

Contribution to Railway Track Maintenance Planning from the Analysis of Dynamic Movements of Trains

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Abstract

Dynamic movements of trains in relation to the track have a significant impact on the displacement stability of rail vehicles, having effects inclusive of operational safety. Although there are numerous approaches to track maintenance planning, most of them are based solely on long-term geometric degradation assessments without taking into account any dynamic parameters in assessing operational safety or establishing means to predict future rolling stock accelerations relative to the track in order to develop safer maintenance plans. This paper introduces a method of track maintenance planning based on geometric degradation modeling and prediction of rolling stock vertical and horizontal acceleration. The goal is to establish how frequent geometric maintenance is necessary to ensure operational safety under geometric and dynamic criteria. This approach is based on regression models defined from geometric and dynamic inspection data. The method was applied in a passenger railway and obtained expressive results that corroborated the need of considering dynamic aspects on maintenance planning, as sections of the analyzed railway were identified with operation becoming unsafe, under the dynamic criterion, before the geometric safety tolerances are reached. This work is intended not only to propose a planning method but also to present to the scientific and technical communities a novel approach to be explored and developed in future research. The obtained results, therefore, do more than confirm quantitatively the relevance of this analysis; they also demonstrate qualitatively how promising the development of this thematic field is. In this regard, this work also presents in its conclusions some research opportunities to be explored for the development of this theme.

Keywords: Railway; Track; Maintenance; Dynamic safety; Accelerations.

1. Introduction

In railway infrastructure management, monitoring and controlling track conditions require significant attention, mainly due to their influence on operational safety. Directly responsible for supporting and driving trains, the railway superstructure (track), formed by rails, sleepers, fasteners, and ballast, degrades influenced by the frequency and intensity of vehicle loads, operating speeds, and environmental conditions, among other factors [1]. For example, abrasion will cause progressive fracture of the ballast material; cracks and breaks will appear in the sleeper structure, damaging the support and fastening of rails; and fatigue and wear of the rails' structures will be developed due to high mechanical stress [2–7]. Such degradations that are developed in the physical structure of components, in conjunction with other factors, such as settlement of the track bed, deterioration of the ballast layer, and the influence of the climate, cause a progressive loss of the ideal spatial arrangement of the track components along the railway section, thus characterizing the geometric degradation [8–10].

Geometric maintenance is typically performed by large-scale equipment that automatically repositions the rails and sleepers and rearranges the ballast (an operation known as tamping) to maintain this position [11]. On the other hand,

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physical degradations require much more incisive interventions, which consist of the renewal (replacement) of components whose degradation has reached levels that characterize the end of their useful lives or their structural failure [12].

To determine the need for maintenance, railway sections are inspected regularly to verify volume and severity of degradations. For each parameter that describes geometry (like gauge, cant, warp, twist, longitudinal level, and alignment), and analogously for physical degradations, there are degradation tolerances defined as a function of operational speed, which determines the limit beyond which operation is considered unsafe. As an example, Table 1 shows the geometric tolerances established by the Federal Railroad Administration (FRA). Thus, deviations recorded during inspections are compared with the standardized tolerances, allowing the identification of places where interventions are needed. For more details on these tolerances, including those related to physical degradation, see FRA (2008) [13].

Table 1. Safety tolerances defined by FRA [14]

Track classification	Maximum allowable speed (km/h)	Gauge not less than (mm)	Gauge not more than (mm)	Profile (surface level) (mm)	Cross level (tangents and curves) (mm)	Warp (over 18.9 m (62 ft) distance) (mm)	Alignment (tangent) (mm)
Class 1	16	1,416.1	1,473.2	76.2	76.2	76.2	127
Class 2	40	1,416.1	1,466.9	69.9	50.8	57.2	76.2
Class 3	64	1,422.4	1,466.9	57.2	44.5	50.8	44.5
Class 4	97	1,422.4	1,460.5	50.8	31.8	44.5	38.1
Class 5	129	1,422.4	1,460.5	31.8	25.4	38.1	19.1

Traditionally, geometric maintenance was planned according to preventive strategies to ensure maximum safety and operational performance, with prominence of planning methods based on time and track conditions [15, 16]. Time-based maintenance, which is planned using inspection data as well as the knowledge of railway technicians, has as its main feature that interventions are performed at constant intervals [17–19].

On the other hand, in condition-based maintenance, services are performed according to the track degradation state, which is more efficient than the time-based model because it considers the differences in the degradation rate of each track section [17, 18]. As a result, geometric tolerances, such as those shown in Table 1, are used to determine the need for maintenance, thereby supporting the intervention planning. An example of an administrative model based on this concept is EN 13848-5 (2008) [20], which establishes three possible measures based on the severity of geometric deviations: the alert limit (AL), which, if reached, requires an analysis of the track geometry to define a future intervention plan, usually to be carried out in a horizon of up to one year; the intervention limit (IL), which, if reached, requires corrective maintenance; and the immediate alert limit (IAL), which, when reached, imposes a speed reduction or track closure for immediate maintenance [21]. As another example of condition-based planning models, several railroads have developed methods for calculating track quality indices (TQI) using inspection data [22]. Such indexes are based on specific formulations that consider, e.g., the standard deviation of geometric parameter readings or the severity of isolated defects to define reference values that represent the track's condition in a summarized way, serving as a reference to indicate the need for maintenance (for examples of these models, see Soleimanmeigouni et al. (2016) [1] and Litherland & Andrews (2019) [23]).

Such maintenance strategies are simplified administrative methods that are widely used, particularly to guide actions over a short time horizon. However, because they are based on a momentary diagnosis of track condition, they require very frequent inspections to ensure that the geometric condition is accurately known because uncertainties in degradation rate can lead to the development of high deviations quickly and unexpectedly. As a result, intervention plans based solely on such approach may imply a large number of corrective interventions or early preventive maintenance. Corrective intervention incurs high costs as a result of unforeseen services and delays on running trains, while preventive maintenance incurs additional costs as a result of unnecessary maintenance [15, 16, 24].

Therefore, as an effort to improve such traditional procedures, management methods have been developed aiming to carry out the so-called predictive maintenance, in which the maintenance is strategically defined so that interventions are not anticipated or postponed, but performed at the optimal time to guarantee a maximum use of the potential life of the track, without tolerating a level of degradation that could compromise safety and performance [25, 26]. Due to the relevance of such predictive maintenance methods, the literature has a significantly large number of works devoted to their development, as can be seen in Sharma et al. (2018) [18], Bakhtiyari et al. (2020) [19], Soleimanmeigouni et al. (2020) [21], Rahimikelarijani et al. (2020) [22], Andrews et al. (2014) [27], Wen et al. (2016) [28], Lee et al. (2017) [29], Khouzani et al. (2017) [30], Khajehei et al. (2019) [31], Nielsen et al. (2018) [32], Andrade & Teixeira (2015) [33], Su et al. (2019) [34], Sadeghi et al. (2017) [35], Neuhold et al. (2020) [36], and Yang et al. (2020) [37]. However, despite the volume and variety of approaches, the most widely used strategy to reduce geometric maintenance costs, in

the planning methods seen in such referred works, consists, in short, in establishing a system capable of predicting the track's behavior over time, so that the moment when the geometric deviations will reach the safety tolerances is identified in advance. This allows maintenance to be scheduled as close as possible to the moment when geometric tolerances would be met and, as a consequence, in the long-term horizon, the number of interventions will be as small as possible, resulting in the lowest possible cost. This predictability minimizes the need for inspections and allows more efficient planning for preparation and routing of maintenance teams. Aspects that also imply the reduction of budgets.

Predictive maintenance policies, based on carrying out maintenance strictly, when necessary, as described above, are the most widely adopted management strategy today, mainly due to the financial benefits achieved. To cite a few examples, in Sharma et al. (2018) [18], Bakhtiary et al. (2020) [19], and Khouzani et al. (2017) [30], implementation of strategies based on predictive maintenance resulted in cost reductions of 4.7, 10, and 40%, respectively, over traditional planning methods, which are relevant values given the high maintenance budgets of the railway infrastructure.

In contrast, some authors have demonstrated that track safety assessment should not be limited to a comparison between geometric parameter values and their tolerances, but should also include an analysis of vehicles' dynamic behavior during traffic [15, 16, 38]. This is based on the fact that accelerations of rolling stock, relative to the track, during displacement have a significant impact on balance and operational safety. Studies show that accelerations and consequent wheel loads, reach significantly high levels when caused by vehicles traveling through sections with geometric changes. This caused accidents even in sections where the geometric deviations were below safety tolerances [15, 16, 39]. In addition, such dynamic loads of high magnitude could reach proportions that would lead to an overload of mechanical stress on the components, causing the development of fatigue in the rails and damage to other components, consequently reducing their lives and increasing costs, mainly due to renewal services [40–43]. Therefore, maintenance policies defined through predictive methods, in which interventions are planned exclusively based on the geometric condition and still aim to postpone them until the safety limit, can tolerate geometric deviations of such an amplitude that they would lead to the development of unsafe dynamic conditions, even capable of compromising the assets life.

It is worth noting that there are dynamic security analysis approaches, such as Nadal's criterion [44]. The operationalization of such analyses depends on dynamic inspections performed with data acquisition equipment, as seen in the works of Guler (2016) [41], Barbosa (2016) [45], Gullers et al. (2008) [46], Bocciolone et al. (2007) [47], and Lee et al. (2011) [48]. Within this scope, several investigations have improved dynamic inspection instruments and the data analysis itself, as can be seen in the works of Lima et al. (2021) [39], Barbosa (2016) [45], Bocciolone et al. (2007) [47], Lee et al. (2011) [48], Weston et al. (2015) [49], and Lederman et al. (2017) [50]. Therefore, managers have tools at their disposal to evaluate dynamic safety and determine whether tracking interventions are necessary to address risky conditions. However, on the whole, such systems are characterized by being reactive, that is, they are instruments that make a punctual evaluation of safety, functioning almost as fault detectors; consequently, to ensure safety, constant inspections are necessary, which also implies high costs. In contrast to the analysis of the geometric condition, no works were seen in the literature that proposed modeling the future dynamic condition or that considered the future accelerations of vehicles relative to the track in the formulation of long-term intervention plans whose moment of intervention is strategically delimited in order to guarantee operational safety from a geometric and dynamic point of view.

Thus, motivated by this finding, this paper presents a method for planning track maintenance whose objective is to define the frequency of geometric maintenance services capable of ensuring operational safety under geometric and dynamic criteria while keeping intervention costs as low as possible. To this end, the proposed planning procedure, in addition to modeling and analyzing geometric degradation as traditional methods do, also establishes a system capable of modeling and predicting the future vertical and horizontal accelerations of vehicles relative to the track, thus allowing the definition of the maintenance frequency capable of ensuring operational safety also under the dynamic criterion.

For this purpose, a literature review was initially developed and presented in Sections 2 and 3, which identified the main characteristics of current maintenance planning methods as well as issues specifically related to dynamic safety and the rolling stock accelerations, and gathered the elementary technical concepts that were used in the development of the proposed method. In Section 4, the maintenance planning method is presented, and, in the next section, its application to data from a Brazilian railway is described. In Section 6, the main results of the practical study were analyzed with evidence of its applicability and relevance, since unsafe dynamic conditions that were observed on the studied railway could be avoided by the application of the maintenance plan defined by the method. Finally, in the conclusions, opportunities for future research identified during the development of this work are presented.

2. Track Maintenance Management

Track conditions over time can be described as shown in Figure 1.

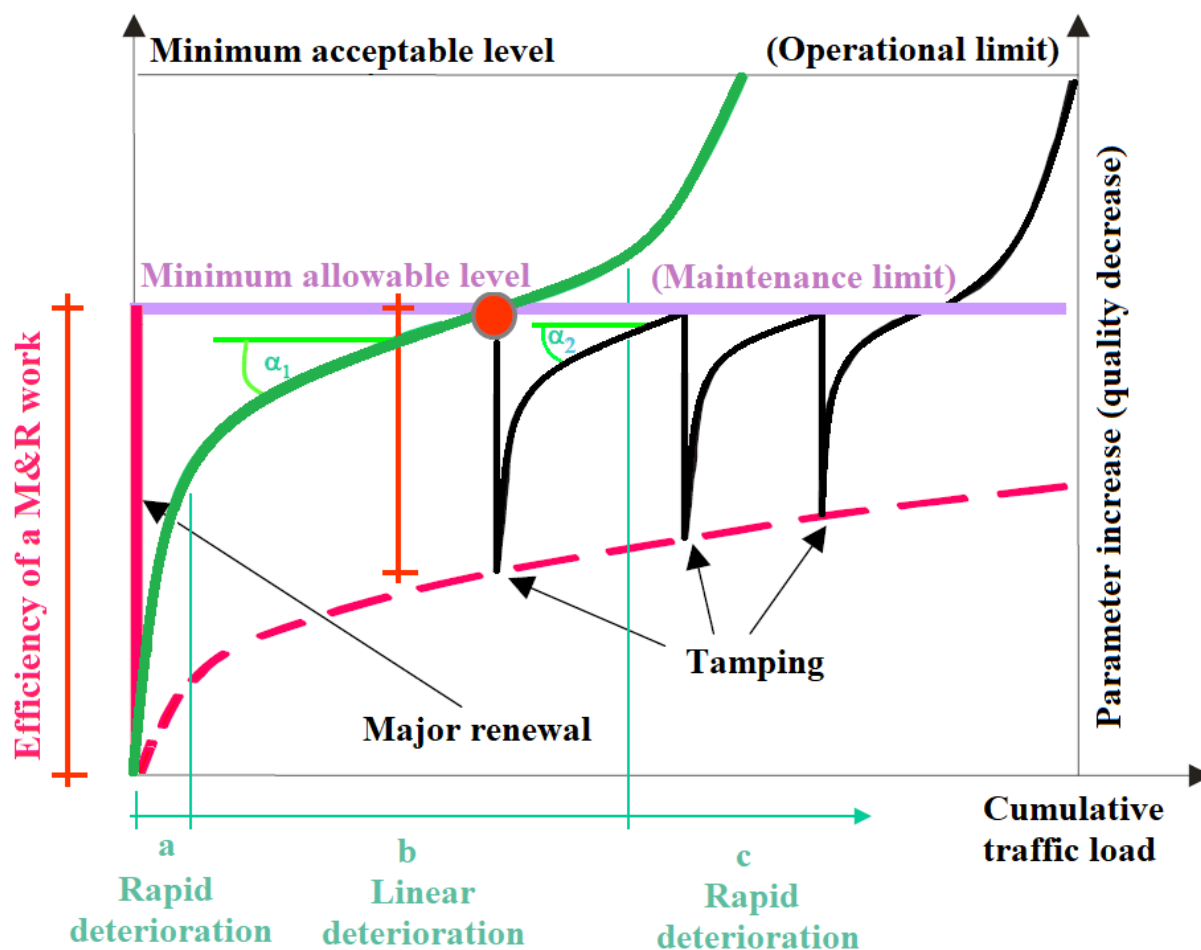


Figure 1. Track degradation over time [51]

The graph in the figure illustrates the degradation of a given geometric parameter. The green curve represents a scenario in which no maintenance is performed, with three degradation stages identified: (a) initially an intense but rapid deterioration occurs; (b) subsequently, the second phase is characterized by a long period during which deviations evolve at a slow and approximately linear rate; and, in the third phase (c), degradation presents an increasingly higher rate with an almost exponential profile. To avoid such last phase, geometric maintenance (tamping) and/or renewal (M&R) actions are carried out when the quality tolerance (horizontal purple line) is reached. Maintenance improves quality (decrease of the parameter value, expressed by the red vertical line), however, due to the progression of physical degradation of components, maintenance efficiency decreases (evolution of dashed line), and degradation intensity increases (change of angle α_1 to α_2). Consequently, the need for geometric maintenance increases to the point where it is no longer justifiable to carry it out, and then more incisive maintenance is required, such as the renewal of ballast or other components (if not done earlier due to the wear limit) [40, 52].

Traditionally, the most elementary techniques for managing track maintenance are based on a comparison between the state of geometric degradation (defined from inspection data) and safety tolerances, as seen in EN 13848-5 (2008) [20], and Roghani & Hendry (2017) [14].

As pointed out in the introduction section, such traditional techniques require constant inspections and can lead to early interventions and many corrective services, which can result in high costs [24]. Predictive management techniques circumvent such problems by performing maintenance strictly when necessary [25, 26]. To do this, managers need tools that allow establishing in-depth knowledge about track behavior over time, particularly to predict the evolution of degradation, which enables the identification of the best interval between inspections, the best maintenance tolerances, and, finally, an accurate interval between maintenance services. The predictability regarding track behavior established by such models facilitates the analysis of the effects that the considered maintenance policies may have on the long-term behavior of the system [1].

Since predictive models are essential components of traditional planning techniques, numerous research works have been devoted to their development. Thus, the literature presents a wide range of models of that kind, but they can be divided into two categories: mechanistic and statistical approaches. Tables 2 and 3 present the main geometric degradation prediction models found in the surveyed literature.

Table 2. Mechanistic models to predict geometric degradation [11, 40]

Model name	Equation	Variables
Shenton	$S = K_s \frac{A_e}{20} (0.69 + 0.028L)N^{0.2} + 2.7 \times 10^{-6}N$	<i>S</i> - track settlement <i>K_s</i> - structure factor <i>A_e</i> - equivalent axle load <i>N</i> - cumulative number of axles <i>L</i> - Lift given by tamping machines
DSM	$S = S_i (1 + K_H \ln N)$	<i>S_i</i> - initial settlement after first loading cycle <i>K_H</i> - coefficient ^a
Hoshino	$\Delta = L_H \cdot J \cdot Z$	<i>Δ</i> - coefficient of track deterioration <i>L_H</i> - load factor <i>J</i> - structure factor <i>Z</i> - condition factor
Sugiyama	$Z_{av} = 2.09 \times 10^{-3} \cdot T^{0.31} \cdot V^{0.98} \cdot J^{1.10} \cdot R^{0.21} \cdot K_p^{0.26}$	<i>Z_{av}</i> - average growth of track irregularities in the section <i>T</i> - cumulative tonnage <i>V</i> - average running speed <i>J</i> - structure factor <i>R</i> - influence factor for jointed rail <i>K_p</i> - influence factor for subgrade
Sato	$BS = \begin{cases} \alpha_s (P_b - P_g \cdot br)^w, & P_b > P_g \cdot br \\ 0, & P_b \leq P_g \cdot br \end{cases}$ $BS = \alpha_s \cdot P_b^w$	<i>BS</i> - ballast settlement <i>α_s</i> - coefficient ^a <i>P_b</i> - sleeper-ballast contact pressure <i>P_g · br</i> - threshold limit value of sleeper-ballast contact pressure <i>w</i> - exponent ^a
Guerin	$\frac{dS}{dN} = \alpha_g \cdot y^{\beta_G}$	<i>y</i> - maximum elastic deflection during a loading cycle <i>α_g</i> , <i>β_G</i> - material parameters
Frohling	$S = \left[\left[K_{F1} + K_{F2} \cdot \left(\frac{D_{2mi}}{K_{F3}} \right) \right] \frac{Q_{tot}}{Q_{ref}} \right]^w \log N$	<i>D_{2mi}</i> - measured track stiffness at a particular sleeper <i>i</i> <i>K_{F1}</i> , <i>K_{F2}</i> , <i>K_{F3}</i> - settlement constants ^a <i>Q_{tot}</i> - prevailing wheel load <i>Q_{ref}</i> - reference wheel load <i>w</i> - exponent ^a
Technical University of Munich	$S_{opt} = 1.57 \cdot p_b \cdot \Delta N + 3.04 \cdot p_b^{1.21} \cdot \ln N_a$ $S_{pess} = 2.33 \cdot p_b \cdot \Delta N + 15.20 \cdot p_b^{1.21} \cdot \ln N_a$ $S_{med} = 1.89 \cdot p_b \cdot \Delta N + 5.15 \cdot p_b^{1.21} \cdot \ln N_a$	<i>S_{opt}</i> - Increase of irregularities in the optimal scenario (mm/100 days) <i>S_{pess}</i> - Increase of irregularities in the pessimistic scenario (mm/100 days) <i>S_{med}</i> - Increase of irregularities in the intermediate scenario (mm/100 days) <i>p_b</i> - Ballast pressure (Zimmermann method, in Pascal) <i>ΔN</i> - Average number of axles passes in the period after geometric maintenance (≤10,000) <i>N_a</i> - Axis passages after <i>ΔN</i>

^a Coefficients determined empirically based on local conditions

Table 3. Statistical models to predict geometric degradation

Author	Purpose				Applied methods	Analyzed geometric parameters
	Track degradation modeling	Maintenance planning	Long term	Short term		
Soleimanmeigouni et al. (2020) [21]	x		x		Piecewise Exponential Model (PCE), ordinal logistic regression, Monte Carlo technique	Longitudinal leveling
Rahimikelarjani et al. (2020) [22]	x		x		Weibull distribution, logistic regression, sensitivity analysis	Longitudinal and horizontal leveling
Nielsen & Li (2018) [32]	x		x		Time-domain model and Empirical Modeling (EM)	Longitudinal leveling
Andrade & Teixeira (2015) [33]	x		x		Hierarchical Bayesian model	Standard deviation of longitudinal leveling and horizontal alignment
Sharma et al. (2018) [18]	x	x	x		Random forests, Markov chains, Bernoulli process, Markov decision process (MDP)	TQI ¹
Su et al. (2019) [34]	x	x	x	x	Chance-constrained Model Predictive Control (MPC), Dantzig-Wolfe decomposition, Arc Routing Problem	Generic, fits any indicator

Sadeghi et al. (2017) [35]		x	x	Statistical analysis	TQI ¹
Neuhold et al. (2020) [36]	x	x	x	Linear regression	Modified standard deviation of longitudinal leveling
Xu et al. (2015) [52]	x			Piecewise linear regression	Gauge, cant, alignment and leveling, twist
Andrews et al. (2014) [27]	x	x	x	Petri net, Weibull distribution, Monte Carlo simulation	Standard deviation of the average vertical alignment
Wen et al. (2016) [28]	x	x	x	Mixed Integer Linear Programming (MILP), Computational experiments	Standard deviation of longitudinal leveling
Litherland & Andrews (2019) [23]	x	x		Petri nets	Horizontal and vertical alignment
Yang et al. (2020) [37]		x		Time–frequency energy density (TFED) method, power spectral density, Hilbert spectral analysis	longitudinal level of left rail, alignment of left rail, track gauge, cross-level
Shafahi & Hakhamaneshi (2009) [53]	x	x	x	Markov process, dynamic programming	TQI ¹

¹ Track Quality Index (TQI) - An index that expresses the geometric quality of a consolidated form. There are several models for its definition. For examples, see Soleimanmeigouni et al. (2016) [1] and Litherland and Andrews (2019) [23].

Mechanistic models are based on laws of physics to establish the track behavior, while statistical models are developed from the treatment and analysis of inspection data. Because they are deterministic, mechanistic models cannot effectively address the uncertainties associated with degradation behavior and the complex association between variables in the development of the track deviations. For this reason, statistical methods are currently the most widely used, given their ability to deal with many variables and analyze the simultaneous effect that each factor has on track conditions. If sufficient and reliable data are available, and if the model validation conditions are satisfied, the estimation made by the statistical models can be quite accurate. Attention is required, however, on interpreting the results, as such models lack a physical foundation and may provide unrealistic results [1, 41].

Most degradation models are part of decision support systems designed to reduce costs from a long-term perspective. Each research has its particularities in terms of strategies to achieve such goal, with approaches varying from the introduction of new technologies into track's physical structure, for example, ballast stabilization systems (as seen in Giunta et al. (2018) [54], Rempelos et al. (2020) [55], and Singh et al. (2020) [26]), to the determination of the best frequency of inspections [31, 56]. Despite this variety, the most common category of works in this field focus on developing guidelines for carrying out interventions over time, as seen in Bakhtiary et al. (2020) [19], Wen et al. (2016) [28], Khouzani et al. (2017) [30], and Khajehei et al. (2019) [31].

It is important to note that, despite the variety of approaches, the reduction in total maintenance cost in the long-term perspective is achieved by rationalizing the volume of activities. Figure 2 illustrates this concept showing how costs can vary depending on the frequency of intervention adopted. Thus, the less frequent the services, as illustrated in point MF 1 of this figure, the lower the geometric maintenance costs (C 1) due to the smaller number of activities performed over time. However, as a consequence, track quality tends to decrease (GQ 1), and can potentially reach unsafe levels for traffic due to geometric deviations that exceeded the safety limit and were not promptly repaired. Due to the necessity of adopting speed restrictions at these locations that harm traffic performance and the unexpected need for corrective work, such conditions may result in financial losses. On the other hand, if managers maintain a high frequency of interventions to ensure maximum traffic performance and safety, as illustrated by point MF 2 in the figure, geometric quality will be maintained at a high level (GQ 2), having as consequence that costs will increase due to the direct costs of maintenance services, as well as the constant stoppage of traffic. Moreover, ballast material will degrade more intensively, thus requiring more renewals.

Thus, by using geometric degradation prediction methods, managers predict the moment when degradations will reach tolerances, thus allowing geometric maintenance being postponed as long as possible, that is, being scheduled to occur as close as possible to the safety limit (point MF 3). In this way, the number of interventions over time is reduced to the lowest possible level and, consequently, direct service costs and penalties due to traffic interference are reduced (C 3), resulting in the lowest life cycle cost (LCC) for maintenance services.

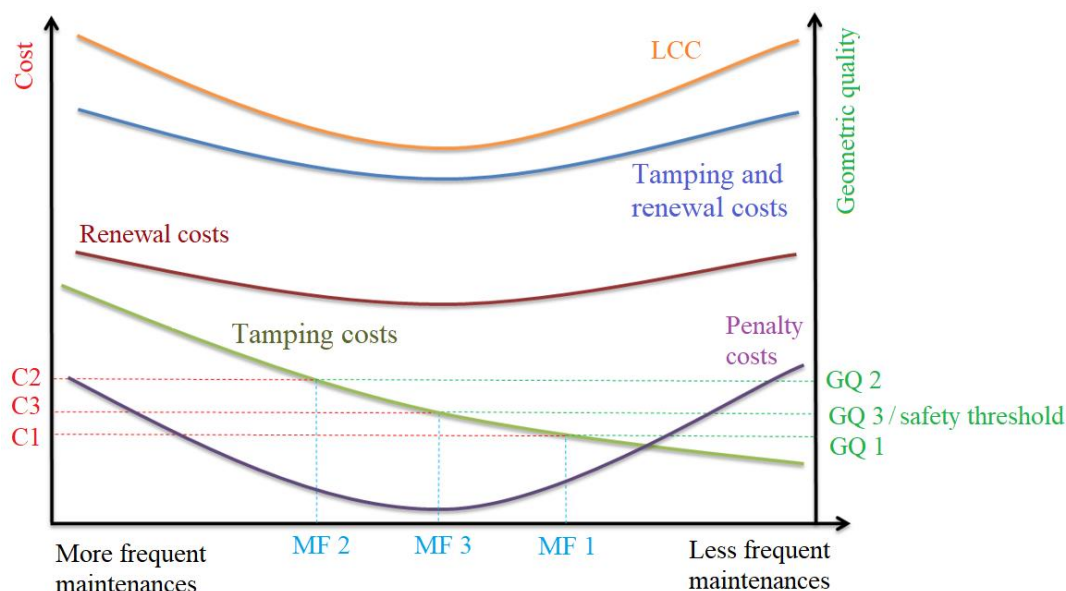


Figure 2. Maintenance cost behavior as a function of intervention strategies

3. Dynamic Safety

The basic force to which the track is exposed is the static force of the vehicle weight. Due to efforts to move vehicles (such as acceleration and braking), forces achieve greater magnitudes, being called dynamic forces. Theoretically, under ideal conditions, such dynamic forces could be considered quasi-static forces, that is, they would remain approximately constant over time [57]. However, this does not happen, and dynamic forces vary greatly as result of a variety of factors, such as aerodynamic concerns, vehicle characteristics, axle load, speed [3, 58], changes in track stiffness [59], as well as due to elements related to the wheel-rail interface, such as flat parts on wheels, rails with faulty joints and welds, corrugations, and geometric problems [3, 59].

Geometric changes or defects in the running surface of rails lead to the development of differential accelerations of vehicles relative to the track, which are function of the length and depth of the deviations as well as the operating speed [49]. With respect to the magnitude of the forces, some investigations indicate that accelerations and the consequent overloads of dynamic forces can reach high levels: in a field study conducted in North Queensland, Australia, You et al. (2017) [60] measured all the dynamic loads exerted by the trains that traveled the section under analysis and, on this railway, the maximum static wheel load was 140 kN. It was found that most loads reached 210 kN/wheel, which correspond to the dynamic design load of the track. However, about 5% of the overall loads reached substantially higher levels, up to 310 kN, 47% higher than the acceptable. In addition, there were also several intermediate loads between this peak and the maximum tolerated force of 210 kN, indicating, therefore, that the track was repeatedly critically stressed.

Remennikov & Kaewunruen (2008) [61] simulated the effects of uneven rail joints and found significant overloads also, up to four times the static load. Molodova & Dollevoet (2011) [62] measured accelerations of about 70 m/s² resulting from unevenness in a welded joint, with dimensions of about 1 meter long by 0.45 mm deep. Gullers et al. (2008) [46] performed a series of inspections on the Swedish railway section between Stockholm and Gothenburg and, in addition to observing dynamic overloads, also verified moments when loss of contact between wheel and rail occurred due to defects in the rail surface.

Such works, therefore, reveal how acute the dynamic effects caused by track defects can be. It has been understood since the 19th century that train accelerations relative to the track are hazardous for security and must be understood and managed. One of the most consolidated operational safety assessment models in this regard is the so-called Nadal criterion [44, 63], which is recommended, for example, by the Federal Railroad Administration (FRA) [64]. Its development is based on the evaluation of the equilibrium between lateral (L) and vertical (V) forces of a wheel at a given moment, with the proposal of a limit value for the imminent climbing of the wheel flange at the rail head, called L/V critical, which is determined by Equation 1 [44, 63, 64].

$$\frac{L}{V} = \frac{\tan \theta - \mu}{1 + \mu \tan \theta} \quad (1)$$

where L is the force in the horizontal direction on the wheel flange; V is the vertical wheel load; θ is the contact angle between the wheel flange and the rail side face; and μ is the friction coefficient between the rail side- and the-wheel flange.

Even when punctual, severe and abrupt changes in wheel-rail contact can have serious consequences on the dynamic balance of trains and even cause derailments [65], which is why railways also carry out dynamic inspections. For this purpose, inspection vehicles or common vehicles equipped with measuring equipment (such as accelerometers) can be used [62]. The data obtained complement geometric inspections in assessing operational safety and can even subsidize maintenance, using the Nadal criterion or through specific guidelines, such as the tolerances adopted by the Turkish State Railways (TCDD) (Table 4).

Table 4. Dynamic tolerances according to TCDD [41]

Accelerations	Unity	AL	IL	IAL
Transverse acceleration of bogie	m/s ²	5,00	7,50	10,00
Transverse acceleration of train floor	m/s ²	2,50	3,25	3,75
Vertical acceleration of train floor	m/s ²	2,50	3,25	3,75

It is important to note that even establishing means to keep the geometry within tolerances, planning techniques based on the postponement of interventions can tolerate geometric deviations that might cause severe traffic overloads and dynamic accelerations. Under such circumstances, dynamic inspections must be more frequent, and managers may be forced to carry out more corrective interventions to maintain safety, which increases costs and goes against one of the objectives of long-term planning methods, which is to seek ways of reducing the LCC.

4. Proposed Method for Track Maintenance Planning

This paper presents a method for determining the frequency of geometric track maintenance whose objective is to guarantee operational safety. To keep maintenance costs as low as possible the proposed procedure uses the same strategy employed by most long-term planning methods, which consists in postponing maintenance until the safety threshold. Its differential lies in the fact that, in addition to estimating future geometric conditions, the method establishes means to predict vehicle accelerations relative to the track, allowing the establishment of intervention guidelines that also guarantee safety under the dynamic criterion. The methodological roadmap for the procedure development is described in Figure 3.

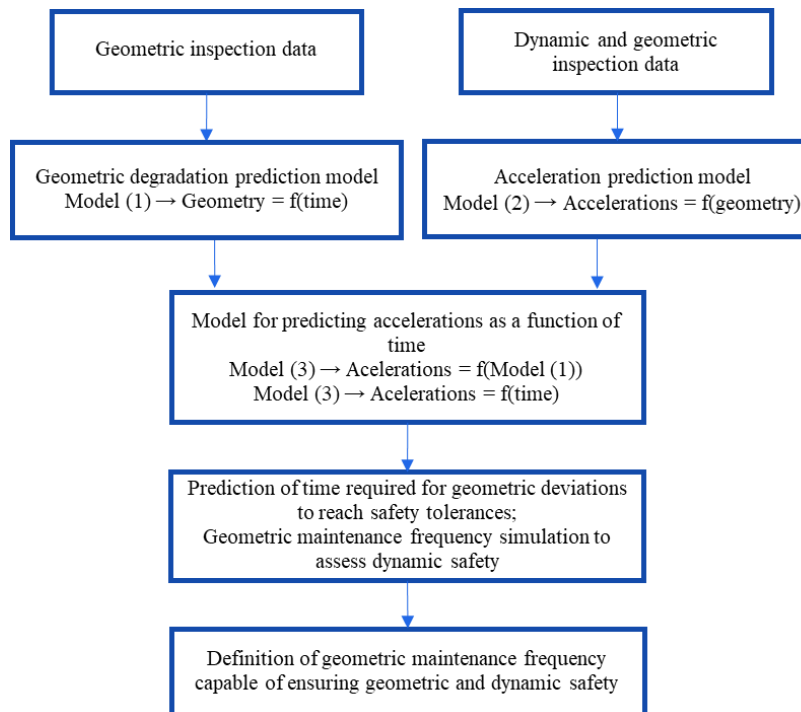


Figure 3. Methodological roadmap of the proposed procedure

4.1. Geometric Degradation Prediction Model

The proposed approach is empirically developed and is based on a statistical analysis of inspection data. The database must contain a series of successive geometric inspection reports, as well as the dates of each inspection, to enable the definition of the time interval between each report.

Following the understanding of several authors [11, 28-30, 66], linear regressions will be used to define the geometric degradation models whose characteristics are described in Equation 2.

$$D_{inT} = Tax_{i_n} \cdot T + VI_{i_n} - EP_{i_n} \quad (2)$$

where D_{inT} is the quality index value (in mm) of geometric parameter i under analysis, on section n , at time T . According to the literature, each railway section may have its degradation rate, then, in this approach, the examined track is divided into 200 m long segments and the geometric degradation condition for each segment is defined (based on Lee et al. (2017) [29], Khouzani et al. (2017) [30], and Caetano and Teixeira (2015) [66]). The standard deviation (SD) of the geometric readings of each segment was used as a quality index to indicate the degradation state of parameter i in segment n throughout the reports under analysis, following Sasidharan et al. (2020) [11], Bakhtiary et al. (2020) [19], Wen et al. (2016) [28], Lee et al. (2017) [29], Khajehei et al. (2019) [31], and Caetano and Teixeira (2015) [66]. Tax_{i_n} is the angular coefficient of the regression curve of geometric parameter i under analysis, on section n , fitted to the inspection data, defined taking the time between inspections as the independent variable of the degradation process (based on Wen et al. (2016) [28] and Khouzani et al. (2017) [30]), and the SD of parameter i in segment n of each of the analyzed inspections as the model dependent variable. This variable expresses the geometric degradation rate in mm/day. T is the elapsed time since last geometric maintenance (in days). VI_{i_n} is the SD value (in mm) verified on the track right after the maintenance services for parameter i in segment n . EP_{i_n} is the standard error of the regression estimate for parameter i in segment n .

4.2. Acceleration Prediction Model

The models for predicting vehicle vertical ($ac_V = f$) and horizontal ($ac_H = f^*$) accelerations relative to the track consider deviations from geometric parameters (g), track planimetric characteristics (p) (curve degrees), and operational speeds (s). As a result, two distinctive equations are defined:

$$ac_V = f(g, p, s) \quad (3)$$

$$ac_H = f^*(g, p, s) \quad (4)$$

These models are empirically defined using data from a geometric inspection and a dynamic inspection performed as closely as possible in time. The track is divided into sections, and the quality indices for each geometric parameter are calculated in the same manner as in the previous step, and each section is assigned the respective acceleration readings measured by the dynamic inspection. Subsequently, the models are obtained through linear regressions using as independent variables the quality indices of each geometric parameter, the maximum and minimum degree of curve, and the speed of the inspection vehicle of each track section under analysis. The accelerations measured in each of the segments under analysis are adopted as dependent variables, with the vertical acceleration readings used for the definition of the model represented in Equation 3 and the horizontal acceleration readings used for the model expressed in Equation 4. The F-statistic and the coefficient of determination (R^2) are used to assess the model representativeness.

4.3. Model for Predicting Accelerations as a Function of Time

The variables of the acceleration prediction models (Equations 3 and 4) related to geometric deviations are replaced by the equations for estimating geometric degradations relative to each of the analyzed geometric parameters established in the first step of this procedure for each track segment, resulting in the model shown in Equation 5:

$$ac_{jnT} = f(D_{1nT} + \dots + D_{inT}, p_n, s_n) \quad (5)$$

where ac_{jnT} is the acceleration j (corresponding to the vertical – V – or horizontal – H – acceleration analyzed), of the segment n under analysis at time T . D_{1nT} is the degradation model corresponding to the first geometric parameter under analysis, which was defined in the first step of the procedure. D_{inT} is the degradation model corresponding to parameter i , the last of the series of geometric parameters available in the geometric inspection reports under analysis. p_n are the maximum and minimum values of the curve degree of segment n , and s_n is the operating speed in segment n .

This step modifies the acceleration prediction models for each segment under analysis, so, from this point on, each track section will have its pair of equations for acceleration prediction.

4.4. Prediction Time to Reach Geometric Tolerances and Dynamic Safety Assessment

To predict the time required for the geometric deviations to evolve from the condition verified immediately after the last maintenance *activity* to the geometric safety tolerances, Equation 2 is modified as presented in Equation 6.

$$TM_{i_n} = \frac{Tsd_i - VI_{i_n} + EP_{i_n}}{Tax_{i_n}} \quad (6)$$

where TM_{i_n} is the time required for the degradation of parameter i in segment n to reach the maintenance threshold (in days). Tsd_i is the SD safety threshold (in mm) for parameter i . VI_{i_n} is the verified SD value (in mm) on the track after maintenance services for parameter i on segment n . EP_{i_n} and Tax_{i_n} are the same variables defined in the first step for parameter i in segment n .

Since TM_{i_n} is calculated for each of the geometric parameters, several possible geometric maintenance frequencies will be defined per segment in an amount equal to the number of available parameters. Among the results, the lowest value of TM_{i_n} found represents the lowest maintenance frequency that can be adopted to ensure safety from a geometric point of view.

For the dynamic safety assessment, the smallest value TM_{i_n} found will be reduced from 0 to 90% at 10% intervals in order to define 10 administrative scenarios for the n segment. Subsequently, each of these frequencies will be used in the $D_{1nT} + \dots + D_{inT}$ portion of the ac_{jnT} models specific of the segment n under analysis to calculate the resulting accelerations for each scenario, allowing, then, to analyze the dynamic safety from the L/V ratio and the tolerance defined by Equation 1.

4.5. Defining the Safe Maintenance Interval

The maintenance frequency for segment n under analysis that can satisfy the safety tolerances will be identified among the 10 administrative scenarios defined in the previous step. Any frequency among the 10 evaluated already guarantees safety under the geometric criterion, since the lowest frequency of the simulated set is relative to the time required for the geometric deviations to reach the safety limit of the reference parameter for the segment n . Therefore, the frequency to be chosen must be the one that ensures a safe L/V ratio. As the 10% variation between the simulated frequencies establishes relatively long intervals between the scenarios, an adjustment may be required to identify the final reference frequency. Thus, the frequency that resulted in the ratio closest to the dynamic tolerance will be adjusted until the resultant L/V ratio is as close as possible to the tolerance defined by Equation 1.

5. Implementation of the Procedure

The planning method proposed in this paper was applied to a Brazilian railway located in Rio de Janeiro. This is a railway system that integrates the passenger transport network of the city, illustrated in Figure 4. Its superstructure is formed by duplicated and ballasted tracks in broad gauge (1,600 mm), and trains have electric traction. The railway company requested confidentiality regarding its identification, so this paper is not authorized to provide further details about the geographical location of the studied railway section.

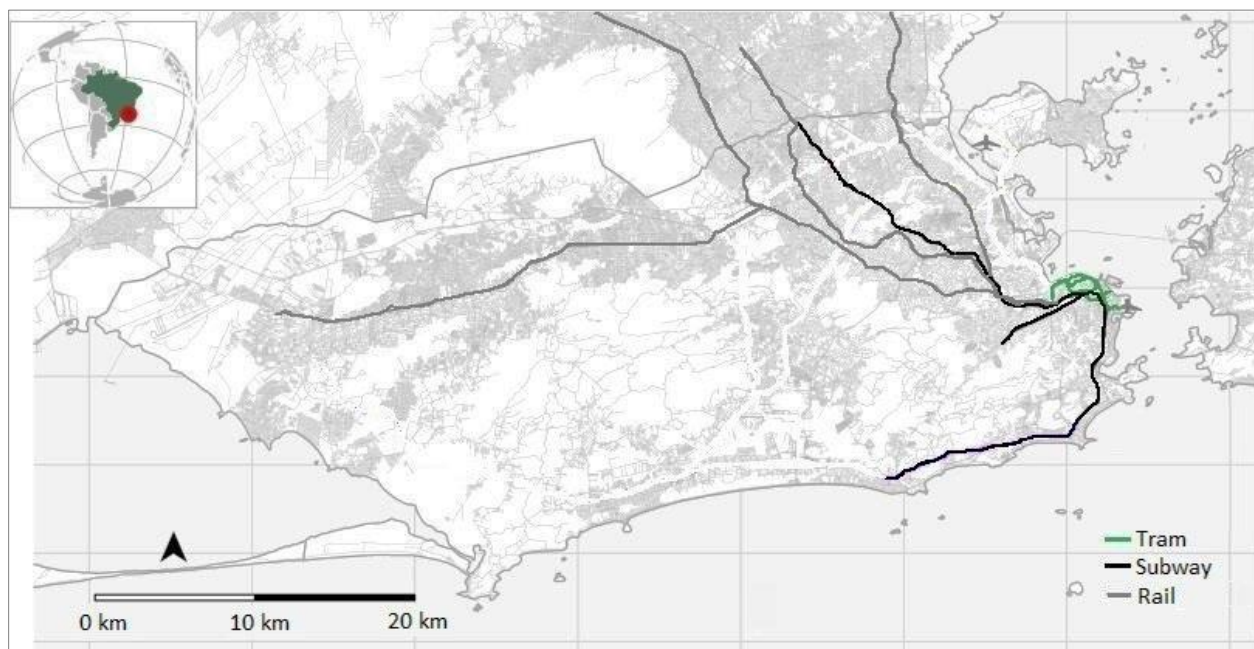


Figure 4. Rail system in the city of Rio de Janeiro. Adapted from Pereira (2019) [67]

5.1. Definition of the Geometric Degradation Prediction Model

To carry out this practical study, 26 consecutive geometric inspection reports from April 2017 to December 2019 were used. Inspections were conducted using inspection cars, with data collected every 25 cm, and geometric parameters measured were as presented in Table 5.

Table 5. Inspected geometric parameters

Parameter i	Parameter name	Measurement details
1	LPROF31	Left rail leveling set from a 31 feet base (≈ 9.5 m)
2	RPROF31	Right rail leveling set from a 31 feet base (≈ 9.5 m)
3	LALIGN31	Left rail alignment set from a 31 feet chord (≈ 9.5 m)
4	RALIGN31	Right rail alignment set from a 31 feet chord (≈ 9.5 m)
5	LPROF62	Left rail leveling set from a 62 feet base (≈ 19 m)
6	RPROF62	Right rail leveling set from a 62 feet base (≈ 19 m)
7	LALIGN62	Left rail alignment set from a 62 feet chord (≈ 19 m)
8	RALIGN62	Right rail alignment set from a 62 feet chord (≈ 19 m)
9	LPROF124	Left rail leveling set from a 124 feet base (≈ 38 m)
10	RPROF124	Right rail leveling set from a 124 feet base (≈ 38 m)
11	LALIGN124	Left rail alignment set from a 124 feet chord (≈ 38 m)
12	RALIGN124	Right rail alignment set from a 124 feet chord (≈ 38 m)
13	GAUGE	-
14	XLEVEL	-
15	WARP_62	Warp set from a base length of 62 feet (≈ 19 m)
16	TWIST_S	Twist defined from a “short” base (≈ 10 m)
17	TWIST_M	Twist defined from an “medium” base (≈ 20 m)
18	TWIST_L	Twist defined from a “long” base (≈ 40 m)

Initially, each inspection report was analyzed individually to identify record errors. All track segments with at least one record error, such as missing data, as well as those segments that generated inconsistent results, were eliminated. Therefore, this procedure analyzed 18 segments corresponding to a 3.6 km stretch.

The interval of inspections in which no maintenance was performed on each segment was identified to define the geometric degradation rate (Tax_{i_n}) in each of the 18 segments. This process was carried out based on Khajehei et al. (2019) [31], who state that reductions in SD values of at least 15% between consecutive geometric inspections records indicate that the geometry has been corrected. The geometric condition of parameter i verified after maintenance services (VI_{i_n}) was assumed to be the SD verified in the first report of the same series of inspections used to calculate the Tax_{i_n} of the evaluated segment. The standard error was determined using the same data that was used to calculate Tax_{i_n} .

The procedure defines individual geometric prediction models for each parameter in each segment. Therefore, as 18 parameters were analyzed in 18 segments, a total of 324 equations were defined in this step.

5.2. Definition of the Acceleration Prediction Model

A dynamic inspection report containing the vertical and horizontal accelerations measured by portable equipment placed on the cabin floor of the trains was utilized to develop this step. Only accelerations greater than a predetermined limit specified by the inspectors were recorded, 0.4 g in the vertical direction and 0.3 g in the horizontal direction ($g =$ gravitational acceleration ≈ 9.81 m/s²). Table 6 illustrates data characteristics by presenting the first five records made in the track under analysis.

Table 6. Sample of the dynamic inspection record

Code	Date	Vertical acceleration (g)	Horizontal acceleration (g)	Line	km	Location inside km (m)
23K090M	March 2021	0.14	0.32	1	23	90
23K110M	March 2021	0.09	0.37	1	23	110
26K740M	March 2021	0.27	0.49	1	26	740
26K750M	March 2021	0.15	0.34	1	26	750
28K310M	March 2021	0.20	0.32	1	28	310
28K460M	March 2021	0.44	0.21	1	28	460

The data used to define the geometric condition indices used in this step were obtained from an inspection carried out in the same month as the dynamic inspection.

To define the acceleration prediction models, linear regressions were performed from the 21 available independent variables (SD of the 18 geometric parameters + maximum curve degree (CV MAX) + minimum curve degree (CV MIN) + speed) associated with vertical and horizontal accelerations readings (dependent variables) measured on the respective segments. To check representativeness of the equations, it was assumed that the model would be rejected if the p-value (probability value) of the F-statistic was greater than 0.05.

The first regression for the ac_V model, based on the 21 independent variables, obtained a valid equation with a p-value of 0.009 for the F-statistic, and R^2 of 0.66. The resulting model is presented below.

$$ac_V = 0.044059281LPROF31 - 0.01392768RPROF31 - 0.024045637LALIGN31 + 0.015647827RALIGN31 + 0.102767307LPROF62 - 0.0791625RPROF62 - 0.089078347LALIGN62 + 0.007001642RALIGN62 - 0.064335625LPROF124 + 0.048105936RPROF124 + 0.147730317LALIGN124 - 0.157234693RALIGN124 + 0.010885111GAUGE + 0.011235503XLEVEL + 0.026305521WARP_62 - 0.136267154TWIST_S + 0.034713059TWIST_M - 0.002739382TWIST_L + 0.004908629SPEED - 0.089002702CV MAX + 0.120039792CV MIN + 0.53134548 \quad (7)$$

As for the ac_H model, the first regression presented a p-value of 0.29 for the F-statistic, and R^2 of 0.48, and, therefore, the set of independent variables was fitted to obtain a valid equation. This was done using the stepwise AIC method, which identified the statistically significant variables for the model (for more details about this technique see Yamashita et al. (2007) [68]). R software was used to perform this operation. At the end of the selection process, a model was obtained with a p-value of 0.002 for the F-statistic, and an R^2 of 0.40. Its equation is:

$$ac_H = -0.019486552LPROF31 + 0.027388788LALIGN31 + 0.032389172RALIGN62 - 0.009833314LALIGN124 - 0.052408375GAUGE + 0.005938666XLEVEL + 0.018097023TWIST_M + 0.221498705 \quad (8)$$

5.3. Definition of the Model for Predicting Accelerations as a Function of Time

In this step, dynamic models were modified to estimate accelerations as a function of time. For this purpose, variables related to the degradation state of each geometric parameter in Equations 7 and 8 were replaced by the equations defined in the first step for the respective parameters. This process was performed for each of the segments under analysis. Therefore, each of the 18 valid segments has its own set of equations to predict the vertical and horizontal accelerations. To illustrate the result, Equation 9 demonstrates the fit of Equation 8 to the geometric degradation models defined for segment 1.

$$ac_{H_1} = -0.019486552(0.019T + 5.85 - 0.61) + 0.027388788(0.002T + 1.73 - 0.49) + 0.032389172(0.009T + 2.76 - 0.61) - 0.009833314(0.087T + 0.59 - 2.57) - 0.052408375(-0.0003T + 2.7 - 0.08) + 0.005938666(0.18T + 4.56 - 12.1) + 0.018097023(0.094T + 4.26 - 6.49) + 0.221498705 \quad (9)$$

where T is the time, in days, corresponding to the geometric maintenance frequency to be analysed.

5.4. Calculation of the Time to Reach Geometric Tolerances, Dynamic Safety Assessment, and Definition of the Safest Maintenance Interval

The time required for the geometry to reach safety tolerances was calculated using Equation 6. Each geometric parameter was analyzed individually in each segment, therefore, 18 possible maintenance frequencies were defined in each segment, taking as reference the Brazilian standard ABNT NBR 16387:2020 [69] to define the SD safety thresholds.

Subsequently, the smallest frequency was identified among the results obtained for each segment, which corresponds to the largest possible interval between the maintenances that could be adopted in order to guarantee operational safety from a geometric point of view.

The lowest frequency obtained in each segment was used as a reference to simulate the ten corresponding administrative scenarios. Then, each of the simulated frequencies was used in the prediction models of the vertical and horizontal accelerations specific to the segment under analysis, allowing future accelerations to be calculated and the dynamic safety to be evaluated from the L/V ratio. Based on the parameters $\theta = 65^\circ$ and $\mu = 0.3$ for the studied railway, the Nadal tolerance index was determined as 1.122.

Table 7 shows the results of this simulation process for Segment 3, whose geometric parameter with the fastest degradation was LALIGN31, which resulted in a TM_{i_n} of 491 days. It was identified that up to the fifth simulated scenario the geometric maintenance frequency resulted in unsafe accelerations, so this frequency was adjusted to find the value that resulted in an L/V ratio of 1.121, in this case, 279 days.

Table 7. Simulated administrative scenarios for Segment 3. The frequency to be adjusted to define the final maintenance frequency for this segment is underlined in bold (Scenario 5)

Segment (n)	Scenario	Maintenance frequency (days)	Reference Parameter (t)	Frequency reduction compared to $TM_{LALIGN313}$ (%)	Accelerations		L/V
					Vertical (V)	Horizontal (L)	
3	1	491	LALIGN31	0	0.490	0.831	1.696
3	2	441.9	LALIGN31	10	0.504	0.782	1.552
3	3	392.8	LALIGN31	20	0.519	0.733	1.412
3	4	343.7	LALIGN31	30	0.533	0.684	1.283
3	5	294.6	LALIGN31	40	0.547	0.635	1.161
3	6	245.5	LALIGN31	50	0.562	0.585	1.041
3	7	196.4	LALIGN31	60	0.576	0.536	0.931
3	8	147.3	LALIGN31	70	0.590	0.487	0.825
3	9	98.2	LALIGN31	80	0.605	0.438	0.724
3	10	49.1	LALIGN31	90	0.619	0.389	0.628

Table 8 summarizes the final results obtained. The value of TM_{i_n} was chosen as the maintenance frequency of reference in segments where there were no scenarios with unsafe L/V ratios up to the geometric safety limit. In the other segments, the scenarios where the dynamic tolerance was reached before the geometric tolerance were identified, and then the corresponding frequency was adjusted until *resulting* in the L/V ratio as close as possible to the Nadal tolerance of 1.122.

Table 8. Minimum maintenance frequencies defined by the procedure able to guarantee operational safety from the geometric and dynamic criteria

Segment (n)	Reference Parameter (t)	TM_{i_n} (days)	Adjusted frequency (days)	Accelerations (g)		L/V
				Vertical (V)	Horizontal (L)	
1	WARP_62	128	-	1.669	0.446	0.267
2	LPROF31	629	317	0.157	0.176	1.121
3	LALIGN31	491	279	0.552	0.619	1.121
4	RPROF62	809	191	0.239	0.267	1.117
5	LPROF62	1918	-	0.961	0.179	0.186
6	GAUGE	1372	926	0.100	0.112	1.120
7	RPROF31	1295	326	0.294	0.330	1.122
8	WARP_62	192	-	7.380	0.085	0.012
9	WARP_62	2647	1977	0.170	0.190	1.118
10	LPROF62	856	-	1.650	0.228	0.138
11	RPROF62	670	364	0.115	0.128	1.113
12	LPROF62	822	-	0.530	0.168	0.317
13	LPROF62	1062	837	0.272	0.305	1.121
14	LPROF62	1499	-	2.230	0.195	0.087
15	WARP_62	161	-	0.682	0.397	0.582
16	WARP_62	131	-	0.893	0.220	0.246
17	TWIST_S	361	-	1.337	0.272	0.203
18	RPROF62	667	186	0.253	0.283	1.119

6. Discussion

The application in Section 5 of the procedure introduced in this work allowed the identification of some relevant aspects regarding its use and development, and the results obtained validated its relevance in achieving safer maintenance standards. Some aspects concerning the characteristics of the used data must be underlined in the application of the method. The database must cover the interval between two successive geometric maintenance on the examined railway sections so that the geometric degradation rate and geometric deviation values verified on the track after the maintenance can be defined more precisely.

The acceleration prediction models, particularly those defined for vertical accelerations, have been developed from a large number of independent variables. Therefore, the database must contain complete and comprehensive geometric inspection reports, through which the readings of several variables, not only geometric but also track characteristics, for example, can be accessed. There must be no reading errors or missing data in the geometric inspection records. In this sense, before starting to apply the procedure, a preliminary analysis is required to identify inconsistencies or reading errors that could lead to distorted results.

Segmenting the rail track into standardized 200-meter segments proved to be essential given the difference in the geometric degradation rate seen between the delimited sections, which even affected the prediction of accelerations. As a result, significantly different maintenance standards were established between the sections. Individual analysis of each parameter, with the development of its own deterioration model, was very relevant, particularly on identifying the reference parameter to estimate the frequency of maintenance under the geometric safety criterion.

Regarding acceleration prediction, it is important to emphasize the relevance of a joint examination of all geometric factors for determining models ac_V and ac_H . It was possible to conclude that the vehicle movements in the vertical direction are significantly affected by track conditions, since the defined model (Equation 7) has as variables the 18 geometric parameters available in the database, in addition to the variables related to the segment's planimetric characteristics and operational speed. Horizontal accelerations, on the other hand, proved to be substantially less sensitive, since the statistically-validated model (Equation 8) has 7 independent variables. It is worth noting that most of the variables in the horizontal model are related to alignment, which is a parameter that indicates the deviation from the ideal longitudinal alignment of the rail in the horizontal plane of the track, therefore, in the same direction as the accelerations under consideration. Likewise for the gauge parameter, relative to the perpendicular distance between rails in the horizontal plane. These observations are important because since the L/V ratio is strongly affected by the horizontal accelerations, and as these horizontal parameters (alignment and gauge) were the most relevant in inducing such accelerations in the model defined for the studied railway, managers can design specific monitoring strategies for them on the analyzed track, as well as seek higher levels of finish for these specific parameters during maintenance.

The accelerations expected at the moment when the geometric deviations would reach the tolerances of the reference parameter did not indicate unsafe operation under the dynamic criterion in Segments 1, 5, 8, 10, 12, 14, 15, 16, and 17, so maintenance strategies in these segments follow the basic understanding described in the literature and adopted by most long-term planning models in this category. This characteristic demonstrates that current planning methods are not inadequate, but limited, since in Segments 2, 3, 4, 6, 7, 9, 11, 13, and 18 the reference for maintenance was determined by dynamic criteria, establishing cycles that were significantly shorter than those defined by geometric criteria, thus demonstrating the importance of these analyses in establishing safer maintenance standards.

In most of the long-term planning methods seen in the literature, the geometric condition is analyzed from just a few parameters, usually leveling and longitudinal alignment. The minimum maintenance frequency determined exclusively by geometric degradation analysis demonstrated that this understanding is coherent, considering that most segments (except Segments 3 and 6) have track leveling parameters (leveling, warp, and twist) that exhibit the highest degradation rates and are, therefore, the reference parameters for defining intervention frequencies. Regarding the segments where maintenance standards were defined under the dynamic criterion, in all of them, the minimum maintenance cycles defined by the geometric criterion were relatively long, with Segment 3 being the only one with a higher frequency (491 days). On the other hand, no correlation was identified between the adoption of the dynamic criterion and a specific geometric parameter, since the dynamic criterion was adopted in segments where TM_{in} was relative to a leveling-based parameter, as well as in segments where these parameters were alignment (Segment 3) and gauge (Segment 6).

The segments that had the maintenance pattern defined by the dynamic criterion had considerable reductions in the frequency of intervention in comparison to the cycle that would be defined under the geometric criterion, indicating, therefore, the relevance of this criterion for the definition of maintenance policies capable of ensuring operational safety. Furthermore, this characteristic demonstrates that vehicle movements, and therefore dynamic safety, are not just affected by the severity of deviation of a single geometric parameter, but that vehicle accelerations relative to the track are a result of the association of geometric changes of all parameters.

The application of the procedure resulted in the definition of specific maintenance frequencies for each segment. Even though segments with close maintenance cycles were obtained (as in 4, 8, 15, and 18), significant differences occurred between most sections (for example between 9 and 4) whether from a geometric or dynamic standpoint. This type of information is important in the implementation of predictive maintenance plans aimed for minimizing LCC because it allows maintenance to be performed only when necessary, according to the needs of each segment, eliminating interventions in unneeded areas.

It should be noted that some segments (1, 4, 8, 15, 16, and 18) had much shorter maintenance cycles than others. This information is particularly important since it may indicate that a change has happened in some layer of the track, such as the subgrade, which was reflected in a faster rate of geometric degradation. Therefore, the relevance of this type

of observation in infrastructure management is highlighted, as it may indicate the need for more accurate inspections at these sites, with the investigation of other layers of the superstructure in order to identify the circumstances that are causing this degradation rate, which may even subsidize specific interventions to correct it.

It is worth noting that the frequencies defined by the procedure, whether based on TM_{i_n} or dynamic tolerance, are not intended to determine a precise schedule of interventions; rather, these results are a reference for managers, allowing them to predict maintenance needs and then define the best strategy capable of satisfying safety and budgets, which may involve other relevant aspects that are not part of the scope of this work (such as the frequency of inspections, operational windows, mobilization and routing of teams, etc.). Furthermore, when considering long-term maintenance plans (for example, with administrative horizons of 20 or 30 years), the defined maintenance frequencies must be revisited over time, examining the need for revisions in function of possible changes in the geometric degradation rate.

Regarding management approaches whose maintenance policy is only based on geometric condition analysis, the results obtained in the implementation of the procedure provided in this study show that maintenance cycles in specific track segments will be similar to those defined by such approaches. Because there are no dangerous accelerations until geometric tolerances are achieved, services can be postponed, following the same idea as approaches typically found in literature. On the other hand, when the dynamic criterion overrides the geometric one, the frequency of intervention must increase, and, as seen, the frequencies are significantly different from those that would be adopted without the consideration of this criterion, with significant service anticipations. Because of this characteristic, the LCC resulting from the maintenance policy of the proposed procedure may have larger values due to the anticipation of interventions in specific segments, resulting in a higher number of interventions. However, safety assessment exclusively through the geometric condition has proven to be limited, and the proposed method, even if it results in an increase in LCC, is capable of ensuring operational safety by avoiding accidents that might imply serious damage to the company, whether material or human, that would compromise the savings eventually obtained by traditional planning methods.

It is also important to compare the characteristics of the results obtained in this paper with those observed in previous works that take dynamic inspections into account in maintenance planning. The most works in literature that propose analyzing maintenance planning considering dynamic issues mainly seek to develop improved techniques to inspect and process the collected data. Consequently, the results obtained are quite accurate and detailed, but they are limited to providing a diagnosis of the current track condition, without establishing any reference to future dynamic behavior. As a result, these techniques typically support short-term planning as well as the identification of immediate intervention points. In this sense, the results obtained demonstrate that the method presented here allows defining predictive and long-term planning, since the prediction of accelerations allowed the maintenance cycles to be defined in advance based also on dynamic criteria, and as a result, managers in possession of this information can adjust the frequency of inspections so that they are not performed in a volume greater than necessary, as well as plan in advance the resources for intervention, with the definition of operational windows and preparation of teams, which will result in the reduction of direct and indirect costs.

7. Conclusions

This paper proposes a method for planning railway track maintenance with the objective of establishing the safest frequency of geometric maintenance from a geometric and dynamic standpoint while still resulting in low intervention costs. Although there are a large number of long-term maintenance planning techniques available in the technical and academic literature, the majority of them are based on modeling geometric degradation and forecasting the achievement of corresponding tolerances. This work differentiates itself from the previous models by establishing means of predicting the vertical and horizontal accelerations of vehicles relative to the track as a function of the estimated future geometric condition, as well as simulating the dynamic conditions for any geometric maintenance frequency that is intended to be analyzed. As a result, such an approach determines the geometric maintenance cycle that allows geometric tolerances and dynamic safety standards to be met, leading to safer maintenance policies.

Moreover, by defining the lowest possible maintenance frequency that ensures operational safety, the technique has the potential to result in the lowest possible budget since it delimits the smallest possible number of interventions that can be performed, thus achieving economy of scale.

This work does not propose to establish a definitive method capable of exhausting the theme but introduces a new analytical perspective that aims to complement the existing approaches in order to develop methods to support planning that are increasingly efficient and complete in terms of addressing operational safety.

Therefore, the application results of the method, in addition to corroborating its applicability and relevance, indicate how promising and viable the research dedicated to the development of this area of analysis is, which may even be inspired by some opportunities for future studies identified throughout the development of this work:

- Investigating if other types of regression models can be used to define dynamic equations, considering, for example, non-linear techniques;

- Using the cumulative traffic load as an independent variable for the geometric degradation process and in dynamic models;
- Analyzing how each geometric parameter influences the generation of accelerations;
- Investigating whether certain geometric parameters should be included in the models, even if they are not statistically significant;
- Repeating the procedure using a continuous recording of accelerations along the inspected section;
- Developing accurate methodologies for defining the efficiency of geometric maintenance services in correcting parameter deviations.

Vertical accelerations in some of the segments analyzed reached notably high levels, which have an effect on the dynamic wheel loads and can result in overloads in these circumstances. Such overloads can damage track components. Therefore, as a matter of future research, it is relevant to investigate how the deterioration rate of components changes as a function of the dynamic loads since if their service life decreases as a function of forces, the demand for renewal tends to rise, implying an increase in costs. Such an approach could lead to significant models to estimate component service life and evaluate whether geometric maintenance should be anticipated in order to decrease the amplitude of accelerations and loads, aiming to reduce long-term costs.

8. Declarations

8.1. Author Contributions

Conceptualization, I.P.S. and H.X.R.N.; methodology, I.P.S. and H.X.R.N.; software, I.P.S.; validation, I.P.S. and H.X.R.N.; formal analysis, I.P.S. and H.X.R.N.; investigation, I.P.S.; resources, I.P.S. and H.X.R.N.; data curation, I.P.S.; writing—original draft preparation, I.P.S.; writing—review and editing, I.P.S. and H.X.R.N.; visualization, I.P.S. and H.X.R.N.; supervision, H.X.R.N. All authors have read and agreed to the published version of the manuscript.

8.2. Data Availability Statement

The data presented in this study are available on request from the corresponding author.

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8.5. Conflicts of Interest

The authors declare no conflict of interest.

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