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〈研究ノート〉

A note on alternate two-way contracts for side payments in two-player games*

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Abstract

We studied side contracting process where two players alternately offer not only schemes for side payment transfer to the other but also schemes for side payment receipt from the other before choosing actions in the underlying game. Then, we found that there may be a kind of the second mover's disadvantage in such side contracting process in contrast to Yamada (2005).

1 Introduction

Yamada (2005b) studied side contracting process where two players alternately offer schemes for side payment transfer to the other before choosing actions in the underlying game, and found that the second offerer's equilibrium payoff can exceed his threat point level to some extent. Then, we noted by the result that there is a kind of the second mover's advantage in such side contracting process.

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In line with Jackson and Wilkie (2005), a canonical paper in this field, Yamada (2005b) focused on voluntarily offered side payments and assumed that such side payments would always be accepted by transferees. This assumption might be thought of as arbitrary, however, since voluntarily offered side payments could be invalidated by spontaneous rejection to receive them.[1]

In contrast to Yamada (2005b), we studied side contracting process where two players alternately offer not only schemes for side payment transfer to the other but also schemes for side payment receipt from the other before choosing actions in the underlying game. Then, we found that the second offerer's equilibrium payoff must be suppressed to his threat point level, and there may be a kind of the second mover's disadvantage in such side contracting process.

In what follows we present the model in Section 2 and the analysis in Section 3. Our concluding remarks appear in Section 4.

2 Model

We consider two-player three-stage games played as follows.

Stage 1: Player 1 announces a transfer function (side-payment transfer scheme) and a receipt function (side-payment acceptance/rejection scheme), each of which is assumed to be binding.
Stage 2: Player 2 announces a transfer function (side-payment transfer scheme) and a receipt function (side-payment transfer acceptance/rejection scheme), each of which is assumed to be binding.
Stage 3: Each player chooses an action.

The players *alternately* promises two-way schemes (transfer and receipt functions) on side payments in the first and second stages. Such an alternate contracting process was analyzed by Yamada (2005b), where the players promise not two-way but one-way (only transfer) schemes. Moreover, the two-way schemes were called *bilateral contracts* in Yamada (2003), which studied the performance of such two-way schemes in not alternate but simultaneous contracting games.

[1] Yamada (2005a) studied another type of two-player three-stage games which are the same as Yamada's (2005b) except that players are allowed to offer negative side payments as well as nonnegative ones. In Yamada's (2005a) setting, the second transfer-offerer takes all the payoff while the first looses everything.

The players are given by a set $N = \{1, 2\}$. Let i or j denote any given one of the two players. When a player is denoted by i, let j denote the other player, and vice versa.

A player i's finite pure strategy space in the third stage game is denoted by X_i, with $X = \times_i X_i$. Let $\Delta(X_i)$ denote the set of mixed strategies for i, and let $\Delta = \times_i \Delta(X_i)$. We denote by x_i, x, μ_i, and μ generic elements of X_i, X, $\Delta(X_i)$, and Δ respectively. For simplicity, we sometimes use x_i and x to denote μ_i and μ respectively that place probability one on x_i and x. A player i's before-side-payment payoffs in the third stage game are given by a von Neumann-Morgenstern utility function $v_i : X \to \mathbb{R}$.

A transfer function announced by i in the first or second stage is denoted by t_i, where $t_i : X \times Z \to \mathbb{R}_+$ with $Z = \{0, 1\}$ represents i's promises to j as a function of actions chosen in the third stage and indicators 0 and 1. Indicator 0 means that according to the transfer and receipt schemes announced in the first and second stages, a player rejects transfer from the other. Indicator 1 means that according to the transfer and receipt schemes announced in the first and second stages, every player accepts transfer from the other. Let T be the set of all possible t_i. Let $t = (t_1, t_2)$.

Note that if $t_i(x, z) = z\tau_i(x)$ for some $\tau_i : X \to \mathbb{R}_+$, then the transfer scheme becomes degenerate, or $t_i(x, z) = 0$ for all x, unless every player accepts transfer from the other. That is, when players are expected to promise acceptance to each other, such transfer function can be sensitive to a player's deviation on the receipt scheme.

A receipt function announced by player i in the first or second stage is denoted by r_i, where $r_i : T^2 \to \{0, 1\}$ represents i's acceptance (1) or rejection (0) of transfer from j as a function of profiles of transfer functions announced in the first and second stages. Let $r = (r_1, r_2)$.

Given a profile t of transfer functions and a profile r of receipt functions in the first and second stages, and a play x in the third stage game, the payoff U_i to player i becomes

$$U_i(x, t, r) = v_i(x) + (r_i(t)\, t_j(x, a(t, r)) - r_j(t)\, t_i(x, a(t, r)))$$

where $a(t, r) = r_1(t) \times r_2(t)$.

Given a profile t of transfer functions and a profile r of receipt functions in the first and second stages, and a play μ in the third stage game, the expected payoff EU_i to player i becomes

$$EU_i(\mu,t,r) = \sum_x \times_k \mu_k(x_k)\left(v_i(x) + (r_i(t)\,t_j(x,a(t,r)) - r_j(t)\,t_i(x,a(t,r)))\right)$$

where $a(t,r) = r_1(t) \times r_2(t)$. Let $EU_i(\mu) = \sum_x \times_k \mu_k(x_k)\, v_i(x)$.

Let $NE(t,r)$ denote the set of (mixed) Nash equilibria of the third stage game given (t,r) in the first and second stages. Let NE represent the set of (mixed) Nash equilibria of the underlying game (the third stage game without side contracts). For sake of simplicity and analyzability, we assume for each i, there exists $\min_{\mu \in NE} EU_i(\mu)$.

A pure strategy profile $x \in X$ of the third stage game is *supportable* if there exists a subgame perfect equilibrium of the three stage game where (i) for t and r announced in the first and second stages and x played in the third stage on the equilibrium path, $U_i(x,t,r) = u_i$ for some $u \in \mathbb{R}^2$ such that $\sum_i u_i = \sum_i v_i(x)$, and moreover, (ii) any 1's deviation in the first stage cannot induce a Nash equilibrium of the following (after-deviation) subgame off the equilibrium path so that 1 may enjoy more payoff than u_1. Note (ii) guarantees a kind of stability of the supportability. This condition requires that there be no incentive for deviation from the beginning of the game.

3 Analysis

We study the model to obtain the next theorem.

Theorem 1. *If* $\overline{x}$ *is supportable, then* 2's equilibrium payoff is no more than $\min_{\mu \in NE} EU_2(\mu)$.

Proof of Theorem 1. Since $\overline{x}$ is supportable, there exists a subgame perfect equilibrium of the three stage game where (i) for $\overline{t}$ and $\overline{r}$ announced in the first and second stages, and $\overline{x}$ played in the third stage on the equilibrium path, $U_i\left(\overline{x},\overline{t},\overline{r}\right) = \overline{u}_i$ for some $\overline{u} \in \mathbb{R}^2$ such that $\sum_i \overline{u}_i = \sum_i v_i(\overline{x})$, and (ii) any 1's deviation in the first stage cannot induce a Nash equilibrium of the following (after-deviation) subgame off the equilibrium path so that 1 may enjoy more payoff than $\overline{u}_1$.

Suppose $\overline{u}_2 > \min_{\mu \in NE} EU_2(\mu)$. Note here that there must be ${}_i\mu$ for all i such that ${}_i\mu \in NE$ and $EU_i({}_i\mu) \leq \overline{u}_i$. Otherwise some i announces degenerate t_i and r_i, or no transfer and no acceptance, and enjoy $EU_i(\mu) > \overline{u}_i$ for any

$\mu \in NE$. That is, i has an incentive to deviate from the equilibrium path. Let ${}_2\widehat{\mu} = \arg\min_{\mu \in NE} EU_2(\mu)$ in particular.

Consider $\tau_i : X \to \mathbb{R}_+$ such that $\tau_i(x) = 0$ for all $x \neq \overline{x}$, $\widehat{u}_i = v_i(\overline{x}) + \tau_j(\overline{x}) - \tau_i(\overline{x})$ where $\widehat{u}_1 = \overline{u}_1 + \frac{\overline{u}_2 - \min_{\mu \in NE}(EU_2(\mu))}{2}$ and $\widehat{u}_2 = \overline{u}_2 - \frac{\overline{u}_2 - \min_{\mu \in NE}(EU_2(\mu))}{2}$, and $\tau_i(\overline{x}) > 0$ implies $\tau_j(\overline{x}) = 0$. Note $\widehat{u}_1 > \overline{u}_1$ and $\widehat{u}_2 > \min_{\mu \in NE}(EU_2(\mu))$ since $\overline{u}_2 > \min_{\mu \in NE}(EU_2(\mu))$. Let $\widehat{t}$ and $\widehat{r}$ be as follows.

$$\widehat{t}_i(x, z) = \begin{cases} z(\tau_i(x) + \max\{0, v_i(x) + \tau_j(x) - \tau_i(x) - \widehat{u}_i\}) & \text{if } x = (x_i, \overline{x}_j) \\ 0 & \text{otherwise} \end{cases}$$

$$\widehat{r}_i(t) = \begin{cases} 1 & \text{if } t = \widehat{t} \\ 0 & \text{otherwise} \end{cases}$$

Suppose 1 deviates to $(\widehat{t}_1, \widehat{r}_1)$ in the first stage, considering the following strategy profile (μ, t_2, r_2) for the subgame afterwards.

(1) $(t_2, r_2) = (\widehat{t}_2, \widehat{r}_2)$;
(2) if $(t_2, r_2) = (\widehat{t}_2, r_2)$, where
$r_2(\widehat{t}) = 1$, then $\mu = \overline{x}$;
(2-1) if $(t_2, r_2) = (\widehat{t}_2, r_2)$, where $r_2(\widehat{t}) = 0$, then ${}_2\widehat{\mu} \in NE$.
(2-2) if $(t_2, r_2) = (t_2, r_2)$, where $t_2 \neq \widehat{t}_2$, then ${}_2\widehat{\mu} \in NE$.

Case (1)-(2). If 2 announces $(\widehat{t}_2, r_2)$ with $r_2(\widehat{t}) = 1$ in the second stage, $\overline{x} \in NE(\widehat{t}, (\widehat{r}_1, r_2))$ and $U_i(\overline{x}, \widehat{t}, (\widehat{r}_1, r_2)) = \widehat{u}_i$ for each i where $\widehat{u}_1 > \overline{u}_1$ and $\widehat{u}_2 > \min_{\mu \in NE} EU_2(\mu)$.

Case (1)-(2-1). Consider 2 announces $(\widehat{t}_2, r_2)$ with $r_2(\widehat{t}) = 0$ in the second stage.
If $\mu = (\mu_1, {}_2\widehat{\mu}_2)$, then

$$\begin{aligned}
&EU_1(\mu, t, r) \\
&= \sum_x \times_k \mu_k(x_k)(v_1(x) + (r_1(t)t_2(x, a(t, r)) - r_2(t)t_1(x, a(t, r)))) \\
&= \sum_x \times_k \mu_k(x_k)(v_1(x) + (1 \cdot 0 - 0 \cdot 0)) \\
&= \sum_x \times_k \mu_k(x_k)v_1(x) = EU_1(\mu_1, {}_2\widehat{\mu}_2) \leq EU_1({}_2\widehat{\mu}),
\end{aligned}$$

If $\mu = ({}_2\widehat{\mu}_1, \mu_2)$, then

$$\begin{aligned}
&EU_2(\mu, t, r) \\
&= \sum_x \times_k \mu_k(x_k)(v_2(x) + (r_2(t)t_1(x, a(t, r)) - r_1(t)t_2(x, a(t, r))))
\end{aligned}$$

$$=\sum_x \times_k \mu_k(x_k)(v_2(x)+(0\cdot 0-1\cdot 0))$$
$$=\sum_x \times_k \mu_k(x_k)\,v_2(x)=EU_2({}_2\widehat{\mu}_1,\mu_2)\leq EU_2({}_2\widehat{\mu})<\widehat{u}_2.$$

Case (1)-(2-2). Consider 2 announces (t_2, r_2) with $t_2 \neq \widehat{t}_2$ in the second stage.
If $\mu = (\mu_{1,2}\,\widehat{\mu}_2)$, then

$$EU_1(\mu,t,r)$$
$$=\sum_x \times_k \mu_k(x_k)(v_1(x)+(r_1(t)\,t_2(x,a(t,r))-r_2(t)\,t_1(x,a(t,r))))$$
$$=\sum_x \times_k \mu_k(x_k)(v_1(x)+(0\cdot t_2(x,a(t,r))-r_2(t)\cdot 0))$$
$$=\sum_x \times_k \mu_k(x_k)\,v_1(x)=EU_1(\mu_{1,2}\,\widehat{\mu}_2)\leq EU_1({}_2\widehat{\mu}).$$

If $\mu = ({}_2\widehat{\mu}_1, \mu_2)$, then

$$EU_2(\mu,t,r)$$
$$=\sum_x \times_k \mu_k(x_k)(v_2(x)+(r_2(t)\,t_1(x,a(t,r))-r_1(t)\,t_2(x,a(t,r))))$$
$$=\sum_x \times_k \mu_k(x_k)(v_2(x)+(r_2(t)\cdot 0-0\cdot t_2(x,a(t,r))))$$
$$=\sum_x \times_k \mu_k(x_k)\,v_2(x)=EU_2({}_2\widehat{\mu}_1,\mu_2)\leq EU_2({}_2\widehat{\mu})<\widehat{u}_2.$$

Thus, (1), (2), (2-1) and (2-2) constitute a Nash equilibrium of the subgame of the three-stage game after $(\widehat{t}_1, \widehat{r}_1)$ is announced in the first stage. In its equilibrium path (1)-(2) 1 can enjoy $\widehat{u}_1 > \overline{u}_1$, which implies 1 has an incentive to deviate to $(\widehat{t}_1, \widehat{r}_1)$ in the first stage. The result contradicts to $\overline{x}$'s supportability in terms of (ii), or its stability condition.

Theorem 1 implies that the second offerer's equilibrium payoff must be suppressed to his threat point level. That is, there may be a kind of the second mover's disadvantage in such side contracting process.

4 Conclusion

We studied side contracting process where two players alternately offer not only schemes for side payment transfer to the other but also schemes for side payment receipt from the other before choosing actions in the underlying game. Then, we found that there may be a kind of the second mover's disadvantage in such side contracting process.

The result seems in contrast to Yamada (2005b). Yamada (2005b) studied side contracting process where two players alternately offer only schemes for side payment transfer to the other before choosing actions in the underlying game, and found that the second offerer's equilibrium payoff can exceed his threat point level to some extent. Then, we noted by the result that there is a kind of the second mover's advantage, not disadvantage, in such side contracting process as in Yamada's (2005b) framework.

In order to make it clear to what extent there really is a kind of the second mover's disadvantage, we must consider in more detail the nature of equilibrium in our setting. Next, we are to fully characterize equilibrium outcomes in side contracting process where two players alternately offer not only schemes for side payment transfer to the other but also schemes for side payment receipt from the other before choosing actions in the underlying game.

5 Reference

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