Russell and Cassirer as Leibniz's Interpreters: On the Analytic and Synthetic Nature of Mathematical and Physical Knowledge

[Russell e Cassirer como intérpretes de Leibniz: sobre a natureza analítica e sintética do conhecimento matemático e físico]

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Abstract

In this paper, I aim to reconstruct the debate on Leibniz to which Russell's and Cassirer's monographs contributed eminently. This task is important both historically and epistemologically. In fact, their interpretations represented the apexes, at least at that time, of the logicist and the neo-Kantian philosophies of mathematics; on the other hand, they also paved the way for later developments of the discussion on the foundations of mathematics. The paper is divided into five sections. The first contains a brief introduction. In the second and third parts, I present Russell's and Cassirer's works to explain why they maintained that mathematics is synthetic. In the fourth section, I compare and contrast the reviews that they wrote about their books. Finally, in the fifth section, I propose a perspectivist interpretation of the dispute.
Keywords: Russell; Cassirer; Logicism; Neo-Kantianism; Leibniz.

Resumo

Neste artigo, pretendo reconstruir o debate sobre Leibniz para o qual as monografias de Russell e Cassirer contribuíram eminentemente. Essa tarefa é importante tanto histórica quanto epistemologicamente. De fato, suas interpretações representavam os ápices, pelo menos naquela época, das filosofias logicista e neokantiana da matemática; por outro lado, também abriram caminho para desenvolvimentos posteriores da discussão sobre os fundamentos da matemática. O trabalho está dividido em cinco seções. A primeira continha uma breve introdução. Na segunda e terceira partes, apresento os trabalhos de Russell e Cassirer para explicar por que eles sustentavam que a matemática é sintética. Na quarta seção, comparo e contraste as resenhas que eles escreveram sobre seus livros. Finalmente, na quinta seção, proponho uma interpretação perspectivista da disputa.
Palavras-chave: Russell; Cassirer; Logicismo; Neokantismo; Leibniz

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1. Introduction

Literature on the comparison between Russell and Cassirer is flourishing today, although scholars preferentially focus on the advanced state of the debate. Less attention is devoted to the dispute over Leibniz's work, which took place in the early years of the XX century. In this paper, I aim to fill this gap.[2]

As we will see, the matter of contention was the distinction between "analytic" and "synthetic" in "logic".[3] This will be the object of the first two sections in particular. In the fourth section, I will address Russell's and Cassirer's reviews of their works, which were issued in 1902 and 1903 respectively. In the fifth section, I will expound the philosophical implications of the discussion, but only as concerns the period we are taking into consideration.

2. Russell on "Analytic" and "Synthetic" Tendencies in Leibniz's Philosophy

Russell's interest in Leibniz's philosophy had early roots: in January and February 1899, while McTaggart intended to be in New Zealand, Russell held lectures on Leibniz in Cambridge and wrote a draft which is now appended to the edition of the *Collected Works* under the title: *Leibniz's Doctrine of Substance as Deduced from His Logic* (1899-1900). In this section, I will discuss both this contribution and the book *A Critical Exposition on the Philosophy of Leibniz* (1900), which contains long excerpts taken from Russell's draft. But before that, it is necessary to contextualise Russell's work at the time.

At the end of the XIX century, Russell was committed to neo-Hegelianism and even to a peculiar recovery of Kantianism, which was the object of *An Essay on the Foundations of Geometry*, published in 1897. The *Essay* is interesting for two reasons. In the first place, it paves the way for the neo-Kantian defence of the apriority of space in light of non-Euclidean geometries by disclosing the *a priori* character of projective against metrical geometry (Russell, 1897, pp. 117-118; pp. 146-147; Cassirer, 1953, pp. 68-111). In the second place, it also endorses anti-psychologism, which was the manifesto of the School of Marburg. By the same token, Russell was dealing with the doctrine of internal relations, to wit, the idea that relations depend on the intrinsic features of the *relata* (Griffin, 2013).

It is thus surprising to see that in that text Russell surrenders to empiristic claims concerning the definition of "distance"[4]. While assuming that projective geometry is *a priori* for it simply tackles the "quality" of spatial relations and does not deal with the metric properties of space, he underlines that we cannot avoid setting up two given points and a straight line that *uncovers* what we call "distance". In fact, if we need four points to determine differences in sets of points as regards projective geometry, among which the choice of two would be but "conventional" (Russell, 1897, pp. 30 ff.), we should not overlook the fact that we start from just two. In Russell's words:

2 To the best of my knowledge one exception, though dated, is Ferrari, 1988, chap. 4.

3 For reasons of space, I cannot delve into Kant's distinction (KrV, A6/B9ff.). Thus, in mentioning Coffa for further enquiry (Coffa, 1991, chap. 1-2-4), I limit myself to what follows. Analysis is given on the basis of the analogy of "representations" with "chemical compounds", that is, of the idea that representations are made of components. Therefore, to know a representation means to identify its constituents. Coffa drew attention to the fact that Kant connected this method to analytic judgements, in order to suggest that this kind of judgements are the result of analysis. Given that a judgement is the relation of two concepts, we thus have analytic judgements when concepts are contained in the subject, and synthetic judgements when they are not. On this distinction and its post-Kantian history, see Hanna, 2001.

4 This was the object, for instance, of Natorp's criticism to Russell (Natorp, 1910, p. 324).

> If *A, B, C,* be three different points on a line, there must be *some* difference between the relation of *A* to *B* and of *A* to C, for otherwise, owing to the qualitative identity of all points, *B* and C could not be distinguished. But such a difference involves a relation, between *A* and *B*, which is independent of other points on the line; for unless we have such a relation, the other points cannot be distinguished as different. Before we can distinguish the two fixed points, therefore, from which the projective definition starts, we must already suppose some relation, between any two points on our line, in which they are independent of other points; and this relation is distance in the ordinary sense (Russell, 1897, p. 35).

Biagioli has efficaciously commented this point by stating that the distinctness of the pair "is not a result of the designation, but its condition" (Biagioli, 2016, p. 133). Hence, two points are set as distinct and their relation designated as independent from any other which will be merely possible. If this, as Biagioli shows, is but a misunderstanding of Klein's standpoint in particular (ibidem), which Russell discussed among others in those pages, it nevertheless reveals what the British philosopher conceives of as relations and *relata* to be. As far as relations are concerned, it appears that he does not mean the pure possibility of the connection of whatever possible objects, but what specifically involves two elements at least. Hence a relation does not annihilate the particularity of its constituents because without them there would be no relation. In this way, both relations and *relata* are supposed to be real. This is why Russell's philosophy was judged to be a commixture of Platonism and empiricism (Smart, 1943, p. 168; Di Francesco, 1991, p. 35).

However, the label "empiricism" is rather vague at the moment. It would be better to listen to Russell himself, when confessing to having been "a realist in the scholastic or platonic sense" (Russell, 1961, p. 44). If we couple the claim for the existence of relations with the proof that space, as a logical *a priori* form, presupposes "externality", we obtain exactly the kind of "empiricism" here at play. That is, an empiricism related to the reform of Kant's intuition. In fact, externality makes possible the "experience of diverse but interrelated things" (Russell, 1897, p. 179) and allows "identity in difference" (ibidem, p. 185). Nevertheless, it is hard to say that externality now stands for our outer sense. In fact, externality does not persist only in our mind: "The *ground* of necessity, we may safely say, arises from the mind; but it by no means follows that the *truth* of what is necessary depends only on the constitution of the mind" (ibidem). In practice, when Russell ventures to ground his neo-Kantian doctrine in pure logics, he finds that externality is essential and that it also deals with "a given element in sense-perception", which can be revealed through "analysis" of the object of perception[5] (ibidem, p. 180).

Another important observation for our purpose stems from the insufficiency of one-dimensional forms of externality to account for diversity in relations. If we assume that things are only given in time, we will not be in the position to assume their plurality: in fact, there would only be things whose complexity is "merely adjectival". In commenting such a puzzle, Russell states:

> The chief difficulty of such a world would lie in the changes of the thing. What could cause these changes, since we should know of nothing external to our thing? It would be like a Leibnitzian monad, without any God outside it to prearrange its changes. Causality, in such a world, could not be applied, and change would be wholly inexplicable (Russell, 1897, 185).

5 The apparent paradox concerning an approach to relations that is both Kantian and realist can be explained by Russell's intent of disentangling intuition from subjectivity. Empiricism, for its part, may be also derived from Russell's opposition to Bradley's conceptual holism: Russell aims in fact to restore knowledge of local realities (Coffa, 1991, p. 89 and ff.; Di Francesco, 1991, chap. 3 and 4). Needless to say, this makes it harder to support Russell's standpoint, which was based on the intention of conceiving of "extension" as a matter whose existence is postulated intellectually. One should ask, indeed, if it is true that "any vestige of thinghood" is removed if sense-perception is involved in a purely "hypothetical" presumption (Russell, 1897, pp. 134 ff.).

Russell cannot thus avoid presuming the existence of a plurality of things if there are to be relations in general. And once that plurality is given, relations must be external to the mind. It is also striking that this thesis is here upheld as opposed to Leibniz's monadism, which reveals to us that this might be related to the rejection of internal relations.

As reported by Di Francesco, 1991, and Griffin, 2013, before reading Leibniz, Russell was indeed working on one of the most controversial consequences of this theory, i.e., the "contradiction of relativity". This contradiction consists of the fact that in mathematics we face entities that are different but fall under the same concept. If the properties of relations are to be ascribed to the nature of *relata*, the contradiction is untenable since it cannot fulfil the requirement that the great part of mathematical relations is asymmetrical (Russell, 1898), to wit, that it is not possible to reverse the simple form aRb[6]. This depends on the fact that order and direction are pivotal in mathematics; if these elements vanished, there would not be important concepts such as "greater than", "less than" etc. However, there are objects that are identical and nonetheless appear to be plural, such as points. How would it thus become plausible that identical objects do not enable symmetrical relations?

It is generally acknowledged, and pinpointed by Russell himself, that in this period he accepted the contradiction, and tackled new concepts that superseded it in vaster definitions, in order to construct a sort of Hegelian encyclopaedia of sciences — the so-called *Tiergarten* Program (Russell, 1961, p. 43; Di Francesco, 1991, p. 41). Is this solution feasible in light of the distinction between the analytic and the synthetic, which comes into play in Russell's interpretation of Leibniz?

Let us start from the above-mentioned 1899-1900 draft on Leibniz's concept of substance. Russell's idea is that in philosophy one should begin with the analysis of propositions and that Leibniz was one of the most respectable precursors of this trend. However, propositions are conceived by Leibniz as implying the attribution of predicates to subjects, in a way that allows us to think that predicates themselves inhere in the subject — henceforth, I will refer to this as the *SP* thesis. Accordingly, the analysis of predicates unfolds the identity of subjects, except for "existence", which, if not referred to God, is excluded because of its "contingency": "There would be no contradiction if the subjects which actually do exist did not exist" (Russell, 1899-1900, p. 516). We thus encounter two types of propositions. In the first case, by discovering the properties of subjects, we address "analytic" judgements; in the second case, when dealing with existence, we have "synthetic" judgments. Russell recapitulates by saying that "necessary propositions are such as are analytic, and synthetic propositions are always contingent" (ibidem).

Both kinds of propositions are related to the concept of substance for, in the first case, we address the properties of given complexes, whereas in the second case we refer the states of a subject to its conditions over time, and need the presupposition of something permanent which exists under the surface of change. We will see that Leibniz differs from tradition on at least one important point, but it is important to notice that a subject-oriented representation does not favour the understanding of "propositions which employ mathematical ideas" (ibidem, p. 517). Evidently, the idea of a subject is moulded on the basis of that of an individual thing, but in mathematics we often take as "subject" concepts such as "three men". This means that we postulate unity as though it implies plurality, but also maintain the faculty to assign predicates to the whole term without dividing it into as many propositions as the number of subjects involved. It should thus be clear that the subject of a mathematical proposition cannot be one thing, and that it should rather involve whole expressions such as: "all men" or "some men", to be treated as "units" (Coffa, 1991, p. 104).

6 Nevertheless, it is possible to express asymmetrical relations in the *SP* structure. For instance, if Paris loves Helen, we can conceive of the "being loved by Paris" as a property of Helen, thus assuming that "Paris is loving insofar as Helen is loved" (Anfray, 2017, p. 175).

However, what disturbs Russell the most is the idea that "propositions may acquire truth by being believed" (Russell, 1899-1900, p. 519). In particular, Russell addresses a famous passage from Leibniz's correspondence with Clark. In case of relations, say, of lesser and greater lines, we should abide by the view that either the ratio is of the greater to the lesser or of the lesser to the greater. The possibility that the relation would concern the purely logical link between the two contravenes the above-mentioned *SP* principle since we would presuppose two subjects in which the same accident partakes (Gerhardt, 1879, p. 486). Hence, trying to reduce each form of proposition to the *SP*, Leibniz condemns relations to be mere "ideal things" (Gerhardt, 1890, pp. 401-402).

This seems to have a crucial consequence. Russell draws our attention to the act of perception which is required by Leibniz for discovering relations. In fact, the "unity" of aggregates seems to be reached by "perception alone", in such a manner as to presuppose "simultaneous perception as a predicate of the percipient" (Russell, 1899-1900, p. 518). In brief, relations are in the mind (Gerhardt, 1879, p. 517) and depend on the fact that we add something to what is simply sensed at the moment in which we sense it. This is particularly ambiguous as far as mathematical knowledge is concerned for, as we have seen, we are not supposed to think of subjects such as "numbers", "all" or "some" as dividable. Therefore, in criticising this psychologistic inclination, Russell refuses to consider the abstractness of mathematics as purely "mental". Moreover, he harshly disapproves the sole assumption that may legitimate this vision, i.e., that an actual world exists outside of us and causes perceptions in our minds (Gerhardt 1885, pp. 598-600).

That being said, I will now explain how such criticisms are expanded in *A Critical Exposition of the Philosophy of Leibniz*, the book published in 1900. To begin with, it is worthwhile recalling that, for Leibniz, judgments in logic, arithmetic and geometry are analytic, while Russell, who was struggling against internal relations, still abided by the idea that mathematical propositions were synthetic (Russell, 2008: 2nd pref.; pp. 21 and ff.). But what does "synthetic" mean here?

Essentially, Russell thinks that a synthetic moment is involved in the general analysis that we make of subjects given that, in analytic propositions, these must be represented by complex ideas. Indeed, if we must reduce simple ideas to predicates, and if simple ideas obey the principle of non-contradiction and trace back to identical propositions, a question is posed about their compatibility which should be assessed as synthetic (ibidem, pp. 17-18). By way of example, if we take the idea of a round square, self-contradiction does not stem from the idea as such; rather, it is due to the proposition "square and round are compatible". Contradiction thus emerges from the fact that the analysis of the predicates shows that the property of having corners is not consistent with the idea of something being round, which has conversely the property of having no corners. If it were not so, any complex idea would be possible. This very principle is also valid for arithmetical propositions (ibidem, p. 21). Russell writes:

> Two simple ideas can never be mutually contradictory in Leibniz's sense, since mere analysis will not reveal any further predicate possessed by the one and denied by the other. Thus a self-contradictory idea, if it be not a mere negative, such as a non-existent existent, must always involve a synthetic relation of incompatibility between two simple notions. The impossible idea, in Leibniz's sense, presupposes the idea which is impossible on account of some synthetic proposition; and conversely, the possible complex idea is possible on account of a synthetic proposition asserting the compatibility of its simple constituents (ibidem).

As is clear, here judgements connect predicates that do not inhere in the same concept. Therefore, analysis elucidates the nature of single terms, but as to their possibility we need to jump out of subjects and point to synthetic judgements. In this sense, analysis is correct, but

the doctrine that discloses "primary principles" as analytic is wrong since the coexistence of predicates is given synthetically. It should be also remembered that the method puts the unity of a given subject between brackets (ibidem, p. 22), so analysis as such does not mirror reality as it is.[7]

Interestingly, Russell's approach also affects the principle of non-contradiction. This law only states "that any proposition must be true or false, but cannot be both" (ibidem), hence in no way does it allow us to pick what specific proposition is true or false. The law does not even seem to give any indication about what a proposition is. Finally, Russell also recalls that in propositions such as "2 is a number" or "number is a concept" the subject is not even complex, so it would be meaningless to say that such propositions are analytic.

Therefore, Russell cannot agree with the statement that if there are any necessary propositions, these are analytic. Besides, he clarifies that even Leibniz's thought is controversial since he considered the laws of motion as synthetic — they refer to the connection of parts in time (ibidem, p. 29). However, for Russell, the introduction of the law of sufficient reason secures at least a partial reduction of the contingent predicates to an analytical scheme.[8] Let us start, Russell comments, from the idea that in Leibniz there are two principles of sufficient reason, one concerning possible existents and the other dealing with actual existents. In the first case, we simply move within the law of causality and metaphysical necessity; it is only in the second case that we enter the reign of contingents, and in particular the idea "that all *actual* causation is determined by desire for the good" (ibidem, p. 30). In Leibniz's parlance, this appears to imply that it is only in dynamics that we learn the difference between "truths of reason", whose necessity is "*brute et geometrique*", and "truths of facts", whose value hinges on "convenience" and "final causes" (Gerhardt, 1887, p. 645).

To get an exemplification of that, we can think of two relations involving points and states of motion respectively (Griffin, 2013, pp. 4-5). If we merely consider mathematical points, we will only face abstractions from which "metaphysical points", that is, monads, represent the universe. By supposing that such points stand for the places occupied by two bodies, it would be impossible to know which of them has moved, which is at rest, or if both of them have moved or are at rest. This is a perfectly understandable consequence in a relational theory of space, where "a change of relative situation is necessarily reciprocal" (Russell, 2008, p. 85). Nevertheless, if we shift from "mathematical" to "metaphysical" points, forces are judged to be causes of motions, and the principle of activity[9] allows us to render relations asymmetrical. In fact, in this way, we would discover in bodies which begin to move a force absent in bodies at rest (ibidem, p. 88). The distinction between primitive and derivative force serves to this purpose: while the former contains certain modifications of the terms of a series of states, the latter designates "the actual present state while tending to the future" (ibidem, p. 95).

As to our main concern, I think that our assessment might be stated this way: the assumption of force, which enables change, motion, and asymmetric relations, is the only way to provide with physical consistency the analytic conviction that complex substances are composed of simple substances. However, Russell himself notes that when we rely on the laws of motions, we exploit "hypothetical necessity", which is not entirely metaphysical — the only one

7 In 1903, in his review of the recent works on the philosophy of Leibniz, which I will analyse in the fourth section, Russell is still convinced that to explain the incompatibility in complex ideas by addressing the simple ones is impossible in light of an analytic theory of truth (Russell, 1903, p. 182). Russell has both himself and Couturat in mind: like him (Russell, 2008, p. 19), for the French philosopher, there is no combination of simple ideas that is contradictory in the possible world (Couturat, 1961, p. 195).

8 Another way to defend analyticity goes through the infinite analysis of contingent predicates. In short, we can conceive of contingency as the impossibility to prove, in a finite number of steps, that a proposition is contradictory. Therefore, if we expand the reasons of contingents as we do with incommensurable magnitudes, there is no end in demonstrations (Arthur, 2014, pp. 94-96). According to Russell, this doctrine yields confusion between the truths "*about* the contingent", which are necessary, and the "contingent truth" that substances exist (Russell, 2008, p. 62).

9 As is known, Leibniz refers to such an activity with the Aristotelian term ἐντελέχεια (Gerhardt, 1885, pp. 608 ff.).

to be called "necessary" *stricto sensu* — ; rather, in this case we have a consequence that "follows with metaphysical necessity from a contingent premiss" (ibidem, p. 69). In fact, Leibniz often points out that the principle of sufficient reason, which should explain the relation between contingent states, "inclines without necessitating" (Gerhardt, 1879, p. 56).

Before moving on, I will conclude this section with a general remark made by Russell about the insufficiency of Kant's position and a summary of what we have found thus far.

Russell is convinced that Kant's doctrine of synthetic judgments paves the way for the extension of necessity to all judgments. It does not matter whether the object of a given knowledge is mathematical or given by perception; of course, these are different, but in considering the modality of their presentation we are not allowed to make any relevant discrimination. As Russell states:

> The distinction of the empirical and the *a priori* seems to depend upon confounding sources of knowledge with grounds of truth. There is no doubt a great difference between *knowledge* gained by perception, and *knowledge* gained by reasoning; but that does not show a corresponding difference as to what is known (Russell, 2008, p. 24).

The full explanation of this is in the sixth chapter of Russell's work. An explanation which is nonetheless paradoxical. Russell raises, indeed, the well-known criticism against the causal character of the thing-in-itself, which is due to a misleading extension of causality beyond experience. He ascribes this to the mistake of conceiving of the source of perceptions as being located elsewhere than in the mind, while Leibniz firmly held to the fact that perceptions are, as it were, spontaneous (Gerhardt, 1882, p. 362). Therefore, Russell thinks that "the ordinary grounds for assuming an external world were thus destroyed by Leibniz" (Russell, 2008, p. 74), although the latter did not maintain that the being external of the world is but "a faulty premiss" (ibidem, p. 75). Yet had not Russell stated that relations and *relata* are external?

I suppose that Russell solely aims to reject a naïve conception of externality, as though what exists outside the mind were a world of things affecting the individual and causing her perceptions. On the contrary, it is important that both relations and *relata* are external,[10] and this is not to say that an external world exists as such. It means that concepts and relations, as well as objects or *relata*, are not mental (Di Francesco, 1991, p. 35). We would inhabit such a world in which concepts, relations and objects coexist, and where there is no dualism between mind and reality, thought and being. Coffa labelled this approach "semantic monism" [11] (Coffa, 1991, p. 93).

In sum, although Russell learnt a lesson that Leibniz did not want to teach, it is undeniable that the study of Leibniz was crucial for him in shifting from Kantianism and neo-Hegelianism to the so-called "analytic realism", to wit, such a philosophy that defends external existence as to relations and *relata*, and thus endorses what we would now call a "referentialist" position in semantics.[12]

10 According to Russell, in Leibniz that might be shown as regards the construction of simultaneity, which would not regard the perceptions within the monad, but simply the relations between monads (Russell, 2008, p. 130).

11 Coffa has extendedly expounded on this aspect of Russell's theory showing that it consists of a reduction of Frege's concept of "sense" to that of "meaning": in this way, there would be, in his intention, no room left for representation, and propositional terms may be assessed as real (Coffa, 1991, pp. 90 ff.).

12 Indeed, the reality of terms will become an important claim of Russell's position in *The Principles of Mathematics* (Russell, 1951, § 50, pp. 46-47). In this work, "terms" are "entities", not merely "thoughts" or "words". Accordingly, for Russell, logic does not deal with inferences, but with what renders such inferences valid in the actual world; it is "the study of the implication between propositions" (Di Francesco, 1991, p. 105). It is thus not by chance that, among terms, Russell includes non-actual objects, to which he attaches some "reality" at least (ibidem, p. 190). According to Di Francesco, this implies the thesis that language works as the "*transparent medium*" of the everlasting reign of propositions (ibidem, pp. 107-108). As is known, though his detachment from Meinong, Russell will depart from such a position to even embrace, in *On Denoting* (1905), the opposite standing of "the intransparency of language" in his theory of descriptions. However, on closer inspection, Russell's strategy still aligns with confined aboutness

3. Cassirer on "Analytic" and "Synthetic" Tendencies in Leibniz's Philosophy

This section is devoted to explaining the apparently obscure claim that relations "generate" their elements and how that impacts to the analytic/synthetic distinction.

The very premise of Cassirer's endeavour consists of negating that analysis and synthesis merely pertain to propositions. For the sake of precision, it is better to say that only analysis seems to fit the *SP* structure, insofar as it is modelled upon the Aristotelian concept of abstraction. In short, Aristotle's theory of knowledge is assumed as a prototype of any empiristic epistemology for it presupposes that universals arise "*via negationis*" from particulars, which are priorly given physically, to wit, in the actual world. In this way, according to Cassirer, logic is destined to always work, as it were, *a posteriori*: it should simply confine itself to remarking which attributes are compatible when the analysis of complex terms is accomplished (Cassirer 1902, pp.105-106).

Considering that such a tendency also characterised the development of Euclidean geometry, Cassirer is not content with developing the "synthetic" against this background. In fact, in contrast with the principle of non-contradiction, this is what we would need:

> The principle which responds to the legitimacy of the logical passage between premises and consequences, seems to have to act, at the same time, as warranty for the value of the content of mathematical assumptions. Mathematical knowledge will be thus conceived of as the progressive and analytic development of an already given material, but not as a moment that, as foundation of a system of posited syntheses (*von Gesetzes-Synthesen*), let emerge from itself the content of being. In fact, the conflation which levels mathematics and logic equally entails that their relations with natural and empirical science loosen; for the latter, another principle must be introduced (ibidem, pp. 107-108).

In the first chapter of his work, Cassirer does not deal with this new principle directly — presumably the law of sufficient reason — , but he explains what is now requested to logic, and in which sense this task is not merely formal. We have seen, however, that Russell's logic contains hypotheses upon the nature of reality, or rather, of what is real. In what sense might it be thus tenable that the debate revolves around the distinction between formal and transcendental logic (Heis, 2010; Amaral, 2017)?

Evidently, the adjective "formal" does not mean that logical relations do not imply hypotheses on the nature of reality. In fact, the contraposition is actually due to the ontological claims involved in the logicist position. These are of two kinds. On the one hand, we face the above-mentioned realistic stances that take place in Russell's philosophy; on the other, we would encounter Couturat's panlogistic assumption, according to which the conventional value of signs and definitions does entail the correspondence with an already given, and thus external reality (Couturat, 1961, p. 105). For this reason, even Cassirer's letter to Natorp risks leading us astray.[13] The point is not that formal logic does not contain an assessment of the nature of reality; rather, it involves a false premise about it, which ultimately relates to the Aristotelean abstraction. At this point, we discover that the order of thought does not mirror reality as it is. Quite the opposite, reality is the product of scientific thought and nothing can be said about it before the application of scientific principles.[14] From this it follows that logic must be the

and transparency since what is given in *On Denoting* is a method to refer untransparent expressions to a subclass of sentences which are conversely transparent (Coffa, 1991, pp. 109-112).

13 For his part, Mahnke noticeably criticized Cassirer's emphasis on functional concepts (Mahnke, 1925, pp. 351-353).

14 In fact Cassirer confessed to Natorp that Couturat's work on Leibniz is "an excellent monograph", which nonetheless mistakes logic and mathematics for mere "doctrines of forms". Indeed, logic would so have no bearing on the "foundation of the content of physics and the problem of reality" (Cassirer, 2009, p. 5). In this letter sent on

"science of objective knowledge" (ibidem, p. 123), that is, a method for discovering the logical principles that regulate the formation of scientific objects, especially those of physics. This claim aligns with the neo-Kantian "purification" of Kant's intuition, in such a way as to refuse the psychologistic prejudice that an external world exists as such. Therefore, the only manner for pointing at objects should be their production according to mathematical and theoretical laws.

Although Cassirer's project will acquire a definite form only in a paper from 1907, i.e., *Kant und die moderne Mathematik* (Cassirer, 2001), in his *Leibniz's System* he is already at work on the emergence of a logic of relations intended for coping both with the autonomy of mathematics, now independent of intuition, and its being a means for determining the objects of physics. To Cassirer, this is the actual meaning of "synthetic" and the very matter of contention:

> As one sees, the thought that the *a priori* "synthesis" of concepts is, at the same time, the proof of the possibility of the objects, is reached. [...] "Causality", to which the A priori is bound, does not address relations of physical becoming; rather, in the concept of causal definition[15] it is the expression of a grounding methodical *lawfulness* (*Gesetzlichkeit*), which is found in mathematical concepts as condition of their objectivity. Therefore, also the A priori becomes the expression of a law that does not derive from things, but from consciousness (ibidem, pp. 114-115).

Limiting ourselves to the comparison between Russell and Cassirer, it is clear that they are at antipodes: while Russell was compelled to embrace both the reality of relations and *relata*, Cassirer steps back from such a realism and, on the contrary, strives to actualise the process of thought, not its products. That is, the product is meaningful only in light of the process from which it develops. There are thus neither formless data nor relations that conjoin these data. Accordingly, the analysis of concepts, which focuses on constituents, is not decisive for it does not tackle "the original act of judgement" (ibidem, p. 117) as the source of the relation itself.[16] And Leibniz's contribution can be expounded thus:

> "Truth" can be appended to ideas only in the metaphorical sense that, with this, it is understood the "truth of the acts of judgments", "which affirms the possibility of the objects of the idea".[17] In this value as criterion, the judgment attains the deepest meaning, in which it does not designate a link, but the spontaneous ideation (*Entwerfen*) of contents. It is only in such a spontaneity that the true idea distinguishes itself from representation and becomes expression of what is "exact" in knowledge (ibidem, p. 118).

As a consequence, it would be a simple inconsistency to assume that mathematical propositions are synthetic and that the analysis reveals something about the reality of terms and relations to us.

Here Cassirer's endeavour aligns with the Marburg neo-Kantian trend, which sees in the recovery of a certain Platonism the key point around which scientific knowledge must revolve. In adapting Plato's terminology to this neo-Leibnizian context, we may say that the point is to conceive of identical propositions as though they mandate the construction of the objects which they try to define. In this way, we ensure both the arbitrariness of definitions,

9 September 1901, Cassirer also manifested the intention to review Couturat's book with a focus on the comparison between "analytic" and "synthetic", which parallels the comparison between "logic" and "critique of knowledge".

15 For this concept, see Leibniz, 1890, p. 295.

16 For Cassirer, this is true even when we come to the definition of identity: indeed, we can conceive of the identity of concepts only when they are replaceable in all judgments in which they can surface as subjects (Cassirer, 1902, p. 131).

17 Cassirer cites Leibniz, 1882, pp. 377-379.

which stands but for the "freedom" of mathematicians (ibidem, pp. 120-121), and the fact that what we conceive of as real is always deduced from such positions. In Cassirer's words:

> We can thus assign an authentic "being" to a singly determined formation of intuition, when we have deduced it conceptually from the first synthetic positions. Nevertheless, these cannot be grounded in an absolute reality anymore, but only in the connection that they have with the immanent object of *knowledge*. The true possibility of the concept guarantees the application of the conceptual content to the object of nature (ibidem, p. 122).

In this type of idealism, we thus uphold: 1) the primitiveness of relations; 2) the production of *relata* according to the law that defines the relation; 3) the synthetic production of contents; 4) the maximum degree of creativity for the arbitrary choice of definitions.

I shall give but one very clear example on this. Cassirer refers to Leibniz's demonstration of arithmetical truths (Gerhardt, 1880, pp. 401-404; Gerhardt, 1882, p. 394 and ff.). In contrast with Russell, Cassirer explains that, for Leibniz, mathematical propositions can be shown to be synthetic. Let us prove the proposition that "two and two is four". Given the law of succession, according to which to count numbers we apply the rule $n + 1$, let us assume as definitions that "two is one and one", "three is two and one", and "four is three and one". By also leveraging the axiom of substitution and the associative principle, we may prove that: "two and two is two and one and one", "two and one and one is three and one", and finally "three and one is four". Since the original proposition is traced back to definitions, it is proved. Nonetheless, we should not be deceived: the truth of the proposition is produced by the definitions, it is not contained in them. Hence, mathematical truths are "the product of synthesis" (Cassirer, 1902, p. 133[18]).

Before coming to the general evaluation of the dispute, I now delve into two topics which were focal for Russell and allow us to tackle the relationship between mathematics and physics.

We have seen that Russell's rejection of internal relations involved the discussion on the concepts of point and force. The dichotomy that surfaced in virtue of internal relations vanishes in light of Cassirer's approach. Cassirer, in the wake of Cohen, conceives of points as genetic elements. Also exploiting Leibniz's identification of points with "situations", to wit, "localities" (Leibniz, 1768, p. 693), Cassirer pinpoints the criticism of the Aristotelian and Cartesian tradition, for which points seem to be determined as results of the analysis of extension. In practice, Cassirer states that the process which led from extension to points is inverted: by disclosing places, points are the origin of extension. No analysis can give a definition of what a point is. Accordingly, extension stems from the unextended (Cassirer, 1902, p. 147) and is synthetic. As is clear, the contradiction of relativity is not even pondered and terms, such as points, contain in themselves the possibility of difference.

To put it briefly, Cassirer thus believes in the *internality* of relations, and not in internal relations. Following this strategy, Cassirer establishes the primacy of topology over metrics by highlighting that spatial relations are prior to *relata*. Indeed, places are defined by the interplay of a given element with all others constituting a system of topological connections. Considering all members of such a system as constant except for one, we can imagine replacing the variable with another element. In this, nothing can be conceded to naïve realism: there are neither subjects which are extrinsic to relations nor relations are externalised as such. Cassirer even recalls the example on proportions from which Russell took Leibniz's incoherence about the mental character of relations. Perhaps unaware of the weight of his words, Cassirer states:

> The element presupposes the synthesis, not vice versa. In this sense, the relation utterly stands outside of subjects as something which is purely ideal.

18 Leibniz, for his part, acknowledges that the relationship between axioms and examples is twofold. It may happen that the example merely embellishes the content of the axiom; nevertheless, it is also possible that what is stated in the example is not subsumable under the universal principles embodied in the axioms (Gerhardt, 1882, pp. 393-394). More or less, this is the distinction between analytic and synthetic judgments in Leibniz's parlance.

> It is interesting to follow up on this development and with which difficulties the conception of a pure, devoid of subjects relation has to struggle. In fact, it remains paradoxical to measure it with the demands of language and its logic: naïve thinking, as it concentrates in linguistic categories, calls for each proposition fixed and given starting points. [...] As we must make clear, scientific method conversely rests on the opposite conception: we can gain being solely in the law, the object only in the synthesis (ibidem, p. 255).

In light of such statements, there remains only one question: how to shift from mathematical abstractions, which especially pertain to space and time as topological forms of order, to experience.

Let us begin with the concept of inertia. According to Cassirer, who refers to Leibniz's correspondence with De Volder, any fixity of the world of substances is overcome with this notion. Each "*situs*" of a body, having a given velocity, cannot be addressed as a mere point in the topological order of relations. Indeed, each of these states is determined in respect of every successive moment (Gerhardt, 1879, II, p. 258). Now, we cannot be content with the mere abstractness of geometry for in thinking of substance as the "*lawfulness of temporal succession*" (Cassirer, 1902, p. 280), we should focus on each state as it does not stem from the law of identity – the states of a substance change over time. Therefore, a state is not a predicate homologous to others and a series is not a subject. Rather, since the "conservation of change" (ibidem, p. 281) is brought into play, particulars are always subjects to law.[19]

For this reason, we do not stick to any substantial or thing-like *relatum*, but only to a "conceptual" determination according to which each and every state is tied to foregoing and successive ones. In short, each state is expressed through relations between changes and each present "now" is dispersed, so as to negate any realism of states or terms. Accordingly, if we simply conceive of possible relations between points, we will lack any criterion for establishing reality in physics; at the same time, the latter does not involve objects as mere things.

Therefore, if we entertained the idea that, given the primacy of relations, these would either transpose the empirical world into an "apparent" one – for it is impossible to denote any content directly – or transform themselves into a part of the real world, we would be wrong either way (ibidem, p. 294). The essential idea is that relations do not annihilate the singularity of particulars, but also that the latter is not presupposed. Cassirer thus points out that the only way to shift from the possibility to the reality of the particular is to endow the law of generation with an additional feature, which works, as it were, as a completion of inertia. This property is to be found in the passage from the primitive to derivative force: while the first addresses the "general causes" of phenomena, the latter delves into a particular state and shows how this contains the mark of the general law.

Bearing in mind that it is difficult to render Leibniz's thought in modern parlance since there is no term which corresponds to primitive force, we can however assert that it is but the function describing the simple mathematical representation of motion. In this sense, the function may still be subject to analysis since it gives a global but static representation of a dynamical system. Nevertheless, we shall assume the reality of change

19 Truth be told, the meaning of Leibniz's sentence from which Cassirer takes his thesis is slightly altered because of the translation of "*successio*" with "*Veränderung*" (change), which, in my opinion, bends the passage to Cassirer's interpretative purpose – that is, that of showing that logic naturally leans towards physics.

in considering force as variable from time to time. So, in each and every point we do not simply encounter the entire system as though it stopped at a given moment, but we will introduce variation which, in turn, would mirror an "external" action (ibidem, pp. 300-301). Therefore, after having limited analysis, we must now pay attention to the meaning of the word "external", and the question of whether what we will find can be assessed as "synthetic".

At first glance, it seems that Cassirer does not illustrate the point intensively. Essentially, we start from the situation in which there are different series of relations between phenomena. At the beginning, these series do not share anything. However, it is possible to suspend diversity and impose on different sets a common principle, which allows us to conjoin such heterogeneous fields. Leibniz would discover this principle in the concept of labour. Therefore, although phenomena remain qualitatively different, we can focus on the quantitative "equivalence" established by the commutation of equal causes into equal effects.

This sets Cassirer's anti-realistic claim: indeed, if we assume that senses always provide our knowledge with diversity, what we establish as homogenous makes phenomenal reality the only one upon which physics imposes its jurisdiction. In this way, Cassirer notes, relations are simply separated from subjects. They rely, as it were, on "a different *logical* dimension". Cassirer might not be any clearer than this: "In this deduction, force is truly the presupposition of homogeneity, although it is not homogeneous to the concrete physical phenomena, as little as the abstract number, which makes objects comparable, thereby becomes the object" (ibidem, p. 307).

Cassirer thus opposes to realism his *prescriptive* idealism. The concept of force can give a complex of "*latent judgements*"; together with the "postulate" of homogeneity introduced by labour, they tell us how to mould our experience. Hence, in Cassirer's own words: "The existence of determined equivalents for given processes must be shown empirically; but the general standpoint of comparison, which strives for the search for stable quantitative relations, is not per se *given* through experience. Starting from it, as we have seen, it is conversely reached via a complex of intellectual operations" (ibidem, p. 308). In this respect, it is thus synthetic *a priori*.

In order to get a completely active generation of contents, we should only avert the possibility that "receptivity" is elsewhere in the system. For this very reason, I conclude my exposition on Cassirer's interpretation of the mass point. In short, together with an active derivative force, we also have a passive derivative force. In fact, the construction of physical reality would be incomplete if we merely attained the description of the change of predicates over time: we also need to mean the unity of body as persistent through changes. However, this persistence is not, as it were, a piece of extension, and must be conceived of as a consequence of the law. Indeed, passive derivative force simply enucleates the countertendency to active derivative force, that is, the proclivity not to change of a state of motion, which we indicate with the symbol *m*.

Cassirer cites, at this stage, Leibniz's *Examen des principes de Malebranche* once again (Leibniz, 1768, p. 693 and ff.). In a nutshell, Cassirer explains that mass is created as an "intensive" magnitude. Indeed, just as extension stems from the "diffusion" of locality, mass arises out of the "diffusion" of materiality. Leibniz is very clear about the fact that the process is the same in both cases. For this reason, we should equally conclude that matter is embodied in the point as well as in the body, and that its diffusion can be

traced through a "material line" which is but the motion of a point mass, and whose results do constitute the volume of bodies, whereas the diffusion of unextended points gave birth to space. We may thus infer, at least as to *Leibniz' System*, that physical reality is related to calculus and the definition of the "integral" and the "differential" (Cassirer 1902, pp. 342-343).

In conclusion, considering that physical reality is but the interplay of different levels of relations,[20] there is no room left for an explanation endorsing the existence of particulars outside of relations. By the same token, relations are not real for they are the means by which to determine the real, not the real itself. In a sense, each act of determination, as it descends from a logic of relations, is thus synthetic. Nevertheless, once the *SP* structure is discarded, we should doubt whether the distinction between analytic and synthetic judgements is tenable. It is based, in fact, on the "categorial" logic of language, which is called into question by the mathematical foundation of reality.

4. When Russell and Cassirer were "Reviewers"

In the fourteenth footnote, I mentioned a letter that Cassirer sent to Natorp in which he promised to review Couturat's work and in particular to tackle the distinction between analytic and synthetic judgements. In the reviews appended to his *Leibniz' System*, it is nonetheless the enquiry into Russell's work which starts from this topic.

To begin with, I will recall Cassirer's definition of analytic and synthetic judgements. He points out that when the predicate is involved in the *concept* of the subject, judgments are analytic; when it is conversely set in the "existence" (*Existenz*) and "subsistence" (*Dasein*) of the subject, but is not derivable from the pure definition of the concept, judgements are synthetic (Cassirer 1902, p. 533).

Cut to the bone, Cassirer's idea is that we must not find it contradictory that Leibniz both upheld the analyticity of mathematical propositions and introduced the law of sufficient reason, and thus a synthetic principle, for contingents. Hence, he agrees with Russell that analysis conceals the synthetic evaluation about the compossibility of simple notions, but he does not find this to be ambivalent. As paradoxical as it might sound, Cassirer is in fact convinced that it would be incorrect to evaluate Leibniz's position through the comparison to Kant's later deliberation over analytic and synthetic judgements (see also ibidem, pp. VII-XI). Rather, it is confirmed that we encounter two trends:

> The decisive aspect of the contrast between synthesis and analysis consists of the emphasis which is put on the function of freely and constructively forming the content, in contrast with the mere *a posteriori* (*nachträglich*) analysis of something which is given. However, in his doctrine of the "causal" definition as condition of possibility of the mathematical object, Leibniz discovered and moulded this concept of "synthetic" (ibidem, p. 534).

In short, Cassirer's strategy may be stated thus: being aware that Kant's logic does not avoid the *SP* structure, he points out that anachronisms would be *a fortiori* misleading. Beyond this conjecture, it is clear that the contraposition entails, on the one hand, the synthetic capacity of establishing any relation between contents, which are thus but contents of thought; on the other hand, the presupposition that something *is given* and only *then* analysed. In this respect, it is easy to see that to overemphasise the second claim, as Russell does, means to introduce the

20 This assumption will become clear when Cassirer expounds his conception of the invariants of experience (Cassirer, 1953, p. 309 ff.).

naïve representation of substance as the simple correlate of a "thing" in the heart of scientific knowledge (ibidem, p. 537 and ff.).

Accordingly, Cassirer also tries to show that Russell's attempt at presenting Leibniz as always busy with the reduction of relations and synthetic judgements to the *SP* structure is a stretch. Indeed, for Cassirer, once the discovery of the ideal character of relations has been accomplished, it would be nonsensical to defuse it. In particular, this entails that empirical reality is actually understandable, as I explained, only in relational terms for it presupposes "the *necessity of the connection* in the appearance" (ibidem, p. 536). In practice, Cassirer pushes forward with the considerations about Leibniz's derivative force and the connection it allows between different states of motions over time, to affirm, against Russell, that this remains safe even though further enquiry into metaphysics will demand that we once again refer to the old-fashioned concept of substance.

This misunderstanding becomes manifest as to the relation of monads with space. We have seen that, according to Russell, it is impossible to conceive of an asymmetrical relation between mathematical points. However, in Leibniz, the relational conception of space is not antithetical to dynamics: they are simply different. The former, as we have seen, points to topology and simply refers to the order of possibility, where relativity and symmetry are admitted, while the latter encompasses the relations between states of motion, and thus asymmetry.

Therefore, Russell, who acknowledged Leibniz's familiarity with relations, to the point that he underlined that "Leibniz could have got rid [...] of the appeal to subject and predicate, and have substituted the unity of the law or series for that of the logical subject" (Russell, 2008, p. 48), would not have understood Leibniz's work as the *trait d'union* between the ancient, ontological meaning of substance, and the new logic of relations. In short, Russell lacked the historical awareness to perceive that, struggling with himself and tradition, Leibniz provided a "theory of *phenomena*" for which he assumed "a system of grounding relations that cannot be exhausted by the relation between subject and predicate" (Cassirer 1902, p. 537).

This very statement marks the distance between two different kinds of Platonism: the one, upheld by Russell, involves external relations and extrinsic *relata* as well; the other, defended by Cassirer, hinges on the Platonism of "saving the appearances".[21] In this latter case, relations are applied to "generate" (*erzeugen*) *termina*. We give definitions arbitrarily; we then derive from them the law. Each single element is thus lawful and cannot stand for an isolated thing. At the same time, it is singularised as that given element within a series of terms unfolded by the law.[22]

I now come to Russell's review. It was published in *Mind* in April 1903. Needless to say, it was also the year in which *The Principles of Mathematics* was published. It is thus not by chance that some criticisms raised in the review are pungent, to say the least, if compared with the positions of the book on Leibniz. In fact, although Russell maintains, as said, that mathematical propositions are synthetic, in the preface to the first edition he nonetheless assumes that logic is autonomous and that, if mathematics be reduced to it, then it is applied to "what exists" (Russell, 1951, p. XVII). Therefore, physical objects cannot be merely phenomenal. Rather, mathematics works, as it were, only insofar as existent objects share some general properties with mathematical ones. Perhaps, even in contrast to what was stated earlier about "semantic monism", we now need dualism between mathematical concepts and something which we must consider as separated from them (Heis, 2010). Conceptual deduction, that is "implication", thus involves "logical constants" and as regards their relation the real world may only participate. More clearly, on this kind of question it is only "the laboratory or the observatory" that makes a final decision (Russell, 1951, p. XVII).

21 With this expression, Cassirer aims to show that the content of physical knowledge is phenomenal in a Kantian sense (Cassirer, 1906, pp. 262 ff.).

22 This conception will eventuate in the doctrine of "concrete universality" defended in: Cassirer, 1953, p. 20 and ff.

With this in mind, it may appear strange that Russell wrote an incendiary comment on a passage of his copy of Cassirer's book (at 238), where Cassirer himself was discussing "induction" and the nature of experiment. Russell noted that: "You can't get over the notion that eternal truths are a kind of human excrement" (Russell, 1903, p. 535). Why, it should be asked, if logic deals with merely conceptual implications, are truths so "human"?

One supposition is that, to Russell, the idea that physical knowledge is only grounded in phenomena may enable both overstatement and misinterpretation of mathematics, which would have to perform too much. Russell's review opens, in fact, with the reference to a passage of Cassirer's work (Cassirer, 1902, p. 264), whereby the neo-Kantian philosopher explained that Kant's merit, in comparison with Leibniz, was to have turned a partial incongruence into a foundation: "Leibniz [...] says that the methods of knowledge, *though* ideal, are valid for the real: Kant's originality lay in turning *though* into *because* in this statement" (Russell, 1903, p. 551).

Russell later clarifies that the disentanglement of mathematics from empirical sciences, which would occur in the case of reduction of mathematics to logic, helps to protect mathematics from any violation or hybridisation. If am not mistaken, Russell seems to suggest that if we assume the neo-Kantian thesis that objects are generated through law, then we should think of an impact of empiricity on pure logic. He so relies on non-Euclidean geometry and current mathematics (Weierstrass, Cantor, Peano) to reinforce his view since it is clear that, in these cases, the transition from pure to applied mathematics demands the addition of supplementary assumptions, and does not touch pure mathematics.

Cassirer's approach would thus be trivial in particular for it overlooks the recent developments in analysis, and in particular the epsilontic method.[23] Simply put, the question is how to interpret the *dx* and its relation to the single "state" of motion in light of derivative force. We have seen that in Leibniz we should hold to the idea that we face inclinations and tendencies in determining the future, but Russell claims that this cannot be conceded: at present, we can solely maintain the concept of limit, not that of the infinitesimal. Therefore, Cassirer's interpretation is weakened by this metaphysical incongruence inherited from Leibniz. Provocatively, Russell even upholds, as he does in *The Principles*, that an arrow in motion is actually at rest at each and every moment (ibidem, pp. 557 ff.). And this bears witness to the profundity of change in Russell's mindset, who in his book on Leibniz even mentioned Cohen as to his explanation of Leibniz's idea that "the momentary increment was real in some way in which the whole sum of increments was not real" (Russell, 2008, p. 88).

Furthermore, the ambiguity encapsulated in the law of progression, as described by Cassirer, also intersects the definition of variable. Russell, in fact, explains that Cassirer's idea about what a term of a series is, is paradoxical. According to Russell, a variable is not something changeable, but a stable logical subject which stands for concepts such as "class", "any", etc. If not, the foundation of logic, which deals with implications, would be impossible since time would be involved in the definition of its objects (Russell, 1903, pp. 553 ff.).

However, as to the law of continuity, in Russell's view it seems nonsensical to focus on a single term of a relation, as the one implied in differential quotients. Rather, "the essential properties of continuity belong primarily to the relation, and belong to the terms composing its field not *qua* class of terms, but only *qua* field of a continuous relation" (ibidem, p. 554). As is clear, the realism of terms coexists with an utterly relationist position. Accordingly, Russell upholds that we should assume, on the one hand, the existence of terms which are "self-identical"; on the other, that the constancy of a law that determines changes in the values of a series is a constant relation, which

23 More or less, this is also Russell's criticism against Cohen (Russell, 1951, pp. 338 ff.). Of course, this point falls away when considering the further development of Cassirer's work, which encompasses, among other things, a critical evaluation of Cantor's transfinite arithmetic (Cassirer, 1953, pp. 54 ff.). In general, Cassirer later replaced Cohen's concept of the differential with the general relational form that he took from Russell's *Principles*, and declined it according to formalist theses (Oberdan, 2014).

is in turn expressed by "absolute timeless self-identity" (ibidem, p. 555).

In sum, as regards his review, and now according to Couturat,[24] Russell does not believe that to prove the possibility of a concept we need experience — as broadly as we can define it — , and that we should be content with the analysis of concepts, if this is complete. Surprisingly, there is nonetheless room for interpreting Russell's claim as reviving Kant's concept of the "synthetic". I will briefly show how in the next section.

5. An Exercise of Perspectivism: Who is Kantian?

It is now time to grant an evaluation of our results. I will do this by asking who among Russell and Cassirer is the real Kantian philosopher.

Apparently, this is not a smart question: Cassirer is the neo-Kantian philosopher, and Russell the one who repudiated idealism. Nevertheless, Russell's realism shares an important aspect with Kant's philosophy. In Russell's analysis there is something relatable to intuition. The point is fairly explicit in the review, and implicit in the book on Leibniz. As we have seen, a relation which holds between two terms urges us to consider the latter as existents. The point is that this claim alludes, above all, to the "acquaintance"[25] that we have with terms. In short, the meanings of terms "are as present to us as an object was for Kant, when given in sensible intuition" (Coffa, 1991, p. 98). Moreover, Russell will later express the idea that acquaintance also concerns the "universal truths" of logic (Russell, 1914, p. 78), thus reinforcing the conviction that primary definitions are available only for intuition (Russell, 2008, p. 171) and are synthetic.[26] Needless to say, this idea of "synthesis" differs from Cassirer's.

Considering Cassirer's criticism of the naïve conception of knowledge as a copy of reality, he could have never accepted such a view, as he would explain in his paper on Kant and mathematics (Ferrari, 1988, pp. 263 ff.; Heis, 2010). In fact, Russell's statements would be untenable if his logic of relations had to be applied. For Cassirer, such logic would be inconsistent with any claim on the reality of objects or things, and thus with each modality that transmits this kind of knowledge receptively.[27] From this, Cassirer's later criticism of Russell's acquaintance naturally follows (Cassirer, 1953, p. 316).

24 Nevertheless, the relationship between Russell and Couturat was not problem-free. In the first place, as we have seen, Russell's early position was inspired by the rejection of idealism, while Couturat was engaged in showing that Leibniz was the very source of the algebraic logic. As a consequence, Couturat shares with Cassirer's idealistic program the view about the importance of the *Characteristica universalis*, despite the fact that his panlogistic approach is unacceptable for Cassirer. In the second place, in his review, Russell corrects his standpoint and upholds that for Leibniz all truths are analytic, but he maintains that logic needs a synthetic foundation. The latter is derivable from the difference between "Universal Mathematics" and the "Logical Calculus", that is, from the existence of logical constants from which synthetic truths originate (Russell, 1903, pp. 546-547). On this point, see: Ferrari, 1988, pp. 257-261; Anfray, 2017.

25 The notion of "acquaintance" has been proved to animate even Russell's early theory of denotation, expounded in his *Points on Denoting* (1903). Since it is maintained throughout Russell's changes of mind, it is intended as a cause and not an effect of Russell's shifts on denoting (Wahl, 2007).

26 Hanna nonetheless noticed that this standpoint is in contrast with the "conceptualism" of the theory of descriptions (Hanna, 2001, chap. 4).

27 In Russell's proposal, this is the case of "*confined aboutness*", to wit, the idea that there is a "link between intuition and propositional knowledge" (Coffa, 1991, p. 103). Russell defends this stance in several ways, but for us suffice it to mention the famous passage from *The Principles*, where it is written of the possibility "to designate a thing which is not a concept". Therefore, "a man" is not the object of a proposition for this is an actual man walking the streets (Russell, 1951, pp. 53-54). In practice, Russell reintroduces existence as "position" (KrV, A592/B620 ff.), after having declared that the existence of an external world is merely a prejudice stemming from common sense. The contradiction was shown by Couturat, who admonished Russell in a letter sent on 4 October 1901. Couturat explained that, in Leibniz, existence cannot be conceived of as "position" since it is derivable from "essence", as well as the infinite analysis of contingents (see *supra*, foot. n. 7; Anfray, 2017, p. 179 and ff.). However, among Russell's scholars, it has been acknowledged that Russell's logicism is intended to broaden our knowledge by shedding light on the structures underlying scientific theories (Patton, 2017). Will that not align with Cassirer's structuralist program in the philosophy of mathematics?

Indeed, Cassirer thought that relations take place between elements of different series; therefore, we need relations of lower and higher sorts, in a way that does not allow us to conceive of any isolated "term".[28] However, by changing perspective, it may be also said that Cassirer's philosophy is analytic, for everything hinges on logical relations (Smart, 1943). In fact Cassirer defends a concept of "synthesis" which excludes that "*Erweiterungsurtheile*" imply an extra-conceptual reference to "intuition". With "synthetic", as I showed, he means the "production" of contents by pure thought (Ferrari, 1988, p. 266).

In a footnote to *Kant und die moderne Mathematik*, where he refers to Couturat, Cassirer argues:

> When he explains that all mathematical judgments are analytic since they are generated a priori and thus cannot involve but what the mind has put in themselves, then the misunderstanding becomes very clear. Indeed, just those acts of the original "putting in" (*Hineinlegen*) are what Kant — to whom we should at least recognise the right to fix and *designate* his problem — has progressively characterised as "synthesis" (Cassirer, 2001, p. 73, foot. n. 63).

In conclusion, depending on which aspect of Kant's philosophy we emphasise, our assessment of what is "analytic" and "synthetic" seems to change, together with our evaluation of Russell's and Cassirer's interpretations.

Bibliographic References

AMARAL, L. *Ernst Cassirer's* sui Generis *Logicism: On the Reception of the Logicist Thesis and its Role in* Substanzbegriff und Funktionsbegriff. *Cognitio-Estudos*, 14(2), pp. 186-198, 2017. DOI: 10.23925/1809-8428.2017v14i2p29-41.

ANFRAY, J.-P. 'Le Leibniz de Couturat et le Leibniz de Russell'. In: FICHANT, M., ROUX, S. (Eds.). *Louis Couturat (1868-1914). Mathématiques, langage, philosophie*, Paris: Garnier, pp. 161-188, 2017.

ARTHUR, R. T. W. *Leibniz*, Cambridge: Polity, 2014.

CASSIRER, E. *Leibniz' System in seinen wissenschaftlichen Grundlagen*, Marburg: Elwert, 1902.

CASSIRER, E. *Das Erkenntnisproblem in der Philosophie und Wissenschaft der neueren Zeit*, I. Berlin: Bruno Cassirer, 1906.

CASSIRER, E. *Substance and Function* (1910) *and Einstein's Theory of Relativity* (1921). Trans.: W. C. Swabey, M. C. Swabey, New York: Dover Publications, 1953.

CASSIRER, E. *Kant und die moderne Mathematik* (1907). In: SIMON, M. (ed.). *Gesammelte Werke*, Bd. 9, Hamburg: Meiner, pp. 37-82, 2001.

CASSIRER, E. *Ausgewählter wissenschaftlicher Briefwechsel.* In: KROIS, J. M. (ed.), *Nachgelassene Manuskripte und Werke*, Bd. 18, Hamburg: Meiner, 2009.

COUTURAT, L. *La logique de Leibniz d'après des documents inédits* (1901), Hildesheim: Olm,

28 Indeed, Cassirer's epistemology is holistic (Ihmig, 2001).

1961.

DI FRANCESCO, M. *Il realismo analitico. Logica, ontologia e significato nel primo Russell*, Milano: Guerini, 1991.

FERRARI, M. *Il giovane Cassirer e la scuola di Marburgo*. Milano: Franco Angeli, 1988.

GRIFFIN, N. 'What did Russell Learn from Leibniz?'. In: *Journal for the History of Analytical Philosophy*, vol. 2(1), pp. 1-11, 2013.

HANNA, R. *Kant and the Foundations of Analytic Philosophy*. Oxford-New York: Oxford University Press, 2004.

HEIS, J. 'Critical Philosophy Begins at the Very Point Where Logistic Leaves off: Cassirer's Response to Frege and Russell'.In: *Perspectives on Science*, 18(4), pp. 383-408, 2001.

IHMIG, K.-N. *Grundzüge einer Philosophie der Wissenschaften bei Ernst Cassirer*. Darmstadt: Wissenschaftliche Buchgesellschaft, 2001.

KANT, I. *Kritik der reinen Vernunft*. In: *Kant's Gesammelte Schriften*. „Akademieausgabe", Königlich Preußische Akademie der Wissenschaften, Berlin 1900 ff., Bd. III, Berlin: Reimer, 1911.

LEIBNIZ, G. W. *Opera omnia*. In: ERDMANN, B. (ed.). Vol. II, Geneve: De Tournes, 1768. In part.: a) *Examen des principes du Malebranche*, pp. 690-697, 1711.

LEIBNIZ, G. W. *Die philosophischen Schriften*. In: GERHARDT, C. I. (ed.), Bd. II, Berlin: Weissman, 1879. In part.: a) *Briefwechsel zwischen Leibniz, Landgraf Ernst von Hessen-Rheinfels und Antoine Arnauld 1686-1690*, pp. 1-138; b) *Briefwechsel zwischen Leibniz und de Volder 1698-1706*, pp. 139-284; c) *Briefwechsel zwischen Leibniz und des Bosses 1706-1716*, pp. 285-522.

LEIBNIZ, G. W. *Die philosophischen Schriften*. In: GERHARDT, C. I. (ed.). Bd. IV, Berlin: Weissman, 1880. In part.: a) *en Schreiben über* Judicium de argumento Cartesii pro existentia Die petito ab eius idea, pp. 401-404.

LEIBNIZ, G. W. *Die philosophischen Schriften*. In: GERHARDT, C. I. (ed.), Bd. V, Berlin: Weissman, 1882. In part.: a) *Nouveaux Essais*, pp. 39-509.

LEIBNIZ, G. W. *Die philosophischen Schriften*. In: GERHARDT, C. I. (ed.), Bd. VI, Berlin: Weissman, 1885. In part.: a) *Principes de la nature et de la grace, fondé en raison*, pp. 598-607; b) *Monadologie*, pp. 607-623.

LEIBNIZ, G. W. *Die philosophischen Schriften*. GERHARDT, C. I. (ed.), Bd. III, Berlin: Weissman, 1887. In part.: a) *Briefwechsel zwischen Leibniz und Remond 1713-1716*, pp. 597-678.

LEIBNIZ, G. W. *Die philosophischen Schriften*, GERHARDT, C. I. (ed.), Bd. VII, Berlin: Weissman, 1890. In part.: a) *De Synthesi et Analysi universali seu arte inveniendi et judicandi*, pp. 292-298; b) *Streitschriften zwischen Leibniz und Clarke 1715-1716*, pp. 347-440.

MAHNKE, D. 'Leibnizens Synthese von Universalmathematik und Individualmetaphysik'. In: *Jahrbuch für phänomenologische Forschung*, 7, pp. 305–609, 1925.

NATORP, P. *Die logischen Grundlagen der exakten Wissenschaften*. Leipzig und Berlin: Teubner,

1910.

OBERDAN, T. 'Russell's *Principles of Mathematics* and the Revolution in Marburg Neo-Kantianism'. In: *Perspectives on Science*, Vol. 22(4), pp. 523-544, 2014.

PATTON, L. 'Russell's Method of Analysis and the Axioms of Mathematics'. In: LAPOINTE, S., PINCOCK, C. (eds.). *Innovations in the History of Analytical Philosophy*, London, Palgrave, pp. 105-126, 2017.

RUSSELL, B. *An Essay on the Foundations of Geometry*. London: Cambridge University Press, 1897.

RUSSELL, B. *Our Knowledge of the External World as a Field for Scientific Method in Philosophy*, London: Allen & Unwind, 1914.

RUSSELL, B. *The Principles of Mathematics* (1903). London: Allen & Unwin, 1951 (repr. of the 2nd edition, 1937).

RUSSELL, B. *The Basic Writings of Bertrand Russell 1903-1959*. In: EGNER, R. E., DENNON, L. E. (eds.). London and New York: Routledge, 1961. In part.: *My Mental Development*, pp. 37-50.

RUSSELL, B. *The Collected Papers of Bertrand Russell*. Vol. 2. In: GRIFFIN, N., LEWIS, A. C. (eds.). *Philosophical Papers 1896-1899*. London and Boston: Unwin Hyman, 1990. In part.: a) *An Analysis of Mathematical Reasoning* (1898), pp. 163-238.

RUSSELL, B. *The Collected Papers of Bertrand Russell*. Vol. 3. In: MOORE, G. H. (ed.), *Towards "The Principles of Mathematics 1900-1902*. London and New York: Routledge, 1993. In part.: *Lebniz's Doctrine of Substance as Deduced from His Logic* (1899-1900), pp. 511-534.

RUSSELL, B. *The Collected Papers of Bertrand Russell*. Vol. 4. In: URQUART, A. (ed.). *Foundations of Logic 1903-1905*. London and New York: Routledge, 1994. In part.: *Recent Work on the Philosophy of Leibniz* (1903), pp. 535-561.

RUSSELL, B. *A Critical Exposition of the Philosophy of Leibniz* (1900). Nottingham: Spokesman, 2008.

SMART, H. 'Cassirer versus Russell'. In: *Philosophy of Science*, vol. 10(3), pp. 167-175, 1943.

WAHL, R. ''On Denoting' and the Principle of Acquaintance'. In: *Russell: Journal of Bertrand Russell Studies*, 27, pp. 7-23, 2007.