Absorption Based Optical Imaging of Dispersion Processes

Absorption-based Optical Imaging of Dispersion Processes

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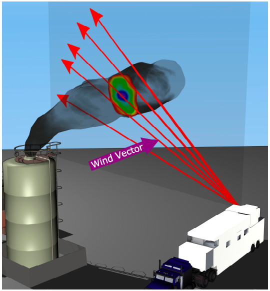
Differential Absorption Lidar (DIAL)

Basics Working Principle

Problem: Determine the 3D spatial concentration profile of a known trace gas using differential absorption Lidar.

- Measure (back-)scattered light at wavelengths, λ_{on} and λ_{off}, with identical scattering but different absorption by the trace gas.
- 3D imaging requires scan of a cone. ($\rightarrow$ DIAL cube)
- Additional atmospheric data is sometimes necessary or useful.



Mobile Lidar scanning a plume cross section[1]

[1] Illustration taken from Innocenti, F and Robinson, R and Gardiner, T and Finlayson, A and Connor, A. Differential Absorption Lidar (DIAL) measurements of landfill methane emissions, *Remote Sensing*, 2017.

Data collected by a time-resolving narrow field-of-view is sufficient to reconstruct a 3D profile but **ill-posedness** along with the **presence of noise** require averaging over long periods of time in order to yield good results.

- **Improving robustness to noise**: Reconstructing the underlying dispersion results in a regularised and well posed problem.

- **Improving signal quality**: The reduced number of parameters merit the use of computationally more demanding but physically more accurate models based on radiative transfer accounting for absorption and scattering.

The Radiative Transfer Equation
Wide vs. Narrow FOV

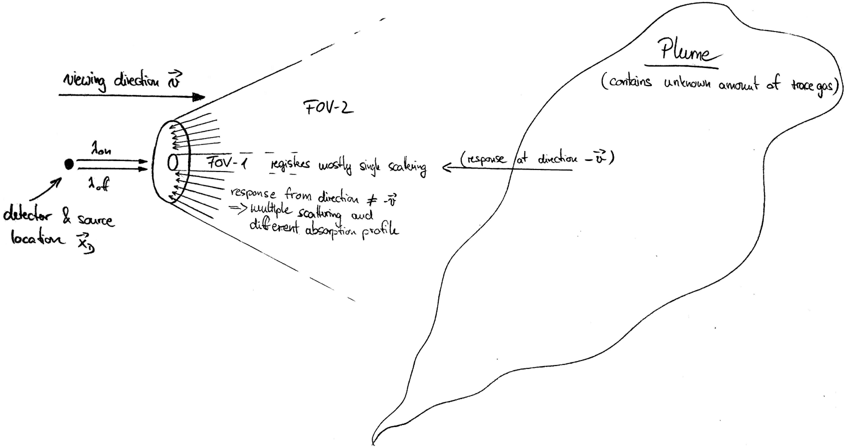


Figure: FOV-1 captures a very narrow cone and thus light that corresponds mostly to single scattering. The wider FOV-2 captures light that scattered multiple times which isn't modelled by the Lidar equation and doesn't have the same absorption profile as single scattering.

The two ingredients

Radiative Transfer

- The dynamics of light in heterogeneous scattering media can be modelled via the Radiative Transfer Equation (RTE)

$$\left(\frac{\partial}{\partial t} + v \cdot \nabla_x + \sigma_a^{\text{on/off}} + \sigma_s\right) H^{\text{on/off}} = \sigma_s \int_{\mathbb{S}^2} H^{\text{on/off}} f_p dv'$$

  where $\sigma_s, \sigma_a^{\text{on/off}}$ are heterogeneous scattering/absorption parameters and f_p is a phase function.

- If we use wider FOVs instead of only a narrow one, then more light is collected but the forward model is considerably more complex and no closed form reconstruction is possible.

The two ingredients
Dispersion

- We consider the advection-diffusion equation given by

$$\frac{\partial}{\partial t} u + \nabla \cdot (\eta u) - Q + \frac{1}{2} \nabla \cdot (\kappa \nabla u) = 0 \tag{1}$$

with $Q = \rho_Q \cdot \delta(\vec{x} - \vec{q})\delta(t)$ is an instantaneous source term at $\vec{q}$ while η, κ model drift and diffusion respectively.

- The plume can be modelled as a superposition of puffs ϕ

$$\sum_{j=1}^{N} w_j \phi \left(\frac{\|x - m_j\|_2}{h_j} \right) \tag{2}$$

for w_j, h_j and m_j which depend on the dispersion quantities and **regularise the inverse problem by imposing PDE based constraints**.

Parameter Uniqueness under RTE Assumption

Single vs. Multiple Scattering

A priori it is unclear whether the inverse problem that results from an RTE model is at all feasible. Under some mild technical conditions we can show the following:

Theorem

Assuming the optical forward model is governed by the RTE, then a differential absorption field $\sigma_a^{\mathrm{on}} - \sigma_a^{\mathrm{off}}$ of the form (2) is uniquely determined by the on and off intensities regardless of the field-of-view.

In other words, there is only a difference between wide and narrow FOVs when we consider noisy data:

- Discrepancies between the average model used in the inverse problem and the true concentration profile
- Optical noise due to limited photon counts in each bin

Noise and Stability Considerations

Information within the Signal: Wide vs. Narrow FOVs

Poisson noise model for the optical yields log-likelihood

$$\begin{aligned} \mathrm{L}(\theta \mid \boldsymbol{m}, \boldsymbol{n}) = \sum_{i,j} & H_{\mathrm{on}}(2t_j, v_i) + H_{\mathrm{off}}(2t_j, v_i) \\ & - \boldsymbol{m}_{v_i, 2t_j} \log(H_{\mathrm{on}}(2t_j, v_i)) - \boldsymbol{n}_{v_i, 2t_j} \log(H_{\mathrm{off}}(2t_j, v_i)) \end{aligned}$$

we may find the quantities of interest by solving a semi-paramteric MLE such as $\theta = (\alpha, H_{\mathrm{off}})$

- The effect of high-dimensional scattering parameters is captured within H_{off} and low-dimensional gradient evaluations for α via expensive RTE evaluations.
- Information content for α, measured by Fisher-Information (or related quantities), is situation dependent but we can use multiple FOVs and combine benefits without shortcomings!

- Simulated reconstruction from $80 \times 20 \times 50$ Lidar scan of 9 parameter dispersion which can be recovered when conventional reconstruction fails due to the low SNR.
- **The reference point**: Low-dimensional (regularised) vs. High-dimensional (noisy) concentration profiles

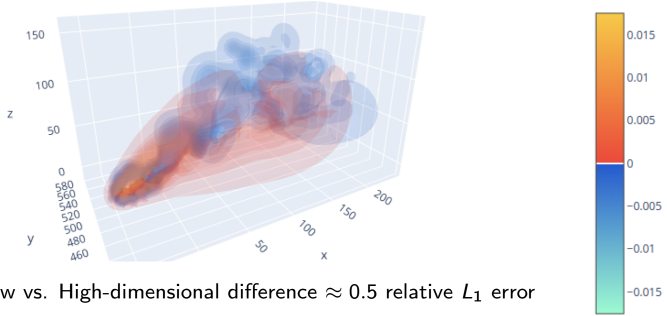
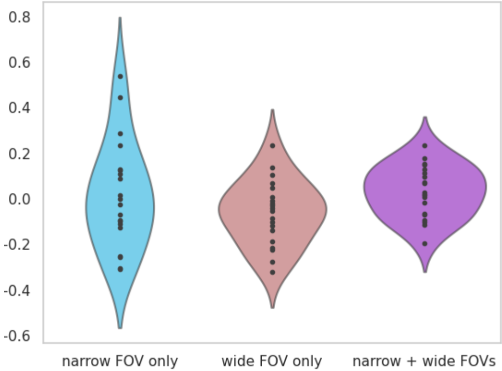


Figure: Low vs. High-dimensional difference ≈ 0.5 relative L_1 error

Simulations

Reconstruction[1] from $80 \times 20 \times 50$ Lidar scan: Release amount ρ_Q

- The parameter that controls the release rate is the ideal case for wide FOVs.
- Most photons are useful and separation of FOVs is of limited use here.

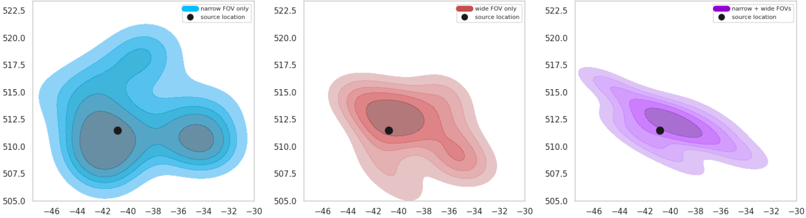


[1] 10 plumes with 2 optical data sets each = 20 runs

Simulations

Reconstruction[1] from $80 \times 20 \times 50$ Lidar scan: Source location $\vec{q}$

- The source parameter controls the overall positioning of the gas plume.
- Most photons are again useful and but separation of FOVs is of use here.
- Different properties of x and y component result in non-isotropic error distribution.



[1] 10 plumes with 2 optical data sets each = 20 runs

- The remaining parameters control the downwind-shape of the plume which is more heavily affected by the random perturbation.
  - Narrow FOV: 0.61 relative L_1 distance.
  - Wide FOV: 0.48 relative L_1 distance.
  - Wide+Narrow FOV: 0.44 relative L_1 distance.
- The difference in L_1 errors[2] is largely determined by the previous two quantities indicating that wide angle light is less useful for localised features.
- In order to make valid statements about gas concentration from low SNR measurements we should also consider UQ.

[2] arguably a sub-optimal measure in this situation