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Connecting Music to the Mathematics Classroom

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HONORS PROJECT

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Introduction to the Problem and Guiding Research Questions

From a young age, it seems that students either love or hate mathematics. While some find value in the numbers on their papers, others feel like they are trying to read a foreign language. I have always felt like mathematics is an area in which I can excel. If nothing else is constant in the world, mathematics always is. I can use different tables, graphs, and charts to express my understanding of the content. I can grow my knowledge through example problems and practice. And, when the time comes for a test, I can study my notes and know what to expect. Other subjects often feel the complete opposite for me. I practice memorizing historical facts for hours, but never retain the details. I cram the parts of a novel into my brain, but to no avail. But, when I return the next day to the mathematics classroom, everything is familiar. No current events, no changing of literature.

Another class I have always felt confident walking into was music. Ever since my elementary teacher taught me how to play the recorder and sing, I have felt drawn towards anything musical. I continued to participate in choir in both middle school and high school. I began to teach myself how to play piano and grow in my musical knowledge and ability. Similar to mathematics, every time I entered the doors to my choir class, everything else slipped away. I felt inspired by what I was learning and desired to learn more. I took pride in my ability to learn the solfege patterns and memorize my part. I loved feeling that success as the entire group sung their harmonies perfectly and it echoed around the auditorium. Parents applauded, friends shouted, and everything I had worked hard for had fallen into place like the pieces of a puzzle.

I always wondered how my brain exceled both musically and mathematically and why some other content areas did not maintain my level of interest. My two favorite classes presented no overlap, and, based on the separation of “core classes” and “electives” outlined in school, choir seemed to be less important for academics. Therefore, I never thought there would be a true connection between the reasoning used in algebra and the songs expressed on stage. This way of thinking would soon change as I graduated from high school and pursued my dreams at BGSU.

As I continued into my college career as a mathematics education major, I started to learn more about how to make mathematics interdisciplinary. A commonly discussed relationship was that between mathematics and science and the other “core” subjects. However, connections between mathematics and the arts were few and far between. It was uncommon for us to be educated on how to teach students how they can see mathematics in things such as music, art, and dance. I was, on the other hand, taking a course with Dr. Rancier on ethnomusicology immediately following Dr. Matney’s geometry education course. Therefore, when I went from experimenting with a mathematics compass to seeing it represented as a connection to music, I was immediately drawn to this interdisciplinary importance of mathematics. I felt the need to combine two of my favorite interests to teach students.

After looking back on the two classes that best shaped me to be the person I am today, music and mathematics, the connections between the two are prominent. While at first glance the similarities may not stand out to an observer, they are expressed in a mindful way of thinking outside the box. To notice them, you must think of mathematics as more than numbers and data, as well as treat music as more than singing and playing instruments. Both content areas allow those who participate to be creative and find passion in their craft. They are both their own form of art that inspires individuality. This overlap helps answer the infamous question asked by mathematics students every day: “when are we ever going to use this?” Most students would probably expect answers involving athletic statistics or tip percentages at restaurants. While these are both important facets of mathematical thinking, students who are interested in the arts would be able to see themselves as mathematicians as well. As a future educator who will aim to treat every student as the successful thinker that they are, the main goal for my project arose from this recognition.

After realizing the extent of the relationship between music and mathematics, it was evident that I did not feel as though my grades 7-12 career had highlighted this real-world connection as much as it could have. I felt the untapped potential to connect with students and build rapport with those involved in musical extra curriculars. Often, I feel as though schools foster an overwhelming emphasis on athletics. Through funding, state tournaments, and district-wide support, the students involved with sports outshine those who are talented by way of music. Having been a student who was involved with both athletic and musical events at my schools, I could see the clear divide of community interest. Other students in my choir classes that I knew were immensely talented musically and academically, even graduating to attend prestigious universities. Their potential in a classroom, however, was overlooked by the school's focus on athletics. This realization gave rise to the project that I wished to create. I feel as though mathematics should be something every student can relate to. There is a need for examples that highlight all facets of students' lives, including music, as they navigate the difficult transition into a career.

As I reflect on these two classes that shaped my academic career, I am inspired by the potential to show their overlap. I wish to be involved in the change of teachers establishing clear connections between artistic and analytic classes, while highlighting the beauty of mathematics. This will better prepare students with a well-rounded education, one lesson at a time. The lack of interdisciplinary conjunction makes me wonder if there is a reason that students are not shown the importance of music in a mathematics classroom setting. It also leads me to wonder if students who are involved in music have increased performance and potential in the mathematics classroom. In this project, I wish to investigate: How are music and mathematics specifically connected? How does a student's musical involvement affect their mathematical ability? How often are music and mathematics combined in schools? How can we involve more music in mathematics classes K-12?

Review of Literature

In order to invest in the connection of music and mathematics, it was critical that I find multiple sources that would highlight this research question. The literature that I gathered for my project includes a wide variety of videos, peer-reviewed articles, journal entries, and more. They each aim to examine the parallels of music and mathematics, while some are interactive activities that could be implemented in a classroom and others are written documentation. This background knowledge gives light to an existence of the relationship that may or may not be shown to students. It also shows the creativity that exists within mathematics when productively coupled with a musical element. The culmination of all these sources serves as the basis for my project, emphasizing the variety of researched materials to advocate for curriculum change.

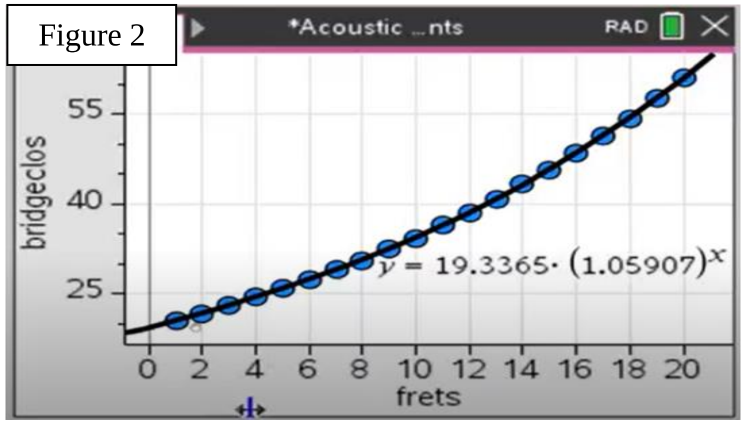
The Overlapping Fields of Mathematics and Music

Since 6th century BC at the school of Pythagoras, music, astronomy, and geometry have been intertwined in an ancient affinity (Tymoczko, 2009). The fusion of rational perspective and emotional sense continued with other famous mathematicians like Euclid and Archimedes. They believed that musical structures could be represented by geometric terms. Some of these are seen in tone progression, macro harmony, octave equivalence, chords, majors, and musical distance. Tymoczko elaborates on each subcategory in a thorough presentation with the Santa Fe Institute. In a section regarding chords, he discusses how when notes of a piano are placed on a circle, major chords, such as C major or D major, can be found by a rigid rotation.

Similarly, Jason Willcoxon of the North Carolina Council of Teachers of Mathematics highlights more connections through stringed instruments and the Pythagoras 2:1 ratio for octaves. Through technology such as GeoGebra and TI-Nspire, it is evident that the relationship between the distance from the bridge to the closest fret and the bridge to the furthest fret of a guitar is exponential. Figure 1 shows the lengths obtained for calculations by measuring in GeoGebra, while Figure 2 shows an exponential graph from TI-Nspire. Any stringed instrument, as it turns out, is a beneficial model of scales and measurement. The twelve notes that are represented on an exponential curve can be found using radicals and roots, and Pythagoras' 2:1 ratio is present when playing. This is when a string of exactly half the length of another plays a pitch exactly an octave higher (Willcoxon, 2019).

Figure 1

Figure 2



Even some of music's fundamentals, such as tone and rhythm, can be highlighted by mathematics. The basic shape of a tone, such as from the vibrations of a tuning fork, are periodic in shape, resembling that of a sine graph. Therefore, the distance between vibrations can be seen as an interval, just we know intervals of whole numbers on a number line (NCTM, 1979). Arithmetic progression of a tone, therefore, leads to a collection of notes separated by equal intervals, which are known as geometric progressions in mathematics. This can be further connected to Pythagoras' viewpoint on the twelve notes connecting octaves. Additionally, there exists a distributive property of rhythm, which has striking similarities to the mathematical distributive property of a(b+c) = ab + ac. In a mathematics sense, this represents multiplication acrost variables, while the musical element is the repetition of a pattern throughout musical variables, which we know as "tone" (NCTM, 1967). Some examples of this concept are in time signatures and beats, which are some of the fundamentals of music. Four "quarter notes" total to one beat, similar to how four "fourths" or four values of 1 divided by 4 equals 1 in mathematics.

Clearly, mathematics as related to music can be seen through both simple and abstract concepts. The connections range from those taught in elementary school, to those investigated in collegiate research. For example, the mathematics of a syncopated "canon" amongst musical participants can be seen in addition and subtraction, derivates and integrals, and even fractals. Canons can be seen though the distance between frequencies of each successive note or the larger lens of direction over long phrases and the overall range of a melody (Sulzer, 2021).

Teaching Mathematics with Music

Other sources highlight music as it pertains directly to mathematics education. There are many available lesson ideas related to the mathematics standards for grades K-6. While these creative components are helpful for younger grades, I find it important to note that these are not true "lesson plans" as we have learned at Bowling Green State University. They are rather outlines for what standards could be used in the classroom. For example, Jeff Todd provides a list of videos teachers could use for instruction on rhythmic patterns for grades 1-6. He provides examples of standards for each grade level, such as "analyze patterns and relationships (OA 3)" for fifth grade (Todd, 2019). The YouTube videos provided are applicable, but not complete, ways to teach each item to the indicated grade level. In addition to mathematics content standards, the National Association for Music Education also provides a list of PK-8 general music standards for educators. One of these, MU:Cn11.0.8a, reads, “demonstrate understanding of the relationships between music and the other arts, other disciplines, varied contexts, and daily life” (NAfME, 2014). This involves the inverse, where music educators can use another subject, like mathematics, to relay their curriculum to students. Either way, the focus lies in students being engaged in a multidisciplinary lesson that stimulates growth and allows class participation.

Other instructional processes, like those derived from the Mathematics Content Standards for California Public Schools are available for teachers through certain written documentation. Alongside these processes are the focus standards, such as NS 2.1 to represent first grade number sense, the overall strategy for teaching, and any materials needed in the classroom. (An, Capraro, & Tillman, 2013). These support teachers’ abilities to create interdisciplinary lessons that engage students through meaningful connections. Again, however, this pertains to elementary students and the integration of music for these students who are already required by “specials” to learn it.

It has been argued that the best teachers are knowledgeable not only about their content, but also about how to best teach their content. There is a noticeable difference, especially with regard to the impact on students. Therefore, teachers should be familiar with theories of child development and student learning in mathematics (Brahier, 2016). One example is Howard Gardner’s theory of multiple intelligences. In this concept, Gardner states that intelligence can be defined amongst seven areas, two of which are logical-mathematical and musical. Therefore, he encourages mathematics teachers to recognize the strengths of students that can be incorporated into planning lessons. Instead of siding with solely logical-mathematic learning, teachers could survey their classes on their areas of intelligence and use those to guide a lesson. After obtaining results to this pre-assessment, teachers could have access to exercises that would stimulate the musically intelligent students' mathematical growth and ability. This serves as a foundation for teachers to best support their students' skill sets and preferred means of learning.

Further classroom activities for integrating music and mathematics can be outlined through patterns, ordering, sorting, classifying, ratio, and fractions. These are all subtopics that elementary students learn in grades PK-6. Even something as simple as patterning symbols for notes and rests could be seen as the perfect combination between these two fields. Students could demonstrate understanding by clapping on notes and spreading arms for rests (Edelson & Johnson, 2003). Students gain beneficial mathematical knowledge through sorting instruments, whether it is grades K-2 classifying them as the sound that they make or grades 5-6 using a Venn diagram to examine rhythm patterns and create discrete sets of groups, just like they do in mathematics. In this article, and many others available upon research, the National Council of Teachers of Mathematics (NCTM) recognizes a beneficial connection between music and mathematics. As a member of the Bowling Green Council of Teachers of Mathematics, an affiliate of NCTM, this relationship holds a strong value in my opinion of the mathematics classroom. These are reputable ideas for teachers to foster this connection in the classroom.

Music in the Classroom Through the Lens of Technology and Classroom Management

In today's world, it is important for students to feel supported in the classroom by engaging materials and curriculum. Incorporating technology and classroom transitions are two resourceful ways to practice this concept. Relevant, stimulating actions from the teacher make students active participants rather than passive listeners. Students with access to technology can go to a given website and watch a video with engaging content to be introduced to the topic, rather than the teacher specifically telling the students what they are going to be learning. When they get to the end of a video, they can see a connection to mathematics and are introduced to a problem-solving activity (Get The Math). Then, they are able to work with the interactive technology to solve the problem. This is the perfect combination of visual, auditory, and bodily-kinesthetic learning. In certain examples, like the "Math in Music" exploratory activity from Get The Math, students can hear the mixed music track and know that their tempo may be wrong due to the sound of it. Then, it is evident that their mathematics could need to be redone in order to find the correct beats per minute. There are even extension tasks for students to do that are also online. This type of resource gives mathematics students a fun and tangible connection to music.

The possibilities are endless for teachers to use instructional videos as a lesson hook to show mathematics using music. Clips are available in many different online websites, including YouTube, and would be beneficial for students to begin working in a creative manner. One website that provides many of these options for teachers is PBS Learning Media. By providing educational and visually descriptive videos, they provide students with the "Math at the Core" experience. Specific music videos, like those with Jake Shimabukuro, typically fall into the 6-8 grade range, accomplishing standards like proportional relationships and unit rates. Teachers are able to transition from a video to an interactive, online musical scale activity (PBS Media, 2013).

While technology tools are abundantly helpful for teaching music in a mathematics classroom, many students are easily distracted by this addition. Whether they are screaming, fighting over a laptop, or frustrated over their device losing battery, this can be difficult for a teacher to manage. A beneficial way to bring the class back together would be to use music to facilitate transitions between activities (Lock & Press, 2006). This could involve the teacher using their own creative patterns, or using others that would be commonly known by the students. When students start to associate the song with a transition, it will bring the class back together into a productive group. In order to continue supporting academic knowledge through these transitions, teachers could also use songs that have mathematical relevance. A common example of this is seen through the “Quadratic Formula Song” that many teachers use to help students remember it (Imbro, 2010).

Finally, as a way of inspiring students who do not enjoy mathematics or do not feel as though they are capable of doing mathematics, teachers can remind students that they are being creative. Even though we normally think of creativity as things such as music, mathematics is just as relevant to this topic. Students who do not normally feel inspired in mathematics class can see that adding the real-world connection of music did not change the creative value of calculations. Science writer Margaret Wertheim argues that “mathematics, in a sense, is logic let loose in the field of the imagination” (Barton, 2019). This inspiration, coupled with a lesson involving musical elements, may be what students need to see their worth in mathematics. Any problem or new approach is done by students in creative way unique to them (Scott-Curran & Scott-Curran, 2019). Even doing their homework strengthens their abilities to be original and create productive work. Reading these books to a younger classroom provides the necessary motivation for future mathematical success. Overall, there are many resources available for teachers, and all professionals, to learn the value of music and mathematics being united to create a positive but unexpected relationship.

Description of Methods Used to Solve the Problem

Upon beginning my research, I centered around my initial research questions. I had to be able to show that there was a worthwhile connection between music and mathematics. Since my project was centered around this concept, it was important to have substantial evidence to show this relationship. When I started by discovering this initial overlap, I was shocked with the amount of research that had previously been done. I was glad to see that other people have investigated this connection, and it made me realize that my project idea was attainable. With so much content available to me, I started by reading as many reputable sources as I could find. Once I had compiled a list of materials that enforced the knowledge I was trying to spread, I had to sort it into my most useful items. The five sources I thought best demonstrated the connection between music and mathematics through a variety of examples were therefore assigned to the literature review. After this step, I felt as if I had obtained the necessary background information to continue forward.

Next, I needed to investigate the relationship between music and mathematics as seen in the academic setting, preferably in grades K-12. I started this with a search in the Curriculum Resource Center on the second floor of the Jerome Library at BGSU. I assumed that I would find books involving both music and mathematics for grades K-5 in this area. What I was not expecting to find, however, were children's books on the power of creativity that implicitly drew the connection I was searching for. These books showed that music and mathematics are both ways of expressing your originality and thoughts, and therefore are intertwined. While this was a beneficial documentation for my research, I consulted the internet for stronger pedagogical overlaps. Similar to my first findings, I was amazed by the abundance of ideas for mathematics teachers that would promote musical intelligence amongst students. I sifted through the articles and interactive activities until I compiled a list that ranged in publication date and technological involvement. I wanted to include sources that were both clearly created in a time that tech intervention would be impossible and those that could use this modern tech for beneficial problem-solving. However, there were several setbacks with these types of sources found.

The first issue I was running into was the lack of "planning" beyond a generic layout of classroom ideas. Even if the article had specific mathematics standards listed, most of them did not provide enough pedagogical context for practical use by teachers. There was not typically a true lesson plan attached to the idea, merely a thought for a mathematics standard and a musical element to be united in the classroom. I searched for further context that would give teachers the instruction needed to guide a classroom in the right direction, to little avail. The other problem I faced during this part of my research was the lack of resources for teachers who are certified to teach grades 7-12. While some of the sources I provided included 7-8 application and standards, most were PK-6. I pondered why the middle to high school age range might not receive the online support for music, and I planned to further examine that when answering my next two research questions. After adding my ten successful pieces, like those from NCTM, to the existing list, this finalized the background literature for my project, and I continued on to further research.

Next, I began to delve deeper into how musical ability affects mathematical performance as well as how often music and mathematics are truly combined in schools. I approached the measurement of ability with a quantitative mindset, looking for raw data that would show the answer to the question. Since I had previously established the connection of the two subject areas, I inferred that the results of tests would indicate that students improve in mathematics classrooms after music intervention. I used many different research websites and libraries to find the data that would best answer the initial question. After extensively looking for the best data sets, it was clear that the results I had found were beneficial to my study. In order to determine the frequency in which schools promote learning mathematics through the medium of music, I had to use multiple strategies. My original idea was to create a survey for K-12 mathematics teachers in multiple different districts to see how often they create interdisciplinary lessons. This would then become more specific to music to see where it is most common to incorporate musical intelligence. However, after several setbacks in the research process, I determined that analyzing three types of existing studies and creating a suggestion for new policy would be a better project approach.

Therefore, with the knowledge that I had obtained from research on specific studies, documents, articles, activities, and much more, I created a lesson plan that could be used in classroom for students in grades 7-12. This original scholarship promotes the understanding that I gained of how to properly connect music and mathematics while developing a pedagogical context. The lesson plan shows an example that teachers can clearly read, understand, and implement in the classroom with or without a musical background. Finally, I created a suggestion for eight grades 7-12 mathematics standards that directly acknowledge not only interdisciplinary learning but learning through music. Both items will be clearly exemplified in the subsequent sections, with more context and reasoning to elaborate on the relevance of these crucial pieces in my project.

Due to the aforementioned setbacks in research, there are some limitations to this project that I would like to address. The main limitation lies in the fact that I am using preexisting data to suggest a change, rather than create my own data set. With the unknown nature of online data, even from reliable sources, the percentage of error in the values could be larger than anticipated. Additionally, in the data used on student performance, there are many unseen variables that have potential to effect results. For example, a student’s general mental health and investment to school are not revealed. Their race and ethnicity are not considered in the results that I used. For these reasons, I present the lack of knowledge of the students studied as another limitation.

Description of results

After extensive research on all things music and mathematics, the answers to the original research questions were evident. Through the three complex studies selected for culminating conclusions and my original contributions, I found ways to fix the problem I set out to solve. All three data sets reflect the result that music involvement improves mathematical ability. In order to perfectly encapsulate this specific conclusion, each study examines a different type of musical intervention amongst students, while still leading to the general positive correlation. Furthermore, the research indicates that the relationship between music and mathematics has more importance in elementary schools than high schools. The lack of musical presence in mathematics classrooms of grades 9-12 is directly reflected by the learning standards for students of these ages. There are very few mathematics standards that specifically encourage interdisciplinary connections to the arts, like music. While some promote real-world connections by way of mathematical modeling, I believe there should be specific content standards for grades 7-12 mathematics that address musical adaptations. In an attempt to eradicate this issue, and answer my final research question, I developed a lesson plan and eight content standards based on the Ohio Learning Standards. I felt that these would adequately reflect the change I wish to see in school curriculum towards an interdisciplinary education for all students.

Mathematics Lesson with a Music Activity Integration in an Elementary School

In this first study, conducted in Southern California in 2012, two teachers designed and implemented music activities as an integrated part of their regular five-week mathematics lesson (An, Capraro, & Tillman, 2013). In totality, the classes taught consisted of 46 students (25 third grade and 21 first grade) ranging in ethnicity, mathematical ability, and English language ability. Each music-mathematics activity, MSA, was designed to determine mathematics ability in modeling, strategy, and application. These lessons were very well detailed and clearly outlined as the teaches used clapping, patterns, songs, handbells, and more in order to teach their content. Before any intervention began, five MSAs were taken by the students labelled Pretests 1-5.

When the intervention occurred, five more MSAs were taken by the students, labeled Posttests 6-10. The results exemplified that the music-mathematics interdisciplinary lessons had positive effects on modeling, strategy, and application. The mean ability level of both age levels of students increased post-activity from every Pretest to Posttest. Figure 3 represents the improvements of tested first grade students, while Figure 4 represents the improvements of third grade students.

MSA Focus N=21		Mean	*SD*	*t* value	*p* value	Cohen's d
Post&Pre Model	Pre 1	2.62	1.02	-7.589	<0.0001	1.66
	Post 1	3.00	.93			
	Pre 2	2.19	.81			
	Post 2	3.24	.86			
	Pre 3	2.38	1.07			
	Post 3	3.43	.51			
	Pre 4	2.57	.75			
	Post 4	3.67	1.04			
	Pre 5	2.09	.83			
	Post 5	3.81	1.20			
Post&Pre Strategy	Pre 1	2.29	.90	-8.939	<0.0001	1.75
	Post 1	2.90	1.0			
	Pre 2	2.24	1.11			
	Post 2	3.14	1.15			
	Pre 3	2.14	.91			
	Post 3	3.29	1.18			
	Pre 4	1.95	1.16			
	Post 4	3.38	.91			
	Pre 5	2.19	1.03			
	Post 5	3.71	.78			
Post&Pre Application	Pre 1	1.90	.77	-8.221	<0.0001	1.70
	Post 1	2.33	1.08			
	Pre 2	1.62	.92			
	Post 2	2.67	1.24			
	Pre 3	2.00	.55			
	Post 3	2.81	.87			
	Pre 4	1.38	.74			
	Post 4	3.00	1.16			
	Pre 5	1.67	.97			
	Post 5	3.38	.79			

Figure 3

MSA Focus N=25		Mean	*SD*	*t* value	*p* value	Cohen's d
Post&Pre Model	Pre 1	2.28	.46	-11.71	<0.0001	3.40
	Post 1	2.80	.95			
	Pre 2	2.12	.60			
	Post 2	3.16	.62			
	Pre 3	2.28	.46			
	Post 3	3.36	.58			
	Pre 4	2.12	.60			
	Post 4	3.68	.48			
	Pre 5	1.96	.54			
	Post 5	3.96	.20			
Post&Pre Strategy	Pre 1	2.04	.68	-11.93	<0.0001	3.44
	Post 1	3.00	.76			
	Pre 2	1.68	.56			
	Post 2	3.20	.71			
	Pre 3	1.92	.64			
	Post 3	3.40	.87			
	Pre 4	1.68	.56			
	Post 4	3.68	.56			
	Pre 5	1.88	.53			
	Post 5	3.68	.69			
Post&Pre Application	Pre 1	2.28	.46	-12.59	<0.0001	3.00
	Post 1	2.88	.82			
	Pre 2	2.24	.44			
	Post 2	3.48	.40			
	Pre 3	2.28	.46			
	Post 3	3.60	.67			
	Pre 4	2.24	.44			
	Post 4	3.84	.87			
	Pre 5	2.12	.33			
	Post 5	3.92	.47			

Figure 4

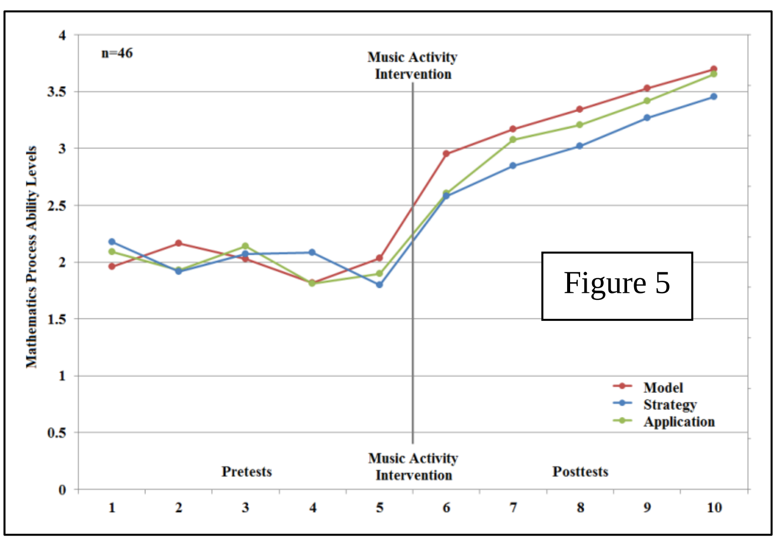


Figure 5

The overarching results of the entire study can be easily depicted in Figure 5, which shows the increase of average ability level for each mathematics process over time for both first and third grade.

Music Infused Mathematics Lesson in an Elementary School

In the second study, the teachers used a double-digit subtraction lesson for a group of second grade students. The experimental group made up of 30 students was taught the concepts using an arts infusion approach, while the control group of 35 students was taught the same concepts without any artistic involvement. For those who received the arts lesson, music involvement consisted of chants, songs, instruments, body percussion, and recorded music. The study took place in an inner-city elementary school in Pennsylvania, with all four classes being comprised of racially diverse subjects. Students were evenly divided amongst the classes based on first grade ability levels to ensure an even distribution of mathematical performance (Habursky & Omniewski, 1998). All groups were taught using the same method of instruction, receiving whole class activities or stations at the same point in the lesson. The test ran for four weeks, using mathematics achievement tests (MTs) as pretests and posttests to analyze data. The results showed that subjects in the experimental group had significantly higher mathematics achievement scores than those in the control group. The pretest mean score for all groups was 43.8 and the posttest mean score was 78.6. Table 1 shows the posttest mean, standard deviation, and number of participants for both the experimental and control group. It is evident that the success of a musical intervention in the arts infused lesson fostered greater success amongst students than the lesson typically taught. The mean of 84.0 is greater than the mean of 73.6 and the standard deviation of 16.03 is less than that of 18.5, supporting this conclusion.

Table 1
T-Test for Independent Samples

	Experimental			Control		
Posttest	*M*	*SD*	*n*	*M*	*SD*	*n*
Scores	84.0	16.03	30	73.6	18.5	35

Note. $t = 2.39$
$*p \leq .05$, two-tailed

School Music Participation and Mathematics Achievement in a High School

The final study in my series of results examined the relationship between student music participation outside of mathematics classes and student success in mathematics. This credible study is found in the Journal of Educational Psychology from the American Psychological Association. 112,916 total student records were analyzed from four Canadian public schools ranging from grades 7-12. For mathematics, grade 10 scores were specifically examined to see if those involved with school music classes scored better than their peers who did not. In order to ensure credible research, the results also show variables such as gender, language background, household income, and exam grades for other courses for all students whose data was collected. The results of the study were that both music participation and higher levels of music engagement were related to higher exam scores (Emerson, Gouzouasis, & Guhn, 2020). The mean score for tenth graders with no music involvement was 67.0, while the mean score for tenth graders with music involvement was 74.7. Table 2 depicts part of a larger graph within the research report that demonstrates mean exam scores for each content area sorted by instrumental music engagement. Overall, the findings of this research show higher academic outcomes to those who took music classes over the course of their 7-12 career, while focusing on tenth grade mathematics score reports.

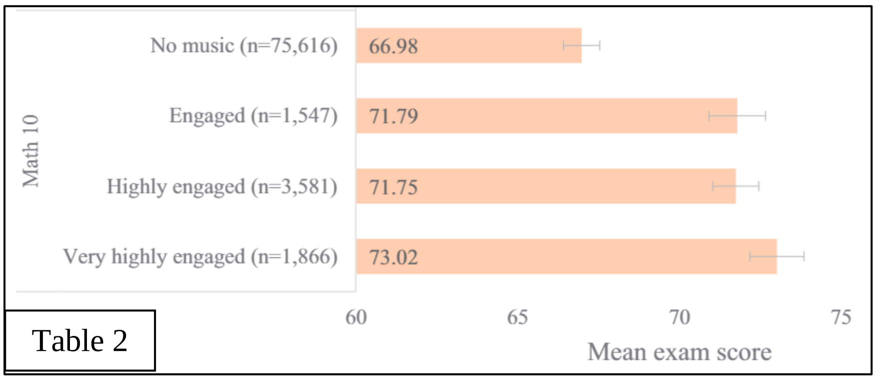


Table 2

While all three of the studies used to sum up my research experience have differences, their conclusions unite them together. Through the provided data, it is evident that music intervention in classes and schools improves mathematical ability for all students K-12. This answer to my second research question leads me to originality regarding the third and fourth.

Original Scholarship: Content Standards

After extensive research regarding the connections of music and mathematics in a school setting, it was clear that this relationship is only enforced amongst young students. While there are many suggestions available for elementary mathematics teachers to use music in the classroom, few exist to support middle school and high school teachers. Each study I found that discussed clear, valuable data to support my specific research questions came from grade K-8 suggestions. The abundance of information was even lower, typically K-6. As exemplified in the research results, I chose to describe, the high school music data discussed music outside of the mathematics classroom, not while specifically applied to grade 9-12 mathematics. The lack of urgency to continue to support musical intelligence while in high school is alarming. Schools are fundamentally designed to include upper grade music as an optional "elective" students can choose, therefore diminishing its potential to be applied to subjects every student learns. As I searched through Ohio's Learning Standards, I analyzed the June 2022 Draft of Fine Arts: Music and the 2017 Mathematics sheet.

The examination of the music standards led me to the conclusion that interdisciplinary connection is only required in K-6 classes. Searching for key words such as "discipline", "content", and "subject" result in the only matches being K.3CO-6.3CO standards for K-8 and beyond. These encourage applying what is learned in music to other subject areas. Therefore, there are no 7-12 music standards that require teachers to discuss mathematics in music classes. Additionally, when searching for the word "mathematics", there are no results on the music standards document. Similarly to this, searching for the word "music" in the mathematics standards yields no matches. The only interdisciplinary word with any beneficial result, including “content” and “discipline”, is “subject”, which only gives a specific example of discussing other subject areas in a Statistics and Probability Standard, S.CP.4. The lack of music or interdisciplinary material outlined in Ohio’s Learning Standards supports the results of the other studies discussed. There are only elementary, K-6, relationships between content areas and high school students are not required to make these fundamental connections through any sort of curriculum required by law.

This leads me to suggest a policy change for the State of Ohio’s Learning Standards for Mathematics. Through my experiences as a leader in the Honors College and a preservice teacher with experience in classrooms, I believe that I am qualified to create this list of mathematics content standards. Furthermore, after seeing the lack of music and mathematics connection, in research and in Ohio’s Learning Standards, this is a productive way to increase student achievement. It also presents as an adapted way to make mathematics classes more engaging and stimulating for all mathematics students. Ideally, middle school and high school mathematics teachers will be able to use the standards in Table 3 to fundamentally connect music to their curriculum. They provide an opportunity to further the interdisciplinary and real-world connections that are crucial to student understanding.

Table 3 – Suggested Ohio Learning Standards for grades 7-12 mathematics with music

Content Standard Title	**Grade Level/Content**	**Music and Mathematics Content Standard**
7.RP.4	Grade 7 Mathematics	Use ratios and proportions to show the meaning of octaves in music.
8.EE.7	Grade 8 Mathematics	Graph musical frequency and tone as a proportional relationship using intervals.

Table 3 (continued)		
M.M.1	High School Modeling	Model mathematics through a creative project using musical elements.
N.RN.3	High School Number and Quantity	Compare the music number system to the real number system using rational numbers.
A.CED.5	High School Algebra	Create musical equations of more than one variable that produce harmony.
F.LE.6	High School Functions	Show that beats in each time signature create a function.
G.CO.15	High School Geometry	Construct congruent sequences and canons using rigid transformations.
S.IC.7	High School Statistics and Probability	Develop a sense of probabilistic prediction in chord progression.

Original Scholarship: Lesson Plan

The research leading to my new mathematics content standards, and the inclusion of the standards themselves, answers the third research question. Students are not shown the overlap between music and mathematics enough in schools, specifically in grades 7-12. It also leads into the potential of the final research question. While standards are a beneficial way to guide teachers' content in their classrooms, there are better ways to help. One of these is an adaptable lesson plan for high school teachers to use. The following lesson plan, Figure 6, is the final step for the culmination of research that has taken place for this project on music and mathematics.

Figure 6

Title of Today's Lesson				
Your Name	Nick Parr		**Date**	4/25/2023
Subject/ Course	Algebra II		**Grade**	9
Unit Topic or Theme	Trigonometric Functions		**# of Students**	20
Class Length	50 minutes	Day 5 of 8	**CMT Initials**	

Central Focus/Concept, Essential Question, or Enduring Understanding

How are European major scales constructed? What is frequency in music as related to mathematics? How do the elements of a European major scale relate to trigonometric functions?

Lesson Rationale and Summary

The students will use the real-world application of music to discuss frequency, amplitude, and period. They will begin by using the different frequencies of overtones to build a scale. This will be done using a printed card-sort. Through this activity, students will understand the making of a scale based on ratios between the notes, as previously discussed in the unit. By graphing the musical frequencies of the notes in the scales, students will develop an understanding of sine graphs and their components. It is important for students to explore this concept on their own, with a partner, and with the entire class.

This lesson serves as a vessel to connect mathematics and music in a high school setting. Throughout exploration, students who are not involved in music can still excel just as must as their peers that are.

Note: All references to scales refer to European major scales that students are typically taught in school music classes such as band and choir.

Learning Standards for Mathematics

Ohio Learning Standards

- F.TF.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- F.TF.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- SMP.5 Use appropriate tools strategically
- SMP.7 Look for and make sense of structure

Parr Learning Standards for Mathematics and Music

- M.M.1 Model mathematics through a creative project using musical elements.

Learning Objectives

Each student will be able to:

- Construct a musical scale using frequency ratios
- Distinguish between musical frequency and mathematical frequency
- Identify amplitude, frequency, and period
- Identify a sine graph
- Accurately match the frequency (in Hz) to its graph
- Replace common phrasing with current academic language (as in 2:1 instead of "double")

Academic Language

- Frequency
- Scale
- Amplitude
- Period
- Graph
- Sine

Planned Assessments

The students will have participated in Pre-Assessments from the other days of the unit. They will have been assessed in their knowledge of trigonometric functions, including how to create them and what they look like with or without using technology.

This lesson's Formative Assessment is an observation of the students' abilities to accurately create a scale using the ratios, as well as the ability to create a trigonometric graph of the frequencies provided. This will take place in the form of a direct check-in with the teacher and an exit ticket.
The unit test Summative Assessment will take place in three days, following the informal Summative Assessment Review Day.

Differentiated Instructional Strategies

- Purposefully seat students in pairs so that those who struggle with trigonometric functions and/or technology have someone who is likely to be able to help them sitting nearby
- Including the options for tangible pencil and paper graphs, as well as including videos and technology such as Desmos to make the lesson adaptable
- Focus attention on students who do not make much progress on the activity after the card-sort

List Resources

Each student will need:
- A partner at a nearby desk
- A set of card-sort frequency cards
- A handheld device with Internet and CANVAS access
- Notebook paper
- A pencil

THE LESSON

1. MOTIVATION Allotted Time: 10 minutes

1. By a show of hands, who is involved with the band or choir here at school? Who has ever played an instrument or knows someone who does? Who is not very involved in music but listens to it in the car or other times throughout the day?
   Anticipated answers: all hands should be raised during at least one question

2. As you can see, we all interact with music at some point in our lives, but we also interact with music while studying mathematics and we do not even know it.

3. Today we will be examining how the frequencies of musical scales are directly tied to what we have been discussing in class.

4. Please take out your devices and find a partner that you would like to work with.

5. Go to the Virtual Piano Keyboard (https://www.onlinepianist.com/virtual-piano) and begin by getting familiar with the keyboard and the names of the notes. You can even try playing Mary Had a Little Lamb (EDCDEEEDDDEGG, EDCDEEEEDDEDC).

6. Once you feel comfortable with this concept, hold a thumbs up so I know when to start. Try building what you think is a "scale" while you wait for your classmates.

ASSESSMENT: The motivation is a class participation activity, so assessment of student participation will take place through informal observation of thumbs up when students are ready.

Transition: Would anyone like to play their version of a scale for the class?

2. LESSON PROCEDURE Allotted Time: 30 minutes

1. As students play their scales, ask students to hold their thumbs up to indicate if they agree or disagree. If many students disagree, ask them to describe why they think so.
   Anticipated answers: it doesn't sound right, one of the notes was off, something is wrong, etc.

2. In order to fix our scales, we are going to look at the ratios between notes. In the card-sort, you are given multiple notes and their overtones, and asked to connect these using ratios based on their frequencies. You should have 2 columns of cards, the left column being the notes and their frequencies and the right column being the ratios between them.

3. Using a frequency pitch tool, play your scale in order from top to bottom to see if it sounds right. When you believe you have it correctly laid out, be ready to show it to me and describe why you did what you did using the ratios given as well.

4. Before you begin working, I will let you know that a standard scale has 8 total notes in it. You may work alone or with your partner.

5. As students work, monitor their progress and answer questions. These are examples of cards that would be in the sort: 220Hz, 260Hz, 520Hz, 780Hz, 1040Hz, 1300Hz, 2:1, 3:2, 4:3, 5:4, 6:5

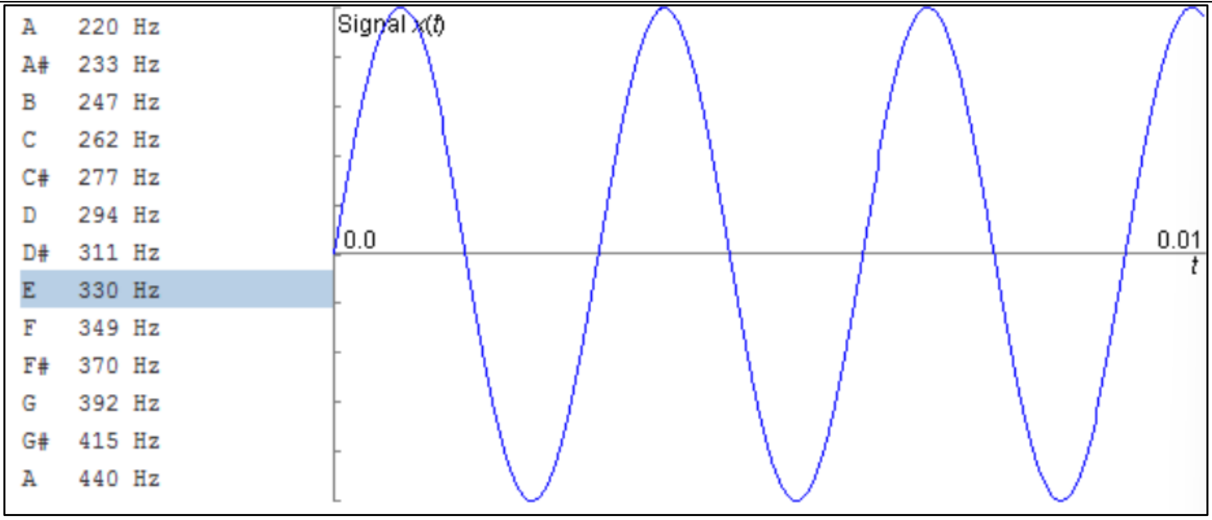
6. As students continue, allow think time, but encourage those who have not finished in 10 minutes to move on to creating graphs with their frequencies.

7. When the students finish the scale, they are to open Desmos on their devices and begin to try to graph what each frequency will look like.

8. Students will initially not understand how to graph a frequency, but they will realize that through trigonometric functions, frequency relates to a "wave shape" like a sine graph. This is also the first hint the teacher can give if students are not comprehending.

9. Additionally, students will have to test multiple different graphs to see if it is sine or cosine and what the period and amplitude should be. When they have graphed multiple frequencies in their scale, they should see that they look to be in rhythm.

10. The higher the frequency, the shorter the period of the graph.



11. Students can check their answer with the teacher and receive guidance for how to continue in the right direction.

12. As students finish, ask them to quietly test more frequencies until the last ten minutes of class.

ASSESSMENT: For their formal assessment, the students have to check-in with the teacher to get approval to move on from the scale to the graph, and from the graph to the exit-ticket.

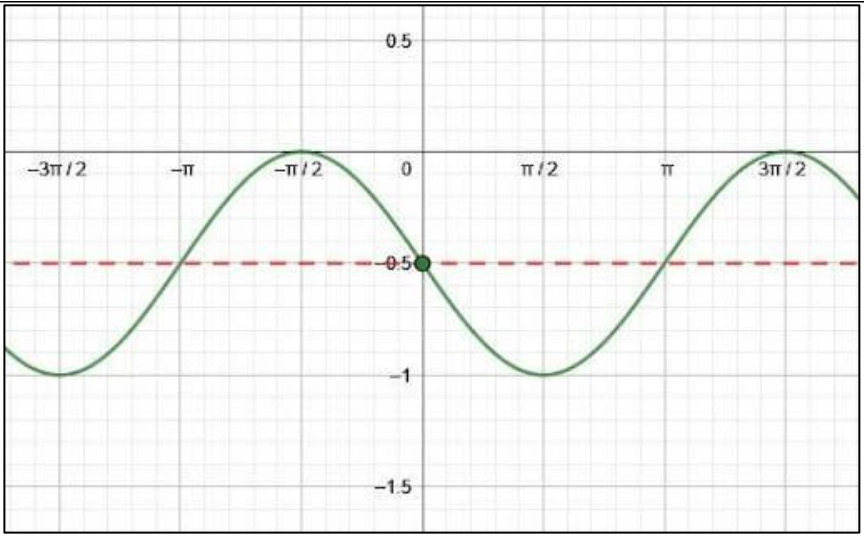
For informal assessment, the students will be asked to show thumbs up based on agreement and if/when they are ready for further instruction.

3. CLOSURE **Allotted Time: 10 minutes**

1. In this class period, you have combined your knowledge of trigonometric functions with real-world music to draw some further conclusions about sine graphs and their properties.

2. Not only were you able to create ideas independently and with others, but you used the activity to relate a fun topic like music to a mathematics concept that we have been studying.

3. Now, I would like for you to complete an exit ticket that is a culmination of what we have learned in this unit. You will be given a graph of a trigonometric function and asked to identify the amplitude, frequency, and period. Please complete this task independently, but feel free to let me know if you have any questions.



4. You must complete this before you leave class to receive credit. If you complete the exit ticket early, please stay quiet for your classmates until the bell rings.

Homework: There will be no homework over this lesson.

4. EXTENSION

If time allows, let students explore Desmos activities on trigonometric functions independently. Since the review day of the lesson is soon, they could also be handed the amplitude, frequency, and period section of the review guide as a silent task to get a head start on.

Additionally, students can investigate other scales outside of European tradition such as *maqams* from Arab culture. Through investigation, they can discover the importance of mathematics in this type of music with regard to intervals. This provides an interesting opportunity to learn more about what ratios create the perfect "scale" and connects music to mathematics across multiple cultures.

Implications for Future Research and Practice

The conclusions made from this research show the importance of connecting music and mathematics in the classroom in order to improve students' mathematical abilities. It is evident that there is a need for change in our education system in order to better integrate the arts with subjects that all students are required to take. In upper grades, the lack of inclusion of music in students' lives calls for added reform of the standards that are being taught. Even with these results, however, there are still unanswered questions to be further explored. How do mathematics scores vary at schools where all students K-12 are required to take a music class? What effects does the type of music that is integrated have on students' abilities? Do the results of these studies pertain to students pursuing higher education? These are the questions that would guide future research on this topic and support a continuation for more answers. This project opens the door for the possibilities on infusing artistic intelligence with logical intelligence, something that would benefit all involved with a well-rounded education. That being said, one implication for future research is the openness of this subject and the many different paths that one could follow to find out more information.

Additionally, a common issue that I envision educators facing is the lack of student engagement from those who are not interested in a mathematical approach to music. While most students listen to music in their daily lives, some may create resistance when asked to examine the more educational components rather than their favorite rap song. In order to combat this issue, I find it important to consider the many different ways that music presents itself. Showing students the benefit of this knowledge while applied to all of their levels of musical ability creates comfort amongst the class. Musical skill has just as wide of a range as mathematical skill, which could create some anxiety amongst students. Therefore, I believe that future research should examine the relationship between success of a music-based mathematics lesson and the classroom rapport. While it is simple for those who are not in front of the classroom to encourage teachers to use this material, it is far more difficult to execute. A proper implementation of this lesson may require a certain level of trust amongst students, which I would be interested in seeing data results on.

With this implication in mind, I believe that with the proper classroom management, mathematics teachers should be able to not only integrate music, but also dance, drama, visual arts, and media arts. These are the subcategories of the fine arts standards that are provided by the state of Ohio (ODE, 2022) and all have a very strong value in a classroom. Through extended curriculum or standards, as provided for music in the results of the project, these all have potential to be showcased as a interdisciplinary vision of art. Therefore, the potential for research on these extended categories is limitless. I believe that this fundamental framework of music through the lens of mathematics would lead to more profound research in this field. It may give way to some more creative approaches from teachers who would have never considered this idea beforehand. By opening the possibilities, many more studies could lead to beneficial practice in the world of education.

Furthermore, this new interdisciplinary development should not be limited to mathematics. Other subjects that are required for students to take during their academic careers could build from of this foundation in order to infuse the arts into content standards. With these newly developed guides for instruction, teachers in content areas such as English, history, and science could benefit from the same concept. As encouraged in schools through STEAM (science, technology, engineering, arts, and mathematics), this would unite the disciplines that are not always thought to overlap, Therefore, the call for research extends beyond the realm of mathematics, in an attempt to foster creativity for all students. Further study could explore these connections as related to the class setting in order to provide educators with the resources to simulate their students' engagement. Fostering this culture of real-world application and understanding is crucial for student success.

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