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# Twenty-Five Years of Sensor Array and Multichannel Signal Processing

Wei Liu, Martin Haardt, Maria S. Greco, Christoph Mecklenbräuker, and Peter Willett

#### Abstract

In this paper, a general introduction to the area of sensor array and multichannel signal processing is provided, including associated activities of the Sensor Array and Multichannel (SAM) Technical Committee (TC) of the IEEE Signal Processing Society (SPS). The main technological advances in five SAM sub-areas made in the past twenty-five years are then presented in detail, including beamforming, direction of arrival estimation, sensor location optimization, target/source localization based on sensor arrays, and multiple input and multiple output arrays. Six recent developments are also provided at the end to indicate possible promising directions for future SAM research, which are graph signal processing for sensor networks, tensor based array signal processing, quaternion valued array signal processing, one-bit and non-coherent sensor array signal processing, machine learning and artificial intelligence for sensor arrays, and array signal processing for next-generation communication systems.

#### **Index Terms**

Sensor array, multichannel signal processing, beamforming, direction of arrival, sparse arrays, MIMO arrays.

#### I. INTRODUCTION

Sensor array and multichannel signal processing has a long history, with typical research topics including beamforming and direction-of-arrival (DOA) estimation at its early stage and corresponding representative algorithms including the Capon beamformer/linearly constrained minimum variance (LCMV) beamformer and the MUSIC/ESPRIT algorithms [1]–[5]. The past twenty-five years have seen an explosive growth of research activities in this area and significant progresses have been made in a wide range of theoretical and application areas of sensor array and multichannel signal processing. Although traditionally their applications have been mainly limited to the defense sector, such as radar and sonar, nowadays, we can see their impact in everyday life, including beamforming for ultrasound imaging, synthetic aperture radar for remote sensing, vehicular radar (ultrasound or electromagnetic)

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for autonomous driving, microphone arrays for human-machine interface (a good example is the Amazon Echo), and multiple input and multiple output (MIMO) antenna arrays for Wi-Fi and mobile communications standards (IEEE 802.11n, IEEE 802.11ac, 3G, WiMAX, and LTE).

As a result, the sensor array and multichannel signal processing research area has expanded significantly in the past years, as reflected by the scope of the Sensor Array and Multichannel (SAM) Technical Committee (TC) of the IEEE Signal Processing Society (SPS). The SAM TC, formed in 2000, aims to promote activities within the technical fields of sensor array processing and multi-channel statistical signal processing [6], including beamforming and space-time adaptive processing, DOA estimation, source separation, target detection, localization and tracking, MIMO signal processing, array processing for radar, sonar and communications, and many other applications of multi-sensor or synthetic aperture systems, as indicated by the list of EDICS covered by the TC (https://signalprocessingsociety.org/community-involvement/sensor-array-and-multichannel/edics).

The SAM TC organizes two biennial workshops dedicated to the SAM research area: the IEEE International Workshop on Computational Advances in Multisensor Adaptive Processing (CAMSAP) organized in December every odd-numbered year since 2005, and the IEEE Sensor Array and Multichannel Signal Processing Workshop organized in June/July every even-numbered year since 2002, each accepting 100-200 research papers. Due to the Covid-19 pandemic, CAMSAP 2021 originally scheduled for December 2021 in Costa Rica has been postponed to December 2023. The next SAM workshop (SAM 2024) will be held in the US with two possible venues: either Oregon State University or Skamania Lodge, Stevenson, Washington. Moreover, at each year's ICASSP conference, the SAM track also receives about 100-200 regular submissions. Currently there is also the Synthetic Aperture Technical Working Group (SA-TWG) which resides under the SAM TC, with the goal of "supporting the maturation of the theoretical framework and the associated empirical techniques that underpin the estimation of parameters of propagating waves through various media using synthetic apertures".

In this paper, as it is not possible to give an exhaustive list of all the advances made in the SAM area, we will focus on five major topics and introduce the corresponding progresses made in tackling their respective technical challenges: beamforming (including robust adaptive beamforming and frequency invariant beamforming), DOA estimation (including sparsity based and underdetermined DOA estimation), sensor location optimization, target/source localization based on sensor arrays, and MIMO arrays (including MIMO radar and MIMO for wireless communications). The first two are classic SAM topics from the very beginning of SAM research as mentioned earlier, while the latter three were only studied systematically in the past decades. Then, six new developments in the SAM area will be presented to give an indication about possible future research directions, including graph signal processing for sensor networks, tensor based array signal processing, quaternion valued array signal processing, one-bit and non-coherent sensor array signal processing, machine learning and artificial intelligence (AI) for sensor

arrays, and array signal processing for next-generation communication systems.

This paper is structured as follows, the five main technological advances will be introduced in detail in Section II, followed by the six new developments in Section III, and some concluding remarks are provided in Section IV.

#### II. MAIN TECHNOLOGICAL ADVANCES IN THE SAM AREA

In this section, advances made in the five major SAM research topics in the past twenty-five years will be presented, including beamforming, DOA estimation, sensor location optimization, target/source localization based on sensor arrays, and MIMO arrays.

## A. Beamforming

Beamforming is a classic sensor array signal processing problem and a core SAM topic [1]–[5], and it has been studied extensively at least for a century. It can be classified into narrowband and wideband beamforming according to the relative bandwidth of the signals, adaptive and fixed beamforming according to its relationship with the received data, analogue and digital beamforming according to its circuits implementation. In the past twenty-five years, three main developments are highlighted, including robust adaptive beamforming [7], frequency invariant beamforming (FIB) [5], and hybrid beamforming [8], which is a combination of digital and analogue beamforming techniques. In this subsection, we will discuss the first two in detail and leave the topic of hybrid beamforming to the subsection about MIMO arrays.

#### 1) Robust Adaptive Beamforming

In general, for the narrowband case, for an M-sensor array with K impinging signals, the received array signals can be formulated into the following form

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where  $\mathbf{x}(t) = [x_1(t), \ldots, x_M(t)]^T$  is the received signal vector,  $\mathbf{A}$  is the steering matrix consisting of K steering vectors  $\mathbf{a}(\theta)$  corresponding to the K source signals ( $\theta$  represents the angle of arrival of an arbitrary impinging signal), and  $\mathbf{n}(t)$  is the noise vector.

Then the beamformer output y(t) is given by an instantaneous linear combination of the received spatial samples  $x_m(t)$  as follows,

$$y(t) = \sum_{m=1}^{M} x_m(t) w_m^* = \mathbf{w}^H \mathbf{x}(t) , \qquad (2)$$

where  $w_m$  is the weight coefficient for the *m*-th received sensor signal with the weight vector  $\mathbf{w} = [w_1, \ldots, w_M]^T$ .

The Capon beamformer, which can be considered as a special case of the more general linearly constrained minimum variance (LCMV) beamformer [1]–[5], can achieve effective adaptive beamforming when the DOA angle

 $\theta_0$  of the desired signal is exactly known, and the following is the standard formulation

$$\min_{\mathbf{w}} \mathbf{w}^H \mathbf{R} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^H \mathbf{a}(\theta_0) = 1$$
(3)

where  $\mathbf{R} = E\{\mathbf{x}(t)\mathbf{x}^{H}(t)\}\$  is the covariance matrix, and  $\mathbf{a}(\theta_{0})\$  is the steering vector of the array at  $\theta_{0}$ . In practice, since  $\mathbf{R}$  is usually not available, as an approximation, it is replaced by the sample covariance matrix  $\hat{\mathbf{R}}$ , which is obtained through the finite number of data samples.

However, the Capon beamformer is very sensitive to model mismatch errors, such as DOA error for the desired signal, mutual coupling, general array manifold errors and finite sample effects in covariance matrix estimation, and therefore various robust adaptive beamforming techniques have been developed [7]. One well-known technique is diagonal loading, with the weight vector expressed as  $\alpha(\hat{\mathbf{R}} + \xi \mathbf{I})^{-1}\mathbf{a}(\theta_0)$ , with  $\alpha$  being a constant,  $\xi$  the diagonal loading factor and  $\mathbf{I}$  the identity matrix.

One prominent development in this area in the past twenty-five years is the worst-case based robust adaptive beamformer [9], where instead of constraining the beamformer response to be unity at the desired signal direction, it is forced to exceed unity within an uncertainty set of steering vectors, which can be expressed as

$$\min_{\mathbf{w}} \mathbf{w}^{H} \hat{\mathbf{R}} \mathbf{w} \quad \text{subject to} \quad |\mathbf{w}^{H} \tilde{\mathbf{a}}| \ge 1, \ \forall \, \tilde{\mathbf{a}} \in \mathcal{A}$$
$$\mathcal{A} = \{ \tilde{\mathbf{a}} \, | \, \tilde{\mathbf{a}} = \mathbf{a}(\theta_{0}) + \mathbf{e}, \|\mathbf{e}\| \le \varepsilon \}$$
(4)

where  $\tilde{\mathbf{a}}$  is the actual steering vector of the desired signal corresponding to the presumed steering vector  $\mathbf{a}(\theta_0)$ ,  $\mathcal{A}$  is the full set that  $\tilde{\mathbf{a}}$  belongs to, and  $\mathbf{e}$  is the steering vector error with its norm bounded by  $\varepsilon$ . The problem is then converted to the following form using the worst-case optimization

$$\min_{\mathbf{w}} \mathbf{w}^{H} \hat{\mathbf{R}} \mathbf{w} \quad \text{subject to} \quad \mathbf{w}^{H} \mathbf{a}(\theta_{0}) \geq \varepsilon \|\mathbf{w}\| + 1$$
$$\operatorname{Im} \{\mathbf{w}^{H} \mathbf{a}(\theta_{0})\} = 0 , \qquad (5)$$

where  $Im\{\cdot\}$  denotes the imaginary part of its argument. As the signal to interference plus noise ratio (SINR) of the beamformer output will not change by rotating the weight vector, an alternative formulation can be derived as

$$\min_{\mathbf{w}} \mathbf{w}^{H} \hat{\mathbf{R}} \mathbf{w} \quad \text{subject to} \quad \operatorname{Re}\{\mathbf{w}^{H} \mathbf{a}(\theta_{0})\} \geq \varepsilon \|\mathbf{w}\| + 1 , \qquad (6)$$

where  $Re\{\cdot\}$  takes the real part of its argument.

Both the Capon beamformer and the worst-case robust beamformer require estimation of the covariance matrix  $\mathbf{R}$  and it is a challenging task when only a small number of snapshots is available; one solution to the problem is the family of iterative adaptive approach (IAA) based methods [10], which can still work for the extreme case with only one snapshot.

Another notable contribution for robust adaptive beamforming is based on interference covariance matrix reconstruction and steering vector estimation [11], which has attracted much attention recently with follow-up works focusing on different ways of reconstructing either or both of the covariance matrices corresponding to the desired signal and interference plus noise, separately.

## 2) Frequency Invariant Beamforming

For wideband arrays, different from the data model in (1), the received array signals are expressed in the form of convolution (represented by  $\star$ ) [5]

$$\mathbf{x}(t) = \mathbf{A} \star \mathbf{s}(t) + \mathbf{n}(t),\tag{7}$$

where the (m, k)-th element of the matrix  $\tilde{\mathbf{A}}$  is given by  $\delta(t - \tau_{m,k})$ , with  $\tau_{m,k}$  being the time delay of the k-th impinging signal at the m-th sensor compared to some reference point.

As a result, wideband beamforming is achieved through a series of tapped delay-lines (TDLs), or FIR/IIR filters in its discrete form [5]. For wideband beamformers, in general the beam width will increase with the decrease of frequency since the relative aperture of the array becomes smaller for lower frequencies and therefore one unique problem for wideband beamforming is how to design a beamformer with a frequency invariant beam response or beam pattern.

In order to achieve a frequency independent beam response, many methods were proposed in the past, and one typical solution is harmonic nesting, where for a number of frequency bands, different subarrays with appropriate aperture and sensor spacing are operated [4]. In a design proposed in [12], each sensor in the array is followed by its own primary filter and the outputs of these primary filters share a common secondary filter to form the final output; although the design for a one-dimensional (1-D) array is relatively simple due to the dilation property of the primary filters, for two-dimensional (2-D) and three-dimensional (3-D) arrays, this property is not guaranteed, which makes the general design case very complicated. In [5], [13], based on a simple Fourier transform relationship, a systematic and consistent approach was developed to design fixed frequency-invariant beamformers for 1-D, 2-D and 3-D arrays and for both continuous and discrete apertures.

Furthermore, a series of least-square based frequency invariant beamformer design methods were proposed with closed-form solutions and applicable to arbitrary array geometries [5]. In its very basic form, given the desired beam pattern  $P_d(\Omega, \theta)$  ( $\Omega$  is the normalized frequency) and designed response  $P(\Omega, \theta)$  (quadratic function of the beamforming weight vector **w**) over the frequency range of interest  $\Omega_I$  and the range of angle of interest  $\Theta$ , the design is to minimize the following cost function:

$$\alpha \int_{\Theta} |P(\Omega_r, \theta) - P_d(\Omega_r, \theta)|^2 d\theta + (1 - \alpha) \int_{\Omega_I} \int_{\Theta} |P(\Omega, \theta) - P(\Omega_r, \theta)|^2 d\Omega d\theta ,$$
(8)

where the first part is the traditional cost function for least-square based design over one reference frequency  $\Omega_r$ , the second part is the term for measuring the difference between the response of the designed beamformer and its response at the reference frequency  $\Omega_r$  over the full range of angle of interest, i.e. the frequency variation of the response, and  $\alpha$  trades these off. Note that the first part of the cost function is only calculated at the reference



Fig. 1: A design example for a frequency invariant beamformer.

frequency, not the whole  $\Omega_I$ , and the reason is that if the response is frequency invariant, then as long as at one single frequency ( $\Omega_r$ ) the designed response is close to the desired one, the whole response will also be close to it. A design example for a frequency invariant beamformer over the normalized frequency range [ $0.3\pi$ ,  $\pi$ ] based on a uniform linear array (ULA) of 10 sensors and a TDL length of 20 is shown in Fig. 1.

The above FIB design techniques can be employed to design an FIB network where multiple frequency invariant beamformers pointing to different directions are placed in parallel to transform the wideband array signal processing problem into a narrowband one, so that traditional narrowband beamforming and DOA estimation solutions can be applied directly to the output of the FIB network [5]; the second part of the cost function in (8) can also be incorporated into the adaptive beamforming process to realize adaptive FIB directly instead of relying on the FIB network [14].

Note that the TDL based wideband beamforming structure could be replaced by the sensor delay-line (SDL) based structure [5], [15], where multiple sensors are placed behind the original array sensors in place of the delay-lines for effective wideband beamforming; such an SDL based structure may prove to be very important for the coming terahertz (THz) or sub-THz communication systems, where the delays required for effective wideband beamforming/beam steering may be too short to be implemented in practice.

## B. Direction of Arrival (DOA) Estimation

DOA estimation is another core SAM research area. Originally, it was realised by various beamforming algorithms in its simplest form, such as the Butler matrix and the Capon beamformer or the LCMV beamformer, and then more advanced super-resolution solutions have been developed under the classic subspace framework. In the past twentyfive years, inspired by developments of compressive sensing (CS) [16], two important advances in this area are the sparsity based DOA estimation framework [17], [18], which, unlike the subspace based framework, can deal with coherent sources directly, and the underdetermined<sup>1</sup> DOA estimation approach based on various signal properties (such as noncircularity and nonGaussianity) and the co-array concept (both sum and difference co-arrays) [17], [19]–[21].

## 1) Sparsity Based DOA Estimation

To introduce the basic idea for sparsity based DOA estimation, consider the following discrete version of the continuous model in (1)

$$\mathbf{x}[i] = \mathbf{A}\mathbf{s}[i] + \mathbf{n}[i] , \qquad (9)$$

where  $\mathbf{x}[i]$  is the array data vector for the *i*-th snapshot,  $\mathbf{s}[i]$  is the source signal vector, and  $\mathbf{n}[i]$  is the noise vector.

For the *i*-th snapshot, to exploit the spatial sparsity property of the source signals, a search grid of  $K_g$  ( $K_g \gg K$ ) potential incident angles  $\theta_{g,0}, \ldots, \theta_{g,K_g-1}$  is first generated, and an overcomplete representation of **A** is then constructed, given by

$$\mathbf{A}(\boldsymbol{\theta}_{\boldsymbol{g}}) = \begin{bmatrix} \mathbf{a}(\theta_{g,0}), \dots, \mathbf{a}(\theta_{g,K_q-1}) \end{bmatrix}.$$
(10)

 $\mathbf{A}(\boldsymbol{\theta}_{g})$  is independent of the actual source directions  $\boldsymbol{\theta}_{k}$ . We also construct a  $K_{g} \times 1$  column vector  $\mathbf{s}_{g}[i]$ , with each entry representing a potential source at the corresponding angle. Then, the model from the perspective of sparse signal reconstruction becomes

$$\mathbf{x}[i] = \mathbf{A}(\boldsymbol{\theta}_{g})\mathbf{s}_{g}[i] + \mathbf{n}[i] .$$
<sup>(11)</sup>

Now the sparsity-based DOA estimation for a single snapshot can be formulated as

min 
$$\|\mathbf{s}_{\mathbf{g}}[i]\|_{0}$$
 (12)  
subject to  $\|\mathbf{x}[i] - \mathbf{A}(\boldsymbol{\theta}_{g})\mathbf{s}_{\mathbf{g}}[i]\|_{2} \leq \varepsilon$ 

where  $\|\cdot\|_0$  is the  $\ell_0$  norm to promote sparsity in  $\mathbf{s_g}[i]$ . Locations of the non-zero entries in the resultant  $\mathbf{s_g}[i]$  represent the corresponding DOA estimation results.

As the  $\ell_0$  norm is non-convex, in practice it is normally replaced by the  $\ell_1$  norm as an approximation. Finally, the sparsity-based DOA estimation for a single snapshot is formulated as

min 
$$\|\mathbf{s}_{\mathbf{g}}[i]\|_{1}$$
(13)
subject to  $\|\mathbf{x}[i] - \mathbf{A}(\boldsymbol{\theta}_{g})\mathbf{s}_{\mathbf{g}}[i]\|_{2} \leq \varepsilon$ 

where  $\left\|\cdot\right\|_1$  is the  $\ell_1$  norm.

When multiple data snapshots are available, we could perform DOA estimation by (13) for each snapshot *i* separately. However, a more effective approach is to jointly estimate the DOAs of the impinging signals across multiple snapshots employing the group sparsity concept, since they all have the same spatial support.

<sup>&</sup>lt;sup>1</sup>Here "underdetermined" means that the number of signals is larger than or equal to the number of physical sensors.

Denote  $\mathbf{X} = [\mathbf{x}[0], \dots, \mathbf{x}[P-1]]$ , where P is the number of snapshots. Similarly, we can define  $\mathbf{S} = [\mathbf{s}[0], \dots, \mathbf{s}[P-1]]$  and  $\mathbf{N} = [\mathbf{n}[0], \dots, \mathbf{n}[P-1]]$ . Then, the signal model for multiple snapshots can be obtained by

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} , \qquad (14)$$

To introduce spatial sparsity, similar to the single-snapshot case, we construct  $\mathbf{S}_{\mathbf{g}} = [\mathbf{s}_{\mathbf{g}}[0], \dots, \mathbf{s}_{\mathbf{g}}[P-1]]$ , and use the row vector  $\mathbf{s}_{\mathbf{g},k_g}$ ,  $0 \le k_g \le K_g - 1$  to represent the  $k_g$ -th row of the matrix  $\mathbf{S}_{\mathbf{g}}$ .

$$\mathbf{X} = \mathbf{A}(\boldsymbol{\theta}_g)\mathbf{S}_g + \overline{\mathbf{N}} . \tag{15}$$

Then a new  $K_g \times 1$  column vector is generated by computing the  $\ell_2$  norm of each row in  $S_g$ , expressed as

$$\hat{\mathbf{s}}_{\mathbf{g}} = \left[ \left\| \mathbf{s}_{\mathbf{g},0} \right\|_{2}, \left\| \mathbf{s}_{\mathbf{g},1} \right\|_{2}, \dots, \left\| \mathbf{s}_{\mathbf{g},K_{g}-1} \right\|_{2} \right]^{T}.$$
(16)

Finally the problem for multiple snapshots can be formulated as

$$\min_{\mathbf{S}_{\mathbf{g}}} \| \hat{\mathbf{s}}_{\mathbf{g}} \|_{1}$$
subject to  $\| \mathbf{X} - \mathbf{A}(\boldsymbol{\theta}_{g}) \mathbf{S}_{\mathbf{g}} \|_{F} \leq \varepsilon$ , (17)

where  $\|\cdot\|_F$  represents the Frobenius norm, and  $\|\hat{\mathbf{s}}_{\mathbf{g}}\|_1$  is also called the  $\ell_{2,1}$  norm of the matrix  $\mathbf{S}_{\mathbf{g}}$ . Locations of the non-zero entries of the resultant column vector  $\hat{\mathbf{s}}_{\mathbf{g}}$  are then the corresponding estimation results.

One problem with the above group sparsity based formulation is its high computational complexity, especially when a large number of snapshots P are available. To reduce the complexity, we can perform singular value decomposition (SVD) to **X** and project the data to a lower dimension signal space, leading to the so-called  $\ell_1$ -SVD method [22], or using the covariance matrix of the data to form a virtual array directly [23].

## 2) Underdetermined DOA Estimation

For underdetermined DOA estimation, although it can be achieved by exploiting nonGaussianity, noncircularity or nonstationarity of the signals, the most important development is through constructing various sparse array structures for virtual co-array generation, such as co-prime arrays, nested arrays and their numerous extensions [24]–[26].

For second-order statistics based co-array generation, one common step is to vectorize the covariance matrix of the physical sparse array. Consider the covariance matrix

$$\mathbf{R}_{\mathbf{x}\mathbf{x}} = \mathbf{E}\left\{\mathbf{x}[i]\mathbf{x}^{H}[i]\right\} = \sum_{k=1}^{K} \sigma_{k}^{2} \mathbf{a}(\theta_{k}) \mathbf{a}^{H}(\theta_{k}) + \sigma_{n}^{2} \mathbf{I}_{N} , \qquad (18)$$

where  $\sigma_k^2$  is the power of the k-th impinging signal.

By vectorizing  $\mathbf{R}_{\mathbf{x}\mathbf{x}}$ , we obtain a virtual array model

$$\mathbf{z} = \operatorname{vec} \left\{ \mathbf{R}_{\mathbf{x}\mathbf{x}} \right\} = \widetilde{\mathbf{A}}(\boldsymbol{\theta})\widetilde{\mathbf{s}} + \sigma_n^2 \widetilde{\mathbf{i}}_{N^2} , \qquad (19)$$





(a) A co-prime microphone array system



Fig. 2: An experimental example for group-sparsity based underdetermined wideband DOA estimation, where 10 uncorrelated acoustic source signals are distributed from around  $-40^{\circ}$  to  $50^{\circ}$  with an approximate step size of  $10^{\circ}$  [27].

where  $\widetilde{\mathbf{A}}(\boldsymbol{\theta}) = [\widetilde{\mathbf{a}}(\theta_1), \dots, \widetilde{\mathbf{a}}(\theta_K)]$  is the equivalent virtual steering matrix, with  $\widetilde{\mathbf{a}}(\theta_k) = \mathbf{a}^*(\theta_k) \otimes \mathbf{a}(\theta_k)$  being the corresponding steering vector ( $\otimes$  denotes the Kronecker product),  $\widetilde{\mathbf{s}} = [\sigma_1^2, \dots, \sigma_K^2]^T$  is the equivalent source signals, and  $\widetilde{\mathbf{i}}_{N^2}$  is obtained by vectorizing  $\mathbf{I}_N$ .

In the above virtual array model, although there are repeated entries in  $\mathbf{R}_{xx}$ , the number of virtual sensors corresponding to the difference co-array is much more than that of physical sensors, and the equivalent source signals share the same spatial sparsity with the original impinging signals. The virtual model in (19) is similar to the single-snapshot array model, and sparsity based DOA estimation methods can be applied here.

Instead of employing a sparse array, it is possible to extend the co-array concept to different frequencies, where a single ULA can be used with two continuous-wave signals of co-prime or other different frequencies and also to the wideband case through frequency decomposition and employing multiple frequency pairs [17].

The group sparsity concept employed for the multiple-snapshot case can be applied to general underdetermined and overdetermined wideband DOA estimation [17]; like in traditional wideband DOA estimation, focusing can also be employed for sparsity based wideband DOA estimation to simplify the problem to a single reference frequency. One interesting observation about the wideband case is, the sensor spacing can be larger than half the wavelength corresponding to the highest frequency of the signal without causing the spatial aliasing problem; on the contrary, an improved estimation performance can be achieved for a larger spacing due to increased aperture.

Fig. 2 shows a real experimental result based on an 8-microphone co-prime array for estimating the directions of 10 speech signals with a bandwidth from 5 kHz to 10 kHz and sampling frequency of 20 kHz [27].

## C. Sensor Location Optimization

In many applications, the array's geometrical layout is assumed to be fixed and given in advance. However, it is possible to change the geometrical layout of the array including adjacent sensor spacing and these additional spatial degrees of freedom (DOFs) can be exploited to improve the performance in terms of either beamforming or DOA estimation or both. For the beamforming side, given the non-convex nature of the optimization problem, traditionally it is solved by genetic algorithms (GAs), simulated annealing, or similar [28]. With the development of compressive sensing or the sparsity maximisation framework, a new CS-based framework with a theoretically optimum solution (given the convex nature of the problem) has been developed for sensor location optimization for fixed beamforming [29], followed by further work in adaptive beamforming [30], [31], with robustness against various array model errors considered too. For the DOA estimation side, the main efforts have been focused on the co-array design to increase the DOFs for underdetermined DOA estimation. As mentioned in the last section, coprime arrays and nested arrays are two representative array structures [24], [25], based on which numerous second-order and fourth-order (or even higher) co-array construction methods have been developed. In this part, we will focus on the sparse array design problem for beamforming.

To illustrate how the design works, consider a narrowband linear array structure consisting of M omni-directional sensors, where the distance from the first sensor to subsequent sensors is denoted as  $d_m$ , for m = 1, 2, ..., M, with  $d_1 = 0$ , i.e. the distance from the first sensor to itself. The output of the beamformer is a weighted sum of the received signals and the weighting coefficients are denoted by  $w_m$ , m = 1, 2, ..., M, which are placed together into the weighting vector  $\mathbf{w}$ . Then, the sparsity based design for sensor location optimization can be described as follows.

First, consider the array geometry being a grid of potential active antenna locations. In this instance,  $d_M$  is the maximum aperture of the array and the values of  $d_m$ , for m = 1, 2, ..., M - 1, are selected to give a uniform grid, with M being a very large number, so that the spacing between adjacent antennas is very small. Through selecting the minimum number of non-zero valued weight coefficients to generate a beam response close to the desired one, a sparse array design result is obtained. In other words, if a weight coefficient is zero-valued, the corresponding sensor will be inactive and therefore can be removed, leading to a sparse or non-uniformly spaced sensor array.

Mathematically, it is formulated as a constrained  $\ell_1$  norm minimization problem

$$\min \quad ||\mathbf{w}||_1 \tag{20}$$

subject to 
$$||\mathbf{p}_r - \mathbf{w}^H \mathbf{A}||_2 \le \varepsilon$$
, (21)

where  $\mathbf{p}_r$  is the vector holding the desired beam responses at the sampled angular range of interest,  $\mathbf{A}$  is the steering matrix corresponding to those angles, with  $\mathbf{w}^H \mathbf{A}$  representing the designed beam responses, and  $\varepsilon$  is the allowed error between the designed and desired beam responses. The minimization of the  $\ell_1$  norm of the weight vector



Fig. 3: Location optimization result and comparison with the ULA.

helps to promote sparsity in the weight vector, and the reweighted  $\ell_1$  norm minimization could be used instead to have a closer approximation to the ideal  $\ell_0$  norm minimization problem, where smaller weighting terms are added to the larger elements of the weight vector **w**, so that smaller values in **w** are penalized more and become closer to zero after minimization [32].

A broadside mainbeam design example is shown in Fig. 3, where the sensor locations are optimized over an overall aperture of  $d_M = 10\lambda$ , which is split into 181 potential sensor locations (M=181). It can be seen that the resultant weight vector is sparse with only 12 non-zero valued coefficients, leading to a sparse array of 12 sensors, and compared to the beam pattern of a standard 12-sensor ULA with half-wavelength spacing, the sparse array has a similar main beamwidth, but a much lower sidelobe level.

Various constraints can be added to the above formulation to deal with more complicated application scenarios. For example, in the above formulation, the steering vector of the array is assumed to be known exactly, which may not be true due to various possible model perturbations, such as errors in sensor locations, mutual coupling and discrepancies in individual sensor responses; then, robust designs can be achieved by applying a norm-bounded error constraint to the weight vector. In another case, it has been assumed that the sensors in the array are of zero size; however, this is not true in a real world, and various size constraints can be added to the design, or some post-processing methods can be introduced to make sure the minimum spacing between adjacent sensors in the result is larger than the size of the sensor. Based on the concept of group sparsity, the design can also be extended to the wideband case with TDLs [29].

#### D. Target/Source Localization Based on Sensor Arrays

This is another important problem in array signal processing and significant progress has been made in this area in the past twenty-five years. Typical solutions include those based on received signal strength (RSS) [33], those based on distance related measurements such as time of arrival (TOA) [34], and those based on the angle of



Fig. 4: A general target/source localization model based on distributed sensor arrays [37].

arrival (AOA) or DOA [35], [36]. The last is also called bearing-only localization, and is an attractive candidate since synchronization among the distributed platforms is not required, and it can be used in both active and passive sensing networks and adopted in a wide range of applications including multistatic radar, distributed massive MIMO, and wireless sensor networks. There are normally two steps in this bearing-only localization: the first is applying existing DOA estimation methods to find the AOAs at all distributed sensor arrays, while the second is to find intersections of those estimated AOAs in order to localize the sources and the maximum likelihood estimator (MLE) has been adopted to minimize the total least square errors of the noise-corrupted angle measurements among all distributed sensor arrays. However, the performance of such a two-step localization approach is dependent on the accuracies of angle measurements obtained at all platforms, and even one bad AOA estimation result can lead to a serious performance degradation.

To tackle the shortcomings of the two-step approach, we could jointly process the collected information across the observation platforms in lieu of fusing the separate angle estimation results at all platforms. One recent advance in this direction is a group sparsity based one-step approach [37], where a common spatial sparsity support corresponding to all distributed sensor arrays is enforced, leading to a better estimation performance, which also avoids the possible pairing and ambiguity problems associated with the two-step AOA based solution.

To show how this idea works, we consider a distributed narrowband sensor array network with M sub-arrays and K targets as shown in Fig. 4, where  $U_m(x_m, y_m)$  and  $T_k(x_{T_k}, y_{T_k})$  represent locations of the receiver platform and the k-th target, respectively. For each receiver, a linear sub-array with  $L_m$  sensors is employed.

For each target located at  $T_k(x_{T_k}, y_{T_k})$ , a unique incident angle  $\theta_{m,k}$  relative to the *m*th sub-array can be obtained. Without loss of generality, a square area of interest in the Cartesian coordinate system is divided into a  $K_x \times K_y$ grid with  $K_x$  and  $K_y$  being the number of grid points along the x-axis and the y-axis, respectively.  $G(x_{k_x}, y_{k_y})$ represents the location of the  $(k_x, k_y)$ -th search grid, and the signal originating from the possible source located at  $G(x_{k_x}, y_{k_y})$  will arrive at the *m*-th sub-array with a DOA angle  $\theta_m^g(k_x, k_y)$ . Since  $(x_{k_x}, y_{k_y})$  is common to all sub-arrays and a source located at  $G(x_{k_x}, y_{k_y})$  will appear to come from the same location with respect to all sub-arrays, we can apply the group sparsity concept to all sub-arrays' source data.

For example, for the mth sub-array, corresponding to the data model in (14), we can have the multiple-snapshot model as

$$\mathbf{X}_m = \mathbf{A}_m \mathbf{S}_m + \mathbf{N}_m , \qquad (22)$$

with m = 1, 2, ..., M. Applying the sparsity based approach, we can construct the following overcomplete data model

$$\mathbf{X}_m = \mathbf{A}_m^g \mathbf{S}_m^g + \mathbf{N}_m \,, \tag{23}$$

where  $\mathbf{A}_m^g$  is the overcomplete steering matrix corresponding to the  $K_x K_y$  potential signal directions  $\theta_m^g(k_x, k_y)$ and  $\mathbf{S}_m^g$  is the potential source matrix. If there is no source located at a particular position  $G(x_{k_x}, y_{k_y})$ , then the corresponding row of  $\mathbf{S}_m^g$  will be zero-valued for all m = 1, 2, ..., M. We can place all the matrices  $\mathbf{S}_m^g$  together to form a new matrix  $\mathbf{S}^g$  as follows

$$\mathbf{S}^{g} = \begin{bmatrix} \mathbf{S}_{1}^{g}, \mathbf{S}_{2}^{g}, \dots, \mathbf{S}_{M}^{g} \end{bmatrix} , \qquad (24)$$

Then the group-sparsity based localization problem can be formulated by minimizing the  $\ell_{2,1}$  norm of  $S^g$  subject to limiting the overall reconstruction error for all sub-arrays to a small value. One main advantage of the group sparsity based approach for direct target localization is that the different sub-arrays are not required to be synchronized and can work on different frequencies, the statistical properties of the sources can be different for different subarrays, and sensor numbers, rotation angles, and corresponding source signals of different sub-arrays do not need to be the same (as long as they come from the same set of target locations). This group-sparsity based one-step direct localization idea can be extended to either the wideband or the underdetermined case – or both – without difficulty [37].

Fig. 5 shows a simulation result for underdetermined wideband localization, where the normalized signal frequency band is from  $0.75\pi$  to  $\pi$ , and there are six sub-arrays and five targets, with each sub-array being a 4-sensor minimum redundancy array [4].

## E. Multiple Input and Multiple Output (MIMO) Arrays

MIMO, which is by its multi-channel implementation at both transmitter and receiver a natural fit within the SAM portfolio, is another significant development in array signal processing in the past twenty-five years. There are mainly two totally different directions. One is MIMO radar which exploits the orthogonality of the transmitted waveforms to increase the DOFs of the system to improve the resolution and capacity of the array [38]–[40], which will play an important part in 4D auto radar imaging in addition to traditional radar detection applications. Note that non-orthogonal waveforms can also be employed for MIMO radar [41]. The other one is MIMO for wireless



Fig. 5: Localization results for five targets with six sub-arrays (20dB SNR, 500 snapshots).

communications to exploit spatial diversity of the channel to improve performance and in particular capacity of the communication system [42]. While MIMO has already been in use for both Wi-Fi and 4G communication systems, its new evolution, the so-called massive MIMO or ultra-massive MIMO (UM-MIMO) will play a crucial role in the next-generation communication systems and beyond [43].

# 1) MIMO Radar

In a MIMO radar, multiple transmit antennas emit orthogonal waveforms and multiple receive antennas then receive the echoes reflected by the targets. Antennas of the MIMO radar can be widely separated [38] or co-located [39], [40], with the latter one more widely studied. For the case with co-located antennas, the transmitting side and the receiving side can be located either at the same site or far away from each other.

Consider a co-located narrowband MIMO radar system, where the transmit and receive antennas are located at the same place. The transmitted multiple orthogonal waveforms are then reflected back by K present targets and received by the receive array. After matched filter processing, the output signal vector  $\mathbf{x}[i]$  at the receiver at the *i*th snapshot can be expressed as

$$\mathbf{x}[i] = \sum_{k=1}^{K} \mathbf{a}_t(\theta_k) \otimes \mathbf{a}_r(\theta_k) b_k[i] + \mathbf{n}[i]$$
  
=  $[\mathbf{a}_t(\theta_1) \otimes \mathbf{a}_r(\theta_1), \cdots, \mathbf{a}_t(\theta_K) \otimes \mathbf{a}_r(\theta_K)] \mathbf{b}[i] + \mathbf{n}[i]$  (25)

where  $\theta_k$  is the DOA of the *k*th target,  $\mathbf{a}_t(\theta_k)$  and  $\mathbf{a}_r(\theta_k)$  are the steering vectors of the transmit and receive array, respectively, and  $b_k[i] = \gamma_k e^{j2\pi f_k i}$ , with  $\gamma_k$  being the complex-valued reflection coefficient of the *k*th target and  $f_k$ being the Doppler frequency for moving targets.

It can be seen that with the MIMO radar configuration, a virtual array with significantly increased aperture has been created due to the effect of the Kronecker product in (25). For example, if both the transmit array and receive array are 3-sensor ULAs with spacing of d and 3d, respectively, the newly generated virtual ULA will consist of 9 sensors. In this way, by exploiting waveform diversity, a virtual array with a much larger aperture and significantly

increased DOFs is formed using a small number of physical sensors, providing enhanced spatial resolution, higher target detection capacity and better performance.

# 2) MIMO for Wireless Communications

On the other hand, MIMO for wireless communications is a huge research area and numerous techniques have been developed centered around this concept, such as space-time coding, MIMO beamforming, spatial multiplexing, and spatial modulation, etc. These days, an element of MIMO can be found in most of the publications in wireless communications. It is impossible to list all the important advances in the area and in this subsection, we will only focus on MIMO beamforming which is playing an increasingly important role in the implementation of MIMO communication systems.

As well known by the array signal processing community and also presented in Section II-A, traditionally beamforming is designed for line of sight (LoS) transmission and reception, and physically a beam will be formed in the process pointing to different directions around the array system. However, in MIMO beamforming, due to a very strong multi-path effect, the result of beamforming between the transmitter and receiver will not necessarily form a beam in space, but rather an overall enhanced signal transmission link between them. Decades of research in MIMO beamforming have pushed the boundaries of beamforming well beyond its traditional meaning, and nowadays any process achieving enhancement of the desired signal while reducing the effect of interference can be considered as beamforming. However, with the introduction of massive MIMO and millimetre wave communications in 5G and beyond, the LoS case is becoming more and more important again in MIMO beamforming and one interesting development in this context is the hybrid beamforming structure proposed for massive MIMO systems [8].

Hybrid beamforming is a combination of analogue beamforming and digital beamforming. Ideally, beamforming could be implemented completely in the digital domain for maximum flexibility and adaptability; however, for extremely large arrays, as in the case of massive/ultra-massive MIMO, the extremely high cost associated with the large number of high-speed analogue to digital converters (ADCs)/digital to analogue converters (DACs) and the high-level power consumption will render it practically infeasible. For hybrid beamforming in the receive mode, analogue beamforming is performed first to reduce the number of analogue channels, which are then converted into digital via a reduced number of ADCs, and after that digital beamforming can then be performed; for the transmit mode, the process is simply reversed. There are many hybrid beamforming structures proposed in the literature, and one representative is the sub-aperture based hybrid beamformer. An interesting recent development in this area is a new class of multi-beam multiplexing designs where the number of analogue coefficients is the same as the number of parallel independent user-beams generated, while the number of subarrays is the same as the number of beams; interested readers can refer to [44], [45] for details.

In the era of artificial intelligence, multi-sensor based systems and techniques are ubiquitous and will play an even greater role in the future. As a result, there has been an exponential increase in research activities in the SAM area in the past few years and in the following, we introduce some new developments which may well indicate promising future research directions.

#### A. Graph signal processing for sensor networks

Graph signal processing (GSP) is an emerging new mathematical tool for analysis of data resident on a largely irregular network of either physical or virtual sensor nodes, where the regular network can be considered as a special case [46], [47]. Examples for the physical sensor network include traffic networks, brain neural networks and energy consumption sensor networks, while for the virtual one, a good example is social networks. In connection with classic signal processing, basic concepts such as frequency, and operations such as shift/delay and filtering have been introduced. However, there is still no unified framework for GSP and it is still an open problem to find the best representations of a graph signal. However, this has not stopped the wide application of GSP and it has been shown to be a powerful data analysis tool providing new insights into the studied problems; for example, the brain signals can be mapped to a graph network to analyze cognitive behavior of the brain. Application of GSP to traditional sensor array signal processing problems such as direction finding and target localization is an emerging but somewhat open area, as traditional sensor arrays and networks normally have a regular structure and traditional sensor array signal processing tools have been extremely successful in tackling those associated problems. It is not clear yet whether GSP can bring any advantage to the traditional sensor array signal processing problems or not.

## B. Tensor based array signal processing

Tensors are an extension of matrices to higher dimensions and have been widely employed for multi-dimensional data analysis and processing with the aid of tensor decomposition tools and algorithms. Many sensor array signals and data can be transformed into a multi-dimensional form or viewed directly as a multi-dimensional structure [48], [49]. For example, the narrowband data received by a rectangular array or multiple sub-arrays are three-dimensional (3-D), the data received by a wideband linear array can be transformed into the 3-D space-time-frequency domain, and the data received by vector sensor arrays is naturally higher dimensional. For MIMO communication systems, the data can be placed into a tensor form by accounting for diversities in space, time, frequency (including Doppler frequency) and polarization. As a result, tensor processing can be applied to solve many array signal processing problems directly without much adaptation. However, although it is recognized that tensors can keep the inherent data structures and therefore have the potential to provide improved performance compared to classic array signal

processing methods and algorithms, further research is needed to demonstrate the clear benefits of tensor processing and fully realize its potential.

# C. Quaternion valued array signal processing

As a higher-dimensional extension of complex numbers, a quaternion has one real part and three imaginary parts, and quaternion calculus has been applied to a range of signal processing problems related to three or fourdimensional signals, such as color image processing, wind profile prediction, vector-sensor array processing, and quaternion-valued wireless communications [50], [51]. In addition to solving the classic array signal processing problems such as DOA estimation and beamforming, one important development is the quaternion-valued MIMO array, where pairs of antennas with orthogonal polarization directions are employed at both the transmitter and the receiver sides, and a four-dimensional (4-D) modulation scheme across the two polarization diversity channels using a quaternion-valued representation is employed. Although the polarization states will change during transmission through the channel, and there may be interferences between these two states, we can employ a quaternion-valued adaptive algorithm to recover the original 4-D signal, which inherently also performs an interference suppression operation to separate the original two 2-D signals. For the MIMO array, reference signal based or blind quaternion-valued adaptive algorithms can be employed for both channel estimation and beamforming. Signal processing has experienced the revolutionary change from real-valued processing to complex-valued processing, and we may be at the doorstep of a quaternion-valued new world and increasing interests for quaternion-valued sensor array signal processing are expected in the near future.

# D. One-bit and non-coherent sensor array signal processing

Given the extremely high data rate and storage requirements for a fully digital large sensor array system, there has been significant work aimed at achieving a reasonable sensor array processing performance with one-bit representation of the array signals, i.e. only signs of the data samples are reserved while the magnitude information is removed [52]. This problem can be simply considered as the normal case but with extremely high quantization noise and we can perform normal array processing irrespective of the number of bits per data sample; however, a more effective way is to try to achieve effective estimation of the statistics of the signals using the one-bit data samples and then, based on the newly obtained statistics information, to perform the corresponding tasks. On the contrary to one-bit array processing, the signs of the data samples are removed and only the magnitude information is kept, which leads to the so-called non-coherent sensor array signal processing problem, with the advantage of being robust against array phase errors. One representative example is non-coherent DOA estimation and target localization [53]–[55], which can be cast into a phase retrieval problem; however, the difference is, there is usually only one snapshot in phase retrieval, while in array signal processing, multiple snapshots are available, which can

be exploited by applying group sparsity to existing phase retrieval algorithms, such as the ToyBar and modified GESPAR algorithms [53], [54].

#### E. Machine learning and AI for sensor arrays

Machine learning and artificial intelligence have been applied to almost all areas of research in the signal processing community and the SAM area is no exception. For example, they have been applied to DOA estimation, beamforming and source separation successfully [56]. There are strong topical connections between sparsity-inspired array processing (cf. Sec. II-B), compressed sensing (cf. Sec. II-C), and machine learning. Unlike in traditional machine learning and AI applications where it is a challenge to acquire sufficient training data, in most of the array signal processing applications, the required training data can be obtained easily by simulation. Nonetheless, their application to array signal processing also faces some similar issues. For example, after training, the system may work very well for the targeted scenario, but it may struggle if there is change to the system or the environment, while the traditional array signal processing methods and algorithms can cope with such changes well. Another challenge is how to apply machine learning and AI to distributed sensor arrays and networks effectively. As a hot topic, federated learning may prove to be a promising direction of research for the SAM community [57].

## F. Array signal processing for next-generation communication systems

Antenna array design and signal processing is one of the fundamental techniques in 5G (and beyond) wireless communication systems since the two underpinning 5G/6G technologies - massive or ultra-massive MIMO and millimetre wave/sub-THz/THz communications, are all based on antenna arrays [58]. It will continue to play a significant role in many other aspects in the future, such as internet of things (IoT) and integrated sensing and communication, both of which are hot topics for 6G wireless communications research, with extensive research activities attracted in the community. Moreover, beamforming is essential to achieve effective communication over the THz or sub-THz frequency band as it is necessary to employ a large number of antennas for such high frequencies, while the widely studied reconfigurable intelligent surfaces (RIS) can be considered as a semi-passive antenna array system [59]. To a great degree, array signal processing will be a main focus of research for next-generation communication systems and for the integration of sensing and communications, particularly at mmWaves [60].

## **IV. CONCLUDING REMARKS**

Accompanied by intensive research activities and significant progresses made in signal processing, the world now has stepped into the new era of artificial intelligence, where multi-sensor based systems and techniques have become ubiquitous and indispensable to our daily life and will play an even greater role in our society in the very near future. This is an exciting time for the SAM community and we welcome new members at different levels to join the TC and work together to promote its activities, make a more extensive and deeper impact in the real world and further enhance its standing in our wider society.

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