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A Microcomputer-Based Data Acquisition System with Hardware Capabilities to Calculate a Fast Fourier Transform

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filter $H(z)$: $S_d(f) = H(z = e^{j2\pi fT})H(z^{-1} = e^{-j2\pi fT})$

$$= \frac{\sum_{k=1}^{n}\sum_{l=1}^{n} b_k b_l e^{j2\pi(l-k)fT}}{\sum_{k=0}^{n}\sum_{l=0}^{n} a_k a_l e^{j2\pi(l-k)fT}}. \tag{18}$$

The double sums in the numerator and denominator may be evaluated on upper and lower triangular grids to write

$$S_d(f) = \frac{\sum_{k=1}^{n} b_k^2 + 2\sum_{l=1}^{n-1}\sum_{k=1}^{n-l} b_k b_{k+l}\cos 2\pi lfT}{\sum_{k=0}^{n} a_k^2 + 2\sum_{k=1}^{n} a_k \cos 2\pi kfT + 2\sum_{l=1}^{n}\sum_{k=1}^{n-l} a_k a_{k+l}\cos 2\pi lfT}. \tag{19}$$

From (12) we may write

$$S_d(f) = \frac{r_0 + 2\sum_{l=1}^{n-1} r_l \cos 2\pi lfT}{\sum_{k=0}^{n} a_k^2 + 2\sum_{k=1}^{n} a_k \cos 2\pi kfT + 2\sum_{l=1}^{n}\sum_{k=1}^{n-l} a_k a_{k+l}\cos 2\pi lfT}. \tag{20}$$

This result shows that the ARMA spectrum may be found in terms of the AR parameters $\{a_l\}_0^n$, and the residual covariance $\{r_l\}_0^{n-1}$, without ever solving for the MA coefficients. Thus, a nonlinear spectral factorization problem is avoided.

VII. Concluding Remarks

When the results discussed here are used for spectrum analysis, then the covariance $\{R_k\}_0^{2n-1}$ is replaced everywhere by estimated variables $\{\hat{R}_k\}_0^{2n-1}$. Gersch [6] has shown that when the order n is known, and covariances are estimated as $\hat{R}_k = N^{-1}\sum_{l=1}^{N} x_l x_{l+k}$, then the AR coefficients found from the normal equations are unbiased and consistent. It does not follow, however, that for finite N the AR coefficients will correspond to a stable filter $H(z)$. That is, the equations of (9) do not enjoy the nice stability property associated with the normal equations

$$\sum_{m=0}^{n} a_m R_{k-m} = 0, \qquad k = 1, 2, \cdots, n.$$

This defect may not be critical from the point of view of spectrum analysis. If a stable filter is desired, unstable poles of $H(z)$ may be reflected inside the unit circle. A more substantial defect is that the estimated residual sequence $\{r_k\}$ may not be a true covariance sequence. This means that the numerator of (20) may be negative for some f. Thus, care must be taken when using (20) to ensure that a negative spectrum is not estimated. This consideration is but a subset of the more general problem of order determination in AR and ARMA spectrum estimation. A thorough study of order determination in ARMA (n, n - 1) spectrum analysis is required before such analysis becomes a well-developed tool. Nonetheless, we feel that ARMA (n, n - 1) spectrum analysis is the natural approach when dealing with data that arise as sampled data from a rational continuous-time process. The calculations are not markedly more complicated than for AR(n) spectrum analysis.

See [6] for a more complete list of references to the statistics literature dealing with ARMA time series.

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A Microcomputer-Based Data Acquisition System with Hardware Capabilities to Calculate a Fast Fourier Transform

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***Abstract*—The fast Fourier transform (FFT) has in recent years become an important tool to the engineer. There are a number of algorithms which calculate the FFT. One such algorithm is the Cooley-**

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Tukey FFT algorithm (radix-2) which lends itself rather easily to a combination software and hardware implementation. With a microcomputer as the controller of a data acquisition module and hardware which computes the butterfly associated with the FFT algorithm, the FFT can be calculated efficiently.

I. Introduction

This paper describes a microcomputer-based system with capabilities for data acquisition and calculation of the Cooley–Tukey FFT algorithm [1]. The system is composed of a Southwest Technical Products Corporation (SWTPC) 6800 computer system [2], an Analogic MP6812-D Data Acquisition Module [3], and dedicated hardware to calculate the butterfly associated with the Cooley–Tukey FFT. In the remainder of this paper the terms "data acquisition module" and "dedicated hardware" are interchanged frequently with the terms "A/D converter" and "FFT hardware," respectively. The basic operation of the system is as follows.

1) Analog data are input to the A/D converter.
2) The A/D converter digitizes the analog input data.
3) The digitized data generated in step 2) are stored in the memory or the microcomputer.
4) The digitized data generated in step 2) are used as the input to the FFT hardware.
5) The FFT hardware performs the butterfly computation with the results stored in the microcomputer.
6) Step 5) is repeated for different sets of input data until the FFT is calculated.

The above sequence of events is controlled by the microcomputer through software.

II. System Hardware Description

The system used to generate the FFT algorithm is shown in block diagram form in Fig. 1. Analog data are input to the A/D converter where they are digitized and stored in the memory of the microprocessor system. These digitized data are later input to the FFT hardware which calculates the butterfly associated with the Cooley–Tukey FFT algorithm. The output of the FFT hardware is then stored in memory, and the processor continues until the algorithm is completed. Referring to Fig. 1, the system hardware can be divided into main sections. One is the peripheral interface adapter (PIA)–A/D converter and the other is the PIA–FFT hardware. These sections are both controlled by the SWTPC 6800 Computer System. A block diagram of the PIA–FFT hardware is shown in Fig. 2.

III. System Software Description

The system software is composed of two main sections. One section is the software program SINAD which controls the PIA–A/D converter interface. The second section is the software program FFT which controls the PIA–FFT hardware interfaces and calculates the Cooley–Tukey FFT algorithm.

Upon execution of microcomputer program FFT, N transformed data points are stored in the same memory locations that the input data occupied. Calculation times for the FFT are shown in Table I. The theoretical best case and worst case is due to the fact that program flow is dependent upon input data. A particular set of input data might result in a faster execution time than another set of data.

IV. Conclusion

This paper has described a microcomputer-based system with capabilities for data acquisition and calculation of the Cooley–Tukey FFT algorithm. The system has demonstrated the feasibility of employing microcomputer-based data acquisition system for applications involving harmonic analysis. The microcomputer approach leads to several benefits, including portability, field data analysis, and a significant savings in cost.

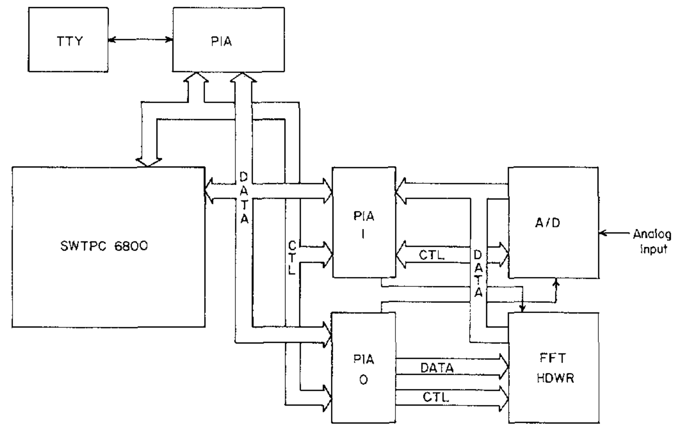

Fig. 1. Block diagram of microcomputer system.

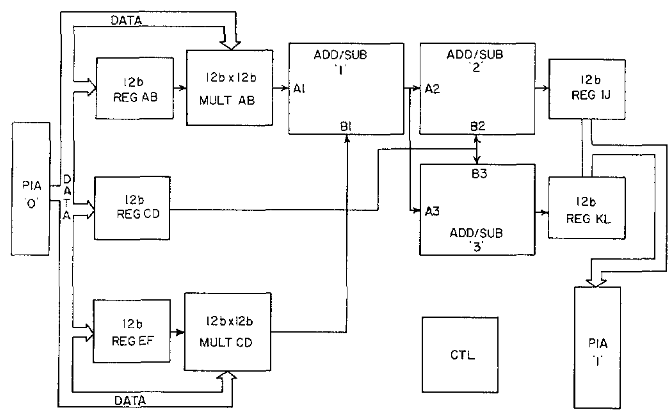

Fig. 2. Block diagram of PIA–FFT hardware.

TABLE I
Calculation Times for Microcomputer Program FFT (12 Bits)

N	Theoretical Best Case (ms)	Observed (ms)	Theoretical Worst Case (ms)
4	10.03496	10.12497	10.195146
8	27.68764	27.94681	28.168193
16	70.90438	71.53526	72.18586
32	174.6379	175.0609	177.8416
64	426.8034	430.2698	434.4923
128	1104.858	1105.696	1122.799

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