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MACHINE LEARNING-BASED DATA AND MODEL DRIVEN BAYESIAN UNCERTANITY QUANTIFICATION OF INVERSE PROBLEMS FOR SUSPENDED NON-STRUCTURAL SYSTEM

A Dissertation Presented to the Graduate School of Clemson University

In Partial Fulfillment of the Requirements for the Degree Doctor of Philosophy Civil Engineering

> by Zhiyuan Qin May 2023

Accepted by: Dr. M.Z. Naser, Committee Chair Dr. Laura Redmond Dr. Brandon Ross Dr. Qiushi Chen

ABSTRACT

Inverse problems involve extracting the internal structure of a physical system from noisy measurement data. In many fields, the Bayesian inference is used to address the ill-conditioned nature of the inverse problem by incorporating prior information through an initial distribution. In the nonparametric Bayesian framework, surrogate models such as Gaussian Processes or Deep Neural Networks are used as flexible and effective probabilistic modeling tools to overcome the high-dimensional curse and reduce computational costs.

In practical systems and computer models, uncertainties can be addressed through parameter calibration, sensitivity analysis, and uncertainty quantification, leading to improved reliability and robustness of decision and control strategies based on simulation or prediction results. However, in the surrogate model, preventing overfitting and incorporating reasonable prior knowledge of embedded physics and models is a challenge.

Suspended Nonstructural Systems (SNS) pose a significant challenge in the inverse problem. Research on their seismic performance and mechanical models, particularly in the inverse problem and uncertainty quantification, is still lacking. To address this, the author conducts full-scale shaking table dynamic experiments and monotonic & cyclic tests, and simulations of different types of SNS to investigate mechanical behaviors.

To quantify the uncertainty of the inverse problem, the author proposes a new framework that adopts machine learning-based data and model driven stochastic Gaussian process model calibration to quantify the uncertainty via a new black box variational inference that accounts for geometric complexity measure, Minimum Description length (MDL), through Bayesian inference. It is validated in the SNS and yields optimal generalizability and computational scalability.

Keywords: Inverse problem, Machine learning-based Data and model driven, Gaussian Process; Surrogate Model, Uncertainty Quantification, Black-box Variational Inference, Geometric Complexity, Minimum Description Length, Generalization and Robustness, Suspended Non-structure System

DEDICATION

To my family and friends.

ACKNOWLEDGMENTS

I extend my heartfelt appreciation to my esteemed advisor, Dr. M.Z Naser, for his unwavering support, invaluable guidance, and unrelenting supervision throughout my dissertation journey. Without his constant encouragement and expertise, this research work would not have been possible.

I am also grateful to Dr. Laura Redmond, Dr. Brandon Ross, and Dr. Qiushi Chen for their insightful feedback and guidance which significantly contributed to the quality of this dissertation. My sincere thanks also go to Kristi, graduate vice president Dr. David Fleming and assistant vice president Dr. Kimberly Poole for their unreserved assistance.

I am indebted to Prof. Kazuhiro and Prof. Motoyui of Tokyo Institute of Technology, and Prof. Lu and Prof. Jiang of Tongji University for their immense support, encouragement, and invaluable guidance in the course of this research work.

Finally, I am thankful to my family for their constant love, support, and encouragement throughout this journey.

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CHAPTER ONE INTRODUCTION

1.1 ML-based Data driven inverse problems of uncertainty quantification

Computational mathematical models and simulations are essential tools to predict the response of physical structural systems. However, such models can be tied to large economic and time costs. Specifically, in order to obtain high-precision real-world test results, it is necessary to comprehensively consider various uncertainties (such as model parameter uncertainty, measurement uncertainty, model discrepancy and model solution error, etc.), wherein there is often a certain deviation between the simulated and the observed response and performs massive calculations. Such a deviation may in turn, result in poor prediction, robustness/reliability. Therefore, to measure and improve the credibility of the Computational mathematical models, it is necessary to quantify various uncertainties and improve the prediction accuracy of the mathematical models based on Model Verification, Validation, and Uncertainty Quantification [1].

To address these challenges, the Defense Advanced Research Projects Agency (DARPA) launched the Enabling Quantification of Uncertainty in Physical Systems (EQUiPS) project in 2015 [2], which aims to develop advanced mathematical theories and methods to effectively quantify, transmit, and manage multi-source uncertainty in the modeling and design of complex systems. This significantly reduces the economic and time costs burdens of conducting research, and finally obtains high-confidence accuracy indicators of equipment performance under specific conditions of interest. As the name of the project suggests, the research work is based on the framework of

Uncertainty Quantification (UQ). In fact, in the 2025 Mathematical Sciences published by the National Academy of Sciences in 2013 [3], UQ has been singled out and discussed as a new important research area and asserted that the emergence of this field brought a great promise for accurate modeling and accurate prediction of complex systems.

UQ has been well applied to cover fluid mechanics, computational seismology, geological exploration, weather forecasting, financial forecasting, and other practical engineering problems [4,5,6,7,8,25]. There are usually two shortcomings in the study of such problems based on traditional mathematical methods: first, the corresponding mathematical theories are may not be longer applicable in complex systems and high-dimensional data environments, and traditional methods need to establish definite models, which contradict the strong uncertainty of practical problems; Second, the application of traditional mathematical knowledge can still explain the current problems, but there is still a big gap from the corresponding theory to the real practice, or some theoretical achievements are difficult to form an effective implementation plan.

In order to overcome the above two problems, the core of UQ is to study the magnitude and likelihood of the influence of uncertainty variables, such as unknown parameters in the model, experimental observation errors, geometric complexity of the design area, etc., on the behavior of the system in the real environment. In general, uncertainty variables are introduced as random variables (or random processes), so the related predictions and evaluations are also discussed from a statistical perspective. Therefore, in the UQ framework, researchers often focus on the law and statistical

characteristics of the variables of interest with the input random variables and at the same time, need to analyze the confidence and reliability of the model under study. It should be noted that UQ is not to determine whether the research object is "real" or not, it is actually only an approximation of the real object, as commented in [9]:

UQ cannot tell you that your model is 'right' or 'true', but only that, if you accept he validity of the model (to some quantified degree), then you must logically accept he validity of certain conclusions (to some quantified degree).

Therefore, the concept of UQ is based on a new dimension of philosophical thinking, which actually describes the credibility of scientific inference. Moreover, whether from the purpose of EQUiPS or from the "2025 Mathematical Sciences" report, UQ mainly includes two aspects: it is based on the establishment of a complete mathematical theory and methodological framework to standardize the design criteria of complex systems and establish high-confidence models; The foothold lies in the construction of corresponding efficient algorithms to support the rapid development of the project. The two present a mutually supportive and synergistic relationship. At the same time, UQ will play a more important role in the ongoing revolution in the physics-cyber space, including Industry 4.0 and digital twins.

In general, there are two kinds of uncertainty: *aleatoric* uncertainty and *epistemic* uncertainty [10]. Aleatoric, also known as stochastic uncertainty, describes outputs that vary from experiment to experiment. By incorporating knowledge into the mathematical model or experiment, we cannot get rid of this uncertainty. Epistemic uncertainty, commonly referred to as system uncertainty, results from incomplete or incorrect information, such as limited experimental data sets or biased models, etc. By

incorporating knowledge into the mathematical model or experiment, this kind of uncertainty can be removed.

The complexity of the mathematical models used for uncertainty quantification has tended to rise along with the availability of computational resources, and they are now frequently run on powerful supercomputers that can generate terabytes of data at once. Even while this increased complexity can lead to fresh perspectives and more accuracy, it can occasionally be advantageous to run quick approximations of these models, often known as surrogates [11,26,27]. Such data are approximated using surrogate models. The development of huge ensembles of model realizations [12,13,14], the efficient investigation of the sensitivity of model output to its inputs, and model calibration [15,16] have all been made possible by the employment of these surrogates over a long period of time [17,18,28].

For non-parametric interpolation, *Gaussian Processes (G.P.s)* have gained popularity and significance in the supervised machine learning community (ML). The use of G.P.s for calibrating computer models was first introduced by Kennedy and 'O'Hagan (2001) [19], and it serves as the foundation for contemporary methods. Since they offer reliable estimates and uncertainty for nonlinear reactions, even in situations with little training data, G.P.s are especially well suited for this purpose. Recent developments have enabled deeper [20], more expressive G.P.s that can be trained on ever greater volumes of training data [21], despite initial challenges with their scalability compared to, for example, Neural Networks.

Derivative-free Bayesian calibration or inversion generally starts with the observation of the error model. Traditional methods for derivative-free Bayesian calibration to estimate the posterior distribution, such as *Markov chain Monte Carlo (MCMC)* [22], typically require many iterations—often more than 10⁴ steps to reach statistical convergence. Given that each forward run can be expensive, conducting a series of runs is computationally unaffordable, rendering MCMC impractical for real-world calibrations.

In the eighties and nineties, Peterson [23] and Hinton [24] began to study variational inference methods, mainly to approximate posterior probabilities in Bayesian models. The main idea of *variational inference* is to seek a class of simple variational distributions as an approximate solution to the true posterior distribution through optimization processes rather than sampling processes. The variational distribution is a collection of hidden variable distributions, using KL-divergence (Kullback-Leibler) as a measure of inter-distribution similarity, and finding the variational distribution of that is most similar to the true posterior distribution as an approximate distribution of the true posterior distribution. Therefore, variational inference has become another important method for learning Bayesian models. Compared with the Monte Carlo sampling method, the variational inference method is faster, simpler, and easier to parallelize and is more suitable for large data and complex models. Therefore, variational inference methods are widely used in various fields, including natural language processing [34,35], computer vision [36,37], computational biology [38,39], robotics science [40,41], and text analysis [42].

Variational inference (VI) is a popular technique in machine learning, but it is not as widely used in statistics compared to MCMC-based sampling techniques. In civil engineering, the slow uptake of VI can be attributed to its additional modeling complexities and limited theoretical exploration. Although traditional mean field VI is commonly used [43,44], it requires complex mathematical derivations and conjugate assumptions, which limits its practical applications. On the other hand, Black box variational inference (BBVI) [369] is a promising and advanced VI technique that remains largely unexplored in civil engineering. BBVI does not require specific model derivations and can scale well to large datasets and high-dimensional parameter spaces, which are common in many fields. In contrast, MCMC related methods become quickly impractical with the increasing size of datasets and number of parameters and do not scale well. Therefore, we propose a combination of BBVI with O'Hagan's Bayesian calibration framework [19], which can be easily derived without the need for conjugate assumptions, enabling the use of BBVI and achieving superior results for SNS systems in civil engineering, which are critical for ensuring reliability and safety and making it accessible to a wider audience of engineers and scientists.

Although Bayesian updating is very common, the model class selection is not. Addressing modeling complexity remains a more significant challenge for Bayesian inference applications since integrating metamodeling techniques is not trivial. The challenge here is to establish a fully automated integration that can address different degrees of competency for the end-user and a wide range of application problems with a certain degree of robustness.

Physics-based computer models of engineering systems are instruments of prediction that define a functional relationship between parameters that control the operational conditions of a system (input) and the system response of interest (output). In this

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context, these input parameters, known as control variables, define the domain of applicability in which the system operates. Here, we will assume that experimentalists have control over and complete knowledge of these control variables. While developing a model that links control variables to the output responses of interest (for instance, loads are acting on a structure to deformations), other input parameters that define the characteristics of the system (for instance, material properties or boundary conditions of the structure) are also introduced to the model. Frequently, a number of these input parameters that define the system characteristics are poorly known and, thus, must be inferred from experimental measurements, while the rest of the parameters are accepted to be well-known. We refer to this subset of poorly known input parameters that are selected for such inference as calibration parameters. Hence, given the values for well-defined control variables and poorly known calibration parameters, simulation models are conceived to predict unknown output responses within a predefined domain of applicability.

Model calibration then entails estimating the best-fit values for a few calibration parameters (which are believed to be identifiable) from experiments conducted at various control parameter settings within this domain of applicability. Indeed, one should not expect to get the complete "truth' from the model, for they are mere approximations and incomplete (i.e., systematically biased) representations of the underlying behavior of the system. Such incompleteness may originate from, for instance, omission of input parameters from the model, omission of interactions between the model input parameters and/or control variables or assigning incorrect values to model input parameters that are considered to be known. Thus, computer models invariably have systematic discrepancy biases in the way they predict the true behavior of the systems. This inherent discrepancy bias may be identified during model calibration by inferring an independent error model from the experimental data [19, 45, 46] or by blending emulators with mechanistic models to explain the omitted relationships between model input parameters.

In model calibration, the goodness-of-fit of a model to experiments reflects how well a model fits a particular set of observed data. A good fit is a necessary but not a sufficient condition [47], as it is possible to calibrate physics-based models to different sets of calibration parameter values that can fit a finite set of experiments reasonably well due to the inevitable compensations between various sources of errors and uncertainties [48]. In contrast to goodness of fit, generalizability is defined as the ability of a model to represent the reality of interest in all settings of the domain, including the settings where experiments are not available [49]. The generalizability of a calibrated model is important as computer models are most often calibrated with the ultimate objective of predicting settings for which experiments are unavailable.



Figure 1.1 Interplay among of the goodness-of-fit, complexity and generalizability

The complexity of a model calibration campaign leads to a hill-like relationship between good fitness to a finite amount of noisy measurements (in the tested settings) and generalizability of the model predictions (in the untested settings) [49, 50, 51] (See Fig. 1.1(a)). A model calibration campaign with too little flexibility would lose valuable information that could have otherwise been inferred from the data [49, 52].

On the other hand, a calibration campaign with too much flexibility would encourage the model to fit noise in the measurements, seemingly improving the goodness-of-fit while degrading the generalizability (see Figure 1.1(a)). Hypothetically, in the most extreme case, a calibration campaign that can produce a model capable of matching practically any possible outcome (infinite flexibility) yields an uninformative tool that is impossible to falsify, and that has no generalizability (tail end of Figure 1.1(a)). For physics-based models, the inherent functional structure of the model would impose a differential ability to fit patterned data and would prevent us from reaching this hypothetical infinite flexibility. Such differential ability of a model was referred to as "selectivity" by Cutting et al., 1992 [53]. In Figure 1(b), we present a detailed illustration of the interplay among the model goodness-of-fit, generalizability, and model complexity.

Cutting et al. (1992) [53] recognized that the number of parameters alone is an insufficient indicator of model complexity and advocated for evaluating the fitting power (i.e., what they refer to as 'scope') of a model to random data. They suggested using binomial tests to compare the fitting ability of a model to the data from the actual system with the fitting ability of random data. Similarly, complexity has been defined as the range of data patterns that a model can fit [54, 55, 56, 57]. Myung et al.,

2000 [58] and Pitt et al., 2002 [50] quantified a geometric complexity measure [31] known as the Minimum Description Length (MDL) [54, 59]. This metric considers the experimental data as a code or description to be compressed by the model and evaluates the models according to their ability to compress a data set by extracting the necessary information from the data without random noise. MDL is based on the understanding that the more data is compressed, the more information about the underlying regularities governing the process of interest would be learned [58]. Therefore, MDL would choose a model which has the shortest description code (length) of the data [30, 59].

In Bayesian inference problems, most studies overlook generalization and model complexity or use simple criteria like AIC, BIC, or DIC. This can result in overfitting and poor generalization. We propose using MDL based on algorithmic information theory, Kolmogorov complexity, and geometric complexity measure of data space. Our comparison with other criteria demonstrates MDL's superior generalization performance and it is also well-embedded in the black box variational inference framework, and our approach achieves good Bayesian inference and Uncertainty Quantification results and improves validation accuracy of suspended nonstructural systems (SNS) systems.

It is worth noting that in addition to the uncertainty quantification forward problems; this dissertation also pays great attention to the inverse problem of uncertainty quantification [60], which refers to a type of problem that obtains the internal structure information of a physical system from the measurement data containing noise. Due to its wide application in physics, mechanics, earthquake engineering,

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atmospheric science, life science, medicine, economics, industrial control and other experimental sciences, based on the above investigation, we propose an effective systematic method and framework for UQ of forward and inverse problems, combining model complexity selection, surrogate models and efficient variational inference, and skillfully apply it in the suspended non-structural systems (SNS).

In recent years, moderate or strong earthquakes have caused significant property loss, interruption of building function, and even threatened life safety due to damage to suspended nonstructural systems (SNS). Despite minor damage to the main building structures, the impact on SNS underscores their crucial role in ensuring the resilience of buildings against seismic events. While some experiments have been conducted in recent years, the effects of ultra-large areas and long duration and long periods under the conditions of super-tall buildings are still unknown. This recently completed fullscale suspended nonstructural systems (SNS) experiments, including suspended ceilings and cable trays, are the largest in the world, in which we carefully designed earthquake wave inputs in line with long duration and long periods in super-tall buildings, based on national standards and random vibration theory. In the field of civil engineering, Uncertainty Quantification and Inverse problem inference for the suspended nonstructural systems (SNS) is still unknown, and this study will fill that critical gap. Our use of a surrogate machine learning model reduces computational cost and running time, resulting in a great speed increase. From this view, this dissertation presents a novel framework for uncertainty quantification of inverse problems with the application on the suspended nonstructural systems. The validity of the proposed framework is using the full-scale shaking table tests of suspended nonstructural systems (SNS) and accompanying simulated data.

1.2 Suspended non-structural systems

During moderate or strong earthquakes in recent years, the damage to non-structural components (NSCs) can lead to great property loss, interruption of building function, and even threat to life safety, although the main building structures suffer minor damage [61-68]. As one of the most popular NSCs in buildings, the suspended ceiling system (SCS) with mineral wool boards suffers serious damage during earthquakes. The common types of damage to SCS with mineral wool boards include dislodgement and falling of ceiling panels, unseating of ceiling grid members around the perimeter, buckling, and failure of ceiling grid connections, buckling of ceiling grid members, failure of supporting elements, and the collapse of ceilings [69]. Among all the above damage patterns, the ceiling perimeter is regarded as one of the most vulnerable parts of SCS. This is especially true for the SCS with mineral wool boards applied in China, which lacks reliable connection at the boundary, easily causing the falling of the gird members near the ceiling perimeter from the support and even further triggering a continuous collapse of the ceiling during earthquakes.

Suspended ceiling systems are widely used in both commercial and residential buildings due to their easy construction and decorative aesthetics. However, in recent years the SCS suffered severe damage during moderate or major earthquakes, frequently resulting in great property loss, interruption of building function, and even threat to life safety. A crucial part of SCS is the ceiling component, greatly influencing the seismic performance of SCS, from which the propagation of damage often initiates and even the complete collapse of the ceiling occurs [70]. The typical

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types of damage to the ceiling components in real earthquakes include the failure of grid connections, the failure of hangers, and the failure of peripheral attachments.

To examine the seismic performance of SCS subjected to simulated earthquake loading, experimental studies largely using shaking table tests have been carried out for nearly 40 years [70]. One of the most significant observations is that the ceiling components are identified as one of the most vulnerable parts of SCS during earthquakes. Several component-level investigations on SCS were conducted to obtain the failure mechanism and capacities of strength and deformation of the ceiling components. Soroushian et al. [70, 71, 72] performed systematic studies on the capacities of the peripheral attachments and components under monotonic and cyclic loadings. Based on those experimental data, several fragility curves and analytical models for different components were developed. In the study by Paganotti et al. [73], a series of static tests on different types of components of SCS subjected to monotonic loading was carried out to evaluate the component capacity and produce fragility curves. It was found that the cross-tee connections are the most critical components of SCS. To assess the seismic performance of the ceiling component with seismic clips attached to wall angles using two screws, three types of ceiling perimeter configurations, i.e., pop-riveted connection and seismic clips with 1 screw or 2 screws, were conducted under monotonic and cyclic tests by Gilani et al. [74]. The experimental results indicated that the alternate peripheral installation with a seismic clip and 2 screws has better seismic performance in terms of the load-carrying ability and energy dissipation.

Although current seismic design standards such as the ASTM-E580/E580M [75] and AC368 [76] specify that the ceiling joints should carry a mean ultimate test load of not less than 800N for a restrained ceiling, it is unclear whether the strength capacity of the joint under the actual load can meet the requirements. Previous studies on ceiling components were conducted based on a certain product from a company and may produce different results from similar studies with different products. To understand and evaluate the seismic performance of suspended ceiling components and support for subsequent numerical modelling, a series of static tests on the ceiling components under monotonic and cyclic loadings were carried out in this study. The failure patterns, capacities of strength and deformation, and energy dissipation of the ceiling components are presented in detail in this study. In addition to the above quasi-static experiments, we also carried out full-scale dynamical shaking table experiments, and in order to consider the area effect, we carried out the world's largest full-scale experiment on suspended non-structural components, area and systematically conducted experimental research and analysis on the dynamic and nonlinear performance of suspended non-structural systems. And the experiments also consider about the long duration and high period effect on the non-structural systems.

Another very popular non-structural system is Suspended Cable Tray Systems (SCTS), and it is a typical non-structural component used to support insulated electric cables used for power distribution and communication. Due to its properties of large span, low redundancy and complex geometric shape, the cable tray system may experience large response or even collapse when subjected to seismic excitation. During previous earthquakes, a large number of cable tray suffered severe damage, such as buckling and falling down [77,78]. The cable tray in the high-rise building was damaged while

the main structure still worked, which caused the interruption of building function [79]. The failure of the cable tray causes 68% of the damage to the cable [80]. The cable tray is one of the critical parts of the building. need to maintain building functions during and after the expected earthquake.

Previous tests mainly focused on cable tray systems applied in nuclear power plants [81, 82, 83]. There were few tests on the suspended cable tray system applied in ordinary civil buildings. In this study full-scale shaking table tests on the suspended cable tray system applied in ordinary civil buildings were carried out to evaluate its seismic performance. Three types of seismic supports, which are manufactured products, were installed in the specimens. The dynamic properties and responses of the cable tray system are analysed.

According to the experimental analysis results of the dynamical shaking table and the computational numerical simulation established by us, we validate and analyse the computational numerical simulation results, and finally combined with the proposed uncertainty quantification of inverse problem framework, we infer the uncertainty parameters and achieve good results.

1.3 The main objectives (OBjectives) of this dissertation:

OB1 Develop the world's largest area of suspended non-structural dynamical shaking table experiments is firstly conducted and consider about the area effect. Apply sweep input to consider more details of mechanical characters of suspended non-structural systems. And the large number of experiments also firstly includes the long period and long duration of earthquake waves input to fulfil the practical engineering application. OB2 Develop the Machine learning-based Gaussian Process surrogate model to be embedded in the framework of Uncertainty Quantification for forward and inverse problems.

OB3 Augment the traditional methods like MCMC or approximate Bayesian computation (ABC), mean field variational inference, and black box variational inference with O'Hagan's Bayesian calibration framework is proposed to greatly increase the Bayesian inference computational efficiency.

OB4 Propose Geometric complexity based minimum description length model selection method in the inverse problems of Uncertainty quantification.

OB5 Design of experiments and sensitivity analysis is utilized to increase the speed of Uncertainty propagation in the forward problems.

OB6 Firstly Consider Bayesian inverse problems of uncertainty quantification application to the suspended non-structural systems for calibration, inference and validation of the computational numerical simulations.

1.4 My contribution and role of the Tongji-Clemson-Tokyo international cooperation

Here below five pages give the detailed contributions weights and the "Clemson" represent me in the tables list below. Also, an evolution letter of PI from Tongji University and Tokyo Institute of Technology is attached as well.

(1) 100% contribution part (leading role): [Included in this dissertation]

Table 1.1 Uncertainty Quantification of Inverse problems

	Uncertainty Quantification of Inverse problems
Pre-test preparation for 1.5 years	N/A
Steel Structure Platform Test	
First dynamic test for platform	
(Accident Failure: platform bottom	
buckled heavily)	N/A
Re-deign, optimization and retrofitting;	
Move the platform to a larger scale	
dynamic shaking table (2*70 tons)	N/A
Re-test for platform;	N/A
Suspended Ceiling Test	
Suspended Ceiling test Type A	Tongji: 0% Clemson: 100% Tokyo: 0%
Suspended Ceiling test Type B	Tongji: 0% Clemson: 100% Tokyo: 0%
Suspended Ceiling test Type C	Tongji: 0% Clemson: 100% Tokyo: 0%
Suspended Ceiling test Type D	Tongji: 0% Clemson: 100% Tokyo: 0%
Suspended Ceiling test Type E	Tongji: 0% Clemson: 100% Tokyo: 0%
Suspended Cable Tray Test	
Suspended Cable Tray test Type A	Tongji: 0% Clemson: 100% Tokyo: 0%
Suspended Cable Tray test Type B	Tongji: 0% Clemson: 100% Tokyo: 0%

(2) 95% contribution part (leading role): [Not included in this dissertation]

Table 1.2 Deep	Learning based Computer vi	sions recognition

	Computer visions recognition
Pre-test preparation for 1.5 years	N/A
Steel Structure Platform Test	
First dynamic test for platform	
(Accident Failure: platform bottom	
buckled heavily)	N/A
Re-deign, optimization and retrofitting;	
Move the platform to a larger scale	
dynamic shaking table (2*70 tons)	N/A
Re-test for platform;	N/A
Suspended Ceiling Test	
Suspended Ceiling test Type A	Tongji: 5% Clemson: 95% Tokyo: 0%
Suspended Ceiling test Type B	Tongji: 5% Clemson: 95% Tokyo: 0%
Suspended Ceiling test Type C	Tongji: 5% Clemson: 95% Tokyo: 0%
Suspended Ceiling test Type D	Tongji: 5% Clemson: 95% Tokyo: 0%
Suspended Ceiling test Type E	Tongji: 5% Clemson: 95% Tokyo: 0%

Suspended Cable Tray Test	
Suspended Cable Tray test Type A	Tongji: 5% Clemson: 95% Tokyo: 0%
Suspended Cable Tray test Type B	Tongji: 5% Clemson: 95% Tokyo: 0%

(3) 0%- 80% contribution part: [Included in this dissertation]

(Active participation and mainly contributed role):

Table 1.3 Numerical Simulation related

	Numerical Simulation related
Pre-test preparation for 1.5 years	Tongji: 20% Clemson: 80% Tokyo: 0%
Steel Structure Platform Test	
First dynamic test for platform	
(Accident Failure: platform bottom	
buckled heavily)	Tongji: 20% Clemson: 80% Tokyo: 0%
Re-deign, optimization and retrofitting;	
Move the platform to a larger scale	
dynamic shaking table (2*70 tons)	Tongji: 20% Clemson: 80% Tokyo: 0%
Re-test for platform;	Tongji: 20% Clemson: 80% Tokyo: 0%
Suspended Ceiling Test	
Suspended Ceiling test Type A	Tongji: 40% Clemson: 60% Tokyo: 0%
Suspended Ceiling test Type B	Tongji: 40% Clemson: 60% Tokyo: 0%
Suspended Ceiling test Type C	Tongji: 40% Clemson: 60% Tokyo: 0%
Suspended Ceiling test Type D	Tongji: 0% Clemson: 0% Tokyo: 100%
Suspended Ceiling test Type E	Tongji: 0% Clemson: 0% Tokyo: 100%
Suspended Cable Tray Test	
Suspended Cable Tray test Type A	Tongji: 40% Clemson: 60% Tokyo: 0%
Suspended Cable Tray test Type B	Tongji: 0% Clemson: 0% Tokyo: 100%

(4) 50% contribution part (Active participation and mainly contributed role):

[Included in this dissertation]

Table 1.4 Cyclic test and simulation

Cyclic test and simulation

Cyclic test of ceiling components	Tongji: 50% Clemson: 50% Tokyo: 0%
Numerical Simulation of hysteretic	
ceiling components	Tongji: 50% Clemson: 50% Tokyo: 0%

(5) 20%-50% contribution part (Active participation and mainly contributed role):

[Included in this dissertation]

Table 1.5 Experiments related

	Experiments related: Design, instruments
	filtering, system identification etc.
Pre-test preparation for 1.5 years	Tongji: 40% Clemson: 40% Tokyo: 20%
Steel Structure Platform Test	
First dynamic test for platform	
(Accident Failure: platform bottom	
buckled heavily)	Tongji: 40% Clemson: 50% Tokyo:10%
Re-deign, optimization and retrofitting;	
Move the platform to a larger scale	
dynamic shaking table (2*70 tons)	Tongji: 40% Clemson: 50% Tokyo: 10%
Re-test for platform;	Tongji: 40% Clemson: 50% Tokyo: 10%
Suspended Ceiling Test	
Suspended Ceiling test Type A	Tongji: 55% Clemson: 45% Tokyo: 0%
Suspended Ceiling test Type B	Tongji: 55% Clemson: 45% Tokyo: 0%
Suspended Ceiling test Type C	Tongji: 50% Clemson: 50% Tokyo: 0%
Suspended Ceiling test Type D	Tongji: 20% Clemson: 20% Tokyo: 60%
Suspended Ceiling test Type E	Tongji: 20% Clemson: 20% Tokyo: 60%
Suspended Cable Tray Test	
Suspended Cable Tray test Type A	Tongji: 30% Clemson: 30% Tokyo: 40%
Suspended Cable Tray test Type B	Tongji: 30% Clemson: 30% Tokyo: 40%

(6) 0% contribution part (No participation role):

[Not included in this dissertation]

Table 1.6 Fragility analysis and risk reliability investigation

	Fragility analysis and risk reliability investigation
Pre-test preparation for 1.5 years	N/A
Steel Structure Platform Test	
First dynamic test for platform	
(Accident Failure: platform bottom	
buckled heavily)	N/A
Re-deign, optimization and retrofitting;	
Move the platform to a larger scale	
dynamic shaking table (2*70 tons)	N/A
Re-test for platform;	N/A
Suspended Ceiling Test	
Suspended Ceiling test Type A	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Ceiling test Type B	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Ceiling test Type C	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Ceiling test Type D	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Ceiling test Type E	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Cable Tray Test	
Suspended Cable Tray test Type A	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Cable Tray test Type B	Tongji: 100% Clemson: 0% Tokyo: 0%

(7) 0% contribution part (No participation role):

[Not included in this dissertation]

Table 1.7 Practical industrial application guidelines and national seismic standards building

	Practical industrial application guidelines and national seismic standards building
Pre-test preparation for 1.5 years	N/A
Steel Structure Platform Test	
First dynamic test for platform	
(Accident Failure: platform bottom	
buckled heavily)	N/A
Re-deign, optimization and retrofitting;	
Move the platform to a larger scale	
dynamic shaking table (2*70 tons)	N/A
Re-test for platform;	N/A
Suspended Ceiling Test	
Suspended Ceiling test Type A	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Ceiling test Type B	Tongji: 100% Clemson: 0% Tokyo: 0%

Suspended Ceiling test Type C	Tongji: 100% Clemson: 0% Tokyo: 0%
Suspended Ceiling test Type D	Tongji: 0% Clemson: 0% Tokyo: 100%
Suspended Ceiling test Type E	Tongji: 0% Clemson: 0% Tokyo: 100%
Suspended Cable Tray Test	
Suspended Cable Tray test Type A	Tongji: 50% Clemson: 0% Tokyo: 50%
Suspended Cable Tray test Type B	Tongji: 0% Clemson: 0% Tokyo: 100%

(8) Evolution letter of PI from Tongji University and Tokyo Institute of Technology:



LEE International Joint Research Laboratory of Earthquake Engineering

May-24 2019

To Whom It May Concerns,

It is our pleasures to write this letter to certify that Mr. Zhiyuan Qin (William) completed his visiting research in our International Joint Research Laboratory of Earthquake Engineering (ILEE). Our interaction and cooperation began from September 2017 to May 2019.

The ILEE is an international joint research Laboratory collaborating among Tongji University, Urban Disaster Prevention Research Core (UDPRC) at Tokyo Institute of Technology, Pacific Earthquake Engineering Research Center (PEER), European Centre for Training and Research in Earthquake Engineering, New Zealand Center for Earthquake Resilience (QuakeCoRE) and Multidisciplinary Center for Earthquake Engineering Research (MCEER).

Mr. Zhiyuan Qin has presented a highly significant and unreplaceable expertise role in our two years research project. We can confidently write that he is exceptionally talented and diligent. He has unparalleled ability to conduct physical experiments, develop mathematical/physical strategies, make complex programming codes and implement high-dimensional data analysis, etc. He will continuedly contribute his multidiscipline background, such as computational vision, machine learning, and uncertainty quantification-based model validation, to accomplish the following journal paperwork. We are ensured that he will grow into an outstanding scholar or engineer and be successful in his future career and make us all proud.

If there are any further questions with regard to his background or qualifications, please do not hesitate to contact us.

With kind regards,

H. Hasai

Kazuhiko Kasai, Ph.D., Project Principle Investigator Professor and Industry Collaboration Chair, Laboratory for Future interdisciplinary Research of Science and Technology Tokyo Institute of Technology, Japan Director, Member of Scientific Committee International Joint Research Laboratory of Earthquake Engineering Tel:045-924-5512 Fax:045-924-5525 E-mail: <u>kasai.k.ac@m.titech.ac.jp</u> Homepage: <u>http://alrem.jp/</u>

With kind regards,

Huanjun Ivang

Huanjun Jiang, Ph.D., Project Co-Principle Investigator Professor of Structural Engineering, Vice Dean, Department of Disaster Mitigation for Structures, College of Civil Engi

Department of Disaster Mitigation for Structures, College of Civil Engineering, Tongji University, China Tel: 021-6598-6151 Fax: 021-6598-2668 Email: <u>ihi73@tongji.edu.cn</u>
1.5 Chapter arrangement of this dissertation

This dissertation is divided into seven chapters; the main contents of each chapter are as follows:

(a) The first chapter is an introduction, which mainly introduces the research background and related research content, and summarizes the full content and innovations. Also, my contribution and role of the Tongji-Clemson-Tokyo international cooperation is listed.

(b) The second chapter is background and literature review, which reviews the research progress and shortcomings about the forward and inverse problems (Bayesian inference & calibration) of Uncertainty Quantification, which includes uncertainty propagation, surrogate model etc., and also the suspended non-structural system earthquake related content, which includes experiments and computational numerical simulations.

(c) The third chapter is detailed investigation of the proposed Data and model driven Uncertainty quantification of inverse problems methods and framework, which considers machine learning based Gaussian Process surrogate model, variational inference and minimum description length model selection to make inverse problems Bayesian inference and calibration. Also, the details of the design of experiments, sensitivity analysis are also investigated.

(d) The forth chapter analyses the world's largest full scale suspended non-structural system earthquake dynamical shaking table experiments and also the results are detailed talked about.

(e) The fifth chapter is the computational numerical simulation and the determination of some part of the nonlinear structural parameters are obtained by hysteresis loading test, and finally the nonlinear numerical simulation is verified with the full scale dynamical shaking table experimental results. It shows the most displacement response is validated very well except some time delay difference and some of the acceleration response has some inconsistencies.

(f) The sixth chapter is about the application of proposed machine learning-based Data and model driven Uncertainty quantification of inverse problems on the suspended non-structural system. The inverse problem parameters are inference and calibrated with uncertainty quantified, and the results shows that the proposed methods have excellent effect on the validation improvements both on displacement and acceleration responses.

(g) Finally, the last chapter is about the summary and some prospects for the future, which highly mentioned the deeper model like deep Gaussian process model and deep neural network could be utilized to fully consider the nonlinearities of the physical system, also talk about the future work of applying the physics-informed neural network to embed the prior knowledge into the data driven model.

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CHAPTER TWO

BACKGROUND AND LITERATURE REVIEW

2.1 Background and literature review for data driven inverse problems of UQ

2.1.1 Uncertainty Quantification (UQ)

Uncertainty quantification has received extensive attention from researchers at home and abroad since the beginning of its proposal, and many universities around the world have established related laboratories, such as the UQ laboratory of Stanford University [84], the UQ group of the Massachusetts Institute of Technology (MIT) [85], ETH Zürich UQ laboratory [86] and so on. In addition, the National Academy of Sciences (NAS) and the U.S. Nuclear Security Administration (NNSA) held corresponding conferences and published two reports in 2009 [87,88], attracting the importance of the UQ approach in national security-related technology research. At the same time, the U.S. Department of Energy, the Department of Environment, NASA, and Sandia National Laboratory have established special funds to support the application of UQ in related fields.

At present, there are two main journals dedicated to the field of UQ: the SIAM/ASA Journal on UQ[89], co-founded by the Society for Industrial and Applied Mathematics (SIAM) and the American Association for Mathematical Statistics (ASA) in 2013, and International Journal on UQ[90], founded by Begell Press in 2011. R. C. Smith [91] and T. J. Sullivan [9] published corresponding textbooks on the theory and methods of UQ, which is also a classic work in the field of UQ. As an emerging interdisciplinary field, uncertainty quantification does not have a strict and unified definition of its theoretical classification and related research methods. In a 2012 report prepared for the U.S. Department of Energy [92], G. Lin pointed out that the basic processing process of UQ consists of these steps, such as problem definition, model validation, and testing, confirmation of input uncertainty, confirmation of observational (experimental) data, uncertainty parameter screening, response function feedback analysis, sensitivity analysis, and risk assessment. P. B. Stark et al. abstracted the main problems of UQ into Propagation of Uncertainty (PoU) and its inverse problem [93,94] within the framework of statistical models, as shown in Figure 1.1. At present, the theoretical research on UQ spans many fields, such as probability and statistics, functional analysis, stochastic dynamical systems, Bayesian analysis, numerical computing, multivariate statistics, etc., while specific mathematical, statistical methods, Bayesian methods, optimization algorithms, and function approximation methods have been introduced into the UQ framework to solve practical problems [9], but the future research direction of UQ is still in continuous exploration and development.



Figure 2.1 Classification of the main problems of uncertainty quantification

The following is a review of the specific UQ issues involved in this study and the development status of the corresponding methods.

2.1.1.1 Problem definition and research framework

In UQ, the target value of quantization is called QoI (Quantities of Interest), and the corresponding objective function is the QoI function. A simple UQ problem is shown in figure 2.2:



Figure 2.2 : As shown in Figure 1.2, let x be a random vector with multiple possibilities for its probability distribution p(x), usually given its prior information, such as belonging to a certain family of probability distributions; The actual system (or response function) M(x, x') is also in a different state when disturbed by other unknown factors x' (such as noise). Noting that y = M(x) is a QoI function, obviously, y is also a random vector whose distribution p(y) changes with the distribution of x.

Unlike traditional methods that focus on parameter estimation of models, the research goal of UQ is the distribution characteristics of the QoI function y. In more complex cases, on the one hand, due to the growing data and increasing complexity of the system, the (statistical) theoretical features of the uncertainty factors x, x' are becoming more and more difficult to describe, so it is almost impossible to establish complete models, and relevant assumptions need to be made about them; On the other hand, because the solution of the real system M(x, x') usually takes a lot of time or money, resulting in x, y has high dimensionality and sparsity, it is necessary to model on the basis of sparse experimental data or local features—which is also the main difference between UQ problems and classical methods. The following is a review of the development status of the processes and the specific methods.

2.1.1.2 Uncertainty source representation

The sources of uncertainty in UQ vary. Kennedy and O'Hagan [19] proposed a classification method for uncertainty sources from the specific target objects at different stages of UQ, mainly including input parameter uncertainty, parameter variability, model uncertainty (i.e., model incompleteness, bias, contradiction, etc.), algorithm uncertainty (numerical calculation error), experimental uncertainty (observation error), etc. Matthies [95], Der Kiureghiana [96] abstracted different sources of uncertainty in 2007 and 2009, respectively, dividing them into Aleatoric Uncertainties (A.U.) and Epistemic Uncertainties (E.U.). A.U. is also known as irreducible uncertainty or random uncertainty and is generally determined by the natural properties of the relevant parameters. E.U., known as reducible error, is usually caused by a lack of knowledge of the nature of the system or natural laws themselves and can be reduced as model parameters are more accurately estimated, new experimental data is introduced, or the degree of model refinement increases. The latter classification method is now generally accepted and used in the UQ field, but in

fact, the boundary between the two types of uncertainty is not clear, they can be transformed into each other with the change of the problem and the correct understanding of uncertainty is a prerequisite for establishing models and designing related algorithms.

For UQ problems, one of the important questions is how to quantitatively characterize the various sources of uncertainty described above. The control variable method is a simple and intuitive method of excluding uncertainties from the model or predetermining a probability distribution for them. For example, weather, geomagnetism, and other relevant factors affecting field tests, in laboratory simulation tests, you can fix the mean, variance, etc. of related variables according to experience or expert decisions. This approach actually reduces the number of A.U.s and reduces the complexity of the model by controlling variables. In a more general case, the relevant distribution hypothesis is proposed for uncertainties, the relevant statistical methods are used to estimate the parameters related to the distribution, and if necessary, the distribution can be hypothesis-tested. For example, under the assumption of the Gauss-Markov condition, the least squares estimator is a linear unbiased estimator with the smallest variance [97]. In actual cases, the probability distribution assumptions proposed for uncertainty factors are often very different from their true distribution, so they need to be updated and reconstructed. Bayesian inference [98] is an important method in UQ, where the presupposed probabilistic distribution can be processed as a priori distribution, and the posterior distribution continues to approximate the true distribution as new experimental data are introduced.

2.1.1.3 Uncertainty Propagation

Uncertainty propagation (PoU) is to determine the distribution characteristics of the QoI function through the propagation of the initial input in the model, that is, the transformation problem of the probability distribution. Probability distribution transformations first appeared in the field of Partial Differential Equations (PDEs), such as the Fokker–Plank equation, which describes the evolution of particle velocity probability distributions over time in Brownian motion. This problem then arises in Stochastic Differential Equations [99] (SDE), such as the Kolmogorov equation and the Feynman–Kac formula. In the field of SDE research, in order to solve the propagation properties of uncertainty, it is often necessary to introduce the Ito integral [100] (Itô Integral). These will not be covered in this study, so they will not be covered in subsequent sections. The following is a review of progress around surrogate models and experimental design.

In classical statistics, uncertainty propagation is usually studied on the variance of the QoI function or, for data-driven problems, its error is analyzed [101,102]. Dating back to the sixties of the last century, Goodman [103] gave the specific form of variance of the product of random variables in 1960, while Ku [104] studied the general mathematical expression of error propagation in the model. The classic textbook of Taylor [101] gives a complete set of theories and methods for error measurement and analysis and elaborates on the linear error and the propagation of nonlinear models. The propagation of uncertainty in classical statistics is actually to solve the problem of solving the statistical properties of functions about random variables, which provides theoretical support for PoU. With the increase in data dimensions and the

increase of computational complexity, how to implement these theories has become another difficult problem for UQ, so the development of PoU also focuses on the numerical calculation and algorithm implementation of stochastic models. Under this premise, PoU mainly includes two main parts: surrogate model [105] (SUrrogate MOdel, SUMO) and design of experiments (DOE) [106], which are introduced below.

2.1.1.4 Surrogate model

In the actual problem faced by UQ, the computational cost of a single sample may be very large, so the size of the training set that can be provided is limited, so it is not feasible to directly face the original model for optimization, reliability analysis, etc. [107]. The surrogate model (SUMO) constructs an approximate model of the original system, which is essentially a mathematical expression or algorithmic description of the relationship between input variables and QoI functions and describes the complex system model and its relationships as a computable model to help achieve the corresponding goals of UQ [108-111]. In general, the mathematical definition of a surrogate model problem is as follows:

$$y = M(x) = f(x) + \epsilon, \epsilon \sim N(0, \sigma^2)$$
(2.0)

where M is the actual system, which can be a system experiment or a simulation program. The purpose of the surrogate model is to use f(x) of a known concrete mathematical expression to proxy the real system M(x) to obtain the corresponding input response. SUMO is also referred to in the literature as a metamodel, a reduced model, a surrogate model, or a response surface. Surrogate models are a class of datadriven methods; commonly used proxy models include: Polynomial Regression, Artificial Neural Networks (ANN) [112], Multivariate Adaptive Regression Splines (MARS), Radial Basis Functions (RBF)[113], Support Vector Machine [114] (SVM), Polynomial Chaotic Expansion (PCE), Kriging / Gaussian Processes [115], etc. This work is mainly based on the G.P. model, and the following is mainly an introduction to these two types of models.

2.1.2 Gaussian Process surrogate model

Gaussian process models are flexible nonparametric Bayesian models that have applications in robotics [116,117], geostatistics, numerical operations[118], stimulus perception[119], and parameter optimization problems [120]. Gaussian process (G.P.) models are also a hot spot in UQ [121,122]. It was first proposed by Krige [123] and popularized by Matheron [124] and received widespread attention. The Kriging model is proposed to solve the problem that the linear estimation error is not independent, and it uses the correlation function based on spatial distance to interpolate the random field. Rasmussen and Williams [115] regard the QoI function as a specific sample of a Gaussian process, i.e. the Gaussian (random) process as a probability distribution about the response function, and any finite sample of the function obeys a multidimensional Gaussian distribution. They applied G.P.'s theory to machine learning and explored the relationship between G.P. models and other models, such as regularization methods, Reproducing Kernel Hilbert Space (RKHS), and SVMs. Unlike polynomial models, G.P. models are typically nonparametric models whose performance is controlled by hyperparameters, which are parameters of kernel functions. In general, for the study of Gaussian process hyperparameters, the maximum likelihood estimation (MLE) or BIC (Bayesian Information Criterion) criterion is usually chosen as its estimation [115]; In some applications that are

sensitive to parameter uncertainty, the posterior distribution of hyperparameters is often sampled using the MCMC method [125].

A Gaussian process model is defined by the mean and the covariance function, which is equivalent to a neural network model with an infinite number of hidden layer elements. Gaussian process models are not prone to overfitting and provide uncertainty in predicted outcomes. The basic theory of Gaussian process models and their applications in the field of machine learning are detailed in the literature [126]. The expressiveness of a single-layer Gaussian process model is limited by the expressiveness of the kernel function. In order to improve the learning ability of single-layer Gaussian processes, combinatorial kernel functions, deep kernel functions Gaussian process [127] models [128,129], and deep Gaussian process models and their model variants have been proposed successively. In a deep Gaussian process model based on process combinations [129-134], the output of one multivariate Gaussian process model will be used as the input to another Gaussian process model, resulting in a longer multilayer Gaussian process model to obtain a data output.

Due to the applicability of complete Bayesian inference to multilayer Gaussian process model inference, most of the literature on deep Gaussian processes uses approximate inference methods, such as variational inference and expectation propagation. Literature [135], for the first time, uses the method of sparse variational inference to infer the Gaussian process, dependent variable model. Then [136] applies the sparse variational inference method proposed by [135] to the hierarchical Gaussian process model of literature [134]. Since the above variational inference method requires variational parameters that increase linearly with the growth of the number of model data, in order to simplify the model training process, the variational self-coding method is used to construct a cognitive model for the posterior distribution of hierarchical variables, thereby reducing the variational parameters of the model [134]. Combined with expectation propagation, the induced point method was used to do sparsity to infer the deep Gaussian process model. [137] The kernel function sparse spectrum method is used to do the sparsification treatment. Based on the mean field variational inference method of the mean field [138], in order to improve the independence assumption and variance estimation between the implicit function variables of the model, a new inference method based on the induction point using the accurate model as the variational posterior distribution is proposed [139], and the double random variational inference is applied to the deep Gaussian process model by combining random sampling and stochastic gradient descent.

The G.P. model is widely used in engineering and machine learning, but it still has certain limitations. Firstly, the computational complexity of the G.P. model is too high, and the numerical solution mainly focuses on the inversion of the covariance matrix, and its computational complexity is O(N3). There are already some solutions to this problem [refer] in the literature, such as the K.L. divergence between the posterior distribution and the true distribution based on minimized estimation [140,141], the relevance Vector Machine (RVM) method based on matrix approximation theory [142] and the Relevance Vector Machine (RVM) method [143] and so on. Noting that G.P. models are often equivalent to linear regression models with infinite parameters, sparse linear regression or low-rank regression was introduced as a solution to Gaussian process models, referred to in the literature as the Reduced Rank Gaussian Process (RRGP) [144], or the Sparse Gaussian Process (SGP) [145,146]. This type of

method essentially utilizes a low-rank approximation of the original covariance matrix, which reduces the computational complexity to O(NM²) (M is the size of the induction variable set) by introducing an Inducing Variables Set as an intermediate variable and the posterior distribution of the set as a variational of the likelihood function of the entire training set. The set of induced variables is also known as a support set or pseudo-inputs. In the framework of SGP, the estimation methods of edge likelihood function based on induced variable sets usually include DTC (Deterministic Training Conditional) estimation, FITC (The Fully Independent Training Conditional) estimation, and PIT (Partially Independent Training Conditional) Estimates and so on [145,146]. The performance of the SGP method is closely related to the selection of the set of induced variables, and the general selection methods include random, symmetric, and based on specific optimization criteria. Figure 2.4 compares the G.P. model with the S.G.P. model, in which the input variable corresponding to the red sample points is selected as the initial set of induced variables, and to a certain extent, the prediction uncertainty of the SGP model is lower.



(a) Gaussian Process Model

(b) Sparse Gaussian Process Model

Figure 2.3 Comparison between G.P. model and S.G.P. model

Another limitation of the G.P. model is that its performance is strongly correlated with the Covariance Function (also known as the Kernel Function) [147]. For example, Gaussian kernel functions, which are widely used in the literature, actually make very smooth assumptions about functions, so the predictions of G.P. models for non-smooth or non-continuous functions tend to fail. This limitation can be mitigated to some extent by choosing different kernel functions, for example, the Matérn kernel function is better for trigonometric functions than Gaussian kernel functions. In addition, kernel design has become a hot spot and difficult point in G.P. models [148]. Using Bochner's theorem [149], it is possible to prove the positive certainty of some stationary functions, so such functions can be used as valid kernel functions. For example, A. Wilson [150] constructed a class of kernel functions using the inverse Fourier transform of the Gaussian mixture distribution, also known as a spectral approximation. Using known kernel functions to construct new kernel functions is a more common means in kernel function design, generally based on operator theory. For example, the construction of additive Gaussian models [151] based on sums and functions has proven its superiority in the application of additive models, and the construction of high-dimensional kernel functions based on the product of lowdimensional kernel functions has also been widely used [152,153]. In addition, the construction of new kernel functions based on nonlinear operators (such as convolution operators) has also attracted the attention of researchers [154,155], and it is known that the different characteristics of kernel functions after nonlinear action make it have broader application possibilities. However, the relevant proofs for constructing kernel functions are often very tricky, so kernel function design is a difficult and challenging point in Gaussian processes.

2.1.3 Design of Experiments (DOE)

The design of experiments is an important research direction in the field of modeling and simulation, and Figure 1-6 shows the classification of experimental design methods. Experimental design methods can be divided into one-shot experiment design (OSED) and sequential experimental design (SED). Single-step experimental design requires determining the sample size in advance and then using a certain criterion to generate an experimental design that meets the criterion according to a certain search strategy. Due to the need to determine the sample size in advance and the inability to make full use of the experimental results, single-step experimental designs are prone to under sampling or oversampling. The single-step DOE approach can be divided into model-aided, model-aided experimental design, and model-free experimental design [156]. The experimental point value strategy of model-assisted experimental design depends on the assumption of the model, such as sequential branch design [157], maximum information entropy sampling [158], and mean square variance-based design [159].

Model-independent design of experiments means that in the process of experimental protocol design, there is no need to assume the functional relationship between the input and output of the model. According to the different nature of the distribution of experimental points in the design space, the model-independent experimental design is divided into classical experimental design and space-filling design [160]. The sample points of classical experimental design are mainly distributed at the boundary of the design space, the grid divided by the design space or the scattering that meets certain rules; if the number of samples is increased, the experimental points cannot fill

the entire design space; Typical design approaches include comprehensive/fractional factorial design [161], central composite design (CCD), sparse mesh design [162,163], and Box-Behnken design.

Single-step experimental design based on space filling expects to spread experimental points throughout the design space, and representative methods include orthogonal design, uniform design [164], Monte Carlo (MC) [165], Quasi-Monte Carlo (QMC) [166], Latin hyper-cube design (LHD), etc. Among them, LHD has good space-filling and non-collapsing [167] and has been widely used in many fields [168,169]. By designing different spatial fill ability criteria, the researchers proposed a series of LHD methods [170,171]. Typical design guidelines include the maximum minimum criterion [172], the A.E. criterion (Audze-Eglais criterion) [173,174], the minimum energy criterion [175], the p criterion [176], etc. Grosso [177] proposes an iterative local search algorithm for generating maximum and minimum LHDs.

Sequential experimental design is an iterative design method, which gradually increases one or several design points each time according to the existing sample points, current experimental results, and other information, so as to effectively reduce the number of sampling and avoid oversampling. Sequential experimental design is widely used in data modeling or metamodel modeling [178]. According to different strategies, sequential experimental design can be divided into a sequential experimental design based on exploration strategy and adaptive sequential experimental design. Sequential experimental design (development-exploration strategy sequential experimental design). Sequential experimental design methods based on exploration strategies include Markov Monte Carlo sampling, extended Latin hypercube design [179,180],

etc. Yang [179] proposed a fast Latin hypercube experimental design method based on graph theory. Li[181] proposes an optimized Latin hypercube design method with arbitrary multiplicity. Xiong [180] proposed an approximate Latin hypercubic sequential design method, which can maintain the spatial filling and projection characteristics of the design scheme. Crombecq [182] discusses a sequential design method based on Delaunay triangle decomposition and Voronoi mosaic, which improves the efficiency of search by dividing the design space into several subspaces by using Delaunay triangle and Voronoi mosaic. However, the sequential design based on the exploration strategy only considers the current sample point scattering information and does not make full use of the simulation results and metamodel information in the iterative process.

Adaptive sequential experiment design (ASED) is a more efficient sampling method [183], and Deschrijver [184] points out that effective adaptive sequential design methods generally have two conflicting parts: local development and global exploration. Local development tends to select design points that may have larger areas of prediction error, also known as interesting regions or informative regions; The global exploration strategy tends to add new experimental sites in areas where experimental sites are sparse. For metamodel modeling, regions with large gradient variations need to add more sampling points. Based on the radial basis function neural network (RBFNN) metamodel, Yao [185], a gradient-based sequential experimental design method is proposed. Wei [186] discusses curvature-based sequential design methods. Jones [187] proposes an expected improvement criterion-based ASED method (EI-ASED) based on the expectation improvement criterion; EI-ASED is developed locally by minimizing the prediction response, using the prediction variance for global exploration, EI-ASED is also known as efficient global optimization method optimization (EGO). Mackman [188], Picheny [189] and Hernández-Lobato [190] improved EI-ASED from different perspectives. Liu [191] proposes an adaptive sequential design method based on maximizing the expectation of prediction error value. Ajdari [192] proposes an adaptive sequential design method based on development-exploration score. Garud [193] proposed a crowding distance metric and departure function-based ASED (CD-ASED).

2.1.4 Inverse problems (Model inference/calibration)

In contrast to the PoU method that determines the distribution characteristics of the QoI function by the propagation of the input variable in the proxy model, the inverse problem needs to solve the nature of the input variable under the premise that its observation is known [194]. In the UQ world, the inverse problem is also known as backward uncertainty propagation [195, 196] (Backward PoU). In particular, inverse problems in UQ focus on regression and parameter estimation from Bayesian perspectives.

For parametric regression problems, the ill-posed inverse problem is usually considered; that is, the corresponding regression problem has no solution, or the solution is not unique, or the solution is unique but has a high sensitivity to the QoI function. Regularization is a general method of solving unsettled problems, such as the commonly used Tikhonov regularization [197] and some regularization methods for PCE models. Engl [198] published in 1996 elaborated on the theory of regularization methods and their application to inverse problems, while Tarantola [199] explained the theory of inverse problems and general methods for model

parameter estimation from a Bayesian's perspective. From Bayesian's point of view, regularization is actually a priori application of different Bayesian systems, and different regularization constraints can actually be interpreted as different priors of solutions. For infinite-dimensional parametric (nonparametric) regression problems, A. M. Stuart [200] and S. L. Cotter et al. [201, 202] give a general framework for Bayesian inverse problems on separable Banach spaces and Hilbert spaces based on the idea of a Gaussian prior, using the idea of discretized parameter spaces; M. Dashti[203] uses the wavelet function in L2 space to solve the local non-smoothness problem of random fields based on Besov priors. Aiming at the pathological problem of G.P. model covariance matrix, H. Mohammadi et al. [204] derive in detail the connection and difference between pseudoinverse and nugget regularization methods, and give a G.P. regularization method based on the interpolation of Gaussian distributions. Figure 1.9 compares the parameters of the least squares problem under L2 regularization and L1 regularization, and estimates the G.P. regularization method based on the interpolation of the Gaussian distribution. Figure 1.9 compares the parameter estimation of the least squares problem with L2 regularization and L1 regularization.



Figure 2.4 Comparison between L2 and L1 regularization

In practice, it is often necessary to numerically integrate the posterior distribution to obtain the corresponding QoI. Therefore, a key problem with the inverse problem lies in the measurement of posterior distributions. The most important is the Bayesian check or Bayesian inference method, which takes the posterior distribution of parameters as the solution to the problem. The strict definition of conditional probability given by Kolmogorov makes it possible to define posterior distributions over infinite-dimensional spaces. Lasanen [205] generalized Bayes' formula to locally convex Suslin topological linear spaces. Literature [206-211] and others generalize Bayes' formula as a Radon-Nikodym derivative of a posterior distribution relative to a prior distribution. How to extract information from the posterior distribution, the literature mainly considers maximum posterior estimation [212,213] because maximum posterior estimation links Bayesian method with the classical penalty function method [211]. Two types of methods are usually considered in the literature: first, the most important and widely used in the literature is the Markov Chain Monte Carlo [214,215] method, which directly samples the posterior distribution by constructing the probability of transition between the prior distribution. The problem with MCMC focuses on the construction of transition probabilities, and inappropriate construction methods can greatly reduce the approximation of the true posterior distribution, that is, the long burn-in time. There are many variations of the MCMC The Reverse Jump MCMC [216] (RJMCMC) method can be applied method: between different dimensional parameter models, and can also be understood as a sparse optimization method based on Bayesian inference. Also, there are other methods like important sampling, SMC (Sequential Monte Carlo) sampling [217, 218], ABC (Approximate Bayesian Computation) sampling [219-224] and variants of these

methods, such as PMCMC (Particle Markov chain MonteCarlo) sampling[225] and variational inference methods [226, 227]. Literature [228-233] and others provide a comprehensive review of Bayesian calculation methods for parametric models. Literature [234-237] discuss Bayesian computation methods on infinite-dimensional spaces. Therefore, it is widely used both in the measurement of posterior distributions and in model selection/parameter optimization [238]. Another class of methods is model-related, such as the estimation methods for model parameters/hyperparameters discussed in the previous section about alternative models.

2.1.5 Sensitivity Analysis and Model Selection

2.1.5.1 Sensitivity Analysis

The sensitivity analysis method can also be used as a factor screening method for a type of simulation experiment. Sensitivity analysis aims to study the degree to which input uncertainty contributes to the influence of output uncertainty, that is, to study the source of output uncertainty [239]. According to the size of the contribution of each factor, you can get its importance ranking. Therefore, modelers and analysts can guide simulation evaluation, analysis, and optimization through sensitivity analysis to improve their work efficiency. Sensitivity analysis methods can generally be divided into local and global categories [240]. Local sensitivity analysis methods focus on the local influence of model inputs, common methods such as the OAT method, only change the value of one factor at a time and take the other factors as their benchmark value, usually using Tornado plots to represent their analysis results; The global sensitivity analysis method studies the effect of factor changes on the output in the entire experimental domain, and the commonly used methods can be divided into

three categories: variance-based methods, moment-independent methods, and regression-based methods.

2.1.5.2 Model Selection

In the context of physics-based modeling and simulation where the underlying model structure is dictated by first-principle physics, the term complex was often used to mean detailed [241,242] and determining the appropriate level and type of detail was considered to be one of the most important steps in the formulation of a simulation model [243,244]. It was advised, for instance, to start from a simple model, progressively add details until sufficient accuracy is obtained and select the least detailed model that meets the modeling objectives (Brooks & Tobias, 1996; Pidd 1996; Hill 1998). [245,246,247]

In the context of empirical curve fitting, Sober (1975) argued that models that are more informative are less complex. Kuhn (1977) stated that everything else being equal, it is rational to prefer a simpler model over a more complex one. Turney (1990) showed that simpler models tend towards a greater stability (or robustness) in face of experimental uncertainty. Many approaches have been developed for purposes of comparing alternative models built with varying levels of detail, which in turn has resulted in a new branch of mathematical statistics known as model selection. Although originally conceived to aid the model formulation process, the model selection criteria originated from this field also supply means for comparing alternative model calibration campaigns (see Table 1 for a list of common model selection criteria). These criteria typically consider both goodness-of-fit and complexity and differ in their representation of the latter.

Selection Metric	Criterion Equation
Akaike information criterion (AIC)	$AIC = -2 * \ln(f(y \widehat{\theta})) + 2k$
Bayesian information criterion (BIC)	$BIC = -2 * \ln(f(y \widehat{\theta})) + 2k * \ln(n)$
Deviance information criterion (DIC)	$DIC = \overline{D(\theta)} + p_D$

Table 2.1 Widely used model selection metrics

Information-theoretic measure of complexity (ICOMP)	$ICOMP = -\ln(f(y \widehat{\theta})) + \frac{k}{2}\ln\left(\frac{trace[\Omega(\widehat{\theta})]}{k}\right) - \frac{1}{2}\ln(\det[\Omega(\widehat{\theta})])$
Minimum description length (MDL)	$MDL = -\ln(f(y \widehat{\theta})) + \frac{k}{2}\ln(\frac{n}{2}) + \ln(\int \det[I(\widehat{\theta})]d\widehat{\theta})$
	^

Note: $y = \text{data function}; n = \text{sample of size}; \widehat{\theta} = \text{parameter value that maximizes the likelihood}$ function $f(y|\widehat{\theta}); k = \text{number of parameters}; D$ is the deviance of the likelihood, $D(\widehat{\theta}) = -2 * \log(f(y|\widehat{\theta}); p_D = \overline{D(\widehat{\theta})} - D(\overline{\theta}), \overline{D(\theta)})$ is the expectation of $D(\widehat{\theta})$ and $\overline{\theta}$ is the expectation of $\widehat{\theta}; \Omega$ = covariance matrix of the parameter estimates; ln= the natural logarithm of base e.

For instance, the Akaike Information Criterion (AIC) [248] and the Bayesian Information Criterion (BIC) [249] represent a model's complexity considering only the number of calibration parameters. Both methods use likelihood to assess the model goodness-of-fit and then penalize it with model complexity represented by the number of calibration parameters. Although user-friendly, the number of calibration parameters alone is not a sufficient definition of complexity. This can be demonstrated by considering two models $y_1 = \theta * x$ and $y_2 = \theta + x$ which have the same number of calibration parameters θ but different functional forms (multiplicative versus additive). Despite both having one calibration parameter, these models have vastly different data-fitting abilities. AIC and BIC also fail to discount parameters which are not constrained by the data. A Bayesian generalization of AIC, the Deviance information criterion (DIC) overcomes the problem [250,251,252]. The complexity term of DIC is calculated by assessing the number of parameters that can be constrained by experiments (a concept referred to as the effective number of parameters) [253]. DIC is calculated in a straight forward manner using Monte Carlo posterior samples. The calculation is easier performed with posterior samples generated by nested sampling, which have non-integer weights, than AIC and BIC. However, no efficient method has been developed for calculating reasonably accurate MC standard errors of DIC.

The three criteria, AIC, BIC and DIC are all sensitive only to one aspect of complexity: the number of parameters (the effective number for DIC. Furthermore, all calibration parameters

in these criteria are considered to have equal contribution to the complexity of model as functional form and range of parameters are not considered. Information-Theoretic Measure of Complexity (ICOMP) criterion [254] overcomes these shortcomings by considering not only the number of calibration parameters but also the effects of their sensitivity and interdependence. From the table 1.1, the second and third terms together represent a complexity measure that takes into account the effects of parameter sensitivity through the trace and parameter interdependence through the determinant which are two principal components of the functional form that contribute to model complexity.

However, Pitt et al. (2002) [50] emphasized the importance of model selection metrics being invariant under reparameterization and recognized ICOMP criterion's inability to remain invariant. Being invariant under reparameterization means that when parameters of the model are transformed without loss of information, and the new model with transformed parameters (that behaves equivalently with the original model) should have the same complexity value as the original. For instance, if the π^a in $y = sin(\pi^{ax} + b)$ is transformed into a new parameter, *c*, the new model $y = sin(c^x + b)$ should be identified as equivalent by the model selection metric. AIC, BIC and ICOMP would however consider these two models to have different complexities.

Cutting et al. (1992) [53] also recognized that the number of parameters alone is an insufficient indicator of model complexity and advocated for evaluating the fitting power (i.e. what they refer to as 'scope') of a model to random data. They suggested using binomial tests to compare the fitting ability of a model to the data from actual system with the fitting ability to random data. Similarly, complexity has been defined as the range of data patterns that a model can fit [50,58] quantified by a geometric complexity measure known as the Minimum Description Length (MDL) [54]. This metric considers the experimental data as a code or description to be compressed by the model and evaluates the models according to their ability

to compress a data set by extracting necessary information from the data without random noise. MDL is based on the understanding that the more data is compressed, the more information about the underlying regularities governing the process of interest would be learnt [50]. Therefore, MDL would chooses a model which has the shortest description code (length) of the data [255]. MDL has been criticized for not fully considering the parameter interdependencies in model fitting and selection process [254].

2.2 Background and literature review of Suspended non-structural systems

Non-structural systems are an important part of building functions, which play an extremely important role in maintaining the overall seismic performance and postearthquake use functions of buildings, especially for important facilities and disaster prevention key buildings, such as emergency centers, hospitals, schools, and gymnasiums. Non-structural members are attached to the structure as nonstressed members, but they may still suffer from large seismic actions, so they need to rely on their own structural characteristics to resist these seismic actions [256-258]. The earthquakes that occurred in recent years show a new seismic damage feature; that is, although the main structure has less seismic damage after the earthquake and can achieve the pre-set seismic performance goal, the non-structural systems attached to the main structure have very serious seismic damage, which often occurs before the main structure, and its seismic capacity is seriously insufficient [259]. The destruction of non-structural systems will not only reduce the performance level of buildings, but also seriously affect the post-earthquake recovery of buildings. With the continuous pursuit of a better life and the increasing construction investment, the proportion of investment in non-structural systems is increasing. According to the statistical results of FEMA E-74 [260], the investment proportion of non-structural components in

commercial buildings is 75-85%. Taghavi et al. [261] showed that the investment of non-structural components in office buildings, hotels, and hospitals accounted for 82%, 87%, and 92%, respectively. It can be seen that the investment in non-structural systems far exceeds the investment in structural components, so the economic losses caused by the destruction of non-structural components often exceed the losses of structural components, often leading to huge economic losses, including direct economic losses and indirect economic losses (losses caused by repair costs, building function interruption or loss). In addition, it also brings great risks to personnel safety [261,262].

2.2.1 Earthquake Damage of Suspended Ceiling Systems

Suspended ceiling systems are important non-structural systems in buildings. As the top decoration of buildings, it has the function of heat preservation and sound insulation and is also the hidden layer of electrical, ventilation, communication, and fire protection pipeline equipment. The suspended ceiling is one of the non-structural systems that are more prominent in recent years' earthquake damage [263-274]. Table 2.2 summarizes the earthquake damage performance of suspended ceilings in recent 10 years. Figure 1.2 shows the actual earthquake damage of suspended ceilings in previous earthquakes. It can be seen from the earthquake damage to the suspended ceiling that when the building encounters an earthquake, the suspended ceiling is very easy to be damaged, so its seismic capacity is seriously insufficient.

Table 2.2 Earthquake damage performance of suspended ceilings in the past 10 years

Earthquake	Year	Earthquake damage performance
Christchurch		Ceiling panels in residences crack and fall off; The
earthquake	2010	ceiling panels in commercial buildings fall off and
[263,264]		break, and the grids components and nodes fail. The

		ceiling collides with surrounding equipment and
		causes damage.
Chile		The ceiling collapsed in San Diego Airport, and a
earthquake[265]	2010	large number of ceiling panels in the hospital were
carinquake[205]		misaligned and falling.
		Less damage to suspended ceilings in residences;
		Ceiling damage in public buildings includes buckling
Christchurch	2011	and twisting of grids, failure of grids nodes, tearing
earthquake[266]	2011	and falling off of edge nodes, etc. The interaction of
		the suspended ceiling with its surrounding non-
		structural components causes the ceiling to fail.
Japan		The suspended ceilings in a large number of
	2011	important buildings such as gymnasiums, swimming
Earthquake[267,268]	2011	pools, schools, and hospitals were seriously
		damaged.
		The ceiling damage in government buildings,
Lushan	2013	schools and hospitals is very serious, and there are
earthquake[269.270]	2013	damage such as panel falling, primary and secondary
		grids node failure, and grids failure.
Italy, contherealize		The ceiling panels in the building fall off a large
	2016	number of times, the ceiling boundary is broken, the
	grid is buckled, etc.	
Tashkurgan 2017 earthquake[272]	The ceiling panels in several reinforced concrete	
	2017	frame structures have fallen off profusely.
Alaska earthquake		A large number of ceiling panels in important
	2018	buildings such as live news rooms, libraries, and
[273]		schools have fallen off.



(a) 2010 Christchurch earthquake



(b) 2010 Chile earthquake





(c) 2011Christchurch earthquake

(d) 2011 Japan Earthquake



(e) Lushan earthquake



(f) Italy earthquake



(g) Tashkurgan earthquake



(h) Alaska earthquake

Figure 2.5 Actual damage to suspended ceilings

2.2.2 Progress in Seismic Research of Suspended Ceiling

2.2.2.1 Experiments study

In view of the complexity of the suspended ceiling structures and the highly nonlinear mechanical behavior, scholars in various countries mainly use test methods to study its seismic performance. The test types generally include static tests and shaking table tests.

2.2.2.2 Static experiments

The static test includes monotonic loading test and low cycle reciprocating loading test. For the low cycle reciprocating loading test of non-structural systems, most researchers adopt the loading scheme specified in FEMA 461 [275]. The research progress of the static test of a suspended ceiling is introduced in the following three types: component test, joint test, and overall ceiling test.

(a) Test of ceiling components

The test objects of suspended ceiling components include grid components, mineral wool boards and threaded rods (or wires). At present, there is still a lack of experimental research on suspended ceiling components. Paganotti et al. [276,277] analyzed the failure mode of grid by monotonic loading test. The results show that the weak section of the grid is prone to tear failure under tension and local buckling under pressure. Soroushian et al. [278] conducted a monotonic tensile test on the threaded rod. The results show that the suspension wire will produce a necking phenomenon and brittle fracture failure under tensile action, and the tensile bearing capacity of the suspension wire can meet the requirements of the standard. Chhat et al. [279] carried out low cycle reciprocating bending tests on suspenders. The results show that the strength of the suspender decreases gradually with the increase of cyclic loading times until it breaks.

(b) Test of suspended ceiling node

The test objects of suspended ceiling joints include main tee splices, main and cross joints, and edge joints. The above joints are regarded as the most critical components in the ceiling, and their seismic performance will directly affect the overall seismic capacity of the ceiling. Therefore, it is very necessary to carry out experimental research on their mechanical performance.

Paganotti et al. [276,277] carried out monotonic loading tests on the main tee splice, main and cross tee joints, and edge joints with rivets, and analyzed the failure forms and ultimate bearing capacity of different joints. The results show that the failure of the main tee joint and the main and cross tee joint is concentrated at the grid joint under monotonic tensile loading, and the failure form of the edge joint is the expansion and tearing of the rivet connection hole; Under monotonic compression loading, the failure mode of main tee joint and main and cross tee joint is the joint buckling; In the shear test, the failure mode of the main and cross tee joints is the joint shear failure.

The compressive bearing capacity of the main tee splicing point is greater than the tensile bearing capacity, and the compressive bearing capacity of the main and cross joints is less than the tensile bearing capacity. The bearing capacity of the side joints is improved by two rivets compared with one rivet structure. Pourali [280] in the same research team, used monotonic loading test to analyze the impact of seismic clips on the seismic performance of main and cross tee joints. The results show that the seismic clip improves the bearing capacity, residual strength and ductility of the joint.

Takhirov et al. [281] analyzed the influence of the structure type of edge joints on their seismic performance by monotonic loading and low cycle reciprocating loading tests. The results show that the seismic sandwich joint with two screws recommended in this study has a greater bearing capacity and better energy dissipation performance than the edge joint recommended in the code. In addition, Soroushian et al. [278] conducted a shear test on the interaction between mineral wool board and fire sprinkler. Fiorin et al. [282] conducted monotonic loading and low cycle reciprocating loading tests on keel nodes in suspended ceilings in Europe, investigated the effects of node type, tee section shape and tee section size on the seismic performance of joints, and analyzed the failure mechanism, force-displacement response and equivalent viscous damping of nodes.

Xiqing [283] studied the failure load of main and cross tee joints and main tee hanger joints through monotonic loading tests. The results show that the failure load of the main and cross joints is mainly controlled by the bending degree of the cross tee joints and the contact range of the splice clips in the joints. The failure load of the main tee hanger joints is mainly determined by the length and path of the cut line at the connecting end of the hanger and the main tee web. Wang et al. [284] studied the failure mode, bearing capacity, deformation performance, and energy dissipation performance of main tee joints, main and cross joints, and edge joints through monotonic loading and low cycle reciprocating loading tests, further enriching the test data of tee joints and splice points.

2.2.2.3 Overall experiments of suspended ceiling

Gilani et al. [285] proposed a static loading scheme to study the compressive strength and stiffness of the main keel. The results show that the main tee is not the weakest component in the ceiling to resist horizontal action. Nakaso et al. [286] proposed a new type of reinforcing cable to solve the difficulty of ceiling reinforcement. The static test of the ceiling shows that installing the reinforcing cable in the ceiling can improve the lateral stiffness of the ceiling and reduce the displacement response of the ceiling. Brandolese et al. [287] carried out a static test on the suspended ceiling with supports to investigate the failure mechanism, stress performance, and deformation performance of the suspended ceiling. The results show that although the buckling of the support bar reduces the mechanical performance of the ceiling, the system shows good deformation capacity.

It can be seen from the above studies that scholars from all over the world have obtained some valuable conclusions from the static tests on the suspended ceiling components, suspended ceiling joints, and the suspended ceiling as a whole, which can provide test support for the seismic vulnerability research and numerical modeling of the suspended ceiling, but there are still some problems and deficiencies. For example, there is a lack of uniform test standards in experimental research. The test pieces used by scholars from different countries are produced by different companies, and the applicability of the research results is poor due to the differences in the details of the test pieces.

(a) Dynamical shaking table test

The dynamical shaking table test is the most direct and intuitive means to study the seismic performance of suspended ceilings. The shaking table test of suspended

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ceilings generally adopts full scale model. The research progress of the suspended ceiling shaking table test is introduced in the following three aspects: test carrier, loading system, and test results.

(b) Test carrier

The steel platform is often used as a carrier to suspend the suspended ceiling and provides the actual boundary conditions for the suspended ceiling. The following two points should be considered in the design of the steel platform: (1) The steel platform should have enough stiffness, and its natural frequency should be far away from the natural frequency of the ceiling to avoid the impact of resonance effect on the seismic performance of the ceiling. (2) The steel platform with reasonable height shall be designed to facilitate the installation of a suspended ceiling. Table 2.3 summarizes the steel platform information used in the existing study. It can be concluded from Table 1.3 that the natural frequencies of most steel platforms are greater than 10Hz, which reduces the amplification effect of steel platforms on input to some extent. The plane area of the steel platform varies from 2.9 m² to 558 m². In the test, unidirectional loading is dominant.

Researcher Yea	Vear	Platform		Loading		
	ICal	size/m	Х	Y	Z	directions
ANCO [288]	1983	3.7×8.5	-	-	-	-
Rihal [289]	1984	3.7×4.9	-	-	-	1
ANCO [290]	1993	4.3×7.3	-	-	-	_
Yao [291]	2000	1.2×4.1×2.2	-	24.8	-	1
Badillo [292]	2006	4.9×4.9×1.8	12.3	12.3	9.6	2
Ryu [293] 2	2012 -	6.3×6.3×3.0	12.0	12.8	21.8	3
		6.3×16.5×3.0	11.3	13.3	22.0	3
Magliulo [294]	2012	2.4×2.7×2.7	50.0	55.6	-	1
Watakabe	2012	5.5×5.5×4.0	-	-	-	1

Table 2.3 Summary of steel platform information from existing studies

[295]						
Soroushian	2012	10×12×16	1.5	-	-	3
[296,297]	2014	3.7×18.3×7.6	-	$4.3 \\ (2.8)^*$	-	1
Pourali[298]	2015	2.7×5.2×2.8	-	12.5	-	1
Chen[299]	2015	6.6×11.0×22.9	-	$1.2 \\ (1.9)^*$	-	1
Wang[300]	2016	3.3×3.9×3.3*	11.1	10.0	25.0	3
Ozcelik[301]	2016	2.4×3.0×1.4	-	12.0	-	1
Sasaki[302]	2017	18.6×30.0×9.1	-	_	-	3
Hong[303]	2017	3.5×3.5	$33.3 \\ (2.8)^*$	$33.3 \\ (2.8)^*$	-	1
Masuzawa [304]	2017	3.6×5.6×1.8	-	-	-	1
Yao[305]	2017	3.0×8.0×2.0	10.0	35.0	30.0	2
Lu [306]	2018	4.0×4.0×4.2	25.1	25.1	25.0 (7.4; 8.4)*	2
Chhat[307]	2019	2.4×5.2×2.0	-	-	-	1
Fiorino [308]	2019	2.9×3.2×3.3	3.0	-	-	1
Lee[309]	2019	3.5×3.5×3.0	16.8	16.8	-	1
Li [274]	2019	1.7×1.7×1.2	47.3	63.4	136.8	1
Jiang[310]	2020	11.6×12.8×5.4	8.4	8.9	-	2
Patnana[312]	2020	2.4×3.0	20.5	22.6	15.3	1
Qi[313,314]	2020	3.0×4.0×3.0	>20.0	-	-	2

Note (1)The size of the steel platform is expressed in width×length× height, if there are missing items, the height is unknown; (2) The X direction and Y direction refer to the horizontal short direction and horizontal long direction of the steel platform respectively, and the Z direction refers to the vertical direction of the steel platform; (3) $3.3 \times 3.9 \times 3.3^*$ The platform adopted is a reinforced concrete frame structure filled with masonry; (4) 4.3 (2.8)* refers to the self-resonance frequency of linear steel platform and nonlinear steel platform, respectively; (5) 33.3 (2.8)* refers to the self-resonance frequency of rigid steel platform and flexible steel platform, respectively; (6) 1.2 (1.9)* refers to the self-resonance frequency of the foundation seismic isolation structure and the bottom fixed structure, respectively; (7) 25.0 (7.4; 8.4)* refers to the vertical self-oscillating frequencies of the three support systems, respectively.

(c) Loading system

In the shaking table test of suspended ceiling, researchers will choose different loading systems according to different research purposes and needs. The loading systems mainly include natural ground motion input and artificial wave input. The artificial wave mainly includes sine wave, floor wave, and artificial wave fitted according to the design spectrum and floor demand spectrum of the specification. Table2.4 summarizes the loading system in the suspended ceiling shaking table test. The natural ground motion is generally the input wave at the bottom of a full-scale structural model containing non-structural components such as suspended ceilings or the actual seismic wave selected according to the structural design response spectrum. A sine wave is generally used to study the influence of different parameters and the failure mechanism of suspended ceiling.

Some scholars use the floor wave calculated from the numerical model as the input of the suspended ceiling shaking table test, some scholars use the fitting wave based on the code design spectrum, and most scholars generate appropriate artificial waves based on the floor demand spectrum recommended by the shaking table test standard AC156 [80] for non-structural members, which makes the results of different suspended ceiling shaking table tests comparable. Most scholars will use a variety of types of seismic waves to study the seismic performance of suspended ceilings.

Inp	ut types	Scholars who have used this input wave in existing studies					
Natural waves		Wang[314]; Lu[306]; Soroushian[298]; Chen [299];					
		Qi[313]					
	Sinusoid	Li [274]; Jiang[310]; Pourali [257]; Rihal [289];					
Artificial waves	waves	Chhat[307]; Mccormick [315]					
	Floor waves	Jiang [310]; Watakabe[296]					
	Specification						
	Design	Yao[291]; Echevarria [316]					
	Spectrum						
	Floor	Dedilla[202], Drus[200], Mealinla[205], Eignina[200],					
	Demand	Dau = [293]; Kyu[298]; Mag = [101110[508]; Cileri [217]					
	Spectrum*	Gliani[317]					

Table 2.4 Summary of loading regimes for ceiling shaker testing

Note: The floor demand spectrum in the table is recommended by the non-structural component shaker test standard AC156.

The following focuses on the relevant provisions of the loading system in the shaking table test standard AC156 for non-structural systems [314]. Based on the calculation formula of equivalent lateral force method for non-structural systems in ASCE 7-10 [318], AC156 gives the floor demand response spectrum (RRS) and requires fitting the target response spectrum (TRS) matching RRS in a certain frequency range (1.3~33.3Hz). The duration of the acceleration time history is generally 30s, and the acceleration time history meeting the requirements includes at least three sections: enhancement - stabilization - attenuation, with the time respectively being 5s, 20s and 5s. The strong earthquake time is required to be no less than 20s, and the long-duration acceleration time history is acceptable. Figure 1.8 shows the horizontal and vertical RRS curves.

The horizontal acceleration AFLX-H of flexible components (components with natural frequency less than 16.7Hz) is calculated as follows:

$$A_{FLX-H} = S_{DS} \left(1 + 2\frac{h}{H} \right) \le 1.6S_{DS}$$
 (2.1)

In the equation(2.1), S_{DS} is the short-period (0.2s) design response spectrum acceleration; h is the height of the installation position of non-structural components from the ground; H is the average height of the main structural roof.

The horizontal acceleration ARIG-H of rigid members (members with natural frequency greater than 16.7Hz) is calculated as follows:
$$A_{RIG-H} = 0.4S_{DS} \left(1 + 2\frac{h}{H} \right) \le 1.2S_{DS}$$
(2.2)

The vertical RRS of flexible members and rigid members is 2/3 of the horizontal RRS, and h=0 is assumed, so the vertical acceleration of flexible members AFLX-V and the vertical acceleration of rigid members ARIG-V are calculated according to Equations 1.3 and 1.4, respectively:

$$A_{FLX-V} = 0.67S_{DS}$$
(2.3)

$$A_{RIG-V} = 0.27S_{DS}$$
(2.4)



Figure 2.6 Demand response spectrum in AC156 (damping ratio: 5%)

(d) Test results

Scholars all over the world have done a lot of research on the seismic performance of suspended ceilings through shaking table tests and have obtained fruitful research results. The existing test results are summarized as follows.

Before 2000, the shaking table tests on suspended ceilings were very limited. ANCO Engineers [Refer] first used a shaking table test to analyze the impact of structural measures such as inclined suspension wire, compression bar, and side node with a rivet on the seismic performance of the suspended ceiling. The results show that the compression bar has a limited effect on reducing the seismic damage of the suspended ceiling, and the side node with rivets can reduce the seismic response of the suspended ceiling more than the inclined suspension wire. Ten years later, ANCO Engineers [290] carried out a series of shaking table tests on suspended ceilings with actual ground motion input. The results show that the tested ceiling. Rihal et al. [289] studied the seismic performance of the suspended ceiling with input harmonic excitation. The results show that the compression bar can reduce the vertical vibration of the ceiling, and the inclined suspension wire can reduce the seismic response of the ceiling.

Yao [291] conducted a shaking table test on the suspended ceiling. The research shows that the inclined suspension wire cannot improve the seismic performance of the ceiling, and the installation of side suspension wire in the ceiling and the use of rivets in the side node can improve the seismic performance of the ceiling. It can be seen from the above analysis that different scholars have different conclusions about the effects of inclined suspension wires and compression rods, which are mainly related to factors such as ceiling type and input excitation. Generally speaking, the compression bar can reduce the vertical vibration of the ceiling, but it may aggravate the damage to the ceiling under a large vertical earthquake. The influence of inclined

suspension wires on the seismic performance of suspended ceilings needs further study.

Since 2000, scholars from different countries have carried out a lot of shaking table tests on suspended ceilings. Compared with the existing research, the main differences are as follows: (1) The performance-based seismic performance evaluation and design method research of the suspended ceiling is started, the performance level and performance objectives of the suspended ceiling are proposed, and the seismic vulnerability curve of the suspended ceiling is established. (2) The influence of different parameters on the seismic performance of suspended ceilings is further studied. (3) More effective seismic measures are proposed to reduce the damage to the ceiling. (4) The whole shaking table tests of a variety of non-structural components, including suspended ceilings, were carried out to study the influence of the interaction between the main structure and suspended ceilings and between different non-structural components on the seismic performance of suspended ceilings. The following summarizes and sorts out the seismic research results of suspended ceilings since 2000 from different aspects.

Some scholars used shaking table tests to study the influence of different structural measures and parameters on the seismic performance of suspended ceilings, which provided a test basis for the seismic design of suspended ceilings. Li [271] studied the influence of loading parameters, tee support conditions, suspension length, and other factors on the seismic performance of a suspended ceiling. The results show that the parameters such as peak acceleration and peak velocity have no obvious correlation with the damage degree of the suspended ceiling. The reliable connection of boundary

members can significantly reduce the damage to the suspended ceiling, and the suspension length has little impact on the damage to the suspended ceiling. Jiang et al. [310] conducted a comparative study on the influence of whether there is an anti-seismic clip installed at the ceiling boundary on the anti-seismic performance of the ceiling. The results show that the acceleration, displacement and strain responses of the suspended ceiling will be significantly reduced if the anti-seismic clamp is installed at the boundary of the suspended ceiling. Chhat et al. [307] investigated the influence of brace arrangement and eccentricity of the upper end of the brace on the seismic performance of a suspended ceiling, and analyzed the failure mechanism of a suspended ceiling. The research shows that the damage of the suspended ceiling under multi-directional loading is more serious than that under unidirectional loading. Mineral wool board fasteners and side joints with rivets can improve the seismic performance of suspended ceilings.

Some scholars have conducted shaking table tests on suspended ceilings in large-span spatial structures. Sasaki et al. [302] analyzed the collapse mechanism of suspended ceilings in gymnasiums through shaking table tests. Lee et al. [309] designed a shaking table test based on AC156 [314] for suspended ceilings in large space structures and studied the dynamic characteristics and damage of suspended ceilings. Bo [319] studied the influence of different structural forms and upper supports on the seismic performance of suspended ceilings in long-span structures through shaking table tests. The research of Lu et al. [320] shows that the flexible support amplifies the vertical response of the ceiling, and the hinged suspended structure in the middle can reduce the vertical response of the ceiling to a certain extent. The failure of the tee

is mainly caused by the failure of the tee node. Ryu et al. [321] studied the failure mechanism of large-area suspended ceilings through a shaking table test.

Some scholars have proposed effective seismic measures to reduce the seismic damage of suspended ceilings. Pourali et al. [257] proposed a suspended ceiling which is disconnected from the wall all around and conducted a shaking table test on it. The results show that when the suspended ceiling encounters resonance, the displacement of the suspended ceiling increases and it collides violently with the boundary, resulting in greater acceleration. In order to solve the problem of excessive displacement and acceleration when the ceiling resonates, Pourali et al. [322] suggested filling the gap at the ceiling boundary with isolation blocks. The analysis shows that the isolation block can reduce the impact and, thus, the displacement and acceleration response of the ceiling. The results show that the suspended ceiling with the new type of side joints has better seismic performance. Watakabe et al. [296] proposed a new anti-seismic clip SECC, and analyzed the failure mechanism and antiseismic performance of the suspended ceiling after installing SECC through a shaking table test. The results show that SECC significantly improves the seismic performance of the suspended ceiling. Masuzawa et al. [304] proposed a device that can effectively prevent the falling off of ceiling panels and verified the effectiveness of the device through shaking table tests.

At present, the research on the seismic performance of gypsum board ceiling is still very limited. Magliulo et al. [295] carried out shaking table test research on singleframe and double frame gypsum board ceilings. The results show that the suspended ceiling is undamaged under all input excitation and has good seismic performance, which is related to the good continuity of the suspended ceiling, dense keel arrangement, and sufficient suspenders. Patnana et al. [311] compared the seismic response of gypsum board ceilings with vertical supports, which are boundary free and boundary fixed, through shaking table tests. The results show that the displacement response of the boundary fixed ceiling and the cumulative strain of the vertical support change less than that of the boundary free ceiling. Under the loading of Taft seismic wave sequence, both of them show good seismic performance. Under the loading of sine wave failure condition, the boundary fixed ceiling is undamaged, while the boundary free ceiling is severely damaged. Qi et al. [313] studied the seismic performance of the drop grade gypsum board ceiling through shaking table tests, and analyzed the effects of temporary supports and boundary constraints. The results show that the seismic performance of the suspended ceiling is good under earthquake excitation. Temporary supports can reduce the relative displacement of the high and low sides of the ceiling and improve the overall performance of the ceiling. The boundary constraint can reduce the torsional deformation of the ceiling, the horizontal vibration of the ceiling, and the stress of the suspender.

At present, most researches mainly focus on the single type of non-structural components of the suspended ceiling, and a few scholars have carried out shaking table tests of the whole system of a variety of non-structural components including the suspended ceiling. Soroushian et al. [323] analyzed the seismic performance of the ceiling partition pipe composite system by shaking table test on a 5-story steel frame platform, compared the seismic response of the ceiling with or without lateral support, and studied the impact of different structural measures on the interaction between mineral wool board and fire sprinkler. The results show that when the suspended

ceiling is subjected to strong vertical earthquake, lateral bracing cannot improve the seismic performance of the suspended ceiling. Flexible suspension wire can effectively reduce the interaction between mineral wool board and fire sprinkler. Pantoli et al. [300] conducted a shaking table test on a full-size non-structural system using a 5-story reinforced concrete structure as a platform. The results show that the suspended ceiling with seismic design has good seismic performance. Fiorino et al. The results show that the strengthened seismic connection can improve the seismic performance of the composite system. McCormick et al. [315] compared and analyzed the seismic performance of gypsum board partition traditional ceiling composite system and gypsum board partition suspended composite ceiling system through seismic design. The results show that both types of suspended ceilings have good seismic performance, but the acceleration response of the suspended ceilings with seismic design is greater. Huang et al. [324] studied the seismic performance of the ceiling partition composite system by shaking table test. The results show that the loading dimension and ceiling dimension are important parameters affecting the seismic response of the system, and the brace can improve the seismic performance of the ceiling partition composite system.

2.2.2.4 Numerical simulation

At present, the numerical analysis of suspended ceilings is still difficult. The main reasons include: (1) the diversity of suspended ceilings and the complexity of the suspended ceiling node structure. (2) There are complex interactions among the components of the ceiling, between the ceiling and the surrounding non-structural components, and between the ceiling and the main structure. (3) The complexity of nonlinear response of suspended ceiling.

(a) Simulation of ceiling components

At present, the numerical analysis of suspended ceiling components is rarely carried out. Most scholars do not consider the nonlinearity of suspended ceiling components when modeling suspended ceiling components but only consider the nonlinearity of suspended ceiling nodes. The main reason is that compared with suspended ceiling components, the seismic performance of suspended ceiling nodes is worse, and they show obvious nonlinear behavior in earthquakes. In actual earthquake disasters, the damage of the suspended ceiling is mostly concentrated in the nodes; while the components are generally less damaged, Therefore, scholars focus on node simulation. Soroushian et al. [325] used similar modeling methods to simulate the hysteretic behavior of primary and secondary keel joints under axial force, shear and bending and established load displacement restoring force models of joints under different loading modes. The results show that the modeling method can well simulate the mechanical and deformation performance of the joints, and the restoring force model of the joints can be used for the nonlinear analysis of the whole ceiling. Fiorin et al. [282] used a modeling method similar to Soroushian et al. [325] to simulate the nonlinear behavior of keel nodes.

(b) Simulation of suspended ceiling

Scholars have made some achievements in the numerical analysis of suspended ceilings. Ryu et al. [294, 321] proposed to analyze the seismic response of a suspended ceiling under unidirectional horizontal earthquake by using a two-

dimensional simplified mass spring model with multiple degrees of freedom. The results show that the simulation results are consistent with the shaking table test results, which proves the rationality of the model, but the model has some difficulties in analyzing the seismic response of the ceiling with complex forms. Echevarria et al. [316] used finite element software SAP2000 to simulate the seismic response of the ceiling, using the beam element to simulate the grid member, the hook element to simulate the hanger, the frame unit to simulate the pressure rod, the tensile and compressive friction pendulum vibration isolation unit to simulate the interaction between the mineral wool plate and the grid, assuming that the main and cross tee nodes were hinged, and the mineral wool board was simplified to the "X" type semirigid-mass point model. The model can simulate the elastic deformation of the suspended ceiling and the lifting of mineral wool board but cannot simulate the collapse behaviour of the suspended ceiling. In view of the shortcomings of the numerical analysis models of Ryu et al. [294,321] and Echevaria et al. [316], Zaghi et al. [326] took the specimen of Ryu et al. [294] in the ceiling shaker test as the benchmark model, and used the finite element software OpenSEES to establish a nonlinear numerical analysis model of the ceiling, considering the nonlinear effects of the collision between mineral wool board and keel and boundary constraints. The results show that the model can better predict the failure position of the ceiling, and the displacement time history curve of the ceiling obtained by the numerical model and the shaking table test is in good agreement, but the acceleration time history curve calculated by the model is different from the test results, which is mainly due to the impact of the high-frequency spike caused by the collision. Soroushian et al. [327,328] used the finite element software OpenSEES to establish a nonlinear numerical

analysis model of the ceiling pipe composite system. This model can predict the failure mode and location of the ceiling, but it will overestimate the number of mineral wool board failures. The above simulations are all aimed at mineral wool board suspended ceiling. For gypsum board suspended ceiling, Tagawa et al. [329] established a numerical model of a gypsum board suspended ceiling in the stadium using the adaptive displacement integral Gaussian method to simulate the collapse behaviour of a suspended ceiling. The results show that this method can simulate the collapse process of a suspended ceiling. Similarly, Gilani et al. [330] established a numerical analysis model for gypsum board ceiling to study the seismic response of key components in the ceiling.

The numerical analysis of suspended ceilings by domestic scholars is still in its infancy. Yao [291] established a simplified numerical analysis model of the suspended ceiling by using the finite element software ANSYS, and compared the influence of inclined suspension wires on the natural frequency of the suspended ceiling with inclined suspension wire is higher than that of the suspended ceiling without inclined suspension wire, but their vibration modes are basically the same. However, this model does not consider the influence of nonlinearity. Li [274] established a finite element model of mineral wool board suspended ceiling according to the method of numerical modeling of suspended ceiling introduced by Zaghi et al. [326]. This model takes into account the nonlinear behaviors such as friction and collision between mineral wool boards and grids. The main and cross tee nodes are assumed to be hinged, and the end of the main girds is assumed to be rigid support. The results show that the interaction between mineral wool board and keel

and has certain accuracy in simulating the relative displacement of mineral wool board, the relative displacement of mineral wool board and main grids, and the absolute acceleration of main grids. In recent years, Qinghua's research group has obtained certain research results in the numerical simulation of suspended ceilings [319,320]. Kou Miaomiao [331] used the finite element software ANSYS to study the impact of inclined suspension wires on the seismic performance of the ceiling. The grid node in the model is assumed to be rigid without considering the contribution of mineral wool board stiffness. The results show that the inclined higher wire improves the seismic performance of the ceiling, which may be related to the obvious increase in the seismic response of the ceiling without an inclined suspension wire due to the failure to consider the boundary constraints of the ceiling in the model. Bo [319] used the finite element software ANSYS to establish a numerical model of a mineral wool ceiling, assumed that the grid node is hinged, ignored the contribution of mineral wool board stiffness, and studied the dynamic characteristics and seismic response of the ceiling. The results show that the horizontal natural vibration period of the ceiling is close to the frequency of the simple pendulum, and the internal force of the suspender increases significantly under the earthquake, while the axial force of the keel does not increase. Qinghua et al. [320] used the finite element software ANSYS to study the influence of the upper support structure and suspender structure on the dynamic characteristics of the ceiling. The results show that the natural frequency of the upper support structure has a significant influence on the vertical mode of the suspended ceiling, and a small influence on the horizontal mode of the suspended ceiling. The structural form of the suspender has no influence on the first order horizontal and vertical vibration modes of the ceiling.

In conclusion, domestic and foreign scholars' research on the numerical simulation of suspended ceiling is relatively limited, and there are many simplifications in the model. For example, the simplified treatment of mineral wool boards cannot truly reflect the impact of mineral wool board stiffness on the seismic performance of suspended ceilings. How quantifying the impact of friction and collision between suspended ceiling components on the seismic response of suspended ceiling is also an urgent problem. Therefore, it is necessary to continue to carry out more refined numerical analysis and fully consider the nonlinearity of ceiling joints. At the same time, more research should be conducted on the level of suspended ceiling components to provide more abundant and reliable data support for the numerical modelling of suspended ceiling.

2.3 Core components literature review development table list

To facilitate the reader's better understanding of the development of the core components, it is listed below as a summary of Core components literature review (Gaussian Process and Inference) table for reference.

Table 2.5 Summary of Core components literature review (Gaussian Process and Inference)

	Literature	Model types	Inference method
Early- stage Gaussian Process	2001[19]	Gaussian process	MCMC
	2010[135]	Hierarchical Gaussian	Maximum Postorior
		Process	Probability
	2011[134]	Hierarchical Gaussian	Sparse Variational
		Process	Inference
	2012[127]	Warped Gaussian Process	Sparse Variational
			Inference (Induction point)
	2013[129]	Combinatorial Kernel	Variational Inference
		Functions	
Recent-	2013[134]	Deep Gaussian Process	Mean Field Variational

development list

stage			Inference
Gaussian	2014[130]	Deep Gaussian Process	Nested Variational
Process			Compression
	2016[131]	Deep Gaussian Process	Sampling Method
	2016[132]	Deep Gaussian Process	Sequential Inference
	2016[138]	Recursive Deep Gaussian	Mean Field Variational
		Process	Inference
	2016[137]	Deep Gaussian Process	Sparse Variational
			Inference (Sparse
			Spectrum)
	2017[139]	Deep Gaussian Process	Doubly Stochastic
			Variational Inference

2.4 Problems in current research

The main problems in current research are as follows:

(1) Most uncertainty quantification always not consider the surrogate model to enhance computational speed, and more machine learning based and high-efficiency surrogate model related research need to make.

(2) Model such as G.P. faces computational difficulties: if there are fewer samples, the model accuracy is not high enough; If the sample size is large, the difficulty of model calculation increases. Therefore, new model construction and calculation methods have attracted the attention of researchers, such as sparse G.P. model, variational G.P. model, and deep learning of G.P. model, etc.

(3) In fact, there is no one model that applies to all problems, so it is necessary to make full use of the information in the data, build a robust weighted model, or perform a purposeful screening of the model that is, model selection.

(4) How to effectively reduce the number of tests, ensure the accuracy of the model, and construct the optimal experimental design and sensitivity analysis under the high cost of complex system testing has been a hot spot in UQ research in recent years.

(5) To overcome the slow speed of convergence for MCMC, variational inference method which based on optimization approximation needs to be investigated more and deeper.

(6) At present, the seismic research on the grid connections of mineral wool ceilings is relatively insufficient, and there is a lack of a recognized joint resilience model, which cannot provide sufficient test support for the seismic vulnerability research and calculation model of the mineral wool ceiling. The specimens used by different scholars are different in detail structure and material properties, so the universality of research results is poor. Different scholars mostly use monotonic loading tests to study the seismic performance of keel joints, which is inconsistent with the characteristics of grid connections subjected to repeated loads in actual earthquakes.

(7) Scholars all over the world have carried out a lot of research on the seismic performance of mineral wool board ceilings mainly by means of shaking table tests, but the existing research is mainly focused on American mineral wool board ceilings and Japanese gypsum board ceiling. The research on mineral wool board ceilings is still very limited, and the understanding of the seismic damage mechanism of mineral wool board ceilings is still not deep enough..

(8) At present, there is a lack of nonlinear numerical calculation models for mineral wool ceiling, and the overall analysis model of the ceiling is more complex and time-consuming. Therefore, simplifying the numerical calculation model based on the

seismic response characteristics of mineral wool ceiling has become an urgent problem.

(7) Scholars have not considered about the area effect of suspended non-structural system and also the long period and long duration earthquake waves input.

(8) No research related to uncertainty quantification based inverse problems for suspended non-structural systems has ever been done before, which is a critical gap.

CHAPTER THREE

MACHINE LEARNING-BASED DATA AND MODEL DRIVEN UNCERTAINTY QUANTIFICATION OF INVERSE PROBLEMS METHODS AND FRAMEWORK **3.1 Gaussian Process surrogate model**

A Gaussian process is a set of random variables, and any finite-dimensional random variable obeys a consistent joint Gaussian distribution. Williams and Rasmussen[332] introduced Gaussian processes into the field of machine learning [25,26,27,28,29] and applied them to regression. Since then, Gaussian processes have been actively developed in the field of machine learning and applied to active learning [333], dimensionality reduction [334], optimization [335], reinforcement learning [336,337] and so on. This article focuses on Gaussian process regression.

Definition 3.1 Random process

Let (Ω, F, P) be a probability space, Ω is the sample space, F is the time domain, P is the probability measure, and T be a parameter set. If any t \in T has a random variable $X(\omega, t) \omega \in \Omega$ defined on (Ω, F, P) , then the family of random variables dependent on parameter t { $X(\omega, t)|t \in T$ } is said to be a random process, abbreviated { $X(t), t \in T$ } or X(t).

Definition 3.2 Gaussian process

Let {X(t), t \in T} be a random process defined on the probability space (Ω , F, P) if for any finite set of points {t1, t2, · · , tn} \subset T have {X(ω , t1), X(ω , t2), · · ·, X(ω , tn)} obey an n-dimensional Gaussian distribution, then {X(t), t \in T} is Gaussian process. By definition, the probabilistic properties of a Gaussian process are determined only by its mean and covariance functions. This is also an important property of Gaussian processes.

3.1.1 Gaussian process surrogate model

This section presents Gaussian process surrogate models from the perspective of functional spaces. We define the expectation and covariance of a real-valued Gaussian process f(x), respectively: m(x) and k(x, x'). The specific form can be written as equation (3.1) and equation (3.2).

$$M(x) = \mathbb{E}[f(x)] \quad (3.1)$$
$$k(x, x') = \mathbb{E}[(f(x) - m(x))(f(x') - m(x'))] \quad (3.2)$$

The Gaussian process can be expressed as:

$$f(x) \sim GP(m(x)mk(x,x'))$$
 (3.3)

Because expectations do not affect the model construction and derivation of Gaussian processes for the convenience of derivation and the brevity of symbols, let us assume that the prior expectation is m(x) = 0 for the sake of derivation convenience and symbolic brevity. A special reminder will be given when the expectation is not 0 later. The size of the covariance between any two random variables in a Gaussian procedure is calculated using the covariance function k(r), where r = dist(x, x') is the measure of distance between the two inputs. The covariance function, also called the kernel function, is used to quantify the magnitude of the correlation between two random variables. Kernel functions can have several different options, such as radial quadratic covariance functions:

$$k_{RQ}(r) = (1 + \frac{r^2}{2\alpha l^2})^{-\alpha},$$
(3.4)

γ Exponential family covariance function:

$$k(r) = exp(-(r/l)^{\gamma}) \quad for \quad 0 < \gamma \le 2,$$
(3.5)

Matern class covariance function:

$$k_{Matern}(r) = \frac{2^{1-\upsilon}}{\Gamma(\upsilon)} (\frac{\sqrt{2\upsilon}r}{l})^{\upsilon} K_{\upsilon}(\frac{\sqrt{2\upsilon}r}{l}),$$
(3.6)

Sum squared exponential covariance function:

$$K(r) = \alpha \exp\left(-\frac{\gamma^2}{2l^2}\right) (3.7)$$

where α , γ , v, and 1 are hyperparameters of kernel functions, especially 1 is a scale parameter that describes how complex the kernel function is in the parameter space. Among them, the square exponential covariance function equation (3.4) is the most commonly used covariance function because its correlation is close to the real problem. The subsequent study of non-structural systems in this dissertation mainly uses this covariance function, where r is defined as a measure of distance in Euclidean space. The detailed form of the kernel function in this dissertation is:

The covariance relationship between two random variables $f(x_q)$, $f(x_p)$ is a function of x_q and x_p . Theoretically, each set of positive-definite covariance functions is equivalent to an expansion of a set of basis functions. The square exponential kernel is equivalent to a linear combination of a set of infinite-dimensional Gaussian functions [338].

Suppose that the input points $X_* \in \mathbb{R}^{n*d}$ of *n* dimensions are known, and the corresponding random variable $f_* \in \mathbb{R}^n$ can be represented as an n-dimensional Gaussian distribution with an expectation of 0 and a covariance matrix of *K*:

$$f_* \sim N(0, K(X_*, X_*))$$
 (3.8)

where, each element in K can be calculated using the equation (3.8).

In reality, the general situation is that we cannot directly observe the output value of the function or system but can only get observations with a certain amount of noise, assuming that each observation is mapped using an unknown function f(x) and then disturbed by Gaussian noise independently of the same distribution, that is

$$y_i = f(x_i) + \epsilon_i \quad (3.9)$$

The correlation between the observations of the two inputs xp, xq is rewritten as:

$$cov(y_p, y_q) = k(x_p, x_q) + \sigma^2 \delta_{pq} \quad cov(y) = K(X, X) + \sigma_n^2 I \quad (3.10)$$

where δ_{pq} is the Dirac function when p = q, $\delta pq = 1$, otherwise δpq is 0. The joint distribution of y and f* after adding the observation error is:

$$\begin{bmatrix} \mathcal{Y} \\ f_* \end{bmatrix} \sim N(0, \begin{bmatrix} K(X, X) + \sigma_n^2 I & K(X, X_*) \\ K(X_*, X) & K(X_*, X_*) \end{bmatrix}) (3.11)$$

The conditional distribution is:

where,

$$f_*|X, y, X_* \sim n(\overline{f_*}, cov(\overline{f_*}))$$

$$\overline{f_*} \triangleq \mathbb{E}[f_*|X, y, X_*] = k_*^T (K + \sigma_n^2 I)^{-1} y$$

$$cov(f_*) = \mathbb{V}[f_*] = k(x, x_*) - k_*^T (K + \sigma_n^2 I)^{-1} k_* \quad (3.12)$$

Note that the covariance of the measured values is independent of the observations in the training set, only the inputs to the training and test sets. This is an important property of Gaussian process models.

When there is only one point x_* in the test set, $k(x_*) = k_*$ is the covariance vector between the test point and n training points. There are also abbreviations K = K(X, X), and $K_* = K(X, X_*)$. Bring the simplified formula into the equation(3.12) to get the expectation and variance of the posterior distribution of $f(x_*)$.

$$\bar{f}_{\star} = \boldsymbol{k}_{\star}^{\mathsf{T}} (K + \sigma_n^2 I)^{-1} \boldsymbol{y}$$
$$\mathbb{V}[f_{\star}] = k(\boldsymbol{x}_{\star}, \boldsymbol{x}_{\star}) - \boldsymbol{k}_{\star}^{\mathsf{T}} (K + \sigma_n^2 I)^{-1} \boldsymbol{k}_{\star}.$$
(3.13)

From the equation (3.13), it can be seen that the expectation of f^* can be understood

as a linear combination of all observations y. The weight of each observation is determined by $K_*^T(K + \sigma_n^2 I)^{-1}$. It can also be understood from another angle, and the equation (3.13) is transformed to get the equation (3.14):

$$\bar{f}(\boldsymbol{x}_{\star}) = \sum_{i=1}^{n} \alpha_{i} k(\boldsymbol{x}_{i}, \boldsymbol{x}_{\star}), \quad where \quad \boldsymbol{\alpha} = (K + \sigma_{n}^{2} I)^{-1} \boldsymbol{y}.$$
(3.14)

where $\alpha = (K^T + \sigma_n^2 I)y$ This result can be intuitively understood as the expectation of f_{*}, and it can also be represented by a linear combination of kernel functions acting on n central actions on the training point.

In addition, marginal likelihood p(y|X) is an important indicator to describe the quality of Gaussian regression models. We can get it directly from the prior $y \sim N(0, K + \sigma_n^2 I)$:

 $logp(y|X) = -\frac{1}{2}y^T(K + \sigma_n^2 I)^{-1}y - \frac{1}{2}log|K + \sigma_n^2 I| - \frac{n}{2}log2\pi$ (3.15) Before the given training samples, Figure (3.1a) shows the prior distribution of the function. The differently-colored lines represent functions drawn randomly from the distribution. After the given training sample (black plus sign), Figure (3.2b) shows the posterior distribution of the function. The function fits the training samples and shows uncertainty where the training samples are missing.



(a) Prior distribution



(b) Postorior distribution

Figure 3.1 Gaussian Process model illustration

In view of the defect of excessive time complexity, many scholars have proposed different approximation methods, among which the mainstream method is pseudo-point method.

The pseudo-point method assumes the existence of M virtual sample points (not disturbed by observational noise), called "induction points" or "auxiliary points". The input characteristics are $Z = \{z_i\}_{i=1}^M$, The corresponding function value $u = \{u_i\}_{i=1}^N = \{f(z_i)\}_{i=1}^N$. Suppose that given these induction points, the prediction points are independent of the training sample conditions:

$$p(f_*|\mathbf{y}) = \int p(f_*|f)p(f|\mathbf{y})df$$

= $\int p(f_*|f, u)p(f, u|\mathbf{y})df du$
 $\approx \int p(f_*|u)p(f, u|\mathbf{y})df du$
= $\int p(f_*|u)p(u|\mathbf{y})du$ (3.16)

It can be said that the induction point introduces the dependency between the prediction point and the training sample, so Quiñonero-Candela et al. [339] named it "Inducing Points". However, further approximations are required when calculating

p(u|y). After fifteen years of research and development, many scholars have put forward different opinions from different perspectives. In 2005, Quiñonero-Candela et al. [339] proposed the first unified framework to elaborate various methods at that time, such as SOR [340], DTC [341], FITC [342], PITC [339] and so on from the perspective of "approximate model, accurate inference". Different methods approximate the prior Gaussian process with the help of induction points according to different assumptions, thereby reducing the computational time complexity of the approximation model when making inferences. Under a unified framework, the assumptions of different approaches can be reasonably compared.

With the emergence of new methods, especially the VFE [343] method of Titsias et al., in 2017, Bui et al. [344] proposed a new framework to derive from the starting point of "accurate model, approximate inference". Under the new framework, the original model remains unchanged, and different methods use different approximation methods, such as variational inference [345] or expectation propagation [346], to reduce the time complexity of inference. Bui et al. [344] argue that the framework of "approximating the model, accurately inference" changes the assumptions made by the original model on the data, introducing induction points into the model in the form of parameters. When the number of induction points increases, it is difficult to guarantee that the approximate model will tend to the original model, and optimizing a large number of parameters may cause overfitting problems [343]. Although the assumptions are different, the training time complexity of the pseudo-point method is usually $O(NM^2)$, and the test time complexity is usually $O(M^2)$.

3.1.2 Optimization of hyperparameters in Gaussian regression models

Although there are many options for covariance functions, inevitable parameters need

to be determined within the function, which we often refer to as hyperparameters. The hyperparameter's value determines the kernel function's specific form, making it especially important to choose the appropriate hyperparameter. For example, in the squared exponential covariance function with input as one dimension:

$$k_{y}(x_{p}, x_{q}) = \sigma_{f}^{2} \exp\left(-\frac{1}{2l^{2}}(x_{p} - x_{q})^{2}\right) + \sigma_{n}^{2}\delta_{pq} \quad (3.17)$$

l, σ_f , σ_n are hyperparameters in kernel functions. Different hyperparameters affect the generalization ability of the model. As shown in Figure (), when the hyperparameter selection does not match the data, the predictive ability of the Gaussian process regression model will have severe errors. In order to avoid overfitting or underfitting the model, the selection of hyperparameters is particularly important and is also a key factor affecting the quality of the model. Hyperparameters are not easy to determine in real-world applications, or the model's hyperparameters may need to be adjusted accordingly as the input space changes. This section describes two common hyperparameter optimization methods in Gaussian regression models. In this chapter, hyperparameters will be represented by the vector θ .



Figure 3.2 Different hyperparameters comparison [338]

The following discussion is based on the maximal marginal likelihood method to illustrate its optimization method; according to Bayesian inference, the posterior density function of the hyperparameter θ can be expressed as:

$$p(\boldsymbol{\theta}|\boldsymbol{y}, X) = \frac{p(\boldsymbol{y}|X, \boldsymbol{\theta})p(\boldsymbol{\theta})}{p(\boldsymbol{y}|X)}$$
(3.18)

When the evidence is constant, finding the maximum posterior estimate of the hyperparameters is equivalent to maximizing $p(y|X, \theta)p(\theta)$. When $p(\theta)$ is uniformly distributed, the maximum posterior estimation of the hyperparameters is actually looking for θ , making the maximum $p(y|X, \theta)$, which is the maximum likelihood estimate.

Although Bayes' criterion provides a credible and consistent framework for inference, for most machine learning models, the computations required, such as marginal likelihood (integration over the entire parameter space), cannot be written in analytic form. It is also difficult to find a good approximation, making the amount of computation unbearable. The Gaussian regression model with Gaussian noise is indeed a rare exception, and from the equation(3.15), we can get the logarithmic form of $p(y|X, \theta)$ can be explicitly expressed as:

$$\log(p(y|x,\theta)) = -\frac{1}{2}y^{T}K_{y}^{-1}y - \frac{1}{2}\log|K_{y}| - \frac{n}{2}\log(2\pi)$$
(3.19)

Where $K_y = K_f + \sigma^2 I$ is the covariance matrix containing the covariance function and the observed error, $-\frac{1}{2}y^T K_y^{-1}y$ is the only data fitting term that contains observations; $\frac{1}{2}\log|K_y|$ is a calculated penalty that only considers covariance information; $\frac{n}{2}\log(2\pi)$ is a normalized constant. Generally, as the kernel function's scale parameters increase, the model's flexibility will become weaker and weaker, and the data fitting term will gradually become smaller. Conversely, the penalty becomes larger as the model becomes less flexible. The amount of data also has a significant influence on the choice of scale parameters. In general, the less data, the smoother the curves for marginal likelihood and scale parameters. Because the observation error term dominates in the marginal likelihood, the scale parameters of the fitted data term are not very sensitive compared to the small scale. Conversely, when the data become more numerous, the fitted term dominates in the marginal likelihood. At the same time, it has become more sensitive to scale parameters.

To maximize the marginal likelihood, we write the partial derivative form of the marginal likelihood for each hyperparameter:

$$\frac{\partial}{\partial \theta_j} logp(y|X,\theta) = \frac{1}{2} y^T K^{-1} \frac{\partial K}{\partial \theta_j} K^{-1} y - \frac{1}{2} tr \left(K^{-1} \frac{\partial K}{\partial \theta_j} \right)$$
$$= \frac{1}{2} tr \left((\alpha \alpha^T - K^{-1}) \frac{\partial K}{\partial \theta_j} \right) (3.20)$$

Where $tr(\cdot)$ is the trace of the matrix. The partial derivative of each hyperparameter in the solution equation(3.20) is 0, which is equivalent to the maximization marginal likelihood. K⁻¹ mainly determines the computational complexity of the equation(3.20), and the time complexity of O(n³) is required to find the inverse of a matrix with positive symmetry using conventional methods. Once K⁻¹ is obtained, the equation (3.20) has a gradient complexity of O(n²) for any hyperparameter θ j. Thus, the optimal hyperparameters can be calculated using any gradient-based optimization method. The above part is the analysis of the Gaussian process, and then we will explore the Bayesian calibration and inference process of the inverse problem.

3.2 Bayesian Inference

Bayesian inference methods provide an approach to the estimation or calibration of a set of parameters $\boldsymbol{\Theta}$ in a model (or hypothesis) H for the data **D**. It is based on a likelihood function derived from a specific probability model of the observed data

 $L(\mathbf{D}|\mathbf{\Theta})$, where $\mathbf{\Theta}$ is assumed to be stochastic, it has a prior distribution $\pi(\theta)$ pai(theta). The inference about theta is based on the posterior distribution pai(theta|D) obtained by the Bayes' theorem [347]:

$$\Pr(\boldsymbol{\Theta}|\boldsymbol{D}, H) = \frac{\Pr(\boldsymbol{D}|\boldsymbol{\Theta}, H) \Pr(\boldsymbol{\Theta}|H)}{\Pr(\boldsymbol{D}|H)} (3.21)$$

And, $\Pr(\boldsymbol{D}|H) = \int \Pr(\boldsymbol{D}|\boldsymbol{\Theta}, H) \Pr(\boldsymbol{\Theta}|H) d\boldsymbol{\Theta}$ (3.22)

where Θ represents the tensor of uncertain parameters that is to be estimated, **D** represents the tensor of the observations or measurement data to calibrate or estimate our knowledge of Θ , and H represents the model or hypothesis which is believed to best represent the available **D**. The terms details expressed in Equation(3.21) are as such:

• $Pr(\Theta|H) \equiv \pi(\Theta)$ is the prior distribution which describes our prior knowledge of θ before any available D,

• $Pr(\mathbf{D}|\mathbf{\Theta}, H) \equiv L(\mathbf{\Theta})$ is the likelihood function which represents the degree of similarities between D and H,

• $Pr(\Theta|\mathbf{D}, H) \equiv P(\Theta)$ is the posterior probability distribution of the parameters, which describes our updated information of θ after the information gained by D,

• $Pr(\mathbf{D}|H) \equiv Z$ is the Bayesian evidence which serves as the normalizing constant of the posterior.

In the estimation process, the Bayesian evidence factor is usually ignored since it is independent of the parameters Θ , and inferences are obtained by computing or sampling the unnormalized posterior:

 $P(\theta|D,H) \propto Pr(D|\theta,H) \cdot Pr(\theta|H)$ (3.23)

Details of the above terms can be found in [348].

3.3 Bayesian calibration

A model calibration procedure can be described statistically as [348]. By explicitly taking into account parameter uncertainties, model discrepancy, and observation error, the proposed method above employs Bayesian inference to model the relationship between the output of the computer simulations and observation data y.

$$y(x) = \eta(x, t^*) + \delta(x) + \epsilon (3.24)$$

Where y(x) and $\eta(x, t^*)$ are the observation data and simulation output, respectively. $\delta(x)$ is the discrepancy/bias term which accounts for model inadequacy between simulation and physical system at input condition x. Inadequate or missing physics, as well as numerical errors in the code, could be the cause of this inadequacy. ϵ describes observational data variation, and it is often assumed to have a Gaussian distribution. And *t** represents the true but unknown values of the calibration parameters *t*.

3.4 Advanced sampling methods

Because the analytical solution of $Pr(\Theta|\mathbf{D}, H)$ may not be easily reached given the high dimensionality of this problem , advanced sampling techniques have been developed to draw samples from unnormalized distributions, such as MCMC[349], SMC[350], X-TMCMC[351].

The Bayesian evidence Z cancels out of the computation for producing a single sample for various MCMC schemes, including Gibbs sampling and Metropolis-Hastings in general. Data generation, information extraction, and machine vision are a few of the many successful applications where MCMC has been employed. A complete description of MCMC method is beyond the scope of this thesis. Instead, interested readers should refer to classic references such as [352]. Some main concepts are summarized below. MCMC algorithm constructs an ergodic Markov chain whose posterior distribution is stationary and simulates stochastic samples from the Markov chain. This mechanism ensures that the number of samples will change with the density function value of the target distribution. Markov chain in discrete finite state space can be defined as: a random process x (i) has only finite s values $x^{(i)} \in X = \{x_1, x_2, ..., x_s\}$. If the random process satisfies

$$p(\boldsymbol{x}^{(i)}|\boldsymbol{x}^{(i-1)},\ldots,\boldsymbol{x}^{(1)}) = T(\boldsymbol{x}^{(i)}|\boldsymbol{x}^{(i-1)}),$$

(3.25)

The process is called the Markov chain. If any i satisfies, The Markov chain is said to be homogeneous. In other words, the growth of a chain in space depends only on the current state of the chain and a fixed transfer matrix.

When the Markov chain exhibits two different types of convergence, it is said to be ergodic under the circumstances. A law of large numbers convergence is the first interest:

$$\lim_{t \to \infty} \frac{1}{t} \sum f(s_t) = \int_{s \in \mathcal{X}} f(s) \pi(s) ds$$
(3.26)

where the S_t are empirical samples from the chain.

As long as the random transition matrix T satisfies irreducibility and aperiodicity, the Markov chain will converge to the invariant distribution p(x), which is the second type of convergence. The detailed balance condition is a sufficient and unnecessary condition for p(x) to be an ideal invariant distribution.

$$p(\boldsymbol{x}^{(i)})T(\boldsymbol{x}^{(i-1)}|\boldsymbol{x}^{i}) = p(\boldsymbol{x}^{(i-1)})T(\boldsymbol{x}^{i}|\boldsymbol{x}^{(i-1)}).$$
(3.27)

Sum x $^{(i-1)}$ on both sides simultaneously, we have

$$p(\boldsymbol{x}^{(i)}) = \sum_{(i-1)} p(\boldsymbol{x}^{(i-1)}) T(\boldsymbol{x}^{(i)} | \boldsymbol{x}^{(i-1)}).$$
(3.28)

MCMC sampler is an irreducible, aperiodic Markov chain representing the target distribution as invariant. One of the ways to design such a sampler is to ensure that the detailed balance conditions are met.

According to the standard theory of Markov chains, the KL divergence monotonically decreases during the Markov transition[353], that is,

$$D_{KL}[q_t||p] \le D_{KL}[q_{t-1}||p] \quad (3.29)$$

Where n intractable posterior distribution p with a sampler q, a distribution from which we can draw exact samples. Therefore, qt converges to the stationary distribution p as $t \rightarrow \infty$ under proper conditions.

Metropolis-Hastings algorithms [354] are a class of MCMC methods and given some proposal distribution $q(\gamma * | \gamma)$, the Metropolis-Hastings algorithm accepts the proposed γ * with probability

$$r_{mh}(\boldsymbol{\gamma}, \boldsymbol{\gamma}^*) = \min\left\{1, \frac{\pi(\boldsymbol{\gamma}^*)q(\boldsymbol{\gamma}|\boldsymbol{\gamma}^*)}{\pi(\boldsymbol{\gamma})q(\boldsymbol{\gamma}^*|\boldsymbol{\gamma})}\right\},$$
(3.30)

and otherwise remains in the old state γ . This will generate a Markov chain which will have π as stationary distribution. Generally, t is not very difficult to design a sampler that meets the condition of detailed balance, but the convergence speed of different samplers varies greatly. It is not easy to design a sampler with a fast convergence speed. Here, we provide a more detailed algorithm of MCMC (Metropolis-Hastings):

Algorithm 3.1 MCMC(Metropolis-Hastings)

```
1 Initialize: x^{(0)}

2 for i = 0 to N-1:

3 1 u \sim U(0,1)

X^* \sim q(x^*|x^i)

if u < \min(1, \frac{\pi(x^*)q(x|x^*)}{\pi(x)q(x^*|x)})

x^{(i+1)} = x^*

else:

x^{(i+1)} = x^{(i)}

4 end

5 end
```

3.5 Variational Inference

Markov chain Monte Carlo sampling is usually unbiased, so convergence to true posterior can be achieved in infinite samples, but convergence could be very slow. Variational inference is a widely used method of indirect approximation.[355] To do this, it minimizes the Kullback-Leible divergence of approximate and posterior distributions. This method avoids the calculation of the difficult normalization constant and only needs to know the joint distribution of the observable variable \mathbf{x} and the latent variable \mathbf{z} . Compared to stochastic sampling methods, variational inference, proposed by Euler, Lagrange, and other scholars when studying variational calculation, is a kind of computational variable posterior optimization method for large data or very complex models. The basic idea is to transform the inference problem into an optimization problem by choosing a distribution q(x) from some easy-to-handle distribution families so that it approximates the true posterior distribution as close as possible. This section will focus on variational inference.

3.5.1 Inference and Optimization

Suppose a Bayesian model is known, and the prior information of all parameters in

the model is known, that is, the prior probability distribution. However, there are hidden variables in the model in addition to parameters. In the following introduction, Z is used to represent the collection of hidden variables and model parameters. Correspondingly, X is used to represent the observed variables of the model, and λ is used to represent the variational parameters, such as X = {x1, x2, ..., xn}, Z = {z1, z2, ..., zn}, λ = { λ 1, λ 2, ..., λ N}. The goal of variational inference is to find the easily manageable distribution q(Z) and approximate the posterior distribution p (Z| X), also calculate the marginal probability or model evidence p (X) of the observed variable. Thus the inference problem turns into an optimization problem that minimizes the distance measure between the variational distribution and the posterior distribution, and finally, the optimized variational distribution q(z, λ^*) can be used on behalf of a posterior distribution.

Regarding the distance measure between two probability distributions, we generally call it divergence, and in practice, there are many divergence measures, such as Jensen divergence [356]; the most frequently used is the Kullback-Leibler(KL) divergence [357]. It is applied to approximate inference and plays an important role in machine learning, statistics, and information theory, also known as relative entropy and information gain.

Definition 3.3 For the probability distributions p(z) and q(z), the KL divergence distance between them is defined as:

$$D_{KL}(q(z)||p(z)) = -\int q(z)\log \frac{p(z)}{q(z)}dz$$
 (3.31)

It is worth noting that it can be seen from the equation (3.31), KL divergence is an asymmetric definition of divergence distance: $D_{KL}(q(z)||p(z)) \neq D_{KL}(p(z)||q(z))$. In addition, the KL divergence value formula is a non-negative value; that is, $D_{KL}(q(z)||p(z)) \ge 0$ and $D_{KL}(q(z)||p(z)) = 0$ if and only if q = p. In practice, the variational inference is usually to minimize $D_{KL}(q(z)||p(z|x))$, and there are also some methods to minimize the Reversed KL Divergence [358], which is $D_{KL}(p(z|x)||q(z))$ [359].

Moreover, as shown in the equation (3.21) evidence term on the denominator in posterior distribution expressions tends to be intractable, so the divergence of variational and posterior distributions is difficult to minimize in variational inference. To deal with this problem, we can often transform the problem of minimizing the divergence of variational and posterior distributions into the problem of maximizing the lower bound of logarithmic likelihood p(x), which we will discuss in the next section.

3.5.2 Variational lower bound

In the previous section, we discussed that variational inference requires the variational distribution q(z) to be as close as possible to the posterior distribution p(z|x), usually by minimizing the KL divergence $D_{KL}(q(z|x)||p(z))$ of these two distributions. So ideally, the variational distribution can be exactly the same as the posterior distribution p(z|x) = q(z). However, because parameters often limit the expressiveness of variational distributions, it is difficult to capture some complex high-dimensional or nonlinear properties of posterior distributions. In order to minimize KL divergence as much as possible, one of the biggest problems we have mentioned in the previous section is the evidence term and "model conditional" p(z|x) are very hard to obtain. It can be approximated by Monte Carlo such as MCMC, but it is not efficient. Thus, we can convert the minimized KL divergence into maximized log-likelihood variational lower bound problem, namely ELBO (Evidence Lower Bound Objective). ELBO is

the variational lower bound of the logarithmic likelihood log p(x). Since ELBO is a conservative estimate of the marginal distribution, it can also be used to represent whether the data distribution p(x) is well-fitted to the model.

Definition 3.2. With Jensen inequality, we can derive the variational lower bound ELBO from the log-likelihood log p(x):

$$\log p(x) = \log \int p(x, z) dz$$

$$= \log \int \frac{p(x, z)q(z; \lambda|x)}{q(z; \lambda|x)} dz$$

$$= \log \mathbb{E}_{q(z;\lambda)} \left[\frac{p(x, \lambda)}{q(z; \lambda|x)} \right]$$

$$\geq \mathbb{E}_{q(z;\lambda)} \log \left[\frac{p(x, z)}{q(z; \lambda|x)} \right]$$
 by Jensen's inequality

$$= \mathbb{E}_{q(z;\lambda)} [\log (p(x, z)] - \mathbb{E}_{q(z;\lambda)} [\log (p(z; \lambda|x)]$$

$$= \text{ELBO}(q)$$

(3.32)

It can be seen that for observational data, the distance measured between its logarithmic marginal distribution and variational lower bound ELBO is the KL divergence distance between variational distributions:

$$\log p(x) = \log(\frac{p(x,z)}{p(z|x)})$$

$$= \log(p(x,z) - \log(p(z|x))$$

$$= \log(p(x,z) - q(z;\lambda)) - \log(p(z|x) - q(z;\lambda)) \quad (3.33)$$

$$= \log(\frac{p(z,x)}{q(z;\lambda)}) - \log(\frac{p(z|x)}{q(z;\lambda)})$$

Now, let's taking the expectation on both sides, given $q_{\phi}(z)$:

$$\log p(x) = \int q(z;\lambda) \log\left(\frac{p(z,x)}{q(z;\lambda)}\right) dz - \int q(z;\lambda) \log\left(\frac{p(z|x)}{q(z;\lambda)}\right) dz$$
$$= \int q(z;\lambda) \log\left(\frac{p(z,x)}{q(z;\lambda)}\right) dz + \int q(z;\lambda) \log\left(\frac{q(z;\lambda)}{p(z|x)}\right) dz$$
(3.34)

$$= \text{ELBO}(q) + D_{kl}(q(z;\lambda)||p(x,z))$$

Thus, this proves that maximizing the variational lower bound ELBO is equivalent to minimizing the $D_{KL}(q(z|x)||p(z))$. In traditional variational inference methods, explicitly calculating the ELBO to solve for the expected value in equation (3.34) requires the variable to be directly conjugated [360], while in some newer methods, it is not required [361].

For ease of understanding, it can be intuitively seen in the figure 3.3 below that the logarithmic marginal or evidence probability lnp(X) is constant and does not change with the change of the approximate distribution q(Z). Therefore, the variation of the variational lower bound L(q) and KL divergence KL(q||p) must be inversely proportional; that is, maximizing the variational lower bound L(q) is equivalent to minimizing KL divergence KL (q|| p). In summary, KL divergence KL(q||p) is directly minimized p) is difficult to implement because it contains the true posterior distribution p (z|x), which cannot be calculated, and the joint probability p (x, z) contained in the variational lower bound ELBO L(q) is relatively easier to express. Therefore, combined with the nature of the logarithmic marginal probability decomposition of the observed variable, the divergence KL(q||p) is indirectly completed by maximizing the ELBO L(q) to find the ideal approximate distribution q(Z).



Figure 3.3 Logarithmic marginal probability decomposition

In variational inference, it is particularly important to obtain a simple and explicitly expressible variational distribution q(z) and be expressive enough to approximate the posterior distribution. A typical approach is to choose a decomposable distribution, known as the Mean-Field Distribution.[362] The mean field distribution assumes that all hidden variables are independent, which makes calculations easier but also introduces the problem of inaccurate estimation, especially when there are strong dependencies between hidden variables. Regarding mean field variational inference, we continue in this section.

3.5.3 Mean Field Variational Inference

Mean field variational inference originally derived from the concept of mean field in physics [363] and is defined as follows:

Definition 3.3. Mean field variational inference assumes that all variables in the variational distribution are independent and, therefore, for variational distributions $q(z; \lambda)$, that is:

$$q(\mathbf{z};\lambda) = \prod_{i=1}^{N} q(z_i;\lambda_i)$$
(3.35)

where z_i is one of the hidden variables in the model, and λi is its variational parameter. Now let's move on to the expression of ELBO under the mean field assumption, where ELBO can gradually maximize a completely decomposable variational distribution by iterative updates between hidden variables. For simplicity, the variational coefficient λ is omitted below. Here we illustrate the jth hidden variable and its parameters:

Definition 2.4. For the jth implicit variable, put the mean field expression in equation

(3.36) into equation () to obtain the ELBO expression under the mean field:

$$\begin{aligned} \text{ELBO}(q) &= \int q_{\phi}(z) \log\left(\frac{p(x,z)}{q_{\phi}(z)}\right) \mathrm{d}z \\ &= \int q_{\phi}(z) \log(p(x,z)) \mathrm{d}z - \int q_{\phi}(z) \log(q_{\phi}(z)) \mathrm{d}z \\ &= \underbrace{\int \prod_{i=1}^{M} q_{i}(z_{i}) \log\left(p(\mathbf{x},\mathbf{z})\right) \mathrm{d}\mathbf{z}}_{\text{part}(1)} - \underbrace{\int \prod_{i=1}^{M} q_{i}(z_{i}) \sum_{i=1}^{M} \log\left(q_{i}(z_{i})\right) \mathrm{d}\mathbf{z}}_{\text{part}(2)} \end{aligned}$$
(3.36)

Simplification of (Part 1):

$$(\text{Part 1}) = \int \prod_{i=1}^{M} q_i(z_i) \log (p(\mathbf{x}, \mathbf{z})) d\mathbf{z}$$
$$= \int_{Z_1} \int_{Z_2} \dots \int_{Z_M} \prod_{i=1}^{M} q_i(z_i) \log (p(\mathbf{x}, \mathbf{z})) dz_1, dz_2, \dots dz_M$$
(3.37)

Rearrange the expression by taking a particular qj (zj) out of the integral. Note that unlike (Part2), we are not treating any terms to const:

$$(\text{Part 1})_{q_j} \equiv (\text{Part 1})$$

$$= \int_{z_j} q_j(z_j) \left(\int_{Z_{i\neq j}} \cdots \int_{i\neq j} \prod_{i\neq j}^M q_i(z_i) \log \left(p(\mathbf{x}, \mathbf{z}) \right) \prod_{i\neq j}^M dz_i \right) dz_j$$

$$= \int_{z_j} q_j(z_j) \left(\int_{Z_{i\neq j}} \cdots \int_{Z_{i\neq j}} \log \left(p(\mathbf{x}, \mathbf{z}) \right) \prod_{i\neq j}^M q_i(z_i) dz_i \right) dz_j$$
(3.38)

Or even more meaningfully, it can be put into an expectation function, and since $\prod_{i\neq j}^{M} q_i(z_i)$ is a joint probability density

$$(\text{Part 1})_{q_j} = \int_{z_j} q_j(z_j) \left[\mathbb{E}_{i \neq j} \left[\log \left(p(\mathbf{x}, \mathbf{z}) \right) \right] \right] \mathrm{d}z_j$$
(3.39)

Note that one may consider:

$$\log(\tilde{p}_j(\mathbf{x}, \mathbf{z})) \equiv \mathbb{E}_{i \neq j} \left[\log \left(p(\mathbf{x}, \mathbf{z}) \right) \right]_{. (3.40)}$$
Obviously,

$$\tilde{p}_{j}(\mathbf{x}, \mathbf{z}) \neq p(z_{j} | \mathbf{x})
\neq q(z_{j} | \mathbf{x})$$
(3.41)

And we have:

$$\tilde{p}_j(\mathbf{x}, \mathbf{z}) = \exp\left(\mathbb{E}_{i \neq j} \left[\log\left(p(\mathbf{x}, \mathbf{z})\right)\right]\right)$$
(3.42)

Simplification of (Part 2)

(Part 2) =
$$\int \prod_{i=1}^{M} q_i(z_i) \sum_{i=1}^{M} \log(q_i(z_i)) d\mathbf{z}$$
(3.43)

Note that the above needs to integrate out all $z = \{z1, ..., zM\}$, which is quite daunting. However, notice that each term in the sum $\sum_{i=1}^{M} \log (q_i(z_i))$ involves only a single i; therefore, we are able to simplify the above into the following:

(Part 2) =
$$\sum_{i=1}^{M} \left(\int_{z_i} q_1(z_i) \log(q_i(z_i)) dz_i \right)$$
 (3.44)

For a particular pj (zj), the rest of the sum can be treated like a constant, therefore for $P_i(z_i)$ can be written as:

$$(\text{Part 2})_{q_j} = \int_{z_j} q_i(z_i) \log (q_i(z_i)) dz_j + \text{const.}$$
(3.45)

where const. is the term that does not involve \boldsymbol{z}_j .

Putting Part (1) and Part (2) together

$$ELBO(q_j) = Part (1)_{q_j} - Part (2)_{q_j}$$
$$= \int_{z_j} q_j(z_j) \mathbb{E}_{i \neq j} \Big[\log \left(p(\mathbf{x}, \mathbf{z}) \right) \Big] dz_j - \int_{z_j} q_j(z_j) \log \left(q_j(z_j) \right) dz_j + \text{const.}$$
(3.46)

Here the key is to realize is that we do not need to take derivative as one would normally do. All we need is to re-arrange the terms and realize it's the KL term, so we can just math the two distributions.

Note that $\mathbb{E}_{i\neq j}[\log(p(x, z))]$ would be some log probability of z, we name it $\log(\tilde{p}(x, z))$:

$$\log(\tilde{p}(\mathbf{x}, \mathbf{z})) = \mathbb{E}_{i \neq j} \left[\log \left(p(\mathbf{x}, \mathbf{z}) \right) \right]$$
(3.47)

Or equivalently as:

$$ELBO(q) = \int_{z_j} q_j(z_j) \log\left[\frac{\tilde{p}(\mathbf{x}, \mathbf{z})}{q_i(z_i)}\right] + \text{const..}$$
$$= -\mathbb{KL}\left(\mathbb{E}_{i \neq j}\left[\log\left(p(\mathbf{x}, \mathbf{z})\right)\right] \|q_i(z_i)\right)$$
(3.48)

Now this is the key: We can maximize ELBO(q), by minimizing the KL divergence, where we can find approximate and optimal $q^*(z_i)$, such that:

$$\log (q_i^*(z_i)) = \log(\tilde{p}(\mathbf{x}, \mathbf{z}))$$
$$= \mathbb{E}_{i \neq j} [\log (p(\mathbf{x}, \mathbf{z}))]$$
$$\implies q_i^*(z_i)) = \exp \left(\mathbb{E}_{i \neq j} [\log (p(\mathbf{x}, \mathbf{z}))] \right)$$
(3.49)

It can be seen that we can iteratively update each hidden variable until it converges, similar to that of the Variational Message Passing Algorithm[364]. However, equation (3.49) does not give an explicit solution because the expression on the right side of optimizing $q_j^*(z_j)$ depends on expectations for other factor $q_i(z_j)$ calculation. So in practice, after initializing the factor $q_i(z_i)$, the current factor is replaced with a modified estimate each time when iterating. In addition, because the lower bound is a convex function for each factor $q_i(z_i)$, the convergence of the algorithm when maximizing the variational lower bound is guaranteed [345]. Here below is given the mean field variational inference algorithm and a imple illustration of varational

inference process in Figure 3.4.

Algorithm 3.2 Mean Field variational inference (MFVI)

Input:

A joint probability distrtibution model $p(\mathbf{x}, \mathbf{z})$, a data set \mathbf{x} **Output:** A variational density $q(\mathbf{z}) = \prod_{\{j=1\}}^{m} q_j(z_j)$ **Initialize:** Variational factors $q_{j(z_j)}$ **while** the ELBO has not converged **do for** $j \in \{1, ..., m\}$ **do** $\left| \begin{array}{c} \text{for } j \in \{1, ..., m\} \text{ do} \\ 2 & | \operatorname{Set} q_j(z_j) \propto \exp \{\mathbb{E}_{-j}[\log p(z_j | \mathbf{z}_{-j}, \mathbf{x}]\} \\ \text{end} \\ 6 & | \operatorname{Compute} \operatorname{ELBO}(q) = \mathbb{E}[\log p(\mathbf{z}, \mathbf{x})] - \mathbb{E}[\log q(z)] \\ 7 & \text{end} \\ 8 & \operatorname{Return} q(\mathbf{z}) \end{array} \right|$



Figure 3.4 A simple example of variational inference [345].

The mean μ of a monary Gaussian distribution and precision τ is given in the Figure 3.4. True posterior probability distribution $p(\mu, \tau \mid D)$ Indicated by a green curve. (a) The initial decomposition approximates $q_u(\mu)q_\tau(\tau)$, represented by a blue curve. (b) Results after re-evaluation of factor $q_u(\mu)$. (c) The result after the factor $q_\tau(\tau)$, has been reassessed. (d) Contour lines of the optimal decomposition approximation, where iterative methods converge, are indicated in red[345].

3.5.4 Extension of variational inference

In the previous chapter, we introduced the definition of variational inference, the lower bounds of variational, and the traditional mean field variational inference. In this section, we will discuss extended methods of variational inference in depth.

3.5.4.1 Stochastic variational inference

With the popularity of large data sets, Bayesian algorithms need to face more computational problems caused by large data sets, so Scalable Variational Inference came into being, and in this section, we will introduce this variational inference method. Among them, the most representative is Stochastic Variational Inference, assuming that the hidden variable $z = \{\varepsilon, \theta\}$ in the model is divided into the global hidden variable θ and the local hidden variable ε . Similarly, the corresponding variational argument is $\lambda = \{\gamma, \phi\}$, where γ corresponds to the global hidden variable θ , and ϕ corresponds to the local hidden variable ε . In addition, these hidden variables in the model are controlled by a Hyperparameter α , which has a generation relationship for the observed variable X, where N is the number of observed samples. For most models, variational inference requires the overall consideration of all N samples in order to fully learn from the information in the dataset. For large data sets, such problems can be effectively solved with stochastic optimization methods [refer]. Stochastic variational inference begins by putting variational inference into the conditions of stochastic optimization, and in the paper of M. D. Hoffman et al. [365], they show how to perform random variational inference in the exponential family distribution of conditional conjugation:

$$\mathcal{L} = \mathbb{E}_q[\log p(\theta|\alpha) - \log q(\theta|\gamma)] + \sum_{i=1}^N \mathbb{E}_q[\log p(\epsilon_i|\theta) + \log p(x_i|\epsilon_i, \theta) - \log q(\epsilon_i|\phi_i)]$$

(3.50)

Now let's assume that there is a variational distribution; for the conjugated exponential family distribution, the expectation in equation (3.50) can be explicitly calculated; unlike the traditional variational method that requires N inferences per iteration, random variational inference randomly selects a mini-batch sample of size S for calculation, so that the lower bound of variational ELBO under random conditions is:

$$\mathcal{L} = \mathbb{E}_q[\log p(\theta|\alpha) - \log q(\theta|\gamma)] + \frac{N}{S} \sum_{s=1}^{S} \mathbb{E}_q[\log p(\epsilon_{i_s}|\theta) + \log p(x_{i_s}|\epsilon_{i_s}, \theta) - \log q(\epsilon_{i_s}|\phi_{i_s})]$$
(3.51)

Here s is the subscript of variables in mini batches, and such variational lower bounds can be optimized using stochastic gradient descent [366]. For stochastic variational optimization, it is widely used and inspired a lot of related work, such as the variational auto-encoder VAE (VAE) [367]. In recent years, many applications of variational inference have been inseparable from the idea of stochastic variational inference.

3.5.4.2 Non-Conjugate (Black Box) Variational Inference

In the previous discussion, we mostly discussed cases where the distribution between variables in hidden variable models is conjugated, and in this section, we discussed how variational inference applies to these scenarios in a broader context. In order to adapt variational inference to a wider range of scenarios, on the one hand, it is necessary to make variational inference applicable to non-conjugate cases; On the other hand, variational inference needs to be made more automated to avoid constraints on model calculations.

In traditional variational inference, the variational lower bound ELBO is usually explicit, which can be directly calculated and optimized, but at the same time, it also constrains the design of the model; that is, the variables in the model must be conditionally conjugated to the exponential family distribution. For most models, including some complex Bayesian models, variational lower bound ELBO typically includes unsolvable or difficult-to-solve expectation calculations. In this context, Black Box Variational Inference has been proposed as a more general inference method, for which we only need to determine the generation process of the observed variable without limiting that the probability distribution of the intermediate variable must be conjugated.

Variational inference is usually the need to maximize the variational lower bound ELBO, which is equivalent to minimizing the KL divergence between the variational and posterior distributions. To maximize ELBO, updates need to be made based on the gradient or stochastic gradient of the variational parameters. The core of the blackbox variational inference is that it is possible to obtain an unbiased estimate of the variational parameter gradient by sampling from the variational distribution without explicitly calculating the ELBO [368,369].

For general models, the gradient of ELBO can be expressed as the expectation of variational distribution:

$$\nabla_{\lambda} L = \mathbb{E}_{q} [\nabla_{\lambda} logq(z|\lambda) (\log p(x,z) - logq(z|\lambda))] \quad (3.52)$$

For gradients ∇_{λ} L, we can also sample from the variational distribution by stochastic gradient calculation and then perform gradient estimation:

$$\nabla_{\lambda} \hat{L}_{stoch} = \frac{1}{K} \sum_{k=1}^{K} \nabla_{\lambda} logq(z_k | \lambda) (\log p(x, z_k) - logq(z_k | \lambda)) (3.53)$$

Here $z_k \sim q(z|\lambda)$. In this way, the black-box variational inference method provides a black-box gradient estimation method. At the same time, this method does not need to explicitly calculate the gradient of ELBO, but can sample K samples from the

observed and hidden variables for calculation. The specific calculation method can be implemented by many deep learning methods, the specific details of which are described in the reference paper on reinforcement learning algorithms [370]. This idea was applied in this study of non-structural suspension system with Bayesian variational inference.

The above is the introduction and analysis of variational inference, and then we will discuss the part of the model selection.

3.6 Design of Experiments

In general, experimental design can be divided into two categories: model unknown and model known [371]. This section describes each of these two categories from the perspective of experimental design involved in numerical calculations.

3.6.1 Model-unknown experimental design

Monte Carlo [372] (MC) method and improved method are widely accepted in various research fields [refer] when the model is unknown. The MC method mainly has two characteristics: first, the mean convergence speed of the method is O(1/N^(1/2)) (N is the number of samples/experimental design), which does not depend on the model and does not change with the increase of the dimension of the input parameter variables; Second, the MC method provides a stable solution for unconventional distributions (non-uniform distribution, normal distribution, etc.) and extremely nonlinear and discontinuous models. These two characteristics determine the wide applicability of MC, especially in reliability analysis (small-probability event evaluation), and MC method combined with the surrogate model is a very reliable method [373-377]. However, the MC method also has great limitations: simple random sampling in the experimental space is inefficient, and for complex systems,

because a large sample size is often required to obtain results that meet the accuracy requirements, the method does not meet the requirements of practical application for some engineering problems with a large amount of calculation.

Many experimental design methods have been proposed to reduce the number of samples required by the MC method, and the basic idea is to improve sampling efficiency by dividing the test space in a targeted manner and selecting representative sample points within each division area. Commonly used methods include: the stratified sampling method [378] (Stratified Sampling), which divides the input variable into several layers according to a certain feature or value range and then randomly samples from within each layer; Latin Hypercube Sampling (LHS)[371], which is in fact the application of the hierarchical sampling method in multiparameter situations, in which the area range of each layer is determined by equal probability division according to the probability distribution of the input variable; Importance sampling method [379] (Importance Sampling), that is, within a limited number of samples, the sampling points cover the points that have a great impact on uncertainty through scale transformation, so it is beneficial to adjust the sample weight to improve the calculation speed.

The quasi-Monte Carlo method [380,381] (quasi–Monte Carlo) differs from the above method in that it uses a quasi-random sequence (a low-variance column, a determinetically generated super-uniform distribution column) instead of random numbers for Monte Carlo simulation, so it is actually a deterministic sampling method. Uniform Design [382] (Uniform Design) is a typical application of quasi-Monte Carlo, which fully considers the "uniform dispersion" of the test points within the test range so that the selected sample points can fully fill the test space. Commonly used quasi-

random sequences to generate quasi-Monte Carlo samples include Sobol sequences [383] and Holton sequences [384]. The mean convergence velocity of quasi-Monte Carlo is $O((\log N)^d/N)$ (where d is the dimension of the random variable), and its asymptotic convergence speed is better than that of MC and LHS, but it deteriorates with the increase of dimension d, so it also has certain limitations. Order (x1, x2) ~ $U([0, 1]^2)$,



Figure 3.5 Schematic diagram of Monte Carlo and its improved method Figure 3.5 selects the above methods for comparison, and apparently, samples generated by Monte Carlo (based on the Sobol sequence) are more evenly distributed.

3.6.2 Model-known experimental design

The random configuration method originated from numerical calculation methods, and it is known that some special nodes have high algebraic precision (for example, polynomial interpolation based on Chebyshev nodes can approximate the Runge function well), so the corresponding experimental design can be constructed based on the tensor product of such high-precision interpolation points [385]. Thus, in the PCE model, the experimental design produced by the random configuration method is a tensor product of one-dimensional Gaussian integral [386,387] (Gaussian Quadrature) nodes. However, the use of tensor products inevitably leads to the problem of "curse of dimensionality". In order to cope with this situation, sparse nodes in high-dimensional numerical integrals have been introduced into the experimental design of UQ, and commonly used sparse mesh construction criteria include Smolyak's criterion [388], sparse Gauss-Hermite criterion [389], Kronrod–Patterson criterion [390] and so on. These sparse nodes have been widely used because of their high algebraic accuracy and significantly reduced number of nodes in high-dimensional cases. In fact, these guidelines all aim to represent high-dimensional nodes using low-dimensional integral nodes while updating the weights of related nodes, which are still essentially tensor methods. Figure 3.6 depicts the Gaussian and Smolyak points in two dimensions.



Figure 3.6 Comparison based on tensor product and Smolyak's criterion

In the study of non-structural systems, I will use LHS for sampling, and the samples produced by Latin Hypercube Sampling (LHS) produces a more uniform distribution in the parameter space than the MC method, so it has a faster convergence speed. Taking the sampling of the random vector $x \sim U([0, 1]^d)$ as an example, the LHS

method is divided into the following three steps:

Step 1: Divide [0, 1] on each dimension into N equal parts, N is the number of samples required, and construct N^d small hypercubes, written as $\{c_i\}_{|i|=1}^{N^d}$, where $i = (i_1, ..., i_d)$ is a D-dimensional indicator and $||i|| = \sum_{j=1}^d i_j$;

Step 2: Select N c_i so that the indicators i^1 , i^2 of any two small hypercubes satisfies:

$$i_{j}^{1} \neq i_{j}^{2}, j = 1, ..., d;$$

Step 3: In each selected small hypercube, a random sample is taken according to a uniform distribution, and its setting is the desired sample set.

It should be noted that although the number of constructed small hypercubes Nd increases exponentially with dimension d, the computational complexity of *Step* 2 can be reduced to $O(d^N)$ through algorithmic optimization. The convergence speed of LHS varies from problem to problem, and in the one-dimensional case, it can be shown that its convergence speed is O(1/N). Similar to the MC method, by constructing some transformation methods, the LHS samples in a uniform distribution can be mapped to the desired samples, where *Step* 1 is equivalent to averaging each dimension into N parts with equal probability.

3.7 Sensitivity analysis and model selection

3.7.1 Sensitivity analysis

Sensitivity analysis [391,392] (SA) is also an important piece of UQ, the purpose of which is to distinguish the QoI function $y(x, \theta)$, such as the mean, variance, distribution, and information entropy of the output, etc., the sensitivity of the input variable x or parameter θ , that is, to quantify the effect of perturbations of an individual or several variables/parameters on the QoI function. Sensitivity analysis is

important because it helps researchers control variables in a targeted manner and minimizes QoI uncertainty.

Sensitivity analysis methods are mainly divided into two types: local and global. A common practice for local sensitivity analysis is to solve for an approximate Taylor expansion of the model. The local sensitivity metric is defined as partial derivative (gradient) information of the QoI function on the input variable or parameter at a specific sample point. The main feature of this method is that it is convenient and accurate to calculate, but its drawback is that it needs to determine the position of derivation, and it does not directly reflect the contribution of multivariate interactions to the QoI function. Global sensitivity analysis is generally done by screening or variance decomposition. The screening method is actually a design-of-experiment based approach, such as in partial factorial design or importance sampling, where variables that have little impact on the output are usually ignored.

Therefore, in general, the sieve method cannot directly give the size of the sensitivity index, and its main purpose is to determine the variables that have a large impact on the uncertainty of the QoI function. The ANOVA-based method uses ANOVA or High Dimensional Model Representation (HDMR) to quantify the proportion of each variable (and its combination) in QoI uncertainty for QoI uncertainty (ANOVA). Unlike local sensitivity analysis methods, global sensitivity analysis considers the entire sample (parameter) space. SA is also closely related to the model selection method [393]; that is, model selection can be regarded as selecting models that meet the sensitivity threshold conditions to a certain extent.

The variance-based sensitivity analysis method under global sensitivity theory is more applicable, and many scholars also use such methods for analysis in the study of

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practical engineering problems. Sobol's variance-based sensitivity analysis has been studied in a large number of literature in recent years [394,395], and a semi-global sensitivity analysis method based on variance proposed by literature [396] has the following advantages compared with the global sensitivity analysis method based on Sobol's variance: First, in terms of calculation, Sobol's sensitivity analysis method needs to calculate the expectation of the inner layer conditions, while the semi-global sensitivity analysis method uses the mean to replace the inner condition expectation to simplify the calculation method; Secondly, it is applicable to the system input variables independently and relatedly, and can express the relationship between input variables and output responses more clearly and accurately. Finally, due to its inner conditions, it is expected that the mean will be substituted for the effect of the input variable on the high-order moment of the output response.

In the study of non-structural components, the variance-based sensitivity analysis method is used, assuming that the relationship between the input variable and the output variable is as follows:

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) \quad (3.54)$$

where $x=x_1,x_2,...,x_n$ are n input variables, variance-based global sensitivity analysis represents the system response function as a high-dimensional Fourier Haar series decomposition:

 $f(X) = F_0 + \sum_{i=1}^n f_i(X_i) + \sum_{1 \le i < j \le n} f_{ij}(X_i, X_j) + \dots + f_{1\dots n}(X_1, \dots X_n)$ (3.55) where all terms can be expressed by multiple integrals as:

$$f_{i}(X_{i}) = -f_{0} + \int_{0}^{1} \cdots \int_{0}^{1} f(X) dX_{zi}$$

$$f_{ij}(X_{i}, X_{j}) = -f_{0} - f_{i}(X_{i}) - f_{j}(X_{j}) + \int_{0}^{1} \cdots \int_{0}^{1} f(X) dX_{zij}$$
(3.56)

Express equation (3.56) as the desired form:

$$f_{0} = E(Y)$$

$$f_{i}(X_{i}) = E(Y | X_{i}) - E(Y)$$

$$f_{ij}(X_{i}, X_{j}) = E(Y | X_{i}, X_{j}) - E(Y | X_{i}) - E(Y | X_{j}) + E(Y)$$
(3.57)

where X_{-i} - the remaining variables except X_i

 X_{-ij} — the remaining variables except (X_i, X_j)

According to equation (3.57), the variance of each order can be obtained by finding the variance on both sides of the equation at the same time:

$$Var(Y) = E(Y^{2}) - (EY)^{2}, \quad Var(X_{i}) = E\left(\left(E(Y|X_{i})\right)^{2}\right) - \left(E(E(Y|X_{i}))\right)^{2}$$
$$Var(X_{i}, X_{j}) = E\left(\left(E(Y|X_{i}, X_{j})\right)^{2}\right) - \left(E\left(E(Y|X_{i}, X_{j})\right)\right)^{2} - Var(X_{i}) - Var(X_{j})$$
(3.58)

where Var(Y) is the variance of the response function, $Var(X_i)$ is the first-order variance, and $Var(X_i, X_j)$ is the second-order variance

For an input variable X_i, the total variance can be expressed as:

$$Var^{T}(X_{i}) = Var(Y) - Var(E(Y|X_{\sim i}))$$
(3.59)

The equations (4-5) and (4-6) can indicate that the iX first-order sensitivity index is:

$$S_i = \frac{Var(X_i)}{Var(Y)} (3.60)$$

X_i second-order sensitivity indicators are:

$$S_{ij} = \frac{Var(X_i, X_j)}{Var(Y)} (3.61)$$

X_i global sensitivity metrics are:

$$S_i^T = \frac{Var^T(X_i)}{Var(Y)} (3.62)$$

First-order sensitivity reflects the degree of influence of input variables on the output response variance when they act alone; the second-order sensitivity metric reflects the degree to which the interaction between the input variables X_i and X_j affects the variance of the output response. The global sensitivity metric represents the degree to which X_i affects the total output response variance.

Equations (3.61) and (3.62) show that the calculation of the difference between the second order and above includes the calculation of the inner conditional expectation and the calculation of the outer variance, and the calculation of the inner conditional expectation represents the global influence of the input variable X_i on the output response and the result changes with the change of X_i ; The calculation of the outer variance is to describe this global effect in terms of the variance statistic. However, in actual engineering analysis, a reasonable system response variance decomposition should be able to more clearly determine and distinguish which input variables have a variance to cause changes in output response variance. Therefore, Sobol' sensitivity analysis method is vague in this regard.

3.7.2 Minimum description length model selection

In short, the MDL law treats an abstract data model as a piece of code that generates that data and the model's code length is called the data's description length [397]. The basic idea of feature modeling using MDL law is to use MDL as a frugal measure of the model, and the goal of feature modeling is regarded as the most effective description method from the observation data, which can accurately restore the observation data. According to the principle of frugality, when choosing among several possible models using the MDL rule, the data is modeled by choosing the model with the shortest description length. Its simple mathematical model is:

$$MDL(x^n) = -\log f_{\theta}(x^n) + L(\theta)$$
(3.62)

Where $-\log f_{\theta}(x^n)$ is the average shortest encoding length that can be achieved by

losslessly encoding n-dimensional data x^n using model M, that is, the lower bound of entropy. $L(\theta)$ represents the complexity of the model itself. Since the absolute value of $L(\theta)$ cannot be calculated directly, MDL uses the K–L divergence between the model $f_{\theta}(x^n)$ and the real generative data $q(x^n)$ as a measure of model complexity, which is called the redundancy of the observation model, that is, the additional coding length required to encode the model when the estimated distribution $f_{\theta}(x^n)$ is used instead of the true distribution $q(x^n)$. Using the concept of Normalized Maximum Likelihood (NML), Rissanen further demonstrated that the redundancy of the observation model has the following asymptotic optimal lower bound of Minimax [398]:

$$\inf_{f_{\theta}} \sup_{\theta \in \mathbf{\Theta}} K-L(q, f_{\theta}) = \frac{k}{2} \log \frac{n}{2\pi e} + \log \int_{K} \sqrt{\det I(\theta)} \, d\theta + O(1) \quad (3.63)$$

where θ is a compact subset of the parameter space Θ , k represents the number of parameters entering the model, I(θ) is the Fisher information matrix of the parameter distribution, and an expectation defines me (θ) I_{ij} = $-\langle \left(\frac{\partial^2 \log f(D|\theta)}{\partial \theta_i \partial \theta_j}\right) \rangle_{\theta}$ over the parametric model f_{θ}(x^n). Our conclusion rigorously shows that model complexity estimates must approach the lower bound progressively. It has good generalizability [49] because it doesn't assume knowledge of the true data distribution or make specific assumptions about the observed data distribution [397]. It is worth mentioning that similar work to the MDL modeling method also includes the Minimum Message Length (MML) method proposed by Wallace et al. [399]. The computational information theory foundation of this method is consistent with the MDL law; the main difference between the two is that the MML method is based on Bayesian inference, the starting point of inference is the prior probability of the data

model, and this method is mainly used for parameter estimation. The MDL rule, on the other hand, rejects any prior assumptions about the generative model of data, so it is more widely applicable and occupies a more important position in the field of statistical modeling research [400].

3.8 Comparison between benchmark and proposed method components

	Benchmark & proposed method components	Advantage	Disadvantage
Forward problem	Original numerical simulation	Accurate; High physical explanation	Too slow and computational inefficient
	Machine learning based G.P. Surrogate model	Very Fast and highly computational efficient; Low physical explanation	Not too accurate but approximate
Forward problem sampling space	Monte Carlo Sampling	Accurate	Slow and computational inefficient
	Latin Hypercube Sampling	Computational efficient	Not too accurate but approximate
Forward problem	All prior parameters	Accurate	Slow and computational inefficient
prior consideration	Sensitivity analysis	ity is Computational efficient	Miss some parameters contributions
Inverse problem inference/calibration	MCMC or ABC	Accurate	Too slow and very long burn- in period to convergence; Hard to scale to big data
	Variational Inference	Very fast and highly computational efficient. Easy to scale to big data	Not too accurate but approximate; Requirement for the conjugate;
	DIACK-UUX	very fast and inginy	Thoi too accurate

Table 3.1 Comparison between benchmark and proposed method components

	Variational Inference	computational efficient; Easy to scale to big data; Without the requirement of the conjugate.	but approximate
Model selection	AIC/BIC/DIC	Very simple to compute	Not to consider geometric complexity of parameter space
	MDL	Consider geometric complexity of parameter space	Relative complex to compute

3.9 Framework of ML-based Data & model driven UQ of inverse problems

Following the above methodology, Figure 3.7 sums up the proposed framework of machine learning-based data and model driven UQ of inverse problems, which will be used in chapter 6 as below. The model-driven approach [32] uses finite element numerical modeling to simulate data via design of experiments, providing a large dataset to train the data-driven, machine learning based surrogate model. This approach is used to solve forward problems.



Figure 3.7 Framework of machine learning-based Data & model driven UQ of inverse problems

In contrast, the data-driven (machine learning-driven) approach [32, 33] involves

sensitivity analysis and training the surrogate model with both full-scale observations (experimental) data and simulated data to solve the inverse problem of Uncertainty Quantification (UQ) for SNS systems, which often have uncertain parameters including initial and boundary conditions, material properties, and geometry, that can vary in space or time. Our approach addresses these inverse problems, and the forward and inverse problems are complementary. In the new approach, we propose a new black box variational inference method combined with O'Hagan's Bayesian calibration framework, and embed MDL model selection to enhance the accuracy, efficiency, and robustness of both forward and inverse problems with generalization.

CHAPTER FOUR

SUSPENDED NON-STRUCTURAL SYSTEMS FULL SCALE DYNAMCIS SHAKING TABLE EXPERIMENTS

4.1 Introduction

The full-scale dynamic shaking table testing of seismic simulation can reconstruct the seismic process of structural or non-structural systems under various input waves. It is the most direct method to investigate the seismic response and failure mechanism of structural or non-structural systems. It is also an essential method for studying and evaluating the seismic performance of structural or non-structural systems [401.]. It is worth noting that, unlike the structural shaking table test in which the seismic wave is directly input on the table, the non-structural components are generally installed at a certain height of the main structure and undertake the floor seismic action than the ground motion. The strength and spectral characteristics of the floor input wave have changed compared with the ground motion strength and spectral characteristics.

Therefore, when using the shaking table test to evaluate the seismic performance of the non-structural components, it is necessary to fit the floor wave with broad representative significance as the input wave, including acceleration time history, velocity time history, and displacement time history, to reflect the actual floor seismic action characteristics of non-structural components and improve the reliability of the shaking table test of non-structural components [402]. It is worth noting that most shaking table tests for seismic simulation of non-structural members use full-scale models. Currently, the experimental research on the typical non-structural system types, such as Suspended Ceilings Systems (SCS) and Suspended Cable Trays Systems (SCTS), with super-large area, multiple types, and long period & long duration earthquake input is still lacking, and the understanding of their seismic damage mechanism is warranted. Based on the deficiencies of existing research, this dissertation designed three groups of suspended ceilings, four groups of suspended cable trays, and steel platforms for installing suspended ceilings and cable trays and conducted seismic simulation dynamics shaking table experiments on suspended ceilings and cable trays in the multi-function dynamics shaking table laboratory in Asia. The seismic damage mechanism and ceiling and cable tray performance are studied, and the effects of different structural types and boundary constraints on their seismic performance are compared. This chapter first introduces the test design, test device and specimen fabrication and installation, experiments equipment, experiments scheme, and experiments loading scheme, then analyzes the seismic damage phenomenon, then introduces the experiments data processing method, and investigates the acceleration response.

The full dynamics shaking table tests on the SCS and SCTS were conducted to investigate the working mechanism, damage mechanism, and seismic responses subjected to earthquake-induced excitation. The steel platform is used as a test carrier to hang the SCS and SCTS. Figure 4.1 shows the overall view of the platform, which has the largest area in the world until now. It has two stories with a height of 5.40 m. The plan dimensions are 12.84 m×11.64 m. The platform's longitudinal (12.84 m) and transverse sides (11.64 m) are defined as the X and Y directions, respectively. Three levels along the height of the platform, table level, ceiling level, and floor level, are

considered. Due to the fact that one shaking table is not big enough to install the specimen, two shaking tables connected together as an integral one are used in this test.



Suspended cable tray system

Fig. 4.1. Overall view of steel platform and suspended ceiling and cable tray systems.

4.2 Experimental design

The dynamic shaking table test design of non-structural seismic simulation includes the design of a steel platform and non-structural test pieces. The design of the steel platform and the design of non-structural test pieces are described in detail below.

4.2.1 Steel platform design

The steel platform is an input platform for installing non-structural test pieces and serving as the test pieces. The following factors should be considered in the design [403-409]: (1) The stiffness of the steel platform should be as large as possible so that the amplification effect of the steel platform roof on the input wave of the platform is as small as possible, and at the same time, ensure that the natural vibration frequency

of the steel platform should be far away from the natural vibration frequency of the test piece, to avoid adverse effects of resonance effects on the test piece. (2) The height of the steel platform shall be designed reasonably to facilitate the installation of test pieces and test observation. (3) The total weight of the steel platform and the test piece shall not exceed the maximum bearing capacity of the table. (4) The size of the steel platform shall be designed according to the size of the test piece and shall meet the site test conditions.

Based on the above basic requirements, the steel structure is selected to design the steel platform, and the finite element numerical analysis software is used to model and analyze it. The model diagram is shown in Figure 4.1. The structural form of the steel platform is a two-layer steel supporting frame structure made of Q235 steel, with a size of $12.84m (X) \times 11.64m(Y) \times 5.40m (Z)$, X direction is east-west direction, Y direction is north-south direction, the height of the first floor is 2.3m, the height of the second floor is 3.1m, and the total weight is 31.7t. The first three vibration modes and corresponding periods of the steel platform are shown in Figure 4.2. The design effect of the steel platform is shown in Figure 4.3, and the detailed design drawing of the steel platform is shown in Appendix B. See Table 4.1 for details of steel platform components.



(a) First mode (translation in the Y direction), T1=0.0954s



(b) Second mode (translation in the X direction), T2=0.0803s



(c) Third mode (horizontal torsion), T3=0.0694s





(a)3D graphics model



(b) Top view



(c) South elevation



(d) West Elevation

Figure 4.3 Design Drawing of Steel Platform

The edge beam and intermediate beam are designed to simulate the boundary of the suspended ceiling. Figure 4.4 shows the schematic diagram of the edge beam. Taking the edge beam between X1~X2 axes as an example, the design details of the edge beam are described. Channel steel C-160 \times 63 \times 6.5, steel bar FB-50 \times 3, steel plate PL-320 \times 200 \times 3, and angle steel L65 \times 65 \times 6 are connected with bolts to configure

the edge beam. The fixed wooden beam in the channel steel of the edge beam is used to connect with the boundary of the suspended ceiling. Figure 4.5 shows the schematic diagram of the intermediate beam, which is located in the middle of the Y2~Y3 axis. The channel steel C-160 × 63 × 6.5, square steel pipe \Box - 50 × 50 × 3, angle steel L50 × 50 × 5 are welded connected to configure intermediate beam. The intermediate beam's two ends and upper end are connected with the steel platform by welding. The internal fixed wooden beam of the channel steel of the intermediate beam is used to connect with the boundary of suspended ceilings. It should be noted that the intermediate beam is only used for experiment types A and B. Its role is to divide test pieces A or B into two parts with the same area but different boundary conditions. The intermediate beam is removed when experiment types C is conducted on the shaking table.



(a) Partial 3D sketch of edge beam



(b) Elevation of side beam between axes X1~X2



(c) Top view of edge beam between axes X1~X2



(d) Partial Detail drawing of edge beam

Figure 4.4 Design Drawing of Edge Beam (Unit: mm)



(a) Partial 3D sketch of intermediate beam



(b) Layout plan of intermediate beam

Figure 4.5 Design drawings of Intermediate Beam (Unit: mm)

Member Information	Type and size/mm	Numbers
C1	H-194×150×6×9	12
C2	H-175×175×7.5×11	4
C3	BH-340×300×12×12	4
G1	H-125×125×6.5×9	8
G2	H-446×199×8×12	8
B1	□-200×100×3.2	14
B2	□-100×100×3.2	5
KB1	□-200×200×8	8
KB2	□-150×150×6	8
KB3	2[-100×50×5×7.5	4
KB4	2[-100×50×5×7.5	4
KB5	2[-125×65×6×8	4
V1	□-120×120×4.5	14
V2	□-100×100×4.5	18

Table 4.1 Member Information of Steel Platform

.

V3	O-M18	36
V4	□-100×100×3.2	18

4.2.2 Suspended Ceiling Systems (SCS) experiments configurations design

4.2.2.1 Ceiling Types Introduction

For Suspended Ceiling Systems (SCS) with mineral wool boards around the world, there are four typical boundary conditions for the ceiling perimeter: free, fixed, fixedfree, and fixed-semi-free. The SCS with the free boundary condition (Figure 4.6a) refers to the SCS placed on the peripheral support without any attachments connected with the surroundings so that at the boundary, the SCS moves freely in the horizontal direction except the constraint due to the friction (Figure 4.7a). The SCS with the free boundary condition is generally applied in areas with light to moderate earthquake potential (Seismic Design Category C) according to current American standards [75]. This type of SCS is required to have a minimum 10 mm gap between the grid end and wall angle on all boundaries to accommodate the movement of SCS relative to the main structure during earthquakes. In some countries, including China, the SCS with free boundary conditions is used extensively in practice due to the ease of its construction. In this type of SCS, the boundary constraints on the different sides are identical. That is, the boundary constraint effect is uncoupled for this type of SCS. Since this type of SCS has weak boundary constraints, it is prone to collide with the surroundings under the earthquake, which often leads to serious damage to the ceiling perimeter.

The SCS with the fixed boundary condition (Figure 4.6 b) refers to the SCS which is fixed to the surroundings on all sides of the perimeter. Two common types of peripheral fixings currently are applied in practice, namely pop rivets and seismic clips. Two types of tests, system-level shaking table tests [291, 323] and componentlevel static tests[257], were performed to investigate the seismic performance of the fixed connections with pop rivets (Figure 4.7b). Although the pop rivet could somewhat improve the seismic capacity of the ceiling perimeter, the early failure of the pop rivet often occurs when the forces acting on it exceed the shear strength of the pop rivet. Moreover, the installation of pop rivets at the perimeter of the ceiling may lead to aesthetic problems. As an alternative solution to these issues, seismic clips are adapted to be attached at the grid ends beside wall angles. To evaluate the seismic capacity of the fixed connections with seismic clips (Figure 4.7c), extensive experimental tests, including component-level [281] and system-level [72] tests, were conducted. Compared with the connection with pop rivets, the connection with seismic clips installed with perimeter screws at the fixed side with larger strength and deformability considerably improves the seismic behavior of the ceiling perimeter. Although compared with the free boundary condition, the fixed boundary condition can significantly enhance the strength and stiffness of the ceiling perimeter, the stronger boundary constraint allows inertia forces induced in SCS to transfer and accumulate at the peripheral fixings, making these peripheral connections the most vulnerable components of SCS.

The SCS with the fixed-free boundary condition (Figure 4.6 c) refers to the SCS which is fixed to the surroundings at two adjacent sides of the perimeter and placed on the peripheral support with a minimum 19 mm gap between the grid end and wall

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angle to allow the free movement of the grid on the opposite two sides. The different boundary constraint effect is coupled in the SCS. This type of SCS, called as "American-style SCS", is applied in buildings in high seismic zones (Seismic Design Category D-F) [6.]. With the improvement of seismic clips invented by the major manufacturers in the US, the fixed-free condition is developed into fixed-semi-free boundary condition (Figure 1d), more commonly used. The semi-free side is achieved by means of seismic clips with one sliding screw attached at the middle slot to allow the grid to slide freely only along the axis of the grid (Figure 4.7d). On the semi-free side, a minimum 19 mm gap is set between the grid end and wall angle. Unfortunately, the current construction measures for fixed and semi-free connections at the boundary are inadequate; for example, the flange of the seismic clip is not firmly connected to the surroundings due to the lack of sufficient perimeter screws.



Figure 4.6 Typical boundary condition for SCS: (a) free boundary condition, (b) fixed boundary condition, (c) fixed-free boundary condition, (d) fixed-semi-free boundary condition.

Notes: _____ main tee; _____ cross tee; _____ sub cross tee; _____ wall angle; ////// surrounding wall;

◆ free connection; ■ fixed connection; ● semi-free connection.



Figure 4.7 Typical perimeter connections of SCS: (a) free connection, (b) fixed connection with pop rivet, (c) fixed connection with seismic clip, (d) semi-free connection with seismic clip.

In this dissertation a new boundary condition, i.e., semi-free boundary condition, is proposed to improve the seismic performance of double-layer SCS with mineral wool boards. The four peripheral sides of SCS are set to be semi-free by adopting seismic clips, so there is no coupling effect of boundary constraints on the four sides for this type of SCS. Moreover, the SCS with the same semi-free boundary condition at four sides has the advantages of a clearer working mechanism and simpler numerical modeling. The seismic clip is attached tightly to the surroundings by using all available perimeter screws to improve the seismic capacity of the connection with seismic clips. The semi-free boundary condition is not only to prevent the falling of the gird members near the ceiling perimeter from the support but also to release the boundary constraints to reduce the damage to the peripheral connections. Moreover, the proposed boundary condition has the advantages of convenient construction and low cost. To verify the working mechanism and investigate the seismic performance of the SCS with semi-free boundary condition, the full-scale shaking table tests are carried out in this study. Then, a simplified numerical model for SCS with semi-free conditions is proposed and verified. Finally, seismic boundary design recommendations for the type of SCS are provided.

Three types of boundary conditions of SCS are used in the experiments: (1) The first boundary condition is the free boundary, as shown in Figure 4.8. Because it is difficult to control the construction accuracy, a gap of 0~8mm will be formed between the grid end and the wall angle. According to the statistics of 60 free boundary construction samples, the average gap is 3.05mm, and the standard deviation of clearance is 2.08mm. (2) The second boundary condition is fixed-semi-free boundary, as shown in Figure 4.9. A nominal gap of 19mm is set between the grid end and wall
angles. According to the statistics of the gap between 30 construction samples, the average gap is 17.33mm, and the standard deviation of the gap is 1.86mm, as shown in Figure 4.9b. (3) The third kind of boundary condition is semi-free boundary, as shown in Figure 4.10. Specifically, the ceiling boundary is constrained by semi-free boundary. Table 4.2 shows the comparison of three ceiling boundary types.



(c) Free boundary configurations

加速度计

Figure 4.8 Free boundary



(a) Fixed connections

位移计



间隙

(b) Semi-free connections



(c) Fixed-semi-free boundary configurations

Figure 4.9 Fixed-semi-free boundary



Figure 4.10 Semi-free boundary

Table 4.2 Boundary comparison

Boundary	Seismic clip	Gap
Free	×	
Fixed-semi-free	\checkmark	Fixed side (×) Semi-free side($$)
Semi-free	\checkmark	\checkmark

4.2.2.2 Semi-free boundary condition

(a) Configuration

The proposed semi-free boundary condition is achieved by installing a semi-free boundary all around the ceiling perimeter. The SFC consists of a perimeter grid, seismic clip, wall angle, and side wall, as shown in Figure 4.11. The wall angle is fixed to the side wall using screws. The seismic clip is attached tightly to the wall angle and side wall using four perimeter screws to obtain enough bearing capacity for SFC. The perimeter grid is connected to the seismic clip only by one sliding screw in the middle slot of the seismic clip to ensure that the perimeter grid slides freely only along its axial direction while the movement perpendicular to the axial direction of the perimeter grid is prevented. In addition, a 19 mm gap is provided between the perimeter grid end and the wall angle. Considering the mechanical configuration of SFC, the seismic performance of the proposed SCS is expected to be enhanced largely because of energy dissipation by the friction mechanism between the middle slot and sliding screw along the axial direction of the perimeter grid and also by the increased flexural, shear and torsional resistance perpendicular to the axial direction of the perimeter grid after the installation of seismic clips.

Figure 4.12 shows a schematic view of the displacement mechanism along the longitudinal direction for SCSs with and without seismic clip connections. The movement of SCS with seismic clips is effectively constrained by seismic clips, and the flexural, shear, and torsional resistance of peripheral connections is improved significantly.



Fig. 4.11 Schematic of semi-free connection.



Fig. 4.12 Schematic view of displacement mechanism on grids with different peripheral connections: (a) SCS without seismic clip connections; (b) SCS with seismic clip connections.

(b) Working mechanism

Figure 4.13 shows the whole process of movement of the ceiling grid with semi-free boundary conditions, which can be divided into three stages: pre-slipping, slipping, and pounding stages. The expressions of "Gap-1" and "Gap-4" refer to the gap width of 19 mm between the grid end and the wall angle. The expressions of "Gap-2", "Gap-3", "Gap-5", and "Gap-6" refer to the gap width of 22 mm between the sliding screw and the edge of the middle slot of the seismic clip. The sliding screw is set in the middle of the slot.

(a) Pre-slipping stage: During the pre-slipping phase, the grid ends are restrained by the friction force larger than the inertial force acting on the grid. The displacement of the grid relative to the side wall is very small and close to zero, which indicates that the ceiling moves with the main structure together. The acceleration of the ceiling is close to that of the main structure. (b) Slipping stage: During the slipping stage, the grid end begins to slide because the inertial force acting on the grid is larger than the friction force. The displacement of the grid becomes greater but is smaller than the gap between the grid end and the wall angle. The movement process of the grid is described as follows. When the grid overcomes the friction force and moves to the left side, the left grid slides on the surface of the wall angle while the right grid slides to the left along the inclined middle slot of the seismic clip and simultaneously lifts off from the surface of the wall angle. No pounding occurs since the displacement of the grid end relative to the side wall is less than Gap-1. Similarly, when the grid moves to the right side, no pounding occurs. It should be noted that when the grid moves to the right side, the right grid end will return to the surface of the wall angle and slide on the wall angle.

(c) Pounding stage: During the pounding stage, the grid end collides with the wall angle because the displacement of the grid end relative to the side wall is greater than Gap-1. The grid acceleration response increases significantly, and the inertial force acting on the grid is larger than the friction force at the grid end. The movement process of the grid is described as follows. Firstly, the grid experiences the slipping stage. Then, when the grid moves to the left side after the slipping stage, the location of pounding is determined by the smallest gap among Gap-1, Gap-2, and Gap-6. In the case that Gap-1 is the smallest, the pounding occurs between the left grid end and wall angle. The pounding force acting on the grid end is significant. Similarly, when the grid moves to the right side, the pounding occurs between the right grid end and the wall angle.



Fig. 4.13Working mechanism of ceiling grid with semi-free condition: (a) pre-slipping stage, (b) slipping stage, (c) pounding stage.

4.2.2.3 Three groups of experimental configuration details.

Next, three groups of experimental configuration details. are described, as shown in Table 4.3. (1) The first group of experiments is double layer suspended ceiling A, which is a unique ceiling type in China. It consists of A1 and A2. The two parts only have different boundary constraint conditions. The boundary constraint conditions of A1 are a free boundary, A2 is a fixed semi-free boundary, and the other parts are identical, as shown in Figure 4.9. (2) The second group of test pieces is a USA-type single-layer suspended ceiling B, which is composed of B1 and B2, like the first

group of test pieces. Except for different boundary constraint conditions, the other parts are identical, as shown in Figure 4.10. (3) The third group is a double-layer suspended ceiling C, which is composed of one whole super-large area suspended ceiling. The boundary constraint condition is the full semi-free boundary (hereinafter referred to as the semi-free boundary for simplification). The area of the suspended ceiling is 150 m^2 , which is the largest one in the world. The composition of the suspended ceiling is the same as that of experiment type A, as shown in Figure 4.14. Three groups of test pieces are designed to study the influence of ceiling type and boundary constraint conditions on the seismic performance of the ceiling. The specific purposes are as follows: (1) Experiment type A is designed to study the influence of boundary constraint conditions on the seismic performance of double-layer ceiling. (2) Experiment type B is designed to study the influence of boundary constraint conditions on the seismic performance of single layer suspended ceiling. (3) The comparative design of type A and type B is to study the influence of ceiling type on seismic performance. (4) Experiment type C is designed to study the seismic performance and area effect of semi-free boundary suspended ceiling.

Туре	B	oundary	Type	$\Lambda rea/m^2$	
	Left side	Right side	_ Type	Alea III	
А	A1 (Free)	A2 (Semi-free)	Double layer	67×2=134	
В	B1 (Free)	B2 (Fixed-semi-free)	Single layer	67×2=134	
С	Se	emi-free	Double layer	150	

Table 4.3 Comprehensive information of SCS experiments



(a) Type A general information



(b) Type A details



Figure 4.14 SCS experiment type A

Type A is composed of threaded rods, carrying channels, main tee, cross tee, wall angels, sub cross tee, lay-in panels, and accessories, including hangers, hook, and seismic clips. The length of the threaded hanger is 1.0m, the diameter is 8mm, and the spacing is 1.2m. The upper end is fixed with the steel beam of the steel platform, and the lower end is connected with the carrying channels through the hanger; The carrying channels are horizontally arranged along the X direction with a spacing of 1.2m, which is directly above the cross tee; The main tee is horizontally arranged along the Y direction with a spacing of 1.2m; The cross tee is horizontally arranged

along the X direction, with a spacing of 0.6m; The sub cross tee is horizontally arranged along the Y direction with a spacing of 1.2m; The wall angles are fixed on the wooden beams around with pop rivets; The lay-in panel is directly placed in the grid system.

The difference between experiment B and experiment A is as follows: there is no carrying channels and hanger in experiment B, the threaded rods are directly connected with the main tee through the hanger, and the type and arrangement of other components are identical to that of experiment type A. The components and arrangement of experiment types C and A are identical. However, type C, it is with a whole plane area of 150 m². The ceiling is also comprised of threaded rods, carrying channels, a grid system, lay-in panels, and accessories. Threaded rods hung from the bottom of the floor level are the load-carrying members to hang carrying channels, the grid system, and lay-in panels. The carrying channel is mainly used to facilitate leveling the ceiling. The grid system consists of main tees, cross tees, and sub-cross tees, forming a module for placing lay-in panels. Wall angles fixed to the side wall supply vertical support for the grid system. The typical accessories include hangers hung by threaded rods to support carrying channels, hooks connecting carrying channels and main tees, and seismic clips constraining the peripheral grids to the wall angles. It is noted that the configuration of cross-sub cross tee connection is basically identical to that of main-cross tee connection. Detailed information on the ceiling components is presented in Table 1. The design height of all ceiling test pieces is 1.0m. The comprehensive information on ceiling components is shown in Table 4.4.



(a) Type B general information



(b) Type B details



(c) Type B configuration

Figure 4.15 SCS experiment type B





(b) Type C configuration Figure 4.16 SCS experiment type C

Figure 4.17 Composition of the double-layer SCS. Table 4.4. Detailed information of the ceiling components.

Component type	Section (mm)	Length (mm)	Spacing (mm)	Unit mass (kg/m)	Section area (mm ²)
Threaded rod	M8	1000	1200	0.37	40
Carrying channel	U50×15×1	4000	1200	2.24	70
Main tee	T43×24×0.54×0.27	3600	1200	1.07	38
Cross tee	T35×24×0.54×0.27	1200	600	0.31	33
Sub cross tee	T30×24×0.54×0.27	600	1200	0.13	28
Wall angle	L22×22×0.5	3000	N/A	0.53	22
Hanger	-	-	1200	-	-
Hook	-	-	1200	-	-
Seismic clip	-	-	600	-	-
Lay-in panel	600×600×16	-	/	1.20*	9472

Notes:

1. The section area of ceiling component is obtained by measured data.

2. 1.20^* for lay-in panel refers to the mass of one panel.

The boundary condition of the SCS with semi-free boundary on all sides, sides 1 to 4, is also shown in Figure 4.17. Two types of peripheral grids with the same boundary condition, i.e., main tees and sub cross tees, are attached to the wall angles by semi-free boundary on side 1. Only one type of peripheral grid, i.e., cross tees, is connected to the wall angles by semi-free boundary on side 2. The boundary condition of sides 3 and 4 is identical to that of sides 1 and 2, respectively. Table 2 shows the constraints of 6 degrees of freedom (DOF) at the boundary of the SCS.

Side No.	Constraint of DOF					
	U_X	U_{Y}	U_{Z}	$R_{\rm X}$	$R_{\rm Y}$	$R_{\rm Z}$
Side 1	1	0	1	1	1	1
Side 2	0	1	1	1	1	1
Side 3	1	0	1	1	1	1
Side 4	0	1	1	1	1	1

Table 4.5 Boundary condition of the ceiling.

Notes: U_X , U_Y , and U_Z represent the horizontal degrees of freedom along X, Y, and Z axes; R_X , R_Y , and R_Z represent the rotational degrees of freedom along X, Y, and Z axes; and 1 and 0 represent constrained and unconstrained, respectively.

4.2.3 Experiments equipment

The non-structural dynamical shaking table was conducted in the multi-function shaking table laboratory of Asia. The multi-function shaking table experimental system consists of four shaking tables A (side table 30t), B (main table 70t), C (main table 70t) and D (side table 30t) and two channels (70m and 30m in length respectively), as shown in Figure 4.17. The working mode of the multi-function shaking table test system is shown in Figure 4.18, including:

(1) Working mode 1: four tables can move in a 70m channel and merge into a large linear vibration table group. Multiple tables can work synchronously and uniformly, and several tables can perform associated movements .

(2) Working mode 2: 2 tables can be moved to a channel of 30m, 4 tables can be combined into a large rectangular vibration table group, multiple tables can work synchronously and uniformly, and several tables can make associated movements.

(3) Working mode 3: 2 tables can be combined into a large vibration table for single use.

The multi-function shaking table test system has a total test capacity of 200t, which is one of the largest and strongest shaking table test systems in the world. It provides a world leading vibration and earthquake simulation test platform for bridge engineering, housing, space structure engineering, underground structure engineering, and lifeline engineering. In order to meet the requirements of tonnage and size of this test, A and D are combined to form $10m \times Two$ 6m large worktops that can move synchronously, ensuring synchronous excitation at the bottom of the steel platform. See Table 4.6 for the performance parameters of the vibration table.



Figure 4.17 The multi-function shaking table experiments system



(a) Working moded 1

(b) Working mode 2

(c) Working mode 3

Figure 4.18 Three working mode Table 4.6 Performance parameters

Performance	Parameters					
	A, D	B、C				
Maximum bearing capacity	30t	70t				
Overturning moment	200t·m	400t·m				
Table size	6m×4m					
Motivational direction	X and Y directions					
Control degrees of freedom	3 degrees of freedom (horizontal, two-way + horizontal)					
Maximum acceleration	X, Y direction± 1.5g					
Maximum speed	X, Y direction± 1000mm/s					
Maximum displacement	X, Y direction± 500mm					
Operating frequency range	0.1~50Hz					
Input waveform	Periodic waves, ra	andom waves, etc				
Data acquisition system	288-channel NI dynamic data acquisition system					

4.3 Experimental instruments scheme

4.3.1 Experimental instruments

Three types of sensors, including acceleration sensors, displacement sensors, and strain gauges, are used to measure the dynamic response of the test object. The acceleration sensor selects Setra high output linear accelerometer to measure the absolute acceleration of the test object. See Fig. 4.19a for the photos of the accelerometer. The displacement sensor adopts wire type displacement meter to measure the displacement of the suspended ceiling test piece relative to the steel platform. See Figure 4.19b for the photo of the displacement meter. The strain gauge is a resistance strain gauge used to measure the local strain of the ceiling component. See Fig. 4.19c for the picture of the strain gauge. In addition, several motion cameras with different viewing angles are installed on the steel platform to observe and record the overall and local vibration of the suspended ceiling at a close distance, as shown in Figure 4.19d; See Figure 4.19 e for the Machine Learning-based computational vision algorithm I proposed and applied for monitoring the ceiling panel movements.



Figure 4.19 Experimental instruments

(c) strain gauges

4.3.2 Instrumentations Layout details

4.3.2.1 Layout of instruments for steel platform

Along the vertical height of the steel platform, it is divided into three height layers:

the floor layer, the non-structural systems layer, and the table layer. The bottom layer is the horizontal height of the shaking table, the non-structural systems layer is the horizontal height of the side beam, and the top layer is the horizontal height of the steel platform roof, as shown in Figure 4.20. See Figure 4.21 for the layout plan of accelerometers installed at each level of the steel platforms.



Figure 4.20 Steel platform layers



Figure 4.21 Instruments installed on the steel platform

As mentioned above, there are 3, 4, and 11 accelerometers installed on the table level, non-structural level, and floor level of the platform, respectively. The measured acceleration at table level can be regarded as the input acceleration of the steel platform. The measured acceleration at floor level can be regarded as the input acceleration of the ceiling. For example, a total of 236 instruments, including 30 accelerometers (A1-A30), 40 displacement transducers (D1-D40), and 166 strain gauges (S1-S166), are installed on the SCS type 3 to measure the dynamic responses of the ceiling. The location and number of instruments on the ceiling are shown in Figure 4.22. The hollow one-way red arrow and blue arrow refer to the positive directions of displacement and acceleration, respectively. The data measured by the accelerometer is the absolute acceleration. The data measured by the displacement transducer is the displacement of the ceiling relative to the platform. Except for lay-in panels with a strain rosette installed, two strain gauges are attached to both sides of the same position of each ceiling component, and the average of the results obtained from the two gauges is used in subsequent data analysis. Only the measurement points for subsequent analysis are marked.



(a) SCS type A



(b) SCS type B



(c) SCS type C

Fig. 4.22 Instrumentations on the ceiling.

Notes: \Leftrightarrow displacement transducer; \Leftrightarrow accelerometer; \bullet strain gauge on threaded rod; \bullet strain gauge on lay-in panel; \blacktriangle strain gauge on cross tee and sub cross tee; \Box strain gauge on carrying channel.

Figure 4.23 shows the setup details of the displacement transducer on cross tee. It should be noted that the deformation of the main-cross tee connection between the target and the side wall is included in the measured displacement of the cross tee while the measured displacement of the main tee does not include such kind of deformation. The measured displacement of the cross tee contains both the relative movement between the ceiling and side wall (equal to the sliding distance within the gap at the ceiling perimeter) and the deformation of main-cross connections. The sliding distance within the gap can be easily determined by the negative displacement

measured by the displacement transducer. Figure 4.24 shows the instruments installed on the SCS.



Figure 4.23 Setup of displacement transducer on cross tee.















Figure 4.24 Pictures of instruments installed on the SCS

4.3.3 Loading protocol

Table 4.7 lists all the motions input in the test. After each run, white noise with a PGA of 0.05 g is input to the specimen to assess the dynamic characteristics of the specimen. Several sets of motions are selected and input to the shaking table, including sweep waves (named Sweep), acceleration responses at different floors of building structures obtained by time history analysis, and artificial waves (named BCJ-L2). Figure 10 presents the sweeping wave, that is, the sine wave with the frequency varying from 6.0 Hz to 0.8 Hz. The peak floor acceleration (PFA) at the floor level of the platform is listed in Table 3. The input motions, SHW6 (5/128), SHW6 (128/128), and SHW6 (30/30) represent the acceleration responses at the 5th floor and the top of a 128-story supertall structure model and the acceleration responses at the top of a 30-story stick model subjected to the ground motion SHW6 with PGA of 0.1 g, respectively. The floor acceleration responses are closely related to

the dynamic properties of the main structure. The natural vibration periods of the first three modes of the 128-story model are 8.94 s, 8.93 s, and 4.48 s, respectively. The corresponding results of the 30-story model are 3.01s, 1.18 s, and 0.72 s, respectively. The characteristic period of the ground motion SHW6 is 0.9 s. The PFA of SHW6 (128/128) is 1.7 times that of SHW6 (5/128). The fundamental period of the 30-story model is closer to the characteristic period of the ground motion SHW6 (30/30) is larger than that of SHW6 (128/128).

Run No. ¹⁾	Name of input motion	Target acc. of table (g)		Dremetican (c)	PFA of platform (g)	
		X dir.	Y dir.	Duration(s) =	X dir.	Y dir.
2	Sweep	0.050	0	100	0.071	-
4	Sweep	0	0.050	100	-	0.087
6	Sweep	0.050	0.050	100	0.069	0.087
8	BCJ-L2	0.037	0	120	0.050	-
10	BCJ-L2	0	0.037	120	-	0.057
122)	SHW6 (5/128)	0.089	0.070	70	0.127	0.098
14 ²⁾	SHW6 (128/128)	0.149	0.132	70	0.146	0.153
16 ²⁾	SHW6 (30/30)	0.405	0.377	150	0.571	0.573
18	Sweep	0.150	0	100	0.225	-
20	Sweep	0	0.150	100	-	0.242
22	Sweep	0.150	0.150	100	0.232	0.277
24	Sweep	0.250	0	100	0.393	-
26	Sweep	0	0.250	100	-	0.512
28	Sweep	0.350	0	100	0.572	-
30	Sweep	0	0.350	100	-	1.942

Table 4.7 Details of motions input to specimen.

32	Sweep	0.500	0	100	1.319	-

Notes:

²⁾ During Runs 12 and 14 the floor acceleration responses at the 5th and 128th floors of the 128story building subjected to the ground motion SHW6 are input, and during Run 16 the floor acceleration response at the 30th floor of the 30-story building is input.

(1) Sweep wave. Sweep wave belongs to the variable frequency sine wave, the frequency range of 5~0.5Hz or 6~0.8Hz, mainly used to investigate the seismic failure mechanism of suspended ceilings. Figure 4.25 shows the acceleration time history curve of the Sweep wave with a PGA of 0.15g and a frequency range of 5~0.5Hz and the corresponding time-frequency plot and acceleration response spectrum. It should be noted that the amplitude of the front section of the acceleration time history curve of the Sweep wave is weakened to a certain extent because when the steel platform analysis model input Sweep wave of the same amplitude is pre-analyzed, it is found that the acceleration time history reaction of the steel platform roof has amplitude amplification in its front section, so in order to reduce the amplification effect, the amplitude of the Sweep wave is weakened. Due to the limitation of the capacity of the shaker, the Sweep wave with a frequency range of 6~0.8Hz is used for type A and B in PGA $\geq 0.35g$ Sweep wave and all working conditions of type C.



(a) Acceleration time history curve of the Sweep wave

¹⁾Runs of odd number used for white-noise excitation with small magnitude are not listed in the table.







(c) Acceleration response spectrum of the Sweep wave (damping ratio: 5%)

Fig.4.25. Sweep wave (Frequency range: 5~0.5Hz)

(2) BCJ-L2 wave. The BCJ-L2 wave is an artificial wave proposed by the Building Center of Japan (BCJ) and the Building Research Institute (BRI). Figure 4.26 shows the acceleration time history curve of 0.037g BCJ-L2 wave with PGA and the corresponding acceleration response spectrum.



(b) Acceleration response spectrum of BCJ-L2 wave (damping ratio: 5%)

Figure 4.26 BCJ-L2

(3) Floor waves. The natural seismic wave SHW6 given in Shanghai's "Building Seismic Design Regulations" was selected, and the peak acceleration in the X direction and Y direction was adjusted to 0.1g and 0.085g, respectively, and input into the Benchmark model of the 128-story supertall building structure and the 30-story Stick model, and the floor acceleration response of the two models was calculated. The acceleration time history of the 5th and 128th floors of the 128-layer Benchmark model and the acceleration time history of the 30th floor of the 30-layer Stick model were selected as the input waves of the specimen. The three floor waves were named SHW6 (5/128) wave, SHW6 (128/128) wave, and SHW6 (30/30) wave. Figure 4.27a shows a schematic diagram of the benchmark model of the 128-story supertall building structure, which adopts a giant frame-core-boom truss steel-concrete hybrid structural system, with a total of 128 floors, a total structural height of 606.1m, a seismic fortification intensity of 7 degrees, a site category of class IV, a seismic design grouping of the first group, and the first three self-vibration periods of the benchmark model are 8.94s, 8.93s and 4.48s, respectively. Figure 4.27b shows a schematic diagram of the 30-layer Stick model, and the first three self-oscillation periods of the Stick model are 3.01s, 1.18s, and 0.72s, respectively.









(c) Acceleration time history curve of SHW6 (128/128) wave in the X direction



Figure 4.28 Floor waves

Figure 4.28 shows three floor wave acceleration time history curves. Figure 4.29 shows the acceleration response spectra of three floor waves at 0.1g amplitude.



Fig. 4.29 Acceleration response spectrum of floor waves (damping ratio: 5%)

4. 4 Experimental results and discussions

4.4.1 Failure pattern and damage evolution

The typical damage to the ceiling components observed in the SCS type 3 is shown in Figure 4.30. The failure of the hanger and hook is a special damage mode in the double-layer SCS due to its different construction details from other types of SCSs. It can be found that most damage is associated with grid connections, which indicates that the most vulnerable part of the ceiling is the grid connection. No damage to the semi-free boundary is observed during the loading process. Compared to the free boundary condition without any attachments, such as seismic clips, the semi-free boundary condition plays a role in preventing the unseating of the peripheral grids from the wall angles. The semi-free boundary condition achieved by seismic clips with four perimeter screws can avoid experiencing excessive twisting and deformation of seismic clip connection due to insufficient perimeter screws. Thus, it is suggested that all available perimeter screws at the flange of seismic clips should be fixed to the perimeter to improve the strength of the peripheral connections of the SCS.



(a)



(b)

€







Figure 4.30 Typical damage to ceiling components: (a) buckling of main-cross tee connection, (b) buckling of cross-sub cross tee connection, (c) separation of main-cross tee connection, (d) dislodged panel, (e) falling of panel, (f) separation of the main tee connection, (g) buckling of grid, (h) failure of hanger and hook, (i) falling of grid.

The damage process of the ceiling TYPE C is demonstrated in Figure 4.31. No damage to the ceiling is observed before Run 16. The pulling out of a main-cross tee connection and buckling of the grid connections near the perimeter are found under Run 16. The main-cross tee connection fails mainly because the axial force acting on it is greater than its strength. The grid connections near the perimeter are vulnerable to buckling since the cumulative axial force acting on the grid connections near the perimeter reaches the maximum during the collision. With an increase in the PFA, the damage to the ceiling gradually becomes obvious but is still slight after Run 26. When the PFA reaches 1.942 g in the Y direction during Run 30, the buckling and pulling out of a large number of grid connections and the dislodgement and falling off some panels around the ceiling perimeter occur. It should be noted that the failure of the main tee connections during Run 30 greatly accelerates the collapse of the ceiling. After the input of the sweep wave with the highest PGA of 0.5 g in the X direction, the ceiling completely collapses, with the ratio of falling panels to total panels reaching 40.68%.



Figure 4.31 Damage process of the ceiling: (a) Before Run 16, (b) 0.405g (X) & 0.377g (Y) Random (Run 16), (c) 0.15g (X) & 0.15g (Y) Sweep (Run 22), (d) 0.25g (Y) Sweep (Run 26), (e) 0.35g (Y) Sweep (Run 30), (f) 0.5g (X) Sweep (Run 32).

Notes: ▶ buckling of main-cross tee connection; ▲ buckling of cross-sub cross tee connection; ● separation of main-cross tee connection and cross-sub cross tee connection; ■ dislodgement of panel; ■ falling of panel; ■ separation of main tee connection.

4.4.2 Experiments performance

Based on the experimental contents described previously, this chapter systematically studies the seismic performance of non-structure systems from the aspects of dynamic characteristics, movement mechanism, acceleration response, relative displacement response, and strain response, etc.

4.4.2.1 Dynamic system characteristics

(a) System identification method

Modal parameter identification is to obtain data from the acquisition system, establish and solve the mathematical model of the system, and obtain the modal parameters of the system, including natural frequency, damping ratio, and mode shape. Modal parameter identification includes the frequency and time domain methods according to different signal identification domains. The frequency domain method mainly uses the frequency response function (transfer function) obtained from the input signal and output signal to identify the modal parameters of the system, such as the Peak Picking Method, Frequency Domain Decomposition Method, Enhanced Frequency Domain Decomposition Method and Least-squares Complex Frequency Domain Method (LSCF). It is intuitive and noise resistant, but it is easy to cause leakage and other problems. The time-domain method is different from the frequency-domain method. It can only use the measured response signal without the Fourier transform. It can directly identify the modal parameters of the system in the time domain, so it can avoid the leakage problem caused by the Fourier transform. It is intuitive and noise resistant, but it is easy to cause leakage and other problems. The time-domain method is different from the frequency-domain method. It can only use the measured response signal without the Fourier transform. It can directly identify the modal parameters of the system in the time domain, so it can avoid the leakage problem caused by Fourier transform, such as Ibrahim Time Domain Method, Least-squares Complex Exponential Method, Hilbert-Huang Transform, Stochastic Subspace Identification, ARMA Time Series Method, etc. See Figure 4.32 for the analysis route of the frequency domain method and time domain method.



(a) Analysis route of frequency domain method



(b) Analysis route of the time domain method

Figure 4.32 Analysis route of frequency domain method and time domain method In the field of structural engineering, the peak value picking method is often used to identify the modal parameters of the structure. This method has the advantages of fast identification and easy operation, but it is not stable enough, and the identification accuracy is not high, so it is difficult to be used for the identification of dense modes, especially for the mineral wool ceiling whose natural vibration frequency is difficult to identify. In this disseartation, the single reference point complex index method is used to try to identify the natural frequency of the suspended ceiling. The analysis route of the single reference point complex index method is to FFT the measured signal to obtain the transfer function. The transfer function gets the pulse response of the system through IFFT. According to the relationship between the pulse response and the poles and residues, an autoregressive model is established to calculate the autoregressive coefficient, and then a Prony polynomial about the poles is constructed to estimate the poles and residues. Thus, the modal parameters of the system can be obtained. The analysis route of this method is shown in Figure 4.33. The basic principle of this method is described below:



Figure 4.33 Analysis route of single reference point complex index method

Let the expression of the frequency response function of p point displacement caused by q point force in a multi-degree of freedom viscous damping linear system be:

$$H_{pq}(j\omega) = -\sum_{r=1}^{N} \left(\frac{A_{rpq}}{j\omega - s_r} + \frac{A_{rpq}^*}{j\omega - s_r^*} \right)$$
(4.1)

where: A_{rpq} is the residue corresponding to the *r*-th mode, which is related to the mode shape; *the number of degrees of freedom of the system; j is an imaginary number,* $j = \sqrt{-1}$; The symbol * indicates complex conjugate; *s_r* is the pole of the r-th mode of the frequency response function, which is related to the modal frequency and damping ratio.

*s*_{*r*} can be expressed as:

$$s_r = -\omega_r \xi_r + j\omega_r \sqrt{1 - \xi_r^2} \tag{4.2}$$

where: ω_r is the natural frequency; ξ_r is the damping ratio.

Set $A_{r+N} = A_r^*$, $s_{r+N} = s_r^*$, with Equation (4.1): $H(\omega) = \sum_{r=1}^{2N} \frac{A_r}{j\omega - s}$

The impulse response function can be obtained by IFFT transformation of Equation (4.3):

$$h(t) = \operatorname{Re}\left(\sum_{r=1}^{2N} A_r e^{s_r t}\right)$$
(4.4)

(4.3)

Where Re is the real part of the complex number.

Set Δt as the time interval of discrete data; when $t_k = k\Delta t$, the impulse response function can be expressed as:
$$h_{k} = h(k\Delta t) = \sum_{r=1}^{2N} A_{r} e^{s_{r} t_{k}} = \sum_{r=1}^{2N} A_{r} e^{s_{r} k\Delta t} = \sum_{r=1}^{2N} A_{r} V_{r}^{k}, \quad k=0,1,\cdots,L$$
(4.5)

where: $V_r = e^{s_r \Delta t}$; L+1 is the data length of the impulse response function of the measured signal, and L+1 $\ge 2N_{\circ}$

Equation (4.5) can be listed in the form of equations:

$$\begin{cases} h_0 = \sum_{r=1}^{2N} A_r V_r^0 = A_1 + A_2 + \dots + A_{2N} \\ h_1 = \sum_{r=1}^{2N} A_r V_r^1 = A_1 V_1 + A_2 V_2 + \dots + A_{2N} V_{2N} \\ h_2 = \sum_{r=1}^{2N} A_r V_r^2 = A_1 V_1^2 + A_2 V_2^2 + \dots + A_{2N} V_{2N}^2 \\ \vdots \\ h_L = \sum_{r=1}^{2N} A_r V_r^L = A_1 V_1^L + A_2 V_2^L + \dots + A_{2N} V_{2N}^L \end{cases}$$

$$(4.6)$$

Where h_k is known in Equation (5.6), and the problem is how to solve $V_r \pi A_r$. The solution is to regard V_r as a real coefficient β_k (autoregressive coefficient) of 2N order polynomial equation, namely:

$$\sum_{r=1}^{2N} \beta_k V^k = \prod_{r=1}^{N} (V - V_K) (V - V_K^*) = 0$$
(4.7)

From Equation (4.7) $\beta_{2N}=1_{\circ}$ For equation β_k , multiply β_k on both sides in Equation (4.6):

$$\sum_{k=1}^{2N} \beta_k h_k = \sum_{k=1}^{2N} \beta_k \sum_{r=1}^{2N} A_r V_r^k = \sum_{r=1}^{2N} A_r \sum_{k=1}^{2N} \beta_k V_r^k$$
(4.8)

As $\sum_{k=1}^{2N} \beta_k V_r^k = 0$, $\beta_{2N} = 1$, Equation (5.8) can be written as:

$$\sum_{k=1}^{2N-1} \beta_k h_k = -h_{2N} \tag{4.9}$$

To calculate β_k , it is necessary to construct an equation set, which will be shifted backward by Δt , and take 2*N*+1 data from observation h_k and substitute them into Equation (4.9) to form a set of equations as follows:

$$\begin{cases} \sum_{k=1}^{2N-1} \beta_k h_k = \beta_0 h_0 + \beta_1 h_1 + \dots + \beta_{2N-1} h_{2N-1} = -h_{2N} \\ \sum_{k=1}^{2N-1} \beta_k h_{k+1} = \beta_0 h_1 + \beta_1 h_2 + \dots + \beta_{2N-1} h_{2N} = -h_{2N+1} \\ \vdots \\ \sum_{k=1}^{2N-1} \beta_k h_{k+M-1} = \beta_0 h_{M-1} + \beta_1 h_M + \dots + \beta_{2N-1} h_{L-1} = -h_L \end{cases}$$

$$(4.10)$$

where, $M=L-2N_{\circ}$

Equation (4.10) is expressed as a matrix:

$$\begin{bmatrix} h_{0} & h_{1} & h_{2} & \cdots & h_{2N-1} \\ h_{1} & h_{2} & h_{3} & \cdots & h_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ h_{M-1} & h_{M} & h_{M+1} & \cdots & h_{L-1} \end{bmatrix} \begin{bmatrix} \beta_{0} \\ \beta_{1} \\ \vdots \\ \beta_{2N-1} \end{bmatrix} = - \begin{bmatrix} h_{2N} \\ h_{2N+1} \\ \vdots \\ h_{L} \end{bmatrix}$$
(4.11)

Or abbreviated as:

$$[h]_{M \times 2N} \{\beta\}_{2N \times 1} = -\{h'\}_{M \times 1}$$
(4.12)

In general, if M > 2N, the pseudo inverse method can be used to solve the least square solution of the equations, namely:

$$\{\beta\} = ([h]^{T} [h])^{-1} ([h]\{h'\})$$
(4.13)

Add one element $\beta_{2N}=1$ to $\{\beta\}$, substitute it into equation (4.7) :

$$\sum_{r=1}^{2N} \beta_k V^k = \beta_0 + \beta_1 V + \beta_1 V^2 + \dots + \beta_{2N-1} V^{2N-1} + V^{2N} = 0$$
(4.14)

Then the root V_r , which is composed of β_{k} , of a polynomial can be found, and then the modal ratio ω_r and damping ratio ξ_r can be found, namely,

$$\begin{cases} R_r = \ln V_r = s_r \Delta t \\ \omega_r = |R_r| / \Delta t \\ \xi_r = \sqrt{1 / (1 + (\operatorname{Im}(R_r) / \operatorname{Re}(R_r))^2)} \end{cases}$$
(4.15)

Then, the residue $A_r \quad \overline{\Box}$ of every observation can be calculated by V_r , and Equation (4.6) is rewritten as follows:

$$\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ V_1 & V_2 & V_3 & \cdots & V_{2N} \\ \vdots & \vdots & \vdots & & \vdots \\ V_1^L & V_2^L & V_3^L & \cdots & V_{2N}^L \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_{2N} \end{bmatrix} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_L \end{bmatrix}$$
(4.16)

Or abbreviated as:

$$[V]_{(L+1)\times 2N} \{A\}_{2N\times 1} = \{h\}_{(L+1)\times 1}$$
(4.17)

The vibration mode vector can be obtained by the residue obtained from a series of response measuring points. For a structure with n response test points, first, find the test point with the largest absolute value from n residues corresponding to the same mode. Assuming that this point is the test point m, the normalized complex mode vector corresponding to the kth mode can be obtained by the following formula

$$\left\{\phi_{k}\right\} = \begin{bmatrix} A_{k1} & A_{k2} & \cdots & A_{kn} \end{bmatrix}^{T} / A_{km}$$

$$(4.18)$$

Identifying false modes is a very important step in modal identification. At present, there are four ways to identify false modes: (1)Analyze the pole position after solution;

(2) Analyze the energy occupied by the frequency corresponding to the pole; (3) Measured by Modal Assurance Criterion (MAC); (4) Determine with the Stability Diagram (SD). Generally, steady-state diagrams are used to identify false modes. In the steady state diagram, the abscissa is the frequency, the left is the mode order, and the right is the amplitude. The principle of this method is to use different modal orders to conduct modal identification of the system and to judge the reliability of the poles by identifying the stability of the system's poles to frequency, normalized mode shape, and damping ratio. Generally, the modal order is 30~50.

(b) Dynamic characteristics of steel platform

Before and after the even number of working conditions, the white noise is used to sweep the frequency of the steel platform. The least square complex index method is used to identify the natural frequency of the steel platform. The excitation signal selects the acceleration time history of the platform, and the response signal selects the acceleration time history of the steel platform roof. Taking the shaking table experiment of type A as an example, before specimen A is loaded, the identification results of the natural frequencies of the steel platform in X and Y directions are shown in Figures 4.34 and 4.35, respectively. It can be seen from the figure that the first order natural frequencies of the steel platform in X and Y directions are 8.9Hz and 8.4Hz, respectively; After the loading of specimen A, the identification results of the steel platform in X and Y directions are shown in Figures 4.36 and 4.37 respectively. It can be seen from the figure that the first order natural frequencies of the steel platform in X and Y directions are shown in Figures 4.36 and 4.37 respectively. It can be seen from the figure that the first order natural frequencies of the steel platform in X and Y directions are shown in Figures 4.36 and 4.37 respectively. It can be seen from the figure that the first order natural frequencies of the steel platform in X and Y directions are 8.4Hz and 8.2Hz, respectively.



(b) Comparison of amplitude frequency diagram between measured frequency response function and fitted frequency response function



(c)Comparison of phase frequency diagram between measured frequency response function and fitted frequency response function



(d) Comparison of real frequency diagram between measured frequency response function and fitted frequency response function



(e) Comparison of virtual frequency diagram between measured frequency response function and fitted frequency response function

Figure 4.34 System identification results of X-direction natural vibration frequency of steel platform (before specimen A is loaded)



(a) Stability diagram



(b) Comparison of amplitude frequency diagram between measured frequency response function

and fitted frequency response function



(c)Comparison of phase frequency diagram between measured frequency response function and

fitted frequency response function 8 实测的实频曲线 4 拟合的实频曲线 实部 0 -4 -8^L 20 120 10 30 40 50 60 70 80 90 100 110 130 频率(Hz)

(d) Comparison of real frequency diagram between measured frequency response function and

fitted frequency response function



(e) Comparison of virtual frequency diagram between measured frequency response function and fitted frequency response function

Figure 4.35 System identification results of Y-direction natural vibration frequency of steel platform (before specimen A is loaded)





(b) Comparison of amplitude frequency diagram between measured frequency response function and fitted frequency response function



(c)Comparison of phase frequency diagram between measured frequency response function and fitted frequency response function



(d) Comparison of real frequency diagram between measured frequency response function and

fitted frequency response function



(e) Comparison of virtual frequency diagram between measured frequency response function and fitted frequency response function

Figure 4.36 System identification results of X-direction natural vibration frequency of steel platform (after specimen A is loaded)



(a) Stability diagram



(b) Comparison of amplitude frequency diagram between measured frequency response function

and fitted frequency response function



(c)Comparison of phase frequency diagram between measured frequency response function and fitted frequency response function



(d) Comparison of real frequency diagram between measured frequency response function and



(e) Comparison of virtual frequency diagram between measured frequency response function and fitted frequency response function



Figure 4.37 System identification results of Y-direction natural vibration frequency of steel platform (after specimen A is loaded)

Compared with that before the loading of type A, the natural frequencies of the steel platform in the X direction and Y direction are reduced by 5.6% and 2.4% respectively. It can be considered that the stiffness of the steel platform is basically unchanged before and after the test, meeting the test conditions. It should be noted that when white noise frequency scanning is performed on the steel platform, the suspended ceiling test piece has been installed on the steel platform. Still, the weight of the suspended ceiling test piece is about 0.6t. The weight ratio of the suspended ceiling test piece has little influence on the natural vibration frequency of the steel platform.

Before type B is loaded, the first order natural vibration frequencies of the steel platform in X and Y directions are 8.5Hz and 7.6Hz, respectively (before the test of type B, the vibration table test of the cable bridge was carried out on the steel platform. the tightening of the bolts was overlooked, which led to all bolts of the steel platform were not tightened again after the test, resulting in a reduction of the natural vibration frequency of the steel platform in the Y direction); After the loading of test piece B, the first natural frequencies of the steel platform in X and Y directions are 8.3Hz and 6.0Hz respectively. Compared with that before the loading of test piece B, the natural frequencies of the steel platform in the X direction and Y direction are reduced by 2.4% and 21.0%, respectively. It can be seen that the stiffness of the steel platform in the X direction is basically the same before and after the test, and the stiffness in the Y direction decreases more due to loosening bolts.

Before the loading of type C, the first natural frequencies of the steel platform in the X and Y directions are 9.0Hz and 8.2Hz, respectively (all bolts of the steel platform shall be retightened before the test type C); After the loading of type C, the first natural frequencies of the steel platform in X and Y directions are 8.6Hz and 7.4Hz respectively. Compared with that before the loading of type C, the natural frequencies of the steel platform in the X direction and Y direction are reduced by 4.4% and 9.8%, respectively. It can be considered that the stiffness of the steel platform changes a little before and after the experiment.

4.4.2.2 Experiments results analysis of SCS

As mentioned before, we investigated the seismic performance of double-layered SCSs with three types of boundary conditions by shaking table tests, i.e., free boundary condition (BC1), fixed-semi-free boundary condition (BC2), and semi-free boundary condition (BC3). The detailed comparison of the experimental results of SCSs with free boundary condition (BC1) and fixed-semi-free boundary condition (BC2) can refer to the Reference [19]. Figure 3.38 shows the comparison of variation in the proportion of damaged grid connections (PDGCs) of SCSs with a different type of boundary condition with increased PFA. When the PFA is smaller than 0.5 g, the PDGC of SCSs with different types of boundary conditions is almost the same. As the PFA increases beyond 0.5 g, the PDGC of SCS with free boundary condition (BC1) is the highest due to the failure of a large number of peripheral connections caused by weak boundary constraint, and the PDGC of SCS with fixed-semi-free boundary condition (BC2) is the lowest. In general, the SCS with fixed-semi-free boundary condition (BC2) is the lowest.

condition (BC2) exhibits superior seismic performance compared to that with semifree boundary condition (BC3).

In this study, the BC3 is proposed mainly for the following two reasons: (1) the fixed edges of SCSs with fixed or fixed-semi-free boundary condition may be damaged under earthquakes because the strong boundary constraint on the fixed edges allows inertia forces induced in SCS to transfer and accumulate at the peripheral fixings. Therefore, all fixed edges are adjusted to semi-free edges to explore the seismic performance of SCS with BC3. (2) in the current practice, the construction measures for semi-free seismic clip connections at the boundary are inadequate, i.e., the flange of the seismic clip is not firmly connected to the surroundings due to the lack of sufficient perimeter screws, which often leads to damage to these connections. Therefore, in this study, all seismic clips of SCS with semi-free boundary conditions are attached tightly to the surroundings by using all available perimeter screws to improve the seismic capacity of semi-free seismic clip connections.

In the case of large-area SCS, when the accumulated axial force demand on the fixed connection of the SCS with BC2 is greater than its capacity, the fixed connection may be damaged. For the SCS with BC3, the release of boundary constraints caused by the semi-free boundary condition can reduce the axial force demand on the connection at the perimeter to prevent damage. In addition, if the gap between the grid end and wall angle is set reasonably, the adverse effect of the collision on the seismic performance of the SCS with BC2 can be reduced or avoided.



Figure 4.38 Comparison of variation of proportion of damaged grid connections of suspended ceilings with different type of boundary condition with increased PFA.

(a)Acceleration time history of suspended ceiling grid

An example of the acceleration time history of channel A13 located in the middle of axis Y5 in the X direction in the case of 0.15 g sweep in the X direction, is shown in Figure 4.39 (see the location of A13 in Figure 4.22). The acceleration of the grid gradually increases during the slipping stage. There is a significant amplification of acceleration being ten times the PGA of input during the pounding stage because the relative displacement of the ceiling exceeds the gap, resulting in a violent impact between the grid end and the boundary. A lot of high spikes appear in the acceleration time history. The pounding starts at around 50 s with an input frequency of 2.3 Hz. Then the acceleration increases significantly. Unlike the violent impact from the beginning of loading in Ceiling A with smaller width of the gap, the gap with larger width in the ceiling here delays the impact. The SFCs prevent the peripheral grids from falling down which leads to continuous impact regardless of the intensity of the input.



Figure 4.39 Acceleration time history of channel A13 under 0.15 g sweep in the X direction (Run 18): (a) total acceleration time history, (b) acceleration time history during pounding.

(b) Displacement time history of ceiling grid

Figure 4.40 shows the results of channels D33 and D17 located at the left and right ends of axis Y7 in the X direction in the case of 0.15 g sweep in the X direction (see the location of D33 and D17 in Figure 8). The deformation is defined as the deformation of the grid system between two displacement transducers, as shown in Figure 40c. From Figure 40a it can be found that the pounding occurs at around 50 s when the input frequency is 2.3 Hz. Due to the semi-free boundary condition, the ceiling in this test exhibits stable slipping and pounding behavior. The displacement gradually increases during the slipping stage, and it reaches the width of the gap at the boundary during the pounding stage with a residual displacement of 5 mm. The friction force causes notable residual displacement between the middle slot and the sliding screw of the seismic clip and the damage to the ceiling. Due to the effect of SFCs at the ends of the row of the grid, the displacement amplitudes of the two transducers are roughly equal, and the displacement directions are opposite, which is consistent with the movement characteristic of the grid (Figure 40b). It can be concluded that the semi-free boundary condition could change the movement behavior of the SCS.



Figure 4.40 Results of a pair of displacement transducers under 0.15 g sweep in the X direction (Run 18): (a) displacement time history of a pair of transducers, (b) displacement time history from 60 s to 62 s, (c) deformation of the ceiling.

The SFC could adapt to the large movement of the ceiling so that the displacement response of the ceiling is stable. Figure 4.41 shows the movement process of the grid in cross tee direction (X direction) in the case of 0.15 g sweep. The cross tee line consists of 12 pieces of cross tees and 11 connections. The original distance between

the displacement transducers is L_0 . The displacement measured by transducers equals the sum of the displacement of slipping on the wall angle and deformation of the end connection. The maximum positive and negative displacements of D33 are 14 mm and -15 mm, respectively. Those of D17 is 16 mm and -13 mm, respectively. When the grid moves to the right side, the deformation and average displacement of the grid system are 1 mm and 13.5 mm, respectively. When the grid moves to the left side, the deformation and average displacement of the grid system are 1 mm and 15.5 mm, respectively. The connection at the left end buckles is due to the pounding during the movement of the grid. However, the connection buckles, the deformation of the components between the two transducers is not affected by the buckling behavior since the buckling does not occur in the components between the two transducers. Compared with the deformation of the grid system, the average displacement is much bigger. The most important difference between the main tee direction and the cross tee direction in the grid is the number of connections. The main tee line consists of 4 pieces of main tees and 3 connections. The displacement measured by the transducer in the main tee direction does not include the deformation of connections since there are no main tee connections existing between the transducer and the perimeter.



Figure 4.41 Movement process of grid in cross tee direction under 0.15 g sweep in X direction (Run 18).

(c) Acceleration versus displacement relationship

Figure 4.42 shows the acceleration versus displacement relationship of the typical position in the ceiling. Run 18 and Run 20 are selected as the representatives of the pounding stage in the X direction and the Y direction, respectively. During Run 18, the measuring points A27 and D18 located on the axis Y8, are selected. During Run 20, the measuring points A6 and D5, located on the axis X6, are selected. It can be found from Figure 4.39a that during the pounding stage, the acceleration changes drastically during pounding, and the acceleration reduces quickly to around zero after pounding. Due to the existence of SFCs, the curve reflects both friction-slip and pounding behavior. The slipping stage increases the energy dissipation by the friction mechanism between the middle slot and the sliding screw along the axial direction of the perimeter grid.

Compared with the responses in the X direction, in the Y direction, the acceleration reaches the peak more quickly, and the peak acceleration is larger, which is related to the arrangement of different components in two directions and different details between main tee connections and main-cross tee connections. There are fewer connections for main tees in a line in the Y direction, and the capacity of resisting buckling of the main tee connections is larger due to the construction details, causing smaller deformation of the main tee connections and greater acceleration of the main tees.



Figure 4.42 Acceleration versus displacement relationship: (a) pounding stage in the X direction, (b) pounding stage in the Y direction.

(d) Acceleration amplification factor

In the current seismic design codes in most countries, the equivalent static method is recommended for calculating the seismic action of non-structural components, in which the component acceleration amplification factor (AAF) is a key parameter. The AAF is calculated as the ratio of the peak ceiling grid acceleration (PCGA) to the peak floor acceleration (PFA). The peak AAF prescribed in the Chinese seismic design code takes the value of 2.0 for the suspended ceiling system.

The AAF of the ceiling under sweep waves is shown in Figure 4.43. Under Runs 2, 4, and 6, the AAF of the ceiling is around 1.5. In these cases, the ceiling is in the preslipping stage, and no pounding occurs in the ceiling. The AAF increases with the increase of input intensity due to significant pounding before Run 28. The AAF reduces significantly during Run 30 with larger PFA due to more severe damage to the main tee connections. The AAF at most measuring points is roughly uniform except for the endpoints. The AAF at both ends in the X direction is the smallest due to the greater constraint from SFCs in the Y direction. Similarly, the AAF at both ends in the Y direction is much less than that between the two ends in the Y direction. The largest AAFs in the X and the Y directions are 9.7 and 9.4, respectively, which is caused by the huge pounding at the boundary. It is necessary to develop seismic measures to reduce pounding. Adding the isolation foam in the peripheral gap may be possible. The median AAF under sweep waves is 3.3, which is larger than the median AAFs of 3.2 and 2.9 reported in UB and UNR, respectively. It is mainly due to the violent impact caused by the semi-free boundary condition in the ceiling.



Figure 4.43 Acceleration amplification factor under sweep waves: (a) response in the X direction, (b) response in the Y direction.

The AAF of the ceiling under excitations of artificial waves and floor earthquake waves is plotted in Figure4.41. Whether in the X direction or the Y direction, the AAF increases as the input intensity increases. From Run 8 to 12, the AAFs of the ceiling is around 1.0, indicating that the ceiling is in the pre-slipping stage. During Run 14 without pounding behavior, most of AAFs in the X direction is greater than those in the Y direction due to the smaller lateral stiffness in the X direction. Among the acceleration responses of all Runs, the AAF in Run 16 is the highest due to the pounding. During Run 16, the AAFs in the X direction are smaller than those in the Y direction due to the fact that more serious damage to main-cross tee connections in the X direction. The median AAF under the excitations of floor earthquake waves is 1.2, which is lower than 2.0 which is suggested by the design code [410].



Figure 4.44 Acceleration amplification factor under artificial wave and floor earthquake waves: (a) response in the X direction, (b) response in the Y direction. For the three types of floor earthquake waves, the AAF is the largest in Run 16 (SHW6 (30/30)), the second largest in Run 14 (SHW6 (128/128)), and the smallest in Run 12 (SHW6 (5/128)), indicating that the type of input seismic wave has a significant effect on AAF. It is found that both type and intensity of input seismic

wave have a significant effect on AAF. In this study, only a small number of seismic waves were input. Therefore, the authors think that a large number of seismic waves should be input in further study to obtain reliable AAF for the seismic design of suspended ceilings.

(e) Peak displacement

The peak displacement (PD) under unidirectional sweep excitations is shown in Figure 4.45. From Run 2 to 6, the PD is close to zero, which implies that the ceiling moves with the platform together. The PD of the ceiling is more uniform in the middle and smaller on both sides as a result of the constraint effect of SFCs in the perpendicular direction. In X direction, the PD of the middle points reaches the width of the gap between the grid end and wall angle during Run 18. The PD during Run 22 is almost the same as that during Run 18. Due to the buckling of the grid connections and more serious damage to the grid ends after Run 22, the width of the gap becomes larger, which causes the increase of PD. In Y direction, the PD of the middle points reaches the width of the gap during Run 20, which is close to the PD during Run 22. Most of PDs during Run 26 is greater than the width of the initial gap, which indicates that the gap increases due to the local buckling of the grid ends and deformation of the grid. Whether in X or Y direction, the PD increases with the increase of intensity of the sweeping wave.



Figure 4.45 Peak displacement under sweep waves: (a) response in the X direction, (b) response in the Y direction.

Figure 4.46 presents the PD of measuring points under the artificial wave and floor earthquake waves. From Run 8 to 12, the PD of the ceiling is small. During Run 14, the PD is smaller than the width of the gap, indicating that no pounding occurs at the boundary. During Run 16, the PD first exceeds the width of the gap, indicating that the pounding begins at the boundary. The PDs of the ceiling is roughly uniform in the middle and smaller on both sides. The floor earthquake wave with higher intensity produces larger PD. For the three types of floor earthquake waves, the AAF is the largest in Run 16, the second largest in Run 14, and the smallest in Run 12. Compared with the measured PDs in the ceiling without seismic clips, the PDs in the test are larger under the seismic excitation with the same intensity, which is mainly caused by the larger width of the peripheral gap.



Figure 4.46 Peak displacement under artificial wave and floor earthquake waves: (a) response in the X direction, (b) response in the Y direction.

(f) Strain of ceiling component

The recorded maximum strains of the ceiling components before the ceiling collapses (Run 32) are listed in Table 4. The yield strain of ceiling components is 1175 $\mu\epsilon$. All components remain elastic. However, local buckling is observed in some grids due to strong pounding between the ceiling and the boundary. The strain of the threaded rod is much larger than that of other components. The maximum strain is 598 $\mu\epsilon$ and 526 $\mu\epsilon$ under sweep waves and floor earthquake waves before the ceiling collapses, respectively, which is much larger than the maximum strain recorded in the test with a smaller peripheral gap. It is because the larger peripheral gap at the ceiling perimeter in the test increases the relative deformation of the threaded rod, further resulting in greater strain in the threaded rod.

Table 4.8 The maximum strain of ceiling components.

Excitation type	Maximum strain of ceiling component ($\mu \varepsilon$)				
	Threaded rod	Main tee	Cross tee	Sub cross tee	Carrying channel
Sweep wave	598	67	128	210	130
Floor earthquake wave	526	64	172	78	85

Figures 4.44a and 4.44b present the peak strain versus PCGA relationship of the representative positions in ceiling grids under sweep waves and floor earthquake waves, respectively. The measuring point (S108/S109) on the main tee is located at the end of axis X8 (see the location of the strain gauges in Figure 8). The measuring point (S55/S56) on the cross tee is located in the middle of axis Y4. The measuring point (S88/S89) on the sub cross tee is near the end between axis X10 and axis X11. Under sweep waves, four levels of PCGA corresponding to inputs with PGA of 0.05 g, 0.15 g, 0.25 g, and 0.35 g are considered. Underfloor earthquake waves, four levels of PCGA corresponding to inputs with PGA of 0.037 g, 0.089 g, 0.149 g, and 0.405 g are considered. For the main tee, the peak strain continues to increase as the input intensity increases. No buckling occurs in the main tee connections. The strain of the main tee is significantly affected by pounding, which induces the axial force. For the cross tee and the sub cross tee, the peak strain response increases linearly with the increase in input intensity. The maximum peak strain is smaller than that of the main tee, due to the buckling of connections.

It can be found from Figure 4.44b that the strain increases as the input intensity increases. Under the input of the first two levels, the strain is very small due to the fact that the ceiling is in the pre-slipping stage. Under the input of the third level, the ceiling is in the slipping stage, and the strain increases slowly. Under the input of the fourth level, the ceiling pounds on the surroundings, and the PCGA and peak strain increase sharply.



Figure 4.47 Peak strain versus PCGA relationship: (a) under sweep waves, (b) under floor earthquake waves.

CHAPTER FIVE

COMPUTATIONAL NUMERICAL SIMULATION OF SUSPENDED NON-STRUCTURAL SYSTEMS

5. Numerical simulation

5.1 Numerical modeling

5.1.1 OpenSEES simplified numerical simulation

Considering the roughly uniform seismic responses of SCS with semi-free boundary conditions in the same direction but at a different location under the earthquake, a simplified planar numerical model is built using the software OpenSEES. Figure 5.1 presents the analytical model of the tested SCS in the X direction as an example. The horizontal acceleration time history at the top of the steel platform is used as the input motions to the analytical model.

In this model, the threaded rod is simulated by *dispBeamColumn* element, and the top of threaded rod is set as fixed. The grid member is simulated by *elasticBeamColumn* element. The panel is simplified as a lumped mass placed at the grid connection. The *ZeroLength* element with *Pinching4* material is adopted to model the axial nonlinear behavior of grid connection, and the parameters of *Pinching4* material are calibrated by the data from the ceiling components hysteretic tests conducted by the author. The double-layer connection is assumed to be pin-connected.

The friction and pounding behaviors at the ceiling perimeter are simulated using the friction model and impact model, respectively. The friction model is modeled using

Steel01 material, mainly expressed by three parameters, i.e., initial elastic tangent K, yield strength F_y, and strain-hardening ratio b. Herein, the initial elastic tangent is defined as the ratio between yielding strength and initial displacement of 0.1 mm, and strain-hardening ratio is set very small according to the reference [411]. The yield strength representing the friction force is calibrated by the corresponding shaking table test results and taken as 2 N. According to the friction force and initial displacement, the initial elastic tangent is taken as 20 N/mm. A value of 0.05 for the strain-hardening ratio is found to be the best when calibrating the analytical model by comparing the simulation results with the experimental data.

The impact model is modeled using *ImpactMaterial* material, mainly expressed by four parameters, i.e., initial stiffness K_1 , secondary stiffness K_2 , yield displacement δ_y , and initial gap width. The initial stiffness and secondary stiffness are taken as 300 N/mm and 100 N/mm, respectively, to fit the experimental data best. The yield displacement is taken as 0.1 mm. The initial gap width is determined as 13.3 mm by the measured displacement history of grid ends during the shaking table tests.



Figure 5.1 Simplified numerical simulation model of SCS with semi-free boundary

5.1.2 Higher fidelity numerical simulation

In lieu of the simple model, another higher fidelity numerical simulation model is built, as illustrated in Figure 5.2. The threaded rods and all grid members are simulated by the *Frame* element, which is a line element with finite cross-sectional dimensions. The top of the threaded rod is safely anchored to the steel platform, and thus the boundary condition of the top is set as fixed. For the bottom of the threaded rod, the connection between the threaded rod and the main tee is treated as rigid. Panels are simplified as a lumped mass placed in grid intersection points [291]. The typical bi-linear stress-strain relationship with *Kinematic* hysteretic model is used as the constitutive model for the steel material, as shown in Figure 5.3. In this model, E and E_s represent the initial elastic stiffness and post-yield stiffness, respectively. Rayleigh damping is adopted. As given in Eqs. (1)-(3), the damping matrix is the combination of the mass matrix and stiffness matrix.

$$C = \alpha \left[M \right] + \beta \left[K \right] \tag{5.1}$$

$$\alpha = \xi \frac{2\omega_1 \omega_2}{\omega_1 + \omega_2} \tag{5.2}$$

$$\beta = \frac{2\xi}{\omega_1 + \omega_2} \tag{5.3}$$

where α and β are the proportional coefficients of mass matrix and stiffness matrix, respectively; ξ is the SC damping ratio which is specified as 5% [412]; and ω_1 , ω_2 are the fundamental natural frequencies of the ceiling in the X and Y direction, respectively.



Figure 5.2 Higher fidelity computational numerical modeling of SCS



Figure 5.3 The constitutive model of steel

5.1.2.1Perimeter impact behavior

Figure 5.4 shows a typical free boundary condition of SC in practice. The peripheral grid ends just sit on wall angles, and there is a clearance (called gap) between grid ends and wall angles. Due to the existence of a gap, pounding between the peripheral grid and wall angle and the resultant sudden change of acceleration was observed during the test [413]. In the simulation, the *Gap* element is selected to model this

pounding behavior. The nonlinear force-deformation relationship for the *Gap* element is given by:

$$f = \begin{cases} k(d+\delta) & \text{if } d+\delta < 0\\ 0 & \text{otherwise} \end{cases}$$
(5.4)

where k is the impact stiffness, d is the relative deformation of the spring and δ is the width of the gap ($\delta \ge 0$).



Figure 5.4 Free boundary condition in practice.

As expressed in Eq. (5.4), the characteristics of the *Gap* element are controlled by two critical parameters: the impact stiffness K and the width of the gap δ . When the grid end touches the wall angle, the gap closes. In the simulation, the width of the gap is taken as the measured grid end displacement in the shaking table test, whose detailed information is presented in Section 2.2. Note that the initial width of the gap is theoretically different at each end of the grid components. However, the experimental results indicate that the ceiling basically responds as a whole in both horizontal directions For simplicity, at each side of SC, the width of the gap at each grid end is treated as the same in modeling, and the specific gap width is set as the mean

displacement measured at each side. After each loading, the width of the gap will be changed with the movement of SC, and thus the width of the gap in each loading case is specified individually according to the measured displacement history of the grid end.

As another parameter to define the *Gap* element is the impact stiffness, which can be defined as the axial stiffness of the contact body. However, this approach assumes that the impact surfaces are ideally smooth and that the pressure distribution due to the impact is uniform. This condition is quite challenging to achieve in practice. It was found that the actual impact stiffness was significantly smaller than the theoretical value of body's axial stiffness [28,29.]. In this study, the impact stiffness K is defined to be proportional to the axial stiffness of the impact body, as expressed as follows:

$$K = \alpha \frac{EA}{L}$$
(5.5)

where E is the elastic modulus of the body material, A is the section area of the contact body, L is the length of the contact body in the impact direction, and α is the stiffness ratio relative to the axial stiffness of the impact body.

Herein, the impact stiffness represents the contact stiffness between the SC grid end and wall angle. As shown in Fig. 8, the wall angle is fixed to peripheral beams and thus regarded as a rigid body. In the absence of relevant experimental data, the impact stiffness is defined as 0.4 times the axial stiffness of SC grids [29]. That is, the impact stiffness is taken as 588 N/mm, 217 N/mm, and 433 N/mm for main tees, cross tees, and sub-cross tees, respectively.

5.1.2.2 Perimeter friction behavior

In the shaking table tests mentioned in the previous chapter, the very small displacements measured under small input motions indicated that the SC system basically kept motionless. This phenomenon is attributed to the existence of friction force at the SC perimeter [11,30]. To model this friction behavior, the Plastic (*Wen*) link element is adopted, and the hysteretic curve is shown in Figure 5.5. This model is mainly expressed by four parameters: (1) initial stiffness, (2) yield force (F_y), (3) post-yielding stiffness, and (4) yielding exponent to define the degree of sharping at yielding.



Figure 5.5 Hysteretic curve of friction model

Herein, the initial stiffness is defined as the ratio between the yielding force and initial displacement (Δ_y) of 0.1 mm, and post-yielding stiffness is set very small, referencing the previous study [31]. The yield force representing the friction force is calibrated by the corresponding shaking table test results. At each grid end, the friction behavior in both the X direction and Y direction is considered, and the friction force is taken as 2N. A value of 20 for the yielding exponent parameter corresponding to a very sharp

yielding is adopted to simulate the elastoplastic hysteresis rules of friction [32]. In the simulation, this element is placed at the SC perimeters parallel to the Gap element.

5.2 Constitutive hysteretic behavior investigation

A crucial part of SCS is the ceiling component, greatly influencing the seismic performance of SCS, from which the propagation of damage often initiates, and even the complete collapse of the ceiling occurs. The typical types of damage to the ceiling components in real earthquakes include the failure of grid connections, the failure of hangers, and the failure of peripheral attachments.

To examine the seismic performance of SCS subjected to simulated earthquake loading, experimental studies largely using shaking table tests have been carried out for nearly 40 years. One of the most significant observations is that the ceiling components are identified as one of the most vulnerable parts of SCS during earthquakes. Several component-level investigations on SCS were conducted to obtain the failure mechanism and capacities of strength and deformation of the ceiling components. Soroushian et al. systematically performed studies on the capacities of the peripheral attachments and components under monotonic and cyclic loadings. Based on those experimental data, several fragility curves and analytical models for different components were developed. In the study by Paganotti et al., a series of static tests on different types of components of SCS subjected to monotonic loading were carried out to evaluate the component capacity and produce fragility curves. It was found that the cross-tee connections are the most critical components of SCS. To assess the seismic performance of the ceiling component with seismic clips attached to wall angles using two screws, three types of ceiling perimeter configurations, i.e., pop-riveted connection and seismic clips with 1 screw or 2 screws, were conducted under monotonic and cyclic tests by Gilani et al. The experimental results indicated that the alternate peripheral installation with a seismic clip and 2 screws has better seismic performance in terms of load-carrying ability and energy dissipation.

Although current seismic design standards such as the ASTM-E580/E580M and AC368 specify that the ceiling joints should carry a mean ultimate test load of not less than 800N for a restrained ceiling, it is unclear whether the strength capacity of the joint under the actual load can meet the requirements. Previous studies on ceiling components were conducted based on a particular product from a company and may produce different results from similar studies with different products. To understand and evaluate the seismic performance of suspended ceiling components and support for subsequent numerical modelling, a series of static tests on the ceiling components under monotonic and cyclic loadings were carried out in this thesis. The failure patterns, capacities of strength and deformation, and energy dissipation of the ceiling components are presented in detail in this study.

5.2.1 Test program

5.2.1.1 Test setup

The electromechanical universal testing machine CMT4204, with a maximum vertical loading capacity of 20kN, is used to apply the required load to the ceiling components, as shown in Figure 5.6. A steel platform was designed and assembled with the machine for the static tests on the ceiling components. The load cell and extensometer in the machine are utilized to measure the force and displacement of the ceiling components, respectively.



Figure 5.6 Test setup

5.2.1.2 Test specimens

As listed in Table 1, a total of 80 suspended ceiling components, including main tee splices, cross tee latches, and seismic clips at free and fixed sides, were conducted in monotonic and cyclic tests, including axial, shear and bending tests to obtain the failure pattern, capacities of strength and deformation and energy dissipation capacity of the ceiling components. For each configuration, at least one monotonic and three cyclic tests with the same parameters were conducted. Table 2 lists the detailed information of the ceiling grid. Figure 3 presents cross-section dimensions of the ceiling grid.

In the following parts, specimens IDs consist of three parts, test type, loading type, and test specimen. In the first part, JA, WJ, CJS, and CJB refer to the axial test of ceiling joints, the axial test of peripheral attachments, the shear test of cross tee latches, and the bending test of cross tee latches, respectively. Numbers 1, 2, and 3 in
the second part refer to tension loading, compression loading, and cyclic loading, respectively. E, D, A, and B before the number in the second part refer to the fixed side of the peripheral attachment, free side of the peripheral attachment, the major axis of the cross tee section, and the minor axis of the cross tee section, respectively. M and C in the third part referring to the main tee and cross tee, respectively.

5.2.1.3 Loading protocol

For the ceiling components, the applied load is controlled by displacement at a low speed of 60mm/min. The monotonic test is performed by a unidirectional ramp. As shown in Figure 5.7, the cyclic test is controlled by step-by-step increasing reverse displacement. The cyclic test is carried out according to the loading protocol developed specifically for determining the seismic performance of non-structural components [8].



(a) Loading protocol of the cyclic test



Figure 5.7 Loading protocol of the cyclic test and cross section dimensions of ceiling grid (unit: mm)

			Number of ceiling components					
Loading protocol	Loading type	Loading direction	Main tee splice	Cross tee latch	Seismic clip at free side	Seismic clip at fixed side		
Monotonic test	Axial	Tension	3	3	4	6		
	test	Compression	3	3	4	6		
	Shear	Major axis	-	6	-	-		
	test	Minor axis	-	6	-	-		
	Bending	Major axis	-	4	-	-		
	test	Minor axis	-	4	-	-		
Cyclic test	Axial test	Tension/Compression	3	3	4	6		
	Shear	Major axis	-	3	-	-		
	test	Minor axis	-	3	-	-		
	Bending	Major axis	-	3	-	-		
	test	Minor axis	-	3	-	-		

Table 5.1 Comprehensive information of test specimens

Table 5.2 Detailed information of ceiling grid

Components	Section (mm)	Unit (kg)	mass	Section (mm2)	area
Main tee	T43×24×0.54×0.27	1.07		38	
Cross tee	T35×24×0.54×0.27	0.31		33	
Wall angle	L22×22×0.5	0.53		22	

5.2.2 Tests results

5.2.2.1 1Axial tests

(a) Axial tests of main tee splices

A total of 9 specimens were tested to obtain the axial capacity of main tee splices under monotonic and cyclic loadings. Two main tee pieces were used to form the test specimens, as shown in Figure 5.8.



Figure 5.8 Axial tests of main tee splices

An example of hysteresis curves is shown in Figure 5.9a. Figures 5.9b shows the skeleton curves obtained from all the axial tests. Figures 5.9c and 5.9d show the damage to the main tee splices. The skeleton curves from the hysteretic curves agree well with the curves from the monotonic test, where the strength capacities of all specimens exceed the threshold of 800N. The damage pattern of the tensile test is similar to that of the cyclic test, both of which are damaged due to the pulling out of the plug and crushing of the locking lance (Figure 5.9c). The results from compressive tests show the main tee splice is prone to the out-of-plane buckling of the plug.



(a) Hysteresis curve



(b) Skeleton curves



- (c) Damage pattern in tensile and cyclic
 - tests

(d) Damage pattern in compressive test



(b) Axial tests of cross tee latches

To evaluate the axial capacity of cross tee latches, a total of 9 specimens were performed as shown in Figure 5.10. The main tee piece is used to simulate the boundary condition.

From Figure 5.11, the tensile strength of cross tee latches is greater than the compressive strength. The damage pattern performs consistently, whether under monotonic or cyclic loading. The damage to the cross tee latches includes the crushing of double lock slot plates, the pushback of the locking lance, buckling of the plug, failure of riveting holes, and even the separation of latches from the cross tee, which is accompanied by the bulging of the socket of the main tee.



(a) Test specimen

(b) Mechanical diagram

(c) Parts of latches



Figure 5.10 Axial tests of cross tee latches

(a) Hysteresis curve

(b) Skeleton curves



(c) Damage pattern of cross tee splices



(d) Damage pattern of main tee piece

Figure 5.11 Results of axial tests of cross tee latches

(c) Axial tests of the peripheral attachments

As shown in Figure 5.12, the axial tests of two types of ceiling peripheral attachments were conducted to obtain and compare the axial capacities of the seismic clip at the fixed and free sides. Two screws at the end of the slot are used for attaching the grid to the seismic clip at the fixed side, while only one screw is put in the middle of the slot at the free side, allowing the grid to slide along its axial direction.



(a) Attachment of seismic clip at the fixed side



(b) Attachment of seismic clip at the free side



(c) Mechanical diagram (d) Parts of seismic clip

Figure 5.12 Axial tests of the peripheral attachments

Figure 5.13 demonstrates the experimental results of main tee configurations subjected to the axial loading. The skeleton curves show that the compressive strength of the peripheral attachment is much larger than the tensile strength, regardless of the boundary conditions. The damage to the peripheral attachment under the tensile loading is accompanied by the pulling out of the clamp of the seismic clip. The damage pattern of the peripheral attachment under the compressive test is the buckling of the grid and wall angle. The test configurations under cyclic loading experience all damage patterns occurring in monotonic tests. It should be noted that the seismic clip remains intact in all tests. Similar to the main tee configurations, the cross tee configurations perform consistently in terms of mechanical responses and damage patterns. It is found that the clamp of the seismic clip without screws used to attach the clip to the wall angle is extremely easy to pull out. The peripheral attachment probably has a better seismic performance by using screws attaching the seismic clip to the wall angle.





(a) Skeleton curves of specimens at

- fixed side
- (b) Skeleton curves of specimens at

free side



(c) Damage pattern of peripheral attachments

Figure 5.13 Results of axial tests of the peripheral attachments

(d) Comparison of energy dissipation

The PA, MT, and CT refer to the peripheral attachment, main tee, and cross tee. The accumulated dissipated energy (ADE) of main tee splices under axial loading is significantly larger than that of cross tee latches. Compared with the cross tee configuration, the main tee configuration performs consistently in terms of ADE either at the fixed or free sides, as shown in Figure 5.14.



(a) Axial tests of grid connections

(b) Axial tests of peripheral attachments

Figure 5.14 Comparison of accumulated dissipated energy for different configurations **5.2.2.2 Shear tests**

(a) Shear tests of cross tee latches in the major axis

Eighteen specimens were conducted to assess the shear capacity of the cross tee latches. A short vertical main tee piece and two cross tee pieces were assembled into the specimens (Figure 5.15a). Figure 5.15c shows the axis definition of the component. Figure 5.16b shows the skeleton curves from the hysteretic curves are in good agreement with the curves collected from the monotonic tests. The cross tee latches are vulnerable to the shear failure of the plug, which is accompanied by the tearing of the main tee socket.



(a) Test specimen



(b) Mechanical diagram (c) Axis definition

Figure 5.15 Shear tests of latches in the major axis



Figure 5.16 Results of shear tests of latches in the major axis

(b) Shear tests of cross tee latches in the minor axis

Similar to the shear tests of cross tee latches in the major axis, nine specimens in total were carried out to obtain the shear capacity of cross tee latches in the minor axis under static tests. A vertical main tee piece and two cross tee pieces were used to form the test configurations, as shown in Figure 5.17.



Figure 5.17 Shear tests of latches in the minor axis

Figure 5.18b presents the skeleton curves from the hysteretic curves that are consistent with the curves from the monotonic tests in initial stiffness of approximately 25N/mm

while are different from the monotonic tests in peak response. The cross tee latches are vulnerable to the buckling of the plug, which is frequently accompanied by the expansion and bulging of the socket of the main tee.





Figure 5.18 Results of shear tests of latches in the minor axis

(c) Comparison of energy dissipation

Figure 5.19 compares the accumulated dissipated energy (ADE) of cross tee latches in the major and minor axes under shear loading. It is noticed that the ADE of test configurations in the major axis is significantly larger than that in the minor axis.



Figure 5.19 Comparison of accumulated dissipated energy for specimens in different axis **5.2.2.3 Bending tests**

(a) Bending tests of cross tee latches in the major axis

A series of test configurations were performed to estimate the bending capacity and energy dissipation capacity of cross tee latches under monotonic and cyclic loadings. A short vertical main tee piece and two cross tee pieces were assembled into the test specimens (Figure 5.20).



(a) Test specimen

(b) Mechanical diagram

Figure 5.20 Bending tests of latches in the major axis

Figure 5.21 shows the mechanical model of cross tee latches in the bending test. The moment and rotation of cross tee latches under bending loading can be expressed as follows:

$$M = \frac{F_y L}{2}$$
(5.6)

$$\theta = \frac{D}{L}$$
(5.7)



Figure 5.21 Mechanical model of cross tee latches in the bending test

Figure 5.22b presents the skeleton curves from the hysteretic curves are in good agreement with the curves from the monotonic tests. The cross tee latches are vulnerable to the crushing of the locking lance, which is frequently accompanied by the tearing of the socket of the main tee piece.



(a) Hysteresis curve



(b) Skeleton curves



(c) Damage pattern of main tee piece



(d) Damage pattern of cross tee piece

Figure 5.22 Results of bending tests of latches in the major axis

(b) Bending tests of cross tee latches in the minor axis

Similar to the bending tests of cross tee latches in the major axis, a total of 7 specimens were carried out to obtain the bending capacity of cross tee latches under static tests, as shown in Figure 5.23.



Figure 5.23. Bending tests of latches in the major axis

Figure 5.24a shows almost no energy dissipation is observed in the moment-rotation response of cross tee latches except a peak due to the engagement of the locking lances. The skeleton curves from the hysteretic curves are basically consistent with that under monotonic tests (Figure 5.24b). It is found that the expansion and bulging of the socket of the main tee piece and the buckling of the plug of the cross tee latch are the most typical damage modes (Figures 5.24c and 5.24d).



(a) Hysteresis curve

(b) Skeleton curves



(c) Damage pattern of main tee piece



(d) Damage pattern of cross tee piece

Figure 5.24. Results of bending tests of latches in the minor axis

(d) Comparison of energy dissipation

Figure 5.25 compares the accumulated dissipated energy (ADE) of cross tee latches in the major and minor axes under bending loading. It is found that the test configurations in the major axis have better energy dissipation and plastic rotation capacity than that of specimens in the minor axis.



Figure 5.25 Comparison of accumulated dissipated energy for specimens in different axis

5.2.2.4 Mechanical property of ceiling components

The mechanical properties of ceiling components are listed in Table 5.3. The loadcarrying ability of cross tee latches under compressive or bending loading and the peripheral attachment applied by tension is less than the threshold value of 800N. The capacities of strength and displacement of cross tee latches in the minor axis under cyclic loading in shear tests are half of that under unidirectional loading. The strength capacity of the peripheral attachment at the fixed side is close to that at the free side, but the corresponding peak displacement at the fixed side is smaller than that at the free side due to the sliding distance of around 19mm.

	Mechanical		Axial test			Shear test		Bending test	
Loading type	property	MTS	CTL	PA_FREE	PA_FIX	CTL_MA	CTL_MI	CTL_MA	CTL_MI
T 11 4 4	TS (N)	1091	976	75	95	1062	928	111	523
Telislie test	TD (mm)	3.0	1.5	30.4	11.7	12.0	32.3	34.0	50.7
Compressive test	CS (N)	1245	578	2200	1856	1064	809	64	332
	CD (mm)	4.6	0.6	27.7	13.9	11.0	27.4	31.8	36.0
	TS (N)	1220	1151	71	86	1012	359	113	271
Cyclic test	TD (mm)	3.8	3.0	30.1	11.5	11.2	15.0	40.3	26.7
	CS (N)	957	475	2220	1990	1222	367	71	298
	CD (mm)	3.1	0.5	28.1	12.4	11.9	16.1	43.1	35.9

Table 5.3 Mechanical properties of ceiling components

Note: TS and TD refer to tensile strength and displacement, respectively; CS and CD refer to compressive strength and displacement, respectively; MTS, CTL, PA, MA and MI refer to main tee splice, cross tee latch, peripheral attachment, major and minor, respectively.

5.2.2.5 Conclusions

The quasi-static tests on the suspended ceiling components were carried out to achieve the failure pattern, strength capacity, and energy dissipation capacity of the ceiling components. Based on the test results, the following conclusions can be drawn:

(1) In the axial tests of main tee splices, the crushing of the locking lance and the buckling of the plug of the splice are the most common type of damage. The strength of all components meets the threshold of 800N. The skeleton curves agree well with the curves obtained from the monotonic tests. The tensile strength of cross-tee latches is greater than the compressive strength. Excluding the damage observed in the main tee splice, the failure patterns of cross tee latches under the axial test include the failure of riveting holes and the bulging of the socket of the main tee. Compared with cross tee latches, the main tee splices have a larger energy dissipation capacity.

(2) The compressive strength of the peripheral attachment is much larger than its tensile strength, regardless of the grid type and boundary condition. The damage to the peripheral attachment includes the pulling out of the clamp, and buckling of the grid and wall angle while the seismic clip remains intact. Compared with cross tee configurations, the main tee configurations perform consistently in terms of ADE either at fixed or free sides.

(3) The cross tee latches in shear tests are vulnerable to the plug of the cross tee latch, which is frequently accompanied by the failure of the socket and flange of the main tee. The response of the monotonic tests is consistent with the cyclic tests in the major axis, while the peak response of the monotonic tests is greater than those of the cyclic tests in

the minor axis. The ADE of specimens in the major axis is significantly larger than that in the minor axis.

(4) The damage pattern of cross tee latches in the major axis under bending loading is similar to that in the minor axis. The specimens in the major axis have better energy dissipation capacity than that in the minor axis.

5.3 Load-displacement hysterical model of suspended ceiling grids

5.3.1 Introduction

The load-displacement recovery force model is a practical mathematical model of the force-displacement relationship curve of the structure or member under reciprocating load loading, which is the basis and key to the seismic analysis of the structure or component. The load-displacement recovery force model includes two parts: the skeleton curve and the hysteresis rule. The skeleton curve defines the control points for the structure or member in different stress stages, and the hysteresis rule expresses the mechanical properties of the structure or member such as strength degradation, stiffness degradation, slip characteristics and energy dissipation properties. The load-displacement restoriing model can be divided into two categories according to the shape of the model curve: one is to describe the curve-type restoring model with more parameters with complex mathematical formulas, such as the Ramberg-Osgood model and the Bouc-Wen model, the curve-type restoring model has the characteristics of high accuracy and continuous change of stiffness, but the model parameter determination and calculation are more complicated. The other type is the simplified polyline restoring model, which is

more widely used in practical engineering because of its simple and practical advantages. Polylinear restoring models are commonly used in bilinear models, degenerate bilinear models, Clough models, Otani models, Takeda models, Munder models and Park models[414]. The methods for establishing load-displacement restoring models of structures or components include theoretical calculation methods, experimental fitting methods and system identification methods, but experimental research is still an important means to determine load-displacement restoring models of structures or components.

Due to the diverse structural forms and complex stress mechanisms of ceiling grid nodes and splicing points, the load-displacement recovery model of ceiling grid nodes and splicing points is still very lacking in domestic and foreign scholars. Soroushian et al. [72] carried out monotonic loading and low cycle reciprocating loading tests on the primary and secondary grid nodes and edge nodes in the American mineral wool board ceiling, and based on the data obtained from the experiment, the load-displacement recovery force model of the nodes was established by using the Pinching4 model. Fiorin et al. [282] carried out monotonic loading and low cycle reciprocating loading tests on grid nodes in suspended ceilings in Europe, and established a load-displacement recovery force model of the joints using the same method as Soroushian et al. [72]. In general, the current research on grid node and splicing point restoring model is relatively insufficient, and China is still in the gap in the research of grid node and splicing point restoring model. Therefore, it is very necessary to study the load-displacement recovery force model of common grid nodes and splicing points in China.

5.3.2 Pinching 4 model

The grid nodes and splicing points of the ceiling exhibit a high degree of nonlinear behavior in seismic action, and it is difficult to analyze them using the commonly used load-displacement recovery force model. The Pinching4 model [415,416] in the finite element software OpenSEES can consider the strength degradation, stiffness degradation, slip characteristics and pinching effect of joints under repeated loading, and can accurately simulate the hysteresis characteristics of various complex nodes, which has good universality, so the Pinching4 model is selected to establish the load-displacement recovery force model of suspended ceiling grid nodes and splicing points.

Figure 5.26 shows a schematic diagram of the Pinching4 model. The Pinching4 model includes a multilinear skeleton curve under monotonic loading, which requires 16 parameter definitions ((ePd1, ePf1), (ePd2, ePf2), (ePd3, ePf3), (ePd4, ePf4), (eNd1, eNf1), (eNd2, eNf2), (eNd3, eNf3), (eNd4, eNf4)); The three-line unload-reload path under reciprocating loading requires 6 parameter definitions (rDispP, rForceP, uForceP, rDispN, rForceN, uForceN); 3 failure criteria, requiring 15 parameter definitions, take into account unloading stiffness degradation (gK1, gK2, gK3, gK4, gKLim), reload stiffness degradation (gD1, gD2, gD3, gD4, gDLim) and strength degradation (gF1, gF2, gF3, gF4, gFLim), energy degradation (gE) and damage type (dmgType).



Figure 5.26 Pinching4 model

The calculation of the three failure criteria in the Pinching4 model, namely unloading stiffness degradation, heavy load stiffness degradation and strength degradation, is based on the generalized damage index theory proposed by Park and Ang [417], and the degradation can be calculated by using damage factors [415,416], as follows:

$$\delta_{i} = \left[\alpha_{1} \cdot (d_{\max})^{2} + \alpha_{2} \cdot \left(\frac{E_{i}}{E_{monotonic}} \right)^{\alpha_{4}} \right] \leq limit \qquad (5.6)$$

where:

$$d_{\max} = \max\left[\frac{\left(d_{\max}\right)_{i}}{def_{\max}}, \frac{\left(d_{\min}\right)_{i}}{def_{\min}}\right]$$
(5.7)

$$E_i = \int_{load \ history} dE \tag{5.8}$$

$$E_{monotonic} = gE \cdot \int_{monotonic \ load \ history} dE \tag{5.9}$$

where δ_i is the damage factor; *i* is the current deformation increment; $\alpha_1 \propto \alpha_2 \propto \alpha_3$ and α_4 are the damage factor correction factors; V is hysteresis energy consumption; $E_{monotonic}$ is monotonic energy consumption when loaded into destruction; def_{max} is the deformation of positive loading failure; def_{min} is the deformation of negative loading failure; $(d_{max})_i$ is the

historical maximum deformation; $(d_{\min})_i$ is the historical minimum deformation The degradation of unloading stiffness (see Figure 5,27) is calculated as follows:

$$k_i = k_0 \cdot (1 - \delta k_i) \tag{5.10}$$

where k_i is the current unloading stiffness; k_0 is the initial unloading stiffness without damage; δk_i is the current unloading stiffness damage factor

The reload stiffness degradation (see Figure 5.28) is calculated as follows:

$$(d_{\max})_i = (d_{\max})_0 \cdot (1 + \delta d_i) \tag{5.11}$$

where $(d_{\text{max}})_i$ is the deformation at the end of the current reload; $(d_{\text{max}})_0$ is the historical maximum deformation; δd_i is the current reload stiffness damage factor The strength degradation (see Figure 5.29) is calculated as follows:

$$\left(f_{\max}\right)_{i} = \left(f_{\max}\right)_{0} \cdot \left(1 - \delta f_{i}\right) \tag{5.12}$$

where $(f_{\text{max}})_i$ is the maximum strength of the current skeleton curve; $(f_{\text{max}})_0$ is the maximum strength of the initial skeleton curve without damage; δf_i is the current intensity damage factor.



Figure 5.27 Degradation of unloading stiffness



Figure 5.28 Reload stiffness degradation



Figure 5.29 Strength degradation

5.3.3 Determination of Pinching4 model parameters

Because of the complexity of the structure of grid nodes and splicing points, it is difficult to determine the parameters of the Pinching4 model of grid nodes and splicing points through the theoretical calculation formula, so this dissertation determined calibrates the model parameters according to the test results of low cycle loading of grid nodes and splicing points, and the principle of calibration is to ensure that the hysteresis curve and cumulative energy dissipation capacity of grid nodes and splicing points obtained according to the Pinching4 model are basically consistent with the test results. The Pinching4 model parameters in this chapter are determined by reference to those used in similar published papers [282]. In addition, based on the literature survey results on the Pinching4 model, it is found that when the load-displacement restoring force model of complex analytical objects is established using the Pinching4 model, the parameters of the Pinching4 model are generally calibrated and determined according to the corresponding test results [418-419].

Based on the my experience and the above literature research conclusions, the Pinching4 model parameters of grid nodes and splicing points can be determined from the following three aspects: (1) The skeleton curve parameters of the Pinching4 model are determined according to the characteristic points of the specimen skeleton curve. (2) The unloading-reloading path parameters of the Pinching4 model are determined according to the shape characteristics of the hysteresis curve of the specimen. (3) The failure criterion parameters of the Pinching4 model are determined according to the unloading stiffness degradation, heavy loading stiffness degradation and strength degradation of the hysteresis curve of the specimen.

5.3.4 Validation of load-displacement hysteretic model

According to the results of the low cyclic loading test of grid nodes and splicing points, the parameters of the Pinching4 model of each specimen were calibrated, and the loaddisplacement hysteretic model of typical specimens was established by using the calibrated Pinching4 model, and the applicability of the Pinching4 model to the grid nodes and splicing points was discussed.

5.3.4.1 Hysteresis curve comparison

Figure 5.30 shows the comparison of the load-displacement hysteresis curve and the test curve obtained by the typical specimen based on the Pinching4 model, and the hysteresis law reflected by the recovery force model of the specimen has some error difference but is basically acceptable consistent with the hysteresis law of the test curve, and the hysteresis curve shape of the two is in good agreement.





Figure 5.30 Comparison of hysteresis curve obtained by typical specimens based on Pinching4 model with test results

5.3.4.1 Comparison of cumulative energy consumption

Figure 5.31 shows the comparison between the cumulative energy consumption obtained by the typical specimen based on the resirestoring lience model and the cumulative energy consumption of the test with the number of loading turns, and the cumulative energy consumption of the two increases with the increase of the number of loading turns, and the results are similar under the same number of loading turns, indicating that the Pinching4 model can better simulate the cumulative energy consumption of grids nodes and splicing points, and has acceptable reliability and accuracy.





(g) Fixed end point axial force (h) Semi end point axial force Figure 5.31 The cumulative energy consumption of a typical specimen based on the Pinching4 model is compared with the test results

5.3.5 Establishment of general load-displacement recovery force model

The feasibility of establishing a grid node and splicing point load-displacement recovery force model according to the Pinching model is verified earlier, but for the same type of grid node or splicing point specimen, the specimen will inevitably produce initial defects in the process of making connections and loading and installation, which will cause certain test errors, and due to the small size of the specimen in this paper, the bearing capacity is weak, and the sensitivity to the production and installation accuracy is strong, resulting in a certain discreteness in the test results of the same type of specimen. Therefore, it is necessary to establish a general load-displacement recovery force model of grid nodes and splicing points.

5.3.5.1 The process of establishing a general recovery force model

Taking the shear specimen of the main and cross tee nodes as an example, the determination process of the skeleton curve and hysteresis rule parameters of the general load-displacement recovery force model of grid nodes and splicing points is illustrated. According to the test results of the main shaft shear test pieces of three main and secondary grid nodes, the load-displacement recovery force model of each specimen is first established by using the Pinching4 model, and when the test results are consistent with the hysteresis curve and cumulative energy consumption curve of the model, the Pinching4 model parameters of each specimen can be determined. Secondly, the model parameters of all specimens in the same group were analyzed, and it was found that the model parameters of all specimens in the same group were better consistent with the test results by taking the median value than the average value. This is because the relative average value of the median value is not affected by the extreme value and the

performance is relatively stable, so the median value corresponding to the restoring model parameters of the same group of specimens is selected as the representative value of the general restoring model parameters. Table 3.3 shows the results of establishing the general load-displacement recovery force model parameters of the main and secondary grid joint spindle shear test pieces.

Pinching4 model	Parameter	CJS-A3-C1	CJS-A3-C2	CJS-A3-C3	Value
	ePf ₁ /N	150	250	150	150
	ePf ₂ /N	550	740	651	651
	ePf ₃ /N	1001	998	1038	1001
Desitive alcolator	ePf ₄ /N	707	458	92	458
Positive skeleton	ePd ₁ /mm	0.5	0.7	0.5	0.5
	ePd ₂ /mm	4.2	5.5	6.5	5.5
	ePd ₃ /mm	10.5	10.4	12.7	10.5
	ePd ₄ /mm	13.8	12.1	17.7	13.8
	eNf ₁ /N	-200	-200	-200	-200
	eNf ₂ /N	-600	-650	-620	-620
	eNf ₃ /N	-1158	-1259	-1248	-1248
Nagativa skalaton	eNf ₄ /N	-1143	-252	-556	-556
Regative skeletoli	eNd ₁ /mm	-0.5	-0.5	-0.5	-0.5
	eNd ₂ /mm	-4.2	-5.0	-5.2	-5.0
	eNd ₃ /mm	-10.4	-12.7	-12.7	-12.7
	eNd ₄ /mm	-12.5	-16.0	-16.2	-16.0
	rDispP	0.6	0.6	0.6	0.6
	rForceP	0.2	0.2	0.2	0.2
Unload-reload	uForceP	-0.01	-0.01	-0.01	-0.01
path	rDispN	0.6	0.3	0.6	0.6
	rForceN	0.2	0.2	0.2	0.2
	uForceN	-0.01	-0.01	-0.01	-0.01
Unloading	$gK_i(i=1-4)$	0	0	0	0
stiffness	σK	0	0	0	0
degradation	gittim	0	0	0	0
Reload stiffness	$gD_i(i=1-4)$	0	0	0	0
degradation	gD_{Lim}	0	0	0	0
Strength	$gF_i(i=1-4)$	0	0	0	0
degradation	gF_{Lim}	0	0	0	0
Energy degradation	gE	1	1	1	1
Type of injury	dmgTvpe	cvcle	cycle	cvcle	cvcle

Table 5.4 Pinching4 model parameters of the main and secondary grid node spindle shear specimens

According to the above calibration method for the parameters of the general recovery force model of the specimen, Figure 5.32 shows the skeleton curve results of the general



load-displacement recovery force model of all specimens.



Figure 5.32 Skeleton curve of general load-displacement restoring force model of grid nodes and splicing points of suspended ceilings

Table 5.5 shows the recommended values for the general load-displacement restoring model parameters of grid nodes and splicing points in the ceiling. It should be noted that the secondary axis shear specimen of the primary and secondary grid nodes is symmetric with respect to the secondary axis, and theoretically its load-displacement recovery force model is symmetric with respect to the origin, so the recommended values of the general restoring force model parameters of the secondary axis shear specimens of the primary and secondary grid nodes given in Table 3.4 are the results of further optimization, and the positive and negative parameters are symmetric with respect to the origin. Similarly, the secondary axis bending specimen of the primary and secondary grid nodes is symmetrical with respect to the secondary axis, but there are two hysteresis modes due to the randomness of its failure, but the theoretical restoring model parameters of these two hysteresis modes correspond positively and negatively, so the recommended values of the general restoring model parameters of the secondary axis bending specimens of the secondary axis bending specimens of the primary and secondary axis bending specimens of the secondary axis bending specimens of the primary and secondary axis bending specimens of the secondary axis bending specimens of the secondary axis bending specimens of the primary and secondary axis bending specimens of the secondary axis bending specimens of the primary and secondary grid nodes given in Table 3.4 are also the results of further

optimization. In addition, due to the influence of test conditions and the number of specimens, the recommended values of the general load-displacement recovery force model parameters of various grid nodes and splicing points may have certain deviations, and a certain number of relevant tests can be supplemented to further optimize them in the later stage.

Pinching4 Model	Paramter	JAM	JAC	CJSA	CJSB	CJBA	CJBB ¹	CJBB ²	WJE	WJD
	ePf ₁	450	472	150	75	0.15^{*}	0.15^{*}	0.15^{*}	0.1	0.1
	ePf ₂	756	889	651	230	4.44^{*}	0.20^{*}	3.20^{*}	30	1
	ePf ₃	1206	1216	1001	350	7.97^{*}	0.30^{*}	19.62*	109	25
Positive	ePf ₄	507	378	458	100	7.90^{*}	0.60^{*}	4.50^{*}	111	50
skeleton	ePd ₁	0.3	0.6	0.5	0.5	0.0004^{*}	0.0002^{*}	0.0002^{*}	0.01	0.01
	ePd ₂	1.7	1.7	5.5	8.5	0.139^{*}	0.033^{*}	0.13^{*}	1.9	18.0
	ePd ₃	3.6	3.1	10.5	17.0	0.24^{*}	0.1^{*}	0.2^*	8.2	20.7
	ePd ₄	4.7	3.6	13.8	21.5	0.268^{*}	0.21^{*}	0.23^{*}	12.6	23.8
	eNf_1	-450	-472	-200	-75	-0.15*	-0.15*	-0.15*	-4	-4
	eNf ₂	-650	-361	-620	-230	-2.43*	-3.20*	-0.20*	-173	-9
	eNf ₃	-979	-290	-1248	-350	-2.93*	-19.62*	-0.30*	-566	-587
Negative	eNf ₄	-196	-261	-556	-100	-5.432*	-4.50*	-0.60*	- 2385	- 2336
skeleton	eNd ₁	-0.3	-0.6	-0.5	-0.5	-0.0007^{*}	-0.0002^{*}	-0.0002*	-0.01	-0.01
	eNd ₂	-1.4	-0.8	-5.0	-8.5	-0.105^{*}	-0.133*	-0.033*	-1.7	-15.0
	eNd ₃	-3.3	-3.3	-12.7	-17.0	-0.204*	-0.2*	-0.1*	-4.4	-19.3
	eNd ₄	-4.7	-5.4	-16.0	-21.5	-0.287^{*}	-0.23*	-0.21*	-12.7	-27.9
	rDispP	-0.55	0.45	0.6	0.5	0.6	0.8	0.8	0.5	0.5
	rForceP	0.01	0.1	0.2	0.2	0.0001	0.13	0.13	0.5	0.01
Unload-	uForceP	-0.01	-0.3	-0.01	-0.1	-0.01	-0.01	-0.01	-0.01	-0.13
reload path	rDispN	-0.55	-0.8	0.6	0.5	0.6	0.8	0.8	0.45	0.95
	rForceN	0.01	0.2	0.2	0.2	0.0001	0.13	0.13	0.03	0.01
	uForceN	-0.01	-0.01	-0.01	-0.1	-0.01	-0.01	-0.01	-0.1	-0.1
Unloading	gK_i	0	0	0	0	0	0	0	0	0
degradation	gK_{Lim}	0	0	0	0	0	0	0	0	0
Reload	gD_i	0	0	0	0	0	0	0	0	0
stiffness degradation	$gD_{Lim} \\$	0	0	0	0	0	0	0	0	0
Strength	gF_i	0	0	0	0	0	0	0	0	0
degradation	gF_{Lim}	0	0	0	0	0	0	0	0	0
Energy degradation	gE	1	1	1	1	1	1	1	1	1
Type of injury	dmgType	cycle	cycle	cycle	cycle	cycle	cycle	cycle	cycle	cycle

Table 5.5 Recommended values for general load-displacement restoring model parameters

Note: i=1-4 in the table. JAM is the axial stress test piece of the main tee splicing point, JAC is the axial stress test piece of the main and cross tee nodes, CJSA is the main and cross tee node main axis shear test piece, CJBB is the main and cross tee node secondary axis shear test piece, CJBA is the main and secondary grid node main axis bending test piece, CJBB is the main and cross tee node secondary axis bending test piece, WJD is the semi-free side node axial force test piece. The units of load and displacement corresponding to the skeleton curve parameters in the Pinching4 model are N and mm, respectively, and the units of bending moment and angle corresponding to the skeleton parameters with the mark "*" in the bending specimen of the main and cross tee nodes are kN·mm and rad, respectively. There are two types of restoring force models for secondary axial bending specimens of main and cross tee nodes, CJBB1 and CJBB2 in the table correspond to the two hysteresis types.

5.3.5.2 Validation of a general load-displacement restoring model

Figure 5.33 shows the comparison of the hysteresis curve obtained by using the general load-displacement recovery force model for the grid nodes and splicing points of the ceiling compared with the test hysteresis curve. It can be seen from the figure that the degree of agreement between the two is acceptable, which proves the rationality of the general load-displacement recovery force model of grid nodes and splicing points. It should be noted that the general restoring model proposed in this article is only suitable for the analysis and application of the same product specimens as this paper.





Figure 5.33 Comparison of hysteresis curve and test hysteresis curve obtained based on the general load-displacement recovery force model

5.4 Numerical simulation (continued)

5.4.1 Main tee connections
According to the hysteretic characteristic of main tee connections investigated above, for the higher-fidelity model, the Takeda model along with the *Multi-Linear Plastic Link* element is used to simulate the axial force-displacement response of MT connections. The Takeda model was first proposed by Takeda [420.], capable of considering pinching, stiffness degradation, and strength degradation. The typical loop shapes generated by this model are plotted in Figure 5.34 Note that the Takeda model available is a simplified version and does not consider the stiffness degradation on its unloading branch. In other words, the unloading stiffness (K_y) always follows the initial loading stiffness (K_0). Herein, the force-displacement relationship of main tee connections is adopted to define the skeleton curve of the Takeda model.



Figure 5.34 Takeda hysteretic model

Figure 35(a) compares the hysteretic curves between the numerical simulation and test results. A fairly good agreement is obtained. As can be seen, there is no stiffness degradation in the unloading branches in the simulation because of the simplified Takeda

model. Furthermore, the force versus loading step is compared with the test data, as depicted in Figure 5.35(b). Good agreement is observed. In addition, the energy dissipation of main tee connections as per each cycle and accumulative energy is analyzed, as shown in Figure 5.36. It can be seen that the simulated result of energy dissipation is in good agreement with the test result. Overall, the adopted Takeda model is suitable to characterize the axial hysteretic behavior of main tee connections.



(a) hysteretic curve and

(b) force-loading step relationship.

Figure 5.35 Comparison between numerical simulation and experimental results:



(b) cumulative

Figure 5.36 Comparison of energy dissipation

5.4.2 Main cross tee connections

Similarly, the Pivot model is employed in this study to simulate the axial nonlinear behavior of main cross tee connections according to the hysteretic characteristic of this type of connection. This model was first proposed by Dowell et al. [420.] to predict the hysteretic behavior of RC members under cyclic loading. The Pivot model has a great flexibility in modeling unsymmetrical tension-compression behavior by specifying the hysteretic parameters separately. The hysteretic model is illustrated in Figure 5.37 (F_{y1} and F_{y2} represent the yielding force in tension and compression, respectively). Basic hysteretic rules of this model are controlled by four parameters including α_1 , α_2 , β_1 and β_2 , which define the pivot points for unloading from positive peak force to zero, unloading from negative peak force to zero, loading from zero to a positive direction, and loading from zero to the negative direction, respectively. In this study, the four parameters are set as $\alpha_1 = 25$, $\alpha_2 = 0.05$, $\beta_1 = 0.01$ and $\beta_2 = 0.9$, respectively, to best fit the experimental hysteresis curves of MCT connections through parameter identification analysis. In addition, the parameter η needs to be defined considering the degradation degree of the elastic slope after inelastic deformation, and its value is usually set to zero [421].



Figure 5.37 Pivot hysteretic model

Similar to the Takeda model, the skeleton curve of the Pivot model is defined based on the defined generic force-displacement relationship of main cross tee connections mentioned above. The comparison of hysteretic curves and force-loading step relationship between numerical simulation and experimental results is shown in Figure 5.38. The noticeable pinching effect in both positive and negative directions is well captured. A comparison of dissipated energy is shown in Figure 5.39. The numerical simulation results as per each cycle and accumulative energy agree well with the test result. In sum, the adopted Pivot hysteretic model is appropriate to characterize the hysteretic behavior of main cross tee connections.



Figure 5.38 Comparison between the numerical simulation results and experimental results



Figure 5.39 Comparison of energy dissipation

Note that the Multi-Linear Plastic Link element is successfully applied to simulate the nonlinear behavior of grid connections. However, it is still a simplified method incapable of simulating the failure mode of grid connections. To comprehensively assess the

seismic behavior of grid connections, developing a more refined model for grid connections is still needed. In the end, the 1st- 4th modes mode shapes is given below (Type A1 suspended ceiling system)



(a) The1st- 4th modes of type A1 suspended ceiling system





(b) The1st- 4th modes of type B suspended cable tray system

Figure 5.40 The1st- 4th modes of suspended nonstructural systems

Also, for systematic consideration, since this dissertation will not talk more about the suspended cable tray system numerical simulation, because the next chapter will applying the proposed Bayesian inverse problem uncertainty quantification to make inferences for suspended ceiling system. Only the 1st- 4th modes of type B of suspended cable tray system are given above in Figure 5.40b.

5.5 Comparison of simulation and experimental results

5.5.1 OpenSEES simplified numerical simulation

Figure 5.41 compare the simulated acceleration time history and experimental results of the tested SCS with semi-free boundary condition. The comparison of the simulated displacement time history and experimental results are shown in Figure 5.42. The mean absolute errors of peak acceleration (the absolute difference between the simulated and test result divided by the test result) within the overall time history for 4 Runs in Figure 5.32 are 43.16%, 6.01%, 9.95%, and 21.99%, respectively. This may be due to the errors caused by model simplification and inaccurate parameters models. The mean absolute errors of peak displacement within the overall time history for 4 Runs in Figure 5.33 are 12.73%, 10.07%, 10.84%, and 17.67%, respectively, and there are some delay effect of the response for displacement. Generally, the proposed simplified analytical model's calculated acceleration and displacement responses agree well with the test results. A more precise three-dimensional numerical model employing accurate parameters needs to be studied in future work. Notably, there are some differences between simulation and experimental results in the high-frequency components of acceleration.



(a) 0.405g (X) & 0.377g (Y) (Run 16)



(b) 0.15g (X) Sweep (Run 18)



(c) 0.15g (X) & 0.15g (Y) Sweep (Run 22)



(d) 0.25g (X) Sweep (Run 24)

Figure 5.41 Comparison of acceleration time history between simulation and experimental results



(a) 0.405g (X) & 0.377g(Y) (Run 16)



(b) 0.15g (X) Sweep (Run 18)



(c) 0.15g (X) & 0.15g (Y) Sweep (Run 22)



(d) 0.25g (X) Sweep (Run 24)

Figure 5.42 Comparison of displacement time history between simulation and experimental results

5.5.2 Higher fidelity numerical simulation

The acceleration responses of A12 in the X direction and A3 in the Y direction measured in the shaking table tests are used to compare with the simulation results. For demonstration, comparisons under Run 14(seismic wave with the largest intensity) and Run 16,18,22 (sine waves in single direction and bi-direction) are shown in Figure 5.43. As can be seen, the simulation results agree good with the test results in most cases. Some of the differences also exist.



(e) 0.15g (X)&0.15g (Y) Sweep (Run 22)(X) (f) 0.15g (X)&0.15g (Y) Sweep (Run 22)
(Y)

Figure 5.43 Acceleration time history under selected input motions

Similarly, for demonstration, the displacement responses of D40 in the X direction and D8 in the Y direction are employed to compare experimental displacement responses with the simulation results. The comparison of experimental and numerical simulation results of displacement responses under Loading No. 7 (seismic wave with the largest intensity) and Loading No. 8-10 (sine waves in a single direction and bi-direction) are shown in Figure 5.44. The simulation results agree well with the test results. However, it also has some delay effect.



(a) 0.149g(X) & 0.132g(Y) (Run 14)(X)



(c) 0.405g (X) & 0.377g (Run 16)





(b)0.149g(X) & 0.132g(Y) (Run 14)(Y)



(d) 0.15g (X) Sweep (Run 18)



(e) 0.15g (X)&0.15g (Y) Sweep (Run 22)(X) (f) 0.15g (X)&0.15g (Y) Sweep (Run 22)
(Y)

Figure 5.44 Displacement time history under selected input motions

5.6 Conclusions

A refined computational numerical model for the SCS is built and its effectiveness is validated by dynamic shaking table tests. The main conclusions are obtained as follows:

- (1) The proposed model of SCS is capable of modeling the nonlinear behavior of grid connections, impact, and friction behavior at the perimeter. The Takeda model and Pivot model are successfully used to model the hysteretic behavior of main tee connections and main-cross tee connections, respectively.
- (2) Both experimental results and numerical simulation results indicate that the suspended ceiling responds as a whole body in both directions. The proposed model of a suspended ceiling with free boundary conditions can almost reproduce both acceleration and displacement responses.
- (3) Even though the most part agreement between simulation and experiments, there are still spaces to calibrate or making inference of it with inverse problems uncertainty quantified by Bayesian inference, which will be investigated in the next chapter 6.

CHAPTER SIX

MACHINE LEARNING-BASED DATA AND MODEL DRIVEN BAYESIAN INVERSE PROBLEMS OF UNCERTAINTY QUANTIFICATION APPLICATION FOR NON-STRUCTURAL SYSTEMS

6.1 Data

Type C suspended ceiling system is taken as the research target to make uncertainty quantification and inverse problem Bayesian inference. As mentioned in the above chapter, the acceleration response simulation performance is relatively worse than the displacement response, which has some lag effect because our goal is to generate a large amount of data based on the established numerical simulation (data and model-driven) combined with the actual seismic table vibration test data (data-driven) to solve the inverse problem of uncertainty parameters.

6.2 Training points

As previously mentioned, the LHS method is used for sampling combinations of training points, which can consider the global picture while minimizing computing resource consumption. For each pair of the two sets of training data, we generated fifteen pairs of 1000 Latin Hypercube samples and selected five pairs of the training point data to be illustrated (also see Figure 6.1). And this will be used for forward running (model-driven) the computational numerical model of SNS systems. It uses the maximin method projected in 2D (input: 0-1 uniform space, output: 0-1 uniform space). This particular design relies on the distance criterion, and the final design is a result of maximizing the minimum distance between points. A detailed account of the method process can be

found in chapter three.



(a) A_cross and E LHS sampling



(b) E and G LHS sampling



(d) K_cc1 and K_cc2 smapling



(e) A_main and K_mc smapling

Figure 6.1 Latin Hypercube samples results

6.3 Priors

Regarding the inverse problem parameters and the prior, the first thing to note is that this Bayesian inference problem does not involve nonlinearities such as collision nonlinearity and connection nonlinearity problems because the training input is an inverse (calibration) parameter, and the output is the first six modes of frequency response data. Concerning the nonlinearities we will discuss in detail in future studies, and also because the selected surrogate model is still the shallow model, and it is still necessary to have a deep Gaussian Process model for high-dimensional nonlinear problems or the help of Deep neural network, so the parameters modified in this section are linear model parameters, according to the discussion and experience of numerical simulation in the previous chapter, the fifteen selected inverse problem parameter names, symbols, prior, and observation prior data are as follows:

Inverse problem parameters prior				
Parameter label	Parameter meaning	Prior distribution		
1. A_cross	Cross tee cross-section area	U~(30, 36)		
2. E	Elastic modulus	U~ (19, 2.3) 10^5		
3. G	Shear modulus	U~ (7.4,8.2) 10^4		
4. J_cross	Cross tee moment of Torsion	U~ (4800,5100)		
5. I_{y} _cross	Y-dir moment of inertia	U~ (4000,4700)		
6. I_{z} cross	Z-dir moment of inertia	U~ (650,750)		
7. K_rod	Threaded rods connection stiffness	U~ (0.5,3)		
8. K_friction	Friction elastic stiffness	U~ (15,25)		
9. K_cc1	X-dir Cross tee connections stiffness	U~ (750,850)		
10. K_cc2	Y-dir Cross tee connections stiffness	U~ (75,125)		
11. K_cc3	Z-dir Cross tee connections stiffness	U~ (10,35)		
12. A_main	Main tee cross-section area	U~ (35,42)		
13. J_ main	Cross tee moment of Torsion	U~ (7900,8500)		
14. I_y _cross	Y-dir moment of inertia	U~ (7250,7800)		
15. K_mc	X-dir Main tee connnections stiffness	U~ (1400,1600)		
Observations data prior				
mode1	Suspended ceiling system frequency mode 1	(7.5 + 0.02)		
mode2	Suspended ceiling system frequency mode 2	(8.1+- 0.006)		
mode3	Suspended ceiling system frequency mode 3	(8.9+-0.04)		
mode4	Suspended ceiling system frequency mode 4	(9.3+-0.05)		
mode5	Suspended ceiling system frequency mode 5	(9.5+-0.01)		
moded6	Suspended ceiling system frequency mode 6	(9.8+-0.02)		

Table 6.1 Inverse problem parameters prior and Observations data prior

6.4 Sensitivity analysis

According to the introduction in the previous section, in order to save computing resources, find the uncertainty parameters that account for the main contribution, and the results are as follows:



Figure 6.2 Sensitivity analysis result

To conserve computing resources, a sensitivity analysis was conducted to identify the key uncertainty parameters (refer to Figure 10). The findings indicate that the 4th and 13th parameters have a negligible effect (less than 1%) on the results. Consequently, we retained the remaining thirteen inverse problem parameters and specified their prior ranges, which are listed in Table 4. A detailed account of the analysis process can be found in chapter 3.

Our sensitivity analysis revealed that certain parameters, such as those associated with

main tee connections, cross tee connections, and boundary connections, significantly impact the accuracy of the model's predictions. During the validation process, for instance, we observed that the acceleration prediction results were not precise between 60-61 seconds. This inaccuracy can be attributed to bias and uncertainty in the parameters, which may have been affected by energy dissipation and friction effects in the connection and friction parts, leading to a notable difference in the amplitude. We will provide further insights into these findings in section 6.8 of this chapter.

Inverse problem parameters prior after sensitivity analysis				
Parameter label	Parameter meaning	Prior distribution		
A_cross	Cross tee cross-section area	U~(30, 36)		
Е	Elastic modulus	U~ (19, 2.3)*100000		
G	Shear modulus	U~ (7.4,8.2) *10000		
I _y _cross	Y-dir moment of inertia	U~ (4000,4700)		
I_z_cross	Z-dir moment of inertia	U~ (650,750)		
K_rod	Threaded rods connection stiffness	U~ (0.5,3)		
K_friction	Friction elastic stiffness	U~ (15,25)		
K_cc1	X-dir Cross tee connections stiffness	U~ (750,850)		
K_cc2	Y-dir Cross tee connnections stiffness	U~ (75,125)		
K_cc3	Z-dir Cross tee connections stiffness	U~ (10,35)		
A_main	Main tee cross-section area	U~ (35,42)		
I _y _cross	Y-dir moment of inertia	U~ (7250,7800)		
K_mc	X-dir Main tee connnections stiffness	U~ (1400,1600)		

Table 6.2 Pre-defined prior box for the 13 inverse problem parameters

6.5 Surrogate Gaussian Process model training

For a non-structural system with n degree of freedom (DOFs) can be written as

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) (6.1)$$

where, M, C, K are the mass, damping, and stiffness matrix, respectively; $\ddot{x}(t)$, $\dot{x}(t)$ and

x(t) are the acceleration, velocity, and displacement of the non-structural system. F(t) is the applied excitation force on the system.

When the accelerometers are installed on the system, the measured acceleration response can be expressed in a discrete form as

$$y_{obs}(x,t) = d\ddot{x}(t) + \epsilon_i (6.2)$$

where, y_{obs} is the observed or measured response at time instant t. $\ddot{x}(t)$ and ϵ_i are the values of acceleration response of the system and observation noise. The matrix d is a sensor placement matrix which associated with the locations of accelerometers.

In general, we now consider the Kennedy and O'Hagan's formulation [19], where data observation (frequency response) are computational numerical simulation are available, namely $y_{obs}(x)$ and $y_{sim}(x,t)$. The design variable x is assumed to be taking values within a feasible design space $X \subset \mathbb{R}^D$, and θ is a set of parameters to be calibrated or inferenced.

As mentioned in the chapter on methods and framework, The relationship between data observation and simulation is assumed to be

$$y_{obs}(x,t) = \alpha y_{sim}(x,\theta,t) + \delta(x,t) + \epsilon(x)$$
(6.3)

Where $\delta(x)$ is a discrepancy term that is statistically independent of $\eta(x, \theta, t)$ and $\epsilon(x)$ which accounts for observation noise. The coefficient α satisfies

$$\alpha = \frac{cov[y_{obs}(x, t), \eta_{sim}(x, \theta, t)]}{var[y_{sim}(x, \theta, t)]}$$
(6.4)

which account the further we take $y_{sim}(x, \theta, t)$, $\delta(x)$ to be Gaussian Process with zero mean and variances $\sigma_{sim}^2 \gamma_{sim}(x, x')$ and $\sigma_{\delta}^2 \gamma_{\delta}(x, x')$, where γ_{η} and γ_{δ} are correlation kernels, as mentioned in chapter 3.

To optimize kernel selection for Gaussian process regression, we explore different kernels as candidates and perform cross-validation to select the best one. The process involves dividing the data into training and validation sets, fitting the model with each candidate kernel on the training set, and evaluating the performance of the mean squared error score on the validation set. After selecting the appropriate kernel, hyperparameter tuning is performed to train the final model. Refer to section 3.1 for further details. By following this process, we can select the most appropriate kernel as squared exponential kernel:

$$\gamma_{\delta}(x, x') = \exp\left[-\sum_{i=1}^{D} \frac{(x_i - x_i')^2}{l_{i,\delta}}\right] (6.5)$$
$$\gamma_{sim}(x, x') = \exp\left[-\sum_{i=1}^{D} \frac{(x_i - x_i')^2}{l_{isim}}\right] (6.6)$$

where $l_{i,sim}$, $l_{i,\delta}$ are the correlation length or length scale for the two kernels.

To optimize the performance of the selected kernel, we can perform hyperparameter tuning using maximum likelihood estimation to find the optimal values for the kernel's hyperparameters. This will help achieve the best performance for the final model. Chapter 3 provides more details on the process of hyperparameter tuning, and its algorithm is shown as follows.

It is essential to note that the chosen splitting of the LHS sample data into training, validation, and testing sets is a widely accepted and well-established approach in the field of machine learning. This approach ensures that the model can generalize well to unseen data, which is crucial for the practical application of the model.

Algorithm 6.1 Suspended non-structural system G.P. Surrogate Model training

Input:

Training data (**X**, **y**); hyperparameters θ , covariance function k; σ_n^2 ; test point (**x**, **y**);

Output:

Log marginal likelihood log p(y); prediction $f_*, \mathbb{V}[f_*]$ 9 Initialization 10 while the training accurancy has not converged do 11 For i={1,...n} do 12 3 $L := \text{cholesky}(K + \sigma_n^2 I)$ $\alpha := L^T \setminus (L \setminus y)$ $f_* = k_*^T \alpha$ $v := L \setminus k_*$ $\mathbb{V}[f_*] := K(x_*, x_*) - v^T v$ $\log p(y|\mathbf{X}) := -\frac{1}{2}y^T \alpha - \sum_i \log L_{i,i} - \frac{n}{2}\log 2 * \pi$ 13 end 14 15 end 16 Return $logp(y|\mathbf{X}), f_*, \mathbb{V}[f_*]$

It is worth noting that in this study, we utilized a standard approach for splitting the LHS sample data into training, validation, and testing sets. Specifically, 70% of the LHS samples were used for training the Gaussian process, 20% for validation, and the remaining 10% for testing. To evaluate the performance of our model, we measured the loss and accuracy of the training process. The final training data had a loss of 0.343, while the test data had a loss of 0.328, indicating good performance and generalization ability of the model. Furthermore, the training accuracy was 90.189%, and the testing accuracy was 90.527%, as shown in Figure 11. These results demonstrate the effectiveness of our methodology in predicting the behavior of the system under study. Compared to traditional physical models, our surrogate model approach is significantly faster while achieving good extrapolation results, where our approach achieved a several

hundred-fold improvements in time. The surrogate models also enable us to consider a larger range of input parameters in the future, which is important for SNS applications.



Figure 6.3 G.P. surrogate model training iteration result

6.6 Black Box Variational Inference

According to the discussion in the previous chapter, in order to avoid inefficiency, we substitute Markov Chain Monte Carlo (MCMC) or Approximate Bayeisan

Computation(ABC) to use variational inference for posterior distribution estimation, and since mean field variational inference has conjugate requirements for the variational distribution q(x), as mentioned before, we propose to use black box variational inference for analysis. First, we determine the analytic solution form of the joint posterior distribution. Finally, it should be noted that in the posterior distribution inference, we construct 5 sets of model types for inference to provide a judgment basis for the subsequent MDL model selection.

Since we will consider the frequency response as the output, then the t in the equation will be omitted in the below content. Assume a set of computational numerical simulations and data observations are available, namely, $D_{sim} = \{x_i, \theta_i, \eta_i\}_{(i=1)}^{N_{sim}}$, $D_{obs} = \{x_i^*, \theta_i, y_i\}_{(i=1)}^{N_{obs}}$, respectively.

Now name $k = sim, \delta$, and $R_k(D_k)$ is the correlation matrix with $\gamma_k(x, x') \in D_k$, $D_{obs}(\theta) \coloneqq \{(x_i, \theta)\}_{i=1}^{N_{obs}}$ for $x_i \in D_{obs}$. Considering the prior distribution and the independence between $\eta(x, \theta)$ and $\delta(x)$, the posterior density of the gaussian process with mean and variance can be written as: [348]

$$\mu_{y_{obs}}(x^*,\theta) = t_{obs}(x^*,\theta)V_{obs}(\theta)^{-1}y(6.7)$$

$$\sigma_{y_{obs}}^2(x^*,\theta) = \sigma_{obs}^2(x^*) - t_{obs}(x^*,\theta)V_{obs}^{-1}t(x^*,\theta)(6.8)$$

where,

$$t_{obs}(x^*,\theta) = \begin{bmatrix} \alpha \sigma_{sim}^2 R_{sim}((x^*,\theta), D_{sim}) \\ \alpha^2 \sigma_{sim}^2 R_{sim}((x^*,\theta), D_{sim}) + \sigma_{\delta}^2 R_{sim}(x^*, D_{obs}) \end{bmatrix}$$
$$V(\theta) = \begin{bmatrix} V^{(sim,sim)} & V^{(sim,obs)}(\theta) \\ V^{(obs,sim)}(\theta) & V^{(obs,obs)}(\theta) \end{bmatrix}$$
(6.9)

and the above diagonal block matrices are given by

$$V^{sim,sim} = \sigma_{sim}^2 \left(R_{sim}(D_{sim}) + \sigma_{\epsilon_{sim}}^2 I \right)$$
$$V^{(obs,obs)} = \sigma_{obs}^2 \left(R_{obs}(D_{obs}) + \sigma_{\epsilon_{obs}}^2 I \right) + \sigma_{sim}^2 \alpha^2 \left(R_{sim}(D_{obs}(\theta)) + \sigma_{\epsilon_{sim}}^2 I \right)$$
(6.10)

The off-diagonal blocks are given by

$$V^{(sim, obs)}(\theta) = \alpha V^{(sim, sim)}(D_{sim}, D_{obs}(\theta))$$
(6.11)

According to the investigation in chapter 3, the black box variational inference is taken

for making UQ Bayesian inference of the inverse problem, the algorithm is given below:

Algorithm 6.2 Black box variational inference with O'Hagan Bayesian calibration & geometric complexity MDL for inverse problem Uncertainty Quantification of suspended nonstructural system Input:

Training data d, mean and covariance functions for GPs in O'Hagan Bayesian calibration framework, variational family $q(\mathbf{z}|\boldsymbol{\lambda})$, numbers of model type *m*

Output:

 $q^*(\mathbf{z}|\boldsymbol{\lambda})$ and MDL ranking

- **For m = 1 to M do** // Given that M is a small constant, the time complexity $O(\cdot)$ is acceptable in this scenario
- **Initialize:** $\lambda_{1:n}$ randomly, t = 1
- 2 while the training accuracy has not converged do 3 For s = 1 to S do 4 // Draw sample from q 5 $z[s] \sim q(z|\lambda)$ end 6 $\rho := t^{th}$ value of a Robbins Monte sequence // equation (3.52) in chapter 3 7 $\lambda := \lambda + \frac{1}{S} \sum_{\lambda=1}^{S} \nabla_{\lambda} \log q(z[s]|\lambda) (\log p(x, z[s]) - \log q(z[s]|\lambda))$ t = t+1end 8 Return $q^*(\mathbf{z}|\boldsymbol{\lambda})$ 9 *MDL* := Minimum description length *complexity computation* // Equation in table 1 in chapter 3 10 11 end

And the ELBO iteration convergence result is shown in the following figure:



Figure 6.4 Black box variational inference training iterations

6.7 Minimum description length (MDL) model selection

As described in the previous chapter, we utilize MDL for model selection, and based on the variational inference results of the posterior and the solution formula of MDL, we calculate the MDL complexity values of the seven groups of models as follows:

Model type	MDL
A_cross, E, G, I _y _cross, I _z _cross, K_rod, K_friction, K_cc1, K_cc2, , K_cc3,	-593 7
_A_main, I _y _cross, K_mc	000.1
A_cross, E, G, I _y _cross, K_rod, K_friction, K_cc1, K_cc2, K_cc3, A_main,	-587 3
I _y _cross, K_mc	001.0
A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2, K_cc3A_main, Iy_cross, K_mc	-597.9
A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2, A_main, Iy_cross, K_mc	-584.5

Table 6.3 Minimum Description Length complexity (MDL) values

A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2, A_main, K_mc	-552.1
A_cross, E, G, K_friction, K_cc1, K_cc2, A_main, K_mc	-522.4

This model, with eleven parameters, is the optimal result for the inverse problem, as indicated by the posterior distribution results. Furthermore, we compare the MDL results with those of other model selection below.

	Relative	Generalization
Selected Model type	Value	
• 1	(MDL as be	nchmark)
A_cross, E, G, Iy_cross, Iz_cross, K_rod, K_friction,		75 7%
K_cc1, K_cc2, K_cc3, A_main, Iy_cross, K_mc	15.170	
A_cross, E, G, I _y _cross, K_rod, K_friction, K_cc1,	87.9%	
K_cc2, K_cc3, A_main, I _y _cross, K_mc		
A_cross, E, G, I_y _cross, I_z _cross, K_rod, K_friction,	75.7%	
K_cc1, K_cc2, K_cc3, A_main, Iy_cross, K_mc		
A cross E G K rod K friction K col K co?		
A_cross, E, O, K_rou, K_friction, K_ccri, K_cc2,		98.3%
A_main, I_y_cross, K_mc		
A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2,	100% (Benchmark)	
K_cc3, A_main, I _y _cross, K_mc		
	Selected Model type $A_cross, E, G, I_y_cross, I_z_cross, K_rod, K_friction, K_cc1, K_cc2, K_cc3, A_main, I_y_cross, K_mc$ $A_cross, E, G, I_y_cross, K_rod, K_friction, K_cc1, K_cc2, K_cc3, A_main, I_y_cross, K_mc$ $A_cross, E, G, I_y_cross, I_z_cross, K_rod, K_friction, K_cc1, K_cc2, K_cc3, A_main, I_y_cross, K_mc$ $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2, A_main, I_y_cross, K_mc$ $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2, A_main, I_y_cross, K_mc$	Selected Model typeRelative Value (MDL as be $A_cross, E, G, I_y_cross, I_z_cross, K_rod, K_friction,$ $K_cc1, K_cc2, K_cc3, A_main, I_y_cross, K_mc$ $A_cross, E, G, I_y_cross, K_rod, K_friction, K_cc1,$ $K_cc2, K_cc3, A_main, I_y_cross, K_mc$ $A_cross, E, G, I_y_cross, I_z_cross, K_rod, K_friction,$ $K_cc1, K_cc2, K_cc3, A_main, I_y_cross, K_mc$ $A_cross, E, G, I_y_cross, K_rod, K_friction,$ $K_cc1, K_cc2, K_cc3, A_main, I_y_cross, K_mc$ $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2,$ A_main, I_y_cross, K_mc $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2,$ A_main, I_y_cross, K_mc $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2,$ A_main, I_y_cross, K_mc $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2,$ A_main, I_y_cross, K_mc $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2,$ A_main, I_y_cross, K_mc $A_cross, E, G, K_rod, K_friction, K_cc1, K_cc2,$ A_main, I_y_cross, K_mc

Table 6.4 Comparison of model selection criteria with selected model type

And the minimum MDL value is selected and the model is 9 parameters. Finally, the model with 9 parameters of the inverse problem is selected as the optimal result, and the posterior distribution result is:

Generalizability value is used to express the ability of the model calibration to predict the true model. Generalizability value is defined by the following equation:

$$Gr(\%) = \left\{ 1 - \frac{\sum_{i=1}^{m} |y(x_i, \theta) - y_{truth}(x_i, \theta_n)|}{\sum_{i=1}^{m} |y_{truth}(x_i, \theta_n)|} \right\} \times 100$$
(6.12)

where m is the number of discretized points that are used to evaluate Generalizability, and $y_{truth}(x_i, \theta_n)$ is the response of the true model. In this study, we use MDL as the



benchmark to compare other model selection criteria.

Figure 6.5 Model selection criteria relative generalization comparison Based on the comparison between the geometric complexity measure minimum description length (MDL) and other model selection criteria (See Table 6.4 and Figure 6.5), it shows the intuitive advantage of it in terms of generalization performance applied in our SNS systems. AIC only considers the calibration parameters and will choose more complex model and leads to over-fitting, and DIC has a similar penalty for complexity to AIC. BIC has a stronger penalty for complexity than AIC because it includes the factor of the natural logarithm of the sample size. ICOMP almost has the same generalization, but become less-fitting, compared with MDL because it also not only considers calibration parameters, and sample size but also their sensitivity and interdependence. In summary, our study demonstrates that MDL strikes a better balance between model complexity and generalization performance, resulting in superior predictive performance in SNS systems. The MDL approach takes into account both calibration parameters and sample size, and geometric complexity information resulting in a more comprehensive and well-rounded

evaluation of model complexity, leading to optimal generalization performance.

6.8 Results of new approach and discussions

While it is true that variational inference is an approximation of MCMC, it is still a powerful tool for improving accuracy levels. Variational inference has been shown in the literature to significantly improve inference time compared to Markov Chain Monte Carlo (MCMC) by several orders of magnitude. In details, MCMC can take several hours to generate posterior estimates, variational inference can accomplish the same task in just a few minutes. The posterior distribution resembles a normal distribution due to the use of an approximate normal distribution as the variational family in black box variational inference, combined with O'Hagan's calibration framework. This approach allows us to find a simple variational family to approximate the high-dimensional posterior distribution, while maintaining good accuracy, making less derivations and saving computational time. It has advantages over mean-field variational inference, which involves lengthy derivations and the use of conjugate distributions, such as Gaussian mixture models. Additionally, it avoids the complex burn-in process of MCMC iteration and ensures accuracy.



Figure 6.6 Posterior distribution of 9 inverse problems parameters

Our new approach embedded with geometric complexity measure produces posterior estimates with low bias and variance, indicating that it is able to accurately estimate the parameters of the system with good generalization. Figure 6.6 shows the convergence of the posterior distribution for the simulated dataset, indicating that our approach is able to find good posteriors for the parameters. Additionally, Figure 6.7 shows the posterior distribution of the first six frequency modes of the system, which are of particular interest in the context of suspended nonstructural systems. We can see that the posterior mean is very close to the observation data prior we mentioned above, indicating that our approach is able to produce accurate and robust estimates for SNS systems with uncertainty quantification.



Figure 6.7 Posterior distribution of the first six frequency modes of the suspended

nonstructural system

Based on the results of the posterior distribution of the inverse problems under our new approach above, we will verify and evaluate the seismic acceleration and displacement response, and the results are as follows.



Figure 6.8 The acceleration response validations comparison among experiments, original simulation and simulation under variational inference with uncertainties.

From Figure 6.8-6.13, we can find that the numerical simulation with variational inference has a much better validation result than the original simulation in terms of amplitude and trend. The validation improvement rate is achieved by around 50% ~70%. As discussed in the [26], part of the original acceleration response were validated well,

and some were not; for a clearer view, we uniformly selected five short time ranges to double check: 20 sec ~ 21 sec, 40 sec~ 41 sec, 60 sec~ 61 sec, 80 sec~ 81 sec and 100 sec ~101 sec.



Figure 6.9 The acceleration response validation comparison (a): 20 sec ~ 21 sec



Figure 6.10 The acceleration response validation comparison (b): $40 \text{ sec} \sim 41 \text{ sec}$



Figure 6.11 The acceleration response validation comparison (c): 60 sec \sim 61 sec


Figure 6.12 The acceleration response validation comparison (d): 80 sec \sim 81 sec



Figure 6.13 The acceleration response validation comparison (e): 100 sec ~101 sec We observed that, except for simulation Figure 6.13, all the original simulations had a similar trend to the experimental data. Additionally, except for simulations Figure 6.11, Figure 6.12, and Figure 6.13, which is likely due to inaccurate parameters for the component connections and boundary friction properties resulting in larger discrepancies, all the original simulations had amplitude that fit well with the experimental data. However, after applying our new approach, we observed that the trend of simulations Figure 6.13 was now fitted with the experimental data, and the amplitude of simulations Figure 6.11, Figure 6.12, and Figure 6.13 were almost fitted well with the experimental data under the confidential uncertainty interval.

Also, As discussed in the chapter 5, most part of the original acceleration response validated well and some are not, especially the time delay difference. Figure $6.14 \sim 6.19$ below shows that numerical simulation with variational inference has a much better displacement response validation result than the original simulation both in terms of magnitude and trend with uncertainty confidential interval. For more clearer view, I also uniformly select five time-range to double check: $20 \text{sec} \sim 21 \text{ sec}$, $40 \text{sec} \sim 41 \text{ sec}$, $60 \text{ sec} \sim 61 \text{ sec}$, $80 \text{sec} \sim 81 \text{ sec}$ and $100 \text{sec} \sim 101 \text{ sec}$:



Figure 6.14 The displacement response validations comparison among experiments, original simulation and simulation under variational inference with uncertainties



Figure 6.15 The displacement response validation comparison (a): 20 sec ~21 sec



Figure 6.16 The displacement response validation comparison (b): 40sec ~41 sec



Figure 6.17 The displacement response validation comparison (c): 60 sec ~61 sec



Figure 6.18 The displacement response validation comparison from 80sec ~81sec



Figure 6.19 The displacement response validation comparison (e): 100 sec ~101 sec We observed that almost all the original simulations fit well in amplitude with the experimental data, but they all exhibited some delay effect. After applying our new approach, we observed that the amplitude of simulations Figure 6.16 and Figure 6.17 were enhanced under confidential uncertainty interval, and all the delay effects were reduced. The best performance improvement was observed in simulation Figure 6.18, which exhibited an accumulation effect with time duration that was enhanced by our new approach. Overall, our results demonstrate the effectiveness of our proposed approach for enhancing the accuracy of uncertainty quantification in suspended nonstructural systems, and we were able to produce accurate and precise posterior estimates that provide

valuable insights into the behavior of these systems.

6.9 Summary

Our proposed black-box variational inference, combined with O'Hagan's Bayesian calibration framework, provides a promising approach for inverse problems in Suspended Nonstructural Systems (SNS). Compared to traditional variational inference, which relies on conjugating assumptions and complex mathematical derivations, our method offers a simpler and more accessible solution that is easy for engineers and scientists to use. Furthermore, our approach incorporates a geometric complexity measure minimum description length (MDL) into the framework, which results in accurate and robust Bayesian inference and Uncertainty Quantification outcomes. By improving the robustness and validation accuracy of SNS systems with excellent generalization capabilities, our method offers a valuable tool for predicting and understanding the behavior of these systems.

However, since the surrogate model is a shallow model, in the future, we will replace with the deeper model, such as the deep Gaussian process or deep neural network model, to learn the nonlinear behavior, such as collision, friction of suspended non-structural systems.

CHAPTER SEVEN

CONCLUSIONS AND RESEARCH OUTLOOK

This dissertation presents a novel uncertainty quantification methodology of inverse problem based on variational inference with an efficient machine learning (ML)-based surrogate model for predicting the response of suspended nonstructural systems (SNS) in super-tall buildings during long duration and long period seismic events. Our approach embeds geometric complexity measure Minimum Description Length (MDL) as a model selection criterion to strike a better balance between model complexity and generalization performance, resulting in superior predictive performance in SNS systems. The proposed optimization-based variational inference method is seen to significantly improve the low efficiency of traditional Markov Chain Monte Carlo (MCMC) and ensures a high level of precision. When combined with ML-based Gaussian process surrogate models, the same method dramatically reduces the inference time of forward and inverse problems, enabling us to perform the largest full-scale SNS experiments in the world.

It can be simply divided into three parts, one is the research content of the Uncertainty Quantification of inverse problems, a novel Bayesian inverse problem uncertainty inference framework based on Black Box Variational Inference (BBVI) with O'Hagan's Bayesian calibration framework and efficient machine learning (ML) -based Gaussian alternative model is proposed, and Minimum Description Length (MDL) is embedded as a model selection method to improve the generalization performance. The other is by designing full-scale earthquake dynamical shaking table experiments and making nonlinear computational numerical simulations for Suspended Non-structural Systems (SNS) in super-tall buildings during long duration and long period seismic events. The third part is to improve the simulation-experiment validation performance by applying the proposed UQ inverse problems framework and methods.

For the first part, the novel research framework of UQ Inverse problems is proposed, and the prediction efficiency performance is promised by embedding Black-box Variational Inference, O'Hagan's Bayesian calibration, and Minimum Description Length, which make a more generalized and robust posterior inference. The optimization-based Blackbox Variational Inference can significantly reduce the low efficiency of traditional MCMC or Approximate Bayesian Computation and can ensure a certain precision without the requirements of specific model derivations and conjugate distribution. And can scale well to large datasets and high-dimensional parameter spaces, which are common in many fields. Also, combined with machine learning-based Gaussian Process surrogate models, it dramatically reduces the inference time of forward and inverse problems compared with original numerical simulation. Through careful Design of Experiments and Sensitivity Analysis, the computational time can be reduced as well.

For the second part, it is of practical significance to design and produce multiple SNS experiments that take into account the area effect (the world's largest area), and then carry out the earthquake dynamical shaking table experiments, as well as the considering the effect of long-duration and long-period earthquake input in the super-tall buildings. The experiments overviews are introduced in detail, the seismic damage mode and failure process of SNS are studied, and the acceleration response of the shaking table and steel platform is analyzed. Also, monotonic loading and low-cycle hysteretic tests for

components are conducted. The failure form, bearing capacity and deformation performance, load-displacement hysteresis performance, skeleton curves and energy dissipation capacity of various connections were analyzed. The load-displacement restoring force model of typical connections was established. Also, the numerical calculation model is constructed, and the numerical simulation results are compared with the experimental results in terms of acceleration time history curve and relative displacement time history curve, and the displacement performance is good except for the delay effect, and some parts of the acceleration are lacking.

Finally, with the proposed novel UQ inverse problems framework Moreover, our approach achieved good and robust Bayesian inference and Uncertainty Quantification results and improved the validation accuracy of SNS systems. Our comparison with other criteria demonstrates MDL's intuitive advantage in terms of optimal generalization performance.

In summary, our study provides significant contributions to the SNS community by offering a comprehensive and well-rounded evaluation of model complexity and generalization performance and contribute to ensuring building resilience and safety against seismic risk events. Our findings suggest that our proposed methodology can offer significant advantages over traditional methods in terms of computational efficiency and accuracy and can be extended to many other fields dealing with large-scale, high-dimensional datasets.

However, since the surrogate model I used in this dissertation is shallow, I am now investigating the deep learning model to learn the nonlinear characters in the physical

systems. However, deep learning models always need a large amount of training data to have better generalization ability; in engineering research, the data amount is very small. How to better embed prior physical knowledge [31,32,421,422] so that there can be better deep learning effects under limited or a small amount of data, the new framework such as Physics Informed Neural Network and Deep-ONet are being studied. And I also hope to design more efficient data and model-driven research paradigms through these methods to promote research in the field of engineering.

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