

PLANE-PARALLEL LAMINAR FLOW OF VISCOUS FLUID IN THE TRANSITION ZONE OF THE INLET SECTION



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Abstract: A study was conducted to analyze how hydrodynamic parameters change in the entrance region of plane-parallel flow under stationary flow conditions, with an initial arbitrary distribution of velocities in the entrance section. This study was based on boundary layer equations, and a boundary problem was formed under the conditions of plane-parallel flow. The boundary conditions were chosen to reflect the pattern of arbitrary velocity distribution in the entrance section. A general solution of the approximating Navier-Stokes equations is provided, which depends on the initial conditions and the Reynolds number. The boundary conditions are established based on the nature of the motion, and the boundary value problem is described. A method for integrating the boundary value problem has been developed, and regularities for the change in velocities along the length of the inlet section have been obtained for both constant and parabolic velocity distributions in the entrance sections. Analytical solutions have been derived to obtain patterns of velocity and pressure changes in any given flow direction. Through computer analysis, velocity change patterns in various sections along the inlet transition area have been constructed, allowing for the determination of fluid velocity at any point on the section and an estimation of the length of the transition area. These proposed solutions provide a framework for accurately constructing individual units of hydromechanical equipment.

Keywords: viscous fluid, inlet section, velocity, velocity distribution, pressure, length.

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Introduction

Studies of velocity distribution patterns after the inlet section in closed beds show that the particles near the static walls perform a decelerated motion, and the particles near the axis experience an acceleration and perform an accelerated motion. These two tendencies result in a rearrangement of the velocity field at a certain point of the movement, causing a change in the distribution pattern of velocities in the inlet section. Within the transition zone, the distribution of the inlet section velocities is rearranged to match the velocity distribution pattern of the stabilized zone in closed beds. The accuracy of the research for the transition zone of the inlet section is determined by the challenges associated with the precise construction of the fluid channels of the machinery. It is necessary to ensure clear and stable operation of the control and regulation systems.

Therefore, the problem discussed is both relevant and of significant practical importance. The vital problem of fluid flow research is the mathematical model construction of the given physical phenomenon, which determines the applicability limits of the selected calculation method. It is vital that the built model more accurately describe the ongoing hydromechanical phenomena and, meanwhile provide the possibility of obtaining analytical solutions.

Literature review and problem statement

Research on the flow patterns of viscous fluid in transition areas of liquid channels, such as inlet and outlet sections, sudden expansion and narrowing, etc., is an applied problem that provides solutions for designing various machine tools. Due to their urgent and applied importance, the investigation of these problems remains highly relevant.

The starting equation for the study is the system of Navier-Stokes equations in a non-deformable medium. For each problem, boundary conditions are defined and the basic equations are simplified. However, obtaining analytical solutions to the problem is often not feasible. In such cases, modern computing techniques can provide an opportunity to obtain approximate solutions, which ensure accuracy in practice.

Many theoretical and approximate calculation methods have been developed for studying the hydrodynamic phenomena in the transition area of the inlet section.

Each calculation method is based on conclusions about the nature of the flow, which are used to conduct theoretical research and summarize the results. These conclusions often refer to certain ranges of motion, due to which the applicability of the obtained results is limited. A method for calculating the entrance transition area of plane-parallel motion was proposed in work [2], in which the author numerically implemented the integration of the boundary layer equations, which makes it possible to calculate the image of the distribution of velocities in any section of the transition area of the entrance section. Similar solutions were also implemented in [1]. In that study, solutions were obtained for velocity distribution and pressure change in the transition zone of the inlet section of the plane-parallel motion. These solutions were obtained using the parabolic law of velocity distribution in the boundary layer and the condition of their constancy at the core. Studies have been conducted to identify patterns of changes in the hydrodynamic parameters of the flow in the transition zone of the inlet section of the plane-parallel motion based on the approximation of the Navier-Stokes equations [3,4]. In these studies, a boundary problem was developed and analytical solutions were obtained to determine the patterns of changes in velocities and pressure. The results of the analytical solutions were compared with those obtained from experimental research, and the comparative analysis confirmed the reliability of the obtained results.

Similar research was conducted in [9], where approximate solutions were obtained for a cylindrical and plane-parallel isothermal laminar motion, providing results with sufficient accuracy. The study also investigated the laws of the velocity field and pressure change under conditions of a four-step change of velocities in the boundary layer of the entrance region [14].

In [11], the viscous fluid flow in the inlet transition zone under the conditions of changing pressure gradient in axisymmetric pipes is considered. The proposed solutions are also applicable to Newtonian fluids.

Under the conditions of varying the kinematic coefficient of viscosity according to an arbitrary law, the proposed solutions to the momentum equation [12] are represented by the Fourier series and the Bessel and Kelvin functions. Reference [13] analyzed flow stability based on the flow rate ratio to the steady-state laminar flow rate. This analysis led to the establishment of stability conditions for unsteady flow in a pipe. In [8], the authors developed a mathematical model to determine the velocity field and pressure distribution in the laminar motion of a viscous incompressible fluid in a two-dimensional change environment. However, the proposed solutions do not apply to determining the hydrodynamic parameters of the flow in areas of sudden expansion.

The studies mentioned mainly focus on the interpretation of phenomena occurring at the entrance of the pipe. However, hydrodynamic parameter rearrangement phenomena also occur at other transition sites in the pipe, for which research is scarce. For instance, at the site of sudden expansion of the cut ($D/d = 4$), numerical integration was utilized to construct current lines using the equations of plastic fluid current movement. This allowed for the determination of velocity and pressure changes in the axial direction [15].

Extensive experimental studies on the area of sudden expansion have been conducted [16]. The authors utilized the magnetic-resonance tomographic method to obtain quantitative estimates of the change in velocities in the transition area. Under conditions of sudden, symmetric and asymmetric expansion of the section, quantitative estimates of the Navier-Stokes equations' members were obtained, and the resulting nonlinear inhomogeneous differential equations were numerically integrated [18]. The integration results were compared with the results of experiments. Remarkable experimental studies on the region of sudden expansion were conducted in [19]. To that end, test equipment was constructed, and the areas of sudden expansion were

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tested for the cases $d/D = 0.22$; 0.5 and 0.85 . The investigations were carried out under both Newtonian and non-Newtonian fluid conditions.

The study [7] examined the patterns of changes in the hydrodynamic parameters of laminar motion in a viscous fluid at the transition zone of the entrance section of a cylindrical pipe with a radius R under the conditions of an initially arbitrary distribution of velocities.

Under these conditions, an axisymmetric, isothermal viscous fluid flow occurs. At the pipe inlet section, the velocity of the fluid $u = \varphi(r)$, which is moving with the velocity profile on the pipe wall, becomes zero. It leads to a deformation of the velocity profile, which extends over a certain distance along the length of the pipe. A boundary layer is formed near the pipe walls, where the velocity gradient $\frac{du}{dn}$ becomes very large. It causes friction forces to take on high values regardless of the viscosity coefficient μ . The boundary layer gradually spreads from near the pipe walls until it covers the entire pipe. Therefore, studies should be conducted in the transition area using boundary layer equations.

The aim and objectives of the study

The theoretical aim of studying the transition zone is to obtain patterns of variation in hydrodynamic parameters along the length of the pipe and to develop methods for calculating energy losses.

The study aims to find patterns in the hydrodynamic parameters of a viscous fluid while flowing through a round pipe, depending on the Reynolds number.

To achieve this goal, the following tasks must be accomplished:

- to develop a boundary value problem and determine the initial and boundary conditions,
- to develop a method for solving the boundary value problem and identifying patterns of changes in hydrodynamic parameters of the viscous flow at the entrance section of the plane-parallel pressure movement,
- to construct graphs of changes in axial velocities in length and Reynolds numbers,
- to identify conditions for determining the length of the entrance section.

Materials and Methods

Choosing a Calculation Scheme

The study observes the plane-parallel laminar flow of the viscous fluid at the transition zone of the entrance section. The origin of the Z -axis is the center of the entrance section (as shown in Fig. 1) and is directed infinitely long in the direction of motion.

Plane-parallel motion in a round pipe will be considered in Cartesian coordinates, starting from the zero point (Fig. 1).

The study assumes that the velocities change according to an arbitrary law at $z=1$ in the entrance section. It is necessary to find patterns of change in the hydrodynamic parameters of a viscous fluid in the transition zone, considering them to be axisymmetric and non-stationary. Mass forces are neglected.

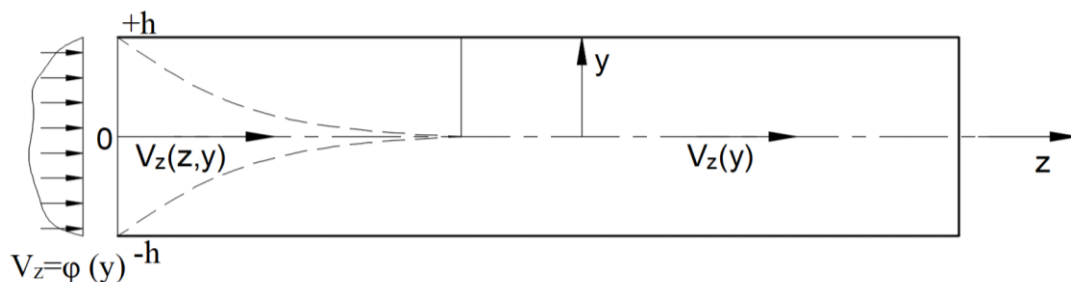


Fig. 1. On the study of a viscous incompressible fluid at the inlet plane-parallel motion

Statement of the problem and formulation of the system of differential equations for the study

Let's assume we have two stationary walls placed at a distance of $2h$ from each other, between which there is a laminar, static viscous fluid flow. The pattern of the distribution of forces in the entrance section is given in the form of an arbitrary function $V_z = \varphi(y)$.

Under these conditions, an axisymmetric, isothermal movement of a viscous fluid flow occurs. Fluid velocity at the pipe inlet is $u = \varphi(y)$, and a deformation of the velocity profile occurs over a certain distance.

A boundary layer occurs near the tube walls, where the velocity gradient $\frac{du}{dn}$ becomes very large, due to which the friction forces assume very high values regardless of the viscosity coefficient μ . The boundary layer gradually spreads from near the stationary walls and covers the entire pipe. Therefore, it is necessary to conduct studies in the transition zone using boundary layer equations.

Prandtl suggested using the Nave-Stokes equations [1] for the boundary layer, which can be simplified to obtain the boundary layer equations. Since the main influencing forces in the boundary layer are the viscous forces, Prandtl simplifies the Nave-Stokes equations by ignoring terms that are very small compared to the viscous forces. These results in simplified equations for the boundary layer.

Let's take the midpoint of the entrance section as the origin of the coordinates and point the oz axis in the direction of movement (as shown in Fig. 1). In this case, the boundary layer equations proposed by L. Prandtl [2] will take the following form:

$$V_y \frac{\partial V_z}{\partial z} + V_z \frac{\partial V_z}{\partial y} = -\frac{1}{\rho} \cdot \frac{\partial P}{\partial z} + \nu \frac{\partial^2 V_z}{\partial y^2}, \tag{1}$$

$$\frac{\partial V_z}{\partial z} + \frac{\partial V_z}{\partial y} = 0. \tag{2}$$

By linearizing this system of equations according to [3,4], we will have:

$$U_0 \frac{\partial V_z}{\partial z} = -\frac{1}{\rho} \frac{\partial P}{\partial z} + \nu \frac{\partial^2 V_z}{\partial y^2}, \tag{3}$$

$$\frac{\partial V_z}{\partial z} + \frac{\partial V_y}{\partial y} = 0 \tag{4},$$

where U_0 is the characteristic velocity of the entrance section, which is equal to the average velocity of the entrance section:

$$U_0 = \frac{1}{2h} \int_{-h}^{+h} \varphi(y) dy. \tag{5}$$

Accepting that the pressures at each point of the hydraulic section have the same values, we will have: $P = P(z)$.

To integrate equations (3) and (4), the boundary conditions of the problem are defined:

$$V_z = 0, \quad V_y = 0, \quad \text{when} \quad y = \pm h, \tag{6}$$

$$V_z = \varphi(y), \quad \text{when} \quad z = 0, \quad -h < y < +h, \tag{7}$$

$$V_z \rightarrow V', \quad \text{when} \quad z \rightarrow \infty, \quad -h < y < +h, \tag{8}$$

where V' is the velocity at the stabilized area, which is determined by the following equation:

$$\frac{1}{\rho} \frac{\partial P}{\partial z} = \nu \frac{\partial^2 V'}{\partial y^2}. \tag{9}$$

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The study assumes that the boundary layers forming near the stationary walls in the stabilized section merge to form a symmetric parabolic velocity profile. The solution to equation (9) will be:

$$V'(y) = \frac{3}{2} U_0 \left(1 - \frac{y^2}{h^2} \right). \quad (10)$$

This corresponds to the uniform distribution of velocities in the case of plane-parallel motion [1].

To obtain appropriate solutions for the changes in hydrodynamic parameters in the transition zone of the entrance section, let's introduce dimensionless variables:

$$\bar{V}_z = \frac{V_z}{U_0}, \quad x = \frac{y}{h}, \quad \sigma = \frac{z}{h}, \quad \bar{p}(\sigma) = \frac{p(z)}{p_0}. \quad (11)$$

Under these conditions, equation (3) will take the following form:

$$\frac{\partial \bar{V}_z(x, \sigma)}{\partial \sigma} = -\frac{p_0}{\rho U_0^2} \frac{\partial \bar{p}(\sigma)}{\partial \sigma} + \frac{1}{\text{Re}} \frac{\partial^2 \bar{V}_z}{\partial x^2}, \quad (12)$$

where $\text{Re} = \frac{U_0 h}{\nu}$ - is the Reynolds number.

The boundary conditions for integrating equations (12) will be:

$$\bar{V}_z(1, \sigma) = 0, \quad \bar{V}_z(x, 0) = \psi(x), \quad \frac{\partial \bar{V}_z(x, \sigma)}{\partial \sigma} \rightarrow 0, \quad \text{when } \sigma \rightarrow \infty, \quad \bar{V}_z = 0, \quad \bar{V}_z(x, \infty) \rightarrow \bar{V}'(x). \quad (13)$$

Results of research to identify patterns of changes in hydrodynamic parameters

The solution of the inhomogeneous equation (12) under the boundary conditions (13), let's look for a solution in the form of a sum [5]:

$$\bar{V}(x, \sigma)_z = U(x, \sigma) + \psi(\sigma), \quad (14)$$

where $U(x, \sigma)$ represents the general solution of the homogeneous equation for inhomogeneous boundary conditions, while $\psi(\sigma)$ is the particular solution of the inhomogeneous equation (12) for zero boundary conditions.

Let's look for the general solution of the homogeneous equation $\frac{\partial U(x, \sigma)}{\partial \sigma} = \frac{1}{\text{Re}} \frac{\partial^2 U(x, \sigma)}{\partial x^2}$ in the form of a sum:

$$U(x, \sigma) = \sum_{k=1}^{\infty} C_k(\sigma) \cos(\gamma_k x). \quad (15)$$

If the function $U(x, \sigma)$ satisfies the equation (15), then the coefficients $C_k(\sigma)$ must satisfy the following equation:

$$C_k'(\sigma) = -\frac{1}{\text{Re}} C_k(\sigma), \quad (16)$$

from where

$$C_k(\sigma) = C_k \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right), \quad (17)$$

where C_k is the arbitrary constant coefficient.

By inserting the value of function $C_k(\sigma)$ into equation (15) and taking into account equation (14), we obtain the following expression:

$$\bar{V}_z(x, \sigma) = \sum_{k=1}^{\infty} C_k \cos(\gamma_k x) \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right) + \psi(\sigma). \quad (18)$$

To determine the value of the unknown function $\psi(\sigma)$, we multiply both sides of equation (4) by bdx and integrate x over the interval $-1 \leq x \leq +1$. Taking into account that the viscous liquid on the static wall has zero velocity, we obtain the following expression:

$$\int_{-1}^{+1} \frac{\partial \bar{V}_z(x, \sigma)}{\partial \sigma} dx = 0. \quad (19)$$

By substituting the value of velocity $\bar{V}_z(x, \sigma)$ from equation (18) into equation (19) for determining the function $\psi(\sigma)$, we obtain the following expression:

$$2\psi'(\sigma) = \frac{2}{\text{Re}} \sum_{k=1}^{\infty} \gamma_k C_k \sin \gamma_k \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right). \quad (20)$$

Integrating the last equation according to σ , we will have:

$$\psi(\sigma) = -\sum_{k=1}^{\infty} \frac{C_k}{\gamma_k} \sin \gamma_k \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right) + C(x), \quad (21)$$

where $C(x)$ is the integration constant.

By substituting the value of function $\psi(\sigma)$ into equation (14), we obtain the following equation for the velocity $\bar{V}_z(x, \sigma)$:

$$\bar{V}_z(x, \sigma) = \sum_{k=1}^{\infty} C_k \left[\cos(\gamma_k x) - \frac{\sin \gamma_k}{\gamma_k} \right] \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right) + C(x). \quad (22)$$

We determine the value of the constant $C(x)$ from the boundary condition of the problem when $\sigma \rightarrow \infty$ $\bar{V}_z(x, \sigma) \rightarrow \frac{3}{2} \left(1 - \frac{y^2}{h^2}\right)$. The value of the constant $C(x)$ will correspond to this boundary condition:

$$C(x) = \frac{3}{2} (1 - x^2). \quad (23)$$

Thus, the final solution to the problem will be:

$$\bar{V}_z(x, \sigma) = \frac{3}{2} (1 - x^2) + \sum_{k=1}^{\infty} C_k \left[\cos(\gamma_k x) - \frac{\sin \gamma_k}{\gamma_k} \right] \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right). \quad (24)$$

This velocity distribution equation must satisfy the boundary condition of the problem (13), according to which $\bar{V}_z(x, 0) = \psi(\sigma)$. Taking into account the equation (24) and the boundary condition, we will have:

$$\psi(\sigma) = \frac{3}{2} (1 - x^2) + \sum_{k=1}^{\infty} C_k \left[\cos(\gamma_k x) - \frac{\sin \gamma_k}{\gamma_k} \right]. \quad (25)$$

To determine the values of the C_k coefficients, we must multiply both parts of the equation (25) by given expression $\left(\cos(\gamma_k x) - \frac{\sin \gamma_k}{\gamma_k} \right)$ and integrate it over the range of $(-1, +1)$ and we will have:

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$$\int_{-1}^1 \psi(x) \left(\cos(\gamma_n x) - \frac{\sin \gamma_n}{\gamma_n} \right) dx - \frac{3}{2} \int_{-1}^1 (1-x^2) \left(\cos(\gamma_n x) - \frac{\sin \gamma_n}{\gamma_n} \right) dx = \sum_{k=1}^{\infty} C_k \int_{-1}^1 \left(\cos(\gamma_n x) - \frac{\sin \gamma_n}{\gamma_n} \right) \left[\cos(\gamma_k x) - \frac{\sin \gamma_k}{\gamma_k} \right] dx \quad (26)$$

As

$$\int_{-1}^1 \left(\cos(\gamma_n x) - \frac{\sin \gamma_n}{\gamma_n} \right) \left[\cos(\gamma_k x) - \frac{\sin \gamma_k}{\gamma_k} \right] dx = \begin{cases} 0, & \gamma_k \neq \gamma_n \\ \sin^2 \gamma_k, & \gamma_k = \gamma_n \end{cases}, \quad (27)$$

we will have:

$$C_k = \frac{L_k^{(1)}}{\sin^2 \gamma_k} - \frac{3L_k^{(2)}}{\sin^2 \gamma_k}, \quad (28)$$

where

$$L_k^{(1)} = \int_{-1}^1 \psi(x) (\cos(\gamma_n x) - \cos(\gamma_n)) dx, \quad (29)$$

$$L_k^{(2)} = \int_0^1 (1-x^2) (\cos(\gamma_n x) - \cos(\gamma_n)) dx, \quad (30)$$

γ_k is the $\cos(\gamma_k) - \frac{\sin \gamma_k}{\gamma_k} = 0$, or the positive roots of the equation $tg \gamma_k = \gamma_k$. Taking into account (28-30), we will have the solution to the problem.

$$\bar{V}_z(x, \sigma) = \frac{3}{2} (1-x^2) + \sum_{k=1}^{\infty} \left[\frac{L_k^{(1)} - 3L_k^{(2)}}{\sin^2 \gamma_k} \right] \left(\cos(\gamma_k x) - \frac{\sin \gamma_k}{\gamma_k} \right) \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma \right). \quad (31)$$

The resulting equation will satisfy the boundary conditions of the problem.

We get the pattern of pressure change from equations (12) and (31), and we will have:

$$\frac{\partial \bar{p}(\sigma)}{\partial \sigma} = -\frac{\rho U_0^2}{\bar{p}_0 \text{Re}} \left\{ \sum_{k=1}^{\infty} \frac{\gamma_k (L_k^{(1)} - 3L_k^{(2)})}{\sin \gamma_k} \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma \right) + 3 \right\}. \quad (32)$$

By integrating the last equation, we can obtain the pattern of pressure variation in the transition area of the inlet section:

$$\bar{p}(\sigma) = \bar{p}(0) - \frac{\rho U_0^2}{\bar{p}_0} \sum_{k=1}^{\infty} \frac{\gamma_k (L_k^{(1)} - 3L_k^{(2)})}{\gamma_k \sin \gamma_k} \cdot \left[1 - \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma \right) \right] - \frac{3\rho U_0^2}{\bar{p}_0 \text{Re}}. \quad (33)$$

Let's consider two special cases:

1. The incoming fluid has a uniformly distributed velocity field $V_z(y, z) = \varphi(y) = U_0$. Under these conditions, we will have $\bar{V}_z(x, 0) = \psi(x) = 1$, corresponding to which we will get:

$$L_k^{(1)} = \int_{-1}^1 \psi(x) (\cos(\gamma_n x) - \cos(\gamma_n)) dx = 2 \int_0^1 (\cos(\gamma_n x) - \cos(\gamma_n)) dx = 0,$$

$$L_k^{(2)} = \int_0^1 (1-x^2) (\cos(\gamma_n x) - \cos(\gamma_n)) dx = -\frac{2 \sin \gamma_k}{3\gamma_k}.$$

Inserting these values into the equation (31):

$$\bar{V}_z(x, \sigma) = \frac{3}{2}(1 - x^2) + \sum_{k=1}^{\infty} \left[\frac{2}{\gamma_k \sin \gamma_k} \right] (\cos(\gamma_k x) - \cos(\gamma_k)) \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right). \quad (34)$$

In this case, the irregularity of pressure distribution according to (33) will be equal to:

$$\bar{p}(\sigma) = \bar{p}(0) - \frac{\rho U_0^2}{\bar{p}_0} \sum_{k=1}^{\infty} \frac{2}{\gamma_k^2} \cdot \left[1 - \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right) \right] - \frac{3\rho U_0^2}{\bar{p}_0 \text{Re}} \sigma. \quad (35)$$

2. Let's imagine the velocity of the liquid entering the reservoir changes according to the parabolic law $V_z(y, z) = \varphi(y) = A(1 - y^2)$, according to which we will have $\bar{V}_z(x, 0) = \psi(x) = (1 - x^2)$,

$$L_k^{(1)} = \int_{-1}^1 (1 - x^2) (\cos(\gamma_n x) - \cos(\gamma_n)) dx = -\frac{4 \sin \gamma_k}{3\gamma_k}, \quad (36)$$

$$L_k^{(2)} = -\frac{4A \sin \gamma_k}{3\gamma_k}. \quad (37)$$

In this case, the velocity change patterns according to (31), (36) and (37) will be:

$$\bar{V}_z(x, \sigma) = \frac{3}{2}(1 - x^2) + \sum_{k=1}^{\infty} \frac{2(3 - A)}{3\gamma_k \sin \gamma_k} (\cos(\gamma_k x) - \cos(\gamma_k)) \cdot \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right). \quad (38)$$

Having the velocity distribution patterns, we can get the pressure distribution pattern in the transition zone:

$$\bar{p}(\sigma) = \bar{p}(0) - \frac{\rho U_0^2}{\bar{p}_0} \sum_{k=1}^{\infty} \frac{2(3 - A)}{3\gamma_k^2} \cdot \left[1 - \exp\left(-\frac{\gamma_k^2}{\text{Re}} \sigma\right) \right] - \frac{3\rho U_0^2}{\bar{p}_0 \text{Re}} \sigma. \quad (39)$$

The resulting patterns of velocity and pressure changes make it possible to fully reveal the physical nature of the ongoing phenomena and make generalizations.

Results and Discussion

The integration of differential equations of viscous fluid flow yielded patterns of changes in the distribution of axial velocities $\bar{V}_z(x, \sigma)$. To visually represent these patterns of changes in the axial velocity $\bar{V}_z(x, \sigma)$ across the cross-section and along the length of the transition zone, depending on the initial distribution of velocities $\bar{V}_z(x, 0) = \psi(x)$ and Reynolds number, their graphs of change were constructed. Figs. 2...5 show the specified calculated graphs for cases $\bar{V}_z(x, 0) = 1$ and $\bar{V}_z(x, 0) = (1 - x^2)$.

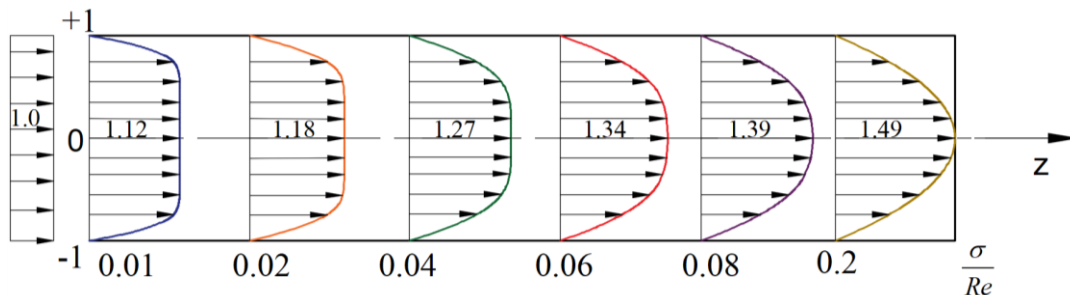


Fig. 2. Graphs of changes in axial velocities $\bar{V}_z(x, \sigma)$ along the cross-section in the transition zone of the entrance section of the plane-parallel pressure flow at $\bar{V}_z(x, 0) = 1$ and

1. $\frac{\sigma}{\text{Re}} = 0.01$; 2. $\frac{\sigma}{\text{Re}} = 0.02$; 3. $\frac{\sigma}{\text{Re}} = 0.04$; 4. $\frac{\sigma}{\text{Re}} = 0.06$; 5. $\frac{\sigma}{\text{Re}} = 0.08$; 6. $\frac{\sigma}{\text{Re}} = 0.2$

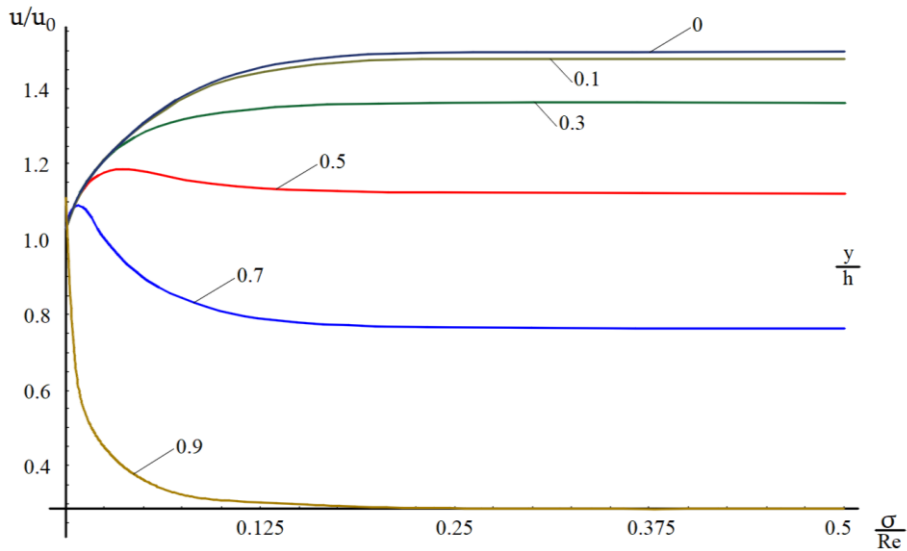


Fig. 3. Graphs of changes in axial velocities $\bar{V}_z(x, \sigma)$ at $\bar{V}_z(x, 0) = 1$ and
 1. $x = 0$; 2. $x = 0.1$; 3. $x = 0.3$; 4. $x = 0.5$; 5. $x = 0.7$; 6. $x = 0.9$

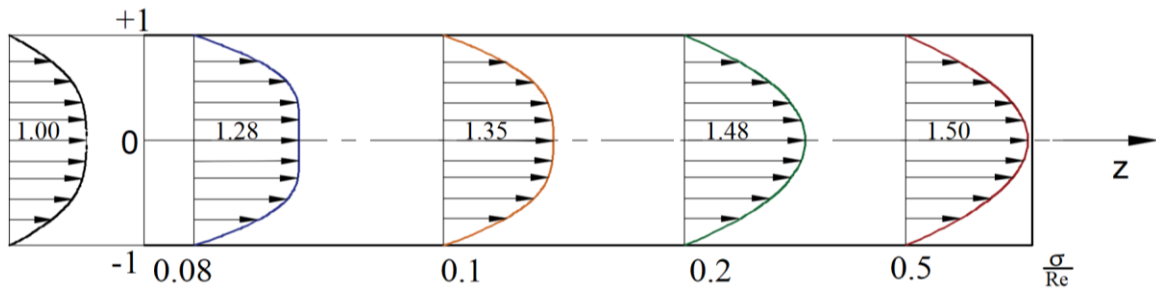


Fig. 4. Graphs of changes of the axial velocities $\bar{V}_z(x, \sigma)$ along the cross-section on the entrance section of the plane-parallel pressure flow at $\bar{V}_z(x, 0) = (1 - x^2)$ and
 1. $\frac{\sigma}{Pe} = 0.08$; 2. $\frac{\sigma}{Pe} = 0.1$; 3. $\frac{\sigma}{Pe} = 0.2$; 4. $\frac{\sigma}{Pe} = 0.5$

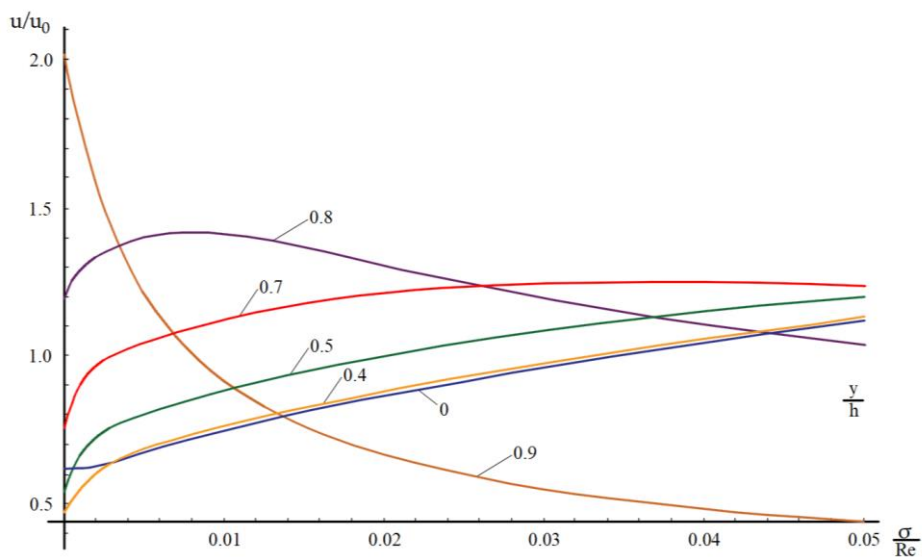


Fig. 5. Graphs of changes in axial velocities $\bar{V}_z(x, \sigma)$ at $\bar{V}_z(x, 0) = (1 - x^2)$ and
 1. $x = 0$; 2. $x = 0.1$; 3. $x = 0.3$; 4. $x = 0.5$; 5. $x = 0.7$; 6. $x = 0.9$.

The dynamics of the transition zone were determined based on the results of numerical calculations and graphs, using the condition that the ratio of axial velocities between the transitional and stabilized areas is 0.99.

From this condition, it was found that $L = \frac{\sigma}{Re} = 0.147$. The proposed method for calculating velocity

rearrangements in the transition area of the entrance section enables the determination of patterns of changes in hydrodynamic parameters of the flow under general boundary conditions. Using the identified relationships, the deformation process of the velocity field at the transition site of the entrance section was determined for both constant and parabolic law distributions of incoming liquid. This enables the calculation of changes in the hydrodynamic parameters of the flow and facilitates generalizations of the results.

The patterns of changes in the velocity field in the transition area of the entrance section of plane-parallel pressure flow, along with the graphs constructed based on these findings, provide valuable information for the accurate design of hydromechanical equipment units.

Conclusion

The study of viscous fluid flow in the transition zone of the entrance area of a plane-parallel pressure flow has revealed the following findings:

- A boundary value problem was formulated to investigate the regularities of changes in hydrodynamic parameters of a viscous incompressible fluid in the transition zone of the entrance section of a plane-parallel pressure flow.
- A method for solving the boundary value problem was developed, and formulas were derived to calculate axial velocities across the cross-section and pressure along the length of the diffuser.
- Graphs were constructed to illustrate the changes in hydrodynamic parameters of the flow across the cross-section and along the length of the transition zone for constant and parabolic distributions of initial velocities at the entrance section.
- A calculation formula was derived to determine the length of the transition zone of the inlet section of a plane-parallel pressure flow.

The solutions derived from the approximating Navier-Stokes equations to identify patterns of changes in hydrodynamic parameters of a plane-parallel pressure flow with constant and parabolic distributions of initial velocities at the entrance section provide a comprehensive understanding of the underlying processes. These results are essential for hydraulic calculations of various systems.

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