Benefits of Open-Mindedness in Vaccination Games on Models of Disease Transmission

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Introduction

Vaccination games are a tool to show mathematicians how behavioral changes in players of the game affect the outcomes of a disease outbreak. In a two-step vaccination game, the player first makes a decision whether or not to vaccinate, then the season occurs and the final results of infection are found. This model is based on the model [3], which explores how changing strategies in the game changes the outcome of the epidemic. The game is made using a general infectious disease, which can be changed on a case-by-case basis. This project focuses on how different efficacies and strategies can potentially lead to reaching "herd immunity."

Vaccination Games

In **vaccination games**, two or more players choose whether or not to vaccinate, with the goal of minimizing their cost. This cost can be getting the vaccine and side effects, or getting the virus in the infection season. The **Nash equilibrium** is a game theory concept which describes the case in which all players are satisfied with their decisions in the game and have no intention of changing their decision.

Background

The goal of the model is to try to reach the closest vaccination coverage to herd immunity as possible. The **herd immunity threshold** is the vaccination coverage needed to stop the disease from spreading. In the classical formulation of the vaccination game , we have that

$$V^* < V_{Nash} < V_{hit}$$

where V^* is the vaccination coverage, V_{Nash} is the Nash equilibrium, and V_{hit} is the herd immunity threshold.

Previous work: Xin et al (2019)

In [3], changes are made to the classical vaccination game to result in a higher value of V^*. One major change is the addition of a new parameter into the functions to account for more behavioral aspects of the model. Overall, this model shows that it is possible for V^* to be above the Nash equilibrium.

Methodology

Fermi Functions

Fermi functions show the probability of a player switching strategies. The classical version of the function comes from [1], which is:

$$p_{switch}(i \to j) = \frac{1}{1 + e^{-\beta(P_j - P_i)}},$$

where i is the player and j is the player being imitated. The model [3] uses modified Fermi functions that take into account the imitation of other players. Player i is free to change strategies to match player j's strategy. The equation is as follows:

$$p_{switch}(i \to j) = \frac{1}{\alpha + e^{-\beta(C(i) - C(j))}}$$

where α is a fixed number ($\alpha \geq 1$) and β is the strength of selection ($0 < \beta < \infty$) [3]. The addition of the α makes important changes to the model, which may lead to a n increased V^*.

What is "Open-Mindedness?"

Open-mindedness is the addition of the parameter α to the Fermi function. While β lends to the strength of perception when making a vaccination decision, α lends to the actual switching of probabilities. Β is simply how accurately the players perceive the cost of each decision per season [3].

This Project

For this new version of the model, there are some important changes to be made to the parameters:

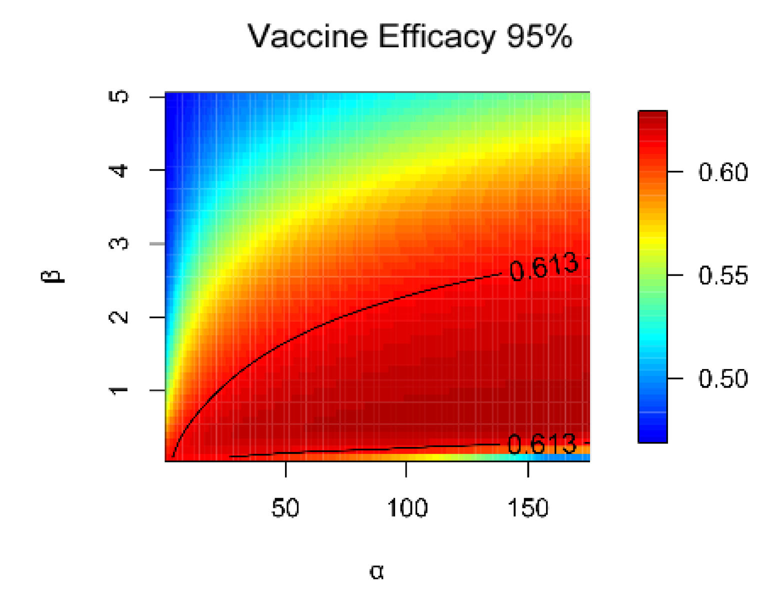
Changes to the Vaccine Efficacy. The efficacy of the vaccine, E, remains "perfect" in [3], meaning E = 1. In this model, E is lowered to represent a more realistic vaccine. Efficacy is changed to E = 0.95 and E = 0.9 in the results.

Changes to the Strategy. P represents the matrix of decisions based on what happened the previous season. The matrix is represented as four scenarios:

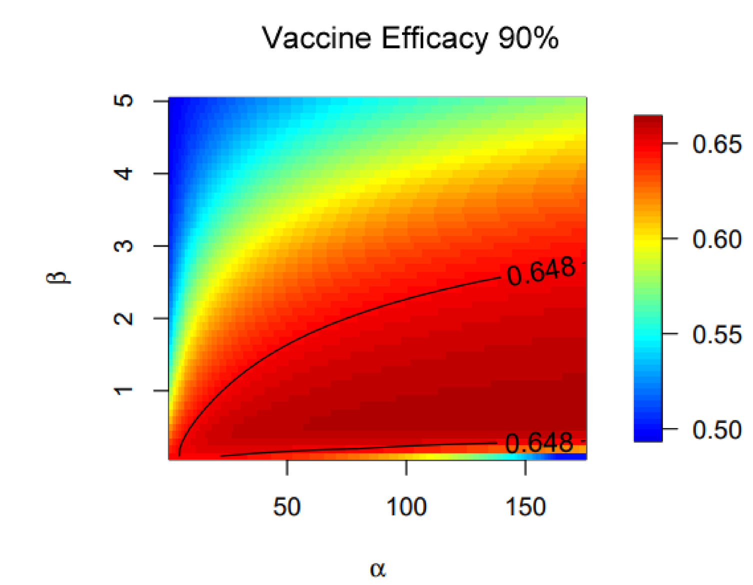
$$P = \begin{bmatrix} p_v^e & p_v^i \\ p_u^e & p_u^i \end{bmatrix}$$

Each value of P can be $0 \leq p \leq 1$. Increasing P means increasing the chance that the player will vaccinate.

Results

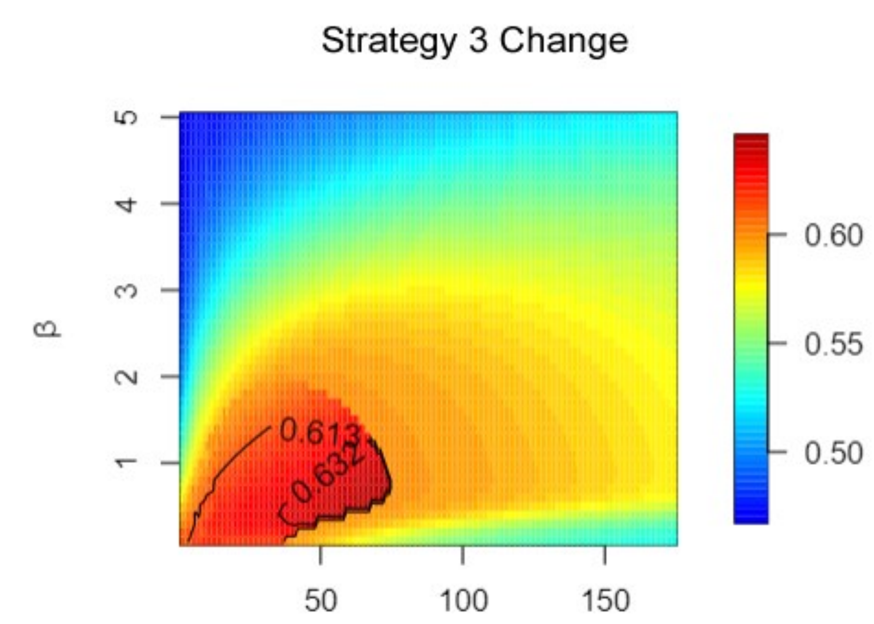


Surface graph of α and β, with E = 0.95, with final Nash equilibrium equal to 0.613. Code original-



Surface graph of α and β, with E = 0.9, with final Nash equilibrium equal to 0.648. Code originally from [3].

In this model, strategy 3 is set at P = [1, .3, 0, 0], meaning that the player will vaccinate in certain cases, being if the player escaped the virus and vaccinated (100%), or got infected and vaccinated (30%) the season before.



Surface graph of α and β, with E = 0.95, and strategy 3 P = [1, .3 , 0, 0]. Code originally from [3].

These results show that there are combinations that lead to $V^* < V_{hit}$, which shows the importance of the strategies.

Discussion

Significance of Results

As seen in the two changes in efficacy, the lower the efficacy, the higher the Nash equilibrium will be. However, changes in the efficacy do not qualitatively change the results.

The change of the strategy yields an interesting result. With this change, there exists a range of values for α and β in which the final size of the outbreak exceed the Nash equilibrium and the herd immunity threshold, i. e., $V_{Nash} < V_{hit} < V^*$.

Societal Impact

This model, with the added element of open-mindedness , is an impactful way of observing human behavior through mathematics. Models of infectious disease by themselves do not consider the behaviors and connections between hosts, while [3] considers the three types of behaviors observed when planning to get a vaccine. Vaccine studies are extremely vital to the current on-going pandemic, and will continue to be studied in the future, in the hopes of making more accurate portrayals of human decision making.

Conclusion

With the power of vaccination games, [3] makes it possible to change parameters to explore the outcomes of many different changes. The results show that even slight changes to parameters such as the efficacy can make significant changes to the vaccination equilibrium. The results we have here show that there are combinations of α and β for which $V^* > V_{hit}$.

References

1. Fu et. al. (2011). "Imitation dynamics of vaccination behaviour on social networks." Proceedings of the Royal Society B, 278, 42-49.

2. Just, Winfried (2021). *What is behavioral epidemiology and what does math have to do with it*? Ohio University.

3. Xin, et. al. (2019). "Open-minded imitation can achieve near-optimal vaccination coverage." Journal of Mathematical Biology, 79, 1491-1514.