Investigating the potentials and limitations of capillary-fed vapor generators: A heat and mass transfer study

SUPPLEMENTAL MATERIAL
Investigating the potentials and limitations of capillary-fed vapor generators: a heat and mass transfer study

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1. SUPPLEMENTAL NOTE 1 - Heat transfer through the evaporator

Considering a generic control volume Ω and its time-fixed boundary $\partial\Omega$, the partial differential equations describing the thermal problem can be derived exploiting the Eulerian approach. In detail, the first law of thermodynamics applied to the control volume (here composed by the pure fluid), assuming zero the contribution related to technical work, is:

$$\frac{d}{dt}\int_{\Omega}\left(\rho c_p\right)_{eff} T = -\oint_{\partial\Omega}\sum_i \mathbf{\Phi}_i \cdot \mathbf{n} \tag{1}$$

where $\left(\rho c_p\right)_{eff}$ and T represent the effective heat capacity and the temperature of the control volume, respectively. Thus, the term on the left-hand side of the equation is the total change of internal energy in the control volume Ω. The term on the right-hand side represents instead the energy which flows through the borders, that is the scalar product $\mathbf{\Phi}_i \cdot \mathbf{n}$ between the generic thermal flow vector $\mathbf{\Phi}_i$ and the local normal versor $\mathbf{n}$ to the border $\partial\Omega$. The latter is defined positive if outgoing. Note that, the tangential component of flow at the border $\partial\Omega$ does not contribute in terms of energy variation in the control volume. Below, the considered thermal contributions are introduced:

$$\mathbf{\Phi}_{diffusive} = -\lambda_{eff}\,\nabla T \tag{2}$$

$$\mathbf{\Phi}_{advective} = \dot{\mathbf{m}}\, c_{p,w}\, T \tag{3}$$

$$\mathbf{\Phi}_{in} = \mathbf{q}_{in} \tag{4}$$

$$\mathbf{\Phi}_{evaporative} = \dot{\mathbf{m}}_{evap}\, h_{gl}(T) \tag{5}$$

where λ_{eff} is the effective thermal conductivity, $h_{gl}(T)$ the enthalpy of vaporization at a given temperature, and $c_{p,w}$ the specific heat of water. Moreover, $\dot{\mathbf{m}}$ and $\dot{\mathbf{m}}_{evap}$ are the specific mass flow rate vector flowing through the porous medium and the specific evaporating mass flow rate evaluated in [kg m^{-2} s^{-1}]. Then, $\mathbf{q}_{in}$ is the incoming absorbed thermal energy. Substituting into the general equation, it is:

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$$\frac{d}{dt}\int_{\Omega}\left(\rho c_p\right)_{eff} T = -\oint_{\partial\Omega}\left(\mathbf{\Phi}_{diffusive} + \mathbf{\Phi}_{advective}\right)\cdot\mathbf{n} + \\ -\oint_{\partial\Omega}\left(\mathbf{\Phi}_{in} + \mathbf{\Phi}_{evaporative}\right)\cdot\mathbf{n} \tag{6}$$

Since the boundary Ω has been assumed fixed over time, the previous equation can be rewritten as:

$$\int_{\Omega}\frac{d}{dt}\left(\left(\rho c_p\right)_{eff} T\right) = -\oint_{\partial\Omega} -\lambda_{eff}\,\nabla T\cdot\mathbf{n} - \oint_{\partial\Omega}\dot{\mathbf{m}}\,c_{p,w}\,T\cdot\mathbf{n} + \\ -\oint_{\partial\Omega}\dot{\mathbf{m}}_{evap}\,h_{gl}(T)\cdot\mathbf{n} - \oint_{\partial\Omega}\mathbf{q}_{in}\cdot\mathbf{n} \tag{7}$$

Exploiting the Gauss's theorem, the surface integrals can be replaced by the volume integrals:

$$\int_{\Omega}\frac{d}{dt}\left(\left(\rho c_p\right)_{eff} T\right) = \int_{\Omega}\nabla\cdot\left(\lambda_{eff}\,\nabla T\right) - \int_{\Omega}\nabla\cdot\left(\dot{\mathbf{m}}\,c_{p,w}\,T\right) + \\ -\int_{\Omega}\nabla\cdot\left(\dot{\mathbf{m}}_{evap}\,h_{gl}(T)\right) - \int_{\Omega}\nabla\cdot\mathbf{q}_{in} \tag{8}$$

Since the derived law is valid for any volume Ω, small at will, it is possible to write:

$$\frac{d}{dt}\left(\left(\rho c_p\right)_{eff} T\right) = \nabla\cdot\left(\lambda_{eff}\,\nabla T\right) - \nabla\cdot\left(\dot{\mathbf{m}}\,c_{p,w}\,T\right) + \\ -\nabla\cdot\left(\dot{\mathbf{m}}_{evap}\,h_{gl}(T)\right) - \nabla\cdot\mathbf{q}_{in} \tag{9}$$

Considering half of the horizontal evaporator illustrated in Fig. 1, thus characterized by a width B, length L and thickness s, completely wet by water, with $\dot{\mathbf{m}}_{evap}$ and $\mathbf{q}_{in}$ purely orthogonal to the horizontal plane, it is possible to write the 1-D thermal balance equation:

$$\left(\rho c_p\right)_{eff}\frac{\partial T}{\partial t} = \lambda_{eff}\frac{\partial^2 T}{\partial x^2} - c_{p,w}\frac{\partial\left(\dot{m}\,T\right)}{\partial x} + \\ -\frac{\dot{m}_{evap}}{s}h_{gl}(T) + \frac{q_{in}}{s} \tag{10}$$

which is further developable as:

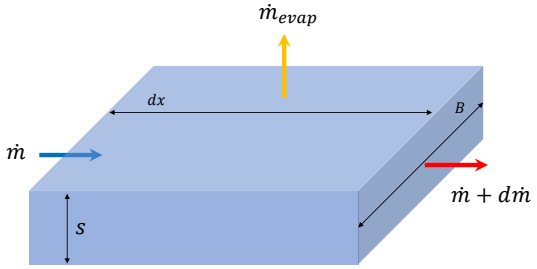
$$\left(\rho c_p\right)_{eff}\frac{\partial T}{\partial t} = \lambda_{eff}\frac{\partial^2 T}{\partial x^2} - \frac{\partial\dot{m}}{\partial x}\,c_{p,w}\,T - \dot{m}\,c_{p,w}\frac{\partial T}{\partial x} + \\ -\frac{\dot{m}_{evap}}{s}h_{gl}(T) + \frac{q_{in}}{s} \tag{11}$$

Then, applying the conservation of mass to the infinitesimal volume shown in Supplemental Fig. 1 we get:

$$\dot{m}\,B\,s = (\dot{m} + d\dot{m})\;B\,s + \dot{m}_{evap}\,B\,dx \tag{12}$$

which becomes:

$$\frac{d\dot{m}}{dx} = -\frac{\dot{m}_{evap}}{s} \tag{13}$$



Supplemental Figure 1: Scheme of an infinitesimal control volume of the evaporator.

Thus, the 1-D energy balance equation finally becomes:

$$\left(\rho c_p\right)_{eff} \frac{\partial T}{\partial t} = \lambda_{eff} \frac{\partial^2 T}{\partial x^2} - \dot{m}\, c_{p,w} \frac{\partial T}{\partial x} + - \frac{\dot{m}_{evap}}{s}\left[h_{gl}\left(T\right) - c_{p,w}\, T\right] + \frac{q_{in}}{s} \tag{14}$$

In order to solve the differential equation obtained, it is necessary to express the specific evaporating mass flow rate $\dot{m}_{evap}$ (kg m^2 s^{-1}) as a function of temperature. Here, the mass evaporation rate can be interpreted via the mass transfer coefficient K_v, due to partial pressure gradient, and the partial pressure driving force:

$$\dot{m}_{evap} = K_v\, \Delta p_v \tag{15}$$

With the aim of establishing whether or not the study can be simplified by assuming a constant temperature throughout the evaporator, the steady-state thermal problem is considered:

$$\lambda_{eff} \frac{\partial^2 T}{\partial x^2} - \dot{m}\, c_{p,w} \frac{\partial T}{\partial x} - \frac{\dot{m}_{evap}}{s}\left[h_{gl}\left(T\right) - c_{p,w}\, T\right] + \frac{q_{in}}{s} = 0 \tag{16}$$

In order to solve the differential equation obtained, we resort to the Matlab's solver ode15s, adequate when dealing with differential equations of type "stiff". The input solver characteristic odefun of the associated differential system is:

$$odefun = \begin{cases} u' = v \\ v' = \frac{c_{p,w}}{\lambda_{eff}}\, w\, v - w' \left(\frac{h_{gl}(u) - c_{p,w}\, u}{\lambda_{eff}}\right) - \frac{1}{\lambda_{eff}} \frac{q_{in}}{s} \\ w' = -\frac{\dot{m}_{evap}}{s} \end{cases} \tag{17}$$

where $u = T$,$v = \frac{dT}{dx}$ and $w = \dot{m}$. Then, it is necessary to impose three boundary conditions:

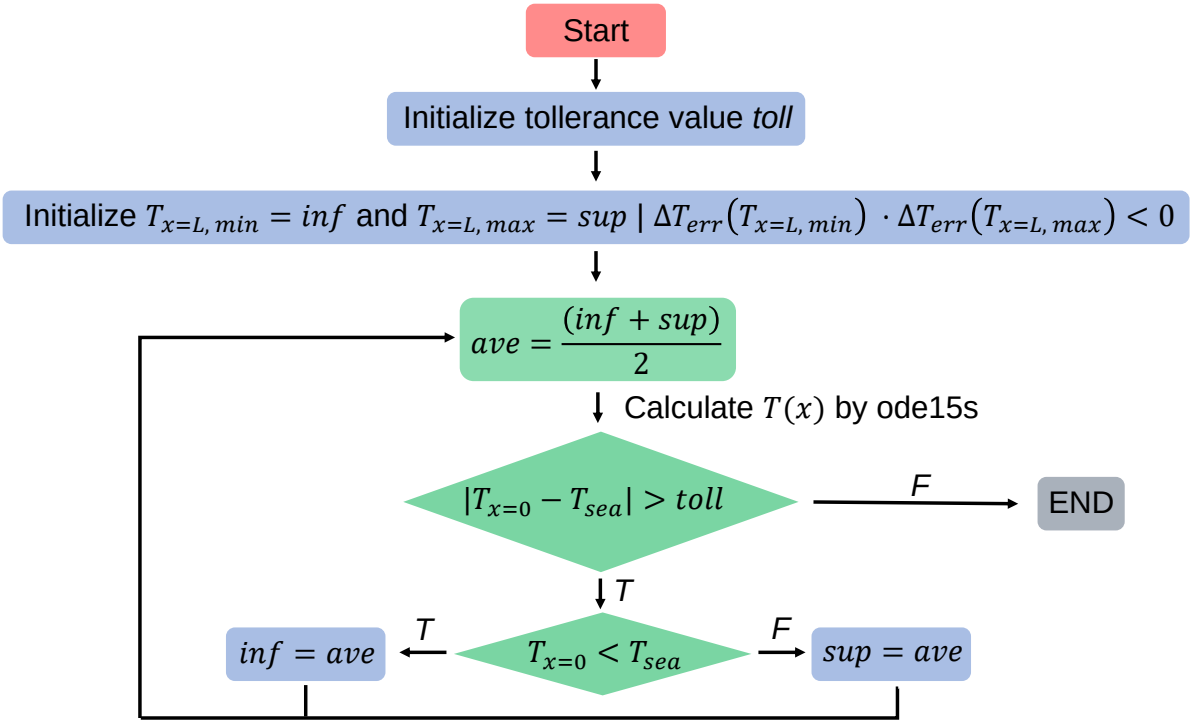
1. Dirichlet boundary condition: fixed temperature at the inlet of the horizontal evaporator, namely $T_{x=0,\ z=H} = T_{sea}$, as the sea is considered an infinite heat capacity;

2. Neumann boundary condition: $\frac{dT}{dx}\Big|_{x=L,\ z=H} = 0$, because of the symmetry;

3. Dirichlet boundary condition: $\dot{m}\Big|_{x=L,\ z=H} = 0$, because of the symmetry.

The numerical integration starts in $x = L$, where guessed initial value of T is imposed. Then, the resulting temperature profile and in particular the solved temperature value at $x = 0$ is compared with the value imposed by

the Dirichlet boundary condition, namely T_{sea}. The guessed value $T_{x=L}$ is iterated until $T_{x=0} \approx T_{sea}$ is respected. The bisection method has been implemented for iterating and finding the solution to the problem, as described below and reported in the flowchart of Supplemental Fig. 2. In detail, the iterative solution is described below:



Supplemental Figure 2: Flowchart showing the iterative solution method employed.

- two guessed temperature values are defined: the lower one (namely, inf) and the upper one (namely, sup);
- the average value ave between inf and sup is calculated. ave is the initial guessed value, applied in $x = L$, which is used as input for ode15s.
- Ode15s numerically calculate the temperature distribution in the evaporator. Then, the error $\Delta T_{err,\%} = \frac{T_{x=0} - T_{sea}}{T_{sea}}\%$, obtained considering the attempted value $T_{x=L}$ equal to ave, is calculated;
- if the value $\Delta T_{err,\%}$ is larger than the fixed tolerance, the moving extremes inf and sup must be updated and the iteration continues. In particular, if the tolerance is not respected and $T_{x=0} < T_{sea}$, the mobile extreme inf is updated and replaced with ave value. If the tolerance is not respected and $T_{x=0} > T_{sea}$, the upper moving extreme sup is set equal to ave.

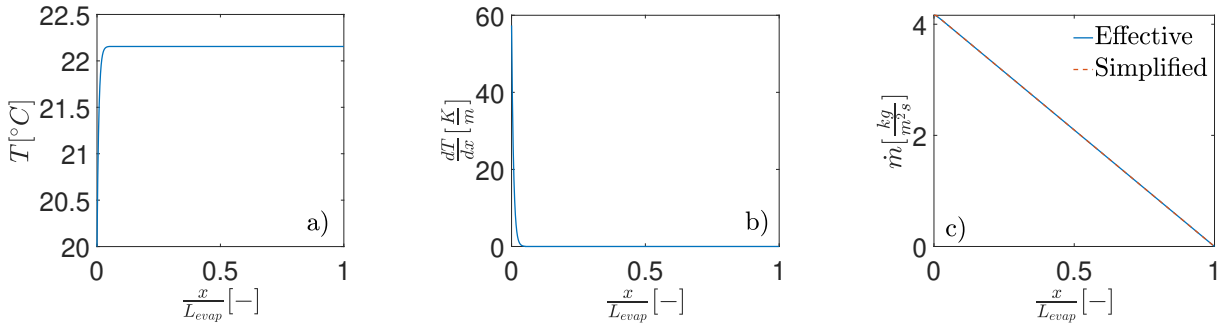
In order to obtain a unique solution, $\Delta T_{err} = T_{x=0} - T_{sea}$ should show a monotonic trend when the value $T_{x=L}$ is iterated. Besides, ΔT_{err} must also assume opposite values in sign at the extremes of the designated temperature range $T_{x=L,min}$ and $T_{x=L,max}$. The two extreme values of temperatures $T_{x=L,min}$ and $T_{x=L,max}$ should first be initialized, so that:

$$\Delta T_{err}(T_{x=L,min}) \cdot \Delta T_{err}(T_{x=L,max}) < 0 \tag{18}$$

The used tolerance value $\Delta T_{err,\%}$, which determines the end of the itarative procedure, is:

$$\frac{T_{x=0} - T_{sea}}{T_{sea}} = \frac{\Delta T_{err}}{T_{sea}} \approx 0.1\% \tag{19}$$

where the value is selected to avoid excessive computational time while still ensuring acceptable accuracy.



Supplemental Figure 3: Heat transfer in the horizontal evaporator. a) Temperature profile versus the normalized distance from the inlet of the horizontal evaporator; b) Temperature gradient versus the normalized distance from the inlet of the horizontal evaporator; c) Effective and simplified (namely evaluated considering a constant temperature throughout the evaporator) specific mass flow rate versus the normalized distance from the inlet of the horizontal evaporator. The distance x is normalized to half of the length of the evaporator L.

2. SUPPLEMENTAL NOTE 2 - Linear momentum balance equation

Having defined a local fixed reference system, characterized by the Eulerian coordinate ζ with its origin at the inlet section of the horizontal evaporator (see Fig. 1), the wicking velocity variation of the fluid $\Delta\dot{x}(\zeta)$ due to the evaporation can be expressed as:

$$\Delta\dot{x}(\zeta) = \frac{\dot{M}_{evap,cumulative}(\zeta)}{\rho B s \epsilon} = \frac{\frac{q_{in} B \zeta}{h_{gl}(T) - c_{p,w}\, T}}{\rho B s \epsilon} = \frac{1}{\rho\epsilon}\frac{\frac{q_{in}}{s}}{h_{gl}(T) - c_{p,w}\, T}\zeta \tag{B1}$$

where the same variables and notation used in Eqs. 13 and 14 are exploited. In this Appendix, the integral terms included in the system of equations 14 able to describe the fluid flow in the L-shaped wick-based component are developed in detail. The development takes into account that $\dot{\zeta} = 0$, being the derivative of an Eulerian coordinate fixed in time. The integral inertial term is:

$$\begin{aligned}\int_0^{x_{front}} \ddot{x}(\zeta) d\zeta = \\ = \int_0^{x_{front}} \frac{d}{dt}\left(\dot{z}_{inlet} - \frac{1}{\epsilon\rho}\frac{\frac{q_{in}}{s}}{h_{gl}(T) - c_{p,w}\, T}\zeta\right) d\zeta \\ = \int_0^{x_{front}} \left(\ddot{z}_{inlet} - \frac{1}{\epsilon\rho}\frac{\frac{q_{in}}{s}}{h_{gl}(T) - c_{p,w}\, T}\dot{\zeta}\right) d\zeta\end{aligned} \tag{B2}$$

which becomes:

$$\int_0^{x_{front}} \ddot{x}(\zeta) d\zeta = \ddot{z}_{inlet} x_{front} \tag{B3}$$

Then, the integral term related to viscous losses in the horizontal evaporator is:

$$\int_0^{x_{front}} \dot{x}(\zeta)d\xi = \tag{B4}$$
$$= \int_0^{x_{front}} \left(\dot{z}_{inlet} - \frac{1}{\epsilon\rho} \frac{\frac{q_{in}}{s}}{h_{gl}(T) - c_{p,w}\, T} \zeta \right) d\zeta$$
$$= \left(\dot{x}_{front} + \frac{1}{\epsilon\rho} \frac{\frac{q_{in}}{s}}{h_{gl}(T) - c_{p,w}\, T} x_{front} \right) x_{front} +$$
$$- \frac{1}{2\epsilon\rho} \frac{\frac{q_{in}}{s}}{h_{gl}(T) - c_{p,w}\, T} x_{front}^2$$

Finally, by expliciting these terms in the system of Eqs. 14, we can derive the equation Eq. 19.

3. SUPPLEMENTAL NOTE 3 - On the dynamic pressure contribution

Here, the purpose is to demonstrate the negligible effect of the dynamic pressure term in Eq. 23:

$$0 = d - \left(\frac{1}{2} \dot{x}_{front} + b x_{front} \right) \dot{x}_{front} - \frac{1}{2} b c x_{front}^2 \tag{C1}$$

where the constant b (assuming that the radius of the capillary is of the order of millimeters) is equal to:

$$b = \frac{\epsilon\mu}{\rho K} = \frac{\epsilon\mu}{\rho\epsilon\Gamma R^2} \approx \frac{1.2 \cdot 10^{-5}}{R^2} \tag{C2}$$

Therefore, b ranges from 50 to 5000 s^{-1}, in case of D = 1 mm and 100 μm, respectively. In turn b multiplies x_{front}, which is of the order of meters. The term $\frac{1}{2}\dot{x}_{front}$, except in the very first moments, is instead of the order of a few mm s^{-1}, i.e. thousandths of m s^{-1}, tending to 0 as the flow progresses. It is therefore negligible for the calculation of impregnation times.