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CROSS PRODUCT OF IDEAL FUZZY SEMIRING

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ABSTRACT

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If one of the axioms in the ring, namely the inverse axiom in the addition operation, is omitted, it will produce another algebraic structure, namely a semiring. Analogous to a ring, there are zero elements, ideal (left/right) in a semiring, and the cross product of the semiring ideal. The analog of the fuzzy semiring has zero elements, ideal (left/right), and the cross product of the semiring fuzzy ideal associated with the membership value. This paper will discuss the cross-product of two (more) fuzzy ideals from a semiring. Furthermore, the cross-product of two (more) fuzzy ideals from a semiring will always be a semiring fuzzy ideal. But the converse is not necessarily true.

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1. INTRODUCTION

One of the extensions of the ring is semiring. A semiring is a non-empty set equipped with two binary operations. The first and second operations are an Abelian semigroup with a zero-identity element, and the left and right distributive laws fulfill a semigroup with a unit element.

The interesting thing in algebraic research is that the object under study has conditions that are not strong, such as the conditions that a semiring has when compared to a ring. Therefore, research related to semiring is interesting, as seen from the research carried out [1]–[5]. In addition, semiring research is also developing by combining other fields, including the fuzzy concept introduced by [6]. The combination of semirings and fuzzy sets produces the concept of fuzzy semirings. Many researchers have researched fuzzy semirings, including [7]–[13].

The fuzzy semiring research above has yet to investigate the cross-product of two (more) ideal fuzzy semirings. This condition motivated the research to be conducted. In this research, we will investigate the characterization of the cross-product of two (more) semiring fuzzy ideals.

We induced the properties we studied in this study from previous semiring fuzzy studies, including [14]–[17]. Furthermore, the properties produced in this study can be used as a basis for research on ideal fuzzy semiring images and preimages under a semiring homomorphism.

2. RESEARCH METHODS

Our theoretical study aims to produce new properties of the fuzzy ideal cross-product of a semiring. At the beginning of the research, we studied related references, and then we built the properties of the fuzzy ideal cross product of semiring by inducing from previous research. In the last step, we prove the correctness of the properties we constructed. To support the construction process of the cross-product properties of fuzzy semiring ideals, we present the definitions of a semiring, semiring ideals, fuzzy sets, and fuzzy ideals semiring.

According to [18], [19], a non-empty set $\mathfrak{D}$ is called a semiring if and only if $\mathfrak{D}$ over two binary operations, addition " $+$ " and multiplication " $\cdot$ ", satisfies the conditions:

(1). $(\mathfrak{D},+)$ is an Abelian semigroup with zero elements $0_{\mathfrak{D}}$;
(2). $(\mathfrak{D},\cdot)$ is a semigroup with a unit element $1_{\mathfrak{D}}$;
(3). Distributive law i.e.

$$a\cdot(w+c)=a\cdot w+a\cdot c \text{ and } (a+w)\cdot c=a\cdot c+w\cdot c$$

for any $a,w,c\in\mathfrak{D}$
(4). $a\cdot 0_{\mathfrak{D}}=0_{\mathfrak{D}}\cdot a=0_{\mathfrak{D}}$ for any $a\in\mathfrak{D}$.

Analogous to rings, on a semiring, there is a semiring ideal. According to [18], [19], a subset $\boldsymbol{\mathcal{A}}(\neq\emptyset)$ of the semiring $\mathfrak{D}$ is called ideal if, for every $\boldsymbol{a},\boldsymbol{w}\in\boldsymbol{\mathcal{A}}$ and $\boldsymbol{d}\in\mathfrak{D}$, the conditions are satisfied $\boldsymbol{a}+\boldsymbol{w}\in\boldsymbol{\mathcal{A}}$ and $\boldsymbol{ad},\boldsymbol{da}\in\boldsymbol{\mathcal{A}}$.

Furthermore, the definition of fuzzy subsets, the cross product of two subsets, and the fuzzy right ideal are presented. The paper [6], [20] defined a fuzzy subset of the non-empty set $\boldsymbol{\mathcal{X}}$, a function of $\boldsymbol{\mathcal{X}}$ to the closed interval $[\mathbf{0},\ \mathbf{1}]$. Let $\boldsymbol{\mathcal{F}}(\boldsymbol{\mathcal{X}})$ represent the collection of all fuzzy subsets of $\boldsymbol{\mathcal{X}}$.

Definition 1. [20] *Let $\sigma,\upsilon\in\mathcal{F}(\boldsymbol{\mathcal{X}})$. The cross product of σ and υ, denoted $\sigma\times\upsilon$, is*

$$\sigma\times\upsilon(a,w)\overset{\text{def}}{=}\sigma(a)\wedge\upsilon(w)$$

for any $(a,w)\in\mathcal{X}\times\mathcal{X}$.

Definition 2. [8], [21] *The fuzzy subset σ of thr semiring $\mathfrak{D}$ is called the fuzzy right ideal semiring of $\mathfrak{D}$ if and only if*

$$\sigma(a+w)\geq\sigma(a)\wedge\sigma(w) \text{ and } \sigma(a\cdot w)\geq\sigma(a),$$

for any $a,w\in\mathfrak{D}$.

Furthermore, in this paper, what is meant by the fuzzy ideal of semiring $\mathfrak{D}$ is the fuzzy right ideal of $\mathfrak{D}$.

3. RESULTS AND DISCUSSION

The following presents the properties of the fuzzy ideal direct product of semiring $\mathfrak{D}$. The properties presented in this section are induced from [11], [15], [16], [21]–[27].

Theorem 1. *If σ is a fuzzy ideal of a semiring $\mathfrak{D}$, then $\sigma(0_{\mathfrak{D}}) \geq \sigma(a)$ For any $a \in \mathfrak{D}$.*

Proof. Since σ is a fuzzy ideal of a semiring $\mathfrak{D}$ and $0_{\mathfrak{D}} \in \mathfrak{D}$, consequently for any $a \in \mathfrak{D}$, we have

$$0_{\mathfrak{D}} = a \cdot 0_{\mathfrak{D}}.$$

Therefore, by Definition 2.2, we have

$$\sigma(0_{\mathfrak{D}}) = \sigma(a \cdot 0_{\mathfrak{D}}) \geq \sigma(a).$$

Hence, $\sigma(0_{\mathfrak{D}}) \geq \sigma(a)$ For any $a \in \mathfrak{D}$. ■

According to the conditions of **Theorem 1**, whether the truth holds for the zero elements $(0_{\mathfrak{D}}, 0_{\mathfrak{D}})$ In semiring $\mathfrak{D} \times \mathfrak{D}$ will be shown. However, before examining these conditions, the following theorem states that $\mathfrak{D}_1 \times \mathfrak{D}_2$ is semiring if $\mathfrak{D}_1$ dan $\mathfrak{D}_2$ are well

Theorem 2. *Let $(\mathfrak{D}_1, +_1, \star_1)$ and $(\mathfrak{D}_2, +_2, \star_2)$ Be two semiring. Suppose that the addition operation $"+"$ and multiplication $"\star"$ on $\mathfrak{D}_1 \times \mathfrak{D}_2$, namely:*

$$(a, w) + (c, z) \stackrel{\text{def}}{=} (a +_1 c, w +_2 z) \text{ and } (a, w) \star (c, z) \stackrel{\text{def}}{=} (a \star_1 c, w \star_2 z)$$

for any $(a, w), (c, z) \in \mathfrak{D}_1 \times \mathfrak{D}_2$. Then $(\mathfrak{D}_1 \times \mathfrak{D}_2, +, \star)$ be a semiring.

Proof. First, it will be shown that the addition operation $"+"$ and multiplication $"\star"$ are binary operations on $\mathfrak{D}_1 \times \mathfrak{D}_2$, namely the closed nature of the operation and the singleness of the operation result (well-defined).

(a). Let $(a, w), (c, z) \in \mathfrak{D}_1 \times \mathfrak{D}_2$. Then

$$(a, w) + (c, z) = \left(\underbrace{a +_1 c}_{\in \mathfrak{D}_1}, \underbrace{w +_2 z}_{\in \mathfrak{D}_2}\right) \in \mathfrak{D}_1 \times \mathfrak{D}_2$$

and

$$(a, w) \star (c, z) = \left(\underbrace{a \star_1 c}_{\in \mathfrak{D}_1}, \underbrace{w \star_2 z}_{\in \mathfrak{D}_2}\right) \in \mathfrak{D}_1 \times \mathfrak{D}_2.$$

(b). Let $(a_1, a_2), (c_1, c_2), (x_1, x_2), (z_1, z_2) \in \mathfrak{D}_1 \times \mathfrak{D}_2$ where

$$(a_1, a_2) = (x_1, x_2) \text{ and } (c_1, c_2) = (z_1, z_2).$$

because of this,

$$a_1 = x_1, a_2 = x_2, c_1 = z_1, \text{ and } c_2 = z_2.$$

Note that,

$$(a_1, a_2) + (c_1, c_2) = (a_1 +_1 c_1, a_2 +_2 c_2) = (x_1 +_1 z_1, x_2 +_2 z_2) = (x_1, x_2) + (z_1, z_2)$$

and

$$(a_1, a_2) \star (c_1, c_2) = (a_1 \star_1 c_1, a_2 \star_2 c_2) = (x_1 \star_1 z_1, x_2 \star_2 z_2) = (x_1, x_2) \star (z_1, z_2).$$

Thus, the operations of addition $"+"$ and multiplication $"\star"$ are well-defined.

So, the addition operations $"+"$ and multiplication $"\star"$ are binary operations on $\mathfrak{D}_1 \times \mathfrak{D}_2$.

Second, it will be shown that the addition operation $"+"$ and multiplication $"\star"$ are associative on $\mathfrak{D}_1 \times \mathfrak{D}_2$.

Let $(a,x),(c,z),(d,w) \in \mathfrak{D}_1 \times \mathfrak{D}_2$. Since operation $+_1$ and $\star_1$, $+_2$ and $\star_2$ are associative on semiring $\mathfrak{D}_1$ and semiring $\mathfrak{D}_2$. We have,

$$\begin{aligned}[(a,x)+(c,z)]+(d,w) &= (a+_1c, x+_2z)+(d,w)\\ &= \big((a+_1c)+_1d, (x+_2z)+_2w\big)\\ &= \big(a+_1(c+_1d), x+_2(z+_2w)\big)\\ &= (a,x)+(c+_1d, z+_2w)\\ &= (a,x)+[(c,z)+(d,w)]\end{aligned}$$

and

$$\begin{aligned}[(a,x)\star(c,z)]\star(d,w) &= (a\star_1 c, x\star_2 z)\star(d,w)\\ &= \big((a+_1c)\star_1 d, (x\star_2 z)\star_2 w\big)\\ &= \big(a\star_1(c\star_1 d), x\star_2(z\star_2 w)\big)\\ &= (a,x)\star(c\star_1 d, z\star_2 w)\\ &= (a,x)\star[(c,z)\star(d,w)]\end{aligned}$$

So, the addition operations " $+$ " and multiplication " $\star$ " are associative on $\mathfrak{D}_1 \times \mathfrak{D}_2$.

Third, it will be shown that the addition operation " $+$ " is commutative on $\mathfrak{D}_1 \times \mathfrak{D}_2$.

Let $(a,x),(c,z) \in \mathfrak{D}_1 \times \mathfrak{D}_2$. Because the operations $+_1$ and $+_2$ are commutative in semiring $\mathfrak{D}_1$ and $\mathfrak{D}_2$. We have,

$$(a,x)+(c,z) = (a+_1c, x+_2z) = (c+_1a, z+_2x) = (c,z)+(a,x).$$

So, the addition operations " $+$ " and multiplication " $\star$ " are commutative on $\mathfrak{D}_1 \times \mathfrak{D}_2$.

Fourth, we will show that $\mathfrak{D}_1 \times \mathfrak{D}_2$ contains a zero element $0_{\mathfrak{D}_1\times\mathfrak{D}_2}$ and a unit element one $1_{\mathfrak{D}_1\times\mathfrak{D}_2}$.

Since $0_{\mathfrak{D}_1}, 1_{\mathfrak{D}_1} \in \mathfrak{D}_1$ and $0_{\mathfrak{D}_2}, 1_{\mathfrak{D}_2} \in \mathfrak{D}_2$. We have,

$$(a,x)+\big(0_{\mathfrak{D}_1},0_{\mathfrak{D}_2}\big) = \big(a+_10_{\mathfrak{D}_1}, x+_20_{\mathfrak{D}_2}\big) = (a,x) = \big(0_{\mathfrak{D}_1}+_1a, 0_{\mathfrak{D}_2}+_2x\big) = \big(0_{\mathfrak{D}_1},0_{\mathfrak{D}_2}\big)+(a,x)$$

and

$$(a,x)\star\big(1_{\mathfrak{D}_1},1_{\mathfrak{D}_2}\big) = \big(a\star_1 1_{\mathfrak{D}_1}, x\star_2 1_{\mathfrak{D}_2}\big) = (a,x) = \big(1_{\mathfrak{D}_1}\star_1 a, 1_{\mathfrak{D}_2}\star_2 x\big) = \big(1_{\mathfrak{D}_1},1_{\mathfrak{D}_2}\big)\star(a,x)$$

for any $(a,w) \in \mathfrak{D}_1 \times \mathfrak{D}_2$.

Hence, $\mathfrak{D}_1 \times \mathfrak{D}_2$ contains the zero element $0_{\mathfrak{D}_1\times\mathfrak{D}_2} = \big(0_{\mathfrak{D}_1},0_{\mathfrak{D}_2}\big)$ and the unit element $1_{\mathfrak{D}_1\times\mathfrak{D}_2} = \big(1_{\mathfrak{D}_1},1_{\mathfrak{D}_2}\big)$.

Fifth, it will be shown in $\mathfrak{D}_1 \times \mathfrak{D}_2$ that the distributive property of multiplication over addition applies. Let $(a,x),(c,z),(d,w) \in \mathfrak{D}_1 \times \mathfrak{D}_2$. Since the operations $\star_1$ on $+_1$, and $\star_2$ on $+_2$ satisfy the distributive law to $\mathfrak{D}_1$ and $\mathfrak{D}_2$. We have

$$\begin{aligned}(a,x)\star[(c,z)+(d,w)] &= (a,x)\star(c+_1d, z+_2w)\\ &= \big(a\star_1(c+_1d), x\star_2(z+_2w)\big)\\ &= (a\star_1 c+_1a\star_1 d, x\star_2 z+_2x\star_2 w)\\ &= (a\star_1 c, x\star_2 z)+(a\star_1 d, x\star_2 w)\\ &= (a,x)\star(c,z)+(a,x)\star(d,w)\end{aligned}$$

and

$$\begin{aligned}[(a,x)+(c,z)]\star(d,w) &= (a+_1c, x+_2z)\star(d,w)\\ &= \big((a+_1c)\star_1 d, (x+_2z)\star_2 w\big)\\ &= (a\star_1 d+_1c\star_1 d, x\star_2 w+_2z\star_2 w)\\ &= (a\star_1 d, x\star_2 w)+(c\star_1 d, z\star_2 w)\end{aligned}$$

$$= (a,x) \star (d,w) + (c,z) \star (d,w)$$

So, on $\mathfrak{D}_1 \times \mathfrak{D}_2$, the distributive property of multiplication over addition applies.

Sixth, it will be shown$(a,x) \star 0_{\mathfrak{D}_1\times\mathfrak{D}_2} = 0_{\mathfrak{D}_1\times\mathfrak{D}_2} \star (a,x) = 0_{\mathfrak{D}_1\times\mathfrak{D}_2}$, for any $(a,w) \in \mathfrak{D}_1 \times \mathfrak{D}_2$. Since, $a \cdot 0_{\mathfrak{D}_1} = 0_{\mathfrak{D}_1} \cdot a = 0_{\mathfrak{D}_1}$ and $w \cdot 0_{\mathfrak{D}_2} = 0_{\mathfrak{D}_2} \cdot w = 0_{\mathfrak{D}_2}$for any $a \in \mathfrak{D}_1$ and $w \in \mathfrak{D}_2$. We have,

$$(a,x) \star 0_{\mathfrak{D}_1\times\mathfrak{D}_2} = (a,x) \star \left(0_{\mathfrak{D}_1}, 0_{\mathfrak{D}_2}\right) = 0_{\mathfrak{D}_1\times\mathfrak{D}_2} = 0_{\mathfrak{D}_1\times\mathfrak{D}_2} \star (a,x).$$

Hence, $(a,x) \star 0_{\mathfrak{D}_1\times\mathfrak{D}_2} = 0_{\mathfrak{D}_1\times\mathfrak{D}_2} \star (a,x) = 0_{\mathfrak{D}_1\times\mathfrak{D}_2}$, for any $(a,w) \in \mathfrak{D}_1 \times \mathfrak{D}_2$.

Based on the analysis results above and the definition of a semiring, it is obtained that $\mathfrak{D}_1 \times \mathfrak{D}_2$ Be a semiring. ■

Condition Consequences **Theorem 2** can be generalized for semirings $(\mathfrak{D}_1, +_1, \star_1)$, $(\mathfrak{D}_2, +_2, \star_2)$, $\cdots$, and $(\mathfrak{D}_n, +_n, \star_n)$ Which results in $(\mathfrak{D}_1 \times \mathfrak{D}_2 \times \cdots \times \mathfrak{D}_n, +, \star)$ Be a semiring.

Theorem 3. *Let's say that σ and υ are fuzzy ideals of semiring $\mathfrak{D}$. Then $\sigma \times \upsilon(0_{\mathfrak{D}}, 0_{\mathfrak{D}}) \geq \sigma \times \upsilon(a,w)$ for any $(a,w) \in \mathfrak{D} \times \mathfrak{D}$.*

Proof. Let $(a,w) \in \mathfrak{D} \times \mathfrak{D}$. Since σ and υ are fuzzy ideals of $\mathfrak{D}$ and $(0_{\mathfrak{D}}, 0_{\mathfrak{D}}) \in \mathfrak{D} \times \mathfrak{D}$. Because of this, by Definisi 2.1 and Theorem 3.1, we have

$$\sigma \times \upsilon(0_{\mathfrak{D}}, 0_{\mathfrak{D}}) = \sigma(0_{\mathfrak{D}}) \wedge \upsilon(0_{\mathfrak{D}}) \geq \sigma(a) \wedge \upsilon(w) = \sigma \times \upsilon(a,w).$$

So, $\sigma \times \upsilon(0_{\mathfrak{D}}, 0_{\mathfrak{D}}) \geq \sigma \times \upsilon(a,w)$ for any $(a,w) \in \mathfrak{D} \times \mathfrak{D}$. ■

In the semi-ring $\mathfrak{D}$ elements, the closed nature applies for the addition operation " + " or the multiplication operation " $\cdot$ ". This condition of closed nature motivates the same thing for the elements in $\mathcal{F}(\mathfrak{D})$. If σ and η are fuzzy ideals of semirings $\mathfrak{D}$, is the cross product of σ and η also a fuzzy ideal?

Theorem 4. *Let's say that σ and υ are fuzzy ideals of semiring $\mathfrak{D}$. Then the cross product σ and υ are the fuzzy ideals of the semiring $\mathfrak{D} \times \mathfrak{D}$.*

Proof. Let $(a,w) \in \mathfrak{D} \times \mathfrak{D}$. Since σ and υ are fuzzy ideals of $\mathfrak{D}$. By **Definition 1** and **Definition 2**, we have

$$\begin{aligned}
\sigma \times \upsilon[(a,w) + (c,z)] &= \sigma \times \upsilon(a+c, w+z) \\
&= \sigma(a+c) \wedge \upsilon(w+z) \\
&\geq \big(\sigma(a) \wedge \sigma(c)\big) \wedge \big(\upsilon(w) \wedge \upsilon(z)\big) \\
&= \big(\sigma(a) \wedge \upsilon(w)\big) \wedge \big(\sigma(c) \wedge \upsilon(z)\big) \\
&= \sigma \times \upsilon(a,w) \wedge \sigma \times \upsilon(c,z)
\end{aligned}$$

and

$$\begin{aligned}
\sigma \times \upsilon[(a,w) \cdot (c,z)] &= \sigma \times \upsilon(a \cdot c, w \cdot z) \\
&= \sigma(a \cdot c) \wedge \upsilon(w \cdot z) \\
&\geq \sigma(a) \wedge \upsilon(w) \\
&= \sigma \times \upsilon(a,w).
\end{aligned}$$

So, $\sigma \times \upsilon$ is the fuzzy ideal of the semiring $\mathfrak{D} \times \mathfrak{D}$. ■

The consequence of the condition of **Theorem 4** is the result that states $\sigma_1 \times \sigma_2 \times \cdots \times \sigma_n$ is a fuzzy ideal of the semiring $\underbrace{\mathfrak{D} \times \mathfrak{D} \times \cdots \times \mathfrak{D}}_{n-times}$ if $\sigma_1, \sigma_2, \cdots$, and σ_n Are fuzzy ideal of the semiring $\mathfrak{D}$.

Corollary 1 *Let $\sigma_1, \sigma_2, \cdots$, and σ_n Are fuzzy ideal fuzzy of the semiring $\mathfrak{D}$. Then $\sigma_1 \times \sigma_2 \times \cdots \times \sigma_n$ is the fuzzy ideal of the semiring $\underbrace{\mathfrak{D} \times \mathfrak{D} \times \cdots \times \mathfrak{D}}_{n-times}$.*

The condition $\sigma \times \upsilon$ is a fuzzy ideal of the semiring $\mathfrak{D}$ in Theorem 3.4 holds if σ and υ are fuzzy ideals of $\mathfrak{D}$. Based on this fact, will the opposite of **Theorem 4** hold? This condition motivates the emergence of the following theorem.

Theorem 5. *Let σ and υ be fuzzy subset ideals of the semiring $\mathfrak{D}$. If $\sigma \times \upsilon$ is a fuzzy ideal fuzzy of $\mathfrak{D}$ then σ or υ is the fuzzy ideal of $\mathfrak{D}$.*

Proof. Let $\sigma \times \upsilon$ be a fuzzy ideal fuzzy of $\mathfrak{D}$. By Theorem 3.3, for each $(a, w) \in \mathfrak{D} \times \mathfrak{D}$, we have

$$\sigma(0_{\mathfrak{D}}) \wedge \upsilon(0_{\mathfrak{D}}) = \sigma \times \upsilon(0_{\mathfrak{D}}, 0_{\mathfrak{D}}) \geq \sigma \times \upsilon(a, w) = \sigma(a) \wedge \upsilon(w).$$

Therefore,

$$\sigma(0_{\mathfrak{D}}) \geq \sigma(a) \text{ and } \sigma(0_{\mathfrak{D}}) \geq \upsilon(w)$$

or

$$\upsilon(0_{\mathfrak{D}}) \geq \sigma(a) \text{ and } \upsilon(0_{\mathfrak{D}}) \geq \upsilon(w).$$

So, for each $a, w \in \mathfrak{D}$, conditions are met:

$$\begin{aligned}
\sigma(a + w) &= \sigma \times \upsilon(a + w, 0_{\mathfrak{D}}) \\
&= \sigma \times \upsilon[(a, 0_{\mathfrak{D}}) + (w, 0_{\mathfrak{D}})] \\
&\geq \sigma \times \upsilon(a, 0_{\mathfrak{D}}) \wedge \sigma \times \upsilon(w, 0_{\mathfrak{D}}) \\
&= \sigma(a) \wedge \upsilon(w)
\end{aligned}$$

and

$$\begin{aligned}
\sigma(a \cdot w) &= \sigma \times \upsilon(a \cdot w, 0_{\mathfrak{D}}) \\
&= \sigma \times \upsilon[(a, 0_{\mathfrak{D}}) \cdot (w, 0_{\mathfrak{D}})] \\
&\geq \sigma \times \upsilon(a, 0_{\mathfrak{D}}) \\
&= \sigma(a).
\end{aligned}$$

Hence, σ is the fuzzy ideal of $\mathfrak{D}$.

Hereafter,

$$\begin{aligned}
\upsilon(a + w) &= \sigma \times \upsilon(0_{\mathfrak{D}}, a + w) \\
&= \sigma \times \upsilon[(0_{\mathfrak{D}}, a) + (0_{\mathfrak{D}}, w)] \\
&\geq \sigma \times \upsilon(0_{\mathfrak{D}}, a) \wedge \sigma \times \upsilon(0_{\mathfrak{D}}, w) \\
&= \upsilon(a) \wedge \upsilon(w)
\end{aligned}$$

and

$$\begin{aligned}
\upsilon(a \cdot w) &= \sigma \times \upsilon(0_{\mathfrak{D}}, a \cdot w) \\
&= \sigma \times \upsilon[(0_{\mathfrak{D}}, a) \cdot (0_{\mathfrak{D}}, w)] \\
&\geq \sigma \times \upsilon(0_{\mathfrak{D}}, a) \\
&= \upsilon(a).
\end{aligned}$$

So, υ is the fuzzy ideal of $\mathfrak{D}$. Hence, based on the results of the analysis above, it is obtained that σ or υ is a fuzzy ideal of semiring $\mathfrak{D}$. ■

According to Theorem 3.6, the converse of Theorem 3.4 does not hold. In addition, the consequences of **Theorem 5** result in the following consequences.

Corollary 2 *Let $\mathfrak{D}$ be a semiring where $\sigma_1, \sigma_2, \cdots, \sigma_n \in \mathcal{F}(\mathfrak{D})$. If $\sigma_1 \times \sigma_2 \times \cdots \times \sigma_n$ Are fuzzy ideals of $\mathfrak{D}$ then σ_1 or σ_2 or $\cdots$ or σ_n It is fuzzy ideals of $\mathfrak{D}$.*

The fact of proving **Theorem 5** and adding the condition $\sigma(0_{\mathfrak{D}}) = \upsilon(0_{\mathfrak{D}})$, results in the following result.

Corollary 3 Let $\mathfrak{D}$ be a semiring where $\sigma, \upsilon \in \mathcal{F}(\mathfrak{D})$ such that $\sigma \times \upsilon$ is fuzzy ideal of $\mathfrak{D} \times \mathfrak{D}$. If $\sigma(0_{\mathfrak{D}}) = \upsilon(0_{\mathfrak{D}})$, $\sigma(a) \leq \sigma(0_{\mathfrak{D}})$, and $\upsilon(w) \leq \upsilon(0_{\mathfrak{D}})$ Then σ and υ are fuzzy ideals of $\mathfrak{D}$, for each $a, w \in \mathfrak{D}$.

The conditions of **Corollary 3** are generalized for n fuzzy subsets of semirings $\mathfrak{D}$ such that they are presented in the following results.

Corollary 4 *Let $\mathfrak{D}$ be a semiring where $\sigma_1, \sigma_2, \cdots, \sigma_n \in \mathcal{F}(\mathfrak{D})$ such that $\sigma_1 \times \sigma_2 \times \cdots \times \sigma_n$ is the fuzzy ideal of semiring $\underbrace{\mathfrak{D} \times \mathfrak{D} \times \cdots \times \mathfrak{D}}_{n-times}$. If $\sigma_1(0_{\mathfrak{D}}) = \sigma_2(0_{\mathfrak{D}}) = \cdots = \sigma_n(0_{\mathfrak{D}})$, $\sigma_1(a) \leq \sigma_1(0_{\mathfrak{D}})$, $\sigma_2(a) \leq \sigma_2(0_{\mathfrak{D}})$, $\cdots$, and $\sigma_n(a) \leq \sigma_n(0_{\mathfrak{D}})$ then σ_1, σ_2, $\cdots$, and σ_n Are fuzzy ideal of $\mathfrak{D}$.*

4. CONCLUSIONS

Based on the results and discussion, the cross product of two (more) fuzzy ideals from a semiring is also always a fuzzy ideal. But the opposite of that nature does not apply. Unless required, as in **Corollary 3** and **Corollary 4**.

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