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An Evaluation of Mathematics Grounding Activity Teachers' Efficacy

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Abstract

Preparing high-quality mathematics teachers to promote students' meaningful learning has gained much attention in Taiwan, where the "Just Do Math (JDM)" Project was correspondently initiated in 2014. This study aimed to evaluate targeted mathematics grounding activity teachers' (MGA-teachers) the efficacy ratings for clarifying the effectiveness of the JDM professional development (PD) programs. A two-year "mixed method approach" was employed: First, a qualitative task analysis of the MGA-teachers PD programs was conducted for both understanding how these PD programs were executed and constructing the MGA-teachers' efficacy instrument for further quantitative investigations. Secondly, the "JDM MGA-Teacher Efficacy Instrument" was employed to examine the status and possible differences of targeted MGA-teachers' efficacy. There were 408 MGA-teachers participated in this study, where the efficacy ratings of these teachers were examined by corresponding statistical analyses. The findings were reported in two parts: Based on the findings of the qualitative task analysis, certain differences practically existed where various teaching approaches were employed by different MGA-instructors associated with a variety of content were provided in every session of the "2-day PD activity". Besides, according to the quantitative findings, it was found that the targeted MGA-teachers' efficacy belief is averagely at an acceptable level (i.e. around 70%~75%). It was also found that the experience and willingness of attending MGA PD activities and hosting fun-math camps had a significant influence on the positive development of their efficacy belief. Finally, discussions of the findings and recommendations were proposed for future study and further improvements of the JDM PD programs.

Keywords—Mathematics Teacher Efficacy, Mathematics Grounding Activity, Professional Development

Introduction

• Potential Problems of Taiwanese Students' Mathematical Learning

Preparing high-quality mathematics teachers to promote students' meaningful learning has gained much attention in Taiwan. However, there are some potential problems while we discuss students' learning achievement, interest, and confidence in mathematics based on the results of recent international assessment comparisons. Ground on the findings of PISA 2012, even though the mathematical performance of Taiwanese 15-year-old students still ranks fourth in all participating countries, the achievement gap between high- and low-achievers is enormous (i.e. amount to 245 points), which is equal to the difference that receiving 6-year education may possess (PISA in Taiwan, 2015; OECD, 2014). Moreover, the phenomenon of low confidence and low interest in mathematics learning, based on the findings of TIMSS 1999~2011, seriously exists, which deserves to be noticed (Lin, 2015; Mullis, Martin, Foy, & Arora, 2012). In fact, previous studies indicated that low learning interest and confidence are critically influential to students' readiness in learning mathematics, which may lead them to give up earlier while learning because of the feeling of helplessness (Brown, Brown, & Bibby, 2008; Fay, Bickerstaff, & Hodara, 2013). Therefore, this large achievement gap and its polarized phenomenon, the highest percentile of low-achievers, and low learning interest and confidence remind us the importance of assisting students with low-readiness in learning mathematics in every classroom. Accordingly, providing better early learning opportunities for these low-readiness students stands at the center of the current educational reform.

- **"Just Do Math" Project as Platform in Building Grounds of Mathematical Learning**

Echoing to this reform movement, the "Just Do Math (JDM)" Project is funded by Ministry of Education, Taiwan in 2014. Professor Fou-Lai Lin (Department of Mathematics, National Taiwan Normal University) initiates this JDM project that intends to build the "ground(s)", by cultivating "Mathematics Grounding Activity (MGA)-designers" and "MGA-teachers" for designing and implementing MGA modules, which will enhance students' learning interest and achievement in mathematics. Eventually, it hopes that every student can learn mathematics successfully (Lin, 2014). Through developing "mathematics fundamental activity modules" from 3rd to 9th grades, this project is implemented for advancing the learning motivation and interest of students with low-readiness in mathematics learning, which in turn may lead to the acquisition of core mathematical concepts and better learning outcomes (Lin, 2015). Mathematics MGA-teachers are trained by the professional development (PD) programs provided by the MGA-instructors and, later on, they are responsible for carrying out after-class learning activities with the designated grounding modules. Thus, it is essential to evaluate the effectiveness of these PD programs for the purpose of providing high-quality MGA-teachers.

- **PD as A Source of MGA-Teachers' Efficacy Development and its Evaluation**

Regarding with the evaluation of teacher quality, teacher efficacy has been considered both as the key indicator for examining the appropriateness and adequacy of a teacher's personal instructional readiness and as a warning of critical problems faced by a teacher PD program and an orientation for future improvement (Bandura, 1997; Cantrell, Young, & Moore, 2003; Chang, 2010; Tschannen-Moran & Woolfolk Hoy, 2001; Woolfolk Hoy & David, 2006). Bandura (1997) claims that a teacher's efficacy beliefs do have a significant influence on her/his task choices, effort, persistence, and achievement. Therefore, high efficacious mathematics teachers are willing to spend more time and efforts in preparing, designing, and implementing appropriate learning activities while students face learning obstacles or have special needs (Cantrell, Young, & Moore, 2003; Chang, 2010; Woolfolk Hoy & David, 2006). Previous research evidences reveal that the more efficacious a mathematics teacher the better her/his students' mathematics self-efficacy, and that, in turn, promotes their mathematical achievement (Chang, 2015). Similar results also showed that mathematics teachers efficacy (MTE) beliefs are powerful for their students' mathematics learning outcomes (Giles, Byrd, & Bendolph, 2016; Fung, et al., 2017; Rutherford, Long, & Farkas, 2017). Regarding the four sources of efficacy information (Bandura, 1997; Tschannen-Moran & Hoy, 2007), verbal persuasion and vicarious experiences yield major effects during PD, where the former deals with the verbal interactions with important others and the latter are those where someone else models the targeted activity (Chang, 2010, 2012, 2015; Chang & Wu, 2009). In these events, teachers can efficaciously learn from those people or activities that involve meaningful or operational interactions. Consequently, evaluating the efficacy ratings of targeted MGA-teachers will help us to clarify the effectiveness of the MGA-teacher PD programs of the "Just Do Math" Project.

- **Purpose**

Based in the motive stated above, there were two main purposes in this study, which were respectively accomplished in two stages. At the first stage (i.e. first half of this study—2016~2017 academic year), a qualitative task analysis of the MGA-teachers' PD program was conducted in order to both acquiring essential information of how this PD program was accomplished and designing the MGA-teachers' efficacy instrument for further quantitative investigations. At the second stage (i.e. second half of this study—2017~2018 academic year), the "JDM MGA-Teacher Efficacy Instrument" was employed to explore the status and possible differences of the targeted MGA-teachers' efficacy beliefs for the sake of evaluating the effectiveness of MGA-teachers' PD program.

Theoretical Framework

- **Mathematics Teacher Efficacy (MTE)**
- Teacher efficacy

Stemmed from the social cognitive theory (Bandura, 1977), perceived "Self-efficacy", defined as individual "beliefs in one's capabilities to organize and execute the courses of action required to produce given attainments" (Bandura, 1997, p. 3), affects one's level of efforts, persistence while working with difficulties or challenges, and resilience when confronting with setbacks. As to teachers, teacher efficacy (TE) has been viewed as "self-efficacy beliefs directed toward a teaching context" (Knoblauch & Woolfolk Hoy, 2008, p.167), and defined by Tschannen-Moran, Woolfolk, Hoy, and Hoy (1998) as a teacher's ''beliefs about his or her capability to organize and execute courses of action required to successfully accomplish a specific teaching task in a particular context" (p. 233) in examining the appropriateness and adequacy of a teacher's personal instructional readiness (Tschannen-Moran & Woolfolk Hoy, 2001). In fact, the significance of TE has been evidently proved and discussed in previous studies (Cantrell, Young, & Moore, 2003; Gibson & Dembo, 1984; Knoblauch & Woolfolk Hoy, 2008; Ross, 1998), as well as being identified as "one of the few teacher characteristics that reliably predicts teacher practice and student outcomes" (Graham, Harris, Fink, & MacArthur, 2001, p. 178).

High efficacious teachers tend to employ new or multiple teaching strategies based on their students' needs (Chang, 2010; Giles, Byrd, & Bendolph, 2016; Nurlu, 2015; Woolfolk Hoy & David, 2006) and establish a safe and fearless learning environment where high levels of interactions occur for motivating students' learning (Chang, 2010; Funga, et al., 2017; Nurlu, 2015). While confronting students' learning problems or difficulties, they also persist longer in discovering alternative ways of resolving possible obstacles, which lead them to pursue more external resources and opportunities of continuous professional development (Chang, 2010; Cantrell, Young, & Moore, 2003; Gersten, Chard, & Baker, 2000; Haney, Lumpe, Czerniak, & Egan, 2002; Tschannen-Moran & Woolfolk Hoy, 2001). This persistence and growth mindset is authentically essential for them to be resilient in furnishing a greater academic focus in classrooms (Gibson & Dembo, 1984). On the contrary, low efficacious teachers are likely to use teacher-centered instructional strategies, such as lecturing only and avoiding operational experiments or inquiry-based instruction (Mulholland & Wallace, 2001). Moreover, TE is evidently influential on teachers' overall effectiveness with students (Pendergast, Garvis, & Keogh, 2011), and, in fact, has a strong impact on students' performance and learning achievement (Chang, 2012; Cantrell, Young, & Moore, 2003; Pajares, Usher, & Johnson, 2007; Woolfolk & Hoy, 1990).

- MTE and its influence on teaching and learning

With regard to the subject domain of mathematics, mathematics teacher efficacy (MTE) is a major factor in mathematics education and the level of MTE a teacher brings to the mathematics classroom will affect the instructional quality students acquire. That is, MTE is a mathematics teacher's perceived ability in the context of mathematics (Pajares, 1996). Woolfolk Hoy and David (2006) indicate that teachers with high efficacious is ready to employ new teaching strategies or different kinds of learning activities in better echoing their students' needs. In Chang's (2010) study, targeted elementary mathematics teachers with higher efficacy beliefs tend to exercise various teaching strategies for improving their students' mathematical problem-solving skills, while low-efficacious mathematics teachers attribute students' lower achievement to their own ability, family background, motivation, or attitude. As Bandura (1997) argues, a teacher's efficacy beliefs do have a significant influence on her/his task choices, effort, persistence, and achievement. Therefore, high efficacious mathematics teachers are willing to spend more time and efforts in preparing, designing, and implementing appropriate learning activities while students face learning obstacles or have special needs.

In addition to the influence of teachers' teaching behaviors, findings of previous studies (Ashton & Webb, 1986; Rosenholtz, 1989) show that MTE has a great impact on students' mathematical achievement in school. Chang (2015) reveals that "the more efficacious a mathematics teacher the better her/his students' mathematics self-efficacy, and that, in turn, promotes their mathematical achievement in school" (p. 1317). In fact, fifth-grade mathematics teachers' efficacy beliefs (MTE) have a significant impact on their students' both mathematics achievement (SMA) and mathematics self-efficacy (SMSE), "where the mediative effect of SMSE on the effect of MTE on SMA is partial" (p. 1317). This finding confirms that MTE is authentically beneficial to students' mathematics learning in the classrooms. Similar results are revealed in recent studies (Giles, Byrd, & Bendolph, 2016; Fung, et al., 2017; Rutherford, Long, & Farkas, 2017) that mathematics teachers' efficacy beliefs are influential to their students' mathematics learning outcomes.

- Measuring MTE is domain-, context-, and task-specific

MTE is defined as a mathematics teacher's beliefs in their ability to teach mathematics efficiently (Enochs, Smith, & Huinker, 2000). Based on Bandura's (1978) reciprocal determinism, "psychological functioning involves a continuous reciprocal interaction between behavioral, cognitive, and environmental influences" (p. 344). Since judgments of self-efficacy are to explore the relationships among an individual's behavior, internal personal factors (in the form of cognitive, affective, and biological events), and external environment (Bandura, 1977, 1986), measuring efficacy beliefs needs to be context-specific (Tschannen-Moran & Woolfolk Hoy, 2007). Because of the limitation of "one measure fits all" of self-efficacy (Bandura, 2006, p. 307), judgments of self-efficacy beliefs are also task-specific (Bandura, 1977; Bong & Skaalvik, 2003; Pajares, 1997). Further, Giles, Byrd, and Bendolph (2016) argue that the construct of self-efficacy is regarded as "situation specific or domain sensitive resulting in the development of multiple instruments to measure the self-efficacy of pre- and in-service teachers in various domains" (p. 4). With regard to teacher efficacy, its measurement is determined based on judgments of how is a teacher's teaching performance and external behavior while facing various instructional contexts and teaching tasks (Pajares, 1996, 2006; Zimmerman, 1995); that is, items for measuring teacher efficacy beliefs are domain-specific, context-specific, and task-specific (Bandura, 2006). In addition, in order to obtain the preferred predictive and conceivably interpretive effects, items have to be constructed in a multidimensional approach (Bandura, 2006; Pajares, 1996).

- Professional development as a source of MGA-teachers' efficacy

Teacher efficacy is "considered not only as the key indicator for examining the appropriateness and adequacy of a teacher's personal instructional readiness but also as a warning of critical problems faced by a teacher education program and an orientation for future directions of its reform movement" (Chang, 2010, p. 271-272). In fact, Bandura (1986) indicates that content knowledge, teacher preparation (professional development), student achievement results, the individual's level of personal efficacy and their own level of mastery are major factors influencing teachers' efficacy beliefs. Borko and Putnam (1996) claim that teacher professional development (PD) furnishes teachers with valuable opportunities of learning new or different teaching practices, refines their understanding of content knowledge or new knowledge and teaching practices. Previous empirical evidences also show that high-quality PD is beneficial to teachers' practices (Karabenick & Conley, 2011; Wallace, 2009).

In addition, Tschannen-Moran et al. (1998) suggested that teacher efficacy is a malleable feature, affected by a teacher's teaching practices, which may change over time with adequate supports. Grounded on the four sources of efficacy information (Bandura, 1997; Tschannen-Moran & Hoy, 2007), verbal persuasion and vicarious experiences provide major effects during PD, where the former has to do with the verbal interactions with important others and the latter are those in which someone else models the designated activity (Chang, 2010, 2012, 2015; Chang & Wu, 2009). In these events, teachers can efficaciously learn from those people or activities that involve meaningful or operational interactions. Several studies also reveal that teacher efficacy can be promoted through PD, where teachers are able to acquire the knowledge from PD that may positively enhance their efficacy beliefs (Anderson et al., 1995; Ross, 1994; Watson, 2006). Besides, Rutherford, Long, & Farkas (2017) mentioned that, "how the specific content of a PD may foster teacher self-efficacy is important, so too is how teachers perceive the PD" (p. 24), as well as "how interesting and useful it is". Based on this argument, in this study, PD is considered as the main source of MGA teachers' efficacy development. Accordingly, it is essential to evaluate the targeted MGA teachers' efficacy to examine the effectiveness of the PD program for training those MGA teachers.

Research Design

- **Methodology and Participants**

A "mixed method approach" research project is conducted to reach the abovementioned goal, which is described as followings: (1) At the first stage (i.e. first half of this study—2016~2017 academic year), by employing an exploratory qualitative approach, involving the MGA-instructors in the process of task analysis to explore the domain-specific, context-specific, and task-specific information about the MGA-teachers' PD Program. (2) At the second stage (i.e. second half of this study—2017~2018 academic year), a survey method was applied, by administering the "JDM MGA-Teacher Efficacy Instrument" to collect targeted MGA-teachers' efficacy ratings, to explore the status and possible differences of the targeted MGA-teachers' efficacy beliefs for the purpose of evaluating the effectiveness of the MGA-teachers' PD Program.

Based on this design, all MGA-teachers who attended the MGA-teachers' PD program (i.e. completing "2-day PD activities" and getting certified) during the first stage were the main population of this study. Consent forms of being willing to participate in this evaluation process were given at the beginning of every "2-day PD activities" after a brief illustration of the core role of this PD evaluation project. Afterwards, their consent forms were gathered at the end of the PD program; 568 copies of consent forms were received totally. Later on, the instrument was sent out at the beginning of 2017~2018 academic year to MGA-teachers who agreed to participated in this evaluation process, and 431 copies were returned (return rate around 76%). By eliminating 23 copies with incomplete background information or unfinished instrument, 408 MGA-teachers were the final samples, where their characteristics were shown in table 1.

Table 1 Characteristics of teacher samples (*N*=408)

Variable	Category and Frequency (%)			
Gender	Male 107 (26.2%)		Female 301 (73.8%)	
Age	30 or less 61 (15.0/%)	31 to 40 153 (37.5%)	41 to 50 158 (38.7%)	51 & up 36 (8.8%)
Years of Teaching	10 or less 136 (33.3%)	11 to 20 193 (47.3%)	21 & up 79 (19.4%)	
Expertise	Math-related 152 (37.3%)		Non-Math 256 (62.7%)	

- **Data Collection and Analysis**

Based on the main purposes, there were two stages during the whole process of data collection and analysis. At the first stage, a qualitative task analysis of the MGA-teachers PD program was conducted for both obtaining fundamental information of how this PD program was executed and constructing the MGA-teachers' efficacy instrument for further quantitative investigations. At the second stage, the "JDM MGA-Teacher Efficacy Instrument" was employed to examine the status and possible differences of the targeted MGA-teachers' efficacy beliefs for the purpose of evaluating the effectiveness of MGA-teachers PD program.

- Task analysis of MGA-teachers' PD program

In order to design the "*Just Do Math MGA-Teachers Efficacy Instrument*" for the quantitative investigations, an exploratory qualitative approach (Creswell, 2014) was employed for gathering data of task analysis to explore the domain-, context-, and task-specific information about the "Just Do Math Project MGA-Teachers' Professional Development Program" at the first half of this study. The PD program designers (i.e. teacher educators) and its executors (i.e. MGA-designers and MGA-instructors) were purposefully selected as main participants during this qualitative data collection process. Data were gathered through observations, interviews, and related documents and then categorized and pre-analyzed by five steps (Thomas, 2000): preparation of raw data files, closed reading of text, creation of categories, overlapping coding and uncoded text, continuing revision and refinement of category system. Both editing and immersion analytic techniques were then employed for further analyses (Crabtree & Miller, 1999). Through applying the organizing code topics (i.e. related to JDM and MGA) mentioned above, the editing analytic system focused on MGA-teachers' professional development in the whole learning process. Because of the exploratory character of this study, the immersion analytic system was used to explore essential task information of the targeted PD programs, where the cycle of immersion was repeated until the described interpretation was approached (Crabtree & Miller, 1999).

- Design and Verification of "JDM MGA-Teachers Efficacy Instrument"

1. Instrument design with domain-, context-, and task-specific principles

Regarding teacher efficacy, its measurement is determined based on judgments of how is a teacher's teaching performance and external behavior while facing various instructional contexts and teaching tasks (Pajares, 1996, 2006; Zimmerman, 1995). Grounded on this argument, for MGA-teachers, items for measuring their efficacy beliefs need to be domain-specific, context-specific, and task-specific. Since the "2-day PD activities" was the main source of these MGA teachers' efficacy development, the design principles of the "JDM MGA-Teacher Efficacy Instrument" were characterized as followings: (a) domain specific to—the mathematical content of all MGA modules that were included in the targeted "2-day PD activities"; (b) context specific to—the teaching and learning environment and its content during the targeted "2-day PD activities" (i.e. how MGA-instructors taught and how MGA-teachers learned); (c) task specific to—the main role and task of MGA-teachers. Grounded on the findings of the task analysis of MGA-teachers' PD program (see findings for details), the context- and task-specific information were generalized from the two parts of activities, "rationale of the MGA activities" and "introduction to MGA activities". Moreover, in order to obtain superiorly predictive and possibly explanatory findings (Bandura, 2006; Pajares, 1996; Zimmerman, 2000), a multidimensional approach should be employed while constructing the instrument. In fact, the concept of self-efficacy consists of two kinds of expectation: efficacy expectation and outcome expectancy (Bandura, 1981). Applying this construct to the subject of mathematics, two cognitive dimensions, i.e. "personal mathematics teaching efficacy (PMTE)" and "mathematics teaching outcome expectancy (MTOE)", were included in a previously adapted "Elementary Mathematics Teacher Efficacy Instrument (EMTEI, traditional Chinese version)" (Chang, 2015). Consequently, the framework of the "JDM MGA-Teacher Efficacy Instrument" continuously used these two cognitive dimensions, as well as contained the context- and task-specific information of the targeted "2-day PD activities". Besides, background information was also collected for later analyses, which included two categories of items: (a) Five "MGA-related" factors were closely relevant to the MGA PD activities or fun-math camps—attending the previous (1st) phase PD before (2015~2016 academic year and before, yes or no), attending how many "2-day PD activities" at the this (2nd) phase PD (2016~2018 academic years, yes or no), hosting fun-math camp before (yes or no), willingness of hosting fun-math camp again (yes or no), and willingness of attending PD again (yes or no). (b) The rest six "personal" factors were these teachers' personal backgrounds or experiences—gender, age, years of teaching experience, expertise (is "Math-related background" or not), area receiving MGA PD (i.e. northern, central, southern, and eastern part of Taiwan), participating in teacher professional development and evaluation program before (yes or no).

(1) Personal mathematics teaching efficacy (PMTE)

This dimension, including two subscales, aimed to examine MGA-teachers' beliefs in their ability to effectively teach MGA modules while hosting fun-math camps.

[Subscale I—Rationale & Theory (PMTET)]

Items were composed of the following four elements to examine MGA-teachers' perception of the rationale of JDM project, the core tasks of JDM project, the design principles of MGA modules, and the theory used and its fundamental content while designing MGA modules; 14 items totally (see table 2).

[Subscale II—Goal Setting, Instructional Design, & Assessment (PMTEG)]

Items were composed of the following four elements to examine MGA-teachers' perception of the learning objectives, learning situations, mathematical concepts, and the focal points and strategies of the instructional design and implementation of the designated MGA modules; 13 items totally.

(2) Mathematics teaching outcome expectancy (MTOE)

This dimension intended to examine MGA-teachers' beliefs in their ability to effectively influence their students' learning by using

MGA modules within the fun-math camps. Items related to how a MGA-teacher's thought and action while implementing MGA modules in fun-math camps were included in this dimension; 12 items totally.

There were 39 items totally, where two negative-stated items were imbedded in PMTEG and MTOE (see figure 1 for sample items of each dimension/subscale). Based on the recommendation of Bandura's (2006) opinion of the response scale, a 100-point scale was employed to rate the targeted MGA-teachers' "degree of confidence" to each item statement in this instrument. For example, "0" means no (0%) confidence or cannot do it at all; "50" means 50% confidence or moderately certain can do it; "100" means 100% confidence or highly certain can do it. This 100-point response scale could provide preferred predictive and conceivably interpretive effects (Pajares, Hartley, & Valiante, 2001). Actually, this 100-point scale is suitable for Taiwanese educational context since teachers employed this 100-point system in most assessment tools (e.g. all kinds of examinations), where students were also used to be graded in their learning experiences. That is, MGA-teachers' responses on each item by using this scale would obtain, comparatively to 5-Likert scale, more authentic reactions reflected to what they learned from the targeted "2-day PD activities".

Table 2. Dimensions and subscales and items of the instrument

Dimensions & Subscales		Items	Total
Personal Mathematics Teaching Efficacy (PMTE)	Rationale & Theory	14	27
	Goal Setting, Instructional Design, & Assessment	13	
Mathematics Teaching Outcome Expectancy (MTOE)		12	12
Total			39

Note: Rated on a 100-point scale (i.e. 0/10/20/….50/…90/100)

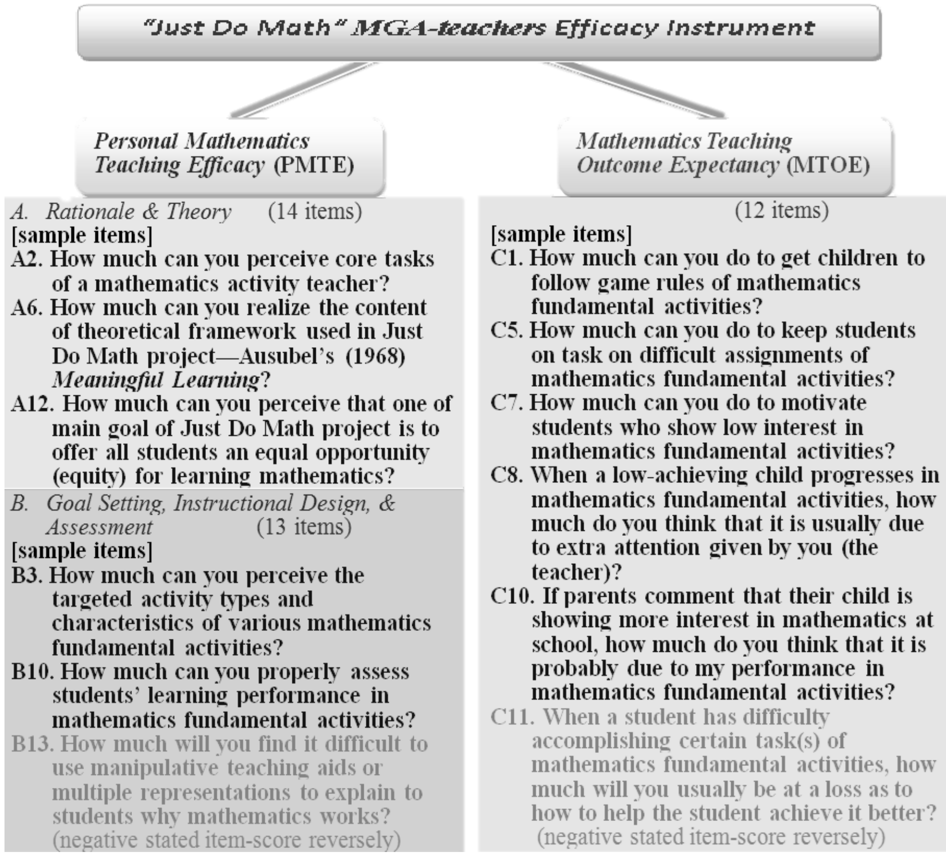

Figure 1. Structure and sample items of the instrument

2. Reliability and validity of the instrument

Within the "JDM MGA-Teachers Efficacy Instrument", 39 items of two dimensions (with three subscales) were included to contextually assess targeted MGA-teachers' efficacy belief, i.e. Personal Mathematics Teaching Efficacy—Rationale & Theory (PMTET), Personal Mathematics Teaching Efficacy—Goal Setting, Instructional Design, Assessment (PMTEG), and Mathematics Teaching Outcome Expectancy (MTOE). Findings of the reliability analyses showed that there were high internal consistency for all subscales and the whole scale (Cronbach's α = .95 for the total scale, see table 3 for details). In this study, PMTET, PMTEG, and MTOE accounted for 33.62%, 32.39%, and 29.25% of variance, respectively. Two dimensions and all three subscales were significantly correlated, r = .53 ~ .75, p< .001.

Table 3. Analyses of reliability and validity

Factor (Subscale)		Item #	Variance	Cronbach's α	
Personal Mathematics Teaching Efficacy (PMTE)	A. (PMTET) Rationale & Theory	1~14	33.62%	.93	.89
	B. (PMTEG) Goal Setting, Instructional	15~27	32.39%	.88	

	Design, Assessment				
Mathematics Teaching Outcome Expectancy (MTOE)		28~39	29.25%	.90	
Total (Whole Scale)		39	95.26%	.95	

Findings

Based on the purposes and the research design, findings were reported in two parts, which were drawn from both the qualitative task analysis and the quantitative survey study: First, there is a brief introduction of the MGA-teachers' PD training program. Secondly, the quantitative findings of the current status and differences of the targeted MGA-teachers' efficacy belief were presented along with the qualitative task analysis.

- **Task Analysis of MGA-Teachers' PD Program—A Brief Introduction**

As briefly mentioned in the introduction, certified elementary teachers and middle school mathematics teachers, including mathematics teachers of the national/central or local/regional advisory group, teachers in those schools of the Wow Math Light-up Program, and all other teachers who are interested in the JDM project, are invited to sign up any training activities of the MGA-teachers PD program that were held in various regions and different dates (usually in the weekends or summer/winter vacation). After completing the PD program, these teachers are certified as qualified MGA-teachers who will be able to host "fun-math camps" in their own or neighboring schools as well as apply those MGA activities to their own classrooms or serve as outreach teachers (see figure 2). The MGA-teacher's certificate is valid for three years. If one's certificate is expired, s/he has to sign up for another training activity again to renew her/his certificate. The periodic design of this three-year training certification is to ensure these MGA-teachers' continuous professional development for quality purposes and further improvements (Lin, 2015).

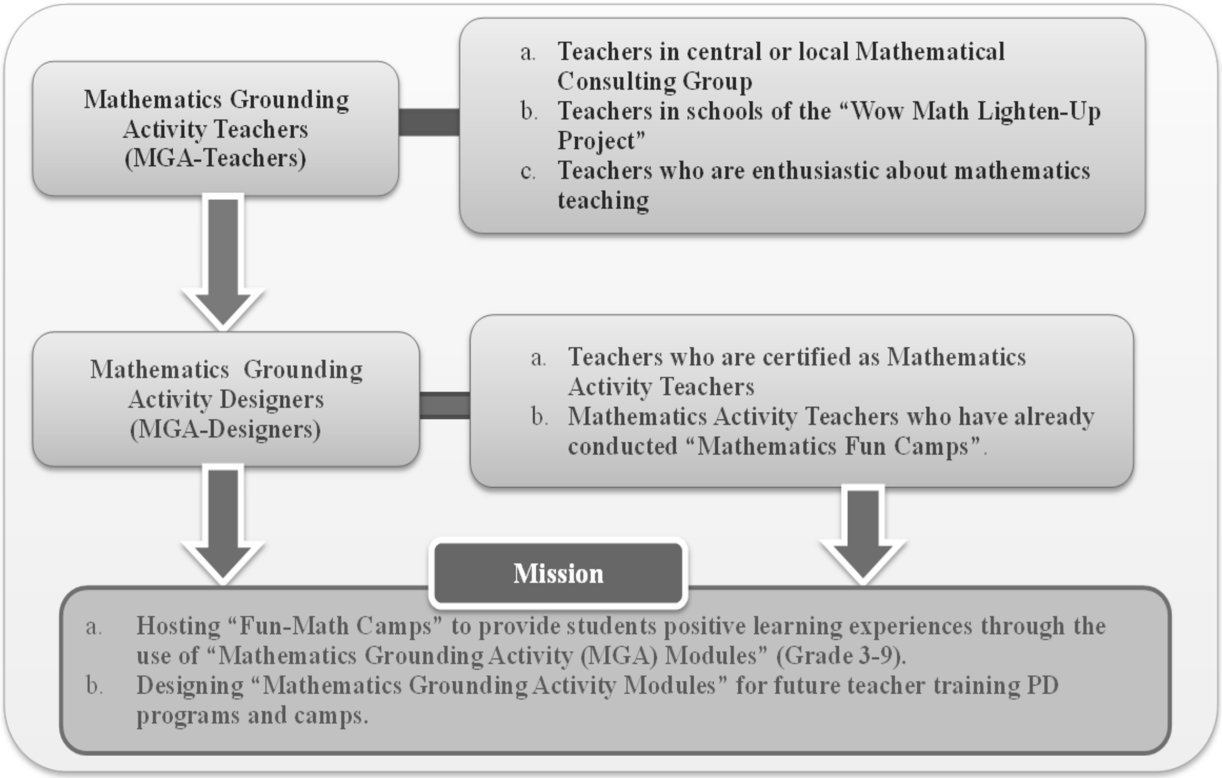

Figure 2. JDM PD program design and its tasks

The two-day MGA-teachers' PD activity comprises two main parts: "rationale of the MGA activities" and "introduction to MGA activities" (an example is shown in figure 3). The first PD class of each two-day PD activity is "rationale of the MGA activities", which is taught by core teacher educators of the JDM project or senior mathematics teachers of the national/central advisory group who closely participate in the JDM project. In this 1.5-hour session, the fundamental rationale of the whole JDM project and the theoretical frameworks and design principles of the MGA activities are briefly introduced, where it will help future MGA-teachers to understand how the JDM project is planned and the MGA activities are designed.

With regard to the second part of the PD activity, five 1.5-hour sessions of "introduction to MGA activities" are scheduled, where two or three MGA activities are presented by MGA-instructors, who are those MGA activities' designers. In fact, these MGA designers are mathematics teachers of the national/central or local/regional advisory group, outstanding mathematics teachers recommended by teacher educators, or experienced MGA-teachers who have hosted fun-math camps. Their responsibility is to demonstrate how these MGA activities are designed, explain the content and instructional strategies, and lead those future MGA-teachers to implement these hands-on modules through actual operations. Two reasons for employing elementary or middle school teachers, they are able to apply "empathy" to lead future MGA-teachers to learn how to implement MGA activities during the whole training process, which can both avoid the possible gap of theories and practices and promote targeted teachers' identification with these MGA modules. (2) Through the use of the interactive model, future MGA-teachers are able to authentically understand the core concept of design principles and operational processes, which may, in turn, result in better teaching and learning outcomes while these activities are implemented in the future. In addition, all participated teachers are divided into three groups based on their backgrounds and choices in these sessions, i.e. middle grades of elementary school (3rd and 4th grades), higher grades of elementary school (5th and 6th grade), and middle school (7th to 9th grades), where they receive the training of diverse MGA activities corresponding to targeted students' grade levels or developmental needs. The design principles of MGA modules and sample modules are introduced in Part II (Ch. 4, 6, 7, 8). In short, after completing the two-day training sessions, these trainees will be certified as MGA-teachers.

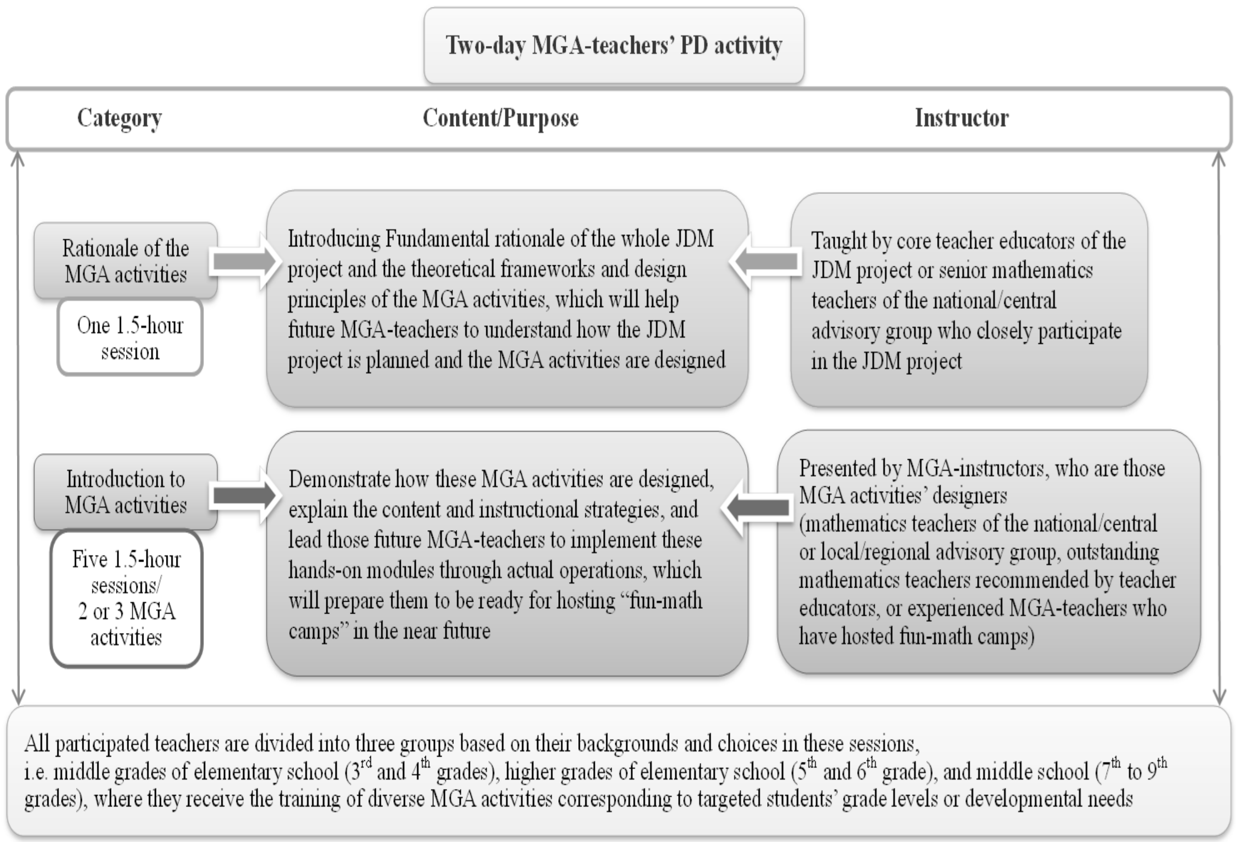

Figure 3. An example of a two-day MGA-teachers' PD activity

- **Current Status and Differences of Targeted MGA-teacher s' Efficacy Belief**

- Descriptive analyses of targeted MGA-teacher s' efficacy belief

First, the mean rating of all 408 MGA-teachers' efficacy belief on the overall scale (i.e. total score of the instrument on a 100-point scale) was 73.48 (*SD*=10.68), which meant that on average they had approximately 73.48% confidence in their own mathematics teaching capabilities regarding to the teaching and learning through MGA modules in fun-math camps in the future (see table 4). The lowest score and largest variance appeared in the subscale PMTET (*M*= 70.43, *SD*=14.17), referring to the targeted MGA-teachers' perception of the rationale and core tasks of JDM project, the design principles of MGA modules, and the theory used and its fundamental content while designing MGA modules, which might be a potential issue for future discussion.

Table 4. MGA-teachers' efficacy status

Subscale	*M*	*SD*	*SE*	*Min.*	*Max.*
PMTET	70.43	14.17	.70	25.00	97.86
PMTEG	75.46	11.71	.58	30.77	100.00
PMTE	72.85	11.38	.56	27.78	98.89
MTOE	74.89	11.13	.55	24.17	100.00
TOTAL	73.48	10.68	.53	26.67	99.23

Based on this descriptive analysis, the average efficacy rating of all targeted MGA-teachers ranged from 70.43 to 75.48, which represented around 70.43% to 75.48% of confidence in their future teaching and possible influential effects to their students' learning through the use of MGA modules in fun-math camps. Corresponding to the findings of the qualitative task analysis of MGA-teachers' PD program, two potential problems were discovered that might echo with these quantitative findings (a bit above 70% of confidence), which were just at an acceptable efficacy level but could be better if possible:

1. Unclear and fragmented delivery of rationale and theoretical framework

The first potential problem refers to unclear and fragmented delivery of rationale and theoretical framework at the first joint training session—rationale of the MGA activities. In fact, grounded on the fundamental scheme of the JDM project, "grounding" and "activity/play-based" are the two core concepts, while six learning theories correspondingly serve as the main theoretical frameworks (see Figure 4 for brief illustrations; also see Part I for details). By applying these core concepts and theoretical frameworks, MGA activities are developed corresponding to targeted students' grade levels (i.e. from 3rd to 9th grades) or based on their developmental needs or possible problems characterized in previous studies. Through implementing these MGA activities, it hopes that students with low learning readiness, interest, or achievement will be motivated to learn mathematics actively; that is, they are willing to learn mathematics because their learning interests are raised. Moreover, they are able to establish preliminary understandings of the designated core mathematical concepts, and, in turn, may help to enhance their official learning in the classrooms.

Learning Theory	Brief Content of Each Theory of "Mathematical Play"
Bruner's (1966) "discovery learning theory"	Discovery Learning is a learning method that encourages students to ask questions and formulate their own tentative answers, and to deduce general principles from practical examples or experiences. In his theory of the stages of cognition: As cognitive growth occurs, students move through three stages of learning: enactive, iconic, and symbolic; where students are able to use mathematical symbols to present mathematical structure or concept they learn eventually.
Ausubel's (1968) "meaningful learning theory"	"Meaningful learning theory" is involved with how individual learn large amounts of "meaningful" material from verbal/texttual lessons in school. Meaningful learning requires the learner to integrate new concepts and propositions with her/his existing concepts and propositions, which is under three conditions: prior knowledge (pre-requisite), learning materials (including language, examples, and strategies), and emotional commitment (motivation to learn meaningfully).
Dienes' (1973) "principles and stages of mathematics teaching and learning"	Dienes believed that the understanding or non-understanding of a mathematical idea depends on the method of communication used by the teacher to transmit the mathematical idea to the student. He refined his four principles by identifying six stages of teaching and learning mathematical concepts. He asserted that mathematical concepts are learned in progressive stages: free play, games, searching for communalities, representation, symbolization, and formalization (p. 6-9). His four principles: Dynamic, Constructivity, Perceptual Variability, and Mathematical Variability Principles.
Skemp's (1989) "two-level model of intelligent learning"	Intelligent learning involves building networks of "schemas" that enable us to achieve our goals. Our schemas are cognitive or intellectual structures that represent the relationships that we have become aware of between concepts and processes, at one level, and between selected schemas, themselves, at another. In *Mathematics in the Primary School* (p. 32-48), he likened the process of schema construction whereby humans build their concepts and processes to that of building a brick wall while periodically testing its alignment. He described intellectual schema construction in terms of building and testing.
Pirie and Kieren's (1994) "mathematical understanding as a dynamic phenomenon"	They propose a theory of the growth of mathematical understanding as a whole, dynamic, leveled but non-linear, transcendently recursive process. This theory attempts to elaborate in detail the constructivist definition of understanding as a continuing process of organizing one's knowledge structures. The process of coming to know starts at a level called "primitive doing", and extends to the levels of "image making, image having, property noticing, formalizing, observing, structuring, and inventing".
Van den Heurrel-Panhuizen's (2000) "realistic mathematics education (RME)"	The features of RME are to use of realistic situations to develop mathematics and well researched activities encourage pupils to move from informal to formal representations. RME involves a number of core principles for teaching mathematics which are inalienably connected to RME. There are six core teaching principles: activity principle, reality principle, reality principle, intertwinement principle, interactivity principle, and guidance principle, which may help students' mathematical learning.

Figure 4. Learning theories of "mathematical play" of the JDM project

However, based on the findings of the qualitative task analysis of the PD program, the first 1.5-hour "rationale of the MGA activities" session in every "2-day PD activity" was taught by various instructors (i.e. core MTEs of the JDM project or senior mathematics teachers of the national/central advisory group who closely participate in the JDM project). Even though the main purpose of this session was to furnish future MGA-teachers with the fundamental rationale of the whole JDM project and the theoretical frameworks and design principles of the MGA activities, every instructor had her/his own teaching style and provided different kinds of PD content. For instance, one instructor initiated this session with Taiwanese students' low mathematical learning interests (based on the findings of PISA and TIMSS), discussed students' potential problems while learning mathematics, and then presented where and how the MGA activities originated from (e.g. design principles of these activities) (OB-102216PD2). Another instructor directly introduced how a MGA activity was designed (by using an example) associated with life-related context, as well as emphasizing the significance of using learning sheets/worksheets that were composed of certain steps (e.g. providing necessary guidance, asking questions, discussing, integrating/reviewing). In fact, only one instructor partially mentioned about Ausubel's (1968) "meaningful learning theory"; instead, less the core rationale of the JDM project and the theoretical framework (e.g. other five learning theories of "mathematical play") were included in their presentations (RR-102216PD2). This discrepancy (i.e. content of every instructor) and fragmental introduction may lead these future MGA-teachers to not fully comprehend the origin and core spirit of the JDM project, such as: "why 'grounding'?", or "what grounds (i.e. fundamental concepts) these MGA activities are going to build?". This defect may also, in turn, result in later confusion of how the MGA activities are designed and how to implement them in "fun-math camps". Especially for those who first contact with the JDM and the MGA activities, lack of authentic understanding of the core rationale and the theoretical framework may jeopardize the possibility of successfully implementing MGA activities in the future.

2. Inconsistency on various teaching approaches of MGA activities

Similar conditions were found in the second sessions—"introduction to MGA activities". The MGA activities included in every two-day PD activity were different, which was based on the availability of the instructors who were able to go to specific regions' PD activity or had been trained as qualifies MGA instructors for teaching particular MGA activities. Therefore, the same MGA activity might be presented by different instructors. In these five 1.5-hour sessions, averagely 12 MGA activities were introduced to each group of future MAG-teachers (i.e. elementary middle grades, elementary higher grades, and middle school). Grounded on this various differences of instructors and activities, two teaching styles were generalized from the qualitative task analysis:

(1) "One-way lecture" instructional activity: Two or three MGA activities were arranged in one 1.5-hour session, depending on the length of the activity that was originally designed. In this style, MGA instructors usually lectured on the design principles as well as the content of the targeted MGA activities. There were two special cases, where two senior mathematics teachers of the national/central advisory group (who was also an experienced MGA instructor that took charge of several activities' training) respectively taught two activities in each session. One instructor lectured to these future MGA-teachers about 80 minutes on the objectives, core mathematical concepts, module structures, and corresponding mathematical items (i.e. word problems and the patterns of these items), as well as when to teach (i.e. appropriate grade levels and timing for teaching them) (OB-100216PD1-T1). Since only around 10 minutes left in this session, there was less opportunity for these

future MGA-teachers to actually practice the whole process of the activity or operate its teaching aids. Another instructor spent more than 80 minutes on lecturing the mathematical concepts (i.e. learning fractions for elementary students—2nd to 6th grades), which echoed to the content of the two MGA activities (OB-100216PD2-T4). Even though it might help to furnish these future MGA-teachers with specific professional content knowledge and perceive possible misconceptions that their students might have while learning fractions, they comparatively lost essential chances of understanding and practicing the two MGA activities. Besides, while training, it is also a beneficial opportunity for these MGA instructors to collect feedbacks from these future MGA-teachers while training them and explore possible problems on implementing these MGA activities in future fun-math camps. However, in these two cases, this "one-way lecture" training style emphasized unidirectional content delivery instead of giving sufficient "hands-on/learning by doing" opportunities for future MGA-teachers to practice how to implement these MGA activities. Within all sessions of "introduction to MGA activities" during 2016-2017 academic year we observed, around 30% of MGA instructors belonged to this one-way lecture style. Under this deficient circumstance, it was unfavorable for these future MGA-teachers to develop their efficacy beliefs.

(2) "Two-way communication or hands-on experience" instructional style: Comparative to the one-way style, around 70% of MGA instructors employed this instructional style, which echoed to the "parallel and interactive PD model" addressed by prof. Lin (Lin, 2015). Through practical operations and hands-on teaching and learning activities, the targeted future MGA-teachers engaged in understanding the designated content, teaching strategies, and teaching aids of every MGA activity, where they learned from the following steps: "initiating" (providing specific life-related situation to motivate students' learning, e.g. story, magic, game), "experiencing" (actually engaging in the MGA activity), "exploring" (exploring the hidden mathematical concept within the MGA activity—not introducing the concept at the beginning steps), "presenting" (the instructor present the mathematical concept used in the MGA activity), and "enhancing" (using learning sheets to practice and review the designated content/concept). In these steps, various types of "two-way communication" occurred, e.g. questioning, guiding, discussing, and sharing. A lot of "hands-on" activities were provided for future MGA-teachers to practice "how to teach and learn" (e.g. they exchanged roles during the process) and operate the teaching aids personally, which truly motivated these teachers to have more positive interactions (OB-121716PD3). Within this interactive processes, they were able to learn by actually doing/experiencing the designated mathematical concept, which, in turn, not only helped them to get better understandings of the targeted MGA activity but also promoted them to discuss and reflect while learning (RR-121716PD3). There was one special case that an instructor employed a magic "poker" game as the initiative action; but, she failed 3 times. However, this failure accidentally involved all teachers to discuss why she was failed and inferred some possible mistakes during the action (OB-100216PD1). In fact, this kind of interactions did help these future MGA–teachers to master how to manage the initiative action while implementing this MGA activity. This mastery experience obtained through the two-way interactive "learning by doing" process was definitely beneficial for their efficacy development.

No matter which potential problem is, this unclear and fragmented delivery of rationale and theoretical framework or inconsistency on various teaching approaches of MGA activities might be a conceivable reason that diminished the possibility of positive development of these MGA-teachers' efficacy belief. On the contrary, the employment of the "two-way communication or hands-on experience" instructional style in the second sessions—"introduction to MGA activities", which conforms to the original design principle of the MGA PD program (i.e. the parallel and interactive PD model), may furnish these

MGA-teachers with more both mastery and vicarious experiences for future implementation of MGA modules in fun-math camps or even in their own classrooms.

- Background analyses of differences of targeted MGA-teachers' efficacy belief

The targeted MGA-teachers' background information (i.e. 11 background factors) was gathered for analyzing the effects on their efficacy belief. Five "MGA-related" factors were reported first since they were closely relevant to the MGA PD activities or fun-math camps, which were correspondingly connected to the effectiveness of the "2-day PD activities".

1. Five "MGA-related" factors

For the first factor "attending the previous (1^{st}) phase PD before (i.e. 2015~2016 academic year and before)", the results showed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) between "yes (attending 1^{st} PD before, M=75.21, N=266)" and "no (not attending 1^{st} PD before, M=70.24, N=142), t (406) = 4.58, p< .001; significant differences were also found on the two dimensions (PMTE and MTOE) and the two subscales of PMTE.

For the second factor "attending how many '2-day PD activities' at the this (2^{nd}) phase PD (2016~2018 academic years)", the findings revealed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) among the three responses of this factor, F (2, 405) = 8.55, p< .001; significant differences were also found on the two dimensions (PMTE and MTOE) and the two subscales of PMTE. The post hoc comparison based on Scheffẻ concluded that MGA-teachers who attended "3 or more times '2-day PD activities' in 2^{nd} phase (M=78.37, N=282)" scored significantly superior in their efficacy belief (Total score) than did those with "twice (M=76.05, N=97)" and "only once (M=72.09, N=29)"; no significant differences were found between the two groups with attendance of twice and only once.

For the third factor "hosting fun-math camp(s) before", the findings indicated that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) between "yes (hosting it before, M=76.57, N=186)" and "no (not hosting it before, M=70.89, N=222), t (406) = 2.95, p< .01; significant differences were also found on the two dimensions (PMTE and MTOE) and the two subscales of PMTE.

For the fourth factor "willingness of attending PD again", the results revealed that there were no significant differences in MGA-teachers' efficacy ratings (Total score) between "yes (willing to attend PD again, M=73.64, N=396)" and "no (not willing to attend PD again, M=68.07, N=12), t (406) = 1.78, p> .05; no significant differences were found in PMTE, MTOE, and PMTET. However, significant differences were found in the subscales PMTEG, t (406) = 2.25, p< .05. In fact, there were only 12 MGA-teachers who did not want to attend PD again. In fact, since items of PMTEG were closely related to the design and implementation of MGA modules, this finding is meaningful for the effectiveness of the second part of "2-day PD activities"—"introduction to MGA activities". Besides, even though no significance was found in the whole scale, PMTE, MTOE, and PMTET, MGA-teachers who were willing to attend PD again had higher average mean scores than those who were not.

For the fifth factor "willingness of hosting fun-math camp again", the findings showed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) between "yes (willing to host fun-math camp again, M=74.87, N=290)" and "no (not willing to host fun-math camp again, M=70.07, N=118), t (406) = 4.20, p< .001; significant differences were also found on the two dimensions (PMTE and MTOE) and the two subscales of PMTE.

2. Six "personal" factors

The rest six "personal" factors were these MGA-teachers' personal backgrounds or previous relevant experiences of professional development. The findings were reported as followings: For the factor "area receiving MGA PD (i.e. northern, central, southern, and eastern parts of Taiwan)", the findings indicated that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) among the four areas, F (3, 404) = 3.06, p< .05; significant differences were also found on the dimensions PMTE and its subscale PMTEG. No significance was found in all post hoc comparisons. However, slight differences did exist, where MGA-teachers who attended PDs at the central and southern parts of Taiwan (i.e. usually they were in-service teachers in surrounding regions) scored a bit higher than those who at the northern and eastern areas.

For the factor "expertise (is "Math-related background" or not)", the results showed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) between "math-related (M=75.35, N=152)" and "non-math-related (M=72.37, N=256), t (406) = 2.75, p< .01; significant differences were also found on the dimension PMTE and its subscale PMTEG.

For the factor "participating in teacher professional development and evaluation program before (yes or no), the results showed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) between "yes (attending before, M=74.38, N=249)" and "no (not attending before, M=72.06, N=159), t (406) = 2.15, p< .05; significant differences were also found on the dimension MTOE and the subscale PMTEG. The teacher professional development and evaluation program was officially promoted by the Ministry of Education since 2009, which encouraged elementary and secondary teachers to participate in various types of PD to advance their teaching capabilities. Therefore, participating this PD before is also influential to their willingness of attending more MGA PDs now and in the future.

For the factor "gender", the results showed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) between "male (M=75.74, N=107)" and "female (M=72.67, N=301), t (406) = 2.57, p< .05; significant differences were also found on the dimension PMTE and its subscales of PMTET and PMTEG. Moreover, for the factor "age", the results showed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) among the four age groups, F (3, 404) = 4.99, p< .01. The post hoc comparison based on Scheffẻ concluded that MGA-teachers who were "41 to 50 years old (M=75.73, N=158)" scored significantly superior in their efficacy belief (Total score) than did those with "30 years old or less (M=70.01, N=61)"; no significant differences were found among other group comparisons. However, MGA-teachers of the two groups "31 to 40 years old" and "51 years old or more" scored a bit higher than those of the youngest group "30 years old or less" who had less teaching experience. Besides, similar significant differences were found on the two dimensions (PMTE and MTOE) and the subscale PMTEG.

For the factor "years of teaching experience", the results showed that there were statistically significant differences in MGA-teachers' efficacy ratings (Total score) among the three groups, F (2, 405) = 5.42, p< .01. The post hoc comparison based on Scheffẻ concluded that MGA-teachers who were "11 to 20 years (M=74.59, N=193)" and "21 years or more (M=74.94, N=79)" scored significantly superior in their efficacy belief (Total score) than did those with "10 years or less (M=71.05, N=136)"; no significant differences were found between the two groups of "11 to 20 years" and "21 years or more". Besides, similar significant differences were found on the two dimensions (PMTE and MTOE) and the subscale PMTEG.

Discussion and Implication

Based on the findings of the qualitative task analysis, certain differences practically existed where various teaching approaches were employed by different MGA-instructors associated with a variety of content were provided in every session of the "2-day PD activity". These differences might cause some potential problems; that is, issues in either "unclear and fragmented delivery of rationale and theoretical framework" or "inconsistency on various teaching approaches of MGA activities" might become conceivably factors that might decrease the possibility of positive development of these MGA-teachers' efficacy belief as well as their future teaching performance in fun-math camps. Two major potential problems may cause from these negative factors that is unfavorable for MGA-teachers' efficacy development; on the contrary, if the issues of unclearness, fragmentation, or inconsistency can be controlled, the PD program will be practically beneficial for these teachers' efficacy development. Here are the two-prong discussions grounded on the findings and the four sources of efficacy information (Bandura, 1997), where concrete recommendations are consequently proposed for future improvement.

- **Effects of Mastery Experiences and Physiological Arousal for Developing MGA-Teachers' Efficacy**

Four sources of efficacy information that would influence the development of teacher efficacy (Bandura, 1997): mastery experiences, verbal persuasion, vicarious experiences, and physiological arousal. Mastery experiences were validated as the most powerful source of efficacy information, which were evidently drawn from one's practical teaching performance attainments with students (Bandura, 1997; Tschannen-Moran & Woolfolk Hoy, 2007). Regarding to mathematical teaching and learning, these successful teaching accomplishments "developed a positive and robust belief in a teacher's efficacy, which then contributed to the expectations of future proficient performance" (Chang, 2010, p. 274). Based on this argument, the more successful mastery experiences a MGA-teacher has the more efficacious s/he is, and that, in turn, advances their teaching performance in fun-math camps or their own mathematics classrooms. Ross and Bruce (2007) claim that PD activities often provide numerous opportunities to enhance mathematics teachers' teaching capabilities. A better PD program may help participating teachers to effectively learn from PD activities and, later on, apply what they learn to the practical situations, which may improve the quality of teaching and learning in their own classrooms. Once they have successful teaching experiences in practical settings, their efficacy belief will be promoted correspondingly and cyclically (Bandura, 1997). In fact, this kind of mastery experiences drawn from the raising "internal factor (teacher profession—teacher's specialty and perspective)" has a powerful impact on the development of mathematics teachers' efficacy (Chang, 2010). In Watson's (2006) study, operational and interactive PD activities are provided in an intensive PD camp to furnish mathematics teachers with technology-based professional knowledge and skills, which intend to promote the targeted teachers' efficacy. Six years after receiving this PD, their efficacy ratings stay at similar levels as six years before (i.e. right after the PD camp). Similarly, in Ingvarson, Meiers, and Beavis's (2005) study, plenty of "understanding and experience" opportunities to learn, including topics of instructional content, active learning, supportive resources, teaching feedback, and collaborative assessment and evaluation, are offered to in-service teachers in PD activities. These enrich learning activities not only authentically assist the targeted teachers to provide more active learning opportunities to their students in the classrooms but also positively promote their own efficacy belief. In short, these empirical evidences show that offering teachers interactive and hand-on learning experiences in the PD may authentically improve their future teaching performance and then positively increase their efficacy belief.

According to the findings, it was found that the targeted MGA-teachers' efficacy belief is averagely at an acceptable level (i.e. around 70-75%). It was also found that the experience and willingness of attending MGA PD activities and hosting fun-math camps had a significant influence on the positive development of their efficacy belief. These results inform us that furnishing MGA-teachers with literally operational and interactive learning experiences in the PD process and then encouraging them to practically exercise what they learn in real settings (e.g. fun-math camps) are able to advance their efficacy belief, which is essential for cyclically promoting their teaching performance in the future (Bandura, 1997; Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998). In addition, positively affective feelings, derived from both efficacious learning experiences obtained in the PD process and successful mastery experience attained in executing fun-math camps, are a powerful source for MGA-teachers' efficacy development. As Bandura (1977) argues, another source on which a teacher depends is physiological arousal that is, "relevant in domains that involve physical accomplishments, health functioning, and coping with stressors" (p. 106). These emotional feelings of joyfulness or pleasure MGA-teachers perceived both from a PD experience and a successful teaching accomplishment may also elevate their efficacy (Bandura, 1997; Chang, 2010; Tschannen-Moran & Woolfolk Hoy, 2007). In fact, around 70% of MGA-instructors employed the "two-way communication or hands-on experience" instructional style, which provided numerous practical operations and hands-on teaching and learning activities to engage MGA-teachers in the PD process. Consistently using the "parallel and interactive PD model" (Lin, 2015) in every PD session will be beneficial to provide sufficient mastery experiences and positive physiological arousal for future development of MGA-Teachers' efficacy.

However, the inconsistency on various teaching approaches of MGA activities leads to the first potential problem, which is the lack of operational and interactive opportunities within those 30% sessions in the "introduction to MGA activities" with the "one-way lecture" instructional approach. In fact, the "2-day PD activities" were the only source that MGA-teachers receive all essential information about the design and implementation of MGA modules before they personally host a fun-math camp. If they do not have any chance to personally experience this teaching and learning process of "doing mathematics", they will not substantially obtain successful master experiences during the PD training period, which may possibly jeopardize the chance to effectively implement those MGA modules in either future fun-math camps or their classrooms.

- **Effects of Vicarious Experiences and Verbal Persuasion for Developing MGA-Teachers' Efficacy**

Bandura (1997) indicated that efficacy estimates are "partly influenced by vicarious experiences mediated through modeled attainments" (p. 86). Garet at al. (2001) claim that high-quality PD activities can make "collectively involving in practical and hands-on learning experiences" available to those teachers who participate in the learning process, which will be favorable to attain new sources of efficacy information for advancing their efficacy belief. In a PD activity with plenty of opportunities of interactive exchanges or positive communications, teachers are able to acquire ample vicarious experiences from observing the instructor's (or peers') teaching demonstration or successful mastery experience (Ross & Bruce, 2007). Simultaneously, this modeling is an effective tool in promoting teachers' efficacy when they are the observers. As Tschannen-Moran and Woolfolk Hoy (2007) argue, "when a model with whom the observer closely identifies performs well, the observer's efficacy would be then enhanced" (p. 945). Besides, teachers will obtain more positive feedback from the interactive process, which may help them believe they will be also successful if they implement what they learn in their own classrooms later (Stevens, et al., 2013). Consequently, those MGA-instructors or peers are considered as "important others" that provide convincible "verbal persuasion" (Bandura, 1997) during the process of verbal interactions. That is, teachers who are verbally persuaded that they possess the essential capabilities to master the teaching tasks are likely to be more efficacious and be able to extend more efforts in the future (Bandura, 1997; Tschannen-Moran & Woolfolk Hoy, 2007).

In this study, those 70% sessions of "introduction to MGA activities" with "two-way communication or hands-on experience" instructional style did provide MGA-teachers adequate opportunities to learn with an interactive and operational mode, where they can observe MGA-instructors' successful teaching demonstration and positively interact with peers. This learning environment furnishes them with enriched mastery experiences, vicarious experiences, and positive physiological arousal, which is beneficial for advancing their efficacy belief. With this assistance from important others, MGA-teachers will be qualified to host future fun-math camps where their students may enjoy in a similar interactive and hand-on learning environment and successfully and meaningfully acquire the designated mathematical concepts. On the contrary, those PD sessions with either "unclear and fragmented delivery of rationale and theoretical framework" or "one-way lecture instructional approach" may imperil MGA-teachers to obtain sufficient sources of efficacy information and, in turn, result in an expectable failure to execute their future MGA teaching tasks. In summary, how to consistently implement the 2-day MGA PD activities corresponding to the original design principles and learning theories stands at the core task for this JDM project. Currently, more MGA modules are designed and introduced in the current PD program, while more MGA-instructors with various backgrounds and perspectives are recruited to lead the two different sessions. Therefore, this inconsistency issue needs to be carefully addressed for future improvements. Moreover, except only encouraging mathematics teachers to attend PD activities, redesigning the certification request by asking MGA-teachers to do the following tasks is essential: for instance, attending 2 or more times of 2-day PD activities every year, requiring to hosting fun-math camps, and/or providing exhibition opportunities for them to share their successful experiences of hosting fun-math camps, etc. By employing diverse strategies to consistently provide interactive and operational opportunities, their internal motivation will be increased that is favorable for their efficacy development and future teaching performance.

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