with various machine configurations and working mechanisms. The manual generations of CNC machining programs are possible for 3-axis CNC machining, but impossible for 5-axis CNC machining; and the 5-axis CNC machining programs must be automatically generated by the CNC postprocessor, of which the main function is to calculate the inverse kinematic data and convert the Cutter Location (CL) data to the CNC machining programs that are used for operating specific 5-axis CNC machines. It is noted that, each record of a CL data generated by the CAM system in the CAD/CAM (Computer Aided Design/Computer Aided Manufacturing) software includes three coordinates of the tooltip position and three directional cosines of the tool axis vector. This commonly used CL data cannot be used directly for the 5-axis CNC controllers, it needs to be transferred into the CNC programs (G-codes files) for operating the 5-axis CNC machines. Fundamentally, the conversion of the CL data into the CNC program is the inverse kinematic transformation, of which the input is the position and the orientation of the tooltip (CL data), and the output is the movements of the 5 axes of a 5-axis CNC machine (G-codes file). Besides such the main task of the inverse kinematic calculation, the CNC postprocessor program also includes some other functions such as the feed rate calculation, the linearization of the cutting trajectory, the singularity avoidance, etc. In this study, the kinematic modeling and the inverse kinematic calculation for creation of a generalized postprocessor is the main scope of the investigation.

There has been a number of studies that focused on the kinematic modeling and the creation of postprocessors for the 5-axis CNC machines.[3–25] The kinematic modeling and the postprocessor creation for specific 5-axis CNC machines with the non-orthogonal rotary axes were investigated.[3–8] The methods for creating CNC postprocessors for the individual 5-axis CNC machines with the orthogonal rotary axes were proposed by Refs.[9–15] My[4] developed the postprocessor for the 5-axis CNC machine DMU 70e to transform the CL data in the form of the ISO format into the CNC machining programs, the G-codes files. Son et al.[5] introduced the postprocessor for the 5-axis CNC machine in which the head is inclined at about 45°, and the rotation and tilting angles are calculated based on inverse kinematics and mapping the part coordinate system with the machine coordinate system. She and Huang[6] proposed the method of creating the postprocessor for the 5-axis CNC machines with the nutating head and the table. Lavernhe et al.[9] presented the predictive model of the kinematic behavior during the 5-axis machining process, with the inverse kinematic modeling and the postprocessor construction for the 5-axis CNC machine MIKRON. Makhanov and Munlin[11] formulated the kinematics model of the 5-axis CNC machine MAHO 600e, with the introduced algorithms to correct the trajectories of the tool tip of a 5-axis milling machine by adjusting the rotation angles in such a way that the kinematics error is reduced. The kinematic modeling of the 5-axis CNC machines was also studied in Refs[16–18] for the purpose of the feed rate scheduling and geometric tolerance analysis. There have been efforts and studies to formulate the kinematics model of a general 5-axis CNC machine, with different machine configurations and working mechanism.[19–25] Yang and Altintas[19] hypothesized the virtual kinematic chain of 7 degrees of freedom (DOF) by adding two more axes to a general 5-axis CNC machine. She and Chang[20] and Liu et al.[22] applied the screw theory, generalized kinematics model and the exponential functions, to describe the kinematics of a machine, then to develop a generic CNC postprocessors, and to compensate geometric errors in 5-axis CNC machines. My et al.[25] proposed a general approach for the kinematic modeling of the two rotary axes of the 5-axis CNC machines. Although there have been investigations, as shown in the above-mentioned attempts,[3–25] that aims to obtain solutions for a generalized kinematic modeling of 5-axis CNC machines, a little attention has been paid to formulation of the kinematic model in a simplified and explicit manner that make it possible to create CNC postprocessors intuitively, without the required strong backgrounds and skills for engineers or CNC operators in mathematical modeling and kinematics of machines, especially mathematical modeling of multibody systems, with the advanced knowledges of kinematic modeling and complex mathematic transformations.

In a perspective of the machine mechanism and theory, a 5-axis CNC machine is a very complex mechanical system; each machine can be considered as a 5 DOFs mechanism with revolute and prismatic joints, which is similar to two robotic arms that collaboratively work in a single system, in which, one arm carries the cutter, and the other arm carries the part. The 5-axis CNC machines can be designed with a variety of sequences of five joints and links, and there are up to 108 possible machine configuration or mechanisms of 5 DOFs, or 5-axis CNC machines.[24] For these reasons, there have been efforts to develop the individual postprocessors and process planning algorithms for CNC machining; however, there has been no universal postprocessor that can be applicable for all types of 5-axis machines with different configurations and working mechanisms. Most of the early-developed postprocessor systems for 5-axis CNC machining were not widely used in the industrial practices; because of limited methodologies and difficulties of use, as well as the limited scope of applications for different types of 5-axis CNC machines; and therefore, they are only relevantly applicable for the specific types of 5-axis CNC machines.

The family of 5-axis CNC machines has a wide spectrum of machine configurations, with hundreds of mechanisms of 5 DOFs; it is therefore practically challenging for the engineers and CNC machine operators to create postprocessors for 5-axis CNC machines. As discussed above, it is easily seen that, it is more challenging in industrial practices to create CNC postprocessors, especially for cases in which engineers or CNC operators do not have a strong background and professional skills in mathematical modeling and kinematics of machines. In addition, the individual postprocessor typically has a limited application, which is restricted to one 5-axis CNC machine only, and sometimes it is only applied for making a specific product to meet expected specifications; and it cannot be applied for other 5-axis CNC machines. Therefore, there has always been an emerging need to develop solutions and frameworks of effectively creating CNC postprocessors that can be conveniently and easily used in industrial practices for a wide range of 5-axis CNC machines; and it should be conveniently be tailored for different types of applications and machine configurations, to meet expected specifications of parts to be machined.

This study proposes a generalized framework of creating CNC postprocessors for three groups of 5-axis CNC machines, based on a novel kinematic modeling method. In order to simplify the kinematic formulation, two parallel coordinate systems, including the cutting tool coordinate system and the workpiece coordinate system, are considered, and the open unified kinematic chain which is constructed from the workpiece to the cutting tool was successfully investigated. The forward and inverse kinematic equations for the three groups of 5-axis CNC machines were explicitly established in the compact and simplified manner. With the proposed solutions resulted from this study, when creating a postprocessor for a given 5-axis CNC machine, the users only need to apply the availably formulated inverse kinematic equations, there is no need to work on complex kinematic and mathematic transformations, that are normally required in the previously documented methods for creating CNC postprocessors.

The rest of the paper is organized as follows. Section “Materials and methods” presents the materials and methods, with the focus on a framework and a methodology for practically effective creation of postprocessors for 5-axis CNC machines with all possible configurations and working mechanisms. Section “Results and discussions” presents the results and discussions, with the details about the inverse kinematics for all 5-axis CNC machines, the framework and practical guidance for creating CNC postprocessors, as well as demonstrations and case studies of creating post-processors for 5-axis CNC machines. Finally, the key summaries and conclusions are presented in Section “Summary and conclusions.”

Materials and methods

A flowchart of methodology for implementation of a study is presented in Figure 1. Firstly, a family of 5-axis CNC machine configurations and related mechanisms of 5 DOFs, generally considered as a family or a class of 5-axis CNC mechanisms or 5-axis mechanisms, are investigated. Secondly, the mathematical modeling is done to formulate the forward kinematic equations for the workpiece-to-tool kinematic chain of the 5-axis CNC mechanism. Each 5-axis CNC mechanism has 5 DOFs, and it is considered as an engineering system with robotic arms or manipulators, in which one arm carries the cutting tool, and the other arm carries the workpiece. Thirdly, the inverse kinematic equations for all 5-axis CNC machines with different configurations are derived based on the formulated forward kinematic equations. Fourthly, a framework and practical guides to create postprocessors for 5-axis CNC machines is developed, with the specific steps. Finally, the findings are checked and validated via the simulations and the actual tests of 5-axis CNC machining, as well as the analysis and comparisons with the published data.

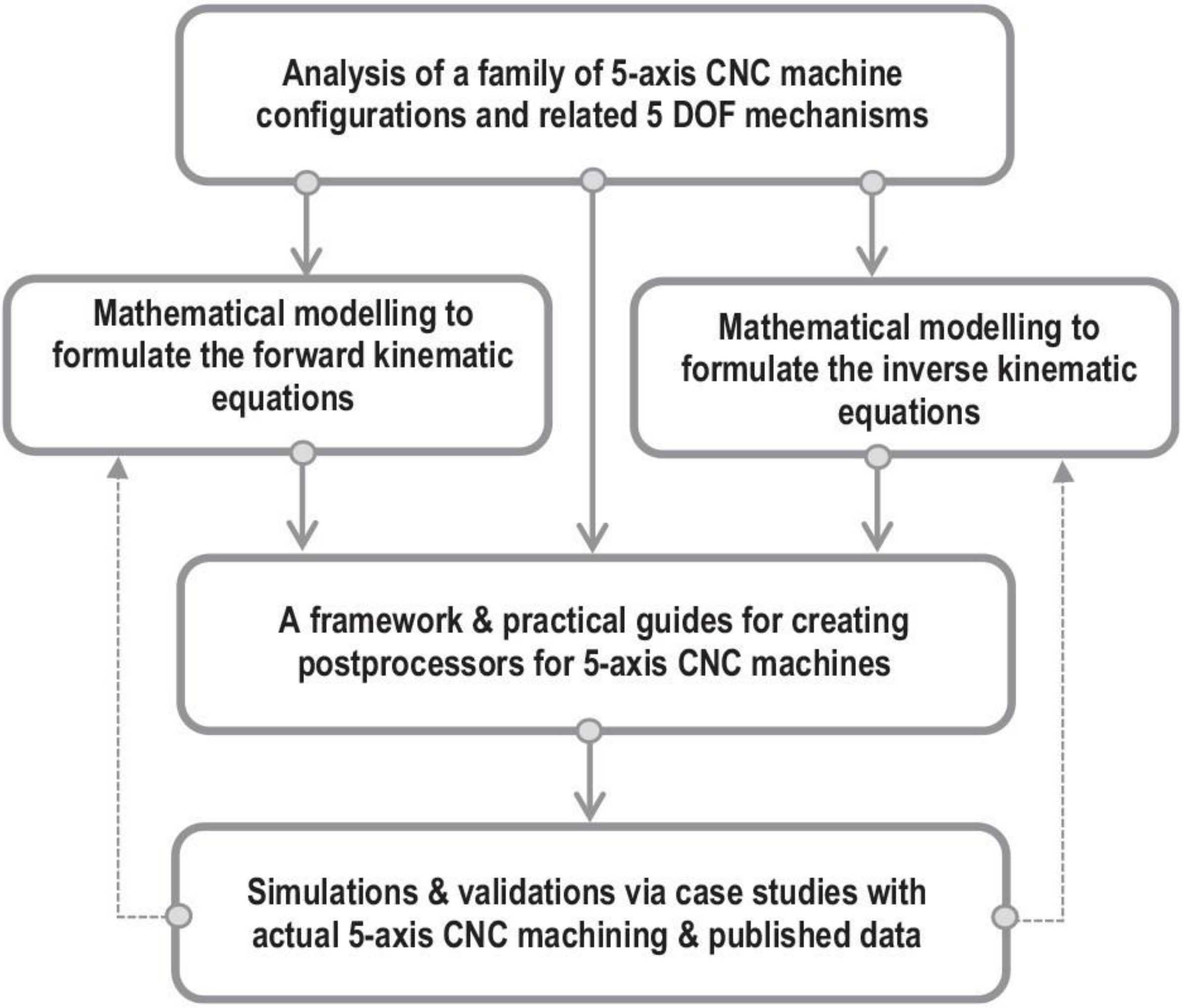

Figure 1. A flowchart of methodology for implementation of a study.OPEN IN VIEWER

Mathematical modeling to formulate the forward kinematic equations for all 5-axis CNC machines

Let's consider the family or the class of 5-axis CNC mechanisms. With the purpose of orienting the cutting tool, or the cutter, relative to the workpiece with any angles in the 3D workspace, each 5-axis CNC mechanism is designed with 5 DOF. As mentioned above, the 5-axis CNC mechanism has 5 DOF, and it is considered as an engineering system with two robotic arms, in which one arm carries the cutting tool, and the other arm carries the workpiece. The robotic arm that carries the workpiece is the so-called the CNC machine table. Figure 2 presents the workpiece-to-tool kinematic chain of the 5-axis mechanism family.

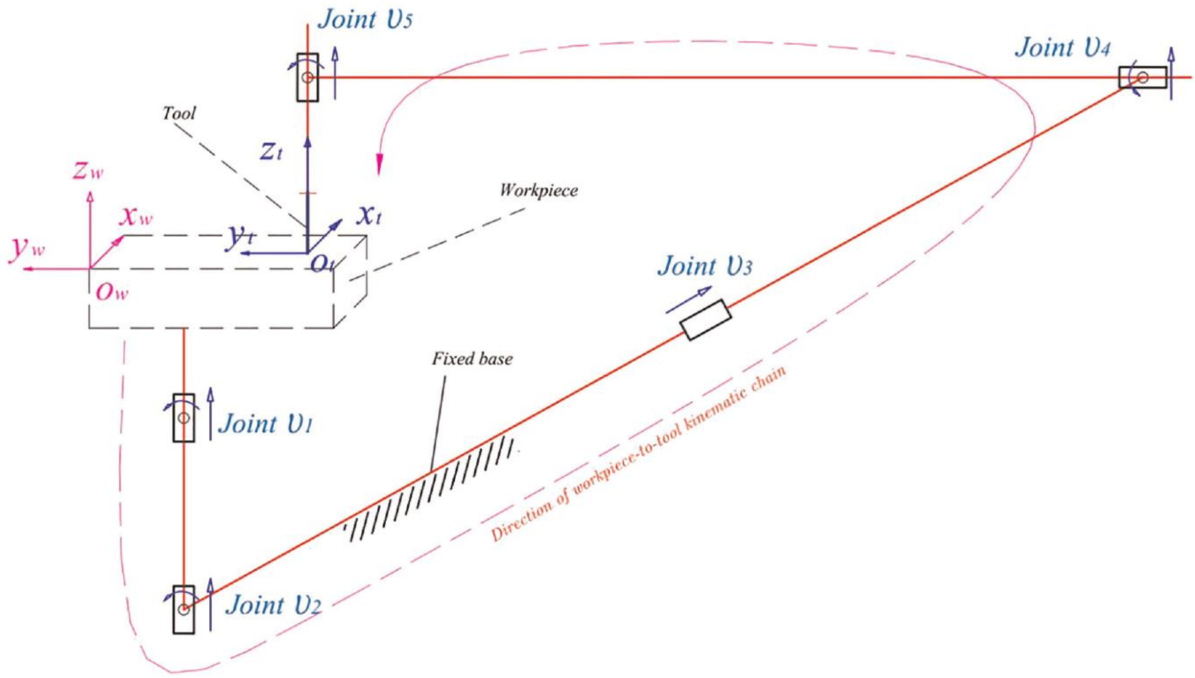

Figure 2. The workpiece-to-tool kinematic chain of the 5-axis mechanism family.

Each 5-axis mechanism of the 5-axis CNC mechanism family has five links and five joints, $\{\vartheta_i\}_{i=1\div5}$, in which there are three prismatic and two revolute joints. The prismatic joints are three linear axes *X*, *Y*, *Z* in accordance with the three orthogonal axes of the reference Cartesian frame in the workspace. The revolute joints are usually the couple of the rotary axes *AB*, *AC*, *BA*, *BC*, *CA*, or *CB*. The notations *A*, *B*, and *C* imply the rotational displacement around the axes *X*, *Y*, *Z*, respectively.

As shown in Figure 2, the kinematic branches of the so-called robotic arms connect with a common fixed base or the machine frame. In the view of the machine mechanism and theory, both the kinematic branches can be combined through the common fixed base to become a common workpiece-to-tool kinematic chain. The common kinematic chain begins with the first link which carries the workpiece and ends with the last link which carries the cutting tool.

In industrial practices, 5-axis CNC machines are normally designed in which the rotary axes are located on the machine table or the machine spindles. The family of 5-axis CNC machines can be categorized into three groups as follows:

- The first group includes 5-axis CNC machines which have both rotary axes located on the machine table. For these machines, the joints ϑ_1 and ϑ_2 are revolute joints (rotary axes), and the joints ϑ_3, ϑ_4, and ϑ_5 are prismatic joints (linear axes).

- The second group includes 5-axis CNC machines which have both rotary axes located on the machine spindles. For these machines, the joints ϑ_1, ϑ_2, and ϑ_3 are prismatic joints (linear axes), and the joints ϑ_4 and ϑ_5 are revolute joints (rotary axes).

- The third group includes 5-axis CNC machines with two rotary axes in which one rotary axis is located on the machine table and the other rotary axis is located on the machine spindle. For these machines, the joints ϑ_2, ϑ_3, and ϑ_4 are prismatic joints (linear axes), and the joints ϑ_1 and ϑ_5 are revolute joints (rotary axes).

When combining and representing all three groups of 5-axis machines into one common kinematic chain as shown in Figure 2, the joints ϑ_1, ϑ_2, ϑ_4, and ϑ_5 can be the prismatic joints or revolute joints.

To simplify the kinematic modeling of the 5-axis mechanism family, the following notations are used.

- $O_t x_t y_t z_t$ and $O_w x_w y_w z_w$ are denoted as the tool coordinate systems and the workpiece coordinate system, respectively. Both of them are parallel each other, $O_t x_t y_t z_t \uparrow\uparrow O_w x_w y_w z_w$.

- $\{\vartheta_i\}_{i=1\div 5} = \{\vartheta_u, \vartheta_v, X, Y, Z\}$ are five joint variables of the general mechanism. $\vartheta_u \text{ and } \vartheta_v$ are the joint variables of two rotary axes, and *X*, *Y*, and *Z* are the joint variables of the three linear axes. ϑ_u is the rotary axis nearest to the workpiece with respect to the forward direction of the workpiece-to-tool kinematic chain as shown in Figure 2. In industrial practices, the couple of rotary axes $\vartheta_u \vartheta_v$ could be labeled as *AB*, *AC*, *BA*, *BC*, *CA*, *CB*.

- $q = [X\ Y\ Z]^T$ is a vector of three linear joint variables.

- $K_u = \begin{bmatrix} R_u & 0 \\ 0 & 1 \end{bmatrix}$ and $K_v = \begin{bmatrix} R_v & 0 \\ 0 & 1 \end{bmatrix}$ are the homogeneous transformation matrices describing rotations of the axes $\vartheta_u \text{ and } \vartheta_v$, respectively.

- $K_{xyz} = \begin{bmatrix} E & q \\ 0 & 1 \end{bmatrix}$ is the homogeneous transformation matrix charactering the translation $q = [X\ Y\ Z]^T$ of three linear axes X, $Y \text{ and } Z$.

- RRTTT stands for the group of 5-axis CNC machines with the table-tilting, called as swivel table machines. Each machine of this group has both rotary axes (RR) located on the machine table. The sequence of joint variables of this machine group is $(\vartheta_u, \vartheta_v, X, Y, Z)$.
Similarly, $\text{TTTRR}(X, Y, Z, \vartheta_u, \vartheta_v)$ is the group of the 5-axis machines with spindle-tilting, called as swivel spindle machines, and $\text{RTTTR}(\vartheta_u, X, Y, Z, \vartheta_v)$ is the group of the 5-axis machines, in which each machine has one rotary axis that is located on the machine table and one rotary that is located on the machine spindle.

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- $\mathbf{f} = [f_x\ f_y\ f_z]^T$ is defined as a constant offset vector, pointing from the origin of the workpiece coordinate system to the pivot point on the machine table, and $W_{offset} = \begin{bmatrix} E & \mathbf{f} \\ 0 & 1 \end{bmatrix}$ is the corresponding transformation matrix. For the configurations RRTTT, the pivot point is the intersection point of two center lines of two rotary axes. For RTTTR machines, the pivot point is the point on the center line of the rotary axis located on the machine table. For TTTRR configurations, $\mathbf{f} = 0$.

- $\mathbf{d} = [d_x\ d_y\ d_z]^T$ is denoted as a constant offset vector pointing from the origin of the tool coordinate system to a pivot point on the machine spindle, and $T_{offset} = \begin{bmatrix} E & \mathbf{d} \\ 0 & 1 \end{bmatrix}$ is the corresponding transformation matrix. For RRTTT configurations, $\mathbf{d} = 0$.

- $K_T = \begin{bmatrix} R_T & p_T \\ 0 & 1 \end{bmatrix}$ is a homogeneous matrix describing the position and orientation of the tool, where R_T is a rotation matrix describing the orientation of the tool axis in $O_w x_w y_w z_w$, and $p_T = [x\ y\ z]^T$ is the position of the tooltip O_t expressed in $O_w x_w y_w z_w$. Note that three entries of the last column of R_T are the direction cosines (i, j, k) of the tool axis vector $\vec{z}_t$.

With respect to three groups of 5-axis CNC machines RRTTT, TTTRR, and RTTTR, the matrix K_T can be calculated and shown in the first row and the second row of Table 1. In the simplified manner, the tool position $p_T = [x\ y\ z]^T$ and the direction cosines (i, j, k) of the tool axis vector can be calculated explicitly and shown in the last row of Table 1 as well.

Table 1. The forward kinematics formulation for the class of the 5-axis mechanisms.

RRTTT (Swivel table)	
$K_T = W_{offset} K_u K_v K_{xyz} T_{offset}$	$K_T = W_{offset} K_{xyz} K_u K_v T_{offset}$
$\begin{bmatrix} R_T & p_T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_u R_v & R_u R_v q + f \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} R_T & p_T \\ 0 & 1 \end{bmatrix} = [R_u R_v\ 0\ R_u R_v d +$
$p_T = R_u R_v q + f$ $[i\ j\ k]^T = R_u R_v [0\ 0\ 1]^T$	$p_T = R_u R_v d + q$

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Note that, in the case that a 5-axis machine that has the rotary axis ϑ_v and it is inclined at an angle α, the rotation matrices R_α and $R_{-\alpha}$ must be multiplied before and after the matrix R_v when calculating the position and orientation of the cutting tool:

$$\begin{bmatrix} R_T & p_T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} R_u R_\alpha R_v R_{-\alpha} & R_u R_\alpha R_v R_{-\alpha} q + f \\ 0 & 1 \end{bmatrix}$$

$$p_T = R_u R_\alpha R_v R_{-\alpha} q + f$$

$$[i\ j\ k]^T = R_u R_\alpha R_v R_{-\alpha} [0\ 0\ 1]^T$$

The last row of Table 1 shows the closed form equations for the forward kinematics of all 5-axis CNC machines.

Mathematical modeling to derive the inverse kinematics for all 5-axis CNC machines

Based on the forward kinematic equations that are established in a simplified and compact manner, the inverse kinematic equations for all 5-axis CNC machine types can be effectively derived. By transforming the forward kinematic equations in last row of Table 1, the rotary joint variables ϑu�� and ϑv��, and the linear joint variables q=[XYZ]T�=[���]� can be easily determined. With respect to the sequences of the two rotary axes ϑu�� and ϑv��, the inverse kinematic equations for the rotary axes and linear axes can be expressed explicitly as shown in Table 2.

Table 2. The inverse kinematic equations for the rotary and linear axes of the 5-axis CNC mechanism family.

ϑu��	ϑv��	Ru��	Rv��	
A	B	⎡⎣⎢1000cosAsinA0−sinAcosA⎤⎦⎥[1000cos�−sin�0sin�cos�]	⎡⎣⎢cosB0−sinB010sinB0cosB⎤⎦⎥[cos�0sin�010−sin�0cos�]	A=arctar
B	A	⎡⎣⎢cosB0−sinB010sinB0cosB⎤⎦⎥[cos�0sin�010−sin�0cos�]	⎡⎣⎢1000cosAsinA0−sinAcosA⎤⎦⎥[1000cos�−sin�0sin�cos�]	B=arctar
C	A	⎡⎣⎢cosCsinC0−sinCcosC0001⎤⎦⎥[cos�−sin�0sin�cosC0001]	⎡⎣⎢1000cosAsinA0−sinAcosA⎤⎦⎥[1000cos�−sin�0sin�cos�]	C=arctar
	B	⎡⎣⎢cosCsinC0−sinCcosC0001⎤⎦⎥[cos�−sin�0sin�cosC0001]	⎡⎣⎢cosB0−sinB010sinB0cosB⎤⎦⎥[cos�0sin�010−sin�0cos�]	C=arctar
	B (inclined at α)	⎡⎣⎢cosCsinC0−sinCcosC0001⎤⎦⎥[cos�−sin�0sin�cosC0001]	⎡⎣⎢cosB0−sinB010sinB0cosB⎤⎦⎥[cos�0sin�010−sin�0cos�]	B=arccos B)

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Important notes

Wherever the rotary axes ϑuandϑv��and�� are in the workpiece-to-tool kinematic chain, the positive direction of them always obey the right-hand rule, with respect to three linear axes *X*, *Y*, and *Z*.

If the axes of a given machine are implemented in the workpiece kinematic chain (on the machine table), the sign of the joint variables that are corresponding to the axes must be changed for all mathematical expressions in Tables

1 and 2 (i.e. $\vartheta_u = -\vartheta_u; \vartheta_v = -\vartheta_v$ $X = -X; Y = -Y; Z = -Z$).

In industrial practices, there are 5-axis CNC machines that have its rotary axes which are located on the machine table; the positive direction of the rotary axes do not obey the right-hand rule. In these cases, the sign of ϑ_u and ϑ_v must be reversed, that is, $\vartheta_u = -\vartheta_u$ and $\vartheta_v = -\vartheta_v$.

Results and discussions

A framework and practical guides for creating CNC postprocessors

Figure 3 presents a proposed framework and practical guides with two fundamental steps for creating the postprocessors for 5-axis CNC machines. The framework and practical guides aim to help engineers and CNC operators to construct a postprocessor for any 5-axis CNC machines, with a specific machine configuration and working mechanism.

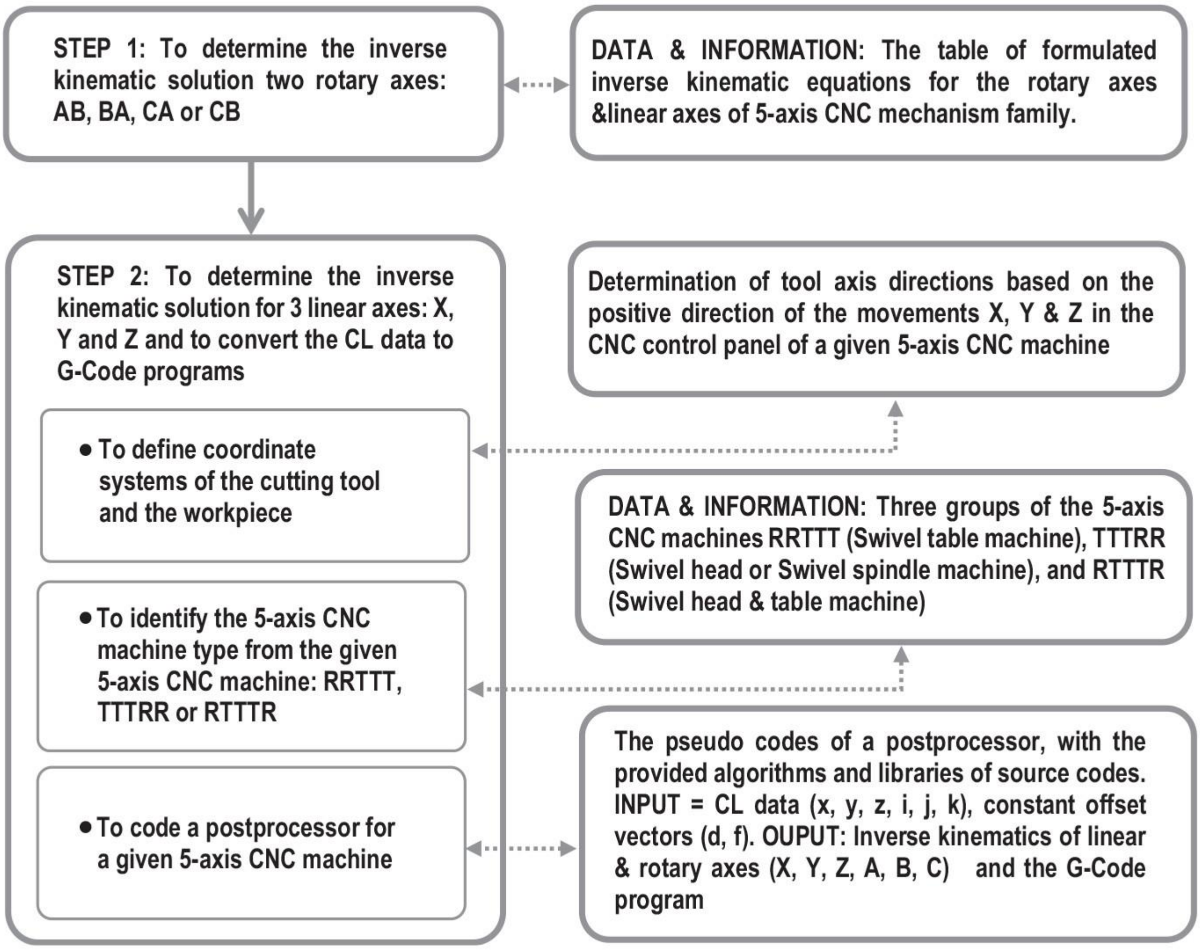

Figure 3. A framework and practical guides for creating postprocessors for 5-axis CNC machines.OPEN IN VIEWER

STEP 1: To determine the inverse kinematic equation for two rotary axes

As shown in Table 2, the inverse kinematic solution of a given 5-axis CNC machine depends on the label of the rotary axes (*AB*, *BA*, *CA*, or *CB*) and the order of the two rotary axes with respect to the forward direction of the workpiece – to-tool kinematic chain of the machine. Thus, both rotary axes of a 5-axis CNC machine must be recognized firstly, and the one that is closest to the workpiece is ϑ_u, and the other one is ϑ_v.

When ϑ_u and ϑ_v are determined, the users are able to look up the corresponding inverse kinematic equations for the rotary axes in Table 2.

It has shown that by using this guideline, the users do not need any mathematical transformations when finding the inverse kinematic equations for the rotary axes. This solution is simple; however, it is practically useful for engineers or CNC operators who do not have strong backgrounds and skills in mathematical modeling and kinematics of machines, especially mathematical modeling of multibody systems, with the advanced knowledges of kinematic modeling and complex mathematic transformations.

STEP 2: To determine the inverse kinematic equation for three linear axes

To define the tool coordinate system

In order to look up the inverse kinematic solution for the linear axes, it is necessary to define a tool coordinate system $O_t x_t y_t z_t$, of which the origin O_t is placed at the center point of the tooltip, and the axes x_t, y_t, and z_t must align with and point in the positive direction of the linear movements *X*, *Y*, and *Z*, respectively. The users should look at the CNC control panel to check the positive direction of the movements *X*, *Y*, and *Z*. As usual, z_t is the tool axis direction; x_t and y_t can be determined with the right-hand rule.

Once the users have defined $O_t x_t y_t z_t$, the constant offset vector $d = [d_x d_y d_z]^T$ from O_t to a pivot point on the machine spindle can be determined easily. For RRTTT configurations, $d = 0$.

Workpiece coordinate system definition

After defining $O_t x_t y_t z_t$, the users must define a workpiece coordinate system $O_w x_w y_w z_w$ (the Part Zero point), of which the origin O_w places at a “center point” of the machine table, when the two rotation axes are set to 0. The vertical axis z_w of the coordinate system points from the table upward to the cutter. The axes x_w and y_w are identified by using the right-hand rule, and they must point in the positive direction of the linear axes X and Y of the machine as well ($O_t x_t y_t z_t \uparrow\uparrow O_w x_w y_w z_w$). Note that the Part Zero point is a reference datum of 0,0,0 coordinates on a design (CAD) drawing. This point is also used for the purpose of the toolpath generation in CAM systems. When setting up the workpiece on the machine table, this point is used to set the zero point in CNC.

After defining $O_w x_w y_w z_w$��������, the constant offset vector $f=[f_x f_y f_z]^T$�=[������]� from O_w�� to a pivot point on the machine table can be determined. For the configurations RRTTT, the pivot point is the intersection point of the two center lines of the rotary axes. For RTTTR machines, the pivot point is a point on the center line of the rotary axis on the machine table. For TTTRR configurations, $f=0$�=0.

To identify the 5-axis CNC machine type

The identification of the type of a given 5-axis CNC machine (RRTTT, TTTRR, or RTTTR) plays an important role when looking up the inverse kinematic solution for linear axes. As shown in Table 2, for the machines with swivel table (RRTTT), the inverse kinematic equation for linear axes is $[XYZ]^T=R_{Tv}R_{Tu}(p^T-f)$[���]�=������(��−�). For the machines TTTRR and RTTTR, the inverse kinematics are calculated as $[XYZ]^T=p^T-R_uR_vd$[���]�=��−����� and $[XYZ]^T=R_{Tu}p^T-R_vd-R_{Tu}f$ [���]�=�����−���−����, respectively.

It is clearly seen that without any mathematical transformations, the inverse kinematic equations for any 5-axis CNC machines can be easily and conveniently determined.

To code a postprocessor for generating the G-Code program

Based on the inverse kinematic equations as shown in Table 2 and presented in the previous steps, a postprocessor for a given 5-axis CNC machine can be coded with the use of any programming languages. The pseudo codes of a postprocessor can be illustrated as follows.

```
// INPUT: CL data (x, y, z, i, j, k), constant offset vectors (d,f)
// OUPUT: Inverse kinematics (X, Y, Z, A, B, C)
q = [X;Y;X];
pt=[x;y;z];
Ra =    [1 0 0;
         0  cos(A)  -sin(A);
        0  sin(A)  cos(A);];
Rb =    [cos(B) 0 sin(B);
         0  1  0;
         -sin(B)  0 cos(B);];
Rc =    [cos(C)  -sin(C) 0;
         sin(C)  cos(C)  0);
         0  0  1;];
if Teta_u==’A’ & Teta_v==’B’
         A=atan2(-j,k);B=asin(i);Ru=Ra; Rv=Rb;
end
if Teta_u==’B’ & Teta_v==’A’
         A=asin(-j);B=atan2(i,k);Ru=Rb; Rv=Ra;
```

```
end
if Teta_u==’C’ & Teta_v==’A’
      A=acos(k);C=atan2(-i,j);Ru=Rc; Rv=Ra;
end
if Teta_u==’C’ & Teta_v==’B’
      B=acos(k);C=atan2(j,i);Ru=Rc; Rv=Rb;
end
// The B axis inclined
if Teta_u==’C’ & Teta_v==’Bi’
     B=acos((k-cos^2(alpha))/sin^2(alpha));
C=atan2((j*sin(alpha)*sin(B)+i*sin(alpha)*cos (alpha))*(1-cos(B)),
     (-i*sin(alpha)*sin(B)+j*sin(alpha)*cos(alpha))*(1-cos(B)));
     Ru=Rc; Rv=Rb;
end
// For the machine type RRTTT
     q=inv(Rv)*inv(Ru)*(pt-f);
// For the machine type TTTRR
     q=pt-Ru*Rv*d;
// For the machine type RTTTR
     q=inv(Ru)*pt-Rv*d-inv(Ru)*f;
```

Case studies and demonstration of creating CNC postprocessors

In this section, the proposed framework and practical guides were applied to create the postprocessors for specific 5-axis CNC machines, with two case studies of creating the postprocessors for the commercially available 5-axis CNC machines: Spinner U5-620[26] and Deckel Maho DMU 70e.[27]

Creation of a CNC postprocessor for the 5-axis CNC machine: Spinner U5-620

Figure 4 presents the 3D model and the configuration of a 5-axis CNC machine tool, Spinner U5-620[26] that has three linear axes {X, Y, Z} and two rotary axes {B, C}.

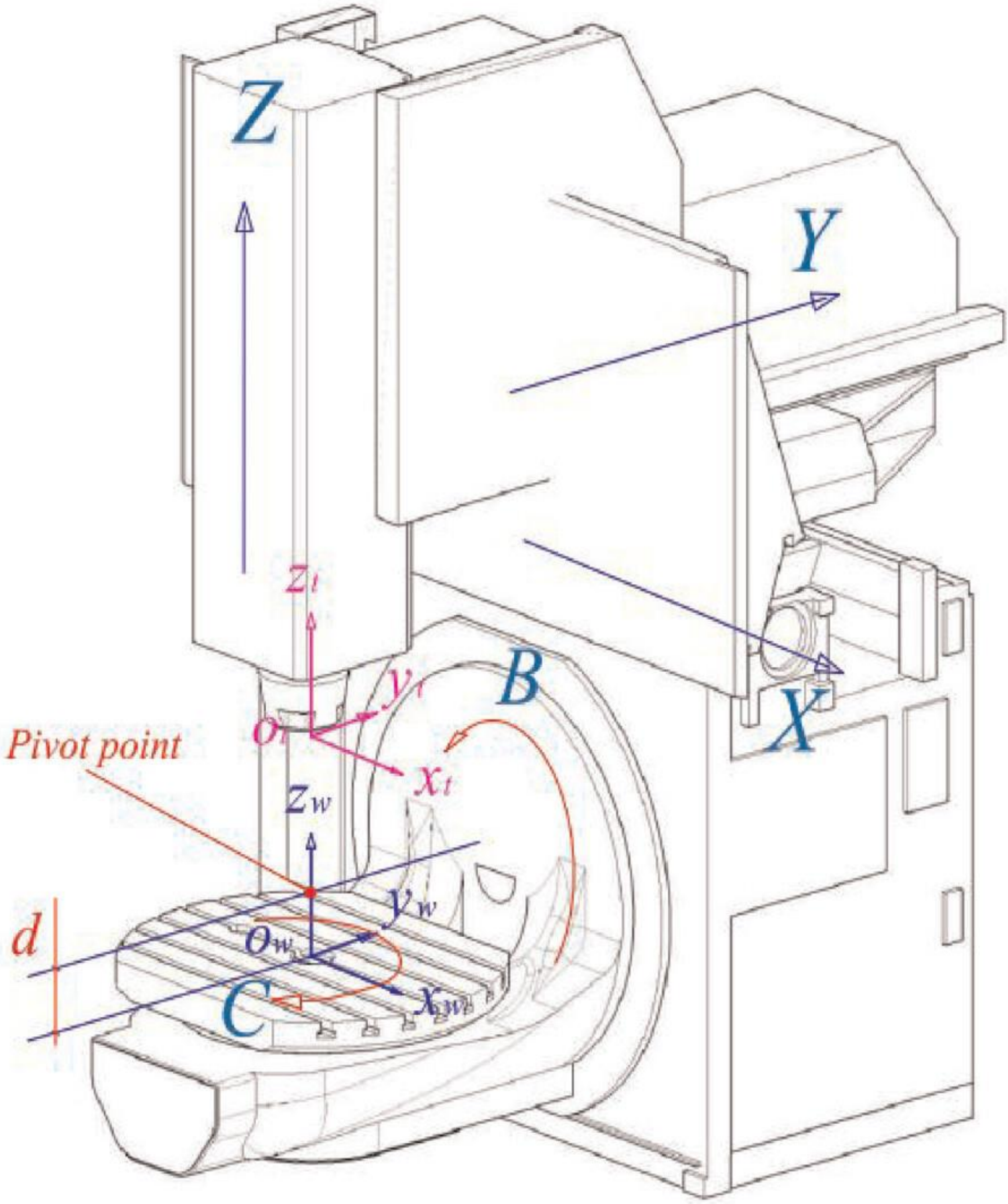

Figure 4. A 3D model and a configuration of the 5-axis CNC machine tool, Spinner U5-620.OPEN IN VIEWER

Following the steps presented in the proposed framework and guides presented in Section "Mathematical modelling to derive the inverse kinematics for all 5-axis CNC machines," the inverse kinematic equation and the postprocessor for a 5-axis CNC machine tool, Spinner U5-620, can be obtained. The following are the detailed demonstrations.

STEP 1: To determine the inverse kinematic equation for two rotary axes

It is clearly seen that the second rotary axis *C* is nearest to the workpiece, according to the order of the five joints (*C*, *B*, *X*, *Y*, and *Z*) in the direction of the workpiece – tool kinematic chain. Therefore, $\vartheta_u = C$ and $\vartheta_v = B$. Look up in Table 2, the inverse kinematic equations for the rotary axes *C* and *B* are obtained as follows.

$$C = \arctan 2(j, i) \quad (1)$$

$$B = \arccos(k) \quad (2)$$

where (i, j, k) are the direction cosines of the tool axis vector.

STEP 2: To determine the inverse kinematic equation for three linear axes

In order to look up the inverse kinematic equation for the three linear axes, we must define the coordinate systems $O_t x_t y_t z_t$ and $O_w x_w y_w z_w$ ($O_t x_t y_t z_t \uparrow\uparrow O_w x_w y_w z_w$) as shown in Figure 4.

The point O_t of the tool coordinate system must coincide with the tooltip point, and the axes x_t, y_t, and z_t must align with the actual movements *X*, *Y*, and *Z*, respectively. It is noted that the positive direction of the axis displacements *X*, *Y*, *Z*, *B*, and *C* can be checked with the CNC control panel.

As shown in Figure 4, the pivot point is the intersection point of the two center lines of two rotary axes, and the constant distant *d* can be determined easily. The user can find this parameter in Technical Documents for the machine operation.

Since Spinner U5-620 is the swivel table 5-axis CNC machine (RRTTT), by looking at in Table 2, the inverse kinematic equation for the three linear axes of the machine is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_{TB} R_{TC}(p_T - f),$$

where $p_T = [x\ y\ z]^T$ and $f = [0\ 0\ d]^T$.

Finally,

$$X = x\cos B\cos C + y\cos B\sin C - z\sin B + d\sin B \quad (3)$$

$$Y = x\sin C + y\cos C \quad (4)$$

$$Z = x\sin B\cos C + y\sin B\sin C + z\cos B - d\cos B + d \quad (5)$$

It is noted that since the axes *B* and *C* are implemented in the workpiece carrying chain (on the machine table), the sign of the variables *B* and *C* in all the inverse kinematic equations should be reversed. However, in industrial practices, by default in the CNC control of Spinner U5-620, the positive direction of the axes *B* and *C* was reversed as shown in Figure 4, with respect to the right-hand rule. Therefore, it is not necessary to change the sign of the variables *B* and *C*.

Based on the inverse kinematic equations ((1)–(5)), the postprocessor for the 5-axis CNC machine Spinner U5-620 was coded easily for generating the G-codes outputs. The simulation scenarios were done via VeriCut which is the CNC machining simulation software developed by CGTech as shown in Figure 5; and the comparisons between the postprocessing data (G-codes files) produced by the postprocessor and the CNC data produced by the commercial CAM systems were taken into account to validate the created postprocessor.

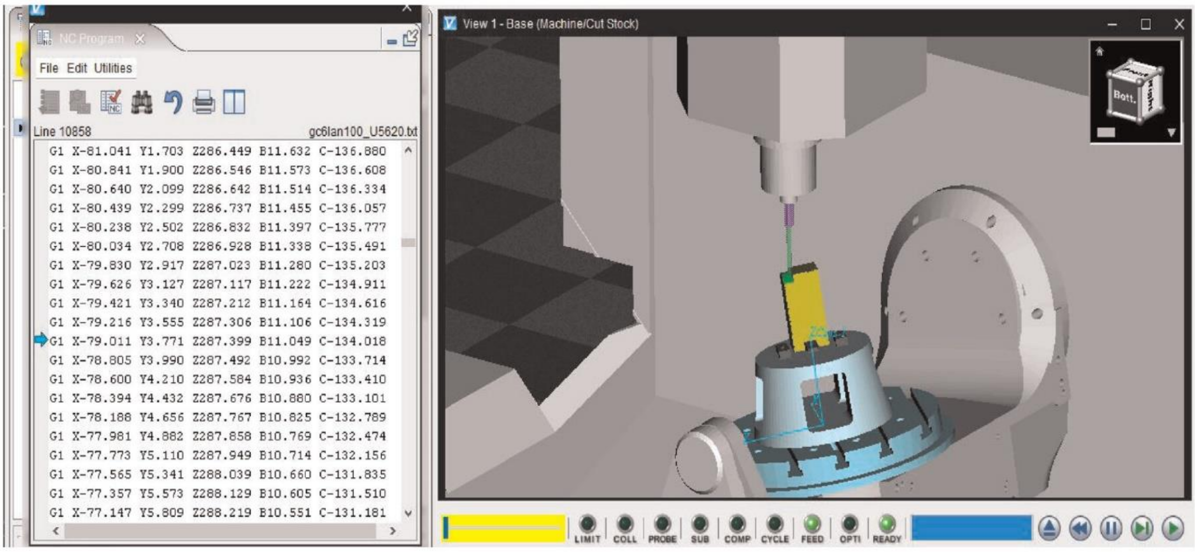

Figure 5. A 5-axis CNC machining simulation with the 5-axis CNC machine Spinner U5-620 via VeriCut using the G-codes files which are produced by the postprocessor created by the proposed framework and practical guides of a study.OPEN IN VIEWER

Figure 6 presents the 5-axis CNC machining demonstration in which a part was machined by the 5-axis CNC machine Spinner U5-620, to verify the postprocessor created by the proposed framework and practical guides of a study. In this experiment, the 3D convex surface that needs to be machined was modeled in the CAD environment, and the toolpath (CL data) for machining this 3D convex surface was generated by the CAM system in the commercially available CAD/CAM software. By using the created postprocessor, the generated CL data was successfully post-processed and converted into the G-codes file that was specifically required for the 5-axis CNC machine Spinner U5-620 to machine the part.

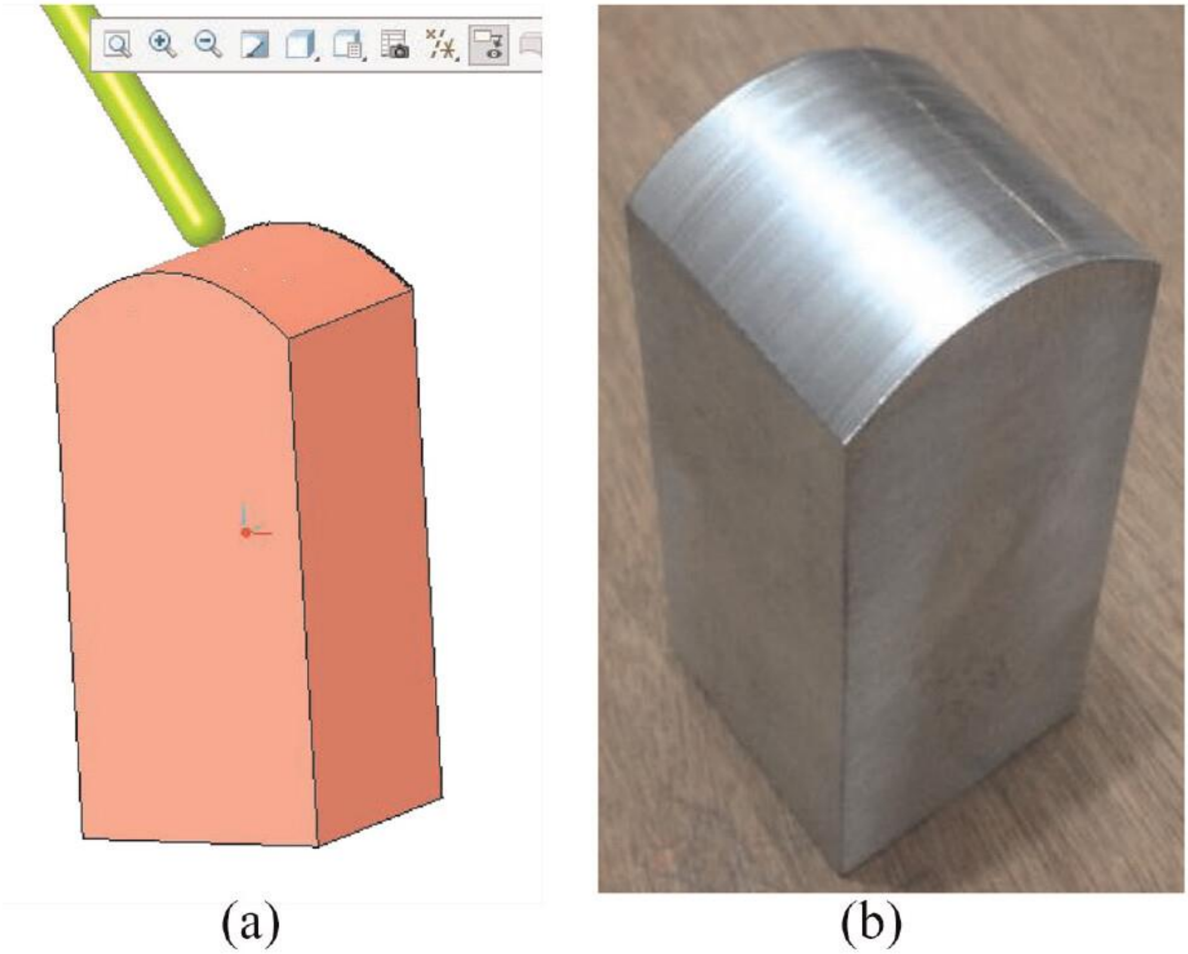

Figure 6. The 5-axis CNC machining demonstration to verify the postprocessor created by the proposed framework and practical guides of a study: (a) a 3D CAD model with the 3D convex surface for the toolpath generation in the CAD/CAM software and (b) a part successfully machined by the 5-axis CNC machine Spinner U5-620.OPEN IN VIEWER

Creation of a CNC postprocessor for the 5-axis CNC machine: Deckel Maho DMU 70e

Figure 7 presents the 3D model and the configuration of the 5-axis CNC machine tool, Deckel Maho DMU 70e[27] that has three linear axes $\{X, Y, Z\}$ and two rotary axes $\{B, C\}$, in which the axis B is inclined with an angle of 45°.

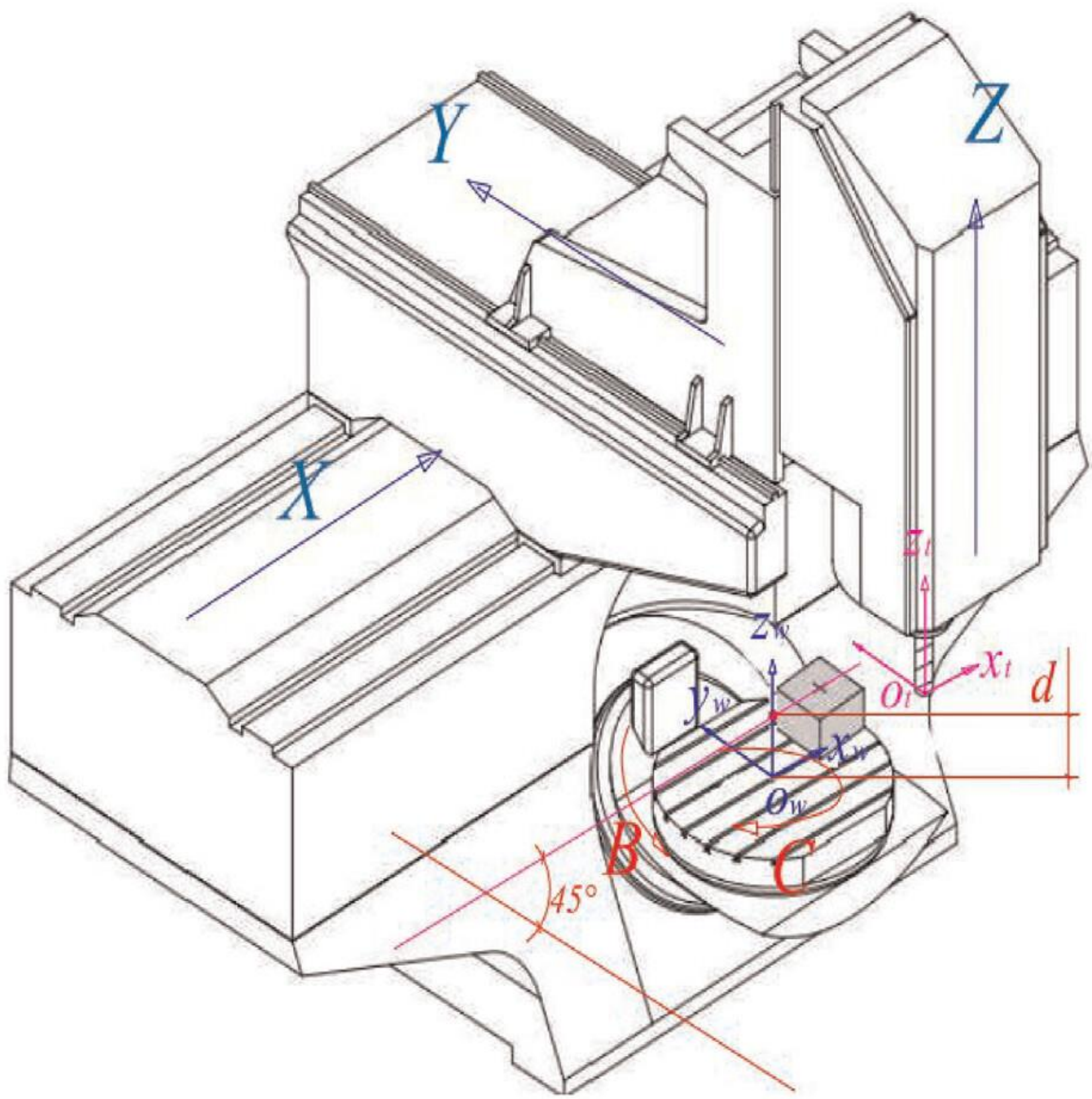

Figure 7. A 3D model and a configuration of the 5-axis CNC machine tool, DMG Deckel Maho DMU 70e.OPEN IN VIEWER

STEP 1: To determine the inverse kinematic equation for two rotary axes

Similar to the demonstration in Section “Creation of a CNC postprocessor for the 5-axis CNC machine: Spinner U5-620,” the second rotary axis *C* is nearest to the workpiece, hence $\vartheta_u = C$ and $\vartheta_v = B$. By looking at Table 2, the inverse kinematic equations for the rotary axes *C* and *B* are obtained as follows.

$$B=\arccos\left((k-\cos^2\alpha)/\sin^2\alpha\right) \tag{6}$$

$$C=\operatorname{arctan2}\left(j\sin\alpha\sin B+i\sin\alpha\cos\alpha(1-\cos B),\,-i\sin\alpha\sin B+j\sin\alpha\cos\alpha(1-\cos B)\right)$$

(7)

Note that $\alpha=-45^{0}$, hence

$$B=\arccos(2k-1)$$

(8)

$$C=\operatorname{arctan2}\left((1-k)i+2\sqrt{(k-k^{2})}\,j,\ 2\sqrt{(k-k^{2})}\,i+(1-k)j\right)$$

(9)

where (i,j,k) are the direction cosines of the tool axis vector.

STEP 2: To determine the inverse kinematic equation for three linear axes

By using the guidance, the coordinate systems $O_t x_t y_t z_t$ and $O_w x_w y_w z_w$ ($O_t x_t y_t z_t \uparrow\uparrow O_w x_w y_w z_w$) are defined as shown in Figure 7.

By looking up in Table 2, the inverse kinematic equation for the three linear axes of the machine is

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}={}^{T}R_{-\alpha}\,{}^{T}R_{B}\,{}^{T}R_{\alpha}\,{}^{T}R_{C}\,(p_{T}-f)$$

(10)

where

$$R_{\alpha}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_{-\alpha}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(-\alpha) & \sin(-\alpha) \\ 0 & -\sin(-\alpha) & \cos(-\alpha) \end{bmatrix}$$

$$f=[0\ 0\ d]^{T}$$

Finally, the inverse kinematic equations for three linear axes can be yielded as follows.

$$X=\frac{1}{\sqrt{2}}\sin B\sin C-\frac{1}{\sqrt{2}}y\sin B\cos C-\frac{1}{\sqrt{2}}z\sin B+\frac{1}{\sqrt{2}}d\sin B+y\cos B\sin C+x\cos B\cos C$$

(11)

$$Y=\frac{1}{\sqrt{2}}y\sin B\sin C+\frac{1}{\sqrt{2}}x\sin B\cos C-\frac{1}{2}x\cos B\sin C+\frac{1}{2}y\cos B\cos C+\frac{1}{2}z\cos B-\frac{1}{2}d\cos B-\frac{1}{2}x\sin C+\frac{1}{2}y\cos C-\frac{1}{2}z+\frac{1}{2}d$$

(12)

$$Z=\frac{1}{\sqrt{2}}y\sin B\sin C+\frac{1}{\sqrt{2}}x\sin B\cos C-\frac{1}{2}x\cos B\sin C+\frac{1}{2}y\cos B\cos C+\frac{1}{2}z\cos B-\frac{1}{2}d\cos B+\frac{1}{2}x\sin C-\frac{1}{2}y\cos C+\frac{1}{2}z-\frac{1}{2}d$$

(13)

By default, in the CNC control of the 5-axis CNC machine Deckel Maho DMU 70e, the positive direction of the axes *B* and *C* is reversed, with respect to the right-hand rule. It is therefore not necessary to change the sign of the variables *B* and *C*.

Summary and conclusions

In this study, a framework and practical guides to create CNC postprocessors for 5-axis CNC machines with all possible configurations and working mechanisms are proposed, with the demonstrations for the cases of commercially available 5-axis CNC machines Spinner U5-620 and Deckel Maho DMU 70e. The CNC programs or G-code data can be conveniently generated, based on the novel mathematical formulation of the inverse kinematics of 5-axis CNC machines with all possible configurations and working mechanisms. The proposed frameworks and guides can be effectively applied in industrial practices without the required strong backgrounds and skills in kinematics and mathematical modeling of multibody systems. A CNC postprocessor for the specific 5-axis CNC machines is created based on (i) the formulated inverse kinematic equations, (ii) the proposed workflow can be easily implemented with specific steps, and (iii) the pseudo codes for generating G-code programs from the calculated CL data.

An open unified kinematic chain which is constructed from the workpiece to the cutting tool was fully investigated for all types of 5-axis CNC machines with different configurations and working mechanisms. Two parallel coordinate systems, including the cutting tool coordinate system and the workpiece coordinate system, were considered, to simplify the kinematic modeling of 5-axis CNC machines. In this way, the analytical solution in a compact and explicit form to the inverse kinematic problem of all 5-axis CNC machines was obtained. Although the unique and closed-form kinematic equation for the whole class of all 5-axis CNC machines has not been obtained, with the successfully developed solutions for the kinematic modeling of the 5-axis CNC machines, the forward and inverse kinematic equations for three groups of 5-axis CNC machines are explicitly established.

The methods proposed in this study for practically effective creation of postprocessors for 5-axis CNC machines were also successfully applied to determine the inverse kinematic equations for the 5-axis CNC machines that include the 5-axis CNC machines with the non-orthogonal rotary axes[3–8] and the 5-axis CNC machines with the orthogonal rotary axes.[9–15] The proposed methods were also verified with the 5-axis CNC machines with the TTTRR configurations,[3,5,6] and the 5-axis CNC machines with the RTTTR configurations.[6,15,20] It is noted that the inverse kinematic equations which are obtained by using the proposed method in this study and the inverse kinematic equations which were documented in Refs[3–15,20] are the same. However, with the use of the formulated inverse kinematic equations as documented in Table 2, and the workflow with specific steps as presented in the proposed framework and practical guides (Figure 3), the inverse kinematic equations of any 5-axis CNC machines can be easily determined, and a postprocessor for a given 5-axis CNC machine can be conveniently constructed.

The proposed framework and practical guides in this study for practically effective creation of postprocessors for 5-axis CNC machines have the following advantages when compared to the documented methods of creating CNC postprocessors. Firstly, there is no requirement for mathematical modeling and formulations with complex matrix operations and transformations, which are normally necessary for determining inverse kinematic equations. Secondly, there is no requirement to have a strong background and skills in kinematic modeling of multibody systems. Only two coordinate systems, including the workpiece coordinate system and the tool coordinate system, are used for the kinematic modeling task, compared to the other methods, such as the use of five coordinate systems in She and Huang,[6] and six coordinate systems in Rooker et al.[10] Finally, with the use of the provided pseudo codes, the CNC programs or G-code data for the specific 5-axis CNC machines can be easily generated from the calculated CL data.

Finally, the methods and solutions proposed in this study can be potentially applied for different manufacturing applications of subtractive and additive manufacturing, especially when considering the trends of moving toward smart manufacturing and hybrid manufacturing to capture and combine the strengths of additive manufacturing and CNC machining, and there is a growing number of innovatively designed CNC machines with different configurations and complex kinematic models, leading to additional challenges for creating post-processors. The further investigations which are well-aligned with this study and development of CNC postprocessors should focus on the following points: optimizations of the machining parameters (feed rate, spindle speed, and depth of cut), tool-path linearization algorithms for 5-axis CNC machining, optimal interpolations of discrete 5-axis machining tool-paths such as circular and conic tool-path interpolation.

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