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Original
An Algorithmic Approach to Palladio’s Design of Stairs / Giovannini, Elisabetta Caterina. - In: NEXUS NETWORK JOURNAL. - ISSN 1522-4600. - (2023). [10.1007/s00004-023-00668-6]

Availability:
This version is available at: 11583/2977601 since: 2023-03-30T08:58:56Z

Publisher:
Birkhäuser

Published
DOI:10.1007/s00004-023-00668-6

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12 June 2023

Nexus Network Journal
https://doi.org/10.1007/s00004-023-00668-6

NEXUS NETWORK JOURNAL
Architecture and Mathematics

CONFERENCE PAPER

An Algorithmic Approach to Palladio's Design of Stairs

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Accepted: 10 March 2023
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Abstract
Book I of Andrea Palladio's *I quattro libri dell'architettura*, contains the rules of Palladio's design. Over the years, many scholars have investigated how to make explicit the reasoning behind the theory of architectural proportions in Palladio. The study addresses the parametric analysis for Stairs described in Chapter XXVIII.

Keywords Rule-based design · Shape grammars · Parametric design

Introduction

Book I of Andrea Palladio's *I quattro libri dell'architettura*, " entitled, *di quelli auertimenti, che sono più necessarij nel fabricare*, contains the rules of Palladio's design for stairs.

Palladio devoted the second-to-last chapter of Book I to the Stairs. His approach in the first part of the chapter (Palladio 1570: 60–63) is to systematize the Stairs according to the geometrical forms of their planimetric representation. The second part of the chapter (Palladio 1570: 64–66) is devoted to the description of complex stairs that he considers "*di bella maniera*": spiral, triangular, and double staircases.

Palladio devotes four pages containing the plan and sections of stairs. This research investigates his approach to the definition of Stairs starting from drawings on page 62 and 63 of Book I, their graphical analysis, geometrical interpretation, and conversion in mathematical syntax to create algorithms for the semi-automatic generation of HBIM Palladio's stairs. Second, this work compiles a comparative analysis offering insight into the use of the diverse types of stairs in Palladio's buildings.

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Published online: 27 March 2023

The Research

The first objective of the present work is to define a set of visual rules to "translate" the language of Palladio's text of Chapter XXVIII of the Book I, entitled "*delle scale, e varie manière di quelle, e del numero, e grandezza de' gradi"*, into a set of predetermined geometrical rules.

The ashlar should have a rise between 4 and 6 *once* as described in the text. An *oncia* is the twelfth part of a *Vicentine foot* or approx. 0.347 m (Zorzi 1957: 65); it is divided into 4 min. The tread's dimensions should be between 1 and 1½ feet. The number of ashlars should be odd to make it easier to climb the staircase: people can ascend on the right foot and finish on the same foot.

Analyzing the drawings of Palladio for a further three-dimensional representation, it is important to notice that stair representations can be read in vertical succession: floor plans and cross-sections are aligned. In cross-sections, the risers of the ramps do not correspond to a section plane passing through the center of the floor plans (spiral or oval) but to a sectioned view possible by removing the front half of the walls and leaving the entire helical development of the ramps in place. (Cirillo 2018: 182).

For each geometrical solution drawn, Palladio proposes two possible configurations: with or without a structural element in the middle (columns or walls).

This study follows the description of diverse types of stairs, creating the first classification based on their geometric aspects that cover eight possible configurations encoded with letters from A to H (Palladio 1570: 60–61):

- spiral staircase
- oval staircase
- straight staircase

In the spiral staircase, the complexity is increased by the presence of spiraling steps for both solutions.

The straight staircase can have two extended flights, which turn into four flights when they are squared. Even if Palladio does not mention them explicitly, we also see straight staircases inscribed in a rectangle and sometimes with an L-shaped configuration in his buildings.

Palladio uses a geometric construction of alignments, recurring intervals, equal subdivisions, and successive partitions to define stairs' characteristics. (Valenti et al. 2012) This is evident in architectural elements and in the definition of classical orders.

This research analyzes his drawings (Fig. 1) and uses a graphic overlay to enlighten the proportional rules used by Palladio in dimensioning the elements that compose the different types of stairs. His proportioning method divides the space between structural elements (columns and walls) and the space left for the steps. Spiral and square staircases have a planimetric ratio of 1:1. The oval

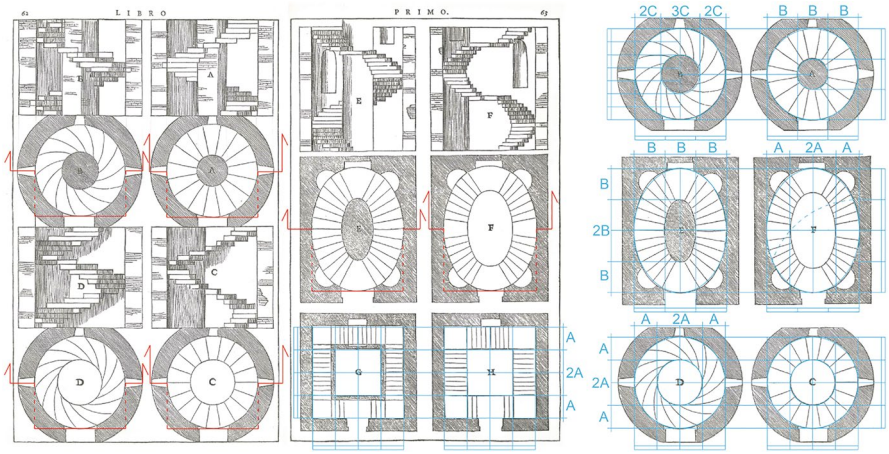

Fig. 1 Annotations of Palladio's stairs and their proportions. Solutions from A to H. Image: graphic overlay by the author on Palladio 1570: 62–63

staircases have a planimetric ratio of 3:4 with a geometric construction based on a polycentric curve (Fig. 2). (Paris 2019: 147–150).

When determining the geometry of the elements that compose a square staircase, solutions G and H divide the square's length into four parts and then leaves a square of two parts as the void in the middle. The lengths of each step consist of one part. When the inner walls are present, their thickness should be comprised inside the steps' partition.

The same partitioning is proposed for configurations C, D, and F corresponding with spiral and oval staircases with a void in the middle.

Solutions A and E have a partition of elements corresponding to steps and the column in the middle, respectively. These can be divided into three parts via the diameter of the spiral and dimensioning of the column with a diameter corresponding to one part.

Solution B divides the diameter of the spiral into seven parts and dimensions the column with a diameter corresponding to three parts.

Subsequently, the analysis of proportions and a geometrical study were conducted to create algorithms that reproduce Palladio's theory of proportions for the semi-automatic construction of stairs solutions A, B, C, D, E, F (Palladio 1570: 62–63) in an HBIM environment using a visual programming language (VPL). The algorithmic approach is similar to Palladio's way of thinking based on proportional rules (algorithms) and a determined set of values (input values).

The creation of 3D objects was modeled according to parametric rules that can make them adaptive and thus reusable in multiple BIM projects. This has been increasingly used in VPL to generate 3D elements for HBIM (Capone and Lanzara 2022; Paris and Wahbeh 2016). The VPL approach to three-dimensional modeling has rapidly grown in recent years. Its popularity is based on the simplicity of the interface that enables complex functions hidden within visual nodes to be solved.

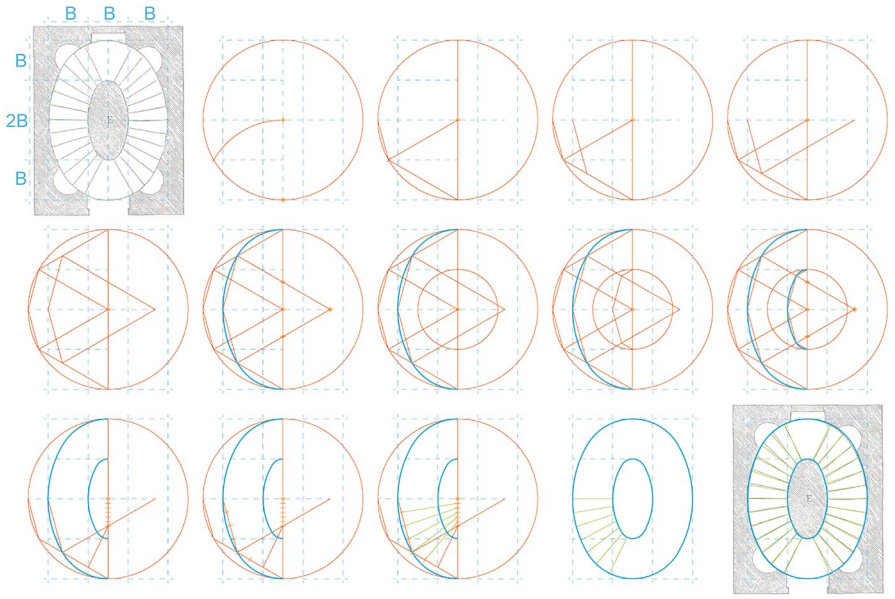

Fig. 2 Annotations of Palladio's oval stair solution E and its geometrical construction. Image: graphic overlay by the author on Palladio 1570: 63

The algorithms were developed using DynamoRevit, a VPL plug-in for Autodesk Revit, to create complex three-dimensional geometries using different inputs.

In the spiral staircase geometric construction, each step is considered a unique ashlar stated and superimposed with the one below. (Migliari 2008: 251–253) Each ashlar is oriented to the central axis of the stair in the case of stairs without a column in the middle. That is, it is anchored to the perimeter wall from one side and embossed on the opposite side.

The ashlar can be converted in a three-dimensional environment consisting of two surfaces belonging to two different cylindrical helicoids for both intrados and extrados of the stair. The surfaces corresponding to the rise and tread were vertical and horizontal surfaces.

In contrast to the illustrated geometric construction, the developed algorithm uses geometric rules of intersections between main geometric elements: lines, circumferences, and arcs of circumferences (Figs. 3, 4). For each proposed solution, the definition of the ashlar and its construction is the consequence of tracing the geometrical elements in the plan and identifying their intersections. The result is a closed polyline that can be extruded vertically to obtain a three-dimensional model. For the creation of the staircase, the ashlar is rotated and translated in height according to the relationships between the number of treads and the height of the floor.

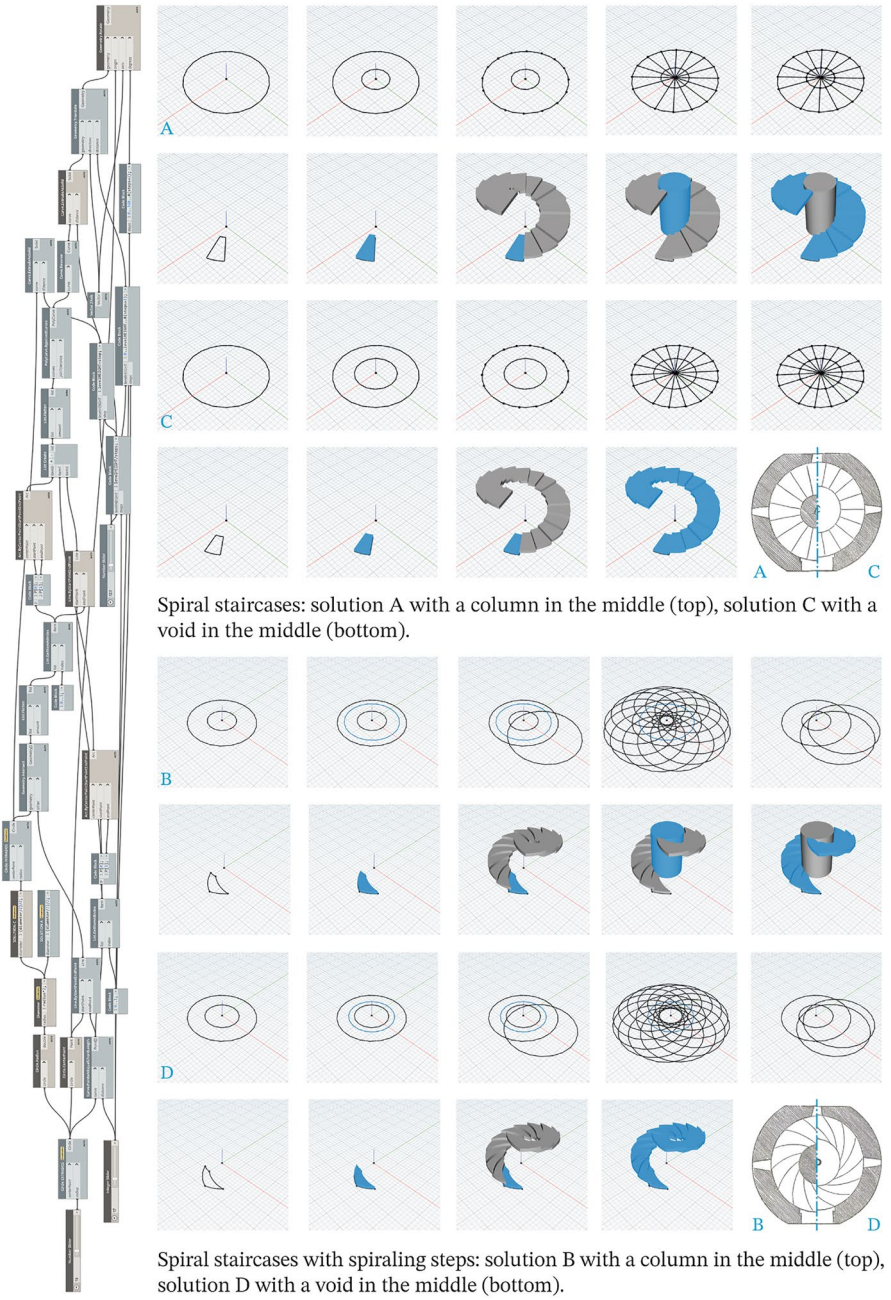

Fig. 3 Algorithmic solutions for the construction of spiral stairs **A**, **B**, **C** and **D**. Image: author

After developing algorithms for spiral and oval staircases, this study analyses Book II to identify and encode stairs in Palladio buildings and villas for 38

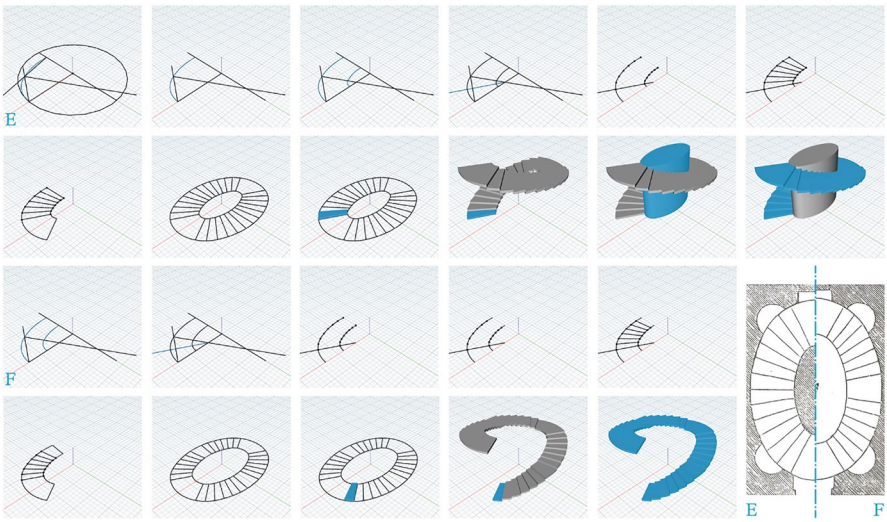

Fig. 4 Algorithmic solutions for the construction of oval stairs E and F. Image: author

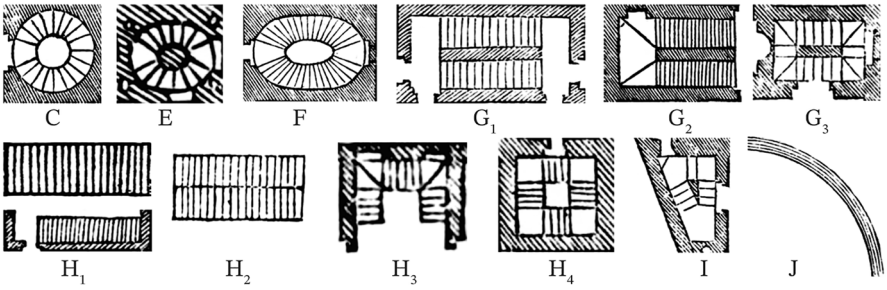

Fig. 5 Graphical encoding of stairs from Palladio's design projects in Book II. Image: author

plans (Fig. 6). The analysis presents considerations on the types of stairs available according to their Palladio's graphical encoding (Fig. 5): (A) spiral staircase with a column in the middle, (B) spiral staircase with a column and spiraling steps, (C) spiral staircase with a void in the middle, (D) spiral staircase with a void in the middle and spiraling steps, (E) oval staircase with a column in the middle, (F) oval staircase without the column, (G_1) straight staircase with a wall inside, (G_2) straight staircase with a wall inside and two flights, (G_3) straight staircase with a wall inside and four flights, (H_1) straight staircase without a wall, (H_2) straight staircase without a wall and two flights, (H_3) straight staircase without a wall and three flights, (H_4) straight staircase without a wall and four flights, (I) triangular staircase, and (J) curved staircase.

Book II	A	B	C	D	E	F	G_1	G_2	G_3	H_1	H_2	H_3	H_4	I	J
p.5								•							
p.6			•								•				
p.8								•					•		
p.11			•			•				•	•				
p.13						•									
p.16							•				•				
p.19														•	
p.21						•			•					•	
p.22						•					•				
p.47							•	•		•	•				
p.48										•	•				
p.49								•		•					
p.50					•					•					
p.51										•		•			
p.52						•					•				
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p.62										•	•				
p.63							•			•	•				
p.64							•			•	•				
p.65								•			•				•
p.67						•									•
p.68							•								
p.70							•								
p.71					•	•	•				•				
p.72						•					•				
p.74						•					•				
p.75							•				•				
p.76							•				•				
p.77						•									
p.78 (66)							•				•				•

Fig. 6 Analysis of different types of stairs and their presence in Book II according to their graphical encoding in Fig. 5. Image: author

Importantly, Palladio's production has no spiral staircases with a column in the middle. This confirms what Palladio makes explicit in the text in which he asserts that he prefers centrally voided staircases that allow better illumination of the space.

Further analysis of Book II will determine the ratio used in each Palladio plan to identify the relationship between the building layout and the role of stairs developed in architecture as independent spatial elements (Leopold 2019).

Three-dimensional reconstructions of drawn elements are based on physical reasoning and offer a different approach to reading the geometric-formal properties of Palladio's architectural work.

One intent of this research is to classify staircases to establish their role in connecting elements of the plan layout and thus identify whether compositional principles govern their definition.

Conclusion

The parametric stairs are based on a predetermined set of algorithms with a fixed geometry that can be adapted to diverse planimetric solutions. This study identified the various types of stairs available in Book I and how they are used within a Palladio building production in Book II.

Sass conducted a similar analysis to develop a Palladian construction grammar for rapidly prototyping 30 villas (Sass 2007). The introduction of BIM platforms in recent years allowed one to construct a Palladian grammar with more flexibility and a parametric approach.

The results obtained in this research and the previous study by the author on chapters XXI–XXVI will contribute to creating the first Palladio HBIM library of architectural elements using visual programming language to parametrize geometrical construction rules for architectural elements.

The algorithms developed here can facilitate a three-dimensional library of oval and spiral stairs. HBIM elements encode the 3D Palladio Grammar, which can be easily findable, accessible, interoperable, and reusable for both didactic activities and research.

Funding Open access funding provided by Politecnico di Torino within the CRUI-CARE Agreement.

Data Availability Statement Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Declarations

Conflict of interest On behalf of all authors, the corresponding author states that there is no conflict of interest.

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