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WRITE, AND WRITE WELL—SPEAK, AND SPEAK WELL: THE GOSPEL ACCORDING TO HALMOS AND ROTA

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Abstract Writing/speaking well—with real intent, focus and clarity—has always been an issue of the utmost importance to some of those working in academia generally, within which mathematicians are no exception. We give consideration to the conventional literature survey and other expository pieces as useful centres of an initial discussion, and develop the broader theme as referenced to two of our very best past communicators and popularisers of mathematics across both facets of dissemination—P.R. Halmos and G.-C. Rota, *de facto* grand seigneurs of the early modern day period.

"Mathematicians, like Proust and everyone else, are at their best when writing about their first love."

Gian-Carlo Rota

"To explain something you must know not only what to put in, but also what to leave out."

Paul Richard Halmos

1 Halmos, Rota, and Writing

The production of engaging, structured and interesting exposition is not easy, even for those to whom it comes rather naturally. For practitioners striving to achieve this some subjects present more of an obstacle than others, and mathematics is one of them. A rather awkward type of piece to deal with—in the sense that an author wants a reader to stay with the disquisition, rather than it be abandoned because of poor content, or because the composition is overly technical, or stilted, or leaden—is the literature survey. The temptation to fill it with jargonistic and esoteric terminologies is best resisted, or at least tempered, if it is to have wide appeal, and on this point I have sympathy with Paul R. Halmos (1916–2006) who held some firm beliefs on the matter and was not afraid to voice his concern about what he felt was the misnomer "research exposition" that had been applied to a new genre of feature surfacing in the then flagship journal *Bulletin of the American Mathematical Society* over 40 years ago; it bothered him. Introduced in the late 1970s to counter the publication's falling approval ratings and marketability at the time, Hungarian-born American Halmos—a prolific communicator in relation to research and teaching, and tireless populariser of mathematics—penned this on what he (somewhat dramatically) referred to as a "lifesaving experiment":

> "Some people like the research-expository articles very much and think that anyone who does not like them belongs to the great body of the anti-intellectual unwashed.
>
> I don't like them.
>
> One reason I don't like them is that fewer than half of the ones that have appeared deserve the name, and I cannot think of a single one that accomplishes the purpose the name was intended to describe." [7, p. 600],

citing editorially sanctioned impostor offerings (such as invited addresses, obituaries and quasi-books) that were also appearing under a title he saw as having a very specific meaning. He was quite within his rights to be a tad miffed, arguing that (p. 600)

> "A survey tells the history of a subject, contains a detailed, scholarly bibliography, and, in between, it defines, it states, it proves, and it is mercilessly complete. A survey is, in effect, a mini-encyclopedia. A good thing—yes, sometimes—but not for exposition, not for learning."

There is no doubt that a literature survey can provide a useful resource for one's academic peers (if comprehensive in range/depth it will be guaranteed citations, have a decent shelf-life, and possibly become a staple reference in the field), but they do tend to be rather dry, formal and rigid (and with it, sometimes monotonous, repetitive and mechanical)—put it like this, those that are agreeably readable lie in a minority group. I've tried to bear this in mind in anything I've written of this ilk, with varying degrees of success, so I suppose I'm pretty much with Halmos on the issue. He'd obviously had enough after four and a half years of reading this "new section" in the *Bulletin*, ending his Letter to the Editor with (p. 601)

> "That's the way things are. Others feel differently, of course, but I am convinced that my conclusions are those of the great majority, even if that word is interpreted in its narrowest, highest, most elitist sense to refer to the majority of active research mathematicians. The "research-expository articles" as they now stand are a failure. They should either be made expository, or, if that cannot be done, they should be abandoned."

These so called "research expository" papers were, for Halmos, neither one thing nor the other, and it is useful to ponder what makes for a good read in this context.

The survey is but one element of a bigger picture when cerebrating exposition. The gifted American-Italian mathematician and commentator Gian-Carlo Rota (1932–1999) also espoused strong convictions about propagating knowledge, being of the view that one is more likely to be remembered by expository work than anything else (he cited Hilbert and Feller as having become household names due in large part to their (respective) tomes on geometry/number theory and probability). Writing

> "Masters will write masterful books. There are few exceptions to this rule. Perhaps the rule is circular, since a great master is better recognized from expository work rather than from research papers." [18, p. 217],

he recalled

> "When I was in graduate school, one of my teachers told me, "When you write a research paper, you are afraid that your result might already be known; but when you write an expository paper, you discover that nothing is known." " (p. 206).

Rota felt that not only does an academic benefit personally in producing such a treatise once in a while, but asserted that this kind of writing is crucial to the survival of mathematics, adding

> "It takes an effort that is likely to go unrewarded and unappreciated to write an interesting exposition for the lay public at the cutting edge of mathematics. Most mathematicians (self-destructive and ungrateful wretches that they are, always ready to bite the hand that feeds them) turn their noses at the very thought. Little do they realize that in our science-eat-science world such expositions are the lifeline of [the subject]." (p. 216),

having previously avowed

> "Gifted expositors of mathematics are rare, indeed rarer than successful researchers. It is unfortunate that they are not rewarded as they deserve, in our present idiotic pecking order." [16, p. 1];

on this last point little, if anything, has altered, indeed the ranking of exposition—a few media 'stars' excepting, most of whom seem to have their own career agenda as a priority—is probably lower now than it was when Rota wrote these things in the 1990s. He was, to his advantage,

also an established philosopher as it happens, offering a course in phenomenology (usually oversubscribed) at M.I.T. where he spent much of his career. Meditating on all components of the mathematical profession was in his blood, and the same can almost be said of Halmos for whom philosophy was originally first choice over mathematics when he attended the University of Illinois graduate school. Mathematics is a holonic subject—inwardly self-contained in its theory, and outward facing in its power of application—enjoying also a cultural heritage that is worthy of critical analysis in its social and occasional political effects; Halmos and Rota were fully cognisant of these things, and wished to educate on their existence and impact as well.

2 Us and Them: Dealing With the Divide

Mathematicians have always been, and still are, disappointingly misapprehended by the general populace who do not really grasp what we do, what spurs us on, and why we act in particular ways at times, yet who feel able to form internalised malrepresentations of which they will not be divested—it's a problem that has always been around. We seem to get an unashamedly bad press by default, vulnerable to deep rooted prejudices and platitudinous stereotyping that are readily refuted but almost impossible to dislodge—these include Confirmation Bias (where people tend to selectively see what validates and in turn reinforces their hackneyed perceptions of us), Hindsight Bias (where people explain our actions and comportment as predictable causal outcomes of natural behaviour patterns), and a so called False Consensus Effect (where people, estimating that their clichéd *prima facie* distortions are predominant, feel safe to air them as normalised opinion). As might be expected, some dismal and very unhelpful misconceptions about our beloved subject abound, too, which is imagined to exist 'somewhere else'—removed and dissociated from everyday life—taking its professional protagonists with it to dwell in an alien bubble of strangeness and oddities. This combined stance is a phenomenon that is not new and on which Halmos was moved to comment, unwilling to tolerate it.

> "Why does mathematics occupy such an isolated position in the intellectual firmament? Why is it good form, for intellectuals, to shudder and announce that they can't bear it, or, at the very least, to giggle and announce that they never could understand it? One reason, perhaps, is that mathematics is a language. Mathematics is a precise and subtle language designed to express certain kinds of ideas more briefly, more accurately, and more usefully than ordinary language. ...
>
> One thing that sometimes upsets and repels the layman is the terminology that mathematicians employ. Mathematical words are intended merely as labels, sometimes suggestive, possibly facetious, but always precisely defined; their everyday connotations must be steadfastly ignored. ...
>
> ... None of us feels insulted when a sinologist uses Chinese phrases, and we are resigned to living without Chinese, or else spend years learning it. Our attitude to mathematics should be the same. It's a language, and it takes years to learn to speak it well. We all speak it a little, just because some of it is in the air all the time, but we speak it with an accent and frequently inaccurately; ... The mathematician sees nothing wrong with this as long as he's not upbraided by the rest of the intellectual community for keeping secrets. It took him a long time to learn his language, and he doesn't look down on the friend who, never having studied it, doesn't speak it. It is however sometimes difficult to keep one's temper with the cocktail party acquaintance who demands that he be taught the language between drinks and who regards failure or refusal to do so as sure signs of stupidity or snobbishness." [5, pp. 386–387].

Rota had a different take on where we stand with those who sit beyond the subject:

> "Flakiness is nowadays creeping into the sciences like a virus through a computer, ... Mathematics can save the world from the invasion of the flakes by unmasking them and by contributing some hard thinking. You and I know that mathematics is not and will never be flaky, by definition. ...
>
> When the going gets rough, we have recourse to a way of salvation that is not available to ordinary mortals: we have that Mighty Fortress that is our Mathematics. This is

> what makes us mathematicians into very special people. The danger is envy from the rest of the world.
>
> When you meet someone who does not know how to differentiate and integrate, be kind, gentle, understanding. Remember, there are lots of people like that out there, and if we are not careful, they will do away with us, as has happened many times before in history to other Very Special People." [18, p. 208].

As a slight digression, the reader is referred to the Appendix where we see other men who chose to address the issues of perception and personality of mathematicians for an audience when afforded the opportunity, or else to write about them.

Using an e-mail sent by him to friends (on October 7th, 1998), a paper was put together and posthumously published, in Rota's name, in which the need to engage with the outside world on practical grounds was forcefully articulated.

> "The question "What is mathematics?", asked to a mathematician by a person ignorant of mathematics, makes mathematicians uneasy. The mathematician senses dishonesty in the abruptness of the question. The questioner believes that an answer can be given, similar to the answers one can give to questions like "What is boeuf bourguignon?", "What is yellow fever?" or "What are Magli shoes?".
>
> The questioner does not want to learn any mathematics when he asks the question "What is mathematics?". The opposite is true: the questioner wants to rid himself of the need of learning any mathematics whatsoever. He wants to add to his conversational repertoire some brilliant answer that will permanently excuse him from any further dealings with the subject.
>
> One cannot escape the duty of giving a nutshell answer to the question ... Non-mathematicians need to have some idea of what mathematics "is" without having to study mathematics. They are dealing with mathematics as outsiders, but their dealings will affect the future of mathematics: mathematics requirements for schools must be determined by professional educators; mathematical proficiency among employees in a firm has to be gauged. Worst of all, the allocation of research funds for mathematics is made by individuals who have at best a fleeting acquaintance with the subject. Mathematics, like all intellectual disciplines, is not economically self-sustaining, and since the beginnings of civilization mathematicians have depended for their survival on the largesse of society or of a few wealthy individuals. Mathematicians, like philosophers and artists, are "kept" persons. In return, the public expects mathematicians to make the results of their work accessible to cultivated persons who may have a passing interest in mathematics, or who deal with the political and economic problems of mathematicians.
>
> We will leave to another occasion the tragedy that has resulted from the mathematicians' failure, going all the way back to Pythagoras, of giving exoteric accounts of their field that the public could appreciate. [A pragmatic] and short answer to the question "What is mathematics?" may be difficult to give, it may turn out to be dishonest and inadequate, but the mathematicians' failure to provide such an answer has been a costly mistake." [20, pp. 3–4].

Interestingly, and unavoidably, Halmos disagreed with Rota on some things. For one, he conceded that there was a need for mathematically literate people to be created (though he regarded the growth in training as little more than forging a trade (i) to assist with the mundanities of everyday life for the masses, and (ii) for professionals, equipping them to combat threats to the country from Eastern Europe and Asia (in keeping pace with their rapid technological progress)), speaking of his desire that all educable human beings should know what mathematics is primarily because their souls would be nurtured and enhanced—they would relish life, they would conceive of life more, and they would have heightened levels of awareness, he thought. Halmos, though, also said

> "I don't think it is vital and important to explain to members of Congress and administrators in the National Science Foundation what mathematics is and how important it is and how much money it must be given. I think we have been given too much

> money. I don't think mathematics needs to be supported. I think the phrase is almost offensive. Mathematics gets along fine, thank you, without money, and I look back with nostalgia to the good old days, ... when only those did mathematics who were willing to do it on their own time. ...
>
> ... If the N.S.F. had never existed, if the government had never funded American mathematics, we would have half as many mathematicians as we now have, and I don't see anything wrong with that." [2, pp. 127–128].

Quite a statement!

Here, then, we see frustration, pride, concern and humour, all feeding into a joint desire (and in turn a driving force) to protect mathematics, extend its compass, and bridge the gap between us mathematicians and those to whom we are always a mystery. Fortunately, a wellspring of passion for the discipline, coupled with irrepressible energy and impeccable credentials, made them model academics and ideally placed to promote it as they saw fit.

With an influence that was felicitous and inspiring in equal measure, it would be nice to imagine that both Halmos and Rota remain relevant today, and I think they do. For one thing, they had both mulled over the infrastructure of an academic department, what should go on inside one, and what makes for a successful set up. Rota, in [18], gave the short Chapter 19 over to 'Ten Lessons for the Survival of a Mathematics Department' presented as '1. Never Wash Your Dirty Linen in Public, 2. Never Go Above the Head of Your Department, 3. Never Compare Fields, 4. Remember That the Grocery Bill is a Piece of Mathematics Too, 5. Do Not Look Down on Good Teachers, 6. Write Expository Papers, 7. Do Not Show Your Questioners the Door, 8. View the Mathematical Community as a United Front, 9. Attack Flakiness, 10. Learn When to Withdraw'; these provide a mixture of droll and sapient deliberations on all manner of professional practices that would ensure a department functions with the right impetuses and remains healthy within the wider university culture. Halmos, in his 1985 so called 'automathography' (where he looked back at, and recounted in detail, his own working life with self-possessed frankness and unusual self-deprecation in places—something for which he received praise in a book review by Rota [18, pp. 235–237]), submitted the following observations which remain acutely pertinent:

> "Faculty members at universities demand, or in any event hope for, self-determination for their departments; *they* should decide with whom and under whom they want to work, not those ignorant deans. Yes, that sounds right; it is right most of the time. It's certainly right for departments of high quality. But what do you do when a department goes bad? André Weil suggested that there is a logarithmic law at work: first-rate people attract other first-rate people, but second-rate people tend to hire third-raters, and third-rate people hire fifth-raters. If a dean or a president is genuinely interested in building and maintaining a high-quality university (and some of them are), then he must not grant complete self-determination to a second-rate department; he must, instead, use his administrative powers to intervene and set things right. That's one of the proper functions of deans and presidents, and pity the poor university in which a large proportion of both the faculty and the administration are second-raters; it is doomed to diverge to minus infinity." [8, p. 123];

senior and 'executive' managers take note.[1]

Halmos and Rota made a lifelong commitment to scholarship across all of its forms, writing and speaking in forthright style—with elegance, enthusiasm and wisdom—about mathematics and pedagogy. Ebullient and exhilarating lecturers who offered the best parts of themselves as established researchers to boot, they were gregarious and lively raconteurs with a touch of the maverick and anecdotes aplenty (to which were added a dash of rhetorical hyperbole if necessary), happily exhibiting mild singularities and heterodox tendencies as forceful character dictates. Their written works and lectures on how to write/talk about, and publish, mathematics helped many mathematicians to convey their postulates and results more effectively, all under-

[1] A point here, if I may. So called 'C-Graders' (a term coined by an ex colleague) are those who—curiously secure in, or else too often blind to, their own professional and personal mediocrity—have somehow clawed their way up a career ladder to hold positions of leverage. In universities, it is now accepted that the institutional band of employment occupied by well salaried directorate and decision makers is heavily littered with these people who happily ride the corporate waves they have created while rightful academics flounder in choppy waters and scramble for the shores of real scholarship; 'Weil's Law', it would seem, is actually true.

pinned by an authority and gravitas as appreciable theoretical analysts and diligent instructors to undergraduate and postgraduate students alike—such people are rare, and stand out from the crowd. They had the ability to communicate with vitality and simplicity when called for, connecting with people at all levels of university education. There was, for them, a splendid inclusivity to the subject that was positive and in which they partook to the benefit of all, sharing their penetrating insights and enriching advice over decades. This picture of our two authentic all rounders sits well with the response by American statistician Herbert E. Robbins (1915–2001) when quizzed, in interview, about the speculation that a decent researcher who makes for a decent teacher (Halmos and Rota were both, in spades[2]) is an exception to the proverbial rule:

> "Good researchers are often poor teachers; bad researchers are almost always poor teachers. The reason that you have poor teachers is that you have poor persons: undeveloped, ignorant, intellectually poverty-stricken individuals who have nothing to offer their students except the subject matter itself. They have no joie de vivre, enthusiasm, or curiosity for learning. They'd be poor in any profession." [15, p. 294].

Robbins, together with German-American mathematician Richard Courant, had published the 1941 text *What Is Mathematics?* which is still in print (as a 1996 second edition updated by I.N. Stewart, a fine expositor himself). He regarded the book as more literary than scientific, promulgating hypotheses, principles and arguments in the self-confessed tradition of what the French have termed *haute vulgarisation*, and continued in this vein by Halmos, Rota, colleagues elsewhere at the time, and more since then (a raft of past and contemporary names roll off the tongue easily, all flexible crafts(wo)men with a message to get across). For anyone interested, I have set down some of my ruminations on the place and function of exposition previously [11, 12].

Lest one misread Robbins as a lone spokesperson, it should be pointed out that others had similar feelings (and the issues surrounding them remain alive and kicking, figuratively speaking). American mathematician Alfred W. Adler (1930–), for instance, had declared this in a hard hitting (and, in places, astringent) article of the early 1970s:

> "There is no reason to expect good mathematicians to be good teachers, any more than to expect them to be good financiers, or even good philosophers. These subjects all rely to a large extent on mathematical reasoning and techniques but involve other talents as well. Nevertheless, almost every good mathematician is also a good teacher, while almost no mediocre mathematician can teach the subject adequately even at an elementary level. This phenomenon is easier to recognize than to explain. Students, even though in most cases they do not know what constitutes good mathematics or which are the best mathematicians they have encountered, will unfailingly pick out the best mathematicians when asked to identify their best mathematics teachers. Love of mathematics and active involvement in its development forge ties between the teacher and his students; the latter are rarely fooled by style or dramatic effect. The usual confusions are absent: confusions between content and presentation, between the subject and the man, between profound inspiration and trivial manipulation—in short, those confusions common in the classrooms of so many other subjects, and common, in fact, in so great a part of life. There is no such thing as a man who does not create mathematics and yet is a fine mathematics teacher. ... What really matters is the communication of the spirit of mathematics. It is a spirit that is active rather than contemplative—a spirit of disciplined search for adventures of the intellect. Only an adventurer can really tell of adventures." [1, pp. 41–42].

[2]Halmos summed up the skills set needed to fulfil the mantle of professional mathematician:

> "To be a scholar of mathematics you must be born with talent, insight, concentration, taste, luck, drive, and the ability to visualize and guess. For teaching you must in addition understand what kinds of obstacles learners are likely to place before themselves, and you must have sympathy for your audience, dedicated selflessness, verbal ability, clear style, and expository skill. To be able, finally, to pull your weight in the profession with the essential clerical and administrative jobs, you must be responsible, conscientious, careful, and organized—it helps if you also have some qualities of leadership and charisma.
>
> You can't be perfect, but if you don't try, you won't be good enough." [8, p. 400];

such versatility is fast becoming redundant now in some of academia's 'high end' institutions—which prefers the specialist over the generalist—whereas it is a severe pressure point of expectation at others; everywhere one looks, there has been caused a teaching versus research (versus administration) schism of strains and frictions where cohesion and continuity should be.

Halmos and Rota were indeed adventurers and, in their own ways, pioneers of sort (that is, self-styled non-conformists with vision, who did things without much constraint), hoping to leave some kind of footprint on the world of mathematics; I aver that they did so.

3 What is Their Legacy?

It takes a lot of patient industry and sedulous care to get a piece of writing polished and properly fit for publication or public delivery, an aim towards which some people are more prepared to invest time than others—yes, writing with credibility and cogency is an art, too, but these will flourish under the kind of self-discipline they embraced. Each gave attention to the relationship we ought to have and want with our own field in order to enlighten others, taking on journal editorships, committee duties, speaking opportunities, mentoring, and much more—all part of a prodigiously productive and rich intellectual benefaction which garnished them with considerable agency.

How successful have they been in making a lasting mark? Essays on professional teaching practices and associated lines of enquiry—things of concern to Halmos and Rota *per se*—are churned out relentlessly these days, reflecting the business-like nature of H.E. and too often insufficiently radical or coherent to be noticed (not, I imagine, something to which they would have taken kindly). Many of these works fight for approbation from those breathing the 'purer' air of technical research who are indifferent to authors not working at the cutting edge of, or in original, mathematics (either by choice or lack of training) and who insist that this affects the tone and substance of discourse accordingly;[3] all we can say is that as *bona fide* researchers and first rate lecturers, Halmos and Rota were free from any justifiable criticism as they set about stimulating dialogue on all things mathematics. More positively, on the other hand, we are obliged to emphasise that exposure of Joe Public to mathematical concepts that moves beyond superficiality has become the preserve of a chosen few for whom Halmos and Rota were forerunners, though the latter were restricted in their reach by the technology available at the time and did not receive the adulation (sometimes misplaced) or financial rewards that the current crop of fashionable torchbearers can savour (nor were they able to make a full time job out of it, as some do now, even if they had wanted to). In research, however, the legacy of this formidable duo is diluted, for standards of writing are questionable, perhaps worse now than in their era.[4] Equations, expressions, and the like, should fit between sentences that carry flow, pace and cohesion as a mandatory *sine qua non* in research papers—having not succumbed to unhelpful shibboleths such as exaggerated brevity of argument, or undue obfuscation, or contrived paths of inferential reasoning, or glittering but deceptive mathematical prestidigitation, into which many authors are locked—but these are properties that have become quietly demoted, sacrificed to proclivities that have formed inside modern day universities where hard pressed academics (save for a sense of personal satisfaction) are neither encouraged nor motivated to craft core parts of a skills set that Halmos and Rota saw as fundamental to university life. The result is a leniency tendered to some of the stale, mundane and spiritless writing which manifests itself and seeps into mathematical exposition[5] in a way Halmos and Rota would not have condoned (even less so the sheer scale on which it is produced). They had allies. N. David Mermin (1935–), currently still Emeritus Professor of Physics at Cornell University, brought together in 1990 a collection of his non-technical and pedagogical essays (in and about physics as it happens), informed by a desire to "cut through [an] atmosphere of verbal dreariness" in which he considered scientists had unnecessarily cloaked themselves. Explaining in the Preface to the book that

> "Over the past fifty years or so, scientists have allowed the conventions of expression available to them to become entirely too confining. The insistence on bland imperson-

[3] There was once a flip side to this situation which is worth highlighting. The British topologist Peter J. Hilton (1923–2010) found, while working in the U.K., that while there was usually a detectable resentment directed towards those who took an interest in secondary and lower level tertiary education, it was welcomed in America to where he subsequently moved. His judgement that one has to earn the right to pursue mathematical research by heartfelt concern for good teaching and outreach engagements is seen now as old guard and unfashionable—Halmos and Rota satisfied this most basic criteria of Hilton.

[4] I have raised the topic [13] in speaking about a mature area of discrete mathematics, though some points made therein apply outwith the immediate scope of the paper.

[5] Electronic platforms haven't helped here. While some forums are fine—with postings and discussions that are eloquent, reasoned, informative, suitably provocative, and so on—many add to a depressing feeling that the world has more or less become one gigantic and unchecked 'Gabfest'.

> ality and the widespread indifference to anything like the display of a unique human author in scientific exposition, have not only transformed the reading of most scientific papers into an act of tedious drudgery, but have also deprived scientists of some powerful tools for enhancing their clarity in communicating matters of great complexity. Scientists wrote beautifully through the 19th century and on into the early 20th. But somewhere after that, coincident with the explosive growth of research, the art of writing science suffered a grave setback, and the stultifying convention descended that the best scientific prose should sound like a non-human author addressing a mechanical reader." [14, pp. xi–xii],

he bemoaned a pious, detached and heavy-handed style which had become a menace to professional academic writing for students, fellow scientists, and the general public, and later reminded us that "... unlike the figures and tables in your article, ..., the equations you display are embedded in your prose, and constitute an inseparable part of it." (p. 71); for him, they function as subordinate clauses, substantive phrases, or merely as objects like quotations, but they have an unambiguous part to play in a paper's assembly and its subsequent readability and appeal.

Wanting to spread the Good Word of Mathematics as evangelists—whilst remaining faithful to the discipline and what it stands for in totality—Halmos and Rota did so as esteemed emissaries with the highest possible qualifications and personal attributes to advance the cause, executing responsibility and service along their journeys. In 1921, the Newbolt Report—*The Teaching of English in England: Being the Report of the Departmental Committee Appointed by the President of the Board of Education to Inquire Into the Position of English in the Educational System of England* (published by H.M. Stationary Office, London, in 1926)—summarised the immediate state, post World War I, of the teaching and profile of English in England,[6] a small part of which (taken from p. 259 in Chapter VIII (titled 'Literature and Adult Education')) stated

> "The rise of modern Universities has accredited an ambassador of poetry to every important capital of industrialism in the country, and upon his shoulders rests a responsibility greater we think than is as yet generally recognised. The Professor of Literature in a University should be—and sometimes is, as we gladly recognise—a missionary in a more real and active sense than any of his colleagues. He has obligations not merely to the students who come to him to read for a degree, but still more towards the teeming population outside the University walls, ... The fulfilment of these obligations means propaganda work, organisation and the building up of a staff of assistant missionaries. But first, and above all, it means a right attitude of mind, a [belief] that literature and life are in fact inseparable, that literature is not just a subject for academic study, but one of the chief temples of the human spirit, in which all should worship."

Although set in its own historical and contextual framework, one could speak of mathematics in the same broad terms and we would have a ready made slogan with which it could be imagined that they mandated themselves to preach their versions of the Gospel of Mathematics to all who would listen both inside and outside its sacred church.

Cassius J. Keyser—American mathematician and philosopher, one time school teacher and principal, and later professor at Columbia University for over twenty years—gave a wonderful address to a meeting of the Michigan School Masters' Club on 28th March 1912 at Ann Arbor. Published in the April 26th issue of *Science* (New Series, **35**, pp. 637–647), he set out a detailed and personal testimony of what it means to humanise the teaching of mathematics in schools and colleges. I draw on his final remarks (pp. 646–647):

> "Finally, I wish to emphasize the fact that the great concepts out of which the so-called higher mathematical branches have grown—the concepts of variable and constant, of function, class and relation, of transformation, invariance, and group, of finite and infinite, of discreteness, limit, and continuity—... these great ideas of the higher mathematics, besides penetrating life, as we have seen, in all its complexity and all its dimensions, are omnipresent, from the very beginning, in the *elements* of mathematics as well. ... Why should the presentation of them have to await the uncertain advent of graduate years of study? For life already abounds, and the great ideas that give it

[6] In May 1919 the President of the Board of Education, H.A.L. Fisher, appointed a Departmental Committee to undertake the task (chaired by Sir Henry Newbolt), and its members presented their extensive findings to him in 1921.

> its interests, order and rationality, that is to say, the focal concepts of the higher mathematics, are everywhere present in the elements of the science as glistening bassets of gold. It is our privilege, in teaching the elements, to avail ourselves of the higher conceptions that are present in them; it is our privilege to have and to give a lively sense of their presence, their human significance, their beauty and their light. I do not advocate the formal presentation, in secondary schools, of the higher conceptions, in the way of printed texts, for the printed text is apt to be arid and the letter killeth. What I wish to recommend is the presentation of them, as opportunity may serve, in Greek fashion, by means of dialectic, face to face, voice answering to voice, animated with the varying moods and motions and accents of life—laughter, if you will, and the lightning of wit to cheer and speed the slower currents of sober thought. Of dialectic excellence, Plato at his best, as in "Phædo" or the "Republic," gives us the ideal model and eternal type. But Plato's ways are frequently circuitous, wearisome and long. They are ill suited to the manners of a direct and undeliberate age; and we must find, each for himself, a shorter course. Somebody imbued with the spirit of the matter, possessed of ample knowledge and having, besides, the requisite skill and verve ought to write a book showing, in so far as the printed page can be made to show, how naturally and swiftly and with what a delightful sense of emancipation and power thought may pass by dialectic paths from the traditional elements of mathematics to both its larger concepts and to a vision of their bearings on the higher interests of life. I need not say that such a handling of ideas implies much more than a verbal knowledge of their definitions. It implies familiarity with the doctrines that unfold the meanings of the ideas defined. It is evident that, in respect of this matter, the scripture must read: Knowing the doctrine is essential to living the life."

I propose that Halmos and Rota met the lofty ambitions of Keyser in both spirit and action—often unconsciously yet fittingly, but always inimitably—as much as anyone in higher education during the last century.

4 Final Thoughts

Perhaps the volume of output demanded of academics is now beginning to override the content that lies within (a huge growth in journals and other outlets has impacted here), the publish-mad mindset induced in us today militating against the kinds of articles—framed around the exchanges Halmos and Rota each had with himself privately and with others—that are needed and to which we might aspire (a colleague reminded me recently that if a professional bar is lowered, it stays in place for the next generation for whom it becomes the norm). I retell the words of a professorial acquaintance of mine who, being quite a bit older than I, proffered these words when I was a young postdoctoral student: "You do realise, don't you Peter, that journals are not for reading but are simply there to lodge our publications and show presence?" He was right in his assessment then, and all of the evidence around us shows that really nothing has changed. In interview, the great Paul Erdős quoted fellow Hungarian Leopold Fejér as saying (back in the 1930s) "Everybody writes and nobody reads." If still true (and I move that it is), we've lost our way and disappeared down a rabbit hole of compromise that puts quantity over quality, as it were, a cultural mantra to which Paul Halmos and Gian-Carlo Rota were not subjected and would have unceremoniously rejected. It could be contended that with the passing of time our arena of education is so divorced from theirs that to make comparisons is a bit futile (we recall those early-mid to late 20th century American institutions where they worked, from which time the nature of mathematical enterprise and publishing has changed appreciably, and the field grown and diversified worldwide). Today's university sector seems to ask different things of its employees both here in the U.K. and abroad, but some aspects of academe are timeless and the value of lucidity, perspicuity and comprehensibility in the written/spoken narrative of mathematics is absolutely a case in point, whether technical, expository, or a mixture of the two.

Referring to new theories in astronomy that (i) threatened to subvert ethical doctrines of the so called stoic movement and (ii) clashed with those contemporary religious predispositions and philosophical traction of the (Hellenistic) period, eclectic intellectual grandee Bertrand

A.W. Russell (1872–1970) wrote something rather observant:

> "It is one of the rarest gifts to be able to hold a view with conviction and detachment at the same time. Philosophers and scientists more than other men strive to train themselves to achieve it, though in the end they are usually no more successful than the layman. Mathematics is admirably suited to foster this kind of attitude. It is by no means accidental that many great philosophers were also mathematicians." [21, p. 101].

A mathematician—if his/her interest is aroused—often makes for more than a decent philosopher, the two occupations being compatible ones. It should come as no surprise, therefore, that Halmos and Rota had powerful attachments to philosophy which shaped their academic comportment and bearing in ways that cannot be dismissed as negligible. They were both intense thinkers, and I am reminded of the words of Stefan Collini, Cambridge Professor of English Literature and Intellectual History, who wrote fairly recently, "I'm not suggesting ... that good thinking is only done, or can only be done, in universities. But universities are, I think, the only *institutions* where pursuing such thinking is in principle not subordinate to any other purpose."[7]

I move towards an end with further musings on writing from Halmos and Rota, each an enthusiastic advocate for the subject whose professional ethos and devotion to it cannot be impugned. Before this, a couple of things of interest to note. Halmos claimed (with justification) to have invented the "iff" notation for the two-way logical implications "if and only if" (that is, $\Leftrightarrow$), and to have been the first to deploy the so called "tombstone" symbol $\square$ (sometimes filled in as a solid $\blacksquare$) in mathematics to denote the end of a proof (since when it has been adopted robustly, and is also termed the "halmos"). Rota had something of a trademark vignette in his writing, producing a series of entertaining pieces with titles such as 'Ten Lessons I Wish I Had Been Taught', 'Ten Lessons I Wish I Had Learned Before I Started Teaching Differential Equations', 'Ten Lessons of an M.I.T. Education', 'Ten Lessons for the Survival of a Mathematics Department' (aforementioned), and 'A Mathematician's Gossip'; a *pot pourri* of quirky statements and idiosyncratic cogitations, tips and pointers that blended the ironic, facetious, pithy, earnest and witty, they were usually thought provoking—as were Halmos' chronicles and conclusions quite often. From Halmos, then,

> "The basic problem in writing mathematics is the same as in writing biology, writing a novel, or writing directions for assembling a harpsichord: the problem is to communicate an idea. To do so, and to do it clearly, you must have something to say, and you must have someone to say it to, you must organize what you want to say, and you must arrange it in the order you want it said in, you must write it, rewrite it, and re-rewrite it several times, and you must be willing to think hard about and work hard on mechanical details such as diction, notation, and punctuation. That's all there is to it." [6, p. 124],

and from Rota,

> "Making mathematics accessible to the educated laymen, while keeping high scientific standards, has always been considered a treacherous navigation between the Scylla of professional contempt and Charybdis of public misunderstanding. Davis and Hersh have sailed across the Strait under full sail [and] opened a discussion of the mathematical experience that is inevitable for survival. Watching from the stern of their ship, we breathe a sigh of relief as the vortex of oversimplification recedes into the distance." [17, p. 155].[8]

[7] S. Collini, *Speaking of Universities*, Verso, London (2017), p. 25.

[8] Reuben Hersh (1927–2020) and Philip J. Davis (1923–2018) were American mathematicians and academic writers. Together they produced the 1980 national award winning text *The Mathematical Experience* that reviewed the prosecution of modern mathematics from a historical and philosophical perspective. Hersh, in an Overture to the book, expressed sentiments that will have chimed with the feelings of Halmos and Rota: "The fact is, ..., that I have come to a point where my wonderment and fascination with the meaning and purpose, if any, of this strange activity we call mathematics is equal to, sometimes even stronger than, my fascination with actually *doing* mathematics." In complementing Davis and Hersh, Rota concorded with Swiss historian Carl Jacob Christoph Burckhardt who had predicted that the 20th century would be "the age of oversimplification", citing examples from the arts, politics, media, philosophy, science, religion and mathematics; surrounding us today, in all of their pernicious materialisations, the many faces of oversimplification feel rife and ingrained across swathes of society, including those grounds that our academic institutions occupy.

Needless to say, neither man was faultless—Halmos had one or two contentious things to divulge about the enduring pure/applied tensions in mathematics, Rota's observations were at times couched using harsh words that could offend, and they each owned large egos (having no problem courting controversy for its own sake (when the mood took them) or simply to make a point)—but they wrote and orated with honest transparency for which they are due so very much more than a mere nod of assent. They were consumed by mathematics, showing occasional shades of dogmatism and peremptoriness that were perhaps due partly to the fact that each was untroubled by the kind of family responsibilities most academics have (both were married (Halmos twice), but neither had children) which enabled them to indulge themselves fully in academic affairs yet with time to form their mathematical *weltanschauung*. These, though, are minor gripes in the larger scheme of things, for they bore their fair portion of teaching and supervision of students gladly, and were acknowledged as prize winning experts, bold P.R. men, and prominent figures of the day. We have some fighting our corner right now, so to speak, on a number of fronts, but we need more learned and erudite artisans like them (and with their calibre) as consummate deliberators on, and sparkling champions of, mathematics—serving the community on which it is based, carrying it forward, and increasing its external visibility as an imposing construct of invention and creativity, the hybrid utilitarian and aesthetic nature of mathematics mirrored in the temperaments and interests of those who take on the challenge—in short, demonstrating an allegiance, loyalty and fidelity to the field in leading by example. Halmos and Rota put themselves above the fierce and petty rivalries which lay (and still lie) embedded inside the mathematical community, to show generosity of time and spirit that—in an occupation with scant acknowledgement that lends itself to insularity, hubris and *folie de grandeur*—present and future mathematicians should endeavour to replicate.[9] I hope I have honoured their memory and the canons they counselled, without straying into homiletic territory, in this essay.

In 2017 Springer launched its timely History of Mathematics Education series, with these objectives: "[It] aims to make available to scholars and interested persons throughout the world the fruits of outstanding research into the history of mathematics education; provide historical syntheses of comparative research on important themes in mathematics education; and establish greater interest in the history of mathematics education." Oxford University's Benjamin Wardhaugh provided a Foreword to the opening title in the series, emphasising that, latterly,

> "... greater attention is being paid to the rich and varied worlds of practitioners and amateurs, teachers and learners. In other words, there is an emerging history of numeracy, and of how members of society have identified, developed and drawn on numerical, geometrical, and logical relationships in their quest not only to survive with dignity but also to become more adept at improving their own lives and those of others around them. That is as it should be. Doing mathematics and "becoming mathematical" were and are part of culture just as much as are reading, writing and becoming literate. And, as parts of culture they are just as transformative, just as disruptive and

[9] Halmos said, when posed the question "What's the worst part [of being a mathematician]?", replied "The worst part has to do with the best part—. . . : competitiveness. I like competitiveness. I am competitive. I want to beat the other guys. At the same time, I don't like it. . . . What else is a bad part? It's a little bad—I wouldn't put it into the worst part—that we are so unrecognized by the world." The feelings are matched by Rota who, when asked "What's it like to be a mathematician?" in interview for the autumn 1998 M.I.T. News, responded "It's the least rewarding profession except one: music. Musicians live an impoverished life. Mathematicians—for what they do—are really poorly rewarded. And it's a very competitive field, almost as bad as being a concert pianist. You've got to be really an egoist. You've got to be terribly self-centered." The view is repeated by others. Successful hedge-fund manager Neil Chriss, for instance—who left a postdoctoral post in mathematics at Harvard University to take up a quantitative research position in New York's Wall Street—wrote some years later that mathematical research

> "... is hard to do well and it is ultimately very competitive. From the outside, academia—where much of the research in pure mathematics takes place—may seem placid or even sleepy, but inside it is anything but. Mathematics may be isolated from the real world, but mathematicians are not isolated from one another. The field of mathematics has a natural hierarchy. Mathematicians generally work on research problems. There are problems and then there are hard problems. Mathematicians look to publish their works in journals. There are good journals and there are great journals. Mathematicians look to get academic jobs. There are good jobs and great jobs. Mathematicians want to do well *relative* to one another. It is hard to do mathematics and not care about what your standing is.
>
> ... In small and big ways, people were always jockeying for position. Whether it was the natural chatter about who got what job or who published in which journal or whose thesis problem was more important, it was definitely competitive." [4, p. 110].

As most of us know, and these quotes confirm, rivalry begins in-house at graduate level and never goes away, merely morphing into more extensive contests fought on battlefields both real and imaginary.

> potentially subversive, just as historically rich and interesting."

Noting that the authors "rightly point out" that "mathematics educators have until recently paid little attention to questions of the "how did we get here?" kind", he added

> "There is an urgent need for better understandings of how mathematics education can be and how it has been: of the variety and the levels of success of different ways of teaching and learning mathematics, as well as of the historical processes that have left us, ultimately, with "math anxiety" and "math wars." " [23, p. v];

one suspects that Halmos and Rota would have been more than happy to be included in the list of contributing writers as men with things to reveal and proclaim about the topic with relish.

One final remark, to conclude. We quoted Halmos in Section 2, who mentioned charisma. Michael Harris has devoted a comprehensive chapter (Chapter 2) to this phenomenon in his 2015 book *Mathematics Without Apologies: Portrait of a Problematic Vocation* [9] (a wide ranging, informative, and high level piece of work). He makes the distinction between socio-colloquial and singularly academic meanings of an enchanting concept that encompasses the contextual attributions of presence, image, expertise, prestige, status, glamour, command, magnetism, mystique, reputation, romance, leadership, and such like, and has some compelling thoughts as to its place, its role, its procurement, and the overlapping channels of symbolic capital and legitimising power it endues within the intellectual aristocracies and structured orders of our mathematical priesthood; whichever way one casts an eye back at the various activities of Halmos and Rota, it is indisputable that they were carried out with aplomb and no little dose of a discernible and uplifting charisma.

"We often hear that mathematics consists mainly in "proving theorems". Is a writer's job mainly that of "writing sentences"?"

Gian-Carlo Rota

"Surveys are hard to write, but good expositions, the low road, are harder still—the lower the harder."

Paul Richard Halmos

Appendix: On a Mathematician's 'Make Up' and Ways We Are Perceived

One of the difficulties mathematicians fight against, and which seems to produce a fog around our practices (both for ourselves and others), is that we have intimate relationships with what Rota called "bounty words" such as mathematical "creativity" and "beauty". A "bounty word", he wrote, is one which "announces and promises some benefit that cannot be controlled or measured, but that can only be attained as the unpredictable byproduct of some concrete activity." [19, p. 177] (others are things such as "happiness" and "saintlihood", Rota defining "the bounty error" as "[bestow]ing a bounty word with concrete content" of which it is devoid in any identifiable sense). Key phenomenon firmly ensconced in our brains, they cannot be taught and are elusively vague but remain problematic to both us (we can beat ourselves up if we don't live up to the venerated principles and beliefs we build around them) and laymen (who appropriate them as they wish in connection to us, fuelling crass partialities and oblique parochialisms). Unravelling and shedding light on the kernel, the psyche, the soul, the ethos, the motivations, the *modus operandi*, *etc.*, of the mathematician (each interwoven with, and organically affected by, 'bounty concepts') for the benefit our own community first and foremost—and also that the external masses might arrive at an understanding of us (even if tenuous and indistinct at times)—is a non-trivial task (in complexity and magnitude), ubiquitous through centuries and as yet unfinished; people like Halmos and Rota considered themselves indentured to it, and did their bit. The mores of mathematical venture (albeit relative to any given period) must indubitably surface in the characters and mentalities of those few who pour their energies into the 'queen of the sciences' and seek its secret disclosures as precious gems to treasure; and put on show—as Halmos and Rota strived to do—for others to marvel at, to applaud, and to even sometimes take ownership of as 'outsiders'; theirs is a truly commendable ambition that we should salute.

The following passages give a snapshot of the way we are perceived from the viewpoint of four academics. The individuals are varied, but recurring themes contained in the words of the

first three are germane even now to mathematicians who still have to wrestle with ways in which non-mathematicians look upon us and the pictures we paint of ourselves. Bell, the final subject, sets down other thoughts on personality types based on men of stature from the past.

Nineteenth century English mathematician James Joseph Sylvester (1814–1897)—in a late 1860s Presidential Address to the Exeter British Association (published as Article No. 100 in Volume II of his *Collected Papers*)—spoke of his wish not

> "... to countenance by my example the too prevailing opinion that mathematical pursuits unfit a person for the discharge of the common duties of life and cut him off from the exercise of Man's highest prerogative, "discourse of reason and faculty of speech divine,"—rather, I say, than favour the notion that we ... are a set of mere calculating-machines endowed with organs of locomotion, or, at best, a sort of poor visionary dumb creatures only capable of communicating by signs and symbols with the outer world, I have resolved to take heart of grace and to say a few words, which I hope to render, if not interesting, at least intelligible, ..." [22, p. 652].

He added, in a footnote on the same page,

> "There is an old adage, "purus mathematicus, purus asinus".[10] On the other hand, I once heard the great Richard Owen[11] say, ..., that he would like to see *Homo Mathematicus* constituted into a distinct subclass, thereby suggesting to my mind sensation, perception, reflection, abstraction, as the successive stages or phases of protoplasm on its way to being made perfect in Mathematicised Man."

German mathematician Wolfgang Krull (1899–1971) opened with these words in his inaugural lecture delivered at the University of Erlangen in January 1930, of which a lightly edited translation (from 1987) is used here:

> "Compared with representatives of most other disciplines, mathematicians suffer from a serious handicap. Lawyers, linguists, biologists, chemists, physicians—all these people can discuss their professions with uninitiated laymen. Perhaps they cannot fully explain the deeper problems with which they themselves are wrestling, but they can easily give a comprehensible account of what lies on the surface, and their listeners will be interested and grateful.
>
> Not so in mathematics! It really seems to be true that a special sixth sense is needed to understand mathematics. The few who possess this sense fling themselves passionately into the subject; the rest stay as far away from it as possible or consider it a necessary evil. Of course, this isolation gives mathematicians one advantage: unlike other professionals, they are seldom tempted to burden the uninitiated with shop-talk at social gatherings. ...
>
> Outsiders usually think of mathematics as an especially dry science. Those who know nothing at all about it picture a mathematician as a kind of calculator. ... Those who know mathematics a little better generally ... believe that the only thing essential for a mathematician is a keen, unerring intellect. But I want to emphasize as strongly as I can that a true mathematician must above all have imagination. I am quite certain that it is precisely [this] that distinguishes the future researcher from the merely talented mathematics student." [10, p. 48].

Time for the views of Adler again (cited in Section 2), who wrote thus:

> "All professions reward accomplishment in part with admiration by peers, but mathematics can reward it with admiration of no other kind. It is, in fact, impossible for a mathematician even to talk intelligently to non-mathematicians about his mathematical work. In the company of friends, writers can discuss their books, economists the

[10] It ridicules by insinuating that someone who is purely a mathematician, doing nothing but load himself down with mathematical problems, is akin to nothing more than a donkey.

[11] Sir Richard Owen (1804–1892) was an English biologist, comparative anatomist and paleontologist, and is known (*inter alia*) for introducing the term "dinosaur". Once marginalised and cast in the role of Charles Darwin's main creationist opponent, he has been re-appraised as a major representative and proponent of credible aspects of non-Darwinian evolution theory.

> state of the economy, lawyers their latest cases, and businessmen their recent acquisitions, but mathematicians cannot discuss their mathematics at all. And the more profound their work, the less understandable it is; a spirited high-school teacher can regale his audience with puzzles and magic squares, but there is no way for the serious mathematician to talk to the non-mathematician ... Few laymen are really interested enough to distinguish between real mathematicians and fools who can multiply six-digit numbers in their heads. Even a well-educated layman is generally willing to grant, at most, an hour's time to the consideration of the implications of the last half century of mathematical discovery, and then only if he is in a benign humor and the explicant is eloquent and talks philosophy rather than mathematics. The listener will then almost certainly leave without any understanding of what is going on in mathematics, because he has not cared to expend any effort on understanding mathematics by first learning a bit about it; that is, by learning it as a language. The most rudimentary requirement for comprehending a language is a knowledge of its vocabulary, and acquiring a vocabulary demands some hard work. The mathematician can take it for granted that acquiring the vocabulary of mathematics is simply out of the question for his friends, acquaintances—nearly everyone.
>
> And yet the language of mathematics is so natural and so simple in comparison to the spoken languages that the resistance it encounters is difficult to understand. ..." [1, p. 42].

We leave the last block of quotations to Eric T. Bell (1883–1960)—a Scottish-born mathematician and science fiction writer—who gave his take on the make up of a mathematician such as he had gleaned from looking at noteworthy personnel in the field. In the introductory chapter of his book *Men of Mathematics*, first released in 1937, he gave us the following which as an absorbing informal synopsis has stood the test of time in its broad accuracy (a word of caution for anyone intending to read the work—Bell's biographies are known for their unreliability in places!):

> "Those who have never known a professional mathematician may be rather surprised on meeting some, for mathematicians as a class are probably less familiar to the general reader than any other group of brain workers. The mathematician is a much rarer character in fiction than his cousin the scientist, and when he does appear in the pages of a novel or on the screen he is only too apt to be a slovenly dreamer totally devoid of common sense—comic relief. What sort of mortal is he in real life? Only by seeing in detail what manner of men some of the *great* mathematicians were and what kind of lives they lived, can we recognize the ludicrous untruth of the traditional portrait of a mathematician.
>
> Strange as it may seem, not all of the great mathematicians have been professors in colleges or universities. Quite a few were soldiers by profession; others went into mathematics from theology, the law, and medicine, ... A few have had no profession at all. Stranger yet, not all professors of mathematics have been mathematicians. But this should not surprise us when we think of the gulf between the average professor of poetry drawing a comfortable salary and the poet starving to death in his garret.
>
> The lives that follow [in the book] will at least suggest that a mathematician can be as human as anyone else—sometimes distressingly more so. In ordinary social contacts the majority have been normal. There have been eccentrics in mathematics, of course; but the percentage is no higher than in commerce or the professions. As a group the great mathematicians have been men of all-round ability, vigorous, alert, keenly interested in many things outside of mathematics and, in a fight, men with their full share of backbone. As a rule mathematicians have been bad customers to persecute; they have usually been capable of returning what they received with compound interest. For the rest they were geniuses of tremendous accomplishment marked off from the majority of their gifted fellowmen only by an irresistible impulse to do mathematics. On occasion mathematicians have been ... extremely able administrators. ...
>
> Returning for a moment to the movie ideal of a mathematician, we note that sloppy clothes have not been the invariable attire of great mathematicians. All through the

long history of mathematics about which we have fairly detailed knowledge, mathematicians have paid the same amount of attention to their personal appearance as any other equally numerous group of men. Some have been fops, others slovens; the majority, decently inconspicuous. . . .

The psychological peculiarities of great mathematicians is another topic in which there is considerable interest. . . . But on the general question not much can be said till psychologists call a truce and agree among themselves as to what is what. On the whole the great mathematicians have lived richer, more virile lives than those that fall to the lot of the ordinary hard-working mortal. Nor has this richness been wholly on the side of intellectual adventuresomeness. Several of the greater mathematicians have had more than their share of physical danger and excitement, and some of them have been implacable haters—or, what is ultimately the same, expert controversialists. Many have known the lust of battle in their prime, reprehensively enough, no doubt, but still humanly enough, . . .

This brings us to what at first sight . . . may seem like a significant trait of mathematicians—their hair-trigger quarrelsomeness. Following the lives of several of these men we get the impression that a great mathematician is more likely than not to think others are stealing his work, or disparaging it, or not doing him sufficient honor, and to start a row to recover imaginary rights. Men who should have been above such brawls seem to have gone out of their way to court battles over priority in discovery and to accuse their competitors of plagiarism. We shall see enough dishonesty to discount the superstition that the pursuit of truth necessarily makes a man truthful, but we shall not find indubitable evidence that mathematics makes a man bad-tempered and quarrelsome.

Another "psychological" detail of a similar sort is more disturbing. Envy is carried up to a higher level. Narrow nationalism and international jealousies, even in impersonal pure mathematics, have marred the history of discovery and invention to such an extent that it is almost impossible in some important instances to get at the facts or to form a just estimate of the significance of a particular man's work for modern thought." [3, pp. 8–10].

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