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1. INTRODUCTION.

In [1] a deterministic routing algorithm for rearrangeable nonblocking interconnection networks using $2^t x 2^t$ switching elements was presented using graph representations of a permutation. However, in order to obtain the required connections for permutations where the required connections are of arbitrary size, a backtracking technique was applied. In this paper, a heuristic algorithm will be introduced that obtains the connections for switches of arbitrary size with less use of backtracking. Furthermore, the algorithm will be applied to three stage rearrangeable Clos networks. The algorithm depends heavily upon some basic theorems about systems of distinct representatives [2,3].

2. SIMULTANEOUS REPRESENTATIVES.

We begin this section by stating the famous theorem of Phillip Hall concerning systems of distinct representatives.

THEOREM 1. Let K be a finite set of indices, K = {1,2,...,n}. For each k in K, let S_k be a subset of a set S. A necessary and sufficient condition for the existence of distinct representatives x_k, k = 1,...,n, x_k in S_k, $x_k \neq x_j$, when $k \neq j$ is Condition C: For every t = 1,...,n and choice of t distinct indices $k_1,\ldots,k_t$, the subsets $S_{k_1},\ldots,S_{k_t}$ contain between them at least t distinct elements.

Slepian and Duguid [3] used this theorem to arrive at necessary and sufficient conditions in order for a three stage Clos network to be rearrangeable nonblocking. As an another immediate consequence of Hall's theorem is the following theorem on simultaneous representatives. It is this result which we will apply to obtain an algorithm for establishing the interconnections in the three stage Clos networks.

THEOREM 2. If a set S is divided into a finite number of subsets in two ways, $S = A_1+A_2+\ldots+A_n = B_1+B_2+\ldots+B_n$, and no k of the A's are contained in fewer than k of the B's for each k = 1,...,n, there will exist elements $x_1,\ldots,x_n$ that are simultaneously representatives of the A's and the B's.

While the above theorem is essentially an existence theorem, the proof however is constructive, and by directly placing our problem in the context of the proof of this theorem, we can arrive at the algorithm we want. We begin by letting P be a permutation of the integers 1,2,...,mn, $n \leq m$ and define a set S to consist of the mn ordered pairs (i,P(i)) where P(i) is the image of i under P. Now decompose S into two collections of m sets (indexed by k = 1,...,m) as follows:

$$A_{k0} = \{(j,P(j)) \mid (k-1)n+1 \leq j \leq kn\}$$
$$B_{k0} = \{(j,P(j)) \mid ((k-1)n+1 \leq P(j) \leq kn\}$$

Clearly, $A_{10}+\ldots+A_{m0} = B_{10}+\ldots+B_{m0}$ and furthermore the A's and the B's are respectively pairwise disjoint since P is a permutation. Hence we can apply Theorem 2. To determine such a collection $\{x_{k0}\}$ of simultaneous representatives, define a collection of sets $\{S_{k0}\}$ where each S_{k0} is the set of all indices j such that the intersection of A_{k0} and B_{j0} is nonempty. The collection $\{S_{k0}\}$ satisfies Condition C of Hall's theorem and let $j_1,\ldots,j_m$ be a system of distinct representatives for the $\{S_{k0}\}$. Thus by choosing x_{k0} such that x_{k0} is in the intersection of A_{k0} and $B_{j_k 0}$, we obtain a system of simultaneous representatives of the A's and B's.

It should be noted that if we delete x_{k0} from A_{k0} and $B_{j_k 0}$ and denote these reduced sets of A's and B's by $\{A_{k1}\}$ and $\{B_{k1}\}$ then the above process can be applied again. As before the solution exists due to Theorem 2 and we can find simultaneous representatives $\{x_{k1}\}$ for the sets $\{A_{k1}\}$ and $\{B_{k1}\}$. This process can be repeated a total of n times at which time all the reduced sets become empty. This fact is important in determining the interconnections in the three stage Clos network.

To illustrate the above process, let us consider the following example for m = n = 3 and P = (1 3)(4 5 7)(6 9 8).

A_{10} = {(1,3),(2,2),(3,1)}
A_{20} = {(4,5),(5,7),(6,9)}
A_{30} = {(7,4),(8,6),(9,8)}

B_{10} = {(1,3),(2,2),(3,1)}
B_{20} = {(4,5),(7,4),(8,6)}
B_{30} = {(5,7),(6,9),(9,8)}

S_{10} = {1}
S_{20} = {2,3}
S_{30} = {2,3}

A system of distinct representatives for the S's is $j_1 = 1$, $j_2 = 2$, and $j_3 = 3$. Now choose x_{10} = (1,3), x_{20} = (4,5), and x_{30} = (9,8) as a system of simultaneous distinct representatives.

3. AN ALGORITHM FOR FINDING REPRESENTATIVES.

In this section we will describe an algorithm for finding a system of distinct representatives for the S sets of the previous section with less backtracking than the algorithm of [1].

ALGORITHM S_D_R

Step 1. Determine the A and B sets for a permutation P.
Step 2. Determine the S sets corresponding to the A and B sets.
Step 3. Repeat thru Step 8 until each S set has a representative.
Step 4. For each value of k = 1,...,m count the number of S sets with no representative which contain K.
Step 5. Let Min_k be the value of k whose count is a minimum and greater than zero. (if zero, then backtrack).
Step 6. Find the S set of smallest cardinality which contains Min_k.
Step 7. Set the value Min_k as the representative for the S set of Step 6.
Step 8. Mark deleted the value Min_k from all S sets which contain it.

With no backtracking the above algorithm has a running time of $O(m^2)$ and when m = n the running time is linear in the length of the permutation P.

The selection of the smallest S set containing the minimum element is essential to the algorithm as the following example illustrates.

S_{10} = {5,4}
S_{20} = {1,3,4}
S_{30} = {3,4}
S_{40} = {1,2}
S_{50} = {2,5}

If we select 5 as the representative for S_{10} and then select 2 as the representative for S_{40} instead of for S_{50}, we force S_{50} to be empty before a representative has been selected for this S set.

4. APPLICATION TO THREE STAGE REARRANGEABLE CLOS NETWORKS

The technique introduced in the previous section can be readily applied to the process of establishing the interconnection network in a three stage rearrangeable Clos network. A network is called rearrangeable nonblocking if it can perform all possible connections between inputs and outputs by rearranging its existing connections so that a connection path for a new input-output pair can always be established [3]. We will be considering networks where the number of inputs equal mn and the switching elements are of size nxn.

Let P be an arbitrary permutation on the mn integers 1,2,...,mn, n ≤ m, and then form the two collections of sets $\{A_{k0}\}$ and $\{B_{k0}\}$ as described above. Using the Algorithm S_D_R, find the system of simultaneous distinct representatives for $\{A_{k0}\}$ and $\{B_{k0}\}$ and label them a_0. These permutation pairs will form the set of connections for the first intermediate switch a_0 of size mxm in the three stage Clos network. Now delete the system of simultaneous distinct representatives from the sets they represent in $\{A_{k0}\}$ and $\{B_{k0}\}$ to form two new classes of sets $\{A_{k1}\}$ and $\{B_{k1}\}$. Again find the system of simultaneous distinct representatives for the classes of sets $\{A_{k1}\}$ and $\{B_{k1}\}$ and these will form the interconnections for the intermediate switch labeled a_2 in the Clos network. This process can be continued to find systems of simultaneous distinct representatives for the sets $\{A_{kj}\}$ and $\{B_{kj}\}$ for $j \le n - 1$. Thus all the interconnections for the intermediate switches $a_0,\ldots,a_{n-1}$ have been established.

It is important to observe that this method allows us to completely determine the interconnections for a given intermediate switch. Furthermore, once the interconnections for a switch are found, they are not affected by what occurs at some later point in the algorithm. To illustrate this technique, consider the example of [1] where m = n = 5 and P = (1 25 17 13 14 5 24 12) (2 16 9 22 4 6 8 3 23 18 7 15 11).
The A and B sets are as follows:

A_{10} = {(1,25),(2,16),(3,23),(4,6),(5,24)}
A_{20} = {(6,8),(7,15),(8,3),(9,22),(10,10)}
A_{30} = {(11,2),(12,1),(13,14),(14,5),(15,11)}
A_{40} = {(16,9),(17,13),(18,7),(19,19),(20,20)}
A_{50} = {(21,21),(22,4),(23,18),(24,12),(25,17)}

B_{10} = {(12,1),(11,2),(8,3),(22,4),(14,5)}

B_{20} = {(4,6),(18,7),(6,8),(16,9),(10,10)}

B_{30} = {(15,11),(24,12),(17,13),(13,14),(7,15)}

B_{40} = {(2,16),(25,17),(23,18),(19,19),(20,20)}

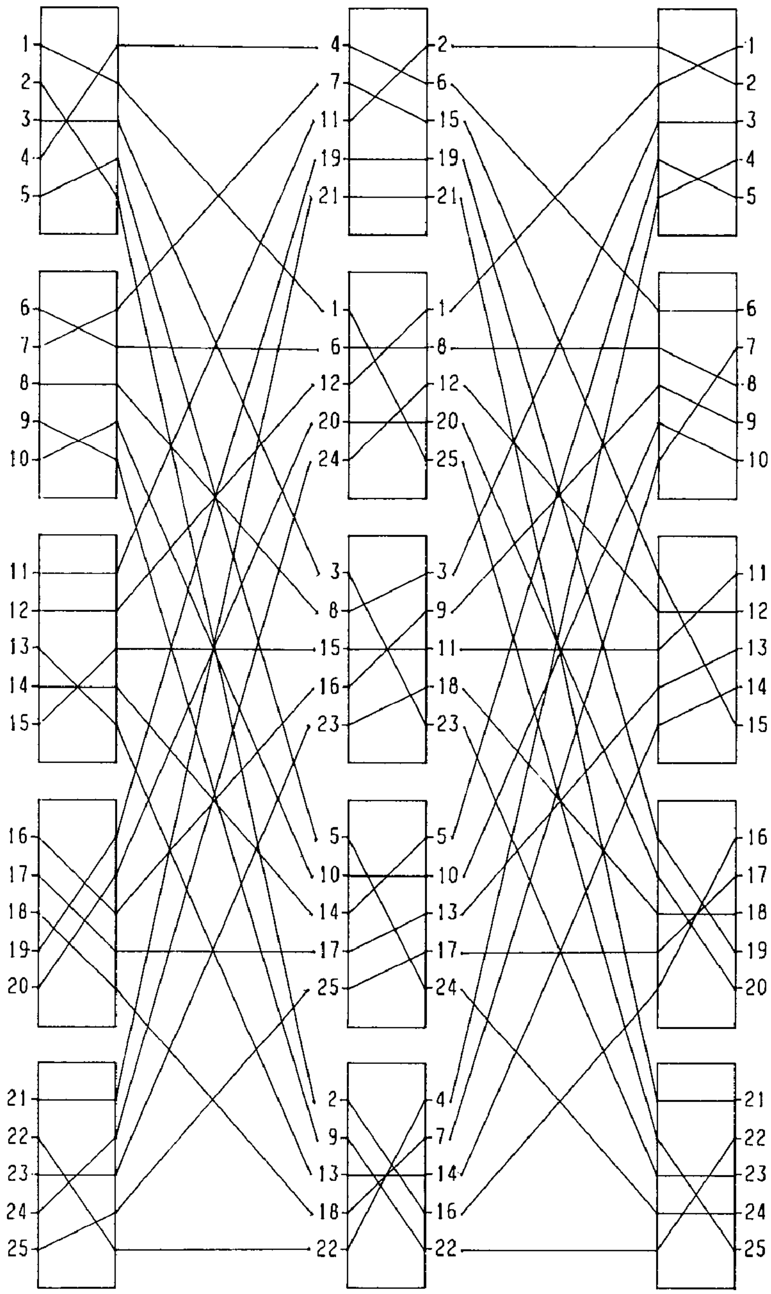
B_{50} = {(21,21),(9,22),(3,23),(5,24),(1,25)}

We now give the sequence of S sets which will be formed as part of the solution. The digit and ordered pair following each S set will be the representative for that set and the simultaneous representative that was selected to represent the corresponding A and B sets. The reduced A and B sets are not shown since they can be found from the information given below.

S_{10} = {5,4,2}	2	(4,6)
S_{20} = {2,3,1,5}	3	(7,15)
S_{30} = {1,3}	1	(11,2)
S_{40} = {2,3,4}	4	(19,19)
S_{50} = {5,1,4,3}	5	(21,21)
S_{11} = {5,4}	5	(1,25)
S_{21} = {2,1,5}	2	(6,8)
S_{31} = {1,3}	1	(12,1)
S_{41} = {2,3,4}	3	(20,20)
S_{51} = {1,4,3}	4	(24,12)
S_{12} = {4,5}	5	(3,23)
S_{22} = {1,5,2}	1	(8,3)
S_{32} = {3,1}	3	(15,11)
S_{42} = {2,3}	2	(16,9)
S_{52} = {1,4}	4	(23,18)
S_{13} = {4,5}	5	(5,24)
S_{23} = {5,2}	2	(10,10)
S_{33} = {3,1}	1	(14,5)
S_{43} = {3,2}	3	(17,13)
S_{53} = {1,4}	4	(25,17)
S_{14} = {4}	4	(2,16)
S_{24} = {5}	5	(9,22)
S_{34} = {3}	3	(13,14)
S_{44} = {2}	2	(18,7)
S_{54} = {1}	1	(22,4)

The Clos network with these interconnections is illustrated below.

It should be noticed that this method finds all the interconnections for each switch and any backtracking only affects the current switch settings which are being calculated. However, the method of [1] does not determine the settings until the entire connection array has been put into its proper form with any backtracking possibly affecting earlier settings. In the above example, no backtracking was required in any intermediate switch calculation using our technique.



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