CG-SCMA Codebook Design Based on Maximized Euclidian Distance

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Abstract — Sparse code multiple access (SCMA) is a multidimensional codebook based on a class of non-orthogonal multiple access (NOMA) technologies enabling the delivery of non-orthogonal resource elements to numerous users in 5G wireless communications without increasing complexity. This paper proposes a computer-generated sparse code multiple access (CG-SCMA) technique, where the minimum Euclidian distance (MED) of a star 16-point quadrature amplitude modulation is maximized by CG-SCMA, thus creating a complex SCMA codebook based on optimizing the difference between the first and other radiuses over rotated constellations. To specify the most suitable values for this constellation, it is divided into four sub-constellations using trellis coded modulation (TCM) in an effort to optimize MED. The new codebook has four sub-constellations with MED values of 3.85, 2.26, 2.26, and 3.85, respectively. Application of the message passing algorithm (MPA) ensures low complexity of the decoding process.

Keywords — codebook, Euclidian distance, MPA, multiple access, sparse code.

1. Introduction

The fifth generation (5G) of mobile communication systems as well as other important technologies, such as the Internet of Things, spread quickly throughout the world, combining various approaches and services to ensure high-throughput, mass-scale connectivity of contemporary networks [1], [2].

A novel code-domain non-orthogonal multiple access (NOMA) technique, referred to as sparse code multiple access (SCMA), has been created to allow for future massive machine-type communication networks. Multiple users in SCMA are distinguished by being assigned with unique sparse codebooks (CBs) serving as an upgraded version of code division multiple access (CDMA). The message passing algorithm (MPA), making use of the sparsity of CBs to produce error-free performance approaching the maximum potential of the receiver, is used at the receiver to perform efficient multiuser identification. To the best of our knowledge, despite multiple research initiatives undertaken in recent years, a thorough tutorial of SCMA covering the background, its fundamental concepts and new advancements, is still to be revealed [3].

The main contributions of this work can be summarized as follows:

- in contrast to the existing CG-SCMA codebook design method, this work uses a dedicated software to optimize the difference values between the first and other radiuses over the rotated constellations,
- a modified CG-SCMA symmetric codebook architecture is suggested to increase the MinED of star 16-QAM by employing TCM to subdivide the constellation,
- the proposed SCMA codebook is designed to maximize the space between star 16-QAM constellations, so that BER is decreased along with the increase in SNR.

2. Related Work

Several papers may be found in the literature concerned with the process of designing an efficient SCMA scheme. Kosasihet *et al*. [4] proposed a low-complexity detector technique for SCMA that is capable of addressing difficulties associated with MPA-SCMA and allows for the use of SCMA in very large MU-MIMO systems. Lou *et al*. [5] proposed a procedure for combining data bits in QAM symbol mapping and spreading, while providing direct mapping of bits to codewords. Liu *et al*. in [6] generated an optimized SCMA codebook by dividing round 16-QAM into several subsets to maximize the MinED. Tabra *et al*. [7] worked with star QAM constellations and proposed an improvement in MinED values using trellis coded modulation, while Chen and Chen [8] created a minimum complexity algorithm for nearly ideal CBs. In [9], Hussain *et al*. examined the effectiveness of various strategies of codebook generation and difficulties of SCMA optimization in real-world applications, across various applications. Zhang *et al*. [10] reviewed selected codebook designs and available SCMA detectors after presenting a comprehensive survey on the state-of-the-art in SCMA and of its fundamental principles. They also used the less complicated max-log-MPA approach instead of the MPA techniques. Waghmare *et al*. in [11] presented an effective method for designing and assigning the SCMA joint codebook based on the optimization of a 32-QAM constellation with large MinMED, high channel capacity and low detection complexity in the downlink network. In [12], Lin *et al*. developed a DL-SCMA deep neural network (DNN) to decode SCMA-modulated signals contaminated by additive white Gaussian noise (AWGN). In [13], Mohamed and Abdullah suggested a new SCMA codebook design method based on chaotic Arnold's cat map interleaving, characterized by a lower degree of complexity compared with the traditional method of interleaving.

3. SCMA Codebook Design

SCMA uses a codebook similar to the signature matrix relied upon in the low-density signatures (LDS) technique. It uses spreading to allow multiple users to share a single resource block. In NOMA, overloading is permitted when factor S/K exceeds 1. Consequently, K resource block (RB) accessed from S sources competes with the $K > S$ rule. The following equation may be used to determine the maximum number of sources for a given number of resources [6], [7], [14]:

$$S = C_k^{dv} = \frac{K.K-1\ldots K-dv+1}{dv.dv-1\ldots 1}\ , \tag{1}$$

where dv represents the number of resources that may be utilized by a single user.

For each user, $\log_2(L)$ bits are transmitted by a K-dimensional complex codebook with L codewords. The expression for the signature matrix for $S=6$ users sharing $K=4$ resource blocks is:

$$\mathbf{F} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}. \tag{2}$$

The amount of non-zero items in each column of the usual SCMA signature matrix (dv) refers to RB as a function node (FN). The number of non-zero elements in each row (df) refers to the number of users that each RB can serve, identifying each user node as a variable node (VN). Matrix (2) is used to map $\log_2(L)$ bits to a K-dimensional complex SCMA codebook with size L. To create the sparser codebook, the number of non-zero values in a codeword dv should be as low as possible, such that $dv < K$ [6], [7], [14]. For the purposes of codebook generation, constellation shaping gain and receiver complexity reduction, SCMA uses multi-dimensional constellations [8]. Even when two users share one resource block, the other resource block used is capable of distinguishing between them. The design of the codebook allows to reduce the number of constellation points used. This leads to lower complexity, hence log-MPA can be used for decoding data at the receiver side [7], [14]. Due to the sparse property of the SCMA codebook, the log-MPA technique can be applied at the receiver to reduce complexity while maintaining the desired performance.

3.1. The Proposed CG-SCMA Codebook Generating Method

To serve a lot of users while relying on a reasonable amount of resources, SCMA was developed – an approach that combines both modulation and multiple access and is capable of operating in a MIMO environment [6], [7], [14]. In a situation in which multiple users require data at the same time, but there are only a few resource blocks available, multiple access techniques allow to coordinate access to these resources by numerous users. Modulation and MA are combined in the proposed SCMA codebook to provide users with access to a small number of resource blocks.

To allow this, an SCMA complex constellation codebook is created by a dedicated algorithm that maximizes the minimal Euclidian distance for better decoding results. Demodulation and demultiplexing are performed at the decoder using the log-MPA block and modified CG-SCMA. As a rule, the number of users in OMA ought to be lower or equal to the number of RBs, but overloading is permitted with NOMA, by a factor of $J/K > 1$. Therefore, there is competition among J mobile sources to provide K with access when $J > K$.

The proposed CG-SCMA scheme is characterized by the following parameters: J, K, L, df, $dv = 6,\ 4,\ 4,\ 3,\ 2$, respectively, and the proposed factor graph is shown in Fig. 1 [6]. Each source can access only dv RBs, with a non-zero dv value in the codeword. Two users are allowed to share two RBs. This is done based on the $\mathbf{F}$ matrix in Eq. (2) that uniquely maps each user to the RBs, where the set of the resources that may be accessed by each user is represented by "1" in $\mathbf{F}$. The star QAM constellation design is generated by extending the standard 16-QAM to four rings, as presented in Fig. 2 [6], [7].

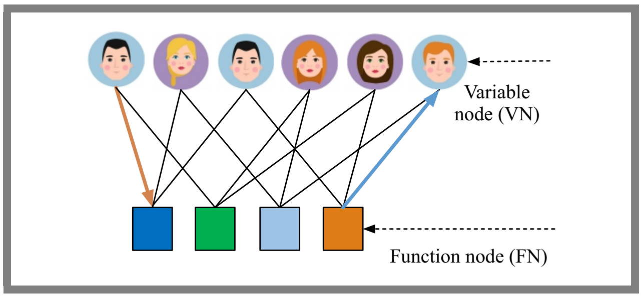

Fig. 1. Factor graph for $J = 6$ and $K = 4$.

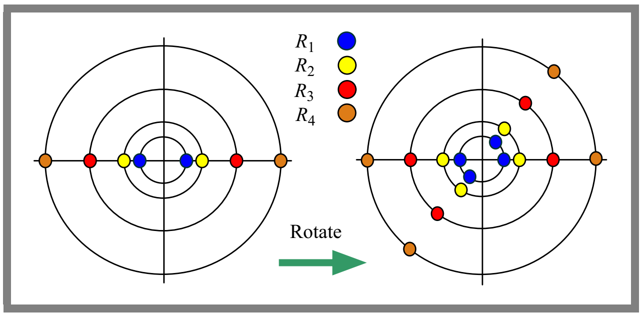

Fig. 2. 16-QAM modulation constellations [6], [7], [14].

The proposed CG-SCMA method follows the procedure presented below.

Let $R_1 = 1$, as proposed in [6], [7].

Let D_1 be the difference between R_2 and R_1, and D_2 be the difference between R_3 and R_1 as:

$$D_1 = R_2 - R_1\ , \tag{3}$$

$$D_2 = R_3 - R_1\ . \tag{4}$$

Let us define the constellation's ratio between two successive points as $\frac{R_2}{R_1} = \frac{R_4}{R_3}$, which is called β [6], [7], and the ratio between non-successive points as $\frac{R_3}{R_1} = \frac{R_4}{R_2}$, which is

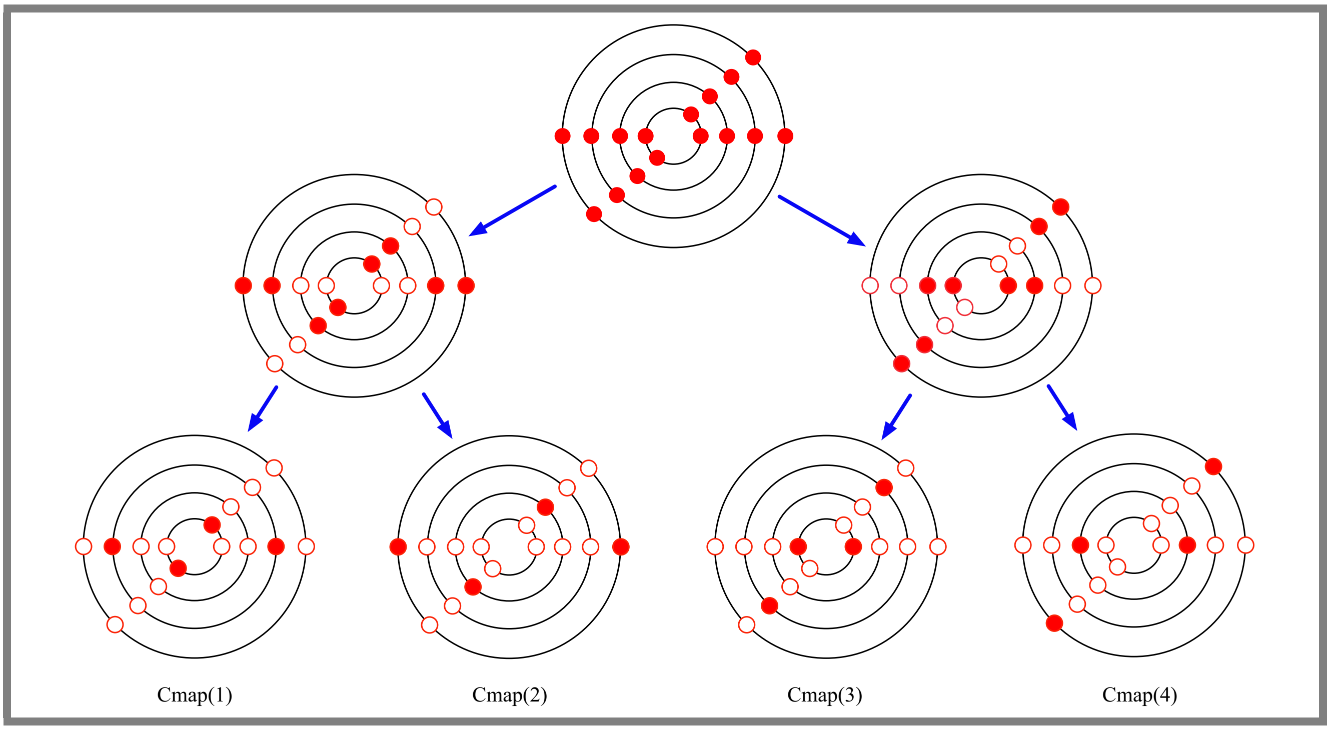

Fig. 3. The mother 16-QAM constellation is divided into four smaller constellations.

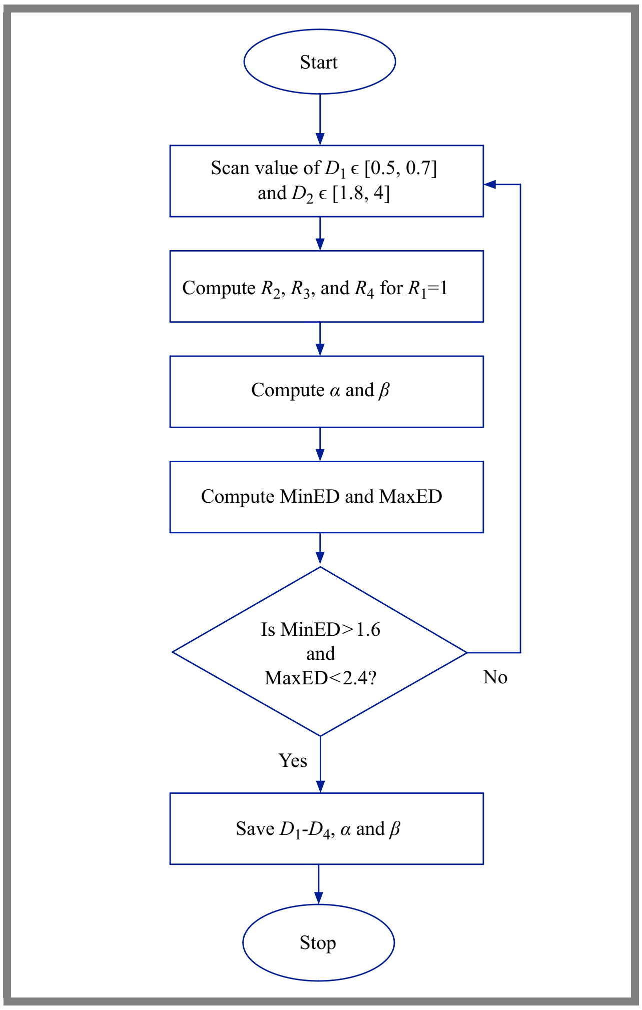

Fig. 4. Flowchart of the proposed CG-SCMA method.

called α. These ratios are then utilized to create the mother codebook [6], [7]. According to Eq. (4), $R_2 = D_1 + R_1$ and $R_3 = D_2 + R_1$, to keep the ratio of α and β constant $R_4 = \frac{R_3 R_2}{R_1}$. The next step is to search for $D_1 \in [0.5, 0.7]$ and $D_2 \in [1.8, 4]$ and then to determine α and β that achieved the minimum Euclidian distance (MinED) and the maximum Euclidian distance (MaxED) using the following formulas:

$$MinED = \frac{d_{\min}}{\sqrt{E}} = \sqrt{\frac{(R_1 - R_1\cos\emptyset)^2 + (R_1\sin\emptyset)^2}{\frac{1}{4}(R_1^2 + R_2^2 + R_3^2 + R_4^2)}}$$

$$= \sqrt{4\frac{2 - 2\cos\emptyset}{(1+\alpha^2)(1+\beta^2)}}, \tag{5}$$

$$MaxED = \frac{d_{\max}}{\sqrt{E}} = \sqrt{\frac{4R4^2}{\frac{1}{4}(R_1^2 + R_2^2 + R_3^2 + R_4^2)}}$$

$$= \sqrt{16\frac{\beta^2}{1+\beta^2} \cdot \frac{1}{1+\frac{1}{\alpha^2}}}, \tag{6}$$

where $\emptyset = \frac{\pi}{L.dv}$ refers to the rotation angle. If MinED is greater than 1.58 and MaxED is lower than 2.4 [6], then these values are selected to generate the mother codebook, or are else used to select another value for D_1 and D_2.

Using the obtained values, the following is a representation of the suggested mother codebook design relying on symmetric mapping [6], [7].

$$\text{Mother codebook} = \begin{bmatrix} a & b & -a & -b \\ c & d & -c & -d \\ e & f & -e & -f \\ g & h & -g & -h \end{bmatrix}. \tag{7}$$

where $a = R_3, b = R_1 \cos\emptyset, c = R_4, d = R_2 \cos\emptyset, e = R_1$, $f = R_3 \cos\emptyset$, $g = R_2$, and $h = R_4 \cos\emptyset$.

Partitioning the star 16-QAM constellation using trellis coded modulation subsets allows to obtain the users' codebooks, as shown in Fig. 3. User codebooks are generated by substituting Cmap(i) of the i-th subset of the mother codebook constellation for each 1 in the $\mathbf{F}$ matrix in Eq. (2), thus resulting in a new $\mathbf{F}'$ matrix, as presented in Eq. (8) [6], [7]:

$$\mathbf{F}' = \begin{bmatrix} \mathrm{Cmap}(2) & \mathrm{Cmap}(1) & \mathrm{Cmap}(4) & 0 & 0 & 0 \\ \mathrm{Cmap}(1) & 0 & 0 & \mathrm{Cmap}(3) & \mathrm{Cmap}(2) & 0 \\ 0 & \mathrm{Cmap}(2) & 0 & \mathrm{Cmap}(4) & 0 & \mathrm{Cmap}(3) \\ 0 & 0 & \mathrm{Cmap}(1) & 0 & \mathrm{Cmap}(3) & \mathrm{Cmap}(2) \end{bmatrix} . \tag{8}$$

Figure 4 shows the flow chart utilized to identify the ideal constellation points needed to create the CG-SCMA mother codebook [6], [7].

4. SCMA Encoder-Decoder Design

The method for encoding and multiplexing data for transmission and using a mother codebook to create user codebooks is illustrated in Fig. 5. A specific codebook is assigned to each user based on sparse matrix $\mathbf{V}$. When all six users provide data concurrently, information bits from user 1 through 6 are relayed in the order shown in Fig. 6 [11]. The user-specific codebook is added to the four time-frequency resource blocks currently in use. A single user owns each codebook, which is a 4×4 complex matrix. Each user's codebook contains four codes [11]. Users that need to send their encoded data should construct codewords by multiplying their codebooks with encoded data. Finally, all users' codewords are multiplexed and sent to the destination [9], [14]. To guarantee good performance of the SCMA design, the potentially highest minimum Euclidian distance needs to be obtained. The general formula for estimating MED is:

$$MED = (\min \text{ or } \max) \, ||x - y|| \, , \\ \forall \, x, y \in \text{mother codebook and } x \neq y \, . \tag{9}$$

4.1. SCMA Decoder

The message passing algorithm (MPA) is an iterative parallel decoding method that works by moving extrinsic data from function nodes (FNs) to variables nodes (VNs) and vice versa [9], as shown in Fig. 1. Each FN of the factor graph computes its outgoing message to a particular VN in each iteration based on the incoming messages from the other VNs. Then, each VN responds with a different message in response to the FNs' reminder messages. The cycle is finally declared complete when one outgoing message has traveled over each edge, in both directions. The log likelihood rates (LLRs) of each coded bit are determined after many iterations to estimate the bits of each user. Algorithm 1 illustrates the three basic processes that make up the MPA [1] for an SCMA system of J users employing M constellation points through K orthogonal R resources.

Algorithm 1. Message passing algorithm

Input: $y, N_0, C_j, h_j, j = 1, \ldots, J$

Result: Calculate each user's coded bit value for each user j, users are referred to as variable nodes (VNs), and resources or subcarriers are referred to as function nodes (FNs),

$U(k) = \{j, \ 1 \leqslant j \leqslant J | \ \mathrm{VN}_j \text{ is connected to } \mathrm{FN}_k\}$,

$R(j) = \{k, \ 1 \leqslant k \leqslant K | \ \mathrm{FN}_k \text{ is connected to } \mathrm{VN}_j\}$.

Step 1. Initialization.

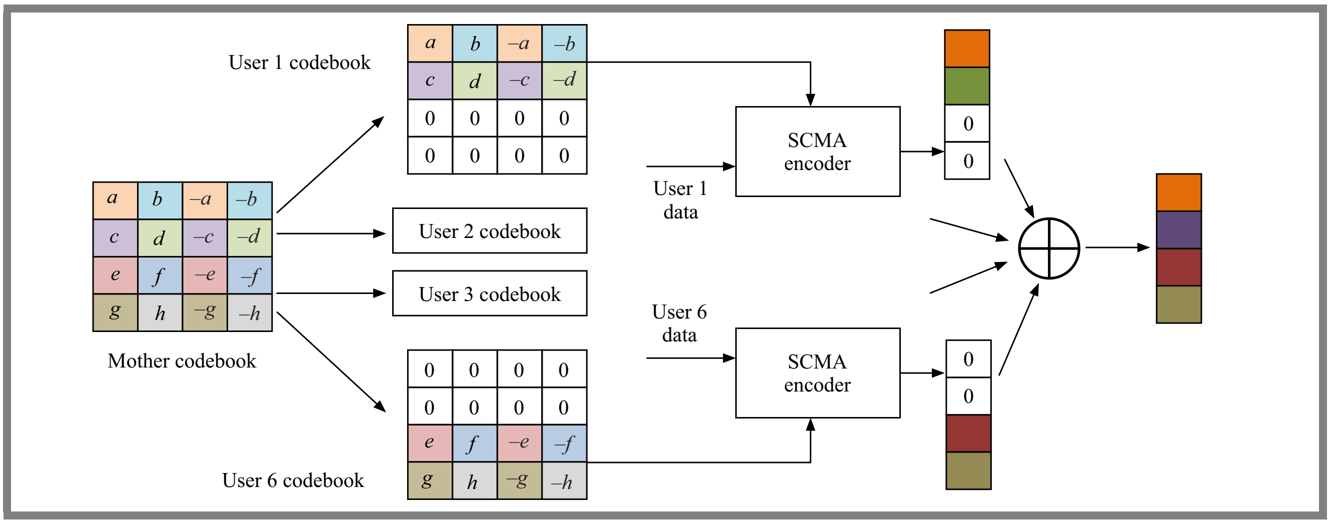

Fig. 5. SCMA encoder scheme.

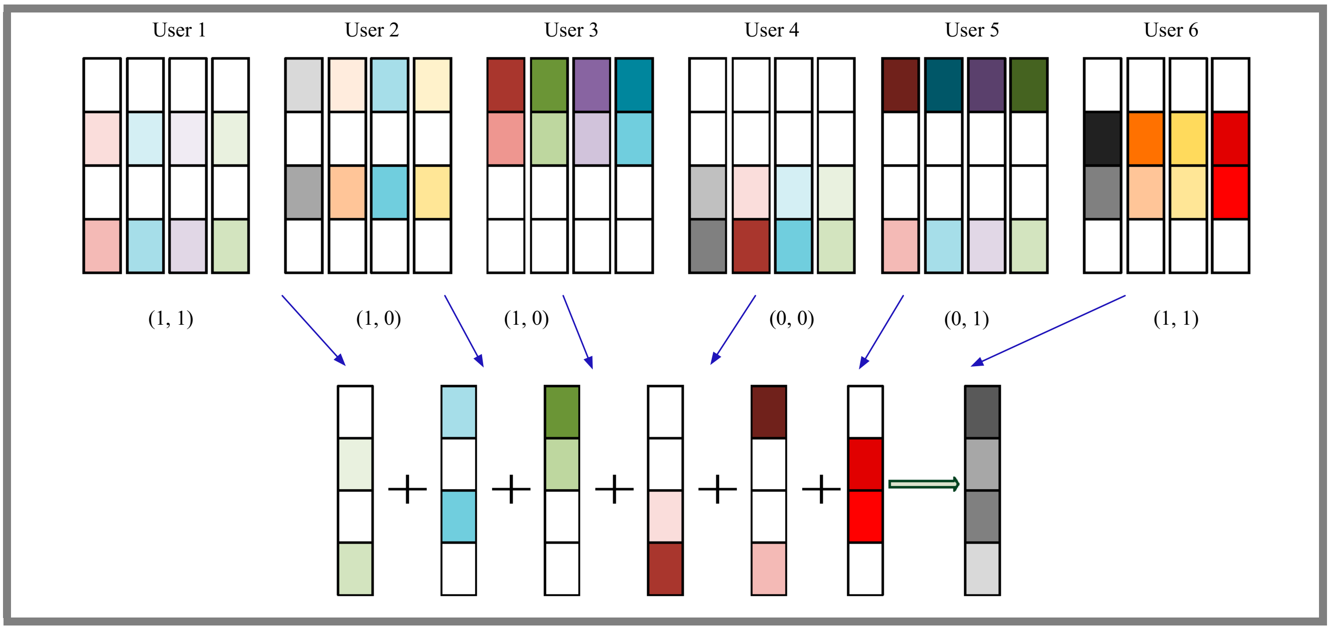

Fig. 6. SCMA encoding principle.

Each codeword's prior probability is given by:

$$V^0_{j\to k}(x_m) = P(x_m) = \frac{1}{M},\ j = 1, \ldots, J,\ k \in R(j)\,.$$

Step 2. Iterative message passing along edges.

While N_{iter} **do**

FN update. The message from FN_k is to be passed to one of its neighbors VN_j, $k = 1, \ldots, K,\ j \in U(k)$.

For a given codeword $x_m \in C_j$, $m = 1, \ldots, M U^t_{j\to k}$ is:

$$U^t_{j\to k} = \left\{ \sum_{x_i | i\in U(k)\backslash j} \mathrm{e}^{-\frac{1}{N_0}} \left\| y_k \right.\right.$$

$$\left.\left. - \sum_j h_{j,k} x_{m,k} \right\|^2 \right\} \prod_{i\in U(k)\backslash j} U^{t-1}_{i\to i}(x_m).$$

VN update. The VN_j has a message that should be delivered to one of its neighbors FN_k, for a given codeword $m \in C_j$, $V^t_{j\to k}(x_m)$ is given by:

$$V^t_{j\to k}(x_m) = \frac{\prod_{i\in R(j)\backslash k} U^{t-1}_{i-j}(x_m)}{\sum\limits_{x_l\in C_j} \prod_{i\in R(j)\backslash k} U^{t-1}_{i\to j}(x_l)}\,.$$

To maintain the method numerically stable, normalization is required.

End while

Step 3. Decision on LLR at each VN.

For each layer j the *a posteriori* probability of codeword x_m is defined as:

$$P(x_m) = \prod_{k\in R(j)} U^{N_{iter}}_{k\to j}(x_m)\,.$$

Each coded bit's log-likelihood rate bi, where $1 \leqslant i \leqslant \log_2(M)$ is represented by:

$$\mathrm{LLR}(bi) = \log \frac{\mathrm{P}(bi = 0)}{\mathrm{P}(bi = 1} = \log \frac{\sum\limits_{\{x_m\in C_j | bi=0\}} \mathrm{P}(x_m)}{\sum\limits_{\{x_m\in C_j | bi+1\}} \mathrm{P}(x_m)}\,.$$

The value of each bit's LLR is determined as follows:

$$bi = \begin{cases} 1 & \text{if } \mathrm{LLR}(bi) \leqslant 0 \\ 0 & \text{otherwise} \end{cases}\,.$$

5. Simulation Results

The best values for the star 16-QAM constellation are determined by using dedicated software. The program outcome is that the best values of D_1 and D_2 produce the optimal ratio values α = 3.153 and β = 1.70. These ratio values result in MinED = 1.60 and MaxED = 2.36 for the mother codebook. According to these ratio values, the four constellation point values are found to be R_1 = 1.00, R_2 = 1.70, R_3 = 3.15, and R_4 = 5.36, respectively. Based on these inputs, the mother codebook is:

$$\begin{bmatrix} 3.15 + i0.00 & 0.92 + i0.38 & -3.15 + i0.00 & -0.92 - i0.38 \\ 5.36 + i0.00 & 1.57 + i0.65 & -5.36 + i0.00 & -1.57 - i0.65 \\ 1.00 + i0.00 & 2.91 + i1.20 & -1.00 + i0.00 & -2.91 - i1.20 \\ 1.70 + i0.00 & 4.95 + i2.05 & -1.70 + i0.00 & -4.95 - i2.05 \end{bmatrix}$$

MinED is increased after the division of the main constellation into sub-constellations. After performing TCM, the four sub-constellations have achieved MinED = $\{3.85,\ 2.26,\ 2.26,\ 3.85\}$ – see Fig. 7.

Two sub-constellations are assigned for each user to generate their codebooks. The improvement in MinED is higher than that of the sub-constellation proposed in [6]–[8]. Table 1 summarizes the obtained values of MinED, MaxED, D_1, D_2 parameters for the proposed CG-SCMA and so-

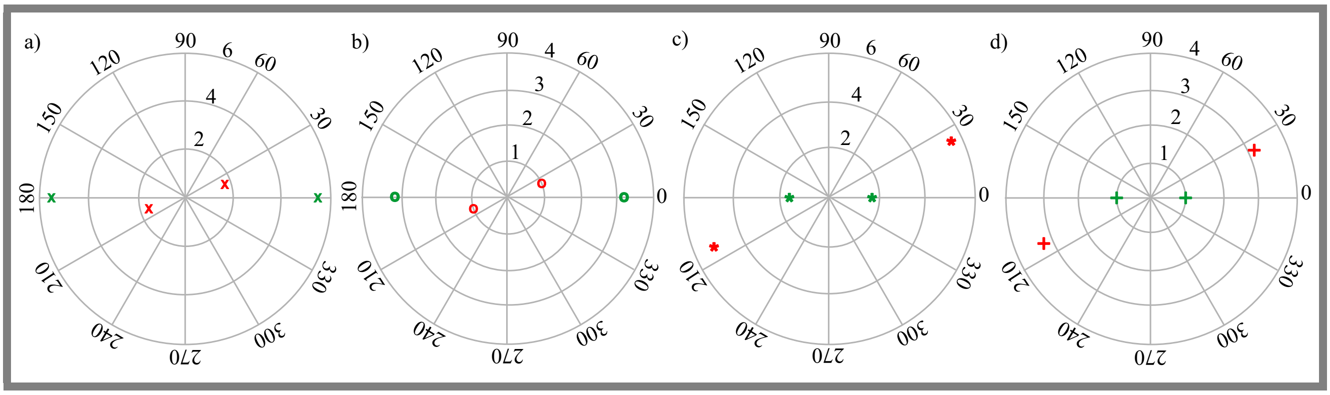

Fig. 7. Users' sub-constellations for: a) Cmap(1), b) Cmap(2), c) Cmap(3), and d) Cmap(4).

Tab. 1. Comparison of MinED and MaxED for the main constellation and sub-constellations.

	MinEd of MC	**MaxEd of MC**	D_1	D_2	**MinEd for sub-constellation**
Optimized SCMA [6]	1.58	2.40	0.58	2.00	3.35, 2.11, 3.35, 2.11
CG-SCMA [7]	1.58	2.33	0.60	2.05	3.46, 2.16, 2.16, 3.46
Proposed CG-SCMA	1.60	2.36	0.70	2.15	3.85, 2.26, 2.26, 3.85

lutions from [6], [7]. Furthermore, Fig. 8 presents the newly generated codebook, the CG-SCMA codebook and two other codebooks from [6], [7] in terms of BER. Figure 8 proves that the proposed method outperforms other known approaches.

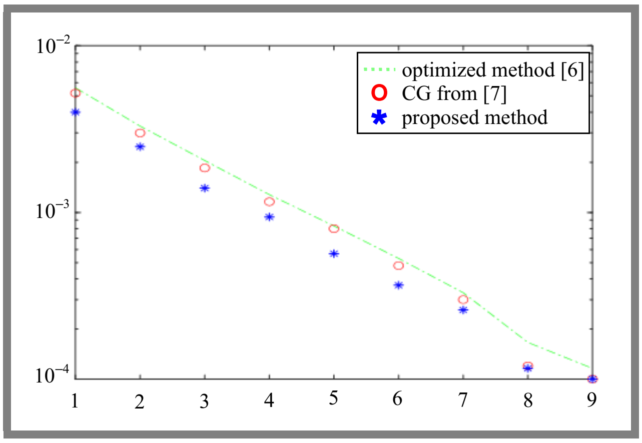

Fig. 8. BER comparison for classic and modified SCMA methods.

6. Conclusions

The paper presents an CG-SCMA codebook featuring maximization of MinED and compares it with other existing codebooks described in the literature. Using the CG-SCMA codebook for encoding data from different users, the differences between the first and others radiuses are optimized over rotated constellations. To increase MinED of a star 16-QAM, a novel CG-SCMA symmetric codebook architecture is proposed by employing TCM to subdivide the constellation. The SCMA codebook maximizes the space between star 16-QAM constellations, simultaneously decreasing BER and improving SNR.

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