Using a mean changing stochastic processes exit-entry model for stock market long-short prediction

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JEL Codes: B26, G01, G12, G15, G41

Keywords: mean changing model, stochastic processes, Apple Computer stock, trend following strategies, bubble asset price exits, stock market crashes, errors in mean estimates, portfolio optimization, Covid-19 2020 era

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1 Introduction

Stochastic processes is one of the key operations research tools for analysis of complex phenomenon. This paper presents a unique application of a mean changing model in stock

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markets. The model is applied to Apple Computer Stock (AAPL), the S&P500 and various equity stock indices in the US, Japan, Iceland and China that had large crash declines. There are various models to predict various stock market falls and rises. What we do here is not predict the fall or rise but actually determine a good time to exit or go long at good prices, so it successfully implements market timing.

To begin we discuss why AAPL is an important stock to analyze, briefly discuss the stochastic process optimization and review the importance of accurate estimates of the mean that will lead us to the stochastic process mean changing model and its applications.

1.1 The importance of Apple Computer stock

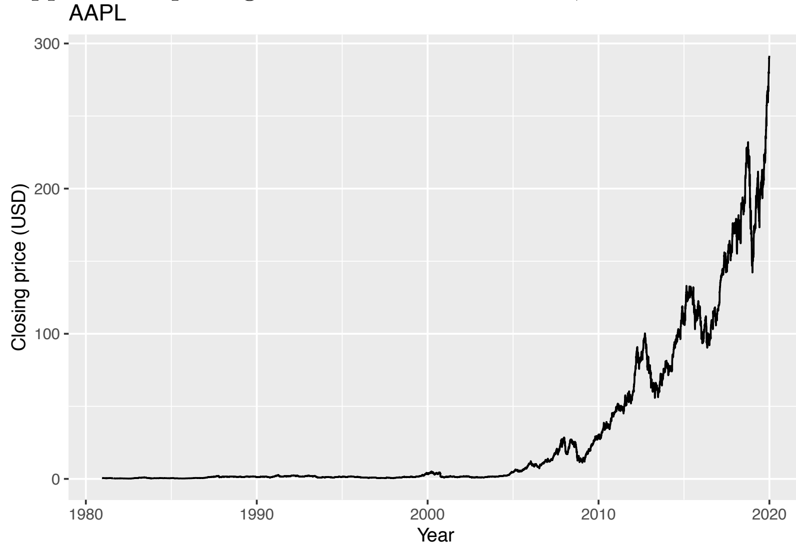
Warren Buffett has said of AAPL that is it ”probably the best business I know in the word” (Buffett, 2020). AAPL's market cap in May 2021, was $2.09 trillion, the most valuable US company in history, only exceeded worldwide by Saudi Aramco. Buffett is a major stock holder through Berkshire Hathaway which holds about 5.4% (905,552,761 shares) of the company's stock worth $120.4 billion. This is about 43% of Berkshire's exchange traded equity holdings and about 18.3% of Berkshire's total assets which were about $658 billion in May 2021. These shares cost $31.09 billion so there is a gain of nearly $90 billion plus the dividends of about 0.7% yearly (data from the 2020 annual report as of December 31, 2020). AAPL is 6% of the S&P500. Ziemba (2005) showed that Buffett acts like a full Kelly investor with very positions in a few stocks like AAPL. This is known to maximize long run wealth almost surely, see MacLean, Thorp and Ziemba (2010).

The price of AAPL increased greatly since its start in April 1976 with shares trading since December 1980. A good decision was to hold the stock forever and its price has increased from $22 at its IPO price on December 12, 1980 ($0.41 adjusted) or splits to $293.65 at the close on December 31, 2019, namely, a gain of 71,918% or 18.4% per year in its geometric mean. See Figure 1 for the graph. In 2020, the stock moved even higher, closing May 31 at $317.94, July 31 at 425.04. And despite the Covid-19, it closed May 15, 2021 at $127.45 equivalent to $509.80 pre 4-1 stock split in August 2020.

AAPL in 2012 had a PE in the 9-10 area. The stock price and PE have risen sharply since then. AAPL in 2019 doubled in price with no real earnings growth but only an expansion in its PE multiple. It went higher in 2021. However, the market has changed and what was a high PE in the past now reflects the low and essentially zero interest environment by a Fed involved with enormous monetary easing due to the Covid-19 pandemic. The earnings yield of AAPL of about 41.33 give an earnings yield of 2.42% plus dividends of about 0.7% is above the interest rate used to price stocks so the 41.33 can be assumed to not be too expensive and the PE by itself may be an inadequate indicator in May 2021.

Along the way, the company and the stock have had many ups and downs. For example,

Figure 1: Apple Inc.'s price growth from December 12, 1980 to December 31, 2019



Shiryaev, Zhitlukhin and Ziemba (2014) showed that in 2012 a bubble exit point model exited the stock around 630-640 depending on the entry price and date, about 90% of its maximum price which was 705.07 on September 21, 2012. The stock later fell to the 380 area and then went to greater highs and other lows. The goal of this paper is to explore whether or not a mean changing exit model was superior to buy and hold and as a strategy to exit bubble like markets at good times and prices. The model is applied to for AAPL and the S&P500 in the Covid era and other times. It is known that mean estimation accuracy is much more crucial to portfolio success than variance or covariance. This dependence is risk aversion sensitive. Low risk aversion utility functions are associated with much more importance of accurate mean estimates, see section 1.4.

1.2 Bubble like markets

We study bubble-like markets, which are ones that are going up just because they are going up and often result eventually in a crash. We discuss a stochastic process mean changing strategy to optimally exit such markets. Trading bubbles is difficult and even the best traders sometimes lose a lot of money by shorting too soon. The famed bubble shorter George Soros made billions successfully shorting overpriced markets. But even he lost shorting too soon the Japanese stock market in 1989 and the Nasdaq 100 in 2000. These were two times when the BSEYD model predicted equity market crashes, see Ziemba et al (2018).

The finance and economics literature on timing bubbles includes Stiglitz (1990), Evanoff et al (2012 and 2018) and Kindleberger and Aliber (2015). Sornette (2003, 2017), Sornette and Ouillon (2012) and Schatz and Sornette (2020) studied bubbles and crashes using physics ideas. Evgenidis and Malliaris (2020), Mishkin (2008) discuss extensive literature and ways to try to eliminate or lessen financial bubbles using interest rates and other measures. Goetzmann et al (2016) used surveys to estimate probabilities of market crashes and showed that actual crashes occur less often than people expect. Gresnigt et al (2015) view stock market crashes as earthquakes. Jarrow and Protter (see Jarrow et al, 2007, 2010, Jarrow, 2016, Protter (2013, 2016) proposed bubble testing theory as do Phillips, Shi and Yu (2015). Altman and Kuehne (2016) and Altman (2020) studied bubbles in credit markets. Reinhart and Rogoff (2009) discuss economic crises that might involve bubbles. Ziemba, Lleo and Zhitlukhin (2018) discuss several models that have predicted large crashes in the past but none of them predicted the 2020 Covid-19 crash in its low interest rate environment. Consigli et al. (2009) and Lleo and Ziemba (2012, 2019a,b) discuss crashes in various markets and possible methods to predict that they will occur.

1.3 Changepoint detection and bubble markets

The idea of the model is sketched in section 2 and is an application of the *changepoint detection* theory for stochastic random processes. The mathematics of the model is in the Appendix.

Changepoint detection methods have been successfully applied in production quality control, radiolocation, information security, and have shown their usefulness. Their history goes back to the pioneering works of Shewhart of 1920s, and the first results by Page, Roberts, Shiryaev and others in 1950-60s. Surveys of the history and the recent developments in this field can be found in the books by Poor and Hadjiliadis (2009) and Shiryaev (2019).

We present a stochastic process mean changing strategy that seems to work well timing when to exit or enter a long or short position. The basic idea is that there is a fast rate of growth in prices, reaching a peak and then a fast decline. The model tries to exit near the peak in prices. The bubble exit model we use for our analysis attempts to determine when the mean return changes direction. The idea of the model is to exit when the mean turns negative and enter a short position until the model indicates a mean change to a positive direction. By a bubble we mean that the rising price is based on the expectation of higher future prices which is observed by a rising PE ratio.

A major strength of this method is that we learn about current price action from past price action. We do not need to know the fundamental cause for a change in drift. In particular, we sidestep entirely the debate about what a bubble is, whether they exist and how to define and classify them and get right to a strategy to exit the market at a good time and price.

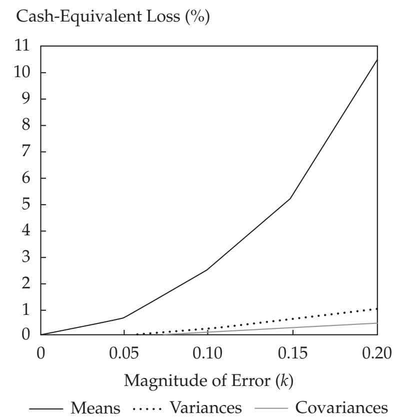
1.4 Review of the importance of accurate mean estimates in portfolio selection problems

There are theoretical and empirical results that indicate that mean estimation is much more important than variance or covariance estimation in portfolio selection problems. Hanoch and Levy (1969) proved that the mean dominates in a choice between differing assets if the cumulative distributions cross only once. Namely, if $X \sim F(\cdot)$ with the higher mean and $Y \sim G(\cdot)$ have cumulative distribution functions that cross only once, but are otherwise arbitrary, then F dominates G for ***all*** concave u. The mean of F must be at least as large as the mean of G to have dominance. Variance and other moments are unimportant. Only the means count. With normal distributions X and Y will cross only once iff the variance of X does not exceed that of Y. That is the basic equivalence of Mean-Variance analysis and Expected Utility Analysis via second order (concave, non-decreasing) stochastic dominance.

In an empirical study, Chopra and Ziemba (1993) showed typical results concerning parameters. Table 1 and Figure 2 show that the errors in means are typically about 20 times the errors in covariances in terms of certainty equivalent (CEL) value and the variances are twice as important as the covariances. Roughly, there is a 20:2:1 ratio in the importance of these errors. This is risk aversion dependent with $T_R = (R_A/2)100$ being the risk tolerance where $R_A = -\frac{u''(w)}{u'(w)}$ for utility of wealth $u(w)$. So for high risk tolerance, that is low risk aversion, the errors in the means are even greater. Hence for utility functions like log of Kelly with essentially zero risk aversion, the errors in the mean can be 100 times as important as the errors in the other parameters.

Table 1: Average Ratio of CEL for Errors in Means, Variances and Covariances

t Risk Tolerance	Errors in Means vs Covariances	Errors in Means vs Variances	Errors in Variances vs Covariances
25	5.38	3.22	1.67
50	22.50	10.98	2.05
75	56.84	21.42	2.68
	↓	↓	↓
	20	10	2
	Error Mean	Error Var	Error Covar
	20	2	1



Source: Based on data from Chopra and Ziemba (1993).

Figure 2: Mean percentage cash equivalent loss due to errors in inputs (Source: Chopra-Ziemba, 1993)

2 A primer on changepoint detection methods in financial time series

Computational statisticians such as Adams and MacKay (2007) routinely test new changepoint detection algorithms on financial time series, which can be in discrete or continuous time. However, financial econometrics has been slow to adopt changepoint methods and consider change points detection as either

- a diagnostic test on parameter stability of fitted models (Brooks 2019, ch 5), or as
- the natural by-product of Markov regime-switching models (Brooks 2019, ch 10).

An exception that we follow here was Shiryaev et al. (2014, 2015) who applied a continuous-time disorder changepoint detection model to identify asset bubbles and determine exit strategies. These were applied to AAPL, the Nasdaq 100 near 2000, and the Japanese stock and golf course memberships bubble in 1989-90.

The parameter changing methods are statistical techniques designed to detect abrupt changes in the statistical properties of a stochastic process or time series, $Y_{t=0}^{T}$ based on a sequence of observed values $y_{0:T} = (y_1, y_2, \ldots, y_T)$ over a given time interval.

A changepoint occurs at time θ if the statistical properties of the sub-sequences $y_0, y_1, \ldots, y_{\theta-1}$ and $y_\theta, y_{\theta+1}, y_{\theta+2}, \ldots, y_T$ differ in their means.

A *changepoint* is a random time when a random sequence (or a random process) changes the structure of its probability distribution. The sequence of independent random variables has one value of the mean before a changepoint and another value of the mean after the changepoint (with all other parameters being the same before and after). The general theory of changepoint detection is not limited to detecting changes in means only; but our model is of this basic type because of its importance for successful portfolio management. This model is derived from one of the pioneering papers, Shiryaev (1963) but adapted to our investigation of bubble-like financial markets. The reader is referred to specialized literature for the state of the art in changepoint detection methods. Tartakovsky, Nikiforov and Basseville (2014), Poor and Hadjiliadis (2009), and Shiryaev (2019) detail the theory.

We consider *online* changepoint detection problems. Observations arrive one by one in a random sequence and it must be determined when a change occurs, if it represents a possible change of the probability distribution. The determination is based on the data available up to the current moment of time. The goal is to recognize the change as soon as possible after it happens, but not earlier.

In the context of bubble-like markets, a changepoint is a moment when the price trend reverses and the observer wants to detect the reversal as soon after it happens.

In the financial context, a changepoint represents a moment when the market starts to decline. It can be identified with a moment of time when the trend of the sequence of the market's index value becomes negative. The objective of the model is to detect this change after it occurs and to close a long position maximizing the gain.

We model bubble-like prices of stock or index values by a random sequence S_t, where time t runs through $t = 0, 1, \ldots, T$; the moment $t = 0$ is the start of observations, $t = T$ is the terminal moment specifying the time horizon of the model. The sequence of prices S_t is assumed to follow a geometric random walk with mean μ_1 and volatility σ_1 before a changepoint, and mean μ_2 and volatility σ_2 after a changepoint. Both pairs (μ_1, σ_1) and (μ_2, σ_2) are assumed to be known or can be estimated. It is assumed there is exactly one changepoint between $t = 0$ and $t = T$, occurring at some time $t = \theta$. Thus, the sequence S_t can be described by its log-returns X_t as follows:

$$X_t := \log \frac{S_t}{S_{t-1}} = \begin{cases} \mu_1 + \sigma_1 \xi_t, & t < \theta, \\ \mu_2 + \sigma_2 \xi_t, & t \geq \theta, \end{cases} \quad \text{for } t = 1, 2, \ldots, T \text{ and } \mu_1 > 0 > \mu_2,$$

where ξ_t are independent standard normal random variables.

Figure 3 shows the idea of a changing mean using typical simulated data illustrating an example of a change in the mean at observation 101.

We work in the Bayesian setting, assuming that θ is a random variable taking values in the set $\{1, 2, \ldots, T+1\}$ with known prior probabilities $p_t = P(\theta = t)$. It is assumed that θ is independent of the sequence ξ_t. The value p_1 is the probability that from the beginning of observation the sequence S follows parameters (μ_2, σ_2) (e.g. the bubble burst before the

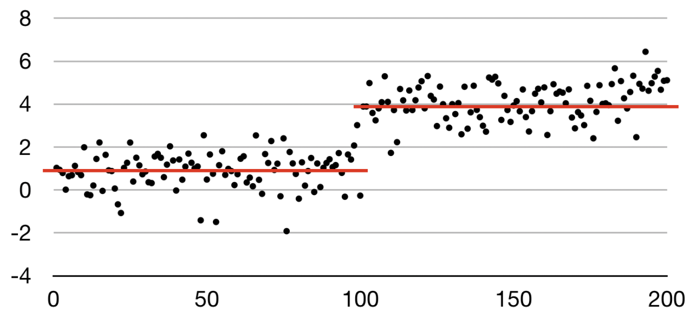


Figure 3: Typical example of a change in mean at observation 101.

beginning of observation), and p_{T+1} is the probability that S_t follows (μ_1, σ_1) until the end of the time horizon of the model (e.g. the bubble continues at least until T).

Under an appropriate criterion of the optimality of a detection rule (the Appendix contains mathematical the details) we show that the optimal detection rule can be expressed through the *Shiryaev-Roberts statistic* ψ_t, which is a sequence constructed recursively as follows:

$$\psi_0 = 0,\ \psi_t = (p_t + \psi_{t-1}) \cdot \frac{\sigma^1}{\sigma^2} \exp\biggl(\frac{(X_t - \mu_1)^2}{2\sigma_1^2} - \frac{(X_t - \mu_2)^2}{2\sigma_2^2}\biggr), \quad t = 1, \ldots, T.$$

The main theorem that we prove in the Appendix states that it is optimal to declare a changepoint when the value of ψ_n exceeds a certain time-dependent threshold $b(t)$:

$$\text{the first time } t \text{ when } \psi_t \geq b(t).$$

where $b(t)$ depends on the parameters of the problem.

The parameters $\mu_1, \mu_2, \sigma_1, \sigma_2$ must be estimated. However, μ_1, σ_1 can be estimated from past data. Since μ_2 and σ_2 cannot be estimated before a changepoint, their values must be chosen in advance, and it might be a difficult problem to predict appropriate values for them. All of our applications shown below indicate that using the simple choice $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$ gives acceptable results.

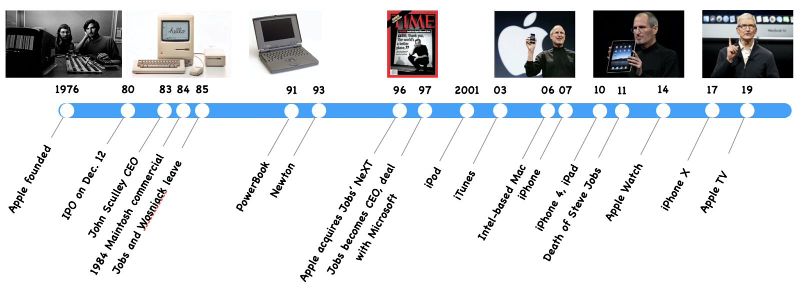
The same applies to the prior distribution of θ. In all the cases where we applied the model it was simply assumed to be uniform over $t = 1, \ldots, T$ with a reasonable choice of probability 0.25 that a there is no changepoint before T. Other choices give similar results.

3 Application of the model to AAPL

3.1 Background on Apple Computer

AAPL has had a tumultuous history with eight CEO's and nearly a bankruptcy in 1997. It had a spectacular rise in price in the 2000's with the firing, return and then death of its founder Steve Jobs. In 2021 the stock is over 5% of the S&P500. Our data includes 9,846 daily share prices to December 2019. Figure 4 has a chronology of some major events from 1976 to current including the sequence of personal computer products and services that it pioneered and is famous as well 5G devices and applications that have begun to come onstream. During the period May 2020 - May 2021 the model was long AAPL and the S&P500.

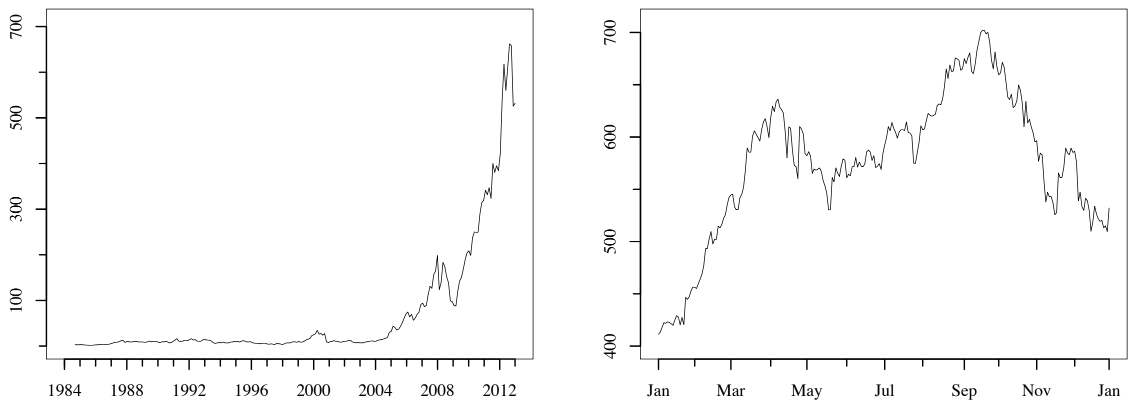
Figure 4: Apple Inc. timeline.



AAPL stock had a spectacular run from the bottom of the 2007-2009 crash in March 2009, see figure 5 which shows the price history from September 1984 to the end of 2012; and the more recent period, from the beginning of 2009 to the end of 2012.

A sequence of valuable and easy to use products inspired by legendary visionary Steve Jobs created huge interest and sales around the world. All of these products had high margins which accumulated large cash levels. In November 2012 they had $121 billion in cash or $128 per share of the 941 million shares outstanding. The company generated cash faster than any corporation in history. The stock was historically at a low price earnings ratio. It was a favorite of hedge funds, open and closed mutual funds, ETFs and various small and large investors. It was traded as a proxy for the market with high liquidity. Its forward price earnings ratio in November 2012 was 10.17 with estimated earnings per share of $49.28. The company had a quarterly dividend of $2.65 per share and a buy back of about $10 million in stock.

Steve Jobs left Apple in 1985 after a power struggle with John Sculley. Jobs recruited Sculley from Pepsi, asking him "do you want to sell sugared water all your life or change



(a) From September 1984 to the end of 2012. (b) In 2009–2012.

Figure 5: The history of AAPL stock price (adjusted for dividends and splits to 2012).

the world". After Sculley came to Apple, he and Jobs had a disagreement on strategy and marketing which stagnated the company. The board favored the marketer over the genius. Jobs sold all but one of his AAPL shares. The company languished while he continued developing his ideas at NeXT and Pixar. When Jobs returned to Apple in 1996, he brought the new NeXT platform and ideas for user-friendly products that had not yet been imagined by the market. He transformed the company into a winner. He held a lot of AAPL stock but more of Pixar which merged with Disney.

After his death on October 5, 2011, many feared that the sequence of great products would cease, that the pace of innovation could not be maintained and that the market cap of about $500 billion, various lawsuits for patent infringement, competition and labour and supply chain issues might slow it down. Some thought it was a bubble but others thought it would continue rallying because it was not expensive not feeding on itself as in a typical bubble. The stock peaked at 705.07 on September 21, 2012 and then fell dramatically to the local low of 505.75 on November 16, 2012. Later, in pre market trading on December 17, 2012, it fell to 499 and later to 380. On December 31, 2012, AAPL closed the year at 532; see Figure 5. A 7-1 split allows one to compare 2012 and 2021 prices (which were much higher).

The concentration of ownership by mutual funds (see figure 6) creates a conundrum for Apple as regulations prohibit ownership to exceed a percentage of a fund's assets, so as AAPL rises relative to other stocks, funds often must sell shares. Some of the selling was tax loss selling in 2012 before expected higher capital gains and dividend rates in 2013 since more gains are in AAPL than in any other stock. Despite the large decline in the latter part of 2012 the stock increased 30% in 2012. Figure 6 shows the 2012 holdings which are similar to those in 2021.

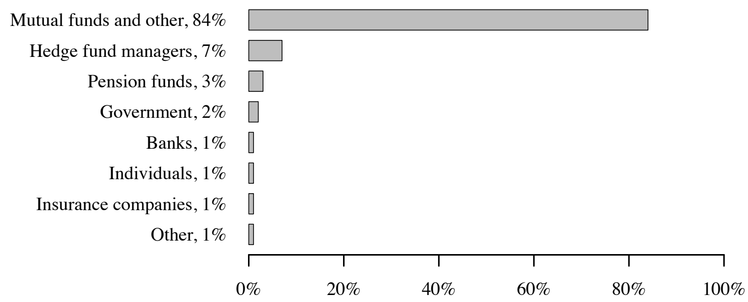


Figure 6: Holders of Apple, April 17, 2012. Source: Bloomberg via Eric Jackson.

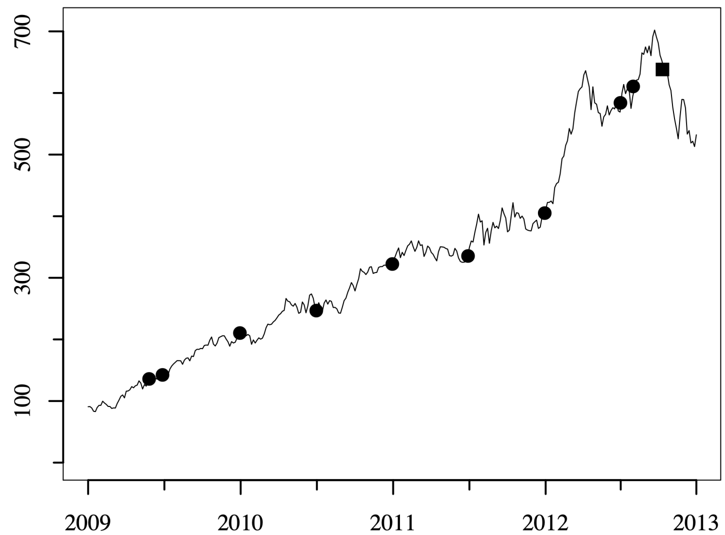
3.2 Applying the model to AAPL

Shiryaev, Zhitlukhin and Ziemba (2014) studied the change in the mean of AAPL in 2012. That is shown in Figure 7. There are several user-chosen entry buy points denoted with circles and the exits are denoted by the squares. The calculations assume that the rate of decrease after the exit point equals the rate of increase, μ_1. It is common knowledge that stock prices tend to rise slowly and fall fast but in the bubble markets studied here we found that using equal rates of ascent and descent is as good as any other assumption we tested. It is also easy to work with, which is why we selected it. The application of the model to the AAPL price bubble started at the local low of 82.33 on March 6, 2009 and considered various entering dates for opening a long position in 2009-2012. It was assumed that the trend reversal would happen before the end of 2012. Higher tax rates on dividends and capital gains were expected in 2013, thus a sale in 2012 was suggested.

Figure 7 presents the result of the changepoint model application. Tests indicate that the choice $\mu_2 = -\mu_1$ is an appropriate one, and for the 2012 time horizon works equally well both for early and late entering dates giving nearly 90% of the maximum price.

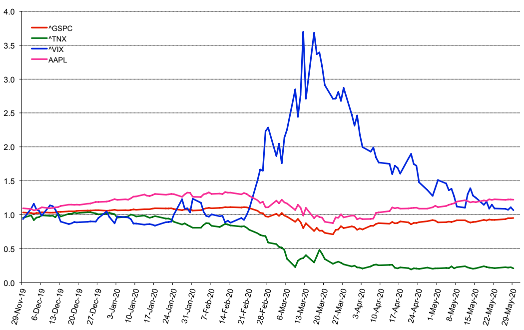
This type of mean change exit model has been successful in the analysis of a number of US stock market crashes in 1929, 1987, 2000, 2002 and 2007, Japanese stock market and golf course membership price crashes in 1990-91, and crashes in China in 2015 and Iceland in 2008 showing that the exit models do exit at good times and prices. These are discussed in section 6. In each crash we have a rise up, then a topping and then a decline over a specified interval and the reverse for re-entries. These points persist using excess returns rather than prices. Conclusions are discussed below.

Figure 7: Buying and selling dates for AAPL when $\mu_2 = -\mu_1$. The dots indicate the eight entering dates, and the square indicates the exit date on October 8, 2012. Source: Shiryaev, Zhitlukhin and Ziemba (2014).

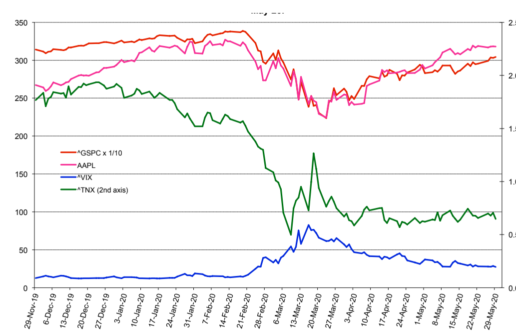


Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
T = end of 2012					
2012-07-01	12-10-19 (87)	12-10-11 (89)	12-11-08 (77)	12-10-09 (91)	12-11-19 (81)
2012-01-01	12-10-11 (89)	12-10-05 (93)	12-04-17 (87)	12-05-17 (76)	12-01-25 (64)
2011-07-01	12-10-25 (87)	12-05-17 (76)	11-08-08 (50)	12-05-17 (76)	11-08-08 (50)
2011-01-01	12-11-02 (82)	12-10-26 (86)	11-08-09 (53)	12-10-26 (86)	11-08-09 (53)
2010-07-01	12-11-02 (82)	12-11-02 (82)	11-08-09 (53)	12-11-02 (82)	11-08-09 (53)
T = end of 2013					
2012-07-01	12-12-14 (73)	12-11-08 (77)	13-01-24 (64)	12-11-08 (77)	13-01-24 (64)
2012-01-01	12-12-14 (73)	12-11-08 (77)	12-04-19 (84)	12-11-07 (79)	12-04-25 (87)
2011-07-01	12-11-08 (77)	12-11-07 (79)	11-08-08 (50)	12-11-07 (79)	11-08-08 (50)
2011-01-01	12-12-14 (73)	12-11-08 (77)	11-08-09 (53)	12-11-08 (77)	11-08-09 (53)
2010-07-01	12-12-14 (73)	12-11-15 (75)	11-08-10 (52)	12-11-08 (77)	12-04-25 (87)

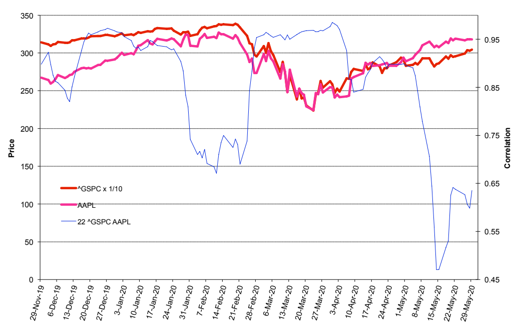
The movements of AAPL up and down are highly correlated with those of the S&P500, the VIX volatility index and the ten year US government bond, see Figures 8(abcd) which covers the period November 29, 2019 to May 31, 2020. One sees the same thing here too with declining VIX associated with increasing S&P500.



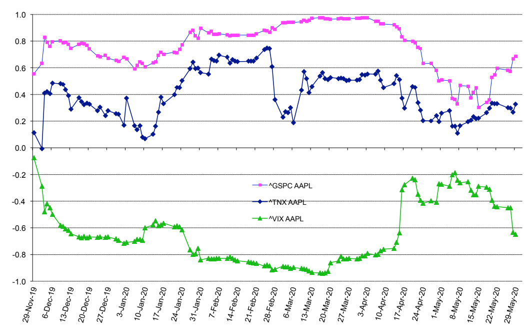
(a) S&P, AAPL, VIX, T



(b) 0.10 S&P, AAPL, VIX, T



(c) price correlations S&P, AAPL, VIX, T



(d) return correlations 0.10 S&P, AAPL, VIX, T

Figure 8: The S&P500, the VIX, AAPL, the ten year government bond (T) closing rates day by day and rolling monthly (22 trading day) correlations from November 29, 2019 to May 31, 2020

Figure 8(a) plots Apple stock and the S&P500 which can be seen to move together fairly closely even though AAPL is about 5% of the S&P500 value weighted index. It is a bellwether and leading indicator. Correlations are in Figure 8(c) for levels and for rates of return in Figure 8(d). In general this latter correlation is about 0.9 on average but varies from nearly 1.0 to 0.4. As the market rallied in late April and May the correlation fell. The VIX-AAPL correlation on rates of return was negative as expected and is closer to -1.0 when the VIX and S&P500 are falling violently. Then as the VIX falls, the S&P500 rises, the VIX-AAPL correlation fell to the -0.3 area. This is even more dramatic in the price correlations, Figure 8(c), which are known to be less interesting in general. But the

AAPL and S&P500 track each other with the correlation varying widely.

Over time the S&P500 volatility measured by the VIX, the average of close to the money near time puts and calls from December 31, 1985 to May 31, 2020 were very variable; see panel (a) in Figure 8. In the 52 weeks ending March 31, 2020, the VIX had a low of 11.3% and a high of 85.47%. Figure 9 shows the two years to May 14, 2021 when the VIX fell during June 2020-May 2021 with occasional brief rises. Meanwhile, AAPL stock had similar movements as shown in Figures 8(abcd) with the price sequences having a negative correlation.

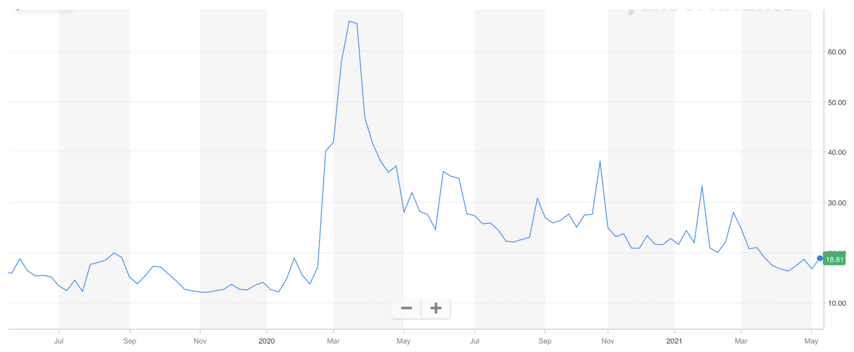


Figure 9: VIX 2 years to May 14, 2021.

4 Using VIX reversals to predict futures S&P500 moves

It is widely thought that the VIX volatility index, which is constructed from close to the money put and call implied volatilities calculated from the option prices, is highly related to the level and direction of the S&P500. Here we investigate the predictions of the S&P500 when the VIX changes direction with three political examples.

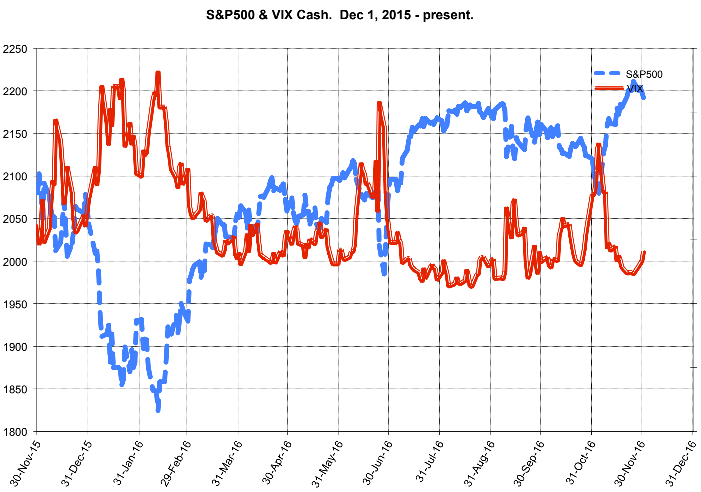
4.1 Brexit vote, June 23 to June 28, 2016

British Prime Minister David Cameron made a huge mistake. To win election in 2015 and appease a wing of his party, he promised a vote, he thought he could win, on remaining in the European Union. Former London mayor Boris Johnson and cabinet member Michael Gove among others campaigned for Brexit promising control over immigration and more money for the National Health Service among other issues. One of the cornerstones of the EU is free movement of people. This has made it more difficult for even highly educated people from non-EU countries, like Ziemba, to get visas. He found getting a UK visa to teach a course more work than actually teaching the course. The UK polls had the vote close but the London bookmakers had it about 70-30 for remain.

During the day of the vote June 23, 2016, the preliminary polls had it looking like remain was winning and the S&P500 rose sharply from 2076.75 to 2105.75 with the VIX falling to 17.25 from 21.17. The market players assumed remain would win and the S&P was rallying upward during the night. Ziemba was following this while in France. He covered most of his short puts which had fallen in value and hedged the portfolio with S&P futures short setting buy back limits on them assuming that there would be trouble at some stage. In the morning he saw that as the vote came in, Brexit was winning and there was a huge reversal. His short futures got bought back at a small gain. Had he not had limits, he could have made more because the VIX went up almost 50% to 25.76 and the S&P500 fell to 2018.5, a drop of 87.25 points or nearly 5%. The short puts that he had went up in value with the higher VIX but he did not too many of them. The next day, June 27, the S&P fell further to 1985 but the VIX dropped to 23.85. Thus the crisis was over and a big rally ensued. The market assumed that the Fed would not raise rates and the low interest rates would favor stocks. Figure 10 shows the S&P500-VIX moves.

4.2 The Trump Election Risk, October 24 to November 10, 2016

Throughout the presidential election campaign Donald Trump used a strategy that tested the boundaries of acceptability: he insulted opponents and called them derogatory names. He argued that only he could save the country. He appealed to the uneducated and poor



Date	Close	Close
15-Jun-16	2,071.75	20.14
16-Jun-16	2,079.25	19.37
17-Jun-16	2,079.12	19.41
20-Jun-16	2,074.25	18.37
21-Jun-16	2,080.50	18.48
22-Jun-16	2,076.75	21.17
23-Jun-16	2,105.75	17.25
24-Jun-16	2,018.50	25.76
27-Jun-16	1,985.00	23.85
28-Jun-16	2,028.50	18.75
29-Jun-16	2,066.75	16.64
30-Jun-16	2,090.25	15.63
1-Jul-16	2,096.25	14.77
4-Jul-16	2,096.25	14.77
5-Jul-16	2,082.75	15.58
6-Jul-16	2,094.00	14.96
7-Jul-16	2,092.00	14.76
8-Jul-16	2,120.50	13.20
11-Jul-16	2,130.25	13.54
12-Jul-16	2,145.75	13.55
13-Jul-16	2,146.00	13.04
14-Jul-16	2,157.25	12.82
15-Jul-16	2,152.75	12.67

Figure 10: S&P500-VIX movements, November 30, 2015 - December 31,2016 .

white while insulting many minority groups. But he developed a following that did not care about his actions and thought he could help them but he did not..

He made markets jittery by threatening to tear up alliances and trade deals. He kept a focus on Hilary Clintons's emails though nothing indictable was found and he had court cases pending, threats to Clinton and a sexually explicit tape that infuriated women.

About a week before the election, the FBI director said there were more emails to be checked and Trump rallied in the polls. Then, near the election day, the FBI director said there was nothing indictable in them but Clinton didn't recover and many had already casted their votes in the interim. Whenever it looked like Trump was winning the market got nervous and fell.

As the vote came in the markets first rallied when it looked like Clinton would win and then fell, especially worldwide as it became clear Trump would win. The Dow Jones was down 800 points. Noted investor Carl Ichan bought $1 billion of S&P500 stock and futures near the bottom. Ziemba bought more as well. Then the market rallied when Trump read a very conciliatory acceptance speech written by an aide praising Clinton. The VIX then fell and the S&P rallied sharply.

Trump won the electoral college and thus the presidency while Clinton won the popular vote by over 2.5 million votes with 4 million who voted for Obama not voting. Thus making for a very divided country.

The VIX-S&P500 movements are in Figure 10.

The market does not like uncertainty and it was uncertain about Trump's policies - some are potentially good like infrastructure and bringing jobs back to the US. The claim is that for the wealthy, the tax would be the same with fewer deductions but this was not realistic given his cabinet choices. He wanted both to reduce taxes on corporations and the wealthy while pursuing a jobs and building program which would vastly increase the deficit and he did this. He also seemed to want to run the presidency in the Oval Office as he ran his business empire - close to the vest and with the help of his children. Conflicts of interest and nepotism were wide spread. We saw the result.

Ziemba's futures fund did well, making 5% on the night of the Trump victory. We can see from the VIX-S&P500 graph, once the VIX turned, the S&P500 turned. Throughout December the market rallied through the options expiry and the Fed FOMC decision to raise short term interest rates by $\frac{1}{4}\%$. The Fed chair Janet Yellen announced that they planned to increase interest rates three more times in 2017. This departure from an expected two raises caused a little selloff before the market resumed its rise.

4.3 The French Election

The election, which drew worldwide attention, was in two rounds: Sunday April 23 and Sunday May 7, 2017. Incumbent president François Hollande of the Socialist Party (PS), though eligible to run for a second term, decided, on December 1, 2016, not to seek reelection, a decision motivated by weak voter approval ratings. With several candidates calling for a referendum on France's future in the European Union, and following an increase in populist sentiments across several elections, the race was important across Europe and thus global markets including the US.

In the first run off there were five major candidates among the ten that were running. One, Melenchon, on the very far left who wanted to Frexit the European Union. One on the far right, master debater lawyer Marine Le Pen, also wanted a Frexit and advocated more restrictions. Le Pen had ousted her father Jean-Marie Le Pen the founder of the nationalist and anti-immigrant National Front (FN) party. Both Melenchon and Le Pen, if elected could have most likely caused a great financial crisis in France and globally given the increased risk of moving back to the old French franc, isolationist policies on immigration likely triggering a stock market steep fall in France, widening of French credit spreads and potentially enough EU instability, to cause US equity markets to fall as well. Just winning the election would not have automatically triggered an exit. The legislature would have needed to allow a vote and then the population would have had to vote yes. Several steps to final transition, but nonetheless much political and market risk.

There were two centrist candidates. François Fillon of the right-leaning establishment Republican Party (LR) won his party nomination but had to contend with the Penelopegate scandal. He was accused of employing his wife in a ficititous position. He was tried, convicted and sentenced in April 2020 (an appeal is currently pending). Finally there was Emmanuel Macron of the new predessor to La Republique En Marche (LRAM)) party. Though a new party, created less than a year before the elections, Macron had a background having served as economy minister in previous governments. Fillon and Le Pen led first round opinion polls in November 2016 and mid January 2017. Polls tightened by late January and after the scandal, Macron passed Fillon and was second in the polls behind Le Pen. At the same time, Hamon won the socialist primary and became fourth in the polls. Because of his debating skills, Jean-Luc Melenchon of the La France Insournise party overtook Hamon and was just slightly behind Fillon.

Ahead of elections, polls had Macron in first place. The first round winners were Macron and Le Pen. When the other centrist candidate Fillon endorsed Macron, Ziemba knew that the risk of breakup of the EU was over and that Macron would win in the final round because other centrists would join Macron. He also knew the US stock market would rally and the VIX collapse, which it did. His four futures accounts made over half a million that night, the best he ever remembered. In the vote, Macron got 20,743,128 (66.1%) to Le Pens 10,638,475 (33.9%). Le Pen only won two small areas in the North of France and conceded defeat. Macron took his presidential office on May 14, 2017 for his five year term and named Edouard Philippe Prime Minister. Macron at 39, a former economy minister, was the youngest candidate in the race.

5 Applying the model in the Covid-19 period in 2020-2021

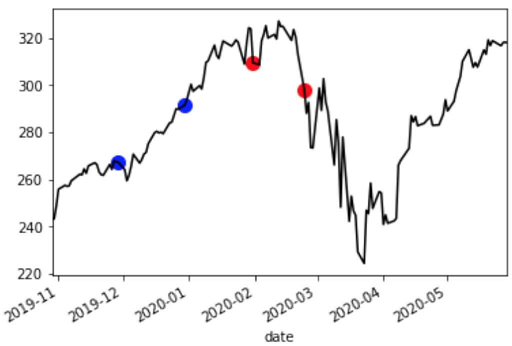
The Covid-19 pandemic that swept the world in 2020 led to *stay at home* orders, massive unemployment and, despite enormous quantitative easing, a violent stock market. It had the largest fall in the shortest time in US market history into mid March falling about 35% in 22 days followed by the largest 50 day rally fueled by unlimited Fed stimulus, some US government stimulus, low interest rates, a low US dollar, large foreign stimulus and buying and exuberance about the reopening of the economy even though the Covid-19 crisis still posed a great danger.

AAPL had a similar path with violent moves up and down daily. There was a huge amount of whipsawing with declines followed by rises and further declines. Figure 8ab shows the S&P500 versus the VIX and the ten-year bond. Figure 8c shows Apple Stock versus the S&P500 and the VIX.

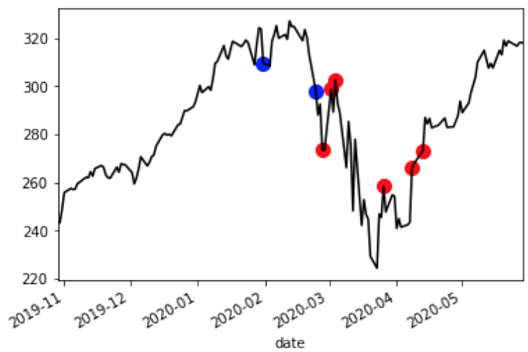
A test of the mean changing exit model during the January 1 - May 31, 2020 Covid-19 period on AAPL and the S&P500

In Figure 11a the blue dots are entries for purchase of APPL. These are from the end of

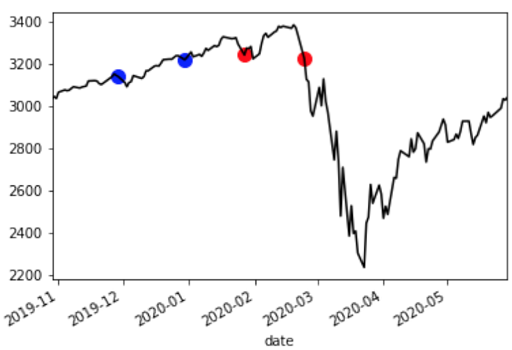
November and December 2019 which we chose to initialize the model. To estimate the parameters of the model: μ the rate of increase and σ the variance we used data from the end of October to the end of November and December, respectively. These entry dates were chosen by the user to test changepoint exit model. The red dots are exits on January 31 and February 24, 2020. Table 2 shows the combinations of parameter values that test if the speed of and variance of the increase and decrease of the price are the same. The numbers in $(\dots)$ give the percent of the highest price when the model exited as well as the actual price on exit. The results show that the model captured 91% and 95% of the highest price at the time of exit.



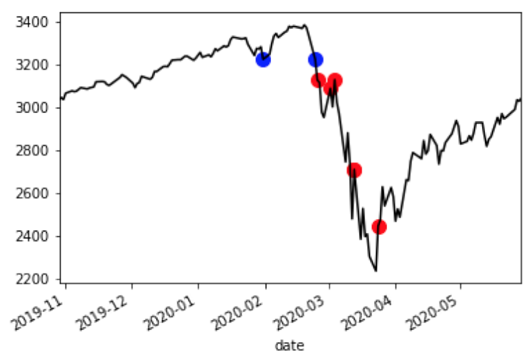
(a) Apple (sell)



(b) Apple (buy)



(c) S&P500 (sell)



(d) S&P500 (buy)

Figure 11: The mean changing model experiment graphs

In Figure 11b we use the exits above as entries to go short. So here the blue circles are the same as the red circles in Figure 11a. We re-estimated the parameters of the model using two weeks of data before each entry (exit). The red dots are the exits from the short positions which were on February 22 and 27, March 2, 4, and 26 and April 8 and 13. Then

Table 2: The mean changing model experiment data

1. Apple (sell)

Buy	Max	a=1, b=1	a=2, b=1	a=2, b=2	a=3, b=1	a=3, b=3
2019-11-29 (267.25)	2020-02-12 (327.20)	2020-02-24 (298.18, 91.1%)	2020-02-24 (298.18, 91.1%)	2020-01-31 (309.51, 94.6%)	2020-01-31 (309.51, 94.6%)	2020-01-31 (309.51, 94.6%)
2019-12-30 (291.52)	2020-02-12 (327.20)	2020-02-24 (298.18, 91.1%)	2020-02-24 (298.18, 91.1%)	2020-01-31 (309.51, 94.6%)	2020-01-31 (309.51, 94.6%)	2020-01-31 (309.51, 94.6%)

2. Apple (buy)

Sell	Min	a=1, b=1	a=2, b=1	a=2, b=2	a=3, b=1	a=3, b=3
2020-1-31 (309.51)	2020-03-23 (224.37)	2020-04-13 (273.25, 121.8%)	2020-04-08 (266.07, 118.6%)	2020-02-27 (273.52, 121.9%)	2020-04-08 (266.07, 118.6%)	2020-02-27 (273.52, 121.9%)
2020-2-24 (298.18)	2020-03-23 (224.37)	2020-03-26 (258.44, 115.2%)	2020-03-04 (302.74, 134.9%)	2020-03-02 (298.81, 133.2%)	2020-03-02 (298.81, 133.2%)	2020-03-02 (298.81, 133.2%))

3. S&P 500 (sell)

Buy	Max	a=1, b=1	a=2, b=1	a=2, b=2	a=3, b=1	a=3, b=3
2019-11-29 (3140.98)	2020-02-19 (3386.15)	2020-01-27 (3243.63, 95.8%)	2020-01-27 (3243.63, 95.8%)	2020-01-27 (3243.63, 95.8%)	2020-01-27 (3243.63, 95.8%)	2020-01-27 (3243.63, 95.8%)
2019-12-30 (3221.29)	2020-02-19 (3386.15)	2020-02-24 (3225.89, 95.3%)	2020-01-27 (3243.63, 95.8%)	2020-01-27 (3243.63, 95.8%)	2020-01-27 (3243.63, 95.8%)	2020-01-27 (3243.63, 95.8%)

4. S&P 500 (buy)

Sell	Min	a=1, b=1	a=2, b=1	a=2, b=2	a=3, b=1	a=3, b=3
2020-1-31 (3225.52)	2020-03-23 (2237.40)	2020-03-13 (2711.02, 121.2%)	2020-03-04 (3130.12, 139.9%)	2020-02-25 (3128.21, 139.8%)	2020-03-02 (3090.23, 138.1%)	2020-02-25 (3128.21, 139.8%)
2020-2-24 (3225.89)	2020-03-23 (2237.40)	2020-03-24 (2447.33, 109.4%)	2020-03-13 (2711.02, 121.2%)	2020-03-02 (3090.23, 138.1%)	2020-03-04 (3130.12, 139.9%)	2020-03-02 (3090.23, 138.1%)

the model was net long until May 31, 2020, the exit date of this experiment To apply the model to approximate the negative drift of the short to a positive re-entry instead of price we use 1 divided by price.

We now compare buy and hold of AAPL to the model. The entries in November 29 and December 30, 2019 were 267.25 and 291.52, respectively. There were many choices of the speed of the increase and the variance as shown in Table 2, but their exits to go long after the buy, exit, short and exit long ere either 266.07 or 273.52 for the original end of November entry long. So there was a slight gain as of that period. Since we have considerable computational experience in calculations suggesting that a=b=1, namely, equal mean rises and falls with the same variance is sufficient to use since other choices have similar results We focus our calculations on that case.

The end of November 2019 entry long in AAPL was at 267.25 and this exited at 298.18 on February 24, 2020 for a gain of +30.93. Then the short at 298.18 exits at 273.25 on April 13, 2020 for a gain of +24.93. Finally the long position on April 13 of 273.25 ended May at 317.94 for an additional gain of +44.69. Buy and hold gained from 267.25 to 317.94 for the same time period for a gain of +50.69. The model's total gain of 30.93+24.93+44.69 = 100.25, about double the buy and hold strategy.

The end of December 2019 entry long in APPL was at 291.52 and this exited at 298.18 on February 24, 2020 for a gain of +6.66. Then the short at 298.18 exits at 258.44 on March 26, 2020 for a gain of +39.74. Finally the long position on March 26 at 258.44 ended on May 31 at 317.94 for an additional gain of +59.50. Buy and hold gained from 291.52 to 317.94 a gain of +26.42. For the same period the model gained 6.66+39.74+59.50=105.92. So the model did work in this period. This experiment does not consider operational risk entering and exiting positions and tax implications on the gains nor dividends, about 0.5% during this period, from buy and hold but it is clear that the model did beat buy and hold in this part of 2020.

For the S&P500, one entry was at the end of November 2019 at 3140.98. That exited on January 27, 2020 at 3243.63 for a gain of +102.65. Then the short was covered at 2711.02 on March 13 for a gain of +532.61. Then exited the new long on May 31 at 3044.31 for a gain of +333.29. The total gain was 102.65+532.61+333.39= 968.55. This compares with a loss for the buy and hold of -96.67.

The other entry for the S&P500 was on December 30, 2019 at 3221.29. That exited on February 24, 2020 at 3225.89 for a gain of +4.06. Then the short was covered at 2237.04 on March 23 for a gain of +988.49. Then the long was exited on May 31 at 3044.31 of another gain of +806.91. The total gain was 4.06+988.49+806.91=1800.72. This compares with the loss for the buy and hold of -178.98.

The exit model and buy and hold remained long in AAPL and the S&P500 at the time of writing in May 2021.

6 Applying the mean changing model to previous stock market crashes: US, Japanese, Chinese, and Icelandic

The stochastic processes mean changing model is also very useful to provide exits at good times and prices for bubble-like markets that did crash. In the figures that follow, the dots are entries long and the squares are exits. These figures start with long positions chosen by the user, the exits and subsequent shorts and longs are chosen by the model, In these applications, there have just longs and exits and not longs and shorts as was done in the example of AAPL and the S&P500 in 2020 above. The examples of Japanese stock and land are from Shiryaev et al (2015). Shiryaev et al (2014) studied the Nasdaq in 2000 and AAPL in 2012. Some of this is modified from Ziemba, Lleo and Zhitlukhin (2018).

In part, the choice of the data was motivated by the fact that these various crashes had somewhat different characteristics, if judged by the pattern of underlying market indices or stock prices. The US crashes in 1929 and 1987 were similar in the behavior before the crash, showing a steady growth and then a sharp decline; the recovery patterns were much different though: it took about two years for the S&P500 to reach the value its pre-crash peak in 1987, while the DJIA needed almost 25 years to recover only in 1954. There were, of course, different macroeconomic environments, but the model is unaware of those because it uses only historical time series of index values. The crash of 2008 was characterized by a slow growth preceding the crash, and then a slow decline. In contrast, the Chinese market in 2014-15 showed the fastest growth and the decline was comparable to the 1929 and 1929 US crashes and like the Covid-19 2020 period. The Japanese stock and land markets crashed and ushered in a twenty plus year depression like atmosphere in the country. Even in 2021 the Nikkei stock average is well below its 1989 value. Iceland fell 95%. Lleo and Ziemba (2012) showed that the bond-stock earnings yield model predicted these crashes. In this paper we attempt to time the crash and exit the markets.

6.1 The Great Crash in 1929

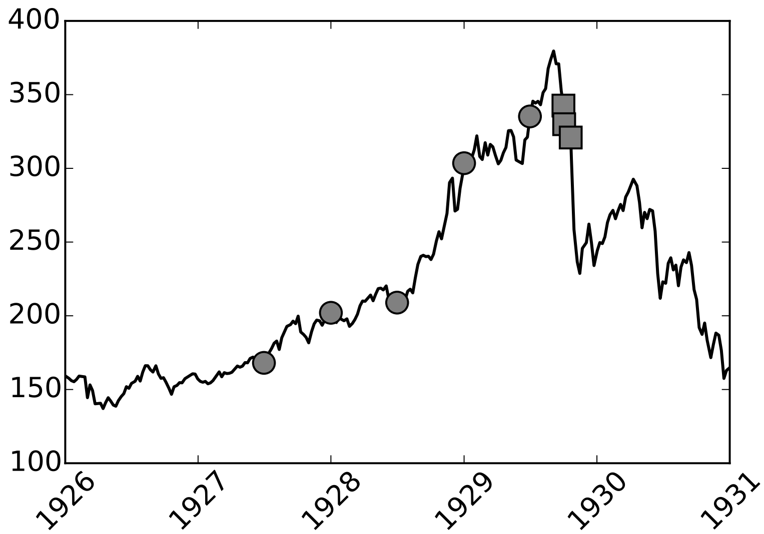
The substantial economic advance of the later half of the 1920s led to the significant growth of the US stock market. That growth was further increased by a speculative bubble, which attracted more people to the stock market in a hope that the prices would continue to increase. The market measured by the DJIA index peaked on September 3, 1929 but then declined by 10% by the end of the month. There was a short stabilization during the first half of October, which was followed by the crashes on Black Thursday October 24 and Black Tuesday October 29, when the market was down almost 40% from its peak.

We applied the changepoint model to the values of the DJIA index with several choices of the parameters: five starting dates throughout 1928-1929, five combinations of the post-changepoint parameters (depending on the values of the pre-changepoint parameters estimated from the data), and two possible terminal dates corresponding to the time horizon T in the model. The parameters of drift and volatility parameters μ_1, σ_1 of the model were estimated from one year of past data before each entry point; based on them the post-changepoint parameters were chosen as shown in Figure 12. In all the combinations of parameters, we took the changepoint probability to be 75% that a change in the drift happens in the given time horizon.

Table 2 shows the exit dates corresponding to the different choices of parameters and the percent ratio of the price on the exit date and the peak value (in brackets). The shaded cells mark the dates after the peak on September 3 and before the Black Monday.

Although the model had some false detections before the crash (caused by short-time declines), and acted too late for particular choices of the parameters after the crash, in most of the cases it was able to provide an exit point well before the local market trough. It follows that with respect to the post-crash volatility, the choice $\sigma_1 = \sigma_2$ gave results better than the other choices. With respect to the drift μ, the choices $\mu_2 = -\mu_1$ and $\mu_2 = -2\mu_1$ are preferable to $\mu_2 = -3\mu_1$, as the latter detected false exit points in May 1929. The exit points for the parameters $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$ are shown by the entry dots and exit squares on the graph in Figure 12.

Figure 12: The DJIA 1929 crash exit points.



Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
T = end of 1929					
1927-07-01	29-10-21 (84)	29-10-03 (87)	27-10-10 (50)	29-05-27 (77)	27-10-10 (50)
1928-01-01	29-10-03 (87)	29-05-27 (77)	28-06-11 (54)	28-12-10 (69)	28-06-12 (53)
1928-07-01	29-10-03 (87)	29-09-27 (90)	28-12-06 (73)	29-05-27 (77)	28-12-06 (73)
1929-01-01	29-10-03 (87)	29-09-27 (90)	29-02-11 (81)	29-05-27 (77)	29-03-25 (78)
1929-07-01	29-10-01 (90)	29-09-27 (90)	29-08-12 (92)	29-09-27 (90)	29-08-12 (92)
T = end of 1930					
1927-07-01	29-10-28 (68)	29-10-23 (80)	27-10-10 (50)	29-10-04 (85)	27-10-10 (50)
1928-01-01	29-10-21 (84)	29-10-03 (87)	28-06-11 (54)	29-10-03 (87)	28-06-13 (55)
1928-07-01	29-10-23 (80)	29-10-21 (84)	28-12-06 (73)	29-10-03 (87)	28-12-07 (71)
1929-01-01	29-10-23 (80)	29-10-04 (85)	29-04-01 (79)	29-10-03 (87)	29-05-22 (79)
1929-07-01	29-10-23 (80)	29-10-23 (80)	29-10-07 (91)	29-10-23 (80)	29-10-07 (91)

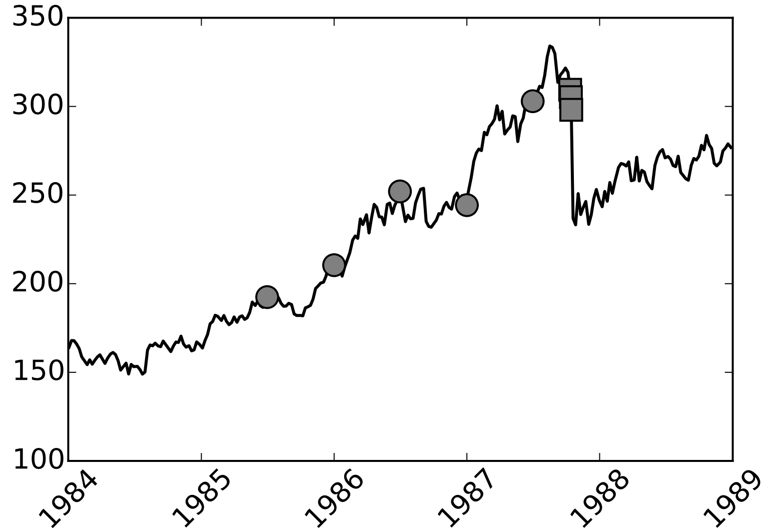
6.2 The 1987 crash of S&P 500

On Monday October 19, 1987 the S&P 500 futures fell 29% and the S&P 500 cash index fell 22%. It was the greatest one-day decline ever in US history. Portfolio insurance which sold futures when they were falling was part of the cause. The BSEYD model based on high interest rates relative to stock earnings discussed in Lleo and Ziemba (2012) did predict the crash. So did the T-model based on high call prices relative to put prices discussed in Ziemba, Lleo and Zhitlukhin (2018). Marty Zweig on *Wall Street Week* predicted the crash on the Friday before, see Swetye and Ziemba (2021). The pre-crash and the crash price patterns resemble those of the 1929 crash. However, while the 1987 crash was very severe in a short term, did not led to a depression as in 1929 or even a recession. The market bounced back, and in about two years the S&P 500 reached its pre-crash values.

The results of the application of the changepoint model to the crash are in Figure 13. The shaded cells in the table shows the exit points after the market peak on August 25 and before Black Monday October 19.

Again, the choice of $\sigma_1 = \sigma_2$ seems to work the best among the settings considered. The choices of $\mu_2 = -\mu_1$ and $\mu_2 = -2\mu_1$ still provide good results (however, the latter reacted exactly on the day of the crash under the assumption that the time horizon of the model is chosen as the end of 1988), and $\mu_2 = -3\mu_1$ works well in that case too. The entry dots and exit squares in figure 13 are for the parameters $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$.

Figure 13: The S&P 500 index in 1984–1988 and the exit points for $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$.

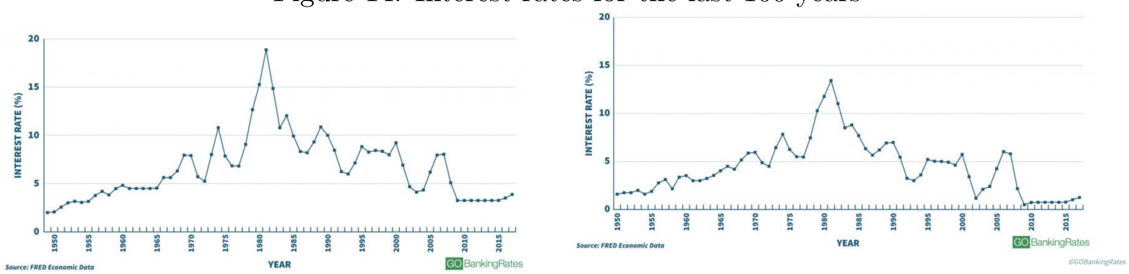


Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
	T = end of 1987				
1985-07-01	87-10-15 (89)	87-10-12 (92)	86-07-08 (72)	86-09-12 (68)	86-07-07 (72)
1986-01-01	87-10-12 (92)	86-09-12 (68)	86-01-08 (62)	86-09-11 (70)	86-01-08 (62)
1986-07-01	87-10-14 (91)	87-09-21 (92)	86-07-15 (69)	86-09-12 (68)	86-09-11 (70)
1987-01-01	87-10-14 (91)	87-09-21 (92)	87-04-14 (83)	87-09-08 (93)	87-04-15 (84)
1987-07-01	87-10-12 (92)	87-09-21 (92)	87-10-14 (91)	87-09-08 (93)	87-10-14 (91)
	T = end of 1988				
1985-07-01	87-10-16 (84)	87-10-16 (84)	86-07-15 (69)	87-10-15 (89)	86-07-15 (69)
1986-01-01	87-10-15 (89)	87-10-15 (89)	86-04-08 (69)	86-09-12 (68)	86-01-08 (62)
1986-07-01	87-10-16 (84)	87-10-15 (89)	86-07-15 (69)	87-10-15 (89)	86-09-11 (70)
1987-01-01	87-10-19 (67)	87-10-16 (84)	87-04-15 (84)	87-10-16 (84)	87-10-15 (89)
1987-07-01	87-10-19 (67)	87-10-16 (84)	87-10-15 (89)	87-10-16 (84)	87-10-16 (84)

6.3 The Internet bubble crash during 2000-2002

The historical movements of the prime rates and discount rates from 1950 to 2015 are shown in Figure 14a,b. The prime rate is a benchmark that banks, credit unions and other financial institutions use to set prices for loans. The discount rate is the cost for financial institutions to borrow short term from the FED. The highest prime rate was 21.5% in 1980 and the highest discount rate was 13.42% in 1981.

Figure 14: Interest rates for the last 100 years



(a) Prime (b) Discount

In the early 1990s, Alan Greenspan, the chairman of the US Federal Reserve System (Fed), began a low interest rate policy that reduced short term rates in a jagged path continuously over a multiyear period. This led to an increase in the S&P500 stock index from 470.42 in January 1995 to 1469.25 at the end of 1999, as shown in figure 15. The price earnings ratios were high and Shiller used these to predict the crash starting in 1996, see Campbell and Shiller (1998) and Shiller (1996, 2000, 2009). It is known that stock price rises usually start with low price earnings ratios and end with high price earnings ratios, see Bertocchi, Schwartz and Ziemba (2010, 2015). But predicting when the market will crash using just price earnings ratios is problematic.

However, Ziemba has found in many markets over many years that the BSEYD model predicts crashes better than just high price-earnings ratios, see Ziemba and Schwartz (1992), Ziemba (2003) and Lleo and Ziemba (2012). For a direct comparison over a long 60 year period in the US see Lleo and Ziemba (2017). Both the high PE and the BSEYD add value with the BSEYD adding more. Also the idea of Graham and Dodd (1934) to use 10 year earnings, an idea championed by Shiller, does add value to the BSEYD.

With high interest rates and high price earnings ratios, the model signalled a crash in the S&P500 in April 1999. It was in the danger zone all of 1999 starting in April and it got deeper in the danger zone as the year progressed, The signal did work but the real decline was not until September 2000 with a temporary fall from the March 24, 2000 high of 1552.87 and a recovery into the September 1, 2000 peak of 1530.09. By October 10, 2002,

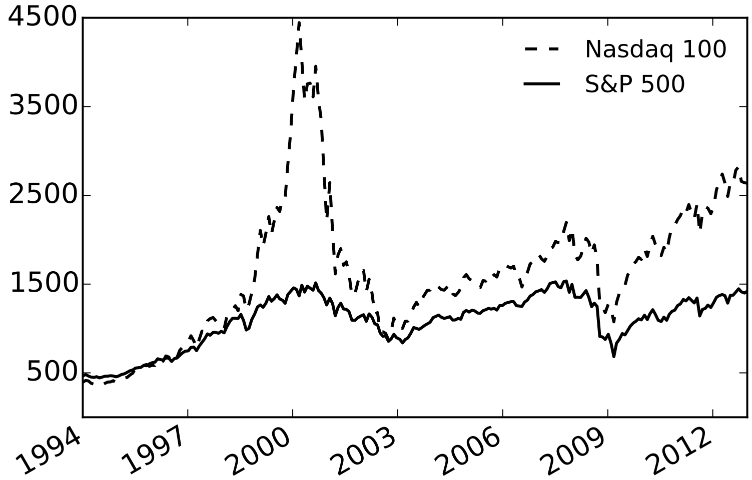


Figure 15: The Internet crash in the US in 2000-2002.

the S&P500 fell to 768.63 having two temporary recoveries from the local lows of 1091.99 on April 4, 2001 and 944.75 on September 21, 2001. There were other signals:

> History shows that a period of shrinking breadth is usually followed by a sharp decline in stock values of the small group of leaders. Then broader market takes a more modest tumble.
> Source: Paul Bagnell in late November 1999 in the *Globe and Mail*.

Ziemba (2003, Chapter 2) describes this episode in stock market history. There was considerable mean-reversion in the eventual crash in 2000, the September 11, 2001 attack and in the subsequent 2002 decline of 22%. This decline was similar to previous crashes.

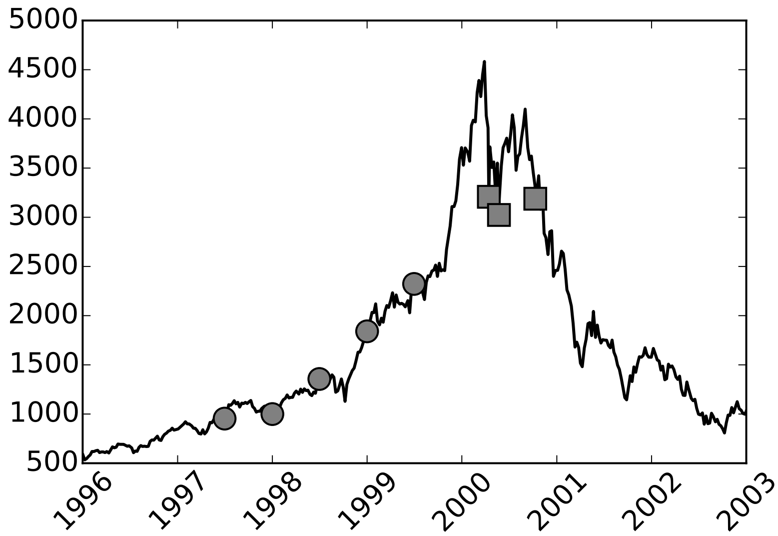
The concentration of stock market gains into very few stocks with momentum and size being the key variables predicting performance was increasing before 1997 in Europe and North America. Table 2.6 in Ziemba (2003) shows that in 1998, the largest cap stocks had the highest return in North America and Europe but small cap stocks outperformed in Asia and Japan. The situation was similar from 1995 to 1999 with 1998 and 1999 the most exaggerated.

Fully 41% of the stocks in the S&P500 did not fall or actually rose during this period and an additional 19% declined by 10% or less annualized. These were small cap stocks with market values of $10 billion of less. The fall in the S&P500 was mainly in three areas: information technology, telecommunications and large cap stocks. Information technology stocks in the S&P500 fell 64% and telecom stocks fell 60% from January 1 to October 31, 2002. The largest cap stocks (with market caps of $50 billion plus) lost 37%. But most other stocks either lost only a little or actually gained. Materials fell 10% but consumer discretionary gained 4.5%, consumer staples gained 21%, energy gained 12%, financial services gained 19%, health care gained 29%, industrials gained 7% and utilities gained 2%. Equally weighted, the S&P500 index lost only 3%. So there was a strong small cap effect. The stocks that gained were the very small cap stocks with market caps below $10 billion. Some 138 companies with market caps between $5-10 billion gained 4% on average and 157 companies with market caps below $5 billion gained on average 23%.

While the BSEYD model has been shown to be useful in predicting S&P500 declines, it is silent on the NASDAQ technology index of the largest 100 stocks by market capitalization, the NDX100. This index with a major Internet component had a spectacular increase during a period where many thought the Internet companies would prosper despite price earnings ratios of 100 plus and many with no earnings at all (see figure 15). Valuation attempts were made to justify these high prices; see Schwartz and Moon (2000) for one such example. Predicting the top of this bubble was not easy as the Internet index (not shown) fell 17% one day and then proceeded to reach new highs. For example, the noted investor George Soros lost some $5 billion of the $12 billion in the Quantum hedge fund during this crash.

The Nasdaq 100 peaked at 4816.35 on March 24, 2000 starting from 398.26 in 1994. In the decline it fell to 795.25 on October 8, 2002. Below we apply the changepoint model to the questions when to close a long short positions on NDX100 for various entering dates. The results appear in Figure 16. Depending upon the long position entry, the exit yielded about 65-75% of the maximum price. Here the parameters $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$ seem to show better performance than the other choices, but however all the three values of μ_2 with $\sigma_2 = \sigma_1$ give similar results.

Figure 16: The Nasdaq 100 index in 1996–2002 and the exit points for $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$.



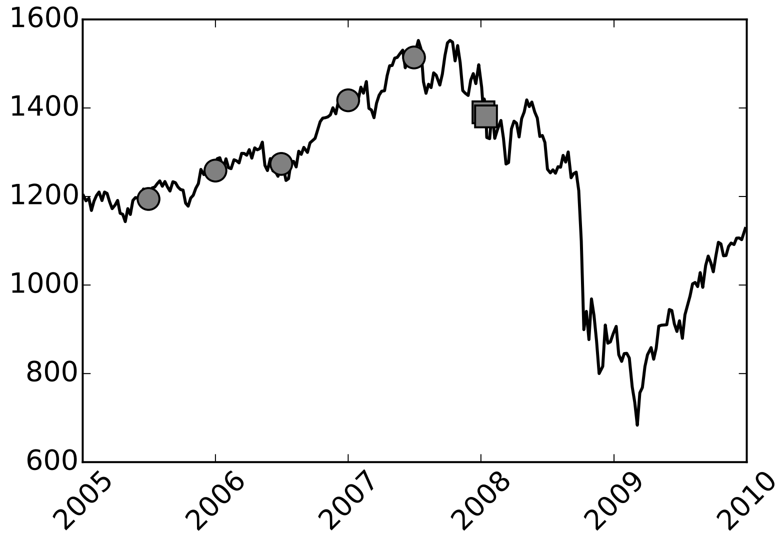
Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
T = end of 2000					
1997-07-01	00-05-23 (64)	00-04-14 (68)	97-10-27 (21)	00-04-12 (77)	97-10-27 (21)
1998-01-01	00-10-10 (68)	00-05-23 (64)	98-08-31 (24)	00-04-14 (68)	98-08-31 (24)
1998-07-01	00-04-14 (68)	00-04-13 (76)	98-08-31 (24)	00-04-12 (77)	98-08-31 (24)
1999-01-01	00-04-14 (68)	00-04-12 (77)	00-01-07 (75)	00-04-12 (77)	00-01-07 (75)
1999-07-01	00-04-14 (68)	00-04-12 (77)	00-01-10 (79)	00-04-12 (77)	00-01-11 (75)
T = end of 2001					
1997-07-01	00-10-12 (64)	00-04-14 (68)	97-10-27 (21)	00-04-14 (68)	97-10-27 (21)
1998-01-01	00-11-30 (53)	00-11-20 (59)	98-08-31 (24)	00-11-10 (61)	98-08-31 (24)
1998-07-01	00-10-11 (66)	00-04-14 (68)	98-08-31 (24)	00-04-14 (68)	98-08-31 (24)
1999-01-01	00-05-23 (64)	00-04-14 (68)	00-01-07 (75)	00-04-13 (76)	00-01-10 (79)
1999-07-01	00-10-10 (68)	00-04-14 (68)	00-01-11 (75)	00-04-14 (68)	00-04-12 (77)

6.4 The 2008 crash

The roots of the 2008 crash can be found in the problems in the mortgage market. US real estate prices peaked in 2005-6 and included loans to many unqualified buyers who were only safe if the real estate prices continued to rise. Unlike the two crashes considered above, this one had a different pattern: the crash was followed for about a year of slowly decreasing equity index prices (which peaked in October 2007) and a decline overall slower than those in 1929 and 1987. The total fall was huge with the index lost more than 57% of its value by March 2009. Lleo and Ziemba (2012) discuss this crash and the application of the BSEYD model to predict it on June 14, 2007.

The changepoint model for any choice of the parameters detected the changepoint well before the actual crash in September 2008 which was pushed lower by the Lehman Brothers bankruptcy in September; though, there were several too early detections as shown in the table in Figure 17. These results again support the choice of the post-crash volatility $\sigma_2 = \sigma_1$ and the post-crash drift $\mu_2 = -\mu_1$ or $\mu_2 = -2\mu_1$ (the shaded cells show exit points after the market peak).

Figure 17: The S&P 500 index in 2005–2010 and the exit points for $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$.



Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
T = end of 2008					
2005-07-01	08-01-15 (88)	07-11-21 (91)	07-02-27 (89)	07-11-19 (92)	07-02-27 (89)
2006-01-01	08-01-08 (89)	07-11-09 (93)	06-06-15 (80)	07-08-14 (91)	07-02-27 (89)
2006-07-01	08-01-08 (89)	07-11-21 (91)	07-02-27 (89)	07-08-15 (90)	07-02-27 (89)
2007-01-01	08-01-08 (89)	07-11-21 (91)	07-02-27 (89)	07-08-15 (90)	07-02-27 (89)
2007-07-01	08-01-08 (89)	07-11-21 (91)	07-07-26 (95)	07-08-15 (90)	07-07-26 (95)
T = end of 09					
2005-07-01	08-03-06 (83)	08-01-17 (85)	07-02-27 (89)	08-01-15 (88)	07-02-27 (89)
2006-01-01	08-03-06 (83)	08-01-08 (89)	06-06-15 (80)	07-11-26 (90)	07-02-27 (89)
2006-07-01	08-03-06 (83)	08-01-16 (88)	07-02-27 (89)	08-01-08 (89)	07-02-27 (89)
2007-01-01	08-01-17 (85)	08-01-15 (88)	07-02-27 (89)	08-01-15 (88)	07-02-27 (89)
2007-07-01	08-01-17 (85)	08-01-15 (88)	07-07-26 (95)	08-01-15 (88)	07-07-26 (95)

6.5 The crash in the Nikkei stock average index

The Japanese stock market was closed after World War II ended in 1945 until its reopening in 1948. From 1948 to 1988 there was a huge rise in the stock market measured by the Nikkei price weighted index of 225 stocks as well as the Topix value weighted index of more than 1000 stocks. A steady increase in quality and quantity of equipment and automobiles of various kinds led to an enormous inflow of financial assets. These in turn were invested primarily in Japanese stocks and land, much of it levered.

Low interest rates in the mid to late 1980s fueled the stock and land prices, and the Nikkei rose 220 times in yen and 550 times in US dollars from 1948 to 1988. There were however 20 corrections/declines of 10% or more from 1949 to 1988 as shown in table 3.

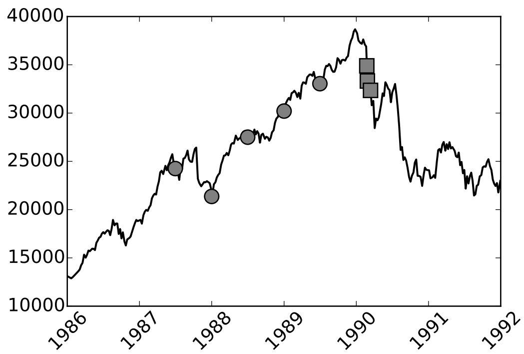
Table 3: The Twenty Corrections of 10% or more on the NSA from 1949 to 1988. Source: Yamaichi Research Institute

	Index Value			Date		
	Peak	Valley	% decrease	Peak	Valley	# Months
1	176.89	85.25	-51.8	01-Sep-49	06-Jul-50	11
2	474.43	295.18	-37.8	4-Feb-53	1-Apr-53	2
3	366.69	321.79	-12.2	6-May-53	3-Jun-53	1
4	595.46	471.53	-20.8	4-May-57	27-Dec-57	8
5	1,829.74	1,258.00	-31.2	18-Jul-61	19-Dec-61	5
6	1,589.76	1,216.04	-23.5	14-Feb-62	29-Oct-62	9
7	1,634.37	1201.26	-26.5	5-Apr-63	18-Dec-63	9
8	1,369.00	1,020.49	-25.5	3-Jul-64	12-Jul-65	13
9	1,588.73	1,364.34	-14.1	1-Apr-66	15-Dec-66	8
10	1,506.27	1,250.40	-17.0	1-Mar-67	11-Dec-67	9
11	2,534.45	1,929.64	-23.9	6-Apr-70	27-May-70	2
12	2,740.98	2,227.25	-18.7	13-Aug-71	20-Oct-71	3
13	5,359.74	3,355.13	-37.4	24-Jan-73	9-Oct-74	21
14	4,564.52	3,814.02	-16.4	12-May-75	29-Sep-75	5
15	5,287.65	4,597.26	-13.1	5-Sep-77	24-Nov-77	3
16	8,019.14	6,849.78	-14.6	17-Aug-81	1-Oct-82	14
17	11,190.17	9,703.35	-13.3	4-May-84	23-Jul-84	3
18	18,936.24	15,819.58	-16.5	20-Aug-86	22-Oct-86	2
19	25,929.42	22,702.74	-12.4	17-Jun-87	22-Jul-87	1
20	26,646.43	21,036.76	-21.1	14-Oct-87	11-Nov-87	1
Average			-0.224			6.5

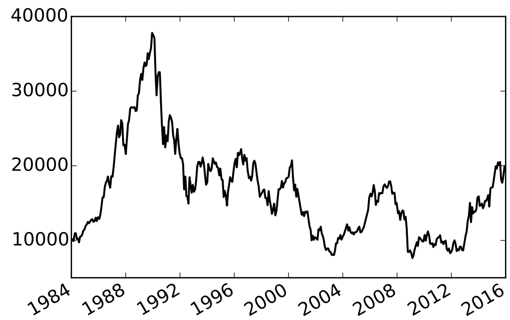
The Nikkei peaked at the end of December 1989 at 39,816. The bond-stock earnings yield model went into the danger zone in April 1989, based on too high interest rates relative to earnings yield, see Lleo and Ziemba (2012). That model suggested that a large decline or crash was coming. The stock marked started to fall on the first trading day of 1990. When the index bottomed, the market had fallen 48% from 38,916 at the end of December 1989 to 20,222 on October 1, 1990. Figure 18 shows the Nikkei stock average index from 1984 to 2016.

Interest rates increased 8 full months till August 1990. It took years and years to recover from this despite dropping interest rates after August 1990 for many years and in 2020

Figure 18: The Nikkei stock average in 1986–1991, and 1984-2016 and the exit points for $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$.



(a) Nikkei, 1986-1991



(b) Nikkei, 1984-2016

Entry	Exit date (% of max. value) $\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
	T = end of 1990				
1987-07-01	90-03-14 (83)	87-11-11 (54)	87-10-20 (56)	87-10-20 (56)	87-10-20 (56)
1988-01-01	90-02-26 (86)	90-02-23 (90)	90-04-02 (72)	90-02-23 (90)	90-04-02 (72)
1988-07-01	90-02-26 (86)	90-02-21 (92)	90-04-02 (72)	90-02-21 (92)	90-04-02 (72)
1989-01-01	90-02-23 (90)	90-02-23 (90)	90-02-21 (92)	90-02-23 (90)	90-02-21 (92)
1989-07-01	90-02-23 (90)	90-02-23 (90)	90-01-16 (95)	90-02-21 (92)	90-01-16 (95)
	T = end of 1991				
1987-07-01	90-03-20 (79)	90-03-19 (80)	87-10-20 (56)	87-10-20 (56)	87-10-20 (56)
1988-01-01	90-04-02 (72)	90-03-20 (79)	90-04-02 (72)	90-03-19 (80)	90-04-02 (72)
1988-07-01	90-04-02 (72)	90-03-19 (80)	90-04-02 (72)	90-03-19 (80)	90-04-02 (72)
1989-01-01	90-02-26 (86)	90-02-26 (86)	90-02-23 (90)	90-02-26 (86)	90-02-23 (90)
1989-07-01	90-02-26 (86)	90-02-26 (86)	90-02-21 (92)	90-02-23 (90)	90-02-21 (92)

(c) Exit points for Nikkei

they are still low. Overall, the crash ushered in three decades of deflation, weak economic markets and a lost generation of young people. Various Japanese policies and regulatory constraints exacerbated the poor economic situation and never resolved the basic problem of over leveraging and excessive debt that was a major part of the 1980s buildup.

We tested the changepoint model on the Japanese crash of 1990 applying it to the Nikkei index with the results shown in Figure 18. For all the choices of the parameters, the model exited near the peak.

6.5.1 The crash in the land market

While the Nikkei stock average in the late 1980s and its -48% crash in 1990 is generally recognized as a financial market bubble, a bigger bubble and crash was in the land market which started to fall in 1990. The land and stock markets were greatly intertwined as discussed by Ziemba (1991) and Stone and Ziemba (1993). To get an idea of the price pressure on Japanese land prices, consider that in the late 1980s:

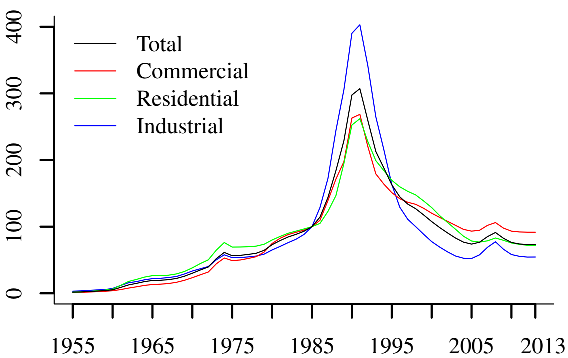
- some 120 million people lived in Japan in an area the size of Montana,
- only 5% of the land was used to house the people, buildings and factories because most of the land is mountainous,
- most of the land was owned by large corporations but 60% of Japanese families and 55% of those in Tokyo owned their own home,
- there was massive savings by households,
- only some 3% of Japanese assets were invested abroad despite great fear in the west and some very public purchases at inflated prices of expensive property such as the Pebble Beach golf course.

Figure 19 gives the Japan Real Estate Institute's land indices for the six largest cities, and for all of Japan for commercial, housing, industrial and total land for each six month period from 1955 to 2013. The graphs also give the yearly rate of changes. The six largest cities are Tokyo, Yokohama, Osaka, Nagoya, Kobe and Kyoto.

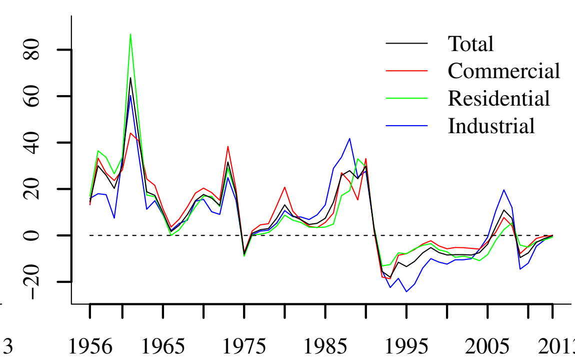
The countrywide indices are based on 140 cities. The data are appraisal-based which tends to smooth the price levels and lag the market. Simple averages of samples of ten lots in each city form the indices which were normalized at 100 as of 1985. The sampling procedure separates land into high, medium and low grades reflecting location, social circumstances, yield, etc. The sampling procedure selects lots randomly and equally from each of these three classes.

Figure 19 indicates that the price increase was largest in the six largest cities. Despite large rises in the 1980s, the relative gain in the period 1955 to 1970 was much larger than from 1970 to the circa 1990 peak. For land in the whole country, the 1955 to 1970 period produced gains of about 15 times the 1955 values. These prices then increased only about four fold in the ensuing twenty years. In the six largest cities, the increase was also much larger in the 1955 to 1970 period versus the next two decades.

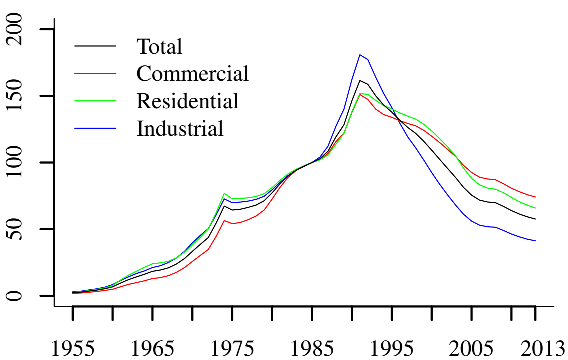
Land values in the six largest cities outpaced the Consumer Prices Index (CPI) by twenty times from 1955 to 1990. In the Ginza district of Tokyo each square meter of land was worth well over US$200,000 with some plots approaching $300,000. Choice downtown land in Tokyo sold for the equivalent of nearly a billion dollars an acre. At neighboring land



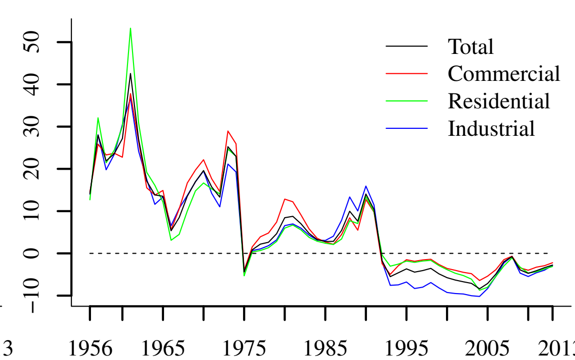
(d) The six largest cities



(e) Average annual rates of price change, % (six largest cities)



(f) All of Japan



(g) Average annual rates of price change, % (all of Japan)

Figure 19: Land price indices for industrial, residential, commercial and all land and annual rates of price change, 1955 to 2013. Source: Japan Real Estate Institute.

Table 4: Increase of land prices, 1955 to 1990, %. Source: Japan Real Estate Institute

	Nationwide				6 largest cities			
	Total	Com	Res	Ind	Total	Com	Res	Ind
1955 to 1990	65.5	59.6	81.2	56.7	178.8	127.7	219.1	150.8
1955 to 1970	15.1	14.5	15.5	15.8	18.7	11.0	20.2	23.6
1970 to 1990	4.3	4.1	5.2	3.6	9.6	11.6	10.8	6.4

prices, the value of land under the Emperor's palace and garden in Tokyo equaled that of all of California or of Canada. The total land value in Japan in 1990 was about 4.1 times that of the whole United States. The average acre of land in Japan was worth fully 100 times the average acre in the U.S. So even though the US has about 25 times more land than Japan, its 1990 total value was less than a fourth as much. Essentially half the world's land value at 1987-90 prices was accounted for by Japanese land!

The price was kept up and bid higher because of the excess of demand over supply. Land turnover was very small as the Japanese believe in holding land whenever possible. This was reinforced by the tax system which encourages the purchase of more land and discourages land sales. High interest rates which led to a sharp fall in stock prices in 1990 did not lead to any decline in land prices until 1991 as shown in Figure 19. However, there was a sharp decline in speculative land such as golf course membership and condos, see Stone and Ziemba (1993). As interest rates rose, land demand fell but in Tokyo, with virtually no new supply, demand still greatly exceeded supply. At the same time supply declined with higher interest rates as development costs were curtailed. All the incentives favored holding land and not even developing it. As Canaway (1990) pointed out, land held less than five years was taxed at fully 52% of its sale value. Meanwhile, yearly taxes paid to hold land were about 0.05 to 0.10% of its current value. Even upon death it paid to borrow money which was deductible at full value while land was valued at about half its market value. Hence inheritance taxes are minimized. Canaway argued that in a major crash the stock market will go first, then the economy and finally the land markets. Ziemba and Schwartz (1992) confirmed this.

6.5.2 The golf course membership index as a proxy for the land market

Japanese land prices are difficult for a direct analysis since only low frequency time series are available. In part, this makes difficult to apply the changepoint model. Nevertheless, the overall land market can be well proxied by speculative land prices. In this section as such a proxy we use data on membership prices in Japanese golf courses.[1]

[1]Merton Miller, the late University of Chicago Economics Nobel Prize winner, encouraged Ziemba on this unique data set so we include it here as a good example of the mean changing model approach to bubble exits.

In 1989 there were more than 400 golf courses in Japan with a total value more than US$300 billion, a value larger than the Australian stock exchange capitalization of A$250 billion. Memberships, which cost as much as US$8 million, allowed play at a reduced cost plus the right to bring guests to play for a higher fee. However, their main value was not the ability to play golf but their share of the land occupied by the course and as an instrument to play the land market for relatively low stake with liquidity. These memberships were actively traded as speculative investments whose market was maintained by six market makers in Tokyo and Osaka. Weekly data were available in various areas of Japan since the beginning of 1982. These data were the best widely available data series on land prices in Japan and formed an ideal source for many types of analyses.

Rachev and Ziemba (1992) modeled the price changes as stable variants. The tails had considerable mass and the distributions were considered to have fat tails with a characteristic exponent about 1.4. This is consistent with the hypothesis that there was a speculative bubble in the late 1980s and the subsequent crash in 1990 to 1992.

Figure 20 shows the golf course membership (GCM) prices in various regions of the country: the western and the eastern parts of Japan, the Tokyo area and the nationwide average. The golf course memberships market was a much bigger bubble than the stock market.

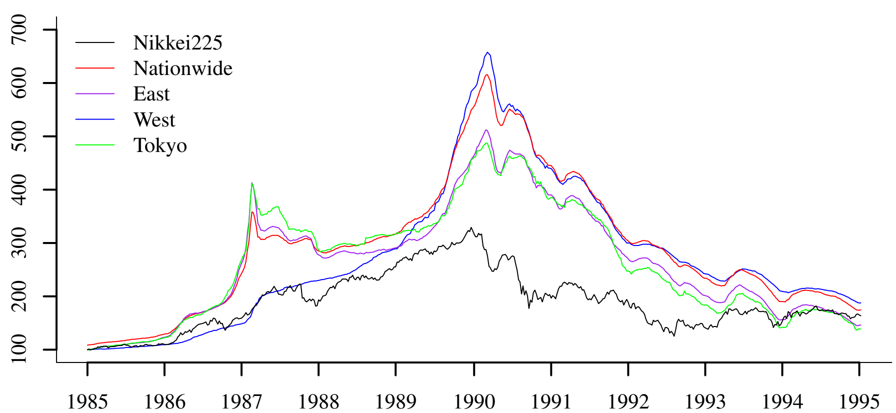
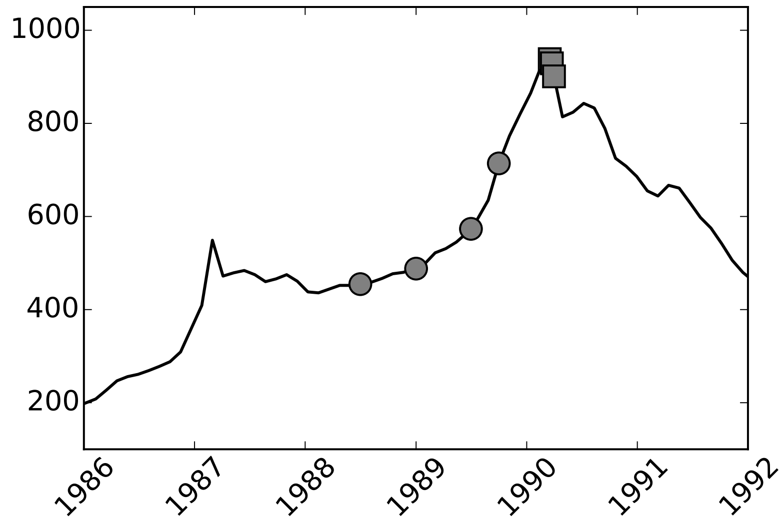


Figure 20: Graphs of the golf course membership prices in various areas of Japan and the Nikkei stock average, 1985-1995, with the 1985 values as 100%.

The results of applying the changepoint model are displayed in Figure 21 with the same notation as the previous sections. We show only the nationwide index, although similar results and exit dates were obtained for the three other indices (the results can be found in Shiryaev, Ziemba, Zhitlukhin, 2015). In all the cases, the model exits well above 90% of the global maximum price, see figure 21. Compared with the stock market bubble, the changepoint model applied to the golf course membership index exits almost immediately after the peak. One reason for that is much higher ratio μ_1/σ of the drift to volatility, which governs the statistic ψ_t from the model.

Figure 21: The Japan Golf Course Membership index and the exit points for $\mu_2 = -\mu_1$, $\sigma_2 = \sigma_1$.



Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
T = end of 1991					
1988-07-01	90-04-01 (95)	90-03-25 (98)	89-07-09 (62)	90-03-25 (98)	89-02-12 (54)
1989-01-01	90-03-18 (99)	90-03-18 (99)	89-07-09 (62)	90-03-18 (99)	89-02-05 (53)
1989-07-01	90-03-18 (99)	90-03-18 (99)	90-03-18 (99)	90-03-25 (98)	89-09-17 (72)
1989-10-01	90-04-01 (95)	90-04-01 (95)	90-04-01 (95)	90-04-01 (95)	90-04-01 (95)
T = end of 1992					
1988-07-01	90-04-01 (95)	90-04-01 (95)	89-07-09 (62)	90-03-25 (98)	89-02-26 (55)
1989-01-01	90-03-18 (99)	90-03-18 (99)	89-07-09 (62)	90-03-18 (99)	89-02-05 (53)
1989-07-01	90-03-25 (98)	90-03-18 (99)	90-03-18 (99)	90-03-25 (98)	89-09-17 (72)
1989-10-01	90-04-01 (95)	90-04-01 (95)	90-04-01 (95)	90-04-01 (95)	90-04-01 (95)

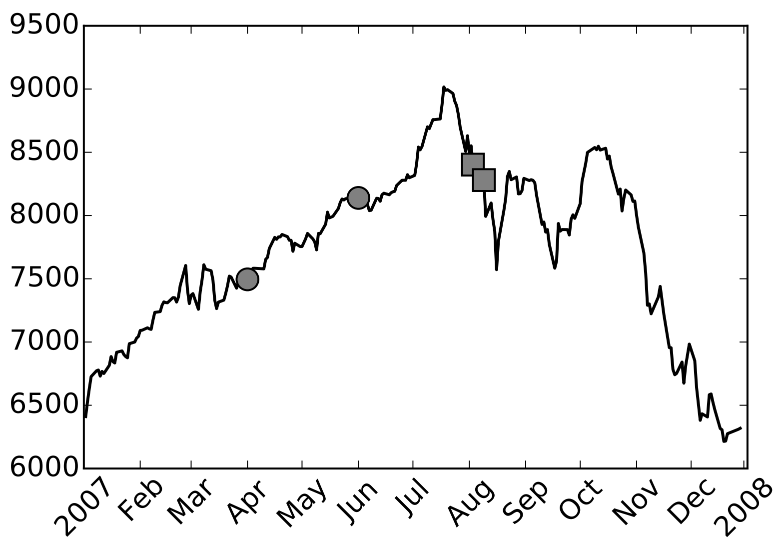
6.6 The crash in Iceland in 2007-2009

Iceland is a small country with only about 300 000 people. From 2002 to 2007, the economy and asset prices rose dramatically, with much leveraging of investments, especially by the banks. This led to high interest rates of about 10% long term and 16% short term. Eventually, it all collapsed in the wake of the 2007-2009 worldwide financial crisis. The decline was a massive crash of -95% in the equity index and a currency collapse.

The crash in Iceland was predicted by the BSEYD model (Lleo and Ziemba, 2012). The decline occurred around the time of the Lehman Brothers collapse in the fall of 2008.

In Figure 22, we show entries and exits using the changepoint model. We consider two entry points in April and June 2007, with the time horizon taken as the end of 2007. The model exits at about 90% of the index's maximum value.

Figure 22: OMX Iceland 15 Index in 2007 and the exit points.



Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$, $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$, $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$, $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$, $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$, $\sigma_1 = 3\sigma_2$
2007-04-01	07-08-10 (89)	07-08-09 (92)	07-08-10 (89)	07-08-09 (92)	07-08-10 (89)
2007-06-01	07-08-09 (92)	07-08-03 (93)	07-08-01 (94)	07-07-30 (94)	07-08-10 (89)

6.7 The crash in China in 2015

The growing Chinese economy attracted many individual investors, whose actions are believed to fuel the bubble in the stock market. They borrowed heavily to buy equities, which resulted in stocks priced beyond their fundamental values. Thus a little decrease could potentially lead to a large sell-off due to investors facing margin calls, which is believed what has happened starting from the summer of 2015.

The Shanghai Stock Exchange Composite Index (SSEC) peaked in the middle of June 2015 showing about a 70% increase from the beginning of the year. By the end of August, the index lost almost a half of its peak value. A local trough was also on July 2, with a loss of about 40% of the maximum value. Lleo and Ziemba (2018) applied the BSEYD model to predict this Chinese stock market crash and the results are discussed there.

The changepoint detection model for all the parameters we consider signaled the changepoint before the July decline, in most cases being able to exit at more than 80% of the maximum value. There were, however, several false detections triggered by the temporary decline in the beginning of 2015. Figure 23 illustrates the results. The shaded cells in the table mark the four highest exit points shown on the graph, all above 80% of the peak value.

7 Conclusions

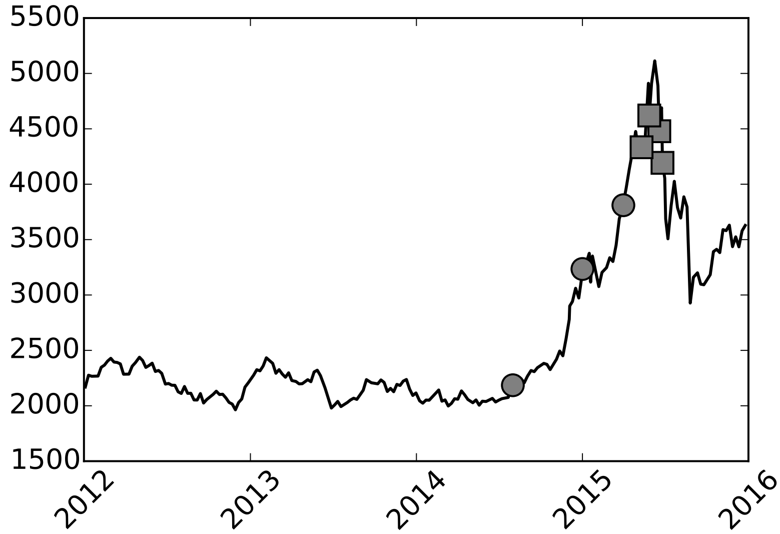
Predicting the end of bubble-like markets is very difficult. The stochastic process mean changing model discussed here has has proved successful in the application to the crashes studied in the US, Japan, Iceland and China. The mean changing model implements the finding that in essentially all portfolio problems, the accuracy of the mean estimates is crucial for success. The model performed well on AAPL and the S&P500 in the Covid-19 era in 2020.

A Appendix: Mathematics of the stopping rule changepoint detection model

The goal of the model is to detect a moment to exit a bubble-like stock market based only on the observation of stock prices (or index values) up to the current time. We focus only on exiting a market; though the model can be applied to the problem when to enter a market by an appropriate change of signs as in the AAPL and S&P500 applications.

We model the evolution of stock prices by a sequence of random variables S_t, $t = 0, 1, \ldots$, which increases in some sense until time θ, called the *changepoint*, and decreases after

Figure 23: The Shanghai Stock Exchange Composite Index in 2012–2015 and the exit point.



Entry	Exit date (% of max. value)				
	$\mu_2 = -\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -2\mu$ $\sigma_1 = 2\sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = \sigma_2$	$\mu_2 = -3\mu$ $\sigma_1 = 3\sigma_2$
2015-04-01	15-06-26 (81)	15-06-19 (87)	15-05-11 (84)	15-06-19 (87)	15-05-28 (89)
2015-01-01	15-06-26 (81)	15-06-19 (87)	15-01-19 (60)	15-06-19 (87)	15-01-19 (60)
2014-08-01	15-07-03 (71)	15-06-26 (81)	14-12-02 (53)	15-06-19 (87)	14-12-04 (56)

θ. The price sequence before θ is interpreted as a bubble, and the moment θ, which is unknown, is interpreted as the moment when the bubble bursts. One would like to exit the market as close as possible to the peak value of the price.

Changepoint detection should be performed in an anticipating way, i.e. a decision to exit the market at time t should be based only on observations of the prices up to time t (including t). We model this by claiming that the exit moment should be a *stopping time* A stopping time is a random variable $\tau \geq 0$ such that for any t the random event $\{\tau = t\}$ belongs to the sigma-algebra $\sigma(S_0, S_1, \ldots, S_t\}$ of the sequence S_t.

There are many methods and criteria of optimality for changepoint detection rules, see e.g. the monographs of Shiryaev (2019), Tartakovsky, Nikiforov and Basseville (2014), Basseville et al. (1993), Poor and Hadjiliadis (2009), and the review papers of Shiryaev (2010), and Polunchenko and Tartakovsky (2012). We use the approach developed and applied in our previous papers (see Shiryaev et al. (2014, 2015)), which assumes that the log-returns are independent and normally distributed with different parameters before and after the changepoint:

$$X_t := \log \frac{S_t}{S_{t-1}} = \begin{cases} \mu_1 + \sigma_1 \xi_t, \ t < \theta, \\ \mu_2 + \sigma_2 \xi_t, \ t \geq \theta, \end{cases}$$

where ξ_t are independent standard normal random variables, $\sigma_1, \sigma_2 > 0$ and $\mu_1 > \mu_2$ are constants. Although such a model is simple and it is well-known that the Gaussian distribution does not capture many empirical facts of log-returns distributions (see Cont, 2001), it has the advantage of having only a small number of parameters.

We apply the model in the following setting. We assume that the market is entered at time $t = 0$ with parameters μ, σ estimated from the past data, and we intend to hold the position until time $t = T$. The changepoint may happen with probability p between $t = 0$ and $t = T$, equally likely for each $t \in [0, T]$. Namely, θ is assumed to be random and independent of ξ, with the probabilities $p_t = P(\theta = t \mid \theta < T) = 1/T$. Parameters $p_t, t = 1, \dots, T$ and T as well as the values of μ_2, σ_2 are chosen before applying the model. In the applications, we analyze and compare several choices for them.

After the parameters have been specified, the criteria of optimality of an exit stopping time is to maximize the expected utility

$$EU(S_\tau) \to \max \qquad \text{over } \tau \in [0, T]$$

where the maximization is over stopping times τ taking values between 0 and T. Intuitively, when $t < \theta$ one should hold the position, as S_t increases on average. When $t \geq \theta$, one should exit. So, if θ occurs before T, τ should ideally signal about the exit close to θ; if a changepoint does not occur until T, then θ should be close to T, meaning the position is held as long as possible.

We use negative power utility which has very good long run optimality properties related to the Kelly capital growth investment criterion, see MacLean, Thorp and Ziemba (2010). Negative power with lognormal assets, so log returns are normally distributed, gives fractional Kelly strategies. If the asset returns are non-log normal, then the fractional Kelly is approximate. So we use the utility function

$$U(x) = \alpha x^\alpha, \qquad \alpha < 0,$$

and it is assumed that

$$\mu_1 > -\frac{\alpha \sigma_1^2}{2}, \qquad \mu_2 < -\frac{\alpha \sigma_2^2}{2}.$$

The above inequalities are a necessary condition for the solution to be non-trivial (if $\mu_1 < -\frac{\alpha\sigma_1^2}{2}$, then always $\theta = T$; if $\mu_2 > -\frac{\alpha\sigma_2^2}{2}$, then $\theta = 0$).

The structure of the optimal stopping time, i.e. which maximizes $EU(S_\tau)$ is found through a sequence ψ_t called the *Shiryaev-Roberts statistic*, which is defined recurrently by $\psi_0 = 0$ and

$$\psi_t = (p_t + \psi_{t-1}) \cdot \frac{\sigma^1}{\sigma^2} \exp\biggl(\frac{(X_t - \mu_1)^2}{2\sigma_1^2} - \frac{(X_t - \mu_2)^2}{2\sigma_2^2} \biggr), \quad t = 1, \dots, T.$$

The optimal stopping time is the first moment τ_α^* when ψ_t exceeds a certain time-dependent threshold

$$\tau_\alpha^* = \inf\{0 \le t \le T : \psi_t \ge b_\alpha^*(t)\}.$$

The threshold b is computed numerically as follows. Introduce the function $V(t,x)$ recurrently by

$$\begin{aligned} &V_\alpha(T,x) \equiv 0, \\ &V_\alpha(t,x) = \max\bigl\{0,\ \alpha\beta^t \bigl[(\gamma-1)(x+p_{t+1}) + (\beta-1)(1-G(t+1))\bigr] \\ &\qquad + f_\alpha(t,x)\bigr\}, \end{aligned}$$

where

$$\beta = \exp\left(\alpha\mu_1 + \frac{\alpha^2\sigma_1^2}{2}\right), \quad \gamma = \exp\left(\alpha\mu_2 + \frac{\alpha^2\sigma_2^2}{2}\right),$$

and

$$\begin{aligned} f_\alpha(t,x) = \int_{\mathbb{R}} V_\alpha\biggl(t+1,\ (p_{t+1}+x) \cdot \frac{\sigma_1}{\sigma_2} \exp\biggl(\frac{(z-\mu_1)^2}{2\sigma_1^2} - \frac{(z-\mu_2)^2}{2\sigma_2^2} \biggr)\biggr) \\ \times \frac{1}{\sigma_1\sqrt{2\pi}} \exp\biggl(-\frac{(z-\mu_1-\alpha\sigma_1^2)^2}{2\sigma_1^2} \biggr) dz. \end{aligned}$$

Then b is the smallest point where $V(x,t)$ turns to zero:

$$b_\alpha^*(t) = \inf\{x \ge 0 : V_\alpha(t,x) = 0\}.$$

Thus, to find the optimal stopping rule one first evaluates the function $V(x,t)$, and then finds the threshold b, which will define the stopping rule.

Remark. The function V is the value function of an optimal stopping problem for the statistic ψ related to the problem of maximizing $EU(S_\tau)$ (see Zhitlukhin, 2013 for details).

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