Optimal Control of Vaccination for Dengue Fever in SIR Model

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Abstract

According to data from The Indonesian ministry of health, many of individuals suffere dengue fever until may 2023 in Indonesia. To reduce its cases, in this article, a single of control strategy of vaccination for infected human by dengue fever has been proposed. To obtain the optimal control, the SIR model has been modificated with single control and the new objective function has been made before the Pontryagin minimum principle is used in this article. According to the differential equation in the model of the dengue fever and the objective function, we made the Hamiltonian equation. Then, from it, the state equation, costate equation, and stationary condition has been made from the Hamiltonian equation so we obtained the optimal control in vaccination. In the end of this article, we did the numerical simulation using the sweep forward-backward method. Through numerical simulation, we find that the control succeed to reduce the infected human by dengue fever and also increase human recovery from this desease. Futhermore, the control of vaccination for infected human should be implemented not only in this mathematical model but also into real life to decrease the dengue fever case.

Keywords: Control, dengue, fordward-backward, sweep, vaccination.

Introduction

According to data from The Indonesian ministry of health, A total of more than 31 thousands confirmed cases of dengue fever have been reported until mei 2023 in Indonesia. The dengue fever is caused by a virus dengue. The main factor in the spread of this dengue fever is immigration where the sufferer can spread this disease into the population [1], [2] . One of the infectious diseases is dengue fever. It is an infectious disease that is caused by a virus that is transmitted by an Aedes aegypti mosquito bite to a human. When bitting humans, the mosquito contains the dengue virus and develops within 4-6 days (intrinsic incubation period) before the disease will occur. The initial symptom will be based on the age of the sufferer. In children usually the symptom like fever with a red stain on the skin. In teenagers and adults, the symptom consists of fever within 2-7 days with headache, vomiting, and a red stain on the skin [3].

Because the worst possibility will occur so we need to conduct preventive action to contain spread of the dengue fever. This action not only can do by medicine but also can do by mathematics with mathematical models [4]. The mathematical model is a process representing and explaining a problem from real life to a mathematical formula, and a tool to solve a problem in real life [5].

The mathematical model which is being used to analyze infectious diseases is the SIR model (susceptible, infected, and Recovered). Form of mathematical model for the spread out of disease has fewer done. For instance, [1] was explaining the optimal control for the epidemic SIR model with vaccination and immigration factors. In this research, the minimum Pontryagin principle has been used for getting optimal control vaccination and optimal therapy. Those control obtained in this research were to minimize the infected population so the spread of the disease can be decreased. Other articles discuss the SIR model and vaccination for various types of diseases [6]–[8]. Another research by [9] discussed the mathematical model for the spread of dengue fever. In [9], it was using state, costate, and stationary condition to get optimal control by using Pontryagin minimum principle. In conclusion in [9], we got that the optimal control could decrease the susceptible human population, the infected human population, and also could increase the recovery of the human population.

In this research, we are interested to develop research by [6] that was minimize the spread of dengue fever with vaccination control in the infected human population. we are going to use the Pontryagin minimum principle to obtain the optimal control and add the sweep method forward-backward into [9] to get a solution for the susceptible human population, the infected human population, and the recovery human population.

Methods

A mathematical model can be created for dengue fever. In this model, there are two parts. The first part is the human population group and the second part is mosquito group. The human population group can be divided into three groups. The groups are the susceptible host (s_h), the infected host (I_h), and the recovery host (R_h). The mosquito population will be divided into two groups. The groups are the susceptible vector (S_v) and the infected vector (I_v).

First of all, the susceptible host (s_h) is a group that fill by all the human not only children, teeneger and adult but also new born will enter to it. Every individual in population will directly add to this group. However, the population of (s_h) not only can increase but also can decrease. Because in this group, there are two factor that will reduce the population. Those factors are the human natural mortality (an individual died is not caused by dengue fever) and the probability contact between human-musquito infected (this factor will move an individual from the susceptible host to the infected host).

Secondly, the infected host is a group filled by all the individuals infected the dengue fever. In this model, the population of (I_h) will only come from individuals in (s_h) who are having symptom of dengue fever. An individual in this group has the ability to infect the mosquito with the dengue virus. In addition, the control $u(t)$ will be added. The $u(t)$ is rate of vaccination only for infected human by dengue fever. Therefore, the number of people in this group can reduce because there are the human natural mortality and the rate of human becomes recovery.

Thirdly, the recovery host (R_h) is a group containing all the people who have been recovered from dengue fever. In this model, the population of (R_h) only comes from an individual who has been given vaccination and healed from dengue fever. However, the population of it can also decrease because the factor of human natural mortality also exists in this group.

Fourthly, the susceptible vector (S_v). All population the mosquito will be added into (S_v). In this group, the mosquito has been assumed not infected by dengue virus. Futhermore, the mosquito bites an individual who infected by dengue fever so that the dengue virus enter into the mosquito. Finally, the musquito has been infected by the dengue virus and becomes a vector for infecting to human. In addition, the infected musquito will exit from (S_v) and move to the next group. Like with the human group, in this group also exists musquito natural mortality.

The next group is the infected vector (I_v). This group contains the infected mosquito by dengue virus. In this model, the infected mosquito will spread out the dengue virus by only biting an individual in the susceptible host (s_h). There is no interaction between (I_v) and (R_h) because every person who recovered from dengue fever will have immune from the dengue virus.

According to the explanation above and from [9], the plot for spread out dengue fever in the SIR model considering both the human population and the mosquito population is

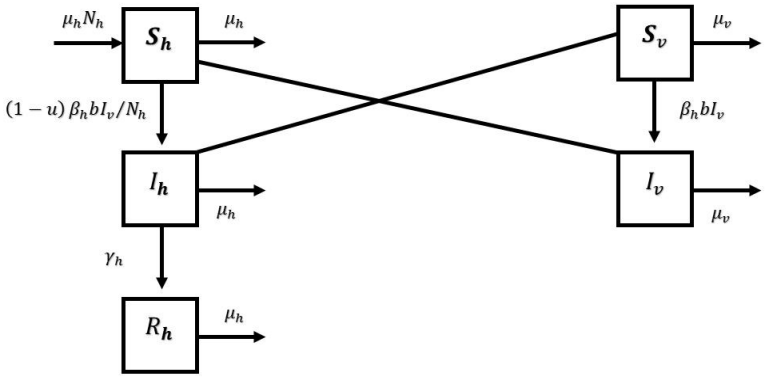

Figure 1. Plot of spread dengue fever in model SIR

According to Figure 1, the mathematical model can be described in a system of ordinary differential equations with human population growth in any group depending on time [10]. Therefore, the differential equation system for the SIR model of dengue fever is,

$$\frac{d}{dt}S_h = \mu_h N_h - \frac{\beta_h b I_v}{N_h} S_h I_v - \mu_h S_h, \quad (1a)$$

$$\frac{d}{dt}I_h = \frac{\beta_h b I_v}{N_h}(1-u) S_h I_v - (\mu_h - \gamma_h) I_h, \quad (1b)$$

$$\frac{d}{dt}R_h = \gamma_h I_h - \mu_h R_h \,, \quad (1c)$$

$$\frac{d}{dt}S_v = \mu_v N_v - \frac{b\beta_v}{N_h} I_h S_v - \mu_v S_v \,, \quad (1d)$$

$$\frac{d}{dt}I_v = \frac{\beta_v b}{N_h} I_h S_v - \mu_v I_v, \quad (1e)$$

Where S_h is the number of human susceptible, I_h is the number of human infected, R_h is the number of human recovery, S_v is the number of musquito susceptible, I_v is the number of musquito infected, N_h is total population of human and N_v is total population of musquito. Furthermore, μ_h and μ_v represent human natural mortality and musquito natural mortality, respectively, and γ_h is rate of human becomes recovery, b is rate of musquito bite. Meanwhile, β_h and β_v represent Probability contact between musquito infected and human susceptible and probability contact between musquito susceptible and human infected, respectively. We assume that all people who infected by dengue fever will recieve the vaccine, and after vaccine administration, the people have immune and will never suffer dengue fever again.

We now will describe step by step process in this article using the Pontryagin minimum principle. We begin by creating the objective function for the SIR model of dengue fever. Because all equations in this mathematical model are the differential equation form, from [11] the objective function will be created for the SIR model of dengue fever for $t \in [0, T_f]$ and from [12] we assume $u(t) \in [0, u_{max}(t)]$ with $u_{max}(t)$ for $u(t)$ as T_f. Then, using both the differential equation and the objective function, we will be made the Hamiltonian function for infinite time. From [11] , we know the Hamiltonian function is,

$$H(x(t), u(t), t) = L(x(t), u(t), t) + \lambda^T(t) f(x(t), u(t), t) \quad (2)$$

Where $L(x(t), u(t), t)$ is integran the objective function, $f(x(t), u(t), t)$ is right side from the differential equation dynamical system, and $\lambda(t)$ is the associated costate for state $x(t)$. Futher we are going to denote $x(t) = x$, $u(t) = u$, and $\lambda(t) = \lambda$. After the Hamiltonian function, we will create the state equation, the costate equation and the stationary equation. The state equation will represent analyze the number population in the susceptible host (s_h), the infected host (I_h), and the recovery host (R_h) at final time T_f, the costate equation is used to complete the control that will obtain from the stasionary equation, and the stationary equation will create the control. Furthermore, after we obtain the control, we perform numerical simulation by using sweep forward-backward with various parameter values [13]–[15]

Results and Discussion

We begin discussion by making the objective function. Because we want to obtain control and to minimaze the infected host (I_h) at time t, we will create the objective function with integran

$$J(u) = \int_0^{T_f} \left(I_h + \frac{c}{2} u^2 \right) dt, \quad (3)$$

consist of $I_h(t)$ and control u.

With the objective function used in this research in quadratic form with $0 < u < 1$, consequently the objective function in this model is positif velue [11] . Next, according to (1a)-(1e) and (3), we using (2) to create The Hamiltonian function,

$$\begin{aligned} H(x, u, t) = & \left(I_h + \frac{c}{2} u^2(t) \right) \\ & + \left((\lambda_1)(\mu_h N_h) - (\lambda_1) \left(\frac{\beta_h b S_h {I_v}^2}{N_h} \right) - (\lambda_1)(\mu_h S_h) \right) \left((\lambda_2) \left(\frac{\beta_h b S_h {I_v}^2}{N_h} \right) \right. \\ & \left. - (\lambda_2) \left(\frac{\beta_h b S_h {I_v}^2 u}{N_h} \right) - (\lambda_2)(\mu_h I_h) + (\lambda_2)(\gamma_h I_h) \right) + \left((\lambda_3)(\gamma_h I_h) - (\lambda_3)(\mu_h R_h) \right) + \\ & \left((\lambda_4)(\mu_v N_v) - (\lambda_4) \left(\frac{b \beta_v I_h S_v}{N_h} \right) - (\lambda_4)(\mu_v S_v) \right) + \left((\lambda_5) \left(\frac{\beta_v b I_h S_v}{N_h} \right) - (\lambda_5)(\mu_v I_v) \right) \quad (4) \end{aligned}$$

Continue, from equation (4), we make the following equations,

a. The state equation, according to [11], we have to perform the partial derivative of H with respect to λ. From equation (4), we know λ_i, $i = 1,2,3,4,5$ so we have to derivative of H respect to $\lambda_1, \lambda_2, \ldots, \lambda_5$.

$$\begin{aligned}
\dot{S_h} &= \frac{\partial H}{\partial \lambda_1} = \mu_h N_h - \frac{\beta_h b S_h {I_v}^2}{N_h} - \mu_h S_h \ , \\
\dot{I_h} &= \frac{\partial H}{\partial \lambda_2} = \left(\frac{\beta_h b S_h {I_v}^2}{N_h}\right) - \left(\frac{\beta_h b S_h {I_v}^2 u}{N_h}\right) - (\mu_h - \gamma_h) I_h \ , \\
\dot{R_h} &= \frac{\partial H}{\partial \lambda_3} = \gamma_h I_h - \mu_h R_h \ , \\
\dot{S_v} &= \frac{\partial H}{\partial \lambda_4} = \mu_v N_v - \frac{b \beta_v I_h S_v}{N_h} - \mu_v S_v \ , \\
\dot{I_v} &= \frac{\partial H}{\partial \lambda_5} = \frac{\beta_v b I_h S_v}{N_h} - \mu_v I_v \ .
\end{aligned} \tag{5}$$

b. The costate equation, according to [11], we have to perform the partial derivative of H with respect to x. From equation (4), we know λ_i, $i = 1,2,3,4,5$ so we have to derivative of H respect to x for λ_i for $i = 1,2,3,4,5$ which satisfying

$$\begin{aligned}
\dot{\lambda}_1 &= -\lambda_1 \frac{\beta_h b {I_v}^2}{N_h} - \lambda_1 \mu_h + \lambda_2 \left(\frac{\beta_h b {I_v}^2}{N_h} - \frac{\beta_h b S_h {I_v}^2 u}{N_h}\right), \\
\dot{\lambda}_2 &= -\lambda_2 \mu_h + \lambda_2 \gamma_h + \lambda_3 \gamma_h - \lambda_4 \frac{b \beta_v S_v}{N_h} + \lambda_5 \frac{b \beta_v S_v}{N_h} \ , \\
\dot{\lambda}_3 &= -\lambda_3 \mu_h \ , \\
\dot{\lambda}_4 &= -\lambda_4 \frac{b \beta_v I_h}{N_h} - \lambda_4 \mu_v + \lambda_5 \frac{\beta_v b I_h}{N_h}, \\
\dot{\lambda}_5 &= -\lambda_1 \frac{2\beta_h b S_h I_v}{N_h} + \lambda_2 \left(\frac{2\beta_h b S_h I_v}{N_h} - \frac{2\beta_h b S_h I_v u}{N_h}\right) - \lambda_5 \mu_v .
\end{aligned} \tag{6}$$

c. The stationary equation, according to [11], we have to perform the partial derivative of H respect to u to obtain control u so From equation (4), control is taken from $\frac{\partial H}{\partial u} = 0$.

$$\begin{aligned}
0 &= cu(t) - (\lambda_2) \frac{\beta_h b S_h {I_v}^2}{N_h} \ , \\
u &= \frac{(\lambda_2) \frac{\beta_h b S_h {I_v}^2}{N_h}}{c} .
\end{aligned} \tag{7}$$

Meanwhile, according to (1a)-(1e), we looked at the differential equation in this model is non linier equation and quadratic form in (1b) and we assume $0 < u < u_{max}$, so that from [12] we can make,

$$u^* = \begin{cases} 0 & u < 0 \\ u & 0 \le u \le u_{max} \\ u_{max} & u > u_{max} \end{cases}$$

Equal with,

$$u^* = \left(0, \frac{(\lambda_2) \frac{\beta_h b S_h {I_v}^2}{N_h}}{c}, u_{max}\right) \tag{8}$$

Where $u^*$is optimal control (optimal vaccination).

Numerical Simulation

In this part, we want to solve the mathematical model SIR problem (1a)-(1e) with the objective function (2). We used an 4th order Runge-Kutta fordward-backward iteration method. We begin by giving an initial guess for control u and solve the state equations (5) in time using the forward Runge–Kutta method. Next, using the backward Runge–Kutta method, we calculate the costate equations (6) in time. In addition, we update the value optimal control (8) and run this iteration until the convergence criteria achieved. Please see [16] for detailed of this iteration method and for more example of this method.

Furthermore, we conduct a numerical simulation to obtain the result for $S, I\ R$ classes after it was given the optimal control. According to the equation obtained from the sweep forward-backward, the numerical simulation will be conducted. The parameter values that will need in this simulation will be taken from [9] and [17]. The parameter values are $N_h = 10000$, $N_v = 50000$, $\gamma_h = 0.1667$, $\mu_h = 0.0000385$, $\mu_v = 0.1, \beta_h = 0.2808, \beta_v = 0.375, b = 0.5$. Futhermore, numerical simulation will be made in two conditions. Firstly, without optimal control, it is not given vaccination effort $(u = 0)$. Secondly, the infectious class will be given rate of vaccination $u_{min} < u(t) < u_{max}$ with $u_{min} = 0{,}1$ and $u_{max} = 0{,}9$. The simulation process do by using Matlab software.

As a result, the human population in the infectious class with control and without control can see in chart in Figure 2. In Figure 2, the simulation result for the population of humans in the infectious class with control optimal is different from the population of humans in the infectious class without control optimal. The cart of this class describing for class with optimal control still increase from $t = 0$ to $t = 12$ days, but it is not higher than without optimal control. Because vaccination control can contain spread of dengue fever, but if humans cannot give the vaccination control, it causes spread of dengue fever can increase that will add to infected human by dengue fever.

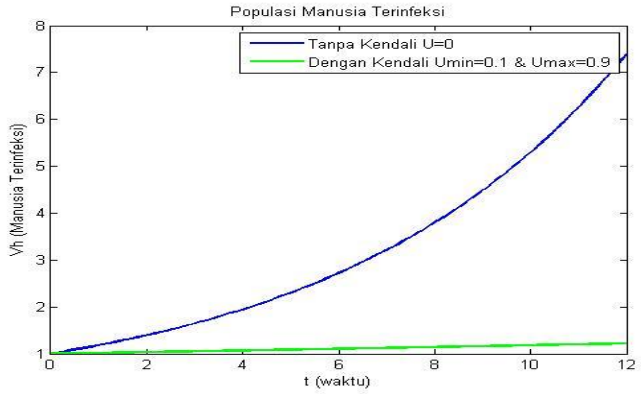

Figure 2. The group of human infected

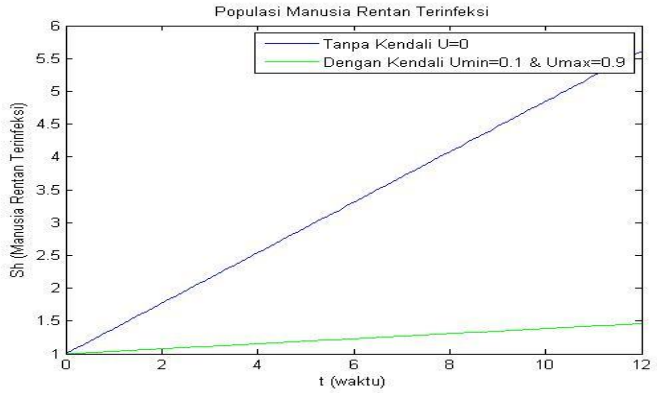

Figure 3. The group of human susceptible

The result for the susceptible class is similar to the infection class. Vaccination control can contain the spread of dengue fever or the number of susceptible individuals infected by dengue fever is lower than without control. According to Figure 3 for $t = 0$ to $t = 12$ days, the number of susceptible population for human is not more than one person after giving optimal control, whereas without optimal control it becomes more than 5 people. Finally, giving vaccination to infected

human will help the effort to recovery human who have been infected by dengue fever. According to Figure 4, we can see by giving optimal control of rate of vaccination the cart of recover for the humans population increased to 7 individuals but without the vaccination the number of recovery humans was only 1 person for $t = 0$ to $t = 12$ days.

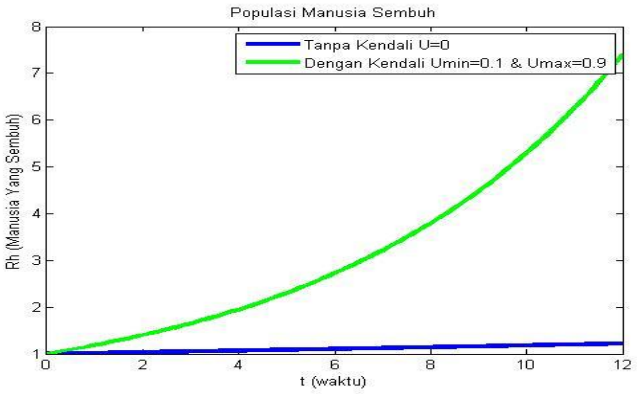

Figure 4. The group of human recovery

Conclusion

In this article, we use the mathematical SIR model for dengue fever and we create the objective function with singel control rate of vaccination. In the objective function, we want to reduce infected human with a control rate of vaccination. The equation in this model conducted a five dimension system of ordinary differential equations (ODE) with two of them groups for mosquito and remain for groups of human. From discussion, we conduct the Pontryagin minimum principle to determine the result this model. We obtain a five sisytem of ODE for the state equations and costate equations, and also optimal control has been obtained. The optimal control is,

$$u^* = \left(0, \frac{(\lambda_2)\frac{\beta_h b S_h {I_v}^2}{N_h}}{c}, u_{max} \right)$$

We do numerical simulation using 4th order Runge-Kutta fordward-backward iteration method after we obtain them.

From simulation, we show how the optimal rate of vaccination could reduce the number of individuals in the susceptible host (s_h) . Figure 2, we see individual who suspected reduce from $t = 0$ to $t = 12$ days with using control of vaccination ($u_{min} = 0.1$ and $u_{max} = 0.9$), but without control of vaccination ($u = 0$) the individuals suspected grow up dramaticaly more than 5 individuals. Futher, in the infected host (I_h) , the control of vaccination ($u_{min} = 0.1$ and $u_{max} = 0.9$) again could decrease the number of individuals who infected dengue fever from $t = 0$ to $t = 12$ days contrary to $u = 0$ (no vaccine for infected individuals) there are 8 individuals will infect by dengue fever in 12 days. From result of numerical simulation, The control rate of vaccine could increase the recovery individuals from 0 to 8 individuals in 12 days, in another word, all infected individuals will recover from dengue fever, contradiction without vaccination only one individual will heal from this desease. Therefore, we could obtain the result, the rate of vaccine could reduce infected individuals and same time to increase recovery indivuals, it mean the vaccine intervention rely heavly on to overcome the dengue fever and greatly effective to increase number of individuals heal from this desease.

Although our result in this article is giving a promise our control (rate of vaccination) having the good result for all group of human, we still have some limitation in this reserach. Our research has

not yet conducted in complete group of mathematical model. Furthermore, we encourage for futher research to add a or some group namely exposed group and therapy group for human, and also we puss to next research to create both group of vaccinated and not vaccinated for this mathematical model.

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