4-27-2023

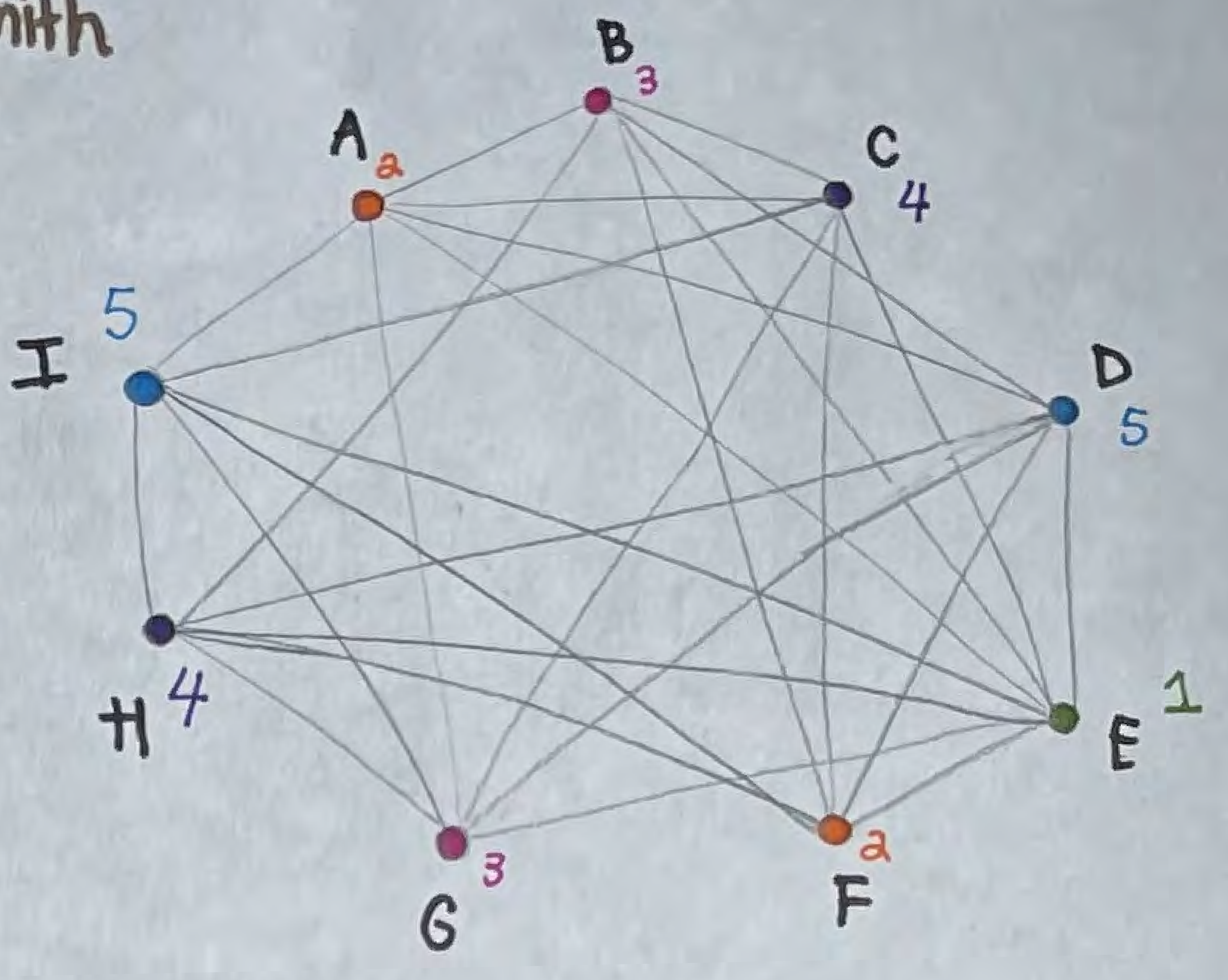
Total Dominator Coloring on the Queen's graph

Fiona Smith
College of Saint Benedict/Saint John's University, fsmith001@csbsju.edu

TOTAL DOMINATOR COLORING

By: Fiona Smith

A 2	B 3	C 4
D 5	E 1	F 2
G 3	H 4	I 5



DEFINITION

$\chi_d^+(Q_{m,n})$ = Total dominator coloring of Queen's graph

↓

proper coloring; each vertex is adjacent to all other colors

*Queen's graph: vertices are adjacent if they are in same row, column, or diagonal

GIVEN THEOREM (Lazuardi, et al)

for $2 \times n$ boards: for $n = 2,3$ $\chi_d^+(Q_{2,n}) = 4$

for $n \geq 4$ $\chi_d^+(Q_{2,n}) = n$

↓ ↓

CONJECTURE

for $3 \times n$ boards: for $n = 3,4$ $\chi_d^+(Q_{3,n}) = 5$

for $n \geq 5$ $\chi_d^+(Q_{3,n}) = n$

for $m \times n$ boards: for $m \leq m+1$ $\chi_d^+(Q_{m,n}) = m+2$

for $m \geq m+2$ $\chi_d^+(Q_{m,n}) = n$

- every "vertex" or "square" on the board has a network of common non-adjacencies
- this network is size m or less
- the total number of vertices divided by the size of the network yields n colors needed, at a minimum

$$n \text{ colors} = \frac{m \times n \text{ vertices}}{m \text{ vertices}}$$

$Q_{4,n}$ Cases

- $Q_{4,4}$: A nonadjacent to GHJLNO : 2
  - B HJKMOP : 5
  - C EJLMNP : 4
  - D EFIKNO : 3
  - E CDKLNP : 3
  - F DLMO : 1
  - G AINP : 2
  - H ABIJMO : 5
  - I BDGHOP : 5
  - J ACHP : 4
  - K BDEM : 3
  - L ACEFMN : 1
  - M BCFHKL : 1
  - N ACDEGL : 2
  - O ABDFHI : 5
  - P BCEGIJ : 4
- $Q_{4,5}$: A : 2, B : 3, C : 4, D : 5, E : 5, F : 1, G : 2, H : 3, I : 3, J : 4, K : 5, L : 1, M : 1, N : 2, O : 3, P : 4, Q : 4, R : 5, S : 1, T : 2

$Q_{3,n}$ Cases

- $Q_{3,3}$: A nonadjacent to F, H : 2
  - B nonadjacent to G, I : 3
  - C nonadjacent to D, H : 4
  - D nonadjacent to C, I : 5
  - E nonadjacent to none : 1
  - F nonadjacent to A, G : 2
  - G nonadjacent to B, F : 3
  - H nonadjacent to A, C : 4
  - I nonadjacent to B, D : 5
- $Q_{3,4}$: A : 1 FHKL, B : 5 GIJL, C : 3 DHJK, D : 4 CIK, E : 2 JL, F : 1 AGK, G : 5 BFL, H : 3 AC, I : 4 BDJ, J : 2 BCEI, K : 1 ACDF, L : 5 ABEG
- $Q_{3,5}$: A : 5 FHKLNO, B : 3 GIJLMO, C : 1 DHJKMN, D : 4 CIKNO, E : 2 JLMO, F : 5 AGKMN, G : 3 BFLN, H : 1 ACMO, I : 4 BDJN, J : 2 BCEIO, K : 5 ACDF, L : 3 ABEGM, M : 1 BCEFHL, N : 4 ACDFGI, O : 2 ABDEHJ
- $Q_{3,6}$: A : 5, B : 2, C : 4, D : 3, E : 1, F : 5, G : 2, H : 6, I : 3, J : 1, K : 4, L : 2, M : 6, N : 3, O : 1, P : 4, Q : 5, R : 6