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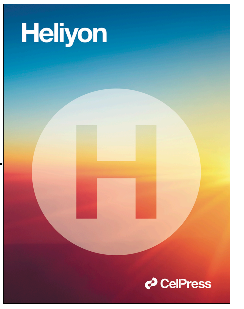

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An Information Geometrical Evaluation of Shannon Information Metrics on a Discrete n-Dimensional Digital Manifold

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Abstract: The definition and nature of information have perplexed scientists due to its dual nature in measurements. The information is discrete and continuous when evaluated on a metric scale, and the Laplace-Beltrami operator and Gauss-Bonnet Theorem can map one to another. On the other hand, defining the information as a discrete entity on the surface area of an n-dimensional discrete digital manifold provides a unique way of calculating the entropy of a manifold. The software simulation shows that the surface area of the discrete n-dimensional digital manifold is an effectively computable function. Moreover, it also provides the information-geometrical evaluation of Shannon information metrics.

Keywords: Planck Level; discrete n-dimensional digital Manifold; Shannon digital information entropy; Information Capacity; Bekenstein-Hawking information entropy; Delaunay triangulation.

1 Introduction

The definition and nature of information remain complex and multifaceted and have been the subject of much debate and exploration in various scientific fields. In a significant attempt to cover human history, from the invention of writing up to memes and the current information and communication technology enabled-enabled information space, James Gleick [1] explores the evolution of our understanding of information, from ancient communication methods such as drums and smoke signals to modern digital technologies, arguing that information is not just a static entity but a force that drives the evolution of the universe and the development of human civilization.

From a formal perspective, information can be viewed as discrete and continuous, depending on the context in which it is being evaluated. Claude Shannon pioneered the study of information theory in the mid-20th century, whose work laid the foundation for modern communication technologies. In recent years, a growing body of literature has examined the implications of quantum mechanics for our understanding of information.

Much work on the potential of the Quantum information entropy for different potential functions has been carried out: In [2], the effects of various parameters such as potential asymmetry, strength, and phase on the entropic measures of the system, including the von Neumann entropy, Tsallis entropy, and Rényi entropy has been studied, and the potential asymmetry was found to have a significant impact on the entropic measures of the system.

For the quantum information entropies associated with a symmetrically trigonometric Rosen-Morse potential, it was found that the entropic measures of the eigenstates are affected by the strength and phase of the potential and that the Tsallis entropy is more sensitive to changes in the potential parameters compared to the von Neumann entropy [3].

Another study shows that the von Neumann entropy and Shannon entropy exhibit a maximum value at a particular parameter value, while the Fisher information entropy displays a minimum value [4]. This study indicates that the system's information content is highest at this specific parameter value for the Poschl-Teller-like potential.

For a hyperbolical potential function, the potential strength and asymmetry significantly impact the system's entropic measures, and the potential phase can affect the entropic measures in different ways [5].

The Shannon entropies of the system were also found to be affected by the potential strength, mass distribution, and potential shape [6], and for an infinite circular well, the Shannon entropy is inversely proportional to the radius of the well and directly proportional to the mass of the particle [7].

These studies have tended to focus on a specific potential function and mass distribution, that the particle is non-relativistic, which may limit its applicability in specific contexts. Additionally, the study only considers a single particle in the potential, which may not accurately reflect the behavior of larger systems.

Although these researches are fascinating, additional research seems necessary to fill the gaps by adding the geometric nature of information through digital n-dimensional manifolds to expand our understanding of this fundamental concept. From information flow in terms of molecular movements to the black hole information storage bounds and the actual fabric of spacetime, the definition and the very nature of information have perplexed humanity for thousands of years. The confusion comes from the fact that the nature of information is abstract and manifold, thus very elusive.

Because of this multifariousness, one may approach information from many different branches of sciences, such as (i) Vector and Tensor Analysis, Geometry, Differential Geometry, (ii) Physics, Statistical Mechanics, and Entropy, (iii) General Theory of Relativity with spacetime and gravitational interactions under information concept, (iv) Quantum Information Theory, (v) Shannon Information Theory towards the quantification of information, (vi) Semantic Information studies towards meaning and truth [8], (vii) Computational aspects with Automata Theory & Turing Machine as information processing machines, (viii) Cellular Automata [9], and (ix) Bekenstein entropy & information bounds. This enumeration of approaches is inconclusive, and representing information on a digital manifold as a geometric entity may be an attempt to resolve this conundrum.

Entropy has been central to many information studies ever since Boltzmann. For example, Shannon entropy quantizes the information and creates a unit for measuring digital information known as a bit [10]. Later studies have accomplished a new definition known as Bekenstein-Hawking Entropy [11,12], which is

$$S_{BH} = k_B \frac{A}{4\ell_p^2} \quad \textbf{(1)}$$

where A is the area of the event horizon, l_p^2 is the square of Planck length, and k_B is Boltzmann's constant.

Moreover, Bekenstein came up with an Information bound [13], which states that there is a maximum information-processing rate for an information system with finite size and energy, which was preceded by Bremermann [14] and later updated by Gorelik [15].

Recent studies have evolved around the concept of "Information Geometry." For example, Amari examined the "dually flat structure of a manifold, highlighted by the generalized Pythagorean theorem" within the information theory [16] and defined the manifold of probability distributions as the origins of information geometry[17].

Nielsen pointed out the fundamental differential-geometric structures of information manifolds in a detailed survey [18].

The main objective of this study is to evaluate the discrete information on metric and Planck levels to highlight its geometric nature over manifolds. We believe that examining the geometric nature of information through digital n-dimensional manifolds will enable us to resolve the problem of the elusiveness of information.

The rationale of this paper is as follows: We briefly summarize the early works and describe the problem in the first section. Then in section two, we first introduce the current framework of the computational potential of information on n-dimensional digital manifolds and the possibility of defining the fabric of information as 1s or 0s of a discrete entity on a Planck scale. Since the spacetime in 2D or 3D near or on the Planck scale can be visualized by Delaunay triangulation algorithm, the aforementioned computational framework is implemented as a simulation. The methodology section, section three, pictures all those simulations and the surface area computations for spheres and manifolds. The evaluation of the simulation results, mainly the Shannon information entropy on a Planck scale, is in section four. Naturally, the discussion of the simulation results and the conclusion are provided in the sections following four.

Therefore, the main contributions of this research work include the following:

i. A novel perspective defines the information as a discrete entity on the surface area of an n-dimensional discrete digital manifold and thus provides a unique way of calculating the entropy of a manifold.
ii. Introducing a geometrical information evaluation of Shannon information metrics on a discrete n-dimensional digital manifold, a new tool for analyzing and understanding digital systems' information content and structure.

After the introduction, this paper is organized as follows. First, in section two, we define the information on metric and Plack levels by computational aspects. Also, this section combines the Bekenstein-Hawking entropic value with Shannon entropy to comply with information entropy requirements. Then, in section three, we define our research methodology used to evaluate Shannon information metrics on a discrete n-dimensional digital manifold. Next, we present the evaluation of Shannon Information Entropy on the Planck Scale in section four. The results we have achieved and their discussion are represented in section five, and finally, the conclusion is in section six.

Due to the computational nature of this study, our simulation codes and tables for Delaunay 2D/3D tetrahedronization, simulation pictures & videos have been uploaded to github for replicability and/or reproducibility.

2 Computational Potentiality of Information

Information is a fundamental concept in computer science and information theory, and understanding its nature is crucial for developing efficient and effective computational algorithms and systems. One of the vital computational aspects of information is its representation. A geometrical information evaluation of Shannon information metrics on a discrete n-dimensional digital manifold is a powerful tool for analyzing and understanding digital systems' information content and structure. The approach involves using geometric and topological techniques to construct a geometric space that represents the information content and structure of the system based on Shannon information metrics such as Entropy and Mutual Information. The computational potentiality of this information lies in its ability to be represented, processed, and analyzed by computational methods, allowing us to make predictions, simulate scenarios, and explore the behavior of complex systems.

Computer simulations can be used to study the entropy of systems at the Planck scale. First, however, a computational model must be developed that accurately represents the behavior of particles at this scale to achieve that. Nevertheless, this has always been a difficult task, as the laws of physics governing particle behavior at the Planck scale are poorly understood.

Our approach to developing a computational model of the Planck scale is to define the information as a discrete entity on the surface area of an n-dimensional discrete digital manifold. In this approach, space is divided into discrete units, and the behavior of particles is simulated on the lattice. Therefore, particles' behavior at the Planck scale can be simulated computationally efficiently. Furthermore, once a computational model has been developed, it can be used to study the entropy of systems at the Planck scale by simulating the behavior of particles under different conditions and measuring the system's entropy. By doing so, we can gain insight into the behavior of particles at the minor length scales and the entropy associated with these systems.

When the nature of information is evaluated computationally on a metric scale (10^{0}m - 10^{-6}m), it presents itself as discrete and continuous. It is shown that mapping information from a discrete domain, in the form of a graph, into a continuous domain as a manifold can be done through the Laplace-Beltrami operator and Gauss-Bonnet theorem [19]. A Laplace-Beltrami operator can measure the curvature of a manifold and is defined as the divergence of the gradient of a function defined on the manifold. The Gauss-Bonnet theorem is a fundamental theorem that relates the curvature of a surface to its topology.

A manifold is a topological space that can be covered by a collection of open subsets O_i, where O_i is isomorphic to some open subset of R_n. Manifolds are suitable differentiable mathematical objects for information to be defined on because they are non-Euclidean in the global view and resemble Euclidean spaces in local scales. A discrete domain, such as a graph, can be considered a collection of discrete points or vertices connected by edges.

The computer simulations and the effectively calculable functions relating to the information entropy are in n-dimensional discrete digital manifolds. A discrete digital manifold means the discretization of a continuous manifold [20] and is defined as a digital manifold as follows:

A connected subset M of $\sum_m$ is an n-dimensional digital manifold
if any point $p \in M$ is included in some n-cell of M, and (i) any two n-cell are $(n-1)$ connected,
(ii) every $(n-1)$ cell in M has only one or two parallel-moves, and (iii) M does not contain any $(n+1)$ cell.
Therefore, the $\sum_m$ is a discretization of Euclidean space where $m \in \mathbb{Z}^+$.

One peculiar way of examining the very nature of information through computer simulations is to consider it a discrete digital entity on the Planck scale (10^{-35}m). Defining information on the Planck scale as a discrete digital entity on the surface area of an n-dimensional digital manifold within the equilateral triangles (2D) or tetrahedrons (3D) uniquely matches it with that of the Bekenstein information metrics.

It is thus possible to define the fabric of information as 1s or 0s of a discrete entity that covers the surface area of an n-dimensional digital manifold on a Planck scale measurable by the Bekenstein number [21]. That is

$$N = 4 \,.\, \ell_p^2 \,.\, \ln 2 \tag{2}$$

whereas ℓ_p^2 is the square of the Planck length, and is equal to $2.612270x10^{-70}\,m^2$.

Considering the spacetime to be 2D (or yet 3D) near or on the Planck scale and thus discretizing it with a digital manifold calls for a triangulation process known as the Causal Dynamical Triangulation (CDT) [22-23-24-25], which is a descendant of quantum Regge calculus [26,27].

The surface area of any given n-dimensional digital manifold is an effectively calculable function by the Delaunay triangulation algorithm. Moreover, the union of multiple n-dimensional digital manifolds can be obtained by point set addition computation. At the same time, the addition of two triangulated manifolds is also obtainable through the wedge product computations.

3 Methodology

In this section, we describe the methodology used to evaluate Shannon information metrics on a discrete n-dimensional digital manifold and the explanation on the steps involved in constructing the digital manifold, selecting the Shannon information metrics, and calculating the metrics and Surface Area Calculations for Sphere and Manifolds using information geometry tools.

In this study, the computer simulations are coded in Processing 3.0 Beta graphics library and Ghull graphics visualization library. First, the triangle approximations of n-dimensional digital manifolds and n-spheres are obtained by point sets randomly generated on objects' surfaces. Then, the Delaunay triangulation algorithm calculates the surface areas of the digital manifold & of the n-sphere. The computations are detailed below.

An n-sphere is an n-dimensional digital manifold. For (n ≥ 2), the n-spheres are simply-connected n-dimensional manifolds of constant, positive curvature. The surface area formula for an n-dimensional digital manifold is then:

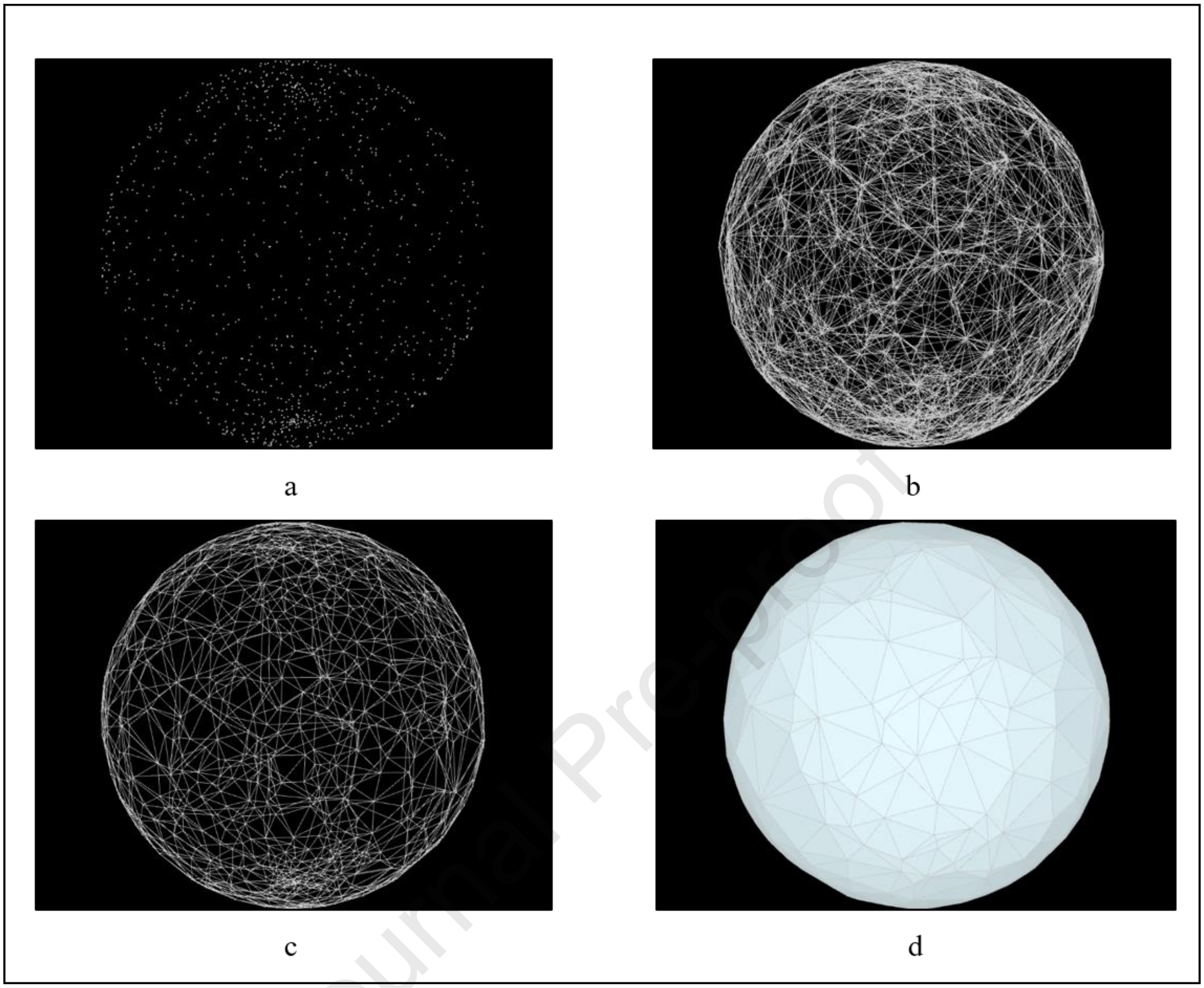
$$S_{n-1}(R) = \frac{n\pi^{n/2}}{\Gamma(\frac{n}{2}+1)} R^{n-1} \quad \textbf{(3)}$$

whereas $n, R \in \mathbb{Z}^+$ and are the number of dimensions and radius respectively.
The Γ is the gamma function which is an extention of the factorial function.
That is, if n is a positive integer then $\Gamma(n) = (n-1)!$.

The amorphous objects that generated a manifold and n-spheres also have positive constant curvature. Therefore, the Euler characteristic of those unstructured manifolds and 3-sphere, 4-sphere is 2.

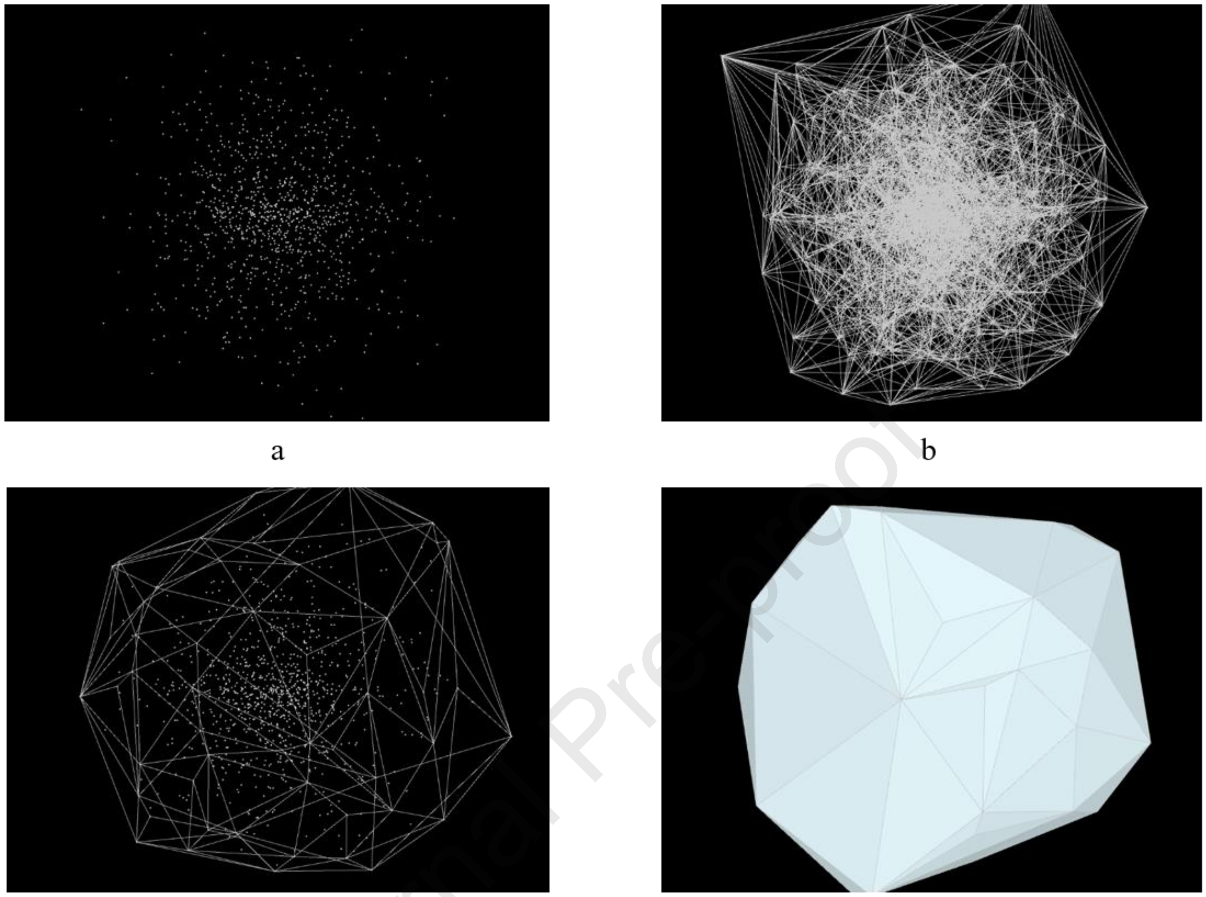
Our computer simulations obtained the below-provided pictures 3.1 and 3.2. They show the triangulation process of a 3-sphere and a 3D manifold.

In picture 3.1, step (a) shows the initial points to form a 3-sphere; steps (b), (c), and (d) establish the point connections, the triangulations, and the wholly formed 3-sphere, respectively.



Picture 3.1 Triangulation of a 3-Sphere. The a-b-c & d show the interim stages of triangulation.

Similarly, in picture 3.2 below, the forming stages of a 3D manifold are provided in 4 specific steps (a) the initial points, (b) connections of the points, (c) the triangulation, and (d) the complete triangulated 3D manifold.



Picture 3.2 Triangulation of a 3D-Amorphous object, a manifold. The a-b-c & d show the interim stages of triangulation of the manifold.

Tables 3.1 & 3.2 below summarize Delaunay triangulation computations for surface area calculations of n-spheres and manifolds.

Table 3.1 Surface Area Calculations for n-Spheres.

Delaunay Triangulations for Surface Area Computation of an n-Sphere					
3-Sphere			**4-Sphere**		
Random Points	**Number of Triangles**	**Area (unit^2)**	**Random Points**	**Number of Triangles**	**Area (unit^2)**
1000	1988	124748.43	1000	23464	18025282.00
2000	3976	125204.76	2000	48304	18620394.00
3000	5961	144430.16	3000	73652	18928320.00
The total area of a sphere		125663.70	The total area of a sphere		19739175.45

Table 3.2 Surface Area Calculations for Manifolds.

Delaunay Triangulations for Surface Area Computation of a Manifold					
3D Manifold			4D manifold		
Random Points	Number of Triangles	Area (unit^2)	Random Points	Number of Triangles	Area (unit^2)
1000	212	214617.36	1000	2976	49536179
2000	302	218472.06	2000	5116	51396624
3000	350	223421.52	3000	6124	52887740

As seen in Table 3.1 above, the n-sphere computations were realized through the formulas $S_{n-1} = 4\pi R^2 \ where \ n = 3, \ R = 100$ for the 3-sphere and $S_{n-1} = 2\pi^2 R^3 \ where \ n = 4, \ R = 100$ 4-sphere, respectively. Each n-sphere is created thrice by 1000, 2000, and 3000 random surface points and has its number of triangulations for the area calculations. Therefore there are six n-spheres with their respective areas to be used for the entropy calculations shown in the next section.

The same approach was taken for the creation of the manifolds. Again, the 3D and 4D manifolds were created thrice by 1000, 2000, and 3000 random surface points, along with their triangles and area calculations. It is imperative to notice that all the manifolds have constant & positive curvature, and their points are between (-100) & (+100) in each dimension.

Thus, these Tables 3.1 and 3.2 show that the surface area of n-spheres and 3D-4D digital manifolds can be obtained by the Delaunay triangulation algorithm computationally. The significance of this methodology lies in the fact that it is now possible to compute the entropy of n-dimensional digital manifolds.

4 Evaluation of Shannon Information Entropy on the Planck Scale

The Shannon metrics for quantizing discrete, digital information on a metric scale are effectively calculable functions. This subsection extends these metrics for information-bearing n-dimensional discrete digital manifolds on the Planck scale. The primary assumption is that the Planck scale is discrete. Therefore, information systems like n-spheres or n-dimensional digital manifolds are independent of measurement.

For simplification in calculations, the unit of the calculated surface area of the n-dimensional digital manifolds and n-spheres is represented by base 10 with the exponent (70) in square meters (x10^{70} m^2).

The information capacity of an n-dimensional digital Manifold (IC_{ndm}) is determined by its surface area (A) divided by the Bekenstein number (N).

$$IC_{ndm} = \frac{A}{N} ln2 \text{ bits} \quad \textbf{(4)}$$

where A is the surface area and N is Bekenstein number which is $N = 4\ell_p^2$

The maximum entropy (H_{max}) for the n-dimensional digital manifold (M_{ndm}) may be defined as a Shannon entropy function. Since this entropy is the maximum, all the probabilities are assumed to be the same and equal to 0.5.

$$H_{\max}(M_{ndm}) = -\sum_{i=1}^{IC_{ndm}} p_i \ \ln p_i \text{ bits per manifold} \quad \textbf{(5)}$$

where $H_{\max}$ is the maximum entropy, M_{ndm} is n-dimensional digital manifold, IC_{ndm} is the information capacity of n-dimensional digital manifold, and p is probability with $p = 0.5$

The formulas (4) and (5) may be applied to n-spheres and n-dimensional digital manifolds on the Planck scale as a second step. An example calculation based on the Delaunay triangulations for surface area computation of a 3-Sphere from Table 3.1 is below.

Example: 3-sphere with 1000 random points:

Area, $A = 124748x10^{70} m^2$

Planck square : $\ell_p^2 = 2.611x10^{-70} m^2$

Bekenstein number, $N = 4\ell_p^2, N = 4x2.611x10^{-70}$

$$IC_{ndm} = \frac{A}{N} \ln 2 = \frac{124748}{4x2.611} 0.693 = 8280.68 \text{ bits.}$$

$$Max\ Entropy : H_{\max}(M_{ndm}) = -\sum_{i=1}^{IC_{ndm}} p_i \ln p_i \text{ where } p = 0.5$$

$$H_{\max}(M_{ndm}) = 8280.68x0.5x0.693 = 2869.02 \text{ bits per manifold.}$$

Enlarging this calculation to cover n-spheres and n-dimensional digital manifolds per the simulation aforementioned in section three is now possible. Tables 4.1 and 4.2 show the results of the analyses exampled above.

Table 4.1 n-Sphere Digital Manifold Entropies on a Plank Scale.

n-Sphere Entropy						
Number of points	**3-Sphere**			**4-Sphere**		
	Area (unit^2)	**IC_{ndm} (bits)**	**$H_{max}(M_{ndm})$ (bits per n-sphere)**	**Area (unit^2)**	**IC_{ndm} (bits)**	**$H_{max}(M_{ndm})$ (bits per n-sphere)**
1000	124748.43	8280.71	2869	18025282.00	1196505.79	414589
2000	125204.76	8311.01	2880	18620394.00	1236008.91	428277
3000	144430.16	9587.17	3322	18928320.00	1256448.83	435360

Table 4.2 n-Dimensional Digital Manifold Entropies on a Plank Scale.

nD Manifold Entropy						
Number of points	**3D Manifold**			**4D manifold**		
	Area (unit^2)	**IC_{ndm} (bits)**	**$H_{max}(M_{ndm})$ (bits per manifold)**	**Area (unit^2)**	**IC_{ndm} (bits)**	**$H_{max}(M_{ndm})$ (bits per manifold)**
1000	214617.36	14246.15	4936	49536179	3288177,40	1139353
2000	218472.06	14502.02	5025	51396624	3411672.46	1182145
3000	223421.52	14830.57	5139	52887740	3510651.71	1216441

Tables 4.1 and 4.2 clearly show that the computation of the maximum entropy for n-dimensional digital manifolds on the Planck scale is possible within the simulation limits.

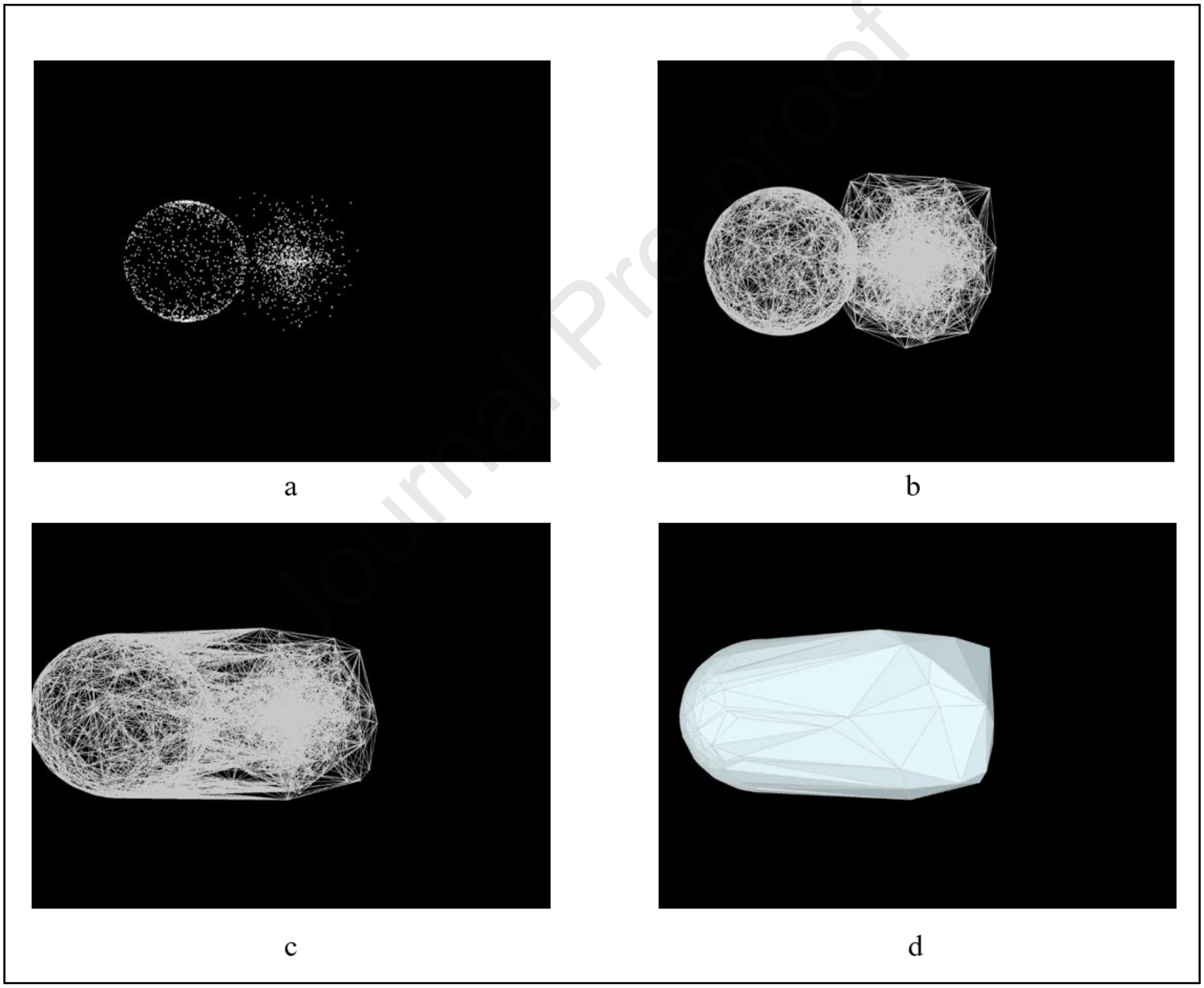
Until this point, we have shown that the surface area of the discrete n-dimensional digital manifold is an effectively computable function; however, more Shannon information metrics are necessary to justify the assumption that the Shannon information axioms and metrics are valid on the Plank scale.

Since the n-spheres and n-dimensional digital manifolds examined in this paper are topological structures, the union (point-to-point) and wedge (by single point only) operators may be applied to create more complex digital topological objects. The Shannon information measures can then be obtained from these complex topological structures.

As a first step, both randomly created 3-sphere and 3D manifold were glued by point-to-point (union) operation; in doing so, all points on both objects were connected one-to-one basis. The union operation was repeated thrice on 1000, 2000, and 3000 point sets.

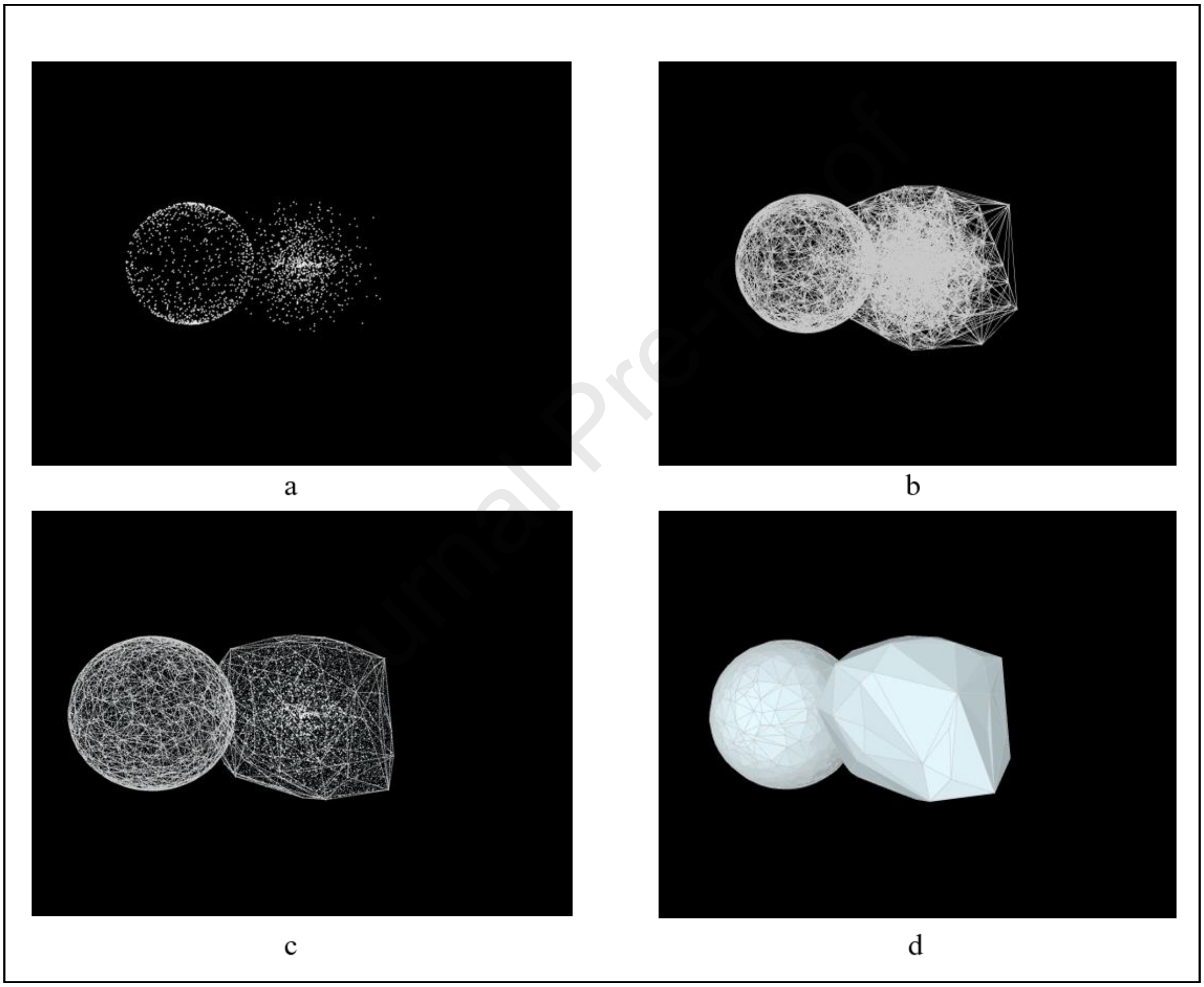
Later, randomly created 3-sphere and 3D manifold were glued by a single point (wedge) only, and this was repeated thrice on 1000, 2000, and 3000 points sets again. The wedge operator connects two n-dimensional digital objects from one point only.

Picture 4.1 below depicts the union operation on a 3-sphere and a 3D manifold in 4 steps progressively. Step (a) shows the formations of a 3-sphere on the left and the 3D manifold on the right. At step (b), the points are connected on both objects, hence the 3-sphere and a 3D manifold. Due to union operation, all points on both entities are combined at step (c). The result of a 3-sphere and a 3D manifold unionization is at step (d).



Picture 4.1 The Union operation on a 3-Sphere and 3D manifold. The a-b-c & d show the interim stages.

Picture 4.2 below depicts the wedge operation on a 3-sphere and a 3D manifold in four distinctive steps. Step (a) shows the formations of a 3-sphere on the left and the 3D manifold on the right. At step (b), the points are connected on both objects, hence the 3-sphere and a 3D manifold. Finally, due to the wedge operation, both objects are connected from one-point-only as seen in step (c). The result of the wedge operation is the one-point-only connected two objects as pictured in step (d)).



Picture 4.2 The Wedge operation on a 3-Sphere and 3D manifold. The a-b-c & d show the interim stages.

Once we have the connected complex topological digital objects, as shown in the above pictures, performing the Shannon information metric on them is possible. By applying formulas (4) & (5), one may calculate the maximum entropy of n-spheres and nD manifolds as the function of their surface areas generated by union and wedge operators for 1000, 2000, and 3000 random point data sets, respectively.

As explained above, Table 4.3 shows the maximum entropies for n-dimensional digital manifolds generated and calculated for three data sets.

On the other hand, Table 4.4 shows the calculated maximum entropies for union and wedge operator generated, complex n-dimensional digital manifolds.

Table 4.3 Shannon Information Measures on Complex Topological Objects.

Random Points	Surface Area of 3-sphere: A_1 (unit^2)	Surface Area of 3D Manifold: A_2 (unit^2)	$H_{max}(A_1)$ (bits per 3-sphere)	$H_{max}(A_2)$ (bits per 3D manifold)
1000	123766,664	210185,560	2847	4834
2000	125117,560	218056,770	2878	5015
3000	125255,540	220274,230	2881	5066

Table 4.4 Shannon Information Measures after the Union & Wedge Operations.

Random Points	$A_{union} = (A_1 \cup A_2)$	$A_{wedge} = (A_1 \wedge A_2)$	$H_{max}(A_{union})$ (bits per union)	$H_{max}(A_{wedge})$ (bits per wedge)
1000	311808,900	333852,220	7172	7679
2000	318368,900	343074,300	7323	7891
3000	319081,440	345429,78	7339	7945

It is clear in Table 4.3 that the steady increase of maximum entropy of the n-dimensional digital manifolds on the Planck scale over the data sets with an increasing number of points is clearly in line with the 2nd law of thermodynamics in all systems in metric space.

Moreover, as seen in Table 4.4, the increase in maximum entropies of the wedged objects compared to unionized objects of the Planck level may also be interpreted as the 2nd law of thermodynamics, again, in action on the Planck level.

At this point, it is imperative to satisfy all the initial axioms of the Shannon information metrics to show that all those axioms are also valid on the Planck scale.

Axiom #1 Entropy is always non-negative for all surfaces of all discrete digital objects. **Proof:** The calculations performed over the data generated for simulation clearly shows this in Table 4.1 to 4.4.

Axiom #2 Entropy is always monotonically increasing and is maximum when all probabilities are equal with a value of p= 0.5.

Proof: By formulas (4) and (5), it is evident that

if all p_i's are equal then it can be written that $\forall i,\ p_i = \dfrac{1}{IC_{ndm}}$

$then\ H_{\max}$ will increase monotonically as a function of IC_{ndm}.

That in turn means that the entropy will increase for an increasing number of equal probabilities reaching to a maximum when all are equal to $\dfrac{1}{2}$.

Axiom #3 That $H_{max}(A_n)$ is symmetrical.

Proof: The ordering of probabilities does not change the result even on independent objects when adding, for instance:

$$H_{\max}\left(A_1\right) + H_{\max}\left(A_2\right) = H_{\max}\left(A_2\right) + H_{\max}\left(A_1\right) = 7893\ bits.$$

where 2000 points on the surface areas of 3-sphere and 3D digital manifold.

Thus Symmetric.

Axiom #4 $H_{max}(A_n)$ is additive.

Proof: The three sets of entropy measures for two objects, (i) the 3-sphere and (ii) the 3-dimensional discrete manifold in an n-dimensional discrete digital manifold, were calculated by effectively calculable functions. Since each object is independent of the others, they both have entropies. Thus, entropies relating to joint events (=objects) were calculated by (i) the ordinary arithmetic summation of the points on surface areas, (ii) by union operation of the whole points of the objects, and (iii) by gluing the objects from one single equilateral triangle of any given tetrahedron on the intersection of the objects named a wedge. An example follows below:

$$H_{\max}(A_1, A_2) = H_{\max}(A_1) + H_{\max}(A_2)$$

Ordinary Summation:

$$H_{\max}(A_1, A_2) = H_{\max}(A_1 + A_2) = H_{\max}(A_1) + H_{\max}(A_2)$$

$7681 = 2847 + 4834$ *bits.*

Union:

$H_{\max}(A_1, A_2) = H_{\max}(\mathrm{A})$ where A is $\left[\mathrm{A} = (A_1 \cup A_2)\right]$.

$7172 \cong 2847 + 4834$ *bits.*

Wedge:

$H_{\max}(A_1, A_2) = H_{\max}$ (A) where A is $\left[\mathrm{A} = (A_1 \wedge A_2)\right]$.

$7679 \cong 2847 + 4834$ *bits.*

thus additive.

Axiom #5 H(P) is continuous in p.
Proof:

By the definition of probability, it is clear that

$H(P) = H(p,\, 1 - p)$ when considered as a function of p.

Where p is probability of a random point on discrete, digital n $-$ dimensional manifold.

Axiom #6. Reproducing the Bekenstein-Hawking Entropy.
Proof: Following formulas (1) and (4), it is clear that

$$S_{BH} = k_B .\, IC_{ndm} \quad \textbf{(6)}$$

where S_{BH} is Bekenstein-Hawking entropy, k_B is Boltzmann's constant and IC_{ndm} is the information capacity of n-dimensional digital manifold.

The Shannon information axioms and their proofs clearly show that all those axioms are valid for n-dimensional digital manifolds defined on the Planck level. This proven concept permits us to consider the information as digital, a discrete entity on the Planck level. However, the unavailability of direct observations from experimental research still blocks the way to definitive conclusions.

5 Results & Discussion

The nature of information on the Planck level spacetime has various competing theories and interpretations in theoretical physics and remains a topic of intense research and debate. Further studies and explorations are needed to determine the most accurate and comprehensive view. On the one hand, information on the Planck scale is discrete and digital, with a finite number of bits of information per unit of space or time. This idea is based on quantum mechanics, which suggests that space and time may be quantized at the Planck scale. Furthermore, that information is encoded in fundamental particles' properties and interactions. On the other hand, information on the Planck scale is continuous and analog, with an infinite number of possible values per unit of space or time. This view is based on theories such as string theory and loop quantum gravity, which propose that space and time are fundamentally continuous and that information is encoded in the geometry of spacetime.

While evaluating these ideas is challenging, there is currently no experimental evidence to support one theory over another.

Through the computerized simulations, we have only shown that defining the information as a discrete entity on the surface area of an n-dimensional discrete digital manifold provides a unique way of calculating the entropy of a manifold. However, the lack of direct experimental data to back our approach is still the main problem, as with other studies concerning the Planck level.

Our software simulation shows that the surface area of the discrete n-dimensional digital manifold is an effectively computable function, justifying the information-geometrical evaluation of Shannon information metrics.

The maximum entropy for n-dimensional digital manifolds [$H_{max}(M_{ndm})$], computed & presented in tables 4.1, 4.2, 4.3, and 4.4., clearly states that the Shannon information metrics are also valid on the Planck level. Moreover, the proven Shannon information axioms in section four offer that all those axioms are reasonable for n-dimensional digital manifolds defined on the Planck level. Therefore, it is now possible to consider the information as digital, a discrete entity on the Planck level.

This approach may provide a better understanding of the nature of information on the Planck scale. In addition, it could offer new insights into the complex systems it involves, including the nature of reality and the universe's origins.

In general, evaluating the nature of the information leads to the discussion:

(i) The information on a Planck level is a physical entity. It is directly measurable, and it can be approached from both discrete (deterministic) and continuous (stochastic) perspectives depending on the scale on which the observations and measurements have been made [19–26]. Therefore, it presents a scale-dependent dual nature. This scale-dependent duality entirely complies with the observations and measurements of the geometric nature of the information, as seen in this study.

(ii) Founding on the assumptions of CDT, as demonstrated earlier in this study, the Planck-level simulation shows that the information may be considered as the triangulation of spacetime or vice versa, for both can be effectively computed as a discrete n-dimensional digital manifold.

(iii) By following formulas **(5)** and **(6)**, the information might now be defined as

$$I_P = k_B . \, IC_{ndm} \tag{7}$$

where I_P is the discrete information on the Planck level, IC_{ndm} is the information capacity of a n-dimensional digital manifold, and k_B is Boltzmann's constant.

(iv) When the spacetime is considered as 2-dimensional manifold on the Planck scale, changing the curvature of a manifold does not affect its net surface area.

6 Conclusion

The nature of information on the Planck level remains a complex and challenging topic of research and debate in theoretical physics, pure mathematics, and computer science. While competing theories propose different perspectives on the discrete or continuous nature of information, we have shown that defining information as a discrete entity on the surface area of an n-dimensional digital manifold provides a unique and effective way to evaluate Shannon information metrics. As such, a better understanding of the nature of information on the Planck scale could provide invaluable insights into the complex systems it involves. However, further research and exploration are needed to determine the most accurate and comprehensive theory and uncover the universe's mysteries at the Planck scale.

Through the calculations proven above and the results that follow, we conclude by below items:

(i) The information, modeled & computed geometrically as a discrete, digital n-dimensional manifold in the shape of tetrahedrons (3D) or equilateral triangles (2D) on a Planck level, can be considered as the very fabric of spacetime itself.

(ii) The axioms of Shannon information entropy for the n-dimensional digital manifold have been satisfied. Therefore, the Shannon information measure is valid on the Planck level.

(iii) The spacetime on a Planck level may be considered as a fluid quantum foam [28–30] with information consisting of 1s and 0s within equilateral triangles (2D) or tetrahedrons (3D), each of which is defined on a Minkowski space [31]. Therefore, spacetime can be considered a 2D flat surface, an information plane, or an information volume in 3D. All that follows the approaches of "At some scale, space, time, and state are discrete. The fundamental process of physics must be a simple deterministic digital process." [32], and "The essence of the universe is information, and the fundamental bits of information that give rise to the universe lives on the Planck scale."[33].

(iv) The information attributes and functions such as confidentiality, integrity, and authenticity may be defined as tensorial operators on the discrete n-dimensional digital manifold thus, are calculable functions. Describing these calculable functions is what we consider future work.

Author contribution statement:

Ahmet Hasan Koltuksuz; Anas Maazu Kademi: Conceived and designed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Cagatay Yucel: Performed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Data availability statement:

Data associated with this study has been deposited at https://github.com/AHK2023/digitalmanifold Mendeley Data Set.

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Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐ The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: