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A new genetic algorithm framework based on Expected Annual Loss for optimizing seismic retrofitting in reinforced concrete frame structures

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Abstract

The design of seismic retrofitting for existing reinforced concrete frame structures concerns the determination of the position and the arrangement of reinforcements. Currently, this design practice is mainly based on trial-and-error attempts and engineers' experience, without a formal implementation of cost/performance optimization. Though, the implementation of this intervention is associated with significant costs, noticeable downtimes, and elevated invasiveness. This paper presents a new genetic algorithm-based framework for the optimization of two different retrofitting techniques (FRP column wrapping and concentric steel braces) that aims at minimizing costs considering indirectly the lessening of expected annual values. The feasibility of each tentative solution is controlled by the outcomes of static pushover analyses in the framework of the N2 method, achieved by a 3D fiber-section model implemented in OpenSees. Application of the framework in a realistic case study structure will show that the sustainability of retrofitting intervention is achievable by employing artificial intelligence aided structural design.

Keywords: Seismic retrofitting, structural optimization, expected annual loss, FRP, steel bracing.

1. Introduction

One of the major issues that structural engineers face in the design of seismic retrofitting of existing structures regards the determination of the optimal position and arrangement of the intervention. The design of this kind of intervention is exclusively entrusted to the engineer's intuition and experience, requiring several trial-and-error

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attempts. This approach may also lead to an overestimation of the retrofitting intervention with a consequent increase in costs, invasiveness, and downtimes. Furthermore, can be observed a growing concern about the impact that a widespread seismic retrofitting of building heritage in earthquake-prone areas has in terms of costs on communities and use of resources. From this standpoint, structural optimization emerges as an effective tool for the suitable employment of funds allocated for seismic retrofitting of existing structures.

In the last years, the scientific interest in structural optimization was mainly focused on the size and shape optimization of new structures. On the contrary, the optimization of seismic retrofitting of existing structures has not been investigated, conspicuous interest is developed in the last years. Few researchers have addressed the problem of the optimization of fiber reinforced polymers (FRP) jackets Chisari et al. (2016) and Seo et al. (2018) or other applications of seismic retrofitting methods for reinforced concrete (RC) buildings by using dissipative bracings Braga et al. (2019), fluid viscous dampers Pollini (2017), or both Lavan et al. (2009).

Only recently different studies tackled the issue of optimization of seismic retrofitting costs. Among them, Falcone et al. (2019) proposed a framework for cost optimization of FRP jacketing and steel bracings for existing reinforced concrete (RC) frame structures through a genetic algorithm. Papavasileiou et al. (2020) faced retrofitting optimization of encased steel-concrete composite columns comparing three different retrofitting devices. A similar approach was followed by Di Trapani et al. (2020) and Di Trapani et al. (2021) who proposed a new framework based on a genetic algorithm aimed at minimizing steel jacketing seismic retrofitting costs for both ductility deficient and shear-critical RC structures. Minafò and Camarda (2021) proposed a GA-based framework to minimize the intervention costs of buckling restrained braces in 2D reinforced concrete frames. Eventually, Di Trapani et al. (2022) provided a genetic algorithm for the optimization of retrofitting interventions that involve two different techniques in RC frame structures by controlling the associated expected annual loss.

In this paper a new optimization framework aimed at optimizing service-life costs of RC frame structures subject to retrofitting interventions. The expected annual loss (EAL) has been proved as a valid parameter for comparing structural seismic performance during service life Calvi (2013). It assesses the overall behaviour of the construction in terms of expected economic annual losses caused by seismic events that could occur during the reference service period of a structure. The main goal of the proposed framework is to determine, for non-seismically compliant RC structures, the best retrofitting configuration in terms of reinforcement design (sizing optimization) and position (topological optimization). Optimization focuses on the minimization of retrofitting costs considering indirectly the resulting EAL value. Since EAL assessment involves different limit states fulfilment, the proposed framework takes into account multiple retrofitting interventions. For the case study of a multistorey RC building, two different techniques are considered: FRP jacketing of RC columns (to increase ductility) and steel bracings (to increase lateral stiffness). The optimization process is carried out by a genetic algorithm (GA) developed in MATLAB® which is linked with a 3D fiber-section model developed in the OpenSees software platform (McKenna et al. (2000)). The structural performance of each solution is evaluated starting from the results of static pushover analyses in the framework of the N2 method (Fajfar (2010)). The validity and efficiency of the proposed method are eventually proved by employing the proposed framework on a case study structure.

2. Optimization framework

The optimization framework herein proposed is based on a genetic algorithm (GA) optimization routine developed in MATLAB®. The optimization algorithm relates a structural model implemented in the OpenSees software platform (McKenna et al. (2000)) with the GA process. Genetic algorithms analyze the search space by calculating the value of the objective function by points and proceeding in the search for minima based on the combining of the set of parameters (called genome) that have had the best results in each iteration. This is implemented by generating populations of tentative solutions (individuals) that in the present case represent different retrofitting arrangements of the structures. Each individual handled by the algorithm is characterized by a design vector gathering all the decision variables to be optimized, in this case, they represent the parameters that define some design characteristics and the position of retrofitting interventions. The optimization involves the definition of a proper objective function that estimates the intervention costs of each tentative solution. The EAL value associated with each retrofitting configuration is indirectly considered during the selection procedures (i.e., parent and survival selection) with a non-penalty approach based on the number of violated constraints. A schematic flowchart of the proposed framework is depicted in Figure 1.

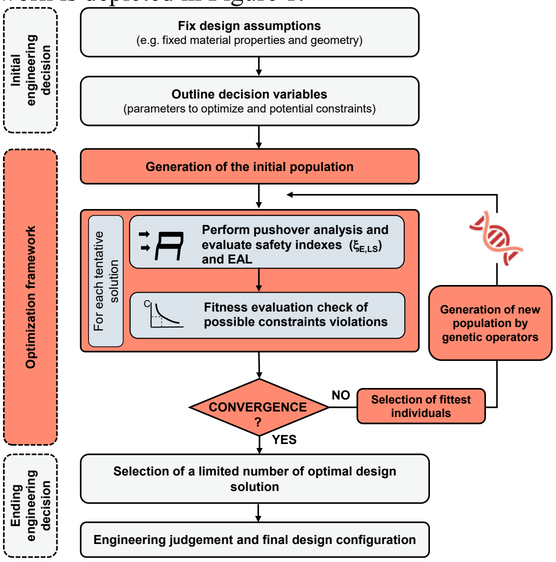

Fig. 1. Schematic flowchart of the proposed optimization framework

2.1. Definition of the design vector

The algorithm aims at optimizing the intervention cost of two different retrofitting systems: FRP wrappings of columns and concentric steel bracings. The decision variables that encode the position and sizing of both retrofits are gathered into a so-called design vector. The assigned design variables are the number of frame fields where the bracings are defined (n_{br}), the braces diameter ($Ø_{br}$), the FRP strips spacing (s_{FRP}), the number of overlapping layers of FRP (n_{FRP}), and the position of the columns retrofitted by the FRP (**p**). All these design variables are gathered in the design vector **b** so defined:

$$\mathbf{b} = \begin{pmatrix} n_{br} & \phi_{br} & s_{FRP} & n_{FRP} & \mathbf{p} \end{pmatrix}^T \tag{1}$$

in which the term **p** is an array of binary numbers representing the position of the FRP retrofitted columns as:

$$\mathbf{p} = \begin{bmatrix} \dots & \dots & c_{ij} & \dots \end{bmatrix}^{\mathrm{T}} \tag{2}$$

where the general element c_{ij}, is a binary assuming the value 1 if the *i-th* column of the *j-th* storey is retrofitted and 0 if it is not. To prevent the premature collapse of columns caused by the additional shear demand induced by the bracings. heuristic repair technique is involved to introduce FRP wrapping on the columns adjoining the bracing.

2.2. Definition of the objective function

The objective function is aimed at evaluating the retrofitting intervention costs considering the implementation of the two retrofitting systems as:

$$F = \sum_{i=1}^{n_{br}} \left(W_{\mathrm{br},i} \cdot c_{\mathrm{br}} \right) + n_{\mathrm{br}} \cdot c_{\mathrm{br,m}} + \sum_{i=1}^{n_c} \left(A_{\mathrm{FRP,i}} \cdot c_{\mathrm{FRP}} \right) + n_c \cdot c_{\mathrm{FRP,m}} \tag{3}$$

where the former summation term is the cost related to the arrangement of bracings, where c_{br} is the material and manpower cost per unit weight (estimated in c_{br} = *6 €/kg*), $c_{br,m}$ is the fixed cost related to the demolition and reconstruction of masonry (2000€ for every braced frame fields), and $W_{br,i}$ is the weight of the bracings in the *i-th* frame. The second summation term represents the cost for the implementation of the FRP wrapping of the columns. The term c_{FRP} is the unit cost of the FRP (estimated in $c_{FRP} = 300$ €/m^2), n_c is the number of retrofitted columns also taking into account the local reinforcement of the columns adjoining the steel bracings systems as presented in the previous section, $c_{FRP,m}$ is the cost per column for the demolition and reconstruction of adjacent masonries and plasters (equal to $c_{FRP,m}$ = 1000 €), and A_{FRP} is the area of the FRP fabric used to retrofit the generic *i-th* column. The latter parameter can be evaluated as:

$$A_{FPR,i} = \left\lfloor \frac{l_{c,i}}{s_{FRP}} \right\rfloor \cdot n_{FRP} \cdot \left[2 \cdot (b_i + h_i) - (4 - \pi) \cdot r_c^2 \right] \cdot l_{FRP} \tag{4}$$

where $l_{c,i}$ is the total length of the *i-th* column, b_i and h_i are respectively width and height of the i-th column cross-section, r_c is the radius of the rounded columns, and l_{FRP} is the width of FRP strips. Both terms consider the material, manpower costs and the necessary works for the demolition and restoration of adjoining plaster and masonry.

2.3. Definition of constrains

The EAL value of each tentative retrofitting arrangement is considered an indirect way as a constraint of the optimization procedure. The EAL evaluate the percentage annual loss of economic value of a structure in its reference life considering the associated seismic risk. The assessment of the EAL value is accomplished by the method proposed in (Cosenza et al. (2018)), according to which, annual losses are expressed as percentages of the repair costs (%RC) concerning the reconstruction cost.

The EAL is evaluated as the area under the curve that connects the points (λ, %RC) for each limit state. For sake of simplicity, the annual rates of failure for the operational and collapse limit states can be evaluated as a function of those evaluated for damage limitation and life safety limit states, thus EAL can be calculated once λ_{DLLS} and λ_{LSLS} are evaluated. For this reason, the feasibility of each solution is restrained by their simultaneous verification of DLLS and LSLS which implies that the EAL of retrofitted structures is minor then the code-compliant building, namely a structure having for each limit state a capacity that is identical to the demand. The minimization problem, thus, can be formalized as:

$$\min F(\mathbf{b}) \qquad \text{s.t.} \begin{cases} \lambda_{DLLS} \le \lambda_{cc_{DLLS}} \\ \lambda_{LSLS} \le \lambda_{cc_{LSLS}} \end{cases} \tag{5}$$

A non-penalty approach is involved to consider the feasibility (or not) of each tentative solution. This is exerted by the survival selection that accomplishes a double sorting process, first arranging the individuals to the number of violated constraints and then, among the individuals with the same number of violations, in ascending way according to the intervention cost value.

3. Analysis and post-processing of the outputs

The feasibility of each solution is assessed by non-linear static analysis. Pushover analysis is carried out in the framework of the N2 method (Fajfar (2002)). The safety factors at the two limit states analysed, useful for the EAL evaluation, can be calculated as:

$$\begin{cases} \zeta_{E,LSLS} = \dfrac{PGA_{c,LSLS}}{PGA_{d,LSLS}} \\ \zeta_{E,DLLS} = \dfrac{d^*_{\mathrm{DLLS},c}}{S_{de,DLLS}\left(T^*\right)} \end{cases} \tag{6}$$

where $PGA_{d,LSLS}$ is the peak ground acceleration demand and $PGA_{c,LSLS}$ is the peak ground acceleration capacity, which is the one associated with the earthquake-inducing LS limit state. The latter can be evaluated from the results of a pushover analysis in the framework of the N2 method (Fajfar (2002)).

In Equation 6, $S_{de,DLLS}(T^*)$ is the displacement demand associated with the elastic DLLS spectrum, and $d^*_{max,DLLS}$ is the top displacement associated with the achievement of the DLLS condition. In infilled frame models, this can be assessed by controlling the stress state inside infill elements. The mean annual rates of exceedance are calculated as reciprocal of the capacity return periods that are evaluated starting from the previously presented safety factors in the simplified approach proposed by Cosenza et al. (2018).

The shear verification of columns is carried out in the post-processing phase in strength terms. The ultimate displacement capacity assumed for the SDOF system is the one associated with the first shear failure of a column. Shear verifications are carried out according to the model by Biskinis et al. (2004), also included in Eurocode 8 (2005) and Italian Technical Code (2018) for the evaluation of the shear strength of elements subjected to cyclic loads. The contribution of the FRP wrapping on RC elements is evaluated as provided by the Italian Technical Code (2018) considering the additional contribution of FRP reinforcement to shear strength. In the case of columns adjoining to infills, the interaction between the infill and the reinforced concrete frame induces an additional shear demand that has been evaluated according to Di Trapani and Malavisi (2019).

4. Case study test of the proposed framework

The proposed framework can be interfaced with any FE software handling non-linear static analysis but, for the current application, the OpenSees software platform has been utilized. Frame elements are modelled by adopting distributed plasticity force-based elements with five Gauss-Lobatto integration points present inside *OpenSees*.

Concrete elements are modelled using a *Concrete02* uniaxial material model. *Concrete02* material is combined with *MinMax* material to model the crushing of the concrete fibers. Steel rebars are modelled using the *Steel02* Giuffrè-Menegotto-Pinto material model. The confined concrete model adopted for RC elements with and without retrofitting is the standard confined parabola-rectangle model, evaluated according to Eurocode 8 (2005) and the Italian Technical Code (2018). The confining effect of FRP retrofitting is introduced by modifying the constitutive model of concrete fibres and, for sake of simplicity, it is assumed that it is extended to the whole cross-section.

Steel bracings are modelled using truss elements available in OpenSees. The steel is modelled by adopting *Steel02* elastic-plastic with isotropic strain hardening (*Giuffrè-Menegotto-Pinto* constitutive model). Steel elements are assumed to have a circular cross-section whose diameter is defined by the decision variable $Ø_{br}$.

Infills are modelled as fibre-section struts according to the model by Di Trapani et al. (2018). Since this model provided that a parabolic linear-softening constitutive law should be used, they are modelled using *Concrete02*. The achievement of the DLLS condition is supposed to take place when the stress inside the most compressed infill exceeds half of the peak stress ($\sigma_{cr} = 0.50 \cdot f_{md0}$).

4.1. Details on the reference structure

The efficiency of the proposed framework is analyzed by executing the retrofitting optimization for an RC building having a typical structural configuration characteristic of constructions designed before the entry into force of seismic guidelines. The building is a five-storey reinforced concrete frame structure presenting unidirectional frames (Fig. 2). Reinforcement details of beams and columns are reported in the following Table 1. Dimensions in-plan of the structure, together with the cross-section sizes of RC elements are represented in Fig.2(b).

Table 1. Geometrical dimensions and reinforcement details of RC elements

RC member	*b* (mm)	*h* (mm)	Longitudinal reinforcement	Transversal reinforcement
Beams	800	500	4+4Ø18	Ø6/200 mm
Inner columns	550	550	8Ø16	Ø6/200 mm
External columns	450	450	8Ø12	Ø6/200 mm

Reinforced concrete elements are assumed to be made of poor resistance concrete having average un-confined cylindrical strength f_{c0} = 20 MPa and steel rebars with nominal average yielding strength f_y = 455 MPa and strain hardening ratio that is supposed to equal to η = 0.01. According to Di Trapani et al. (2018), a parabolic-linear softening constitutive law is involved to model the mechanical behaviour of the infills. These are located on side frames and they are supposed to be made of clay hollow masonry having thickness t = 250 mm, elastic modulus E_m = 6400 MPa, compressive strength fm = 8.66 MPa and shear strength f_{vm} = 1.07 MPa. Thus, the peak stress is supposed to be equal to f_{md0} = 1.88 MPa, peak strain ε_{md0} = 0.0013, ultimate stress f_{mdu} = 0.86 MPa, and ultimate strain ε_{mdu} = 0.0073.

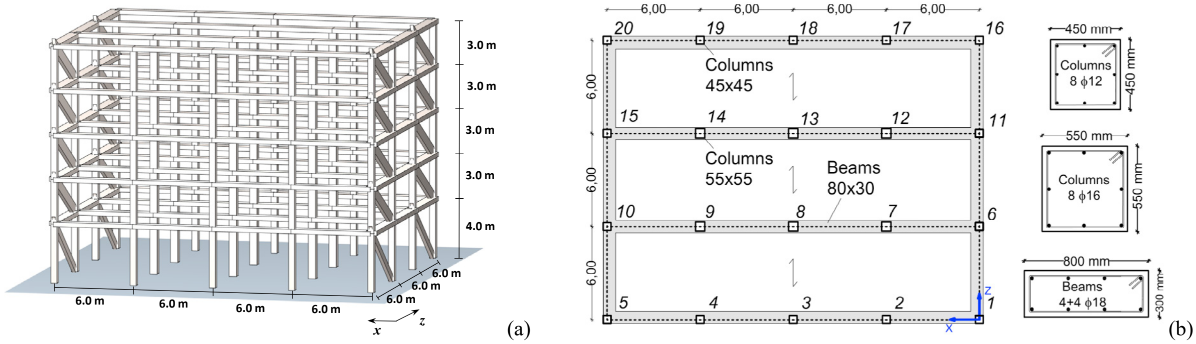

Fig. 2. Geometrical dimensions of the reference structural model

As regards seismic hazards, the building is supposed to be located in Cosenza (Italy), soil type C, and nominal life (V_N) is 50 years (design ground acceleration a_g = 0.271g). Vertical loads are modelled as point loads applied to top nodes of columns as a function of the respective tributary areas in-plan. Rigid diaphragm behaviour is imposed on every floor. Pushover analysis is performed by considering only a uniform profile of lateral loads acting along the z-direction of the structure, which is supposed to be the most vulnerable to seismic actions.

4.2. Preliminary analyses

A preliminary assessment of the as-built structure has been performed to define the reference structural performance for comparing optimization results. Results are shown in Table 2, showing that the as-built configuration safety factors related to DLLS and LSLS are smaller than unity ($\zeta_{E,DLLS}$ = 0.57 and $\zeta_{E,LSLS}$ = 0.15). This leads to an EAL value that is equal to 8.1625 %RC which is significantly greater than the one associated with the code-compliant building (EAL_{ccb} = 1.13%). The structure shows both reduced ductility and vulnerability on damage limit states, therefore seismic retrofitting interventions are needed. The significant reduced life-safety structural performance is caused by the relevant shear deficiency caused by the insufficient transverse reinforcement in columns and the increase in shear demand exerted by the interaction between concrete frame and infills.

Table 2. Structural performance of the as-built configuration

$\zeta_{E,DLLS}$	$\zeta_{E,LSLS}$	λ_{DLLS}	λ_{LSLS}	EAL (%RC)
0.5688	0.15	0.0975	0.1	8.1625

The retrofitting system is composed of FRP wrapping of columns and concentric steel bracings. The FRP fabric has a thickness of $t_{f,1}$ = 0.337 *mm* per layer, elastic modulus E_f = 230 *GPa*, ultimate stress referred to as the net area of the fibres $f_{fib,k}$ = 3250 *MPa* and ultimate strain ε_{fib} = 1,3%. For the implementation of FRP wrapping, it is assumed that rounding of the column edges with a radius equal to r_c = 25 *mm*.

The bracings are supposed to be made of S275 structural steel with f_{yb} = 275 *MPa*, elastic modulus E_{sb} = 210 *GPa*, and strain hardening ratio η = 0.01.

Since the structure has a double symmetry in-plane, the bracings are defined symmetrically on the two external transversal frames. In this way the n_{br} is the number of floors where the bracing systems are defined, starting from the ground floor. To reduce the design space dimension decreasing the computational burden of the analysis, the optimization processes have been restricted to a limited number of columns and a limited number of frames for the bracings.

4.3. Optimization results

The analysis was carried out starting from a first-generation containing 100 tentative solutions generated according to Di Trapani et al. (2021).

Table 3. Parameters of the framework set up for the case study analysis

Population size	Number of offspring	Tournament size	Mutation ratio	Max number generations	Max stall	Dimension of p_{pr} space	Prob. retrofitting element in p_{pr} space
100	100	3	2%	25	5	50	90%

The first half of these individuals are generated randomly, the second half is generated with a high probability of FRP retrofitted elements (p_{pr} = 90%) and other parameters (n_{br}, $Ø_{br}$, s_{FRP}, n_{FRP}) that are randomly selected. The algorithm proceeds by generating 100 new offspring every generation choosing the parents through a tournament selection of three randomly selected parents. In the following Table 3, a summary of the GA framework parameters is reported.

Results of the optimization are shown in Figure 3 in terms of pushover and EAL curves. The optimal solution is characterized by steel bracing retrofitting on the external frames for the first two floors.

Table 4. Case studies optimization analysis results

n_{br} (#)	$Ø_{br}$ (mm)	n_c (#)	s_{FRP} (mm)	n_{FRP} (#)	$\zeta_{E,DLLS}$ (-)	$\zeta_{E,LSLS}$ (-)	EAL (%RC)	Fitness (€)
2	50	8	300	1	1.024	2.297	1.016	28 686

All the columns of these external frames are also retrofitted by 1 layer of FRP with a spacing of 200 mm. The bracings of the optimal configuration have a diameter ($Ø_{br}$ = 50 mm). Among the two intervention systems considered for this application, the bracings are those designed to increase the lateral deformation stiffness, so they are the only ones that allow increasing the DLLS safety factor. In addition, the FRP on the columns of the external frames where the higher shear demand is required due to the interaction between concrete frames and infills.

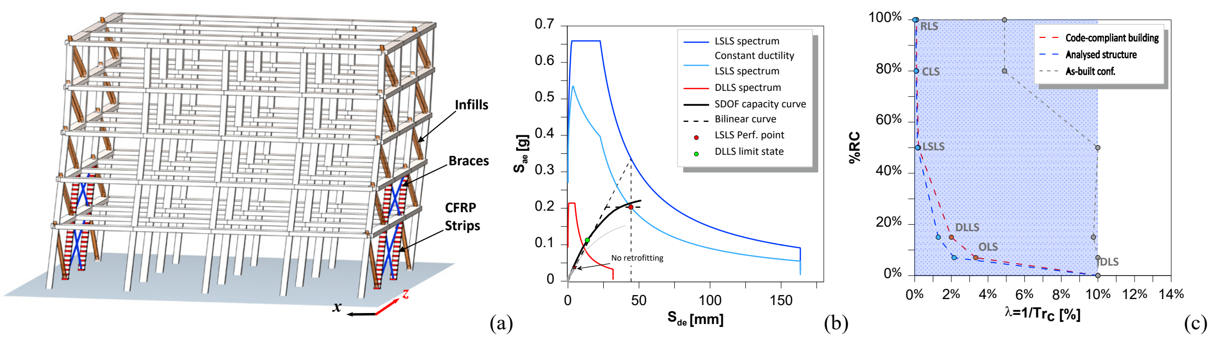

Fig. 3. Optimal solution: (a) retrofitting configuration, (b) pushover curve in ADRS plane, (c) EAL curve

The overall cost of this retrofitting intervention arrangement is 56 981€. The increase in stiffness due to the retrofitting system leads to a reduced displacement demand, which combined with the ductility provided by the steel bracing and FRP on the columns of the external frames allows the structure to satisfy both LS and DL limit states.

As reported in Table 4, the safety factor related to the damage limit state is barely close to the unity ($\zeta_{E,DLLS}$ = 1.024) whereas the safety factor related to LSLS is $\zeta_{E,LSLS}$ = 2.297. The EAL curve displayed in Fig. 4.c shows a significant reduction concerning as-built configuration, resulting in EAL = 1.016%. So, the proposed framework has significantly improved the quality of the retrofitting design by providing a cost-optimized intervention with a control on the EAL.

5. Conclusions

The paper has presented a novel optimization framework that aims to minimize costs for the implementation of seismic retrofitting in RC frame structures. The framework is based on a genetic algorithm developed in MATLAB® which is connected with a 3D fibre-section model developed in OpenSees. Two different typologies of the retrofitting system are considered: FRP jacketing of columns and steel bracings. The main target of the algorithm is to seek the retrofitting arrangement that optimizes the intervention costs and, in an indirect way, takes into account the expected annual loss value referring to that requested by the reference technical codes. The performance of each tentative solution is evaluated starting from the results of pushover analysis in the framework of the N2 method. Through a case study implementation, it has been proved that the proposed framework can efficiently pinpoint optimal retrofitting configuration. Wide usage of optimization techniques for the retrofitting of a single structure leads to better management of the funds allocated to seismic reinforcement of existing structures enhancing the overall structural safety of building heritages.

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