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## Total Dominator Coloring on the Queen's graph

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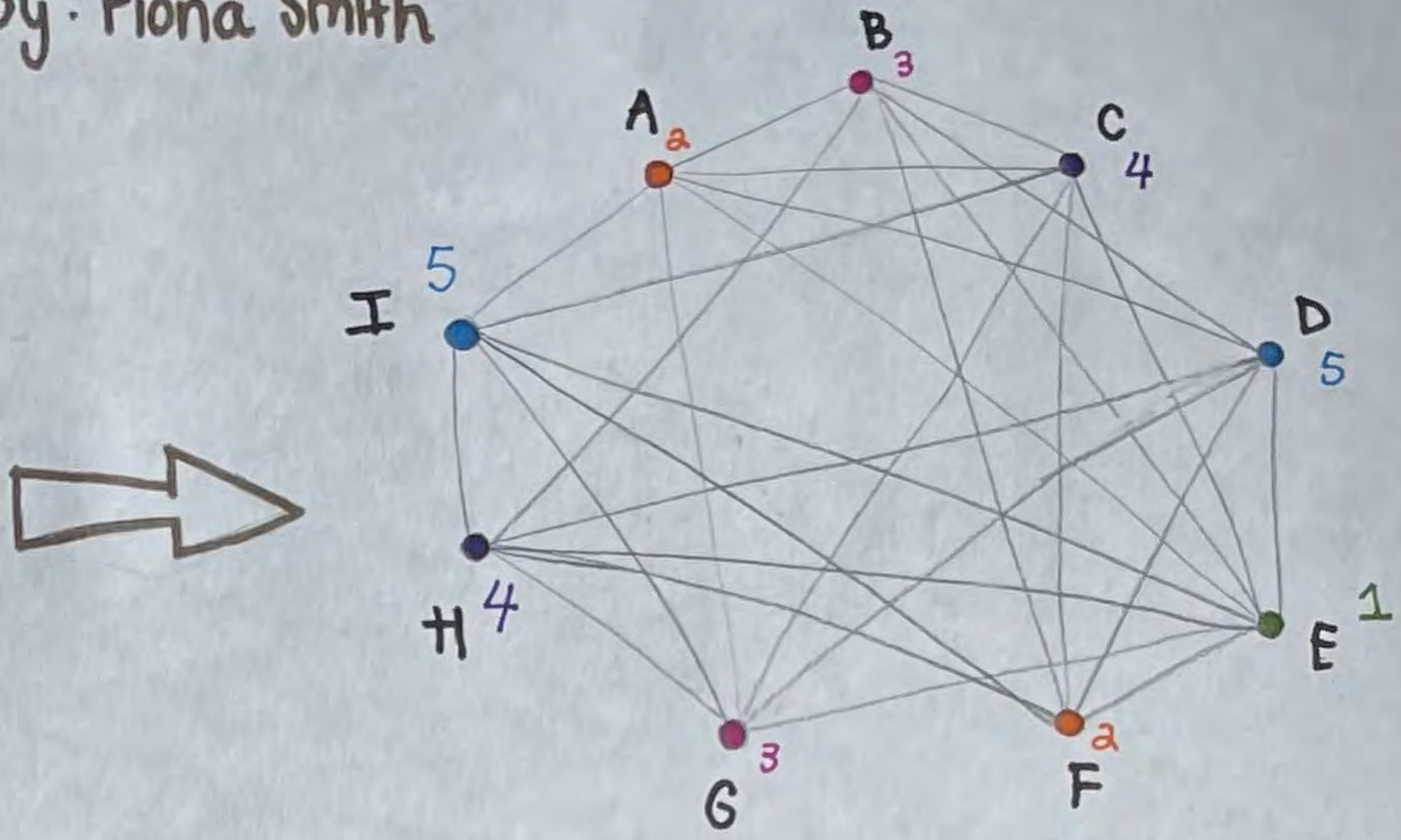
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# TOTAL DOMINATOR COLORING

By: Fiona Smith

A 2	B 3	C 4
D 5	E 1	F 2
G 3	H 4	I 5



## Q<sub>4,n</sub> Cases

- Q<sub>4,4</sub>: A nonadjacent to GHJLNO : 2  
 B HIKMOP : 5 I BDGHOP : 5  
 C ELMNP : 4 J ACHP : 4  
 D EFIKNO : 3 K BDEM : 3  
 E CDKLN : 3 L ACEFMN : 1  
 F DLMO : 1 M BCFHKL : 1  
 G AINP : 2 N ACDEGL : 2  
 H ABIJMO : 5 O ABDFHI : 5  
 P BCEGIJ : 4

- Q<sub>4,5</sub>: A: 2 I: 3 Q: 4  
 B: 3 J: 4 R: 5  
 C: 4 K: 5 S: 1  
 D: 5 L: 1 T: 2  
 E: 5 M: 1  
 F: 1 N: 2  
 G: 2 O: 3  
 H: 3 P: 4

## DEFINITION

$\chi_d^+(Q_{m,n})$  = Total dominator coloring of Queen's graph  
 ↓  
 proper coloring; each vertex is adjacent to all other colors  
 \*Queen's graph: vertices are adjacent if they are in same row, column, or diagonal

## GIVEN THEOREM (Lazuardi, et al)

for  $2 \times n$  boards: for  $n=2,3$   $\chi_d^+(Q_{2,n})=4$   
 for  $n \geq 4$   $\chi_d^+(Q_{2,n})=n$

## CONJECTURE

for  $3 \times n$  boards: for  $n=3,4$   $\chi_d^+(Q_{3,n})=5$   
 for  $n \geq 5$   $\chi_d^+(Q_{3,n})=n$

for  $m \times n$  boards: for  $m \leq m+1$   $\chi_d^+(Q_{m,n})=m+2$   
 for  $m \geq m+2$   $\chi_d^+(Q_{m,n})=n$

## Q<sub>3,n</sub> Cases

- Q<sub>3,3</sub>: A nonadjacent to F, H : 2  
 B nonadjacent to G, I : 3  
 C nonadjacent to D, H : 4  
 D nonadjacent to C, I : 5  
 E nonadjacent to none : 1  
 F nonadjacent to A, G : 2  
 G nonadjacent to B, F : 3  
 H nonadjacent to A, C : 4  
 I nonadjacent to B, D : 5
- Q<sub>3,4</sub>: A: 1 FHKL G: 5 BFL  
 B: 5 GIJL H: 3 AC  
 C: 3 DHJK I: 4 BDJ  
 D: 4 CIK J: 2 BCEI  
 E: 2 JL K: 1 ACDF  
 F: 1 AGK L: 5 ABEG
- Q<sub>3,5</sub>: A: 5 FHKNO F: 5 AGK MN K: 5 ACDF  
 B: 3 GIS LMO G: 3 BFLN L: 3 ABEGM  
 C: 1 DHT KNN H: 1 ACMO M: 1 BCEFHL  
 D: 4 CIK NO I: 4 BDJN N: 4 ACDFGI  
 E: 2 JLMO J: 2 BCEIO O: 2 ABDEHT

- Q<sub>3,6</sub>: A: 5 G: 2 M: 6  
 B: 2 H: 6 N: 3  
 C: 4 I: 3 O: 1  
 D: 3 J: 1 P: 4  
 E: 1 K: 4 Q: 5  
 F: 5 L: 2 R: 6

- every "vertex" or "square" on the board has a network of common non-adjacencies
- this network is size  $m$  or less
- the total number of vertices divided by the size of the network yields  $n$  colors needed, at a minimum

$$n \text{ colors} = \frac{m \times n \text{ vertices}}{m \text{ vertices}}$$