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A proposed sampling design for a timber inventory of a Tennessee forest

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To the Graduate Council:

I am submitting herewith a thesis written by Bernhardt L. Geldmeier entitled "A proposed sampling design for a timber inventory of a Tennessee forest." I have examined the final electronic copy of this thesis for form and content and recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Forestry.

John C. Rennie, Major Professor

We have read this thesis and recommend its acceptance:

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Carolyn R. Hodges

Vice Provost and Dean of the Graduate School

(Original signatures are on file with official student records.)

To the Graduate Council:

I am submitting herewith a thesis written by Bernhardt L. Geldmeier, III, entitled "A Proposed Sampling Design for a Timber Inventory of a Tennessee Forest." I recommend that it be accepted in partial fulfillment of the requirements for the degree of Master of Science, with a major in Forestry.

John C Rennie

John C. Rennie, Major Professor

We have read this thesis
and recommend its acceptance:

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Vice Chancellor
Graduate Studies and Research

A PROPOSED SAMPLING DESIGN FOR A TIMBER INVENTORY OF A TENNESSEE FOREST

A Thesis
Presented for the
Master of Science
Degree
The University of Tennessee

Bernhardt L. Geldmeier, III

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ABSTRACT

The purpose of this investigation was threefold. First, an extensive review of aerial forest parameters was undertaken. This was done in order to establish which parameters might serve as realistic indicators of volume for the Central Forest Region into which most of Tennessee's forests are classified. The four aerial forest parameters that were selected were *forest type*, *crown closure*, *crown diameter*, and *stand height*.

Second, a sampling design was proposed for the Central Forest Region of Tennessee based on the aerial forest volume parameters above, small scale photography (1:120,000) available from the National Aeronautics and Space Administration (NASA), medium altitude photography, and some previous knowledge and subsequent determination of timber volume from the ground. The contributions of such a design are that for the first time: (a) small scale photography is proposed for a timber inventory in Tennessee; (b) photogrammetric parameters at any other scale are proposed as an integral part of a timber inventory in Tennessee; and (c) "3-P" statistical sampling theory is the basis for determination of volume at the ground level of an aerial timber volume inventory.

Third, the problem of optimization of the proposed sampling design was considered with respect to the two conventional optimization methods: (a) maximization of precision for a given cost or (b) minimization of cost for a given precision. It was suggested that a new and

different form of optimization method may evolve as a consequence of the possibility of several, equal results of minimum cost for a given, precision or as a consequence of the possibility of several, equal results of maximum precision for a given cost.

TABLE OF CONTENTS

CHAPTER PAGE

CHAPTER I

INTRODUCTION

A. GENERAL

The objective of a forest inventory is the determination of an *estimate* of merchantable volume for all trees on a particular tract of land. An estimate is involved because the true volume of all such trees cannot be known unless they are felled, cut up, and measured in their finished raw product form. Commercial timber is grown for two basic purposes: pulpwood and sawtimber. The subject of the investigation presented in this paper is concerned only with one of these two purposes: trees grown for sawtimber. Therefore, the problem here is to estimate the merchantable volume of all sawtimber trees on a forested tract.

One feasible solution is to simply cut down all usable trees, cut them up into boards, and then measure merchantable volume. However, this procedure would defeat all the aims of forest management. Sawtimber inventories, or "timber cruises," are undertaken to determine the financial maturity of the standing forest; to serve as the basis for the sale of "stumpage" (the trees as they stand in the forest "as is"); to determine the extent of damage done by insects, disease, weather, and other agents; and to assess the value of forest land for tax purposes. There are other reasons for timber cruises in addition to these examples. One should easily understand why it is important that a reasonably precise estimate of total standing forest sawtimber be available to the forest manager.

If the area of the forested tract to be inventoried is relatively small, no larger than 30 acres for sake of illustration, it might be desirable and comparatively inexpensive to visit and measure all the trees therein. The advantage of complete measurement becomes a distinct liability with larger areas, however. Consider a forest inventory for a tract of 1,200 acres. An attempt to measure every tree on this area would be prohibitively expensive because of the time and effort involved. With an average of 40 trees per acre, for example, there would be an approximate total of 48,000 trees to measure. Each tree would have to be visited and its merchantable volume estimated individually.

Realizing that visiting and measuring every tree on a forested tract is usually impractical, foresters have learned that it is best to sacrifice the precision of the total volume estimate in order to complete a timber cruise within reasonable time and expense. Foresters have accomplished this goal by *sampling* the forested tract to be inventoried. This involves the predetermined selection of a number of temporary plots in the forest. For each tree in the plot, the volume is determined and summed over the plot. Each plot is then summed to give a total volume for the samples taken. Finally, the total volume for all the trees sampled is multiplied by a constant, or "blow-up factor," to get the estimate of total merchantable volume on the entire tract. This description of forest sampling is intended to be elementary for discussion purposes. Exactly how many plots to be sampled, specifically how the plots are located, what is the size of plot to be used, and other considerations are matters which the reader will find well discussed in other sources (30, 26, 38).

Until approximately 1934 (92) in the United States, all forest sampling for merchantable volume was carried out by ground survey. That is, forest plots were located, visited, and measured all on the ground. It was not until comparatively recently that aerial photographs of forested lands were used, even partially, in timber cruises. There are two major gains for the forester in using aerial photography for estimating timber volume. In the first instance, he is able to recognize and specify *certain boundaries*. These are most often forest and non-forest boundaries. In some cases, broad species and forest-type boundaries may also be delineated. In the second instance, the forester is able to use photographs for the *location* of photo plots and the *estimation* of timber volume directly from a photo plot. Thus a forester may use photogrammetric techniques in expediting the completion of a timber cruise, even though some subsequent ground inventory will probably be necessary.

In reference to the latter benefit, this is particularly true for very large forest inventories. Assume that one has to estimate the average volume of merchantable sawtimber per acre on a tract of land which contains some 40,000 acres of commercial timber. A field sample of only one percent of the total area would still mean that 400 acres would have to be measured on the ground. With one-fifth acre ground plots, there would be a total of 2,000 plots to be located and measured. The timber survey of such a large area over a short period of time must necessarily involve considerably less attention to overall detail. (The term "detail" in this sense refers to the number of possible variables available for measurement.) On the other hand, aerial photos for timber inventory are not generally capable of providing any great detail without great expense either. Therefore, the result is that the economic application of aerial timber volume surveys is greatest in those cases where a high intensity of detail is not required (80, 84). One may wonder at this point why aerial photographs are used at all. The answer was given by Spurr, the first person to ever construct an aerial volume table in the United States (in 1946). He stated simply: ". . . the chief value of the photographs in forest inventory lies in their use in rendering ground surveys more efficient" (91). Exactly how ground surveys are made more efficient in this respect will be discussed in the chapters to follow.

Aerial photographs taken from fixed wing aircraft and with scales ranging from 1:1,000 to 1:20,000 have received the most attention up to the present time for use in aerial timber cruising. Only lately have considerably smaller scale photos been considered for this application. Recently, color and color infrared photographs with a scale of 1:120,000 have been taken which cover the entire State of Tennessee. Because this type of photography is available at the present time, an effort was undertaken to determine its suitability for a combined aerial-ground timber cruise.

The first objective of this investigation, therefore, was to consider the possible use of 1:120,000 aerial photography in the timber volume inventory of a large tract of forested land in Tennessee. The possible utilization of such photography depends directly and indirectly upon forest parameters which are observed and measured from the air. At this time, such parameters have not been evaluated and used for any conventional aerial timber inventory whatsoever anywhere in Tennessee.

Therefore, a continuation of the first objective of this study is to evaluate and recommend those aerial forest parameters which show promise of being useful in the aerial photography aspect of timber cruising in Tennessee. Since acceptable photographs for an objective determination of such aerial forest parameters are not now available other than the 1:120,000 scale photos, an intensive review of literature was undertaken for the purpose of completing this first objective.

The second objective of the study was to propose a sampling design which would effectively utilize the recommended aerial forest parameters determined from the first objective. Previously used sampling designs were evaluated for this purpose. Those which offered promise both in their statistical structure and in the parameters utilized were considered for this objective. The sampling design to be proposed incorporates at least 1:120,000 scale photography and a ground survey aspect. A sampling design is essential in order that the fundamental usefulness of the aerial photographs may be achieved: an increase in the efficiency of a ground survey.

The third and final objective of this investigation was to consider how the proposed sampling scheme may be most efficiently implemented considering the precision of the estimate of the average volume per acre and various survey costs. Efficient utilization, or "optimization," of the sampling design will not be analytically analyzed in this study. Therefore, the culmination of this objective will be to illustrate a subjective, but feasible, general technique for the problem of optimization.

B. FOREST CONDITIONS IN TENNESSEE

The forests of the State of Tennessee belong to the Central Forest Region of the continental United States. Accordingly, on dry sites in Tennessee one might expect to find oak-hickory, shortleaf pine-Virginia pine-oak, and eastern redcedar stands. On fresh-to-moist sites one would expect to see oak-hickory, northern red oak, and yellow-poplar stands (87). The major stand designation of "pitch pine-oak" listed under the Central Forest Region is only rarely found in Tennessee. Tennessee has a rich flora and many other tree species are found in the State also. These include sweetgum, sycamore, black walnut, buckeye, American elm, eastern white pine, eastern cottonwood, black cherry, hackberry, boxelder, red maple, sugar maple, eastern hemlock, American beech, balsam fir, and many others in addition to numerous varieties of oak and hickory. It is true that not all of Tennessee is covered by the Central Forest Region description. The extreme western end of the State as well as a small southern portion of the State would not fit in this category. In the discussion to follow in the next chapters, the designation "Central Forest Region" will be used as the geographical region with which this investigation is concerned. The results of study of the Central Forest Region will be applied in the experimental sampling design proposal for Tennessee to be described later.

Topographically, Tennessee is a diverse, interesting state. The western one-third of the State is relatively flat as this area was once the bottom of a shallow sea. The middle one-third of the State is essentially a large basin. This basin encompasses nearly the width of the

State and is flanked by low, rolling hills on the west and the higher Cumberland Plateau on the east. East of the Cumberland Plateau is the Great Tennessee Valley that stretches completely across the State from north to south. At the eastern edge of this tremendous valley lie the Appalachian Mountains with some peaks rising over 6,000 feet above sea level. Much of the land found in the State has at least moderate relief. This is particularly true for the Cumberland Mountains, which adjoin the Cumberland Plateau to the north and east, and the Appalachian Mountains.

With respect to site conditions, there is a wide variation in Tennessee even within a geographical region. Oaks, hickories, and pines typically occupy the drier sites that are found on ridge crests and moderate to steep slopes. In the moist sites such as stream courses, coves, and bottomlands, one will find the more water intolerant hardwoods such as sycamore, yellow-poplar, and sweetgum. Soil types and rainfall in Tennessee play important roles in site quality which, however, will not be discussed here.

In considering the previously published literature dealing with aerial photography as applied to the timber volume inventory field, close attention has been paid to those studies which have been concerned with forest conditions of the Central Forest Region. There was more than enough relevant information for this study. To be more comprehensive, however, the whole of North America exclusive of Mexico was the geographical basis for the complete literature review. Besides the limited geographical scope of this investigation, unavailability of many foreign sources has also contributed to the elimination of similar studies conducted outside of the United States and Canada.

CHAPTER II

LITERATURE REVIEW

For aerial photographs to be useful in timber inventory, essentially two basic steps must be executed. The first step is to describe and measure some variable or group of variables on the aerial photograph which will, hopefully, be related to the volume that is present in the standing forest. This step will be considered in Section A. The second step is to take the measurements of these parameters on the photo imagery and subject them to statistical techniques in order to obtain the final results of the inventory. The second step itself is divided into two broad statistical categories for the purpose of discussion. The first category is "regression analysis" which will be considered in Section B. The second category is "sampling design" and will be discussed in Section C. Section D considers a specific sampling design that will be incorporated into the major sampling design of Chapter III. Finally, Section E considers those methods of combining statistical sampling precision with considerations of survey expenses for the purpose of optimizing the sampling design.

A. AERIAL PHOTO FOREST PARAMETERS AND APPLICATIONS

The Basic Technique

The most common technique of estimating timber volume from aerial photographs is to select plots on the photographs, measure certain parameters on these plots, translate the measurements obtained by use of an "aerial volume table" into an estimate of volume for the plot, sum up all of the plot volumes so obtained to get a volume subtotal for total plots taken on all photographs, and then multiply the subtotal by a "blow-up" factor to get the estimated total volume for the whole tract of land. In many respects it is similar to a ground inventory of the timber in a forest. However, there are minor and major complicating factors. The next section describes some of these factors with regard to the aerial forest parameter to which they apply. The two subsequent sections consider complicating factors not directly associated with specific forest parameters measured on aerial photographs. The last section briefly describes the use of high altitude aerial photographs for timber inventory, a comparatively recent development.

Before proceeding further, a final point needs to be made. The variables discussed below are assumed to be seen and measured on "vertical" aerial photographs only. A vertical aerial photograph is one which is taken with the camera axis in a vertical position. An "oblique" aerial photograph is one which is taken with the camera axis tilted, or at an angle, with respect to the vertical. Oblique aerial photographs have been successfully used in timber volume estimation from the air, particularly in the extreme latitudes. However, only vertical aerial photography will be considered here since it is the most commonly used in forestry in the United States. It is hereafter assumed that *vertical* aerial photography is implied when mention of aerial photographs is made.

Forest Parameters on Aerial Photographs

In reviewing previous studies, several aerial forest parameters were found to be predominantly analyzed and utilized as the basis of the use of aerial photography imagery for timber volume estimation. In the construction of nearly all aerial volume tables, one or more of four *major* aerial forest parameters have been used. Therefore, it is appropriate in the discussion to follow to consider these four major parameters first. After doing so, the remaining *minor* parameters will be briefly mentioned.

Perhaps the most widely used forest-related variable or parameter measured on an aerial photograph of a forest is "crown diameter." Some researchers refer to crown diameter as "visible crown diameter" or "average crown diameter" among other phrases. Generally, crown diameter is taken as the average diameter of an individual tree crown and is the result of two measurements on the image of the same tree. The measurements are taken at right angles to each other. When many crown diameters are to be measured, a template of some type may be useful. The two best templates known are the "micrometer wedge" and the "dot-type scale" (39). Crown diameters of forest trees are usually capable of being measured with comparative uniformity. They may be consistently placed into two-foot classes on 1:12,000 and three-foot classes on 1:15,840 scale aerial photographs (92). In interpreting a crown diameter, there is the possibility of making several types of error. One crown may actually be composed of two crowns of a forked tree. Each fork may represent a single tree of merchantable value. Likewise, two entirely different trees may have interlocking crowns. Several small trees may also grow together to produce one crown. Even if this latter condition is recognized on an aerial photo, it may still be virtually impossible to distinguish the individual crowns. Finally, an individual crown that is too small will not even be seen on a given photograph. This is a problem which is the result of "resolution." According to the American Society of Photogrammetry (6), resolution is "the ability of the entire photographic system, including lens, exposure, processing, and other factors, to render a sharply defined image." However, considering these drawbacks, crown diameter is probably the one variable which may be reliably measured on any aerial photo of a forest. Studies conducted by Aldrich and Norick (4) in North Carolina; Avery (7) in Mississippi; Moessner, Brunson, and Jensen (64) in Kentucky; Hanks and Thomsen (37) in Iowa; Lamont, Trochlil, and Meyer (46) in Minnesota; and Worley and Meyer (99) in Pennsylvania have all incorporated crown diameter readings in constructing aerial volume tables or found that crown diameters could be successfully measured. Outside of the immediate vicinity of the Central Forest Region, the following investigators have studied the crown diameter variable: Aldrich (3); Draeger (29); Moessner (65, 72); Avery and Myhre (9); Avery and Meyer (11); Pope (79); Gingrich and Meyer (34); Spurr (92); Rogers, Avery, and Chapman (83); and Seely (84). This second group of researchers considered the effect of crown diameter in their respective studies, even though this variable was not always a contributing factor in the results.

The next variable to be considered is "crown closure." If one visually projects the crown of a tree vertically onto the forest floor, then the area within the projected crown may be said to be "covered" or "closed over" or simply "closed" by the crown. When all the crowns of the trees in the canopy on an aerial photo plot have thus been projected, the ratio of the amount of closed forest floor to the total amount of forest floor gives a measure of crown closure for the entire plot. Usually this ratio is multiplied by 100 in order to put the measure into a percentage form. Crown closure may also be known as "crown closure percent," "crown density," or "crown coverage."

As one may suppose, an accurate determination of crown closure for one photo plot may be time consuming. For a large number of photo plots or for an entire forest area on photographs the task may become insurmountable. However, there are measuring aids for estimating this variable. Such a device is the simple dot grid. The grid is placed over a photograph and the dots which fall over any part of a crown within the area of interest are counted. The counted dots divided by the total number of dots within the area and multiplied by 100 gives the estimate of crown closure. Another measuring technique has been developed by Moessner (60) which has become rather widely used. It involves the use of white squares with given percentages of smaller, black squares in them. Still another method which has been accepted by photo interpreters is that originated by Losee (49). This method is based on the number of times that tree crowns intersect imaginary lines connecting a set of points.

Careful use of any of the techniques mentioned above can result in reasonably "good" estimates of crown closure. Problems may arise, however. Some inexperienced interpreters may not always consider small openings in the stand. Shadows sometimes make for unclear distinction of crown or they may hide adjoining crowns. The same is true if understory vegetation can be seen on the aerial photograph. Necessarily, scale and the related factor of resolution of the aerial photograph will contribute to these error possibilities. Therefore, even careful measurement of crown closure involves some element of subjectivity which in turn contributes to a lessening of precision. Taking into account the above drawbacks, it appears that crown closure is probably inferior to crown diameter in reliability of determination. Aldrich and Norick (4) in North Carolina; Avery (7) in Mississippi; Avery and Myhre (9) in Arkansas; Avery and Meyer (11) in Minnesota; Gingrich and Meyer (34) in Pennsylvania; Meyer and Worley (53) in Pennsylvania; Meyer (55) in Minnesota; Meyer and Erickson (57) in Minnesota; Hanks and Thomsen (37) in Iowa; Moessner, Brunsen, and Jensen (64) in Kentucky; and Worley and Meyer (99) in Pennsylvania have utilized crown closure in constructing aerial volume tables or, in the case of Meyer and Worley (53, 99) in an intensive analysis of forest variables. Those who have investigated crown closure outside the Central Forest Region or have found that this variable was not a contributing factor in subsequent usefulness include: Aldrich (3); Bonnor (22); Draeger (29); Moessner (70, 72); Nielsen (75); Nielsen and Wightman (77); Pope (79); and Rogers, Avery, and Chapman (83).

The third of the four major variables that are commonly measured on many aerial photographs is "stand height." This term also has its synonyms: "average total height of three tallest trees," "average total height of the dominant stand," "total height of tallest trees in the stand," "average total height of the stand," and "average stand height." In order to perceive heights of any object on photographs, an observer must be able to see the objects in three-dimension, or "stereoscopically." This simply involves the use of aerial photographs taken with areas of overlap in imagery and a "stereoscope." This elementary concept is discussed in detail in most introductory aerial photogrammetry books (16, 50, 92). Unlike crown diameter and crown closure which can be measured relatively easily, stand height is often difficult to measure. It may not even be possible to measure stand heights in some portions of heavily wooded areas. In the latter case, one must take advantage of small openings in the stand or be able to interpolate heights from the edge of the forest itself. The reason for this difficulty is that the observer must be able to see the forest floor.

There are four methods for measuring stand height. They are "parallax," "oblique," "displacement," and "shadow." Only the parallax method will be considered here. The other methods are limited in their application because of photographic restrictions (oblique photography) or because of their applicability to forest stand characteristics (open-grown forests on flat or gently rolling terrain) which are not common to Tennessee. With regard to the parallax method of measuring stand height, there are basically two types of devices upon which the photo interpreter must rely. One of these employs a "floating line" and is known as the "parallax wedge." The other device is based on the "floating dot" principle and may be generally referred to as a "parallax bar." Several different varieties of parallax wedges and parallax bars are available, with some more widely known than others. Both parallax bars and parallax wedges are employed with stereoscopic imagery to give a "spot elevation" or a relative elevation at a given point on the ground as photographed in the overlap area of two aerial photographs. For forestry purposes, the spot at which relative elevation is determined is at a tree base. The change in elevation from this spot is measured to the tree top to give tree height and subsequently stand height. Most investigators prefer the parallax wedge to the parallax bar for reasons of cost and ease of operation. However, researchers who have compared the two types of devices for accuracy have found that there is no clear difference between them (69, 98).

The one primary drawback to measuring stand height is the inability, sometimes or oftentimes present, to see the forest floor. Therefore, stand height is more commonly taken than tree height as a photo-measured variable. Other contributing factors to errors in stand height measurements include scale or resolution. Spurr (92) declared that image sharpness is the most limiting factor in accuracy of height measurements. In effect, he considered the possibility that different degrees of photo quality will exist at a given scale to produce different values of image clarity. The resolution and image sharpness on an aerial photograph are important because they enable the observer to see the narrow tops of some trees (i.e., conifers). This enables the photo interpreter to arrive at more accurate height measurements. The character of the forest in itself may also be a source for stand height error. Local variation in topography such as knolls or ravines may result in an over-estimation or under-estimation, respectively, of stand heights associated with them. Understory vegetation and deep snow may obscure the bases of trees and consequently serve as another source for height measurement errors. And finally, the ability of the interpreter will serve as yet another source for error. The consideration of the interpreter's ability will be discussed in greater detail in the next section. Although the sources for error in stand height measurements are considerable, stand height is frequently measured because of its importance in timber volume estimation from aerial photographs.

The following researchers have studied stand height for its usefulness: Avery (7) in Mississippi; Avery and Myhre (9) in Arkansas; Avery and Meyer (7) in Minnesota; Gingrich and Meyer (34) in Pennsylvania; Meyer and Worley (53) in Pennsylvania; Meyer and Erickson (57) in Minnesota; Minor (58) in Mississippi and Louisiana; Moessner, Brunson, and Jensen (63, 64) in Kentucky; and Lamont, Trochlil, and Meyer (46) in Minnesota. For studies conducted outside the vicinity of the Central Forest Region, the following references will apply: Moessner (66, 70, 72); Nielsen (75); Nielsen and Wightman (77); Pope (78, 79); Rogers, Avery, and Chapman (83); and Seely (84).

The last major variable to be measured on aerial photographs is "forest type." Forest type is also referred to by some investigators as "forest classifications," "stand composition," or "forest composition." For simplicity once again, the term "forest type" will be used to represent all possible descriptions. Forest type, as encountered in the literature, may represent either broad species composition or broad tree size classifications. An example of the former classification would be "pine" or "pine-hardwood." Examples of the latter would be "poletimber," "sawtimber," and "sapling." It is the broad species composition of a forest which is usually of interest to a photo interpreter with regard to forest type.

Forest type by broad species composition can be comparatively simple to delineate on aerial photographs. Forest type ranks with crown closure in ease of determination. The broadest classification involves simply pine and hardwood. A stand may be relatively pure pine or pure hardwood, or it may be composed of a certain percentage of each, such as 30 percent pine and 70 percent hardwood. More detailed classifications of a forested area into species groups is generally time-consuming and is reserved for the compilation of detailed stand maps. The benefit of the forest type variable in aerial timber inventory is to enable the "stratification" of the forest into areas which will hopefully contain significantly different average volumes per acre. This, in turn, will result in a more efficient inventory of the forest. There have been no technical aids developed for use in forest type classification schemes outside of "stereograms" of known forest types and some species identification "keys." Stereograms are stereo paired aerial photographs of a known species or species group. Reference to these photos will enable an interpreter to have some idea of what the different forest types look like on aerial photos. Keys to species or species-group identification are a means by which one may take observed features of a forest and, by a process of elimination, determine the forest type. Avery (16) provided good examples of these technical aids. After forest type boundaries are determined, a dot grid template may be overlaid on the aerial photograph and the forest type tallied dot by dot. The proportion of dots per type is then multiplied by total area to give the number of acres in each forest type. Without this aid, the photo interpreter may elect to delineate the boundaries of each forest type and compute the area within the boundaries by planimeter or other more sophisticated device. Either procedure would also enable the interpreter to confine the assignment of photo plots or ground plots within type or "stratum" if each possible plot in the stratum is to have an equal chance of being selected.

The sources of error for forest type classification are similar to those for crown closure. There is a certain amount of subjectivity in the estimation of forest type unless it is very carefully done. The greatest asset in this respect is for the photo interpreter to have had considerable ground experience in the area to be studied. This experience gives the interpreter knowledge of tree and site associations which he may relate to what he sees on an aerial photo. Scale of the photograph has its influence as in crown closure. Shadows and understory vegetation may be influencing factors also.

Aldrich and Norick (4) in North Carolina, Avery and Meyer (13), and Avery (12) have all conducted studies concerning forest type which may be applied to the Central Forest Region. Other researchers who have investigated the forest type parameter include Nielsen (75), Nielsen and Wightman (76), and Seely (84).

The first of the minor parameters is "tree count." Basically, a tree count, or "stem count" is carried out by simply counting the number of crowns within a given area. The area is commonly taken as the area within a number of photo plots. Therefore, tree counts will be made on photo plots with areas commonly varying from one-fifth acre to one acre in size. Photo plots are utilized in estimating tree counts over large areas, because to do otherwise (i.e., count all the trees) would be quite laborious. In many respects, a tree count is similar to crown closure. Both variables give an estimate of stocking, and both variables have similar errors of determination. Even when confined to a set of photo plots, tree counts are often slow to be measured. The result is that many researchers will use crown closure as the variable for estimating stocking, or the density of trees per acre. Gingrich and Meyer (34) in Pennsylvania and Seely (84) in Canada have considered tree counts in their studies.

The parameter known as "tree height" has been rather intensively studied by some investigators. The same principles underlying the basis for stand height measurement also support the foundation for tree height measurements. In fact, stand height is actually an average for a number of tree heights or the largest of a set of tree heights. Because not all tree heights, like tree counts, may be time-consuming measurements, stand height is taken as the commonly measured height variable. Tree heights are also similar to tree counts in that they will mainly be confined to an area the size of a photo plot and that they will involve every tree on the plot. Many of the errors made in stand height estimation are dependent upon errors in tree height estimation. Some of the studies in tree height determinations were made by Aldred and Kippen (1), Aldred and Sayn-Wittgenstein (2), Bonnor (23), and Seely (84) all in Canada; Hanks and Thomsen (37) in Iowa; Rogers (81); Burks and Wilson (24); Worley and Landis (98); Spurr and Brown (90); Minor (59) in Louisiana and Mississippi; Spurr (88) in Massachusetts; and Johnson (41) in Tennessee.

The next variable to be considered is "species composition." In this context, species composition refers to *individual* tree species. This forest parameter has met with only moderate success in being identified from aerial photographs. Perhaps the major reason that this is true is because of the great similarity of many tree species, especially hardwood tree species. A further drawback to the desirability of identifying tree species is the large scale that sometimes must be employed. Aldred and Kippen (1); Avery (16, 17); Becking (18); Pope (78); Latham and McCarty (48); Krumpe, Deselm, and Amundsen (44, 45); and Spurr and Brown (89) are a few of those who have investigated species identification on aerial photographs.

The variables "crown area" and "stem diameter" have been investigated by Aldred and Kippin (1) in Canada. Aldred and Sayn-Wittgenstein (2), also in Canada, have investigated "crown area" and "expressions of relationship to neighbors" variables. The variable "stand size class," a variation of forest type, was discussed by Moessner (61). Moessner (62, 71) also considered the importance of "site" in its capacity as a parameter in estimating timber volume.

Additional Sources of Error

This section is collectively concerned with additional sources of error in the measurement of forest parameters on aerial photographs. These sources of error were not discussed previously because they must be considered, generally, in the estimation of *all* values of forest photo parameters. The topics below are discussed in a general manner. To give these subjects the space necessary for complete analysis is beyond the scope of this work.

Perhaps the greatest source of error for interpretation of forest imagery is the photo interpreter himself (85). Three major factors contribute to the photo interpreter's ability in measuring forest parameters: his experience with regard to familiarity of measuring instruments and with regard to the results of his earlier work; his on-the-ground knowledge of the forested area under study; and his image perception. To date, there has been no precise method developed which can train a photo interpreter "to specifications." This has been partly due to the subjective nature of some measurements and partly due to the interplay of the three factors described above. Different photo interpreters just see things differently. In some instances, it is possible to achieve a degree of consistency with a group of interpreters in measuring some forest photo variables. This has been done in a number of studies (9, 53, 88). However, in other instances, it may be necessary to use a correction factor in adjusting photo-interpreted measurements to give more accurate results. Additional information on the role of the photo-interpreter in forestry is presented by Spurr (92).

The effect of scale variation should be considered as a source of error for determining the values of forest parameters on aerial photos. In measuring crown diameters, for instance, on aerial photos, crowns directly under the plane at time of photography are larger in scale than those which are further away. Thus, in looking at an aerial photograph, crowns in the center will appear larger than crowns of the same actual size that are seen toward the edges of the photograph. The result is that photo measurements may be significantly biased with respect to distance from photo center if a constant scale is used for interpretation.

In addition, the physical attitude of the airplane during photography can cause scale variations on the subsequent photographs which are not concentric about the center of the photograph. These effects are known as "tip" and "tilt." The nature of the terrain also has an effect on photo scale. Areas of high relief, such as are found in mountainous terrain, have the greatest effect. This is true because trees at relatively high elevations in the area photographed appear at larger scale. The reverse is true for trees photographed at relatively low elevations. On level or moderately hilly terrain, local scale variation due to relief is slight and is usually ignored.

Before proceeding further, an important point should be made. If a template is used for laying out photo plots, the area assigned to each photo plot will vary in indirect proportion to scale. For example, photo plots established somewhat near the edge of an aerial photograph will cover an area photographed at smaller scale than a similar photo plot located nearer the center. The plot having the smaller scale will actually be representative of a larger area. The reverse is true for a photo plot taken near the center of a photograph. Remember, this happens when a template is used for defining the boundaries of a plot. If the entire photograph was taken at a small scale, these effects may be negligible. In general, however, the topics mentioned above which affect scale may also affect the area represented in photo plots (67, 96). This may not seem important, but if bias occurs in the number of trees measured with respect to variable plot areas, the result may be that another significant source of error is introduced.

Image quality should be listed as a source of error. Image quality may be said to be composed of "image distortion" (shape) and "image sharpness" (distinctiveness). Image distortion may be caused by a number of factors. Briefly, here are a few: imperfections in the camera lens, camera speed (image motion), type and quality of films and prints taken therefrom, radial displacement of the image from the "nadir" (the point on the ground intercepted by the camera axis), and focal length of the camera. Image sharpness is dependent upon such considerations as film type, film filters, camera motion during exposure, resolution capacity of the film used, quality of developing and printing methods, and tonal contrasts actually present in the forest canopy. The use of stereoscopic imagery has, as its greatest advantage over non-stereo imagery, the addition of the third dimension to improve image distinctiveness. With the development of modern, superior cameras and the continued improvement of films and printing techniques, image quality is becoming more negligible as a source of significant error in timber inventory.

The final all-inclusive source of error to be considered in this section is "plot size." Nielsen (75) has investigated this source of error. He found that: (a) consistency of photo interpretation deteriorates with decreasing plot size; (b) the estimated proportion of pure forested and non-forested land increases with a decrease of plot size; (c) changes in canopy density results are essentially "spurious"; and (d) mean estimated heights generally decrease with a decrease of plot size. Photo plots of forested areas commonly range in size from one-fifth acre to one acre. However, there is no set rule that determines photo plot size. It is recommended that a photo plot size of one acre be used as a consequence of the investigation by Nielsen (75).

Additional Considerations

Consideration is given below to those aspects of aerial photography which may also influence the success of any aerial inventory operation. Some of these factors (i.e., film type) have been indicated above. The remaining factors, although not a distinct source of error, must be evaluated in planning an aerial flight mission for the purpose of photo-inventory. The objective of the discussion of these factors is to make specific recommendations for inclusion in the planning of a flight mission in the Central Forest Region--more specifically, in Tennessee.

The first consideration will be the season of the year. Some seasons are better for photographing certain species-groups than others. Probably the best times of the year for photography of composite stands of hardwoods and softwoods is early spring of fall (32). Fall coloration is the best time for hardwood aerial photography (93). Winter is probably the most desirable for the study of softwood stands. At this time of year, the hardwood trees and deciduous understory will be defoliated and there may be snow on the ground to provide greater contrast (36, 93). These optimum seasons may be modified somewhat when different films are taken into account, but the best seasons, in general, are those stated above. The least desirable season of the year is the middle or late summer season. In that season, hardwood and softwood crowns are the most nearly alike regardless of the film type used.

Closely allied with the seasons of the year are the number of clear days, weather-wise, available for the taking of aerial photographs. Avery and Meyer (13) give the expected number of clear days per month for the whole State of Tennessee regardless of possible regional variations. The clear days range from a low of 4.5 days and 4.2 days in June and July, respectively, to a high of 11.2 days in October. Grumbine (36) stated that the best season for photography in Tennessee is August through November with October being optimum.

Scale of photography plays a vital role in successful photo reconnaissance missions. Scale determines the number of photos to be taken (a cost consideration) and the amount of accuracy to be obtained. In the South in 1961, 57 of 89 forest industries interviewed in one study (14) used U.S. Department of Agriculture prints for two primary purposes: planning and forest administration and timber-type mapping. Of these predominantly large (over 100,000 acres) companies, 45 used scales of 1:20,000 and 32 used scales of 1:15,840 (one ground mile equals four photo inches). These are the two most popular scales in general forest aerial inventory work, as well. Garver (32) and Rogers, Avery, and Chapman (83) concluded that a scale of 1:15,840 is preferable for photo inventory work. Larger scales will mean that more photographs will be taken for a constant camera focal length. If 1:7,920 scale photographs are taken in lieu of 1:15,840 scale photos, *four* times as many photos will have to be taken at roughly *four* times the cost (13). Larger scale photographs provide an advantage in accuracy over smaller scale photos. This advantage is negligible for scales larger than 1:7,920 unless extremely large scale photographs are employed (1). However, extremely large scale photos are only used in research at present because of their high cost (8, 10) although there is some promise that they may become useful at a later date (1, 17, 27).

In close association with the scale of photography is the focal length of the camera. Specifically, focal length contributes to an effect on the finished photograph which is known as "displacement." Displacement is both a blessing and a curse to photo-interpreters. For a discussion of displacement in detail, one is referred to Spurr (92). For simplicity, displacement may be said to be the photographic distortion of an image's true position in space as would be seen at the time of the photograph by an unbiased observer. Although other factors contribute to distortion, the focal length of the camera is a concern. In general, short focal lengths will cause an increase in displacement and long focal lengths will bring about a decrease. By distorting the images to be seen on a photograph, one may be able to see more of them. This is a valuable asset in measuring crown diameter. If the displacement factor is sufficiently serious, then it may come to be a source of error itself. For Tennessee and the Central Forest Region, a camera focal length of six inches is recommended in areas of moderate relief such as are commonly found in Tennessee (14). More mountainous areas would require a longer focal length to offset the effects of topography upon displacement.

The type of film to be used in aerial photography for forest inventory purposes has been well investigated in the last two decades. The types of film normally available for this purpose are panchromatic (black and white), panchromatic infrared (black and white infrared), natural color, and color infrared. One may rightly conclude that different films with their different costs may be used for specific purposes. A notable example is the use of natural color during the fall coloration period for identification of hardwood tree species (18). No one film type has been proven vastly superior to another for the measurement of *all* forest parameters if all films can produce pictures of good quality. Even so, many investigators have stated that the panchromatic and infrared film types are what they recommend for general aerial volume inventory work (36, 56, 73, 89). Panchromatic and infrared films, though perhaps not overly superior, are easier to evaluate with more consistent measurements as the result. Therefore, it is recommended that panchromatic or infrared film be used for aerial timber inventories. Either of these film types should be combined with the use of a "minus-blue" camera lens filter.

Which is the best photographic paper used in aerial photography for forest inventory has not been a source of widespread controversy among those concerned. Actually, although one might suppose that the matter could be important, aerial photo interpreters have been easily pleased with one or two basic types (73, 94). More recently and frequently, the use of positive film transparencies has been employed in analyzing forest variables from the air. However, there is no clear advantage for photo interpreters in using such transparencies over conventionally printed photographs (85). Perhaps the best advantage of the transparencies over printed photos is the savings in time and cost in not having to transfer the imagery to paper.

Finally, one must consider the variable that cannot be measured well or cannot be measured at all on aerial photographs. By considering what can be measured as compared with what should be measured, one will be in a position to determine whether or not an aerial survey should be made in the first place. Those factors which one cannot measure well on aerial photography are tree stem diameter (DBH), basal area (BA) of a plot, tree species, and stand composition. Basal area is closely related to stem diameter, but they are considered as separate variables here. Those variables which have not been capable of measurement from aerial photos are quality of timber, amount of cull, seedling and sapling counts, under-story timber, "site index," growth rate, taper, and merchantable height (25, 32, 58). In addition, the marking of trees for future cutting cannot be done from the air. On the other hand, Moessner (62) was able to achieve a very good estimate of topographic site in aerial forest surveys in hardwood regions. This has been an accomplishment not reported by others. In order that one can estimate the values of some of these "ground" parameters, it is necessary to select and measure a number of ground plots. The ground aspect of aerial timber inventory will be described in more detail in the following discussion of aerial volume tables.

High Altitude Imagery

It is appropriate at this point to briefly consider a comparatively new aspect of timber volume estimation using aerial photographs. This recent development is "high altitude," or, as some investigators prefer to call it, "space imagery." High altitude imagery, as the term implies, is photography taken from either a high-flying aircraft or from a space satellite. The scale of such photographs taken for agricultural and land-use studies to date has been extremely small. For the purpose of establishing a reference point, Wilson (97) set the photo scale of 1:50,000 as the largest scale to still be considered as high altitude imagery.

High altitude imagery study for land-use analysis was accelerated in the United States with the establishment of the Earth Resources Orbiting Satellite (EROS) program in 1966. The first photographs available under this program unfortunately had poor resolution. Aldrich (5) found that the minimum resolution available in his study was 300 feet with satellite imagery at a scale of 1:1,000,000. What, then, is the advantage of using high altitude imagery? Such photographs are useful in imaging large areas of forested land at one time and thus are superior to conventional "mosaic" images (97). A mosaic image, or photograph, is actually composed of matched and attached smaller-sized photographs of the same scale which together cover a wide area. By being able to cover large areas of land, high altitude photographs lend themselves readily as a basis for "multi-stage" sampling. Studies utilizing this sampling technique and high altitude photos in timber inventories have already been completed (5, 40). The results have at least been encouraging, even though there was not a reduction in sampling error in one of the two inventories conducted in one study (5).

A second advantage is that ultra-small scale aerial photographs can be both suitable and advantageous for broad forest type classification and broad forest type mapping (76). By being able to determine forest type, one would be able to "stratify" the forested tract on this basis with the aim of increasing the sampling efficiency. Stratification will be referenced in greater detail in Section C of this chapter.

The advantages listed above may be considered as the only true assets of high altitude photos as the present time. However, there is promise that more detailed measurements can be made concerning those aerial photo forest parameters of most concern for aerial volume inventories. At a scale of 1:160,000, Nielsen and Wightman (77) have found that it would be feasible to estimate 25 foot stand height classes with about 70 percent confidence and that it would be possible to classify crown closure into three classes with about 65 percent confidence. These authors further concluded that color infrared film was best for forest type identification.

B. CONSTRUCTION AND USE OF AERIAL VOLUME TABLES

The preceding subject material of this review of literature has been concerned with the forest parameters that may be seen and measured on aerial photographs. Additional parts of Section A dealt with errors of measurement and factors influencing the degree of success of a photographic mission. Assuming that good quality photographs are available for the study of a particular forested tract, that a number of photo plots have been established on the imagery of this tract, and that all desirable parameters have been evaluated and measured, how does one convert the information gained into a final estimate of mean volume per acre? Conversion of aerial photography data into a volume per acre estimate is the final step in using aerial photographs for timber inventory.

In order to convert data taken from trees on ground plots in timber inventories to tree volumes, a "volume table" must be used. Perhaps the most common measurements for volume purposes which are taken from trees on the ground are diameter breast height (DBH), and merchantable height (MH). These individual tree measurements are then entered in a volume table to arrive at an estimate of tree volume. Tree volumes are then summed over the plot to arrive at a plot volume estimate. The total of all plot volumes is expanded by a "blow-up" factor to arrive at a total volume figure per tract of land. The volume per tract is divided by the total tract area in acres to determine the average volume per acre (38).

Aerial volume tables are used in much the same way as the ground volume tables which are based on ground measurements. However, ground volume tables usually have as their basis the measurements of individual trees. Aerial volume tables may or may not have individual tree measurements as their basis. Aerial volume tables, therefore, are one of two types: an "aerial tree volume table" or an "aerial stand volume table." An aerial *tree* volume table is used in exactly the same manner as a standard volume table. Measurements are taken on individual trees from aerial photographs which are then entered into the aerial tree volume table. The individual tree volumes are then summed over the whole photo plot. Photo plot volumes are then handled in the same manner as described previously. An aerial *stand* volume table is utilized by taking appropriate "gross" measurements associated with a whole photo plot or an entire photo stand of trees as the basic variables to be entered. Photo plot volumes or photo stand volumes estimated by use of the aerial stand volume table are then utilized in a manner similar to the use of plot volumes derived from aerial tree volume tables or ground volume tables (16).

Aerial stand volume tables are essentially based on the gross characteristics of a photo plot and therefore effectively ignore the single tree. Aerial stand volume tables are commonly used with small-scale photographs where the precision of the average volume per acre estimate is sacrificed for the lower cost of the photography and for the lower cost and speed of its interpretation. The variables commonly measured on photographs for use with aerial stand volume tables are stand height, crown diameter classes, crown closure, and forest type (see Table I). Aerial stand parameters may be estimated with stereo or non-stereo techniques of viewing with no significant advantages to either method (4). However, stereoscopic coverage must be present in order to estimate stand height--a very important variable in some aerial stand volume tables. Aerial stand volume tables are particularly useful when it is desired to assign the volume in a photo plot to a broad volume class. These broad volume classes may be used at a later time to efficiently assign ground plots within the study area. If an aerial stand volume table is based on measurements taken from softwood trees only or from hardwood trees only, then it will be referred to as a "specific" aerial stand volume table. Otherwise, if measurements made on both softwood and hardwood trees are the basis for the aerial stand volume table, the table will be referred to as a "composite" aerial stand volume table. The investigators listed in Table I have all compiled aerial stand volume tables for specific geographic areas. This list is given in the form of a table in order that it may be more useful.

TABLE I

SOME AERIAL STAND VOLUME TABLES FOR THE UNITED STATES AND CANADA COMPILED AS OF 1974

Compiler(s)	Number of Reference	Variables Utilized*	Locale of Derivation	Table Type(s)**
Aldrich and Norick	4	CC, CD	North Carolina Piedmont	S,C
Avery	7	CC, CD, SH	Northeast Mississippi	C
Avery and Myhre	9	CC, SH	Southern Arkansas	C
Avery and Meyer	11	CC, SH	Northern Minnesota	C
Bonnor	21	CC, SH	Canada	S,C
Gingrich and Meyer	34	CC, SH	Central Pennsylvania	S
Hanks and Thomson	37	CC, CD, SH	Iowa	S
Losee	43	CC, CD, SH	Ontario, Canada	S,C
Meyer	54	CC, SH	Northern Minnesota	S
Moessner, Brunson, and Jensen	64	CC, CD, SH	Kentucky	S
Moessner	68	CC, CD, SH	Idaho, Wyoming, Utah	S
Moessner	70	CC, SH	Mountain States	S
Pope	79	CC, SH	Pacific Northwest	S
Rogers, Avery, and Chapman	82	CC	Southern Maine	C

*These variables were used in the construction of the aerial stand volume table(s). Therefore, these same variables must be measured on a given set of photo plots if one is to estimate volumes for the plots with the aid of any given aerial stand volume table. The code used in this column represents all-inclusive definitions where: CC is crown closure; CD is crown diameter; and SH is stand height.

**The letter "S" indicates that one or more "specific" tables have been constructed. Likewise, the letter "C" signifies that one or more "composite" tables were the result of the study.

Aerial tree volume tables have not been constructed in nearly as great a number as aerial stand volume tables. Because individual tree measurements, rather than stand measurements, are involved, the computation of plot volumes is more laborious and expensive. The computations are laborious because many more measurements must usually be taken, because these measurements must be entered in sets in the aerial tree volume tables for as many trees in the photo plot, and because the resultant tree volumes must then be summed to get a plot volume. The compilation of this type of plot volume is more expensive because large-scale aerial photography must invariably be used and because of the added labor described above. Perhaps the only advantage of aerial timber inventorying with aerial tree volume tables over conventional ground cruising with standard volume tables is speed. Minor (53) in southeast Louisiana and southwest Mississippi and Aldred and Kippen (1) in Ontario, Canada, have been able to devise a type of aerial tree volume tables. Other researchers who have studied the relation of photo-measured parameters of single trees to their volumes without necessarily constructing an aerial tree volume table are Aldred and Sayn-Wittgenstein (2). Differences between aerial stand volume tables and aerial tree volume tables are further discussed by Moessner, Brunson, and Jensen (64).

There are two methods to construct aerial volume tables: the "regression analysis" method and the "alinement chart" method. Both are based on a statistical analysis of the data involved. The measured parameters on aerial photos are considered "independent variables" and the resultant tree or stand volume is the "dependent variable." The independent variables form an equation in the regression analysis method which gives the "best" prediction of the tree or stand volume. Measured volumes on the ground or, in some cases, on larger scale photography serve as a norm upon which the prediction equation is based. This does not imply that predicted volumes will always match actual volumes. Different sources of error and the exclusion of variables that may contribute in a minor way to volume estimation make up the difference. As this method of prediction is widely used in many areas, one is referred to any basic statistical work (86) which deals with this subject. Pope (79) in compiling a Douglas-fir volume table has given an excellent description of regression analysis technique.

The alinement chart method is not as commonly used as regression analysis in constructing aerial volume tables. This method involves the use of "alinement curves" based on correlations between independent and dependent variables to a predicted value of the dependent variable (volume). Differences between predicted and actual volumes may exist for the reasons given above. The alinement chart method is obviously more detailed than this simple description. Losee (49) and Moessner, Brunson, and Jensen (64) make use of the alinement chart method in their studies.

In compiling an aerial volume table, not all independent variables are necessarily measured on aerial photographs. Some or all independent variables may be measured on the ground (7, 34, 79, 82). The major reason that independent variables are measured on the ground is because of photo-interpreter error. As indicated in Section A, photo interpreters make measurements on aerial photos which can vary area must be inventoried by either method, a reduction in expenditure might be achieved by concentrating photo plots in different "locales." The locales themselves are selected from the whole tract by systematic or random means. Thus one would, in effect, be taking a "two-stage" sample by first selecting locales and then selecting photo plots within locales. Inventories of very large areas might involve three or more sampling stages. As before, the random method of selection is recommended over the systematic method in the establishment of locales. The use of multi-stage sampling in aerial timber inventories has not been extensive. Mainly, investigators have used this method with high altitude imagery (5, 47).

The last sampling technique employed with aerial photos is that of "two-stage sampling with stratification." This method has not been used as a means for selecting photo plots themselves, but primarily for efficiently selecting ground plots based upon results obtained from photo plots. The first stage of this sampling method involves the determination of "strata." Strata are groupings of individuals in a population based on some characteristic which, it is hoped, is associated in a significant degree with the final estimate to be achieved. By specifying strata and taking measurements accordingly, one may be able to account for and thus reduce the amount of variability of the final estimate. A second advantage is that a proportionately larger stratum can be assigned more plots than a smaller stratum. Therefore, ground plots that are distributed accordingly will have a greater overall efficiency. Since this is a much simplified explanation, the reader is referred to Cochran (26) and others for a more detailed discussion.

Strata on aerial photographs might be defined by forest type (4, 76), broad volume class (3, 4, 11, 15), or topographic site (62, 71) among others. Occasionally, there may be strata within strata (51). Any specific stratification scheme is justified so long as it increases the degree of precision of the final estimate for a given total cost of the project. One investigator has concluded that the smallest area in which strata will invariably be present is 40,000 acres (20).

One note should be made here. The use of the word "stage" is sometimes confusing. In the first place, the term "stage" may be used in a statistical sense to refer to respective levels of sampling. This is the context in which this term has been used previously in this section. Secondly, the orientation of the observer is often referred to as a "stage." Thus, there may be a "ground stage," a "low altitude stage," and a "high altitude stage." However, orientation stages do not necessarily coincide with sampling stages. For example, on one set of photos both major sampling units (locales) and sub-sampling units (photo plots) may be selected. Thus, at the medium altitude observation "stage" for further illustration, two sampling stages may occur. This is a source of confusion. It is recommended that the term "level" be taken to refer to observational stage in order to clarify descriptions.

D. SAMPLING WITH PROBABILITY PROPORTIONAL TO PREDICTION

The probability proportional to prediction ("3-P") sampling scheme is relatively new to forestry. "Three-P" sampling was first introduced for use in the United States about 1964 (35). It is a modification of variable probability sampling. In the paragraphs to follow, "3-P" will be considered in some detail, as this particular sampling technique is an integral part of the sampling design proposed in Chapter III.

For the purpose of illustration, assume one owns a forested tract of timberland that is 400 acres in area. All of the timber must be appraised, marked, and sold by the owner. One may wonder if there is a method by which a timber cruiser, hired by the owner, can visit the forest one time and do all that must be done. With the "3-P" sampling scheme, the answer is "yes." All of the trees will be visited and marked. Some of the trees will be measured for volume with standard forestry equipment such as the "D-tape" for diameter breast height (DBH) and the "Abney level" for merchantable height (MH). The remainder of the trees will be ocularly (visually) placed in a volume class by the timber cruiser. *All* of the trees will be marked for cutting and be assigned a measured or estimated volume. Being able to mark trees when they are measured is an advantage of using "3-P." However, as a sampling technique, "3-P" stands on its own merits.

In the 400 acres of forest to be inventoried, a 10 percent sample of all trees will be measured in detail with the forestry equipment described above. But how should this be done? Should the timber cruiser measure every tenth acre? Should one-tenth acre plots be established in the centers of one-acre squares as the cruise proceeds? With "3-P" the answer is "no" in either case. In the first instance, bias may occur in tree selection. Considerations of efficiency would eliminate the second suggestion. Suppose that every tenth tree could be selected for measurement without bias. That is, suppose that the timber cruiser carried a bag of marbles. Nine marbles would be white and one marble would be black. Upon encountering a tree, the timber cruiser would draw a marble from the bag. The draw would be made with no possibility of bias. If a black marble is drawn, the tree would be measured. A white marble drawn would mean that an ocularly estimated volume class for the tree would be recorded and the next tree considered.

The sampling scheme just outlined might well be acceptable in an even-aged forest where all trees are of approximately the same merchantable value. In an uneven-aged forest, on the other hand, it may be that the owner will want to give greater attention to his more valuable trees. Thus, it would be desirable for a large proportion of the trees sampled to be in the high value categories. Now, one might say that we are back where we started--being biased in the trees selected for measurement. This situation is where "3-P" sampling has its greatest advantage. "Three-P" sampling enables one to give more attention to the high value trees without introducing a source of bias.

Assume that there are five possible value classes for any given merchantable tree in the 400 acre forest. Class A is the most valuable, followed by Class B, and so on with Class E being the least valuable. For this example, the value of Class A will be "5," the value of Class B will be "4," and so forth with Class E having a value of "1." Referring back to the bag of marbles, there was only one value class--any tree deemed worthy of measurement. Now there are five value classes. In order to keep a 10 percent sampling intensity there must now be five times 10, or 50 total marbles in the bag. Five of the marbles will be black. Furthermore, each black marble will now have a number from one to five etched in it. Thus there will be 45 white marbles in the bag and five numbered black marbles.

The procedure now is for the timber cruiser to approach a given tree, ocularly estimate its worth (represented by volume), and draw a marble from the bag. If the marble is white, the timber cruiser tallies the tree in the proper value class (volume class) and proceeds to the next tree. If the marble drawn is black, the number on the marble is compared to the assigned value class of the tree. If the number on the marble is *larger* than the value class assigned, the tree is tallied by class as above and the timber cruiser moves on to the next tree. If the number on the marble is *equal to* or *smaller than* the assigned value, the tree is measured and the resulting "true" value (volume) is recorded. A measured tree would also be tallied in the appropriate value class. Thus, a tree in Class A theoretically has five chances of being measured for every one chance that a tree in Class E theoretically would have.

Now that one has inventoried the entire 400 acres in this manner, how are meaningful results obtained from the information collected? This is done in such a manner so as not to bias the outcome by the influence of the different sizes of the value classes. First of all, the number of all trees tallied for a given value class is determined. This number is then multiplied by the value of its value class. This last procedure "weights" the number of trees per value class by the assigned value of the value class. The weighted value class tree totals are then summed over all the value classes to give a *total value* of all trees visited in the survey tract. Now, what about the trees that were actually measured? The total value of all trees divided by the value of a given class value serves as the "blow-up" factor for any measured tree that is assigned to that value class. Once the blow-up factor is obtained for each value class, it is then multiplied by the value of every measured tree in that value class. When this has been done for every measured tree, the results are added to give the estimated total value of all trees visited in the forest tract *based on the values of all the trees measured* on the tract. The purpose of a tree tally by volume class was necessary to arrive at the proportion of total trees in each estimated value class. The purpose of the value classes was to give greater attention to the more valuable trees. The purpose of tree measurements is to serve as a means of refinement of the total value of the forest tract as derived by ocular estimate. By manipulating the results as described above, bias by value class is eliminated.

A rather simple illustration was picked for purposes of discussion above. Usually, there is one value or volume class for each integer of value or volume. Also, one must have some reasonably accurate advance knowledge about the number and size of value classes on the land to be surveyed. This information may be obtained in a low intensity sample. With such a small number of trees being measured, some foresters elect to take very careful, precise measurements in order to reduce the effect of error that would be magnified through the calculations. A bag of marbles is a nuisance to carry around, so a list of random numbers printed by an electronic computer is carried instead. When the computer senses that a selected random number would ensure automatic rejection, it prints a line of x's or some other such symbol for the number. A line of x's would be the computer's equivalent of a white marble. A printed random number would then represent the drawing of a black marble with an assigned value class. The computer is commonly used again for reduction of data obtained from a "3-P" inventory. Due to lack of space, this discussion of "3-P" is simplified. For further information, one is referred to Johnson (42), Van Hooser (95), Dilworth and Bell (28), and Johnson, Dahms, and Hightree (43).

E. OPTIMIZATION OF SAMPLING DESIGNS

There is an old American political phrase that was once popular in this country. "The greatest good for the greatest number" was a catchy slogan, but unfortunately it implied a promise which was (and is) impossible to meet. The "greatest good" may be understood to mean the "best possible economic and social way of life." The "greatest number" would be everyone. It is not possible to give everyone the best possible economic and social way of life. At least, not everyone can be wealthy--or even financially independent. The political phrase would have been more properly worded if it had said, "The greatest good for a small number" or "The best possible good for the greatest number."

The point that is made here is that the existence of two extremes simultaneously is often impossible. Consider a sampling design which gives the greatest precision for the least cost. The greatest precision would mean that the variance of the mean would be zero and, consequently, at the very least a 100 percent sample would have to be taken. Of course, the least cost is zero--that is, the inventory would cost nothing. How, then, may one take a complete inventory without any expense whatsoever? Relative extremes considered together may be just as impossible. Consider a sampling design which involves the greatest precision for an expense of only \$25.00 if 100,000 people are to be contacted and interviewed. It may cost \$25.00 just to collect data from one person.

The actual costs of the different aspects of aerial timber volume inventories of forests are not considered in this paper. This is because costs vary with the equipment used, the techniques available, the different geographical regions of the country, the year-to-year price changes of products and labor, and other similar considerations. The only manner in which costs can be, and have been, effectively analyzed is in considering them as mathematical constants. Thus, for example, there may be a total cost for a survey (C_T) which may be determined in part by the cost for a photo plot (C_P) and the cost for a ground plot (C_G).

As indicated in Section C, the objective of statistical sampling is to sacrifice some precision of the final estimate for a compensating savings in time and survey expenses. Generally speaking, the time it takes to complete an aerial timber survey has not been a direct concern among investigators up to the present time. Total time is indirectly considered when one speaks of the expenses associated with such a survey. Therefore, the efficiency of a sampling design may be said to resolve down to the concern of precision versus expense. There are two fundamental means by which sampling precision and cost are reconciled. Bearing in mind the dangers of combining extremes and near-extremes described above, these two methods must be approached in a realistic manner. One method is to determine the least cost for a given precision. That is, if a confidence limit of 10 percent of the mean for standard error is given, what combination of sample sizes at various levels will minimize the cost subsequently involved? The second method is to determine the greatest precision for a given total cost of the survey. Assuming that the cost of an aerial timber volume survey is C_T, what sample sizes at various levels involved will give the greatest possible precision in determining the mean value of the final estimate?

Only the latter technique has been found in the aerial timber inventory literature. Avery and Meyer (11) and Bickford (19) have applied formulas developed in an original study by Neyman (74) to determine the number of ground plots and photo plots needed for conducting an aerial timber cruise at a given cost. Marcuse (52) has also investigated the means by which to maximize precision for a given cost. In Section B of the next chapter, the subject of cost and precision will be discussed to some further extent.

CHAPTER III

III. SAMPLING DESIGN PROPOSED FOR TENNESSEE

A. STATISTICAL MODEL

Chapter II covered the basic concepts which must be understood and appraised before one attempts an aerial timber inventory. It was also pointed out that a certain amount of ground work would be necessary for an evaluation of the aerial volume estimate and for an appraisal of factors not measured on aerial photographs. The ultimate and basic usefulness of aerial photographs is, once again, the ability to increase the efficiency of ground timber inventories. How this is accomplished was briefly indicated in Section C of Chapter II. It is the purpose of this section to take these considerations and combine them with appropriate mathematical concepts to arrive at an aerial timber inventory design for the Central Forest Region of Tennessee.

This design must necessarily utilize a certain number of ground plots. The ground plots will constitute the basic data for the inventory. The application of ground plot volumes to correct aerial plot volumes is not essential in this design, but it may be done for the benefit of photo-interpreter training. Aerial photography will be used in the proposed design for the efficient allocation of the ground plots. Other than the necessity for subsequent ground work, the following particular prerequisites must be fulfilled *before* the proposed sampling design can be implemented for a specific tract of land:

1. High altitude imagery on the scale of 1:120,000.

2. Medium altitude imagery with a scale in the range from 1:7,920 to 1:10,560.

3. Some previous knowledge of the merchantable volume of individual trees.

Furthermore, it should be remembered that the tract must be large (for instance, 40,000 acres or more from Section C of Chapter II). If one considers the large size of the forested tract and the necessity for a ground cruise of timber as well as the prerequisites listed above, then there are five basic requirements for the sampling design.

High altitude imagery less than three years old and at a scale of 1:120,000 is presently available for the entire State of Tennessee. This imagery is provided by the National Aeronautic and Space Administration (NASA) from photographs taken from an Air Force RB-57 reconnaissance aircraft flown at an altitude of approximately 60,000 feet. The imagery format is nine-by-nine positive transparencies. The film types employed are color and color infrared, with both types offering stereoscopic coverage. The seasonal time of the photography is early spring. The minimum resolution of these photographs is estimated to be four feet.

The medium altitude imagery necessary for the sampling design may come from one of three sources. The imagery may have been a requirement for past forest management decisions not necessarily concerning volume inventories and thus may already be available. The photographs, or imagery, might be purchased from a federal or state agency if they are available in this scale range. Finally, if neither of these two alternatives offer promise for the desired coverage, the imagery may be contracted from a private aerial photography company. If the last option is necessary, it would be advisable to specify coverage at the smallest scale of 1:10,560 in order to keep costs to a minimum. Photographs in the scale range given above must be available in order to ensure that photo plot centers can be accurately located on the ground. If one believes that a scale smaller than 1:10,560 is sufficient for this purpose, then he should by all means use the smaller scale.

The final prerequisite, previous information about the merchantable volume of individual trees, is necessary in order that a "3-P" sample may be obtained in the ground phase of the sampling scheme. Hopefully, fairly recent results of a previous ground timber inventory on at least a portion of the tract in question will be available for the construction of a "3-P" portion of the sampling plan. If such information is not available, then a very low-intensity, conventional, systematic ground cruise must be carefully conducted. At the very most, this should not involve over 100 one-fifth acre plots. This low-intensity pre-sample is vitally important if no previous information exists, but it should not require a major effort for its execution.

Assuming that the three prerequisites above have been met, a complete description of the proposed sample design will now follow. The discussion will begin by considering the high altitude imagery, then incorporating the medium altitude imagery, and finally presenting the ground inventory. Statistically speaking, this sample design may be referred to as "double stratification with '3-P' subsampling." The first stratification is taken at the high altitude imagery level, the second stratification is taken at the medium altitude imagery level, and the subsampling is the ground timber cruise. It should be remembered that this design, although statistically general and therefore applicable to any inventory of this type, will incorporate those aerial forest parameters which are most likely to be measured on aerial photos of forested land in the Central Forest Region and therefore in Tennessee.

The sampling design begins by utilizing the high altitude imagery described above. On these photographs one will stratify broad forest type within the tract by delineating type boundaries and computing the area within each type. The color infrared film type is recommended for this purpose (76). The author also recommended these three basic strata: pine (80 percent or more of the crown cover is pine); hardwood (20 percent or less of the crown cover is pine); and pine-hardwood (21 percent to 79 percent of the crown cover is pine). The percentages of pine and hardwood in each stratum are not intended to be inflexible, but only specified here for the purpose of illustration. Thus, use is made of the two major advantages of high altitude photographs: the ability to see large areas at one time and the ability to classify broad forest types. To reemphasize, on the high altitude photographs the entire area to be inventoried will be stratified into broad forest types. There is no sampling involved at this stage. The area computed within each forest type in relation to the entire inventory area will serve as a weighting factor for the efficient allocation of the total number of photo plots in the next stage of the sampling design.

The next step, then, is to allocate the total, given number of photo plots to each forest type stratum. There are two techniques by which this may be accomplished: "proportional allocation" and "optimum allocation." Proportional allocation is the simplest method of the two. Optimum allocation considers estimates of cost and variability in assigning units to strata in addition to stratum size, whereas proportional allocation only considers stratum size. For the purpose of simplicity, proportional allocation will be utilized for assignment of the total number of photo plots to forest type strata. Optimum allocation may well be used with this proposed design if so desired.

Assuming that the total number of photo plots has been properly distributed to each stratum, the next step is to *randomly* locate photo plots within each forest type stratum. A random manner of location is desirable for the reason stated in Section C of Chapter II. The photo plots are first located on the high altitude imagery. This is done so that the photo plots are not inadvertantly assigned outside of their type boundaries as defined on the high altitude photos. Also, type boundaries will not have to be reestablished on the medium altitude pohtographs if photo plots are located on the high altitude imagery first. Photo plots are to be circular and are to represent one acre on the ground in keeping with standard photo interpretation practice.

With the establishment of type strata, the proportional allocation of photo plots, and the random location on medium altitude photos of such plots in each type stratum; the next procedure is to estimate the volume in each photo plot by means of an aerial volume table. Several aerial volume tables, discussed in Chapter II, are available for this task. Specifically, the following aerial volume tables are recommended: Moessner, Brunson, and Jensen (64) for the hardwood stratum; Avery (7) for the pine-hardwood stratum; and Meyer (54) for the pine stratum. All of these tables use the variables of stand height, crown closure, and crown diameter with the exception of Meyer's tables which do not utilize crown diameter. Alternatively, one might consider using the tables compiled by Aldrich and Norick (4). There are four advantages to using the tables of Aldrich and Norick. In the first instance, these tables are based on a single geographical locale, the Piedmont region of North Carolina. Thus, only one source of geographical error is to be considered. Secondly, the study area of this Piedmont region closely approximates the Central Forest Region of Tennessee. Thirdly, these tables do not require that the variable of stand height be measured. Finally, because there is not a requirement for stand height measurements, there is not a need for stereoscopic coverage of the survey area at the medium altitude level. This last point, specifically, may mean that a savings in cost could be obtained if one must contract for imagery at this level.

After estimating the volume in each photo plot, the next step is to assign photo plot volumes to broad photo volume *classes*. These volume classes will serve as strata for assignment of ground plots in the next step of the sampling design and will reduce the dependence upon, and consequently the importance of, individual plot volumes. There is a certain amount of subjectivity in choosing the number of photo volume classes for each forest type stratum and the limits for each class. Conceivably, a forest type stratum which is essentially homogeneous with regard to volume per acre may be assigned several photo volume classes. Perhaps the best answer to give at this point is to say that, by *arbitrarily* assigning volume classes, one will hopefully concentrate ground plots in those areas which contribute a greater share (area-wise) to the true total volume in a given forest type stratum. If so desired, one may base the determination of volume classes upon a simple graphical representation of individual plot volumes versus frequency. Even this method involves the subjectivity encountered in determining the graduation of a volume axis. With careful consideration of the data, however, reasonable volume classes can be assigned even if arbitrary to a certain degree. As a last note, cubic foot volumes are to be employed in the stratification of forest type into volume strata since that is what is used in the tables recommended above.

What has been accomplished up to this point is a "double stratification." First, the forest inventory area as recorded on the high altitude photographs is stratified by forest type. Forest type is a broad classification of tree species as the reader will remember from Section A of Chapter II. Second, each forest type stratum is stratified itself by use of the medium altitude photos into broad volume classes, or simply "volume strata." The reason stratification is employed here is to improve efficiency in the next stage of sampling. Total number of ground plots will be proportionally allocated to each type stratum and then proportionally allocated to each volume stratum within a type stratum. The *relative* size of each volume stratum will be considered in the assignment of number of ground plots (within the type stratum) to that volume stratum. The relative size of a particular volume stratum is determined by the number of individual photo plots which have been assigned to that stratum compared to the number of photo plots which have been located within a type stratum. Since the photo plots themselves were a random sample within each type stratum, stratification of each forest type stratum into volume strata is not accomplished through a 100 percent assessment of the type stratum. Stratification of this last type is known as "stratification by proportion."

The third, and final, stage or the sampling design is to select and locate ground plots. The total number of ground plots is not given at the beginning of the inventory, but rather it is determined from a combination of several factors to be discussed in more detail in the next section. The closest approximation to the total *a priori* number of ground plots is to say that it is a given, but somewhat flexible, percentage of the total number of photo plots. That is, one may specify that 10 percent of all photo plots will be located on the ground. However, there should be at least two ground plots in each volume stratum.

In order that ground plots are taken within the stratum that they are assigned, ground plots will *always* be located coincident with photo plots. Therefore, it must be possible to locate photo plot centers on the ground. Prerequisite number 2, relative to the scale range of the medium altitude photos, is for this purpose only. Ground plots will be circular in shape and one acre in area. This shape and area are specified for two reasons: that there will be a means for direct comparison to photo plot volumes of the same area for construction of a local aerial volume table, if desired, and that there will not be a source of statistical error related to shape and size between ground and photo plots.

After ground plots are randomly selected from the photo plots within each volume stratum, they will be located on the ground and measured. A random selection is desirable here in keeping with the theoretical basis of the statistical formulas to be presented shortly. The "3-P" method of ground inventory will be used for this last stage of the sample design. Prerequisite number 3 was necessary in order that value per individual ("KPI" values) and total value ("K") could be determined. A variable factor not associated with the field determination of KPI and K is "Z," the factor which controls sampling intensity within the "3-P" ground sample.

The verbal description of the entire sampling proposal is now complete. It remains now to give a mathematical description of this statistical sampling design. It is assumed that the reader is familiar with basic mathematical notation for comprehension of the continued discussion below. In contrast to the breakdown of the design given above, the mathematical description begins with the ground level first and then considers the medium and high altitude levels in that order.

Computations for Total Volume

Third Stage (Ground Level)

Let KPI_1 = class label for a tree in Class A

KPI_2 = class label for a tree in Class B

. . . .

KPI_x = class label for a tree in Class X

Let n_4 = number of trees on all sample plots in the *j*th volume stratum

Let YI_m = the individual volume *in board feet* of a tree selected for measurement

Then, $TI = \dfrac{\sum\limits_{l=1}^{n_4} KPI_l}{KPI_m}(YI_m) =$ estimated *total* volume in all ground plots selected in the *j*th volume stratum based upon a measured tree (m). (1)

Now, let n_3 = number of all trees measured in the *j*th volume stratum.

Therefore, $\overline{T}_j = \dfrac{\sum\limits_{k=1}^{n_3} TI_k}{n_3} =$ mean estimated *total* volume in all ground plots selected in the *j*th volume stratum. (2)

Let M_j = total number of acres in the *j*th volume stratum,

N_j = number of all possible plots in the *j*th volume stratum ($N_j = M_j$/plot area),

and n_j = number of all measured plots in the *j*th volume stratum.

Then $V_j = \dfrac{N_j}{n_j}(\overline{T}_j) =$ estimated *total* board foot volume for the *j*th stratum. (3)

Second Stage (Medium Altitude Level)

Let n_2 = number of strata in the *i*th forest type.

Then $\dfrac{V_j}{M_j}$ = estimated board foot volume *per acre* for the *j*th stratum.

Subsequently, let M_i = number of acres in all strata of *i*th forest type stratum.

Then, $\overline{V}_i = \dfrac{\sum\limits_{j=1}^{n_2} M_j\left(\dfrac{V_j}{M_j}\right)}{M_i} = \dfrac{\sum\limits_{j=1}^{n_2} V_j}{M_i} =$ estimated board foot volume *per acre* for the *i*th stratum. (4)

First Stage (High Altitude Level)

Let M = number of acres in the entire survey tract, and

n_1 = number of *i* strata.

Then $$\overline{V} = \frac{\sum_{i=1}^{n_1} M_i\overline{V}_i}{M} = \frac{\sum_{i=1}^{n_1} M_i\left(\frac{\sum_{j=1}^{n_2} V_j}{M_i}\right)}{M} = \frac{\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} V_j}{M} = \tag{5}$$ estimated board foot volume *per acre* for the entire survey tract.

Therefore, $$V_T = M\overline{V} = \frac{M\sum_{i=1}^{n_1}\sum_{j=2}^{n_2} V_j}{M} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} V_j = \tag{6}$$ estimated *total* board foot volume for the entire survey tract.

Expressing this final equation in its most basic form, one would have:

$$V_T = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} V_j = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \frac{N_j}{n_j}(\overline{T}_j) = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \frac{N_j}{n_j}\frac{\sum_{k=1}^{n_3} TI_k}{n_3}$$

$$= \sum_{i=1}^{n_1}\sum_{j=2}^{n_2} \frac{N_j}{n_j}\sum_{k=1}^{n_3}\frac{1}{n_3}\sum_{l=1}^{n_4}\frac{KPI_l}{KPI_m}(YI_m). \tag{7}$$

Computations for Standard Error of the Total Volume

From Equations (1) and (2) above:

(1) $TI = \frac{\sum_{l=1}^{n_4} KPI_l}{KPI_m}(YI_m) =$ estimated total volume in all ground plots selected in the *j*th volume stratum based upon a measured tree (YI_m).

(2) $\overline{T}_j = \frac{\sum_{k=1}^{n_3} TI_k}{n_3} =$ mean estimated *total* volume in all ground plots selected in the *j*th volume stratum.

By conventional means for computation of a variance, one has:

$$S^2_{TI} = \sum_{k=1}^{n_3} \frac{(TI - \bar{T}_j)^2}{n_3-1} =$$ *variance* of the *total* board foot volume in all plots selected for ground measurement in the *j*th stratum, and (8)

$$S^2_{\bar{T}_j} = \frac{S^2_{TI}}{n_3}\left(1 - \frac{n_3}{n_4}\right) =$$ *variance* of the *mean* of the estimated *total* board foot volume in all plots selected for ground measurement in the *j*th stratum. (9)

$S_{\bar{T}_j}$ is, therefore, the standard error of the variance of $S^2_{\bar{T}_j}$.

Then $S_{V_j} = \frac{N_j}{n_j} S_{\bar{T}_j} =$ *standard error* of the *mean* of the estimated *total* board foot volume in the entire *j*th stratum, and (10)

$$\frac{S_{V_j}}{M_j} =$$ *standard error* of the *mean* of the estimated *per acre* board foot volume in the entire *j*th stratum. (11)

Therefore,

$$S^2_{\bar{V}_i} = \frac{\sum_{j=1}^{n_2} (M_j)^2 \left[\frac{S^2_{V_j}}{M_j^2}\right]}{M_i^2} = \frac{\sum_{j=1}^{n_2} S^2_{V_j}}{M_i^2} =$$ *variance* of the *mean* of the estimated *per acre* board foot volume for the *i*th type stratum. (12)

Similarly,

$$S^2_{\bar{V}} = \frac{\sum_{i=1}^{n_1} M_i^2 (S^2_{\bar{V}_i})}{M^2} = \frac{\sum_{i=1}^{n_1} M_i^2 \left[\frac{\sum_{j=1}^{n_2} S^2_{\bar{V}_j}}{M_i^2}\right]}{M^2} = \frac{\sum_{i=1}^{n_1} \sum_{j=1}^{n_2} S^2_{\bar{V}_j}}{M^2} \qquad (13)$$

= *variance* of the *mean estimate* of the *per acre* board foot *volume* for the entire tract.

Therefore, $S_{\bar{V}}$ is the *standard error* of the *mean estimate* of the *per acre* board foot *volume* for the entire tract.

$$\text{Let } S_{V_T} = MS_{\overline{V}} = \frac{M\left[\sum_{i=1}^{n_1}\sum_{j=1}^{n_2} S^2_{\overline{V}_j}\right]^{\frac{1}{2}}}{M} = \text{the } \textit{standard error} \text{ of the } \textit{mean estimate} \text{ of the } \textit{total} \text{ board foot } \textit{volume} \text{ for the entire tract.} \tag{14}$$

$$\text{then } S^2_{V_T} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} S^2_{\overline{V}_j} = \text{the } \textit{variance} \text{ of the } \textit{mean estimate} \text{ of the } \textit{total} \text{ board foot } \textit{volume} \text{ for the entire tract.} \tag{15}$$

Expressing this final equation in its most basic form, one would have:

$$S^2_{V_T} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} S^2_{\overline{V}_j} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \left(\frac{N_j}{n_j}\right) S^2_{\overline{T}_j} = \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \left(\frac{N_j}{n_j}\right)^2 \frac{S^2_{TI}}{n_3}\left(1 - \frac{n_3}{n_4}\right)$$

$$= \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \left(\frac{N_j}{n_j}\right)^2 \sum_{k\ 1}^{n_3} \frac{(TI - \overline{T}_j^2)}{n_3(n_3-1)}\left(1 - \frac{n_3}{n_4}\right) \tag{16}$$

$$= \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \left(\frac{N_j}{n_j}\right)^2 \sum_{k=1}^{n_3} \frac{\left[\sum_{l=1}^{n_4} \frac{KPI_l}{KPI_m}(YI_m) - \sum_{k=1}^{n_3} \frac{\sum_{l=1}^{n_4} \frac{KPI_l}{KPI_m}(YI_m)}{n_3}\right]}{n_3(n_3-1)}\left(1 - \frac{n_3}{n_4}\right)$$

$$= \sum_{i=1}^{n_1}\sum_{j=1}^{n_2} \left(\frac{N_j}{n_j}\right)^2 \sum_{k=1}^{n_3} \left[\left(\sum_{l=1}^{n_4} \frac{KPI_l}{KPI_m}(YI_m) - \sum_{k=1}^{n_3} \frac{1}{n_3} \sum_{l=1}^{n_4} \frac{KPI_l}{KPI_m}(YI_m)\right)\left(\frac{n_4-n_3}{n_4 n_3(n_3-1)}\right)\right]$$

As final note, if the size of a ground plot in one acre (as in the example above, then $M_j \equiv N_j$.

B. APPROACH TO OPTIMIZATION OF THE MODEL

With the sampling design proposed in Section A of this chapter, one must somehow consider its implementation with respect to costs. The two general methods of evaluating a statistical sampling proposal and its associated expenses were discussed in Section E of the last chapter. Once again, these two optimization methods are: (a) for a given total inventory cost, maximize the precision of the final estimate and (b) for a given, required precision of the final estimate, minimize the cost to be incurred. These two approaches to optimization can be analyzed for general purposes if one assumes a certain cost function is applicable and if specific costs are considered as mathematical constants.

Either optimization technique could be applied to the sampling design proposed in Section A. However, this proposed sampling design *could* represent an alternative to either of the two optimization techniques described above. Assume that one wishes to optimize the sampling design by first maximizing the precision for a given expenditure. This may be the approach that one must take if he is allocated only so much money for a forest inventory as the result of budgeting over which he has no control. Also, one will probably want to take this course of action if he has little or no knowledge of prior cost and precision relationships. This is because it would be a financially safer move to keep a tight control on expenses and to do one's best to improve precision rather than to specify a given level of precision that may drive expenses beyond reasonable limits. The precision of the tract's total board foot volume is reflected by the variance of the estimated volume. For this study, the estimate's variance is $S^2_{V_T}$ (Equation 16). Referring to the final form of Equation 16, one will observe that several components of the equation can be considered constant as a consequence of a preliminary ground survey. These are the class values (KPI values) per volume stratum and the number of trees that are estimated to be in each volume stratum. The parameter n_4 above gives the closest equivalent to this value on a volume stratum basis. The number of forest type strata (n_1 above) is a constant determined by photo interpretation of the forest tract to be surveyed. The number of volume strata within a forest type stratum is denoted by n_2 and is also determined by photogrammetric means. This last parameter of n_2 should be considered as a constant once it has been established for each type stratum. To allow this parameter to change once established in order to achieve desired results would be an artificial manipulation and would unduly complicate further analysis. Closely related to the parameter n_2 is the number of plots that can be located within each volume stratum. This parameter is designated as N_j in Equation 16 and is also considered a constant once n_2 and plot size are established.

With the determination of the constants for use in Equation 16 above, it is simple enough to identify the variables. In this case they are the number of all measured plots in the jth volume stratum (n_j) and the number of all trees selected for measurement in the jth volume stratum (n_3). These are variables which may be combined in any number of different ways. One should remember that Equation 16 deals only with total variability derived from ground measurements. Therefore, the influence of n_3 and n_j will affect only the ground aspect of a total cost function. The number of photo plots is not reflected in Equation 16. However, as was illustrated in the example at the beginning of Section C of the last chapter, a 100 percent photo cruise can be quite expensive. With 40,000 acres to measure with one-acre photo plots, for example, it would be prohibitively expensive. The point to be made here is that the number of photo plots for the sampling design proposed in the previous section of this chapter is a significant aspect of the total cost of the survey. Should one take 400 photo plots, 800 photo plots, or 1,200 photo plots? The number may, and probably will, greatly concern the mensurationist in charge of the timber inventory. Exactly how many photo plots must be taken for each stratum is not an easy question to answer. The best means of approach to the question of total precision may be to consider the number of photo plots per type stratum ("p_i"), n_3, and n_j as three dimensions to the total variance of the estimated mean. It is therefore conceivable that, for a given total survey cost, it is possible to arrive at several equally, or nearly so, optimum levels of precision.

The solution to the problem of several optimum levels of precision for a given total survey cost may be intricate. One approach to this problem is to consider the second method by which one can optimize a sampling design: for a given, required precision of the final estimate, minimize the cost to be incurred. One may wish to take this approach if he is capable of some influence over the budget for total forestry operations. A researcher may investigate the problem of optimization by considering this approach also. In both cases, information from a previous inventory is required. However, information from a previous inventory is based on fixed values of p_i, n_3, and n_j. This does not give one the ability to define a precision function in terms of cost. Costs at each stage of sampling are the variables in this instance (with respect to a photo or ground plot). The result is that one now has the possibility of having several optimum levels of cost.

The best solution to the problem of several optimum levels of precision or cost may be to consider the optimization of a third factor in addition to cost and precision. Total survey *time* may be such a factor. In any event, there is no straightforward mathematical approach to the selection of the values of p_i, n_3, and n_j for a given n_1, n_2, n_4, and N_j at the present time. The last four parameters above depend upon conditions at a specific locale.

The possible outcomes of optimization of the sampling design proposed above in Section A of this chapter have not been investigated here in detail with regard to the suggestions made in this section. There has been neither available data nor sufficient time for this to have been done at this point. However, for future work some field data can be combined with aerial forest parameter measurements and the resulting aerial forest volume determinations to serve as a basis for statistical analysis of a specific and realistic forest inventory problem. This suggestion will be further explored in Section C of this chapter.

C. IMPLEMENTATION OF THE MODEL

Sections A and B of this chapter have been abstract in their approach. How does one go about using the information in these last two sections in a practical sense? How can one at least determine whether or not the proposed sampling design of Section A is even suitable to his needs and geographic locale?

Consider the forester who must manage a hypothetical forested area containing some 12,000 acres. This forester is called upon to inventory this whole tract in preparation for, the sale of the tract and the commercial timber thereon. The forester knows from past experience that the merchantable volume of the tract may be stratified by broad species composition. He is reasonably certain that at least some of the broad species types can be further stratified by volume classes. For example, the forester believes that the volume per acre of a pine-hardwood stand on a ridge top is significantly different from the volume per acre of a pine-hardwood stand in a bottomland area. Or in the case of the pine stratum, a pine stand in one area has lower stocking (and lower volume) compared to a pine stand in another area--both stands having the same sized trees.

The forester has the high altitude photos available from NASA for his use. The next step is to obtain medium altitude photos described in Prerequisite number 2 of Section A, Chapter III. This could be a rather expensive undertaking if photos in the desired scale range are not available from past operations or from outside sources. Additionally, the forester has not decided that he actually does want to use the particular sampling design that is set forth in this paper. There is an alternative to this situation. Since photos in the photo scale range of Prerequisite number 2 are necessary only for location of ground plots, photos at a smaller scale will suffice for determination of aerial photo plot volumes. Thus, the forester may analyze smaller scale photos to determine the practicality of the sampling design than otherwise would be required for complete implementation of the sampling design. If he then chooses to utilize the sampling design, he must obtain photos of a scale large enough for accurate location of photo plots on the ground. If smaller scale aerial photos are not available from past practices, they may be obtained from the federal government at scales ranging from 1:12,000 to 1:20,000 (16).

With both the high altitude and the medium altitude imagery available, the photo-interpretation aspect of the sampling design is begun. Upon analysis of the high altitude imagery, the forester decides that two broad forest type strata exist. These he classifies as a pine stratum and a pine-hardwood stratum. Further, it is determined that the pine stratum comprises 70 percent of the forest area and that the pine-hardwood stratum comprises the remainder.

At the medium altitude level of imagery it is decided that a relatively small number of 400 one-acre, circular photo plots will be taken. With proportional allocation, then, 70 percent of 400 (or 280) photo plots will be taken in the pine stratum and 30 percent of 400 (or 120) photo plots will be taken in the pine-hardwood stratum. Within each stratum the photo plots are randomly located on the high altitude imagery and then transferred to the medium altitude imagery. This is done for reasons specified in Section A of this chapter. Once all photo plots are located on medium altitude imagery, measurement of specified aerial forest parameters takes place. These measurements are entered in an aerial volume table by photo plot and the plot volume determined and recorded.

The forester now considers the dispersion of photo plot volumes within each forest type stratum and determines the number of volume classes and the limits of each. This is a subjective assignment to a greater or lesser degree for reasons outlined previously in Section A. The forester decides that there are two volume classes in the pine-hardwood forest type stratum and four volume classes in the pine forest type stratum. The *numerical* size of a given volume is then determined by computing the number of photo plot volumes within the type stratum which can be assigned to that volume stratum. The *relative* size of a volume stratum is then calculated by comparison of its numerical size to the total number (size) of photo plots taken within the type stratum.

With the determination of the numbers and relative sizes of type strata and volume strata, the next procedure is to assign ground plots to volume strata. The total number of ground plots will be proportionally allocated to each volume stratum based upon the relative sizes of type and volume strata. Once the number of ground plots per volume stratum is determined, ground plots will be randomly located in each volume stratum, visited in the field, and measured. Ground plots are to be one acre in size and circular in shape.

However, the forester is not concerned at this point with the *accurate* location and visitation of ground plots as per volume stratum. He wants to know if the sampling design is practical in the first place. This is the question with which this section is concerned. To carry out the accurate location of ground plots would require a given scale imagery (if not already available) and would therefore defeat the purpose of a feasibility study. Therefore, the forester has three possible courses of action if he does not desire to accurately locate photo plots on the ground.

The first possibility is to consider the available results from previous timber cruises of the area in question. Assume that such an earlier cruise has been made and that data from some 200 one-fifth acre plots are available as to volume by species. The volume per plot must first be put on a one-acre basis by multiplying each plot volume by a constant of "5." Furthermore, aerial photo volume classes will likely be expressed in cubic feet, whereas ground plot volumes will probably be given in board feet. Thus, ground plot volume must be further converted into cubic foot volumes by use of a "conversion" factor. Once all 200 one-fifth acre ground plots in board feet are converted into one-acre ground plots in cubic feet, they will be stratified into the type and volume strata previously determined from the aerial photo imagery. This must be done as best as possible under the circumstances if one considers the likelihood that the ground plot forest types and volume classes will not completely fit the aerial plot forest types and volume classes in true proportions. A subsample of the expanded and converted ground plots is taken to serve as a basis of information for the "3-P" sampling to be done later. The plots selected in this subsample are "returned" to the remainder of the ground plots for later sampling. The basic information for the "3-P" ground sample will involve the variables "KPI_x," "K," and "Z." These variables are used to generate the "3-P" tree selection list that is described in greater detail in Section D, Chapter II.

With the "3-P" tree selection list available, the forester now commences the timber cruise of the forest area "on paper." First he decided to measure a total of 60 "ground" plots of the 200 plots available with a minimum of two plots per volume stratum. The plots will be taken in a random manner in each volume stratum. The number of plots taken in each volume stratum will have been determined by the proportional allocation method described above. Once a plot is selected for measurement, the forester consults the "3-P" listing to determine which trees to record as to their true volume and which trees to record just as to their volume class. After measurement of all 60 "ground" plots, the data collected is reduced through the formulas in Section B of this chapter to arrive at a total volume per tract estimate and a variance of the total volume per tract estimate. The standard deviation (the square root of the variance) of the total tract volume divided by the total volume per tract and multiplied by 100 gives an "expression of variability on a relative basis" (30) and is known as the Coefficient of Variation. This entire procedure has two major inherent sources of error: (a) information from the previous inventory is artificially manipulated into volume classes and (b) randomization is greatly restricted since the whole tract is only represented by a few hundred plots.

The second possible course of action is to use the information from a previous forest inventory as the basis of information for "3-P" sampling to be conducted on the ground. Once the "3-P" sampling list has been determined for a small number of ground plots, the forester locates, visits, and measures them. The information is handled in the same manner as in the above paragraph and a coefficient of variation is determined. Since proper scale photography is not otherwise available for accurate location of photo plots on the ground, the ground plots will be located in the best manner possible using smaller scale photography. This could be a significant source of error.

The final possible course of action must be utilized if no previous ground inventory information exists. In this instance, the forester must efficiently sample a number of ground plots to serve as the basis of "3-P" information. This may be accomplished best through a multistage sample or a "cluster" sample (30). For example, 10 plots may be located and measured at each of five major locales. With a "3-P" sampling list so derived, the forester then locates, visits, and measures a small number of ground plots as in the second possible course of action. Information from these is treated in the same manner as in the above two alternative courses of action and a coefficient of variation calculated. As before, a source of error is introduced for the inaccurate location of photo plots on the ground.

The results of a feasibility study now available, the forester examines the magnitude of the coefficient of variation. If, for a small number of ground plots, it is not too high, the forester may conclude that complete implementation of the sampling design is justified. If the coefficient of variation is abnormally high, the forester would elect to try a different sampling design from the one proposed in this paper. Only the forester himself, from past experience, can have a reasonably good idea of what the size of an acceptable coefficient of variation should be.

CHAPTER IV

EVALUATION AND CONCLUSIONS

This chapter concludes the presentation of this investigation into the concepts and implementation of aerial photography for forest inventory evaluation. As indicated in the last chapter, no practical test of the sampling design has been undertaken. Therefore, results of such a test and the evaluation of the results cannot be considered There is the alternative of evaluation in a general sense of the sampling design proposed in Chapter III. However, no conclusive results could be derived from such an evaluation. One could point out the "correctness" of the sampling design. One could further conclude that reliable, if somewhat gross, aerial volume estimates can be made of a forest based on the results of earlier studies. It could then be deduced that such aerial volume estimates could be made on a forest tract in the Central Forest Region of Tennessee incorporating the aerial forest parameters and sampling design proposed above. Yet, this is a subjective argument *for* the usefulness of this investigation. Other investigators may offer reasonable arguments to satisfy their own minds that the tentative results of this investigation may *not* be valid. Any evaluation of the design proposal at this point will be subjective to some measure. It remains for future researchers to accurately confirm or deny the basic applicability and economic usefulness of the proposals in this paper.

The accomplishments of this investigation were threefold. First, an extensive review of aerial forest parameters was undertaken. This was done to establish which parameters might serve as realistic indicators of forest volume for the Central Forest Region into which most of Tennessee's forests are classified. The four aerial forest parameters that were selected were forest type, crown closure, crown diameter, and stand height. However, provision was made for using an aerial volume table as a part of the sampling design proposal which did not contain the stand height parameter. This was done in the event that stand height was not capable of measurement in photos of dense stands of timber.

Secondly, a sampling design was proposed for the Central Forest Region based on the aerial forest volume parameters above, small scale photography (1:120,000) available from NASA, medium altitude imagery, and some previous knowledge and subsequent determination of timber volume from the ground. The contributions of such a design are that for the first time: (a) small scale photography is proposed for a timber inventory in Tennessee; (b) photogrammetric parameters at any other scale are proposed as an integral part of a timber inventory in Tennessee; and (c) "3-P" sampling theory is the basis for determination of volume in the final stage (ground level) of an aerial timber volume inventory. In addition to the verbal description of the sampling design, mathematical formulas were developed which will give an estimate of the total board foot volume and its variance for a hypothetical forest tract. The basic purpose of the entire sampling design as proposed in the last chapter is to inventory a large forested area more efficiently than could be done by commercial ground inventory means alone.

Finally, the problem of optimization of the proposed sampling design was considered with respect to the two conventional optimization methods: (a) maximization of precision for a given cost; or (b) minimization of cost for a given precision. It was suggested that a new and different form of optimization technique may evolve as: (a) a consequence of the possibility of several equal results of a maximum precision for a given cost or (b) a consequence of the possibility of several equal results of minimum cost for a given desired precision.

It is not meant to be inferred here that the specific sampling design proposed in this investigation will solve *all* of the problems of inventorying a large tract of forested land in the Central Forest Region of Tennessee. Other sampling designs might perform just as well or better. However, to date there have been no other such designs proposed which incorporate photogrammetric means and "3-P" subsampling as primary ingredients. Thus, what has been accomplished with respect to aerial photography as applied to timber inventory is to have established a starting point. Further investigation into the usefulness of the sampling design which is the subject of this paper and other similar designs is encouraged because of its new and innovative approach to the timber inventory of large areas of forest land.

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