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## Citation for published version:

Hoffman, T \& Belle, V 2023, Abstracting Noisy Robot Programs. in Proceedings of the 2023 International Conference on Autonomous Agents and Multiagent Systems. International Foundation for Autonomous Agents and Multiagent Systems, Richland, SC, pp. 534-542, The 22nd International Conference on Autonomous Agents and Multiagent Systems, 2023, London, United Kingdom, 29/05/23.
https://doi.org/10.5555/3545946.3598681

Digital Object Identifier (DOI):
10.5555/3545946.3598681

## Link:

Link to publication record in Edinburgh Research Explorer

## Document Version:

Peer reviewed version

## Published In

Proceedings of the 2023 International Conference on Autonomous Agents and Multiagent Systems

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# Abstracting Noisy Robot Programs 

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#### Abstract

Abstraction is a commonly used process to represent some low-level system by a more coarse specification with the goal to omit unnecessary details while preserving important aspects. While recent work on abstraction in the situation calculus has focused on non-probabilistic domains, we describe an approach to abstraction of probabilistic and dynamic systems. Based on a variant of the situation calculus with probabilistic belief, we define a notion of bisimulation that allows to abstract a detailed probabilistic basic action theory with noisy actuators and sensors by a possibly non-stochastic basic action theory. By doing so, we obtain abstract Golog programs that omit unnecessary details and which can be translated to detailed programs for execution. This simplifies the implementation of noisy robot programs, opens up the possibility of using non-stochastic reasoning methods (e.g., planning) on probabilistic problems, and provides domain descriptions that are more easily interpretable.


## KEYWORDS

Logic; Robot Programs; Noise; Abstraction

## ACM Reference Format:

Till Hofmann and Vaishak Belle. 2023. Abstracting Noisy Robot Programs. In Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), London, United Kingdom, May 29 - June 2, 2023, IFAAMAS, 9 pages.

## 1 INTRODUCTION

Abstraction - the "process of mapping a representation of a problem onto a new representation" [18] - is a ubiquitous concept both in human behavior and in computing systems, e.g., a simple activity such as buying milk involves dozens of actions that a human conveniently abstracts into a single task, and machine instructions (which itself are abstractions of physical processes) are abstracted by higher programming languages. It has also seen widespread usage in several areas of artificial intelligence research [31], in particular in task planning. Abstraction typically involves suppressing irrelevant information and therefore allows reasoning about complex problems that would otherwise be infeasible. In the context of intelligent agents, abstraction typically serves three purposes [6]: (1) it provides a way to structure knowledge, (2) it allows reasoning about larger problems by abstracting

[^0]the problem domain, resulting in a smaller search space, (3) it may provide more meaningful explanations and is therefore critical for explainable AI. The need for abstraction becomes particularly apparent when dealing with robotic systems: as a robot acts in a dynamic environment with imperfect sensors and actuators, its actions are inherently noisy. As an example, a robot may intend to move but may get stuck with some probability. However, when programming such a robot, it is desirable to ignore those probabilistic aspects and instead work with a high-level and non-stochastic system, where the move action always succeeds, for all of the reasons above: Correctly designing a probabilistic domain is challenging, reasoning on such a domain is hard, and understanding how such a system operates is difficult. This becomes even more important when considering a robot that may get a hardware upgrade: while the low-level behavior changes (e.g., a new sensor has a different noise profile), the high-level behavior should not be affected. By using abstraction, we only need to update the low-level model and may keep the high-level program as is.

In this paper, we present an abstraction framework for robot programs with probabilistic belief: Starting with the logic $\mathcal{D S}$ [7], a modal variant of the situation calculus with probabilistic belief, we describe a transition semantics for noisy Golog programs in Section 3. Based on this logic, we propose a notion of abstraction of noisy programs, building on top of abstraction of probabilistic static models [6] and non-stochastic dynamic models [2]. We do so by defining a notion of bisimulation of probabilistic dynamic systems in Section 4 and we show that the notions of sound and complete abstraction carry over. We also demonstrate how this abstraction framework can be used to define a high-level domain, where noisy actions are abstracted away and thus, no probabilistic reasoning is necessary. We conclude in Section 5.

## 2 RELATED WORK

Reasoning about actions. The situation calculus [27, 29] is a logical formalism for reasoning about dynamical domains based on first-order logic. In the situation calculus, world states are represened explicitly as first-order terms called situations, where fluents describe (possibly changing) properties of the world and actions are axiomatized in basic action theorys (BATs). Golog [16, 24] is a programming language based on the situation calculus that allows to control the high-level behavior of robots. $\mathcal{E S}$ [22] is a modal variant and epistemic extension of the situation calculus, where situations are part of the semantics but do not appear as terms in the language. $\mathcal{E S G}$ extends $\mathcal{E S}$ with a transition semantics for Golog programs, which has been used for program
verification [11]. The situation calculus, $\mathcal{E S}$, and $\mathcal{E S G}$ are all deterministic and non-stochastic, i.e., the execution of an action always results in a unique successor state. De Giacomo and Lespérance [15] extend the situation calculus with nondeterministic actions, where the environment chooses one of several possible outcomes of an action. Bacchus et al. [1] extend the classical situation calculus with degrees of belief and noisy actions. In a similar fashion, $\mathcal{D S}[7]$ extends $\mathcal{E S}$ with degrees of belief and probabilistic actions, where the environment again may choose an outcome (possibly from an unbounded domain) with some pre-defined probability, allowing probabilistic representations of robot actions, in particular noisy sensors and actuators. More recently, reasoning about actions in $\mathcal{D S}$ has been shown to be amenable to regression [26] and progression [25] analogous to regression and progression in $\mathcal{E S}$ and the classical situation calculus [29].

Abstraction. Giunchiglia and Walsh [18] define abstraction generally as a mapping between a ground and an abstract formal system, such that the abstract representation preserves desirable properties while omitting unnecessary details to make it simpler to handle. Abstraction has been widely used in several fields of AI [31]. Hierarchical task network (HTN) planning systems such as SHOP2 [28] decompose tasks into subtasks to accomplish some overall objective, which has also been used in the situation calculus [17]. Macro planners such as MacrofF [9] combine action sequences into macro operators to improve planner performance, e.g., by collecting action traces from plan executions on robots [20], or by learning them from training problems [10]. Similarly, Saribatur and Eiter [32] use abstraction in Answer Set Programming to reduce the search space, improving solver performance. Cui et al. [13] leverage abstraction for generalized planning, i.e., for finding general solutions for a set of similar planning problems. Abstraction has also been used to analyze causal models [4, 30]. Of particular interest for this work is the notion of constructive abstraction [5], where the refinement mapping partitions the low-level variables such that each cell has a unique corresponding high-level variable. Holtzen et al. [21] describe an abstraction framework for probabilistic programs and also describe an algorithm to generate abstractions. REBA [33] is a framework for robot planning that uses abstract and determistic ASP programs to determine a course of action, which are then translated to POMDPs for execution. Banihashemi et al. [2] describe a general abstraction framework based on the situation calculus, where a refinement mapping maps a high-level BAT to a low-level BAT and which is capable of online execution with sensing actions [3]. The framework has been used to effectively synthesize plan process controllers in a smart factory scenario [14]. In contrast to this work, they assume non-probabilistic and deterministic actions. On the other hand, Belle [6] defines abstraction in a probabilistic but static propositional language and describes a search algorithm to derive such abstractions. In this paper, we build on the two approaches to obtain abstraction in a probabilistic and dynamic first-order language with an unbounded domain.

## 3 THE LOGIC $\mathcal{D S G}$

We start by introducing the logic $\mathcal{D S G}$, which we will then use to define abstraction over noisy programs in Section 4. $\mathcal{D S G}$ extends $\mathcal{D S}[7]$ with a transition semantics for Golog, analogous to how $\mathcal{E S G}$ [12] extends $\mathcal{E S}$ [22]. In the same way as $\mathcal{D S}$, the logic uses a countably infinite set of rigid designators $\mathcal{R}$, which allows to define quantification substitutionally. Similar to $\mathcal{D S}, \mathcal{E S}$, and $\mathcal{E S G}$, it uses a possible-worlds semantics, where a world defines the state of the world not only initially but after any sequence of actions. It uses the modal operator [.] to refer to the state after executing some program, e.g., $[\delta] \alpha$ states that $\alpha$ is true after every possible execution of the program $\delta$. Additionally, it uses the modal operator $\mathbf{B}$ to describe the agent's belief, e.g., $\mathbf{B}(\operatorname{Loc}(2): 0.5)$ states the the agent believes with degree 0.5 to be in location 2 .

### 3.1 Syntax

Definition 1 (Symbols of $\mathcal{D S G}$ ). The symbols of the language are from the following vocabulary:
(1) infinitely many variables $x, y, \ldots, u, v, \ldots, a, a_{1}, \ldots$;
(2) rigid function symbols of every arity, e.g., near, goto $(x, y)$;
(3) fluent predicates of every arity, such as $A t(l)$; we assume that this list contains the following distinguished predicates:

- Poss to denote the executability of an action;
- oi to denote that two actions are indistinguishable from the agent's viewpoint; and
- l that takes an action as its first argument and the action's likelihood as its second argument;
(4) connectives and other symbols: $=, \wedge, \neg, \forall, \square,[\cdot], \mathbf{B}$.

Definition 2 (Terms of $\mathcal{D S G}$ ). The set of terms of $\mathcal{D S G}$ is the least set such that (1) every variable is a term, (2) if $t_{1}, \ldots, t_{k}$ are terms and $f$ is a $k$-ary function symbol, then $f\left(t_{1}, \ldots, t_{k}\right)$ is a term.

As in $\mathcal{D S}, \mathcal{R}$ denotes the set of all ground rigid terms. We assume that they contain the rational numbers, i.e., $\mathbb{Q} \subseteq \mathcal{R}$.

Definition 3 (Formulas). The formulas of $\mathcal{D S G}$ are the least set such that
(1) if $t_{1}, \ldots, t_{k}$ are terms and $P$ is a $k$-ary predicate symbol, then $P\left(t_{1}, \ldots, t_{k}\right)$ is a formula,
(2) if $t_{1}$ and $t_{2}$ are terms, then $\left(t_{1}=t_{2}\right)$ is a formula,
(3) if $\alpha$ and $\beta$ are formulas, $x$ is a variable, $\delta$ is a program (defined below), ${ }^{1}$ and $r \in \mathbb{Q}$, then $\alpha \wedge \beta, \neg \alpha, \forall x . \alpha, \square \alpha$, $[\delta] \alpha$, and $\mathbf{B}(\alpha: r)$ are formulas.
We read $\square \alpha$ as " $\alpha$ holds after executing any sequence of actions", $[\delta] \alpha$ as " $\alpha$ holds after the execution of program $\delta$ " and $\mathbf{B}(\alpha: r)$ as " $\alpha$ is believed with probability $r$ ". ${ }^{2}$ We also

[^1]write $\mathbf{K} \alpha$ for $\mathbf{B}(\alpha: 1)$, to be read as " $\alpha$ is known". ${ }^{3}$ We use True as abbreviation for $\forall x(x=x)$ to denote truth. For a formula $\alpha$, we write $\alpha_{r}^{x}$ for the formula resulting from $\alpha$ by substituting every occurrence of $x$ with $r$. For a finite set of formulas $\Sigma=\left\{\alpha_{1}, \ldots, \alpha_{n}\right\}$, we may just write $\Sigma$ for the conjunction $\alpha_{1} \wedge \ldots \wedge \alpha_{n}$, e.g., $\mathbf{K} \Sigma$ for $\mathbf{K}\left(\alpha_{1} \wedge \cdots \wedge \alpha_{n}\right)$. A predicate symbol with terms from $\mathcal{R}$ as arguments is called a atomic formula, and we denote the set of atomic formulas with $\mathcal{P}$. Furthermore, a formula is called bounded if it contains no $\square$ operator, static if it contains no [•] or $\square$ operators, objective if it contains no $\mathbf{B}$ or $\mathbf{K}$, and fluent if it is static and does not mention Poss, B, or K.

We define the syntax of programs used by the operator $[\delta]$ :
Definition 4 (Programs).

$$
\delta::=t|\alpha ?| \delta_{1} ; \delta_{2}\left|\delta_{1}\right| \delta_{2}|\pi x . \delta| \delta^{*}
$$

where $t$ is a ground rigid term and $\alpha$ is a static formula. A program consists of actions $t$, tests $\alpha$ ?, sequences $\delta_{1} ; \delta_{2}$, nondeterministic branching $\delta_{1} \mid \delta_{2}$, nondeterministic choice of argument $\pi x . \delta$, and nondeterministic iteration $\delta^{*}$.

Note that we do not allow interleaved concurrency $\delta_{1} \| \delta_{2}$ known from ConGolog [16]. ${ }^{4}$ We also use nil as abbreviation for True?, the empty program that always succeeds. Similarly to formulas, $\delta_{r}^{x}$ denotes the program resulting from $\delta$ by substituting every $x$ with $r$. Furthermore, we define:

$$
\begin{aligned}
\text { if } \phi \text { then } \delta_{1} \text { else } \delta_{2} \text { fi } & :=\left(\phi ? ; \delta_{1}\right) \mid\left(\neg \phi ? ; \delta_{2}\right) \\
\text { if } \phi_{1} \text { then } \delta_{1} \text { elif } \phi_{2} \text { then } \delta_{2} \mathbf{f i}: & :\left(\phi_{1} ? ; \delta_{1}\right) \mid\left(\neg \phi_{1} \wedge \phi_{2} ? ; \delta_{2}\right) \\
\text { while } \phi \text { do } \delta \text { done } & :=(\phi ? ; \delta)^{*} ; \neg \phi ?
\end{aligned}
$$

### 3.2 Semantics

As described above, the operator $\mathbf{B}$ describes the degree of belief. In order to capture noisy actions and sensors, we need to talk about the likelihood of possible outcomes as well as the fact that when a noisy action is executed, the intended outcome may not be the same as the desired outcome. The latter is captured using the notion of observational indistinguishability. Both likelihood of possible outcomes and observational indistinguishability are built into the worlds using distinguished symbols and then modelled using basic action theories, as described in Section 3.3.

Similar to $\mathcal{D S}$, the semantics of $\mathcal{D S G}$ is given in terms of possible worlds, where a world defines the truth of each fluent both initially and after any sequence of actions:

Definition 5 (Trace). A trace $z=\left\langle a_{1}, \ldots, a_{n}\right\rangle$ is a finite sequence of $\mathcal{R}$. We denote the set of traces as $\mathcal{Z}$ and the empty trace with $\rangle$.

A world defines the truth of each ground atom from $\mathcal{P}$ not only initially but after any sequence of actions:

[^2]Definition 6 (World). A world is mapping $w: \mathcal{P} \times \mathcal{Z} \rightarrow$ $\{0,1\}$. The set of all worlds is denoted as $\mathcal{W}$.

We require that every world $w \in \mathcal{W}$ defines a unary predicate Poss, a binary predicate $l$ that behaves like a function (i.e., there is exactly one $q \in \mathbb{Q}$ such that $w[l(a, q), z]=1$ for any $a, z$ ), as well as an equivalence relation oi $\subseteq \mathcal{R} \times \mathcal{R}$, which define the possibility, the likelihood, and the observational indistinguishability of actions.

We call a pair $(w, z) \in \mathcal{W} \times \mathcal{Z}$ a state, we denote the set of all states with $\mathcal{S}$, and we use $S, S_{i}, \ldots \subseteq \mathcal{S}$ to denote sets of states. Given a state $(w, z)$, the predicate $l(a, q)$ states that the action likelihood of action $a$ in state $(w, z)$ is $q$. We can inductively apply $l$ to compute the likelihood of a sequence:

Definition 7 (Action Sequence Likelihood). The action sequence likelihood $l^{*}: \mathcal{W} \times \mathcal{Z} \rightarrow \mathbb{Q}^{\geq 0}$ is defined inductively:

- $l^{*}(w,\langle \rangle)=1$ for every $w \in \mathcal{W}$,
- $l^{*}(w, z \cdot r)=l^{*}(w, z) \times q$ where $w[l(r, q), z]=1$.

Next, to deal with partially observable states, we define: Definition 8 (ObSERVATIONAL indistinguishability).
(1) Given a world $w \in \mathcal{W}$, we define $\sim_{w} \subset \mathcal{Z} \times \mathcal{Z}$ inductively: - $\left\rangle \sim_{w} z^{\prime}\right.$ iff $z^{\prime}=\langle \rangle$

- $z \cdot r \sim_{w} z^{\prime}$ iff $z^{\prime}=z^{*} \cdot r^{*}, z \sim_{w} z^{*}, w\left[o i\left(r, r^{*}\right), z\right]=1$
(2) We say $w$ is observationally indistinguishable (oi) from $w^{\prime}$, written $w \approx_{o i} w^{\prime}$ iff for all $a, a^{\prime} \in \mathcal{R}, z \in \mathcal{Z}$ : $w\left[o i\left(a, a^{\prime}\right), z\right]=w^{\prime}\left[o i\left(a, a^{\prime}\right), z\right]$.
(3) For $w, w^{\prime} \in \mathcal{W}, z, z^{\prime} \in \mathcal{Z}$, we say $(w, z)$ is oi from $\left(w^{\prime}, z^{\prime}\right)$, written $(w, z) \approx_{o i}\left(w^{\prime}, z^{\prime}\right)$, iff $w \approx_{o i} w^{\prime}$ and $z \sim_{w} z^{\prime}$.
Intuitively, $z \sim_{w} z^{\prime}$ means that the agent cannot distinguish whether it executed $z$ or $z^{\prime}$. For states, $(w, z) \approx_{\text {oi }}$ ( $w^{\prime}, z^{\prime}$ ) is to be understood as "if the agent believes to be in state ( $w, z$ ), it may also be in state $\left(w^{\prime}, z^{\prime}\right)$ ", i.e., it cannot distinguish the worlds $w, w^{\prime}$ and traces $z, z^{\prime}$. As $\approx_{\text {oi }}$ is an equivalence relation, the set of its equivalence classes on a set of states $S$ induces a partition, which we denote with $S / \approx_{\text {oi }}$.

We extend the executability of an action to traces:
Definition 9 (Executable trace). For a trace $z$, we define exec(z) inductively:

- for $z=\langle \rangle, \operatorname{exec}(z):=$ True
- for $z=a \cdot z^{\prime}, \operatorname{exec}(z):=\operatorname{Poss}(a) \wedge[a] \operatorname{exec}\left(z^{\prime}\right)$

As in BHL and $\mathcal{D S}$, it is possible to permit the agent to entertain any set of initial distributions. As an example, the initial theory could say that $\mathbf{B}(p: 0.5) \vee \mathbf{B}(p: 0.6)$, which says that the agent is not sure about the distribution of $p$. In this case, there would be at least two distributions in the epistemic state $e$. As another example, if we say $\mathbf{B}(p \vee q: 1)$, then this says that the disjunction is believed with probability 1 , but it does not specify the probability of $p$ or $q$, resulting in infinitely many distributions that are compatible with this constraint. Thus, not committing to a single distribution results in higher expressivity in the representation of uncertainty.

Definition 10 (Compatible States). Given an epistemic state e, a world $w$, a trace $z$, and a formula $\alpha$, we
define the states $S_{\alpha}^{e, w, z}$ compatible to $(e, w, z)$ wrt to $\alpha$ :

$$
S_{\alpha}^{e, w, z}=\left\{\left(w^{\prime}, z^{\prime}\right) \mid\left(w^{\prime}, z^{\prime}\right) \approx_{o i}(w, z), e, w^{\prime} \vDash \operatorname{exec}\left(z^{\prime}\right) \wedge\left[z^{\prime}\right] \alpha\right\}
$$

We write $S_{\alpha}$ for $S_{\alpha}^{e, w, z}$ if $e, w, z$ are clear from the context.
To define the semantics of belief, we first define epistemic states, which assign probabilities to worlds:

Definition 11 (Epistemic state). A distribution is a mapping $\mathcal{W} \rightarrow \mathbb{R}^{\geq 0}$. An epistemic state is any set of distributions.

This notion of distribution is not directly a probability distribution. To obtain probability distributions, we define:

DEFINITION 12 (Normalization).
For any distribution $d$ and any set $\mathcal{V}=\left\{\left(w_{1}, z_{1}\right),\left(w_{2}, z_{2}\right), \ldots\right\}$, we define:
(1) $\operatorname{BND}(d, \mathcal{V}, r)$ iff there is no $k$ such that $\sum_{i=1}^{k} d\left(w_{i}\right) \times l^{*}\left(w_{i}, z_{i}\right)>r$.
(2) $\operatorname{EQ}(d, \mathcal{V}, r)$ iff $\operatorname{BND}(d, \mathcal{V}, r)$ and there is no $r^{\prime}<r$ such that $\operatorname{BND}\left(d, \mathcal{V}, r^{\prime}\right)$ holds.
(3) For any $\mathcal{U} \subseteq \mathcal{V}: \operatorname{Norm}(d, \mathcal{U}, \mathcal{V}, r)$ iff $\exists b \neq 0$ such that $\mathrm{EQ}(d, \mathcal{U}, b \times r)$ and $\mathrm{EQ}(d, \mathcal{V}, b)$.

Intuitively, given $\operatorname{Norm}(d, \mathcal{V}, r), r$ can be seen as the normalization of the weights of worlds in $\mathcal{V}$ in relation to the set of all worlds $\mathcal{W}$ as accorded by $d$. The conditions Bnd and EQ are auxiliary conditions to define Norm, where $\operatorname{BND}(d, \mathcal{V}, r)$ states that the weight of worlds in $\mathcal{V}$ is bounded by $b$ and $\mathrm{EQ}(d, \mathcal{V}, r)$ expresses that the weight of worlds in $\mathcal{V}$ is equal to $b$. Belle et al. [8] have shown that although the set of worlds $\mathcal{W}$ is in general uncountable, this leads to a well-defined summation over the weights of worlds.

To simplify notation, we also write $\operatorname{Norm}(d, \mathcal{U}, \mathcal{V})=r$ for $\operatorname{Norm}(d, \mathcal{U}, \mathcal{V}, r)$. Furthermore, we write $\operatorname{Norm}\left(d_{1}, \mathcal{U}_{1}, \mathcal{V}_{1}\right)=$ $\operatorname{Norm}\left(d_{2}, \mathcal{U}_{2}, \mathcal{V}_{2}\right)$ if there is an $r$ such that $\operatorname{Norm}\left(d_{1}, \mathcal{U}_{1}, \mathcal{V}_{1}, r\right)$ and $\operatorname{Norm}\left(d_{2}, \mathcal{U}_{2}, \mathcal{V}_{2}, r\right)$. Finally, we write

$$
\operatorname{Norm}\left(d, \mathcal{U}_{1}, \mathcal{V}\right)+\operatorname{Norm}\left(d, \mathcal{U}_{2}, \mathcal{V}\right)=r
$$

if $\operatorname{Norm}\left(d, \mathcal{U}_{1}, \mathcal{V}, r_{1}\right), \operatorname{Norm}\left(d, \mathcal{U}_{2}, \mathcal{V}, r_{2}\right)$, and $r=r_{1}+r_{2}$.
We continue with the program transition semantics, which defines the traces resulting from executing some program. The transition semantics is defined in terms of configurations $\langle z, \delta\rangle$, where $z$ is a trace describing the actions executed so far and $\delta$ is the remaining program. In some places, the transition semantics refers to the truth of formulas (see Definition 15 below). ${ }^{5}$

Definition 13 (Program Transition Semantics). The transition relation $\xrightarrow{e, w}$ among configurations, given an epistemic state $e$ and a world $w$, is the least set satisfying

$$
\begin{aligned}
& \text { (1) }\langle z, a\rangle \xrightarrow{e, w}\langle z \cdot a, \text { nil }\rangle \text { if } w, z \models \operatorname{Poss}(a) \\
& \text { (2) }\left\langle z, \delta_{1} ; \delta_{2}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \gamma ; \delta_{2}\right\rangle \text {, if }\left\langle z, \delta_{1}\right\rangle \xrightarrow{e, w}\langle z \cdot a, \gamma\rangle \text {, } \\
& \text { (3) }\left\langle z, \delta_{1} ; \delta_{2}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \delta^{\prime}\right\rangle \text { if } \\
& \left\langle z, \delta_{1}\right\rangle \in \mathcal{F}^{e, w} \text { and }\left\langle z, \delta_{2}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \delta^{\prime}\right\rangle
\end{aligned}
$$

[^3](4) $\left\langle z, \delta_{1} \mid \delta_{2}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \delta^{\prime}\right\rangle$ if
$$
\left\langle z, \delta_{1}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \delta^{\prime}\right\rangle \text { or }\left\langle z, \delta_{2}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \delta^{\prime}\right\rangle
$$
(5) $\langle z, \pi x . \delta\rangle \xrightarrow{e, w}\left\langle z \cdot a, \delta^{\prime}\right\rangle$, if
$\left\langle z, \delta_{r}^{x}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \delta^{\prime}\right\rangle$ for some $r \in \mathcal{R}$
(6) $\left\langle z, \delta^{*}\right\rangle \xrightarrow{e, w}\left\langle z \cdot a, \gamma ; \delta^{*}\right\rangle$ if $\langle z, \delta\rangle \xrightarrow{e, w}\langle z \cdot a, \gamma\rangle$

The set of final configurations $\mathcal{F}^{e, w}$ is the smallest set s.t.
(1) $\langle z, \alpha ?\rangle \in \mathcal{F}^{e, w}$ if $e, w, z \models \alpha$,
(2) $\left\langle z, \delta_{1} ; \delta_{2}\right\rangle \in \mathcal{F}^{e, w}$ if $\left\langle z, \delta_{1}\right\rangle \in \mathcal{F}^{e, w}$ and $\left\langle z, \delta_{2}\right\rangle \in \mathcal{F}^{e, w}$
(3) $\left\langle z, \delta_{1} \mid \delta_{2}\right\rangle \in \mathcal{F}^{e, w}$ if $\left\langle z, \delta_{1}\right\rangle \in \mathcal{F}^{e, w}$, or $\left\langle z, \delta_{2}\right\rangle \in \mathcal{F}^{e, w}$
(4) $\langle z, \pi x . \delta\rangle \in \mathcal{F}^{e, w}$ if $\left\langle z, \delta_{r}^{x}\right\rangle \in \mathcal{F}^{e, w}$ for some $r \in \mathcal{R}$
(5) $\left\langle z, \delta^{*}\right\rangle \in \mathcal{F}^{e, w}$

We also write $\xrightarrow{e, w} *$ for the transitive closure of $\xrightarrow{e, w}$.
Following the transition semantics for a given program $\delta$, we obtain a set of program traces:

Definition 14 (Program Traces).
Given an epistemic state $e$, a world $w$, and a trace $z$, the set $\|\delta\|_{e, w}^{z}$ of traces of program $\delta$ is defined as the following set:

$$
\begin{aligned}
& \|\delta\|_{e, w}^{z}= \\
& \left\{z^{\prime} \in \mathcal{Z} \mid\langle z, \delta\rangle \xrightarrow{e, w} *\left\langle z \cdot z^{\prime}, \delta^{\prime}\right\rangle \text { and }\left\langle z \cdot z^{\prime}, \delta^{\prime}\right\rangle \in \mathcal{F}^{e, w}\right\}
\end{aligned}
$$

Compared to $\mathcal{E S G}$, this transition semantics also refers to the epistemic state $e$, as test formulas can also mention belief operators. Additionally, in contrast to $\mathcal{E S G}$, it only allows a transition for an atomic action if the action is possible in the current state. Also, while $\mathcal{E S G}$ allows infinite traces, we only allow finite traces, as we focus on terminating programs.

Finally, we can define the semantics for $\mathcal{D S G}$ formulas:
Definition 15 (Truth of Formulas). Given an epistemic state e, a world $w$, and a formula $\alpha$, we define for every $z \in \mathcal{Z}$ :
(1) $e, w, z \mid=F\left(t_{1}, \ldots, t_{k}\right)$ iff $w\left[F\left(t_{1}, \ldots, t_{k}\right), z\right]=1$
(2) $e, w, z \mid=\mathbf{B}(\alpha: r)$ iff $\forall d \in e: \operatorname{Norm}\left(d, S_{\alpha}, S_{\text {TRUE }}, r\right)$
(3) $e, w, z \mid=\left(t_{1}=t_{2}\right)$ iff $t_{1}$ and $t_{2}$ are identical
(4) $e, w, z \mid=\alpha \wedge \beta$ iff $e, w, z \vDash \alpha$ and $e, w, z \models \beta$
(5) $e, w, z \mid=\neg \alpha$ iff $e, w, z \mid \neq \alpha$
(6) $e, w, z \mid=\forall x$. $\alpha$ iff $e, w, z \models \alpha_{r}^{x}$ for all $r \in \mathcal{R}$.
(7) $e, w, z \mid=\square \alpha$ iff $e, w, z \cdot z^{\prime} \models \alpha$ for all $z^{\prime} \in \mathcal{Z}$
(8) $e, w, z \mid=[\delta] \alpha$ iff $e, w, z \cdot z^{\prime} \models \alpha$ for all $z^{\prime} \in\|\delta\|_{e, w}^{z}$.

Note in particular that Item 2 states that the degree of belief in a formula is obtained by looking at the normalized weight of the possible worlds that satisfy the formula.

We write $e, w \vDash \alpha$ for $e, w,\langle \rangle \vDash \alpha$. Also, if $\alpha$ is objective, we write $w, z \models \alpha$ for $e, w, z \models \alpha$ and $w \models \alpha$ for $w,\langle \rangle \vDash \alpha$. Additionally, for a set of sentences $\Sigma$, we write $e, w, z \models \Sigma$ if $e, w, z \models \phi$ for all $\phi \in \Sigma$, and $\Sigma \models \alpha$ if $e, w \mid=\Sigma$ entails $e, w \mid=\alpha$ for every model $(e, w)$.

### 3.3 Basic Action Theories

A basic action theory (BAT) defines the effects of all actions of the domain, as well as the initial state:

Definition 16 (Basic Action Theory). Given a finite set of predicates $\mathcal{F}$ including oi and $l$, a set $\Sigma$ of sentences
only mentioning fluent predicates in $\mathcal{F}$ is called a basic action theory (BAT) over $\mathcal{F}$ iff $\Sigma=\Sigma_{0} \cup \Sigma_{\text {pre }} \cup \Sigma_{\text {post }}$ and
(1) $\Sigma_{0}$ is any set of fluent sentences,
(2) $\Sigma_{\text {pre }}$ consists of a single sentence of the form $\square \operatorname{Poss}(a) \equiv$ $\pi$, where $\pi$ is a fluent formula with free variable $a,{ }^{6}$
(3) $\Sigma_{\text {post }}$ is a set of successor state axioms (SSAs) of the form $\square \operatorname{Poss}(a) \supset\left([a] F(\vec{x}) \equiv \gamma_{F}\right)$, one for each fluent predicate $F \in \mathcal{F}$ and where $\gamma_{F}$ is a fluent formula with free variables among a and $\vec{x}$.

Given a BAT $\Sigma$, we say that a program $\delta$ is a program over $\Sigma$ if it only mentions fluents and actions from $\Sigma$.

Note that the SSAs slightly differ from $\mathcal{E S}$ and $\mathcal{E S G}$, where they have the form $\square[a] F(\vec{x}) \equiv \gamma_{F}$. In contrast to $\mathcal{E S}$ and $\mathcal{E S G}$, the SSAs in $\mathcal{D S G}$ only define the effects of an action if the action is currently possible and otherwise do not say anything about the action's effects. This is necessary because we include $\operatorname{Poss}(a)$ in the transition semantics (Definition 13). To understand why, consider the following example: if $w, z \models \neg \operatorname{Poss}(a)$, then by Definition 15.8, $w, z \models[a] \neg F()$ is vacuously true for any fluent $F$ because there is no trace $z^{\prime} \in\|a\|_{e, w}^{z}$, contradicting to a SSA $\square[a] F() \equiv \gamma_{F}$. Restricting the SSA to possible actions avoids this issue. ${ }^{7}$
3.3.1 A Noisy BAT. We present a BAT for a simple robotics scenario with noisy actions, inspired from [1, 7]. In this scenario, a robot moves towards a wall and it is equipped with a sonar sensor that can measure the distance to the wall. A BAT $\Sigma_{\text {move }}$ defining this scenario may look as follows:

- A move action is possible if the robot moves one step to the back or to the front. A sonar action is always possible:

$$
\begin{aligned}
& \square \operatorname{Poss}(a) \equiv \exists x, y(a=\operatorname{move}(x, y) \wedge(x=1 \vee x=-1)) \\
& \vee \exists z(a=\operatorname{sonar}(z))
\end{aligned}
$$

- After doing action $a$, the robot is at position $x$ if $a$ is a move action that moves the robot to location $x$, if $a$ is a sonar action that measures distance $x$, or if $a$ is neither of the two actions and the robot was at location $x$ before

$$
\begin{aligned}
& \square \operatorname{Poss}(a) \supset([a] \operatorname{Loc}(x) \equiv \\
& \quad \exists y, z,(a=\operatorname{move}(y, z) \wedge \operatorname{Loc}(l) \wedge x=l+z) \\
& \quad \vee a=\operatorname{sonar}(x) \\
& \quad \vee \neg \exists y, z(a=\operatorname{move}(y, z) \vee a=\operatorname{sonar}(y)) \wedge \operatorname{Loc}(x))
\end{aligned}
$$

- For the sonar action, the likelihood that the robot measures the correct distance is 0.8 , the likelihood that it measures a distance with an error of $\pm 1$ is 0.1 . Furthermore, for the move action, the likelihood that the robot

[^4]moves the intended distance $x$ is 0.6 , the likelihood that the actual movement $y$ is off by $\pm 1$ is 0.2 :
\[

$$
\begin{aligned}
& \square l(a, u) \equiv \\
& \quad \exists z(a=\operatorname{sonar}(z) \wedge \operatorname{Loc}(x) \wedge u=\Theta(x, z, .8, .1)) \\
& \quad \vee \exists x, y(a=\operatorname{move}(x, y) \wedge u=\Theta(x, y, .6, .2)) \\
& \vee \neg \exists x, y, z(a=\operatorname{move}(x, y) \vee a=\operatorname{sonar}(z)) \wedge u=.0 \\
& \text { where } \Theta(u, v, c, d)= \begin{cases}c & \text { if } u=v \\
d & \text { if }|u-v|=1 \\
0 & \text { otherwise }\end{cases}
\end{aligned}
$$
\]

- The robot cannot detect the distance that it has actually moved, i.e., any actions move $(x, y)$ and move $(x, z)$ are o.i.:

$$
\begin{aligned}
\square o i\left(a, a^{\prime}\right) \equiv & a=a^{\prime} \vee \\
& \exists x, y, z\left(a=\operatorname{move}(x, y) \wedge a^{\prime}=\operatorname{move}(x, z)\right)
\end{aligned}
$$

- Initially, it is 3 units away from the wall: $\operatorname{Loc}(x) \equiv x=3$

Based on this BAT, we define a program that first moves the robot close to the wall and then back: ${ }^{8}$
sonar ();
while $\neg \mathbf{K} \exists x(\operatorname{Loc}(x) \wedge x \leq 2)$ do move $(-1)$; sonar () done;
while $\neg \mathbf{K} \exists x(\operatorname{Loc}(x) \wedge x>5)$ do move(1); sonar() done
The robot first measures its distance to the wall and then moves closer until it knows that its distance to the wall is less than two units. Afterwards, it moves away until it knows that is more than five units away from the wall. As the robot's move action is noisy, each move is followed by sonar to measure how far it is away from the wall. One possible execution trace of this program may look as follows:

$$
\begin{align*}
z_{l}= & \langle\operatorname{sonar}(3), \text { move }(-1,0), \text { sonar }(3), \text { move }(-1,-1), \\
& \operatorname{sonar}(2), \text { move }(-1,-1), \text { sonar }(1), \text { move }(1,1), \\
& \operatorname{sonar}(3), \text { move }(1,1), \text { sonar }(2), \text { move }(1,1),  \tag{1}\\
& \operatorname{sonar}(4), \text { move }(1,1), \text { sonar }(6)\rangle
\end{align*}
$$

First, the robot (correctly) senses that it is three units away from the wall and starts moving. However, the first move does not have the desired effect: the robot intended to move by one unit but actually did not move (indicated by the second argument being 0 ). After the second move, the robot is at $\operatorname{Loc}(2)$, as it started at $\operatorname{Loc}(3)$ and moved successfully once. However, as its sensor is noisy and it measured sonar(2), it believes that it could also be at $\operatorname{Loc}(3)$. For safe measure, it executes another move and then senses sonar(1), after which it knows for sure that it is at most two units away from the wall. In the second part, the robot moves back until it knows that it is further than five units away from the wall. As this simple example shows, the trace $z_{l}$ is already quite hard to understand. While it is clear from the BAT what each action does, the robot's intent is not immediately obvious and the trace is cluttered with noise.

[^5]3.3.2 An Abstract BAT. We present a second, more abstract BAT for the same scenario without noisy actions:

- Initially, the robot is in the middle: $A t(l) \equiv l=$ mid
- The robot may goto the locations near and far: ${ }^{9}$

$$
\square \operatorname{Poss}(a) \equiv a=\operatorname{goto}(\text { near }) \vee a=\operatorname{goto}(\text { far })
$$

- After doing action $a$, the robot is at location $l$ if $a$ is goto( $l$ ) or if $a$ is no goto action and the robot has been at $l$ before:

```
\square P o s s ( a ) ~ \supset
```

$$
([a] A t(l) \equiv a=g o t o(l) \vee \neg \exists x(a=g \circ t o(x)) \wedge A t(l))
$$

- The action likelihood axiom states that no action is noisy:

$$
\begin{aligned}
\square l(a, u) \equiv & (a=\operatorname{goto}(\text { near }) \vee a=\operatorname{goto}(\text { far })) \wedge u=1.0 \\
& \vee \neg \exists x(a=\operatorname{goto}(x)) \wedge u=0.0
\end{aligned}
$$

- The agent can distinguish all actions: $\square o i\left(a, a^{\prime}\right) \equiv a=a^{\prime}$ In the following, we will connect the low-level BAT $\Sigma_{\text {move }}$ with the high-level BAT $\Sigma_{g o t o}$ by using abstraction.


## 4 ABSTRACTION

In this section, we define the abstraction of a low-level BAT $\Sigma_{l}$ with a high-level BAT $\Sigma_{h}$. This will allow us to construct abstract Golog programs over the high-level BAT, which are equivalent and can be translated to some program over the low-level BAT. For the sake of simplicity ${ }^{10}$, we assume in the following that an epistemic state $e$ is always a singleton, i.e., $e_{h}=\left\{d_{h}\right\}$ and $e_{l}=\left\{d_{l}\right\}$. To translate the high-level BAT $\Sigma_{h}$ into the low-level BAT $\Sigma_{l}$, we map $\Sigma_{h}$ to $\Sigma_{l}$ by mapping each high-level fluent to a low-level formula, and every high-level action to a low-level program:

Definition 17 (Refinement Mapping). Given two basic action theories $\Sigma_{l}$ over $\mathcal{F}_{l}$ and $\Sigma_{h}$ over $\mathcal{F}_{h}$. The function $m$ is a refinement mapping from $\Sigma_{h}$ to $\Sigma_{l}$ iff:
(1) For every action $a(\vec{x})$ mentioned in $\Sigma_{h}, m(a(\vec{x}))=$ $\delta_{a}(\vec{x})$, where $\delta_{a}(\vec{x})$ is a Golog program over the lowlevel theory $\Sigma_{l}$ with free variables among $\vec{x}$.
(2) For every fluent predicate $F \in \mathcal{F}_{h}, m(F(\vec{x}))=\phi_{F}(\vec{x})$, where $\phi_{F}(\vec{x})$ is a static formula over $\mathcal{F}_{l}$ with free variables among $\vec{x}$.

For a formula $\alpha$ over $\mathcal{F}_{h}$, we also write $m(\alpha)$ for the formula obtained by applying $m$ to each fluent predicate and action mentioned in $\alpha$. For a trace $z=\left\langle a_{1}, a_{2}, \ldots\right\rangle$ of actions from $\Sigma_{h}$, we also write $m(z)$ for $\left\langle m\left(a_{1}\right), m\left(a_{2}\right), \ldots\right\rangle$. For a program $\delta$ over $\Sigma_{h}$, the program $m(\delta)$ is the same program as $\delta$ with each primitive action $a$ replaced by $m(a)$ and each formula $\alpha$ replaced by $m(\alpha)$.

Continuing our example, we define a refinement that maps $\Sigma_{\text {goto }}$ to $\Sigma_{\text {move }}$ by mapping each high-level fluent to a low-level formula and each high-level action to a low-level program:

[^6]

Figure 1: An example for epistemic isomorphism.

- The high-level fluent $A t(l)$ is mapped to a low-level formula by translating the distance to near, mid, and far:

$$
\begin{aligned}
A t(l) \mapsto l & =\operatorname{near} \wedge \exists x(\operatorname{Loc}(x) \wedge x \leq 2) \\
\vee l & =\operatorname{mid} \wedge \exists x(\operatorname{Loc}(x) \wedge x>2 \wedge x \leq 5) \\
\vee l & =\operatorname{far} \wedge \exists x(\operatorname{Loc}(x) \wedge x>5)
\end{aligned}
$$

- The action goto is mapped to a program that guarantees that the robot reaches the right position:
$\operatorname{goto}(x) \mapsto \operatorname{sonar}()$;
if $x=$ near then
while $\neg \mathbf{K} \exists x(\operatorname{Loc}(x) \wedge x \leq 2)$ do move $(-1)$; sonar () done elif $x=$ far then
while $\neg \mathbf{K} \exists x(\operatorname{Loc}(x) \wedge x>5)$ do move (1); sonar () done fi
To show that a high-level BAT indeed abstracts a low-level BAT, we first define a notion of isomorphism, intuitively stating that two states satisfy the same fluents:

Definition 18 (Objective Isomorphism).
We say $\left(w_{h}, z_{h}\right)$ is objectively $m$-isomorphic to $\left(w_{l}, z_{l}\right)$, written $\left(w_{h}, z_{h}\right) \sim_{m}\left(w_{l}, z_{l}\right)$ iff for every atomic formula $\alpha$ mentioned in $\Sigma_{h}$ :

$$
w_{h}, z_{h} \models \alpha \text { iff } w_{l}, z_{l} \models m(\alpha)
$$

Additionally, because we need to relate degrees of belief, we need to connect the two BATs in terms of epistemic states. To do so, we define epistemic isomorphism as follows:

Definition 19 (Epistemic Isomorphism).
For every $\left(w_{h}, z_{h}\right) \in \mathcal{S}$ and $S_{l} \subseteq \mathcal{S}$, we say that $\left(d_{h}, w_{h}, z_{h}\right)$ is epistemically m-isomorphic to $\left(d_{l}, S_{l}\right)$, written $\left(d_{l}, w_{h}, z_{h}\right) \sim_{e}$ $\left(d_{l}, S_{l}\right)$ iff for the partition $P=S_{l} / \approx_{o i}$, for each $S_{l}^{i} \in P$ and $\left(w_{l}^{i}, z_{l}^{i}\right) \in S_{l}^{i}:$

$$
\operatorname{Norm}\left(d_{h},\left\{\left(w_{h}, z_{h}\right)\right\}, S_{\text {TRUE }}^{e_{h}, w_{h}, z_{h}}\right)=\operatorname{Norm}\left(d_{l}, S_{l}^{i}, S_{\text {TRUE }}^{e_{l}, w_{l}^{i}, z_{l}^{i}}\right)
$$

The intuition of epistemic isomorphism is illustrated in Figure 1: As the high-level state $\left(w_{h}^{1}, z_{h}^{1}\right)$ is more abstract than the low-level states $S_{l}$, multiple low-level states (highlighted in orange) may be isomorphic to the same high-level state. Hence, each high-level state is mapped to a set of low-level states. To be epistemically isomorphic, they must entail the same beliefs, so the corresponding normalized weights must be equal. However, we do not require the low-level states $S_{l}$ to be oi. Indeed, since we have a high-level action corresponding
to many low-level actions, low-level states are typically not oi. Thus, we partition $S_{l}$ according to $\approx_{\text {oi }}$, obtaining $S_{\text {TRUE }}^{w_{l}^{1}, z_{l}^{1}}$ and $S_{\text {TRUE }}^{w_{l}^{3}, z_{l}^{3}}$, and we require the NORM over $\left(w_{h}, z_{h}\right)$ to be the same as the Norm over each member of the partition.

Having established objective and epistemic isomorphisms, we can now define a suitable notion of bisimulation:

Definition 20 (Bisimulation).
$A$ relation $B \subseteq \mathcal{S} \times \mathcal{S}$ is an $m$-bisimulation between $\left(e_{h}, w_{h}\right)$ and $\left(e_{l}, w_{l}\right)$ if $\left(\left(w_{h}, z_{h}\right),\left(w_{l}, z_{l}\right)\right) \in B$ implies that
(1) $\left(w_{h}, z_{h}\right) \sim_{m}\left(w_{l}, z_{l}\right)$,
(2) $\left(d_{h}, w_{h}, z_{h}\right) \sim_{e}\left(d_{l},\left\{\left(w_{l}^{\prime}, z_{l}^{\prime}\right) \mid\left(\left(w_{h}, z_{h}\right),\left(w_{l}^{\prime}, z_{l}^{\prime}\right)\right) \in B\right\}\right)$,
(3) $w_{h} \models \operatorname{exec}\left(z_{h}\right)$ and $w_{l} \models \operatorname{exec}\left(z_{l}\right)$,
(4) for every high-level action $a$, if $w_{h}, z_{h} \models \operatorname{Poss}(a)$, then there is $z_{l}^{\prime} \in\|m(a)\|_{e_{l}, w_{l}}^{z_{l}}$ s.t. $\left(\left(w_{h}, z_{h} \cdot a\right),\left(w_{l}, z_{l} \cdot z_{l}^{\prime}\right)\right) \in B$,
(5) for every high-level action $a$, if there is $z_{l}^{\prime} \in\|m(a)\|_{e_{l}, w_{l}}^{z_{l}}$, then $w_{h}, z_{h} \models \operatorname{Poss}(a)$ and $\left(\left(w_{h}, z_{h} \cdot a\right),\left(w_{l}, z_{l} \cdot z_{l}^{\prime}\right)\right) \in B$,
(6) for every $\left(w_{h}^{\prime}, z_{h}^{\prime}\right) \approx_{o i}\left(w_{h}, z_{h}\right)$ with $d_{h}\left(w_{h}^{\prime}\right)>0$ and $e_{h}, w_{h}^{\prime}=\operatorname{exec}\left(z_{h}^{\prime}\right)$, there is $\left(w_{l}^{\prime}, z_{l}^{\prime}\right) \approx_{o i}\left(w_{l}, z_{l}\right)$ such that $\left(\left(w_{h}^{\prime}, z_{h}^{\prime}\right),\left(w_{l}^{\prime}, z_{l}^{\prime}\right)\right) \in B$,
(7) for every $\left(w_{l}^{\prime}, z_{l}^{\prime}\right) \approx_{o i}\left(w_{l}, z_{l}\right)$ with $d_{l}\left(w_{l}^{\prime}\right)>0$ and $e_{l}, w_{l}^{\prime} \models \operatorname{exec}\left(z_{l}^{\prime}\right)$, there is $\left(w_{h}^{\prime}, z_{h}^{\prime}\right) \approx_{o i}\left(w_{h}, z_{h}\right)$ such that $\left(\left(w_{h}^{\prime}, z_{h}^{\prime}\right),\left(w_{l}^{\prime}, z_{l}^{\prime}\right)\right) \in B$.
We call a bisimulation $B$ definite if $\left(\left(w_{h}, z_{h}\right),\left(w_{l}, z_{l}\right)\right) \in B$ and $\left(\left(w_{h}^{\prime}, z_{h}^{\prime}\right),\left(w_{l}, z_{l}\right)\right) \in B$ implies $\left(w_{h}, z_{h}\right)=\left(w_{h}^{\prime}, z_{h}^{\prime}\right)$.

We say that $\left(e_{h}, w_{h}\right)$ is bisimilar to $\left(e_{l}, w_{l}\right)$ relative to refinement mapping $m$, written $\left(e_{h}, w_{h}\right) \sim_{m}\left(e_{l}, w_{l}\right)$, if and only if there exists a definite $m$-bisimulation relation $B$ between $\left(e_{h}, w_{h}\right)$ and $\left(e_{l}, w_{l}\right)$ such that $\left(\left(w_{h},\langle \rangle\right),\left(w_{l},\langle \rangle\right)\right) \in B$.

The general idea of bisimulation is that two states are bisimilar if they have the same local properties (i.e., they are isomorphic) and each reachable state from the first state has a corresponding reachable state from the second state (and vice versa) such that the two successors are again bisimilar. Here, properties 1,2 , and 3 refer to static properties of $\left(w_{h}, z_{h}\right)$ and ( $w_{l}, z_{l}$ ). While property 1 directly establishes objective isomorphism of $\left(w_{h}, z_{h}\right)$ and $\left(w_{l}, z_{l}\right)$, property 2 establishes epistemic isomorphism between $\left(w_{h}, z_{h}\right)$ and all states $\left(w_{l}^{\prime}, z_{l}^{\prime}\right)$ that occur in $B$. As usual in bisimulations, we also require that if we follow a high-level transition of the system, there is a corresponding low-level transition (and vice versa). Here, such a transition may be an action that is executed (properties 4 and 5), or it may be an epistemic transition from the current state to another oi state (properties 6 and 7).

Our notion of bisimulation is similar to bisimulation for abstracting non-stochastic and objective basic action theories [2]. In comparison, the notion of objective isomorphism (property 1) and reachable states via actions (properties 4 and 5) are analogous, while epistemic isomorphism (property 2) and reachable states via observational indistinguishability (properties 6 and 7) have no corresponding counterparts.

Given a corresponding $m$-bisimulation, we want to show that $\left(e_{h}, w_{h}\right)$ is a model of a formula $\alpha$ iff $\left(e_{l}, w_{l}\right)$ is a model of the mapped formula $m(\alpha)$. To do so, we first show that this is true for static formulas, not considering programs. In the second step, we will show that the high-level and low-level
models induce the same program traces, which will then allow us to extend the statement to bounded formulas, which may refer to programs. We start with static formulas: ${ }^{11}$

Theorem 1. Let $\left(e_{h}, w_{h}\right) \sim_{m}\left(e_{l}, w_{l}\right)$ with definite $m$ bisimulation B. For every static formula $\alpha$ and traces $z_{h}, z_{l}$ with $\left(\left(w_{h}, z_{h}\right),\left(w_{l}, z_{l}\right)\right) \in B$ :

$$
e_{h}, w_{h}, z_{h} \models \alpha \text { iff } e_{l}, w_{l}, z_{l} \models m(\alpha)
$$

Using Theorem 1, we show that if $\left(e_{h}, w_{h}\right)$ is bisimilar to $\left(e_{l}, w_{l}\right)$, then they induce the same traces of a program $\delta$ :

Lemma 1. Let $\left(e_{h}, w_{h}\right) \sim_{m}\left(e_{l}, w_{l}\right)$ with $m$-bisimulation $B$, $\left(\left(w_{h}, z_{h}\right),\left(w_{l}, z_{l}\right)\right) \in B$, and $\delta$ be an arbitrary program.
(1) If $z_{l}^{\prime} \in\|m(\delta)\|_{e_{l}, w_{l}}^{z_{l}}$ is a low-level trace, then there is a high-level trace $z_{h}^{\prime} \in\|\delta\|_{e_{h}, w_{h}}^{z_{h}}$ such that $z_{h}^{\prime}=\left\langle a_{1}, \ldots, a_{n}\right\rangle$, $z_{l}^{\prime}=\left\langle m\left(a_{1}\right), \ldots, m\left(a_{n}\right)\right\rangle$, and $\left(z_{h} \cdot z_{h}^{\prime}, z_{l} \cdot z_{l}^{\prime}\right) \in B$.
(2) If $z_{h}^{\prime}=\left\langle a_{1}, \ldots, a_{n}\right\rangle \in\|\delta\|_{e_{h}, w_{h}}^{z_{h}}$ is a high-level trace, then there is a low-level trace $z_{l}^{\prime} \in\|m(\delta)\|_{e_{l}, w_{l}}^{z_{l}}$ such that $z_{l}^{\prime}=\left\langle m\left(a_{1}\right), \ldots, m\left(a_{n}\right)\right\rangle$ and $\left(z_{h} \cdot z_{h}^{\prime}, z_{l} \cdot z_{l}^{\prime}\right) \in B$.
Note that Lemma 1 would not hold if $\delta$ contained interleaved concurrency. Intuitively, this is because for a high-level program such as $a_{h}^{1} \| a_{h}^{2}$, the only valid high-level traces would be $\left\langle a_{h}^{1}, a_{h}^{2}\right\rangle$ and $\left\langle a_{h}^{2}, a_{h}^{1}\right\rangle$, i.e., one action is completely executed before the other action is started. On the other hand, with $m\left(a_{h}^{1}\right)=a_{l}^{1} ; a_{l}^{2}$ and $m\left(a_{h}^{2}\right)=a_{l}^{3} ; a_{l}^{4}$, we may obtain interleaved traces such as $\left\langle a_{l}^{1}, a_{l}^{3}, a_{l}^{2}, a_{l}^{4}\right\rangle$, which does not have a corresponding high-level trace. ${ }^{12}$

With Lemma 1, we extend Theorem 1 to bounded formulas:

Theorem 2. Let $\left(e_{h}, w_{h}\right) \sim_{m}\left(e_{l}, w_{l}\right)$. For all bounded formulas $\alpha$ and traces $z_{h}$, $z_{l}$ with $\left(z_{h}, z_{l}\right) \in B$ :

$$
e_{h}, w_{h}, z_{h} \models \alpha \text { iff } e_{l}, w_{l}, z_{l} \models m(\alpha)
$$

It directly follows that the high- and low-level models entail the same formulas after executing some program $\delta$ :

Corollary 1. Let $\left(e_{h}, w_{h}\right) \sim_{m}\left(e_{l}, w_{l}\right)$. Then for any high-level Golog program $\delta$ and static high-level formula $\beta$ :

$$
e_{l}, w_{l} \models[m(\delta)] m(\beta) \text { iff } e_{h}, w_{h} \models[\delta] \beta
$$

### 4.1 Sound and Complete Abstraction

In the previous section, we described properties of abstraction with respect to particular models $\left(e_{h}, w_{h}\right)$ and $\left(e_{l}, w_{l}\right)$. However, we are usually more interested in the relationship between a high-level BAT $\Sigma_{h}$ and a low-level BAT $\Sigma_{l}:{ }^{13}$

Definition 21 (Sound Abstraction). We say that $\Sigma_{h}$ is a sound abstraction of $\Sigma_{l}$ relative to refinement mapping $m$

[^7]

Figure 2: Bisimulation for the running example, where sets of states are summarized by the belief that they entail.
if and only if for each model $\left(e_{l}, w_{l}\right) \models \mathbf{K} \Sigma_{l} \wedge \Sigma_{l}$, there exists $a \operatorname{model}\left(e_{h}, w_{h}\right) \models \mathbf{K} \Sigma_{h} \wedge \Sigma_{h}$ such that $\left(e_{h}, w_{h}\right) \sim_{m}\left(e_{l}, w_{l}\right)$.

Conclusions by $\Sigma_{h}$ are consistent with $\Sigma_{l}$ :
Theorem 3. Let $\Sigma_{h}$ be a sound abstraction of $\Sigma_{l}$ relative to mapping $m$. Then, for every bounded formula $\alpha$, if $\mathbf{K} \Sigma_{h} \wedge$ $\Sigma_{h} \models \alpha$, then $\mathbf{K} \Sigma_{l} \wedge \Sigma_{l} \models m(\alpha)$.

While a sound abstraction ensures that any entailment of the high-level BAT $\Sigma_{h}$ is consistent with the low-level BAT $\Sigma_{l}$, the $\Sigma_{h}$ may have less information than $\Sigma_{l}$, e.g., $\Sigma_{h}$ may consider it possible that some program $\delta$ is executable, while $\Sigma_{l}$ knows that it is not. This leads to a second notion of abstraction:

Definition 22 (Complete Abstraction). We say that $\Sigma_{h}$ is a complete abstraction of $\Sigma_{l}$ relative to refinement mapping $m$ if and only if for each model $\left(e_{h}, w_{h}\right) \models \mathbf{K} \Sigma_{h} \wedge \Sigma_{h}$, there exists a model $\left(e_{l}, w_{l}\right) \models \mathbf{K} \Sigma_{l} \wedge \Sigma_{l}$ such that $\left(e_{h}, w_{h}\right) \sim_{m}$ $\left(e_{l}, w_{l}\right)$.

Indeed, if we have a complete abstraction, then $\Sigma_{h}$ must entail everything that $\Sigma_{l}$ entails:

Theorem 4. Let $\Sigma_{h}$ be a complete abstraction of $\Sigma_{l}$ relative to mapping $m$. Then, for every bounded formula $\alpha$, if $\mathbf{K} \Sigma_{l} \wedge \Sigma_{l} \models m(\alpha)$, then $\mathbf{K} \Sigma_{h} \wedge \Sigma_{h}=\alpha$.

The strongest notion is the combination of both:
Definition 23 (Sound and Complete Abstraction). We say that $\Sigma_{h}$ is a sound and complete abstraction of $\Sigma_{l}$ relative to refinement mapping $m$ if $\Sigma_{h}$ is both a sound and a complete abstraction of $\Sigma_{l}$ wrt $m$.

Theorem 5. Let $\Sigma_{h}$ be a sound and complete abstraction of $\Sigma_{l}$ relative to refinement mapping $m$. Then, for every bounded formula $\alpha, \mathbf{K} \Sigma_{h} \wedge \Sigma_{h} \models \alpha$ iff $\mathbf{K} \Sigma_{l} \wedge \Sigma_{l} \models m(\alpha)$.

Coming back to our example, we can show that $\Sigma_{g o t o}$ is indeed a sound and complete abstraction of $\Sigma_{\text {move }}$. Figure 2 shows an exemplary bisimulation for the running example. The single transition for goto of the high-level BAT is shown on the left. The agent knows that it is initially in the middle
and after doing goto(far), it is far away from the wall. Some corresponding transitions of the low-level BAT are shown on the right: Initially, the agent knows that it is at $\operatorname{Loc}(3)$, which is a bisimilar state to the initial high-level state (blue). Eventually, it reaches a state where it knows that it is at $\operatorname{Loc}(5)$, which is again a bisimilar state to the corresponding high-level state (orange). With Theorem 5, it follows that both BATs entail the same (mapped) formulas. Therefore, we can use $\Sigma_{\text {goto }}$ for reasoning and planning, e.g., we may write a high-level Golog program in terms of $\Sigma_{\text {goto }}$ and then use a classical Golog interpreter to find a ground action sequence that realizes the program. To continue the example, we may write a very simple abstract program $\delta_{h}$ that first moves to the wall if necessary and then moves back:

$$
\text { if } \neg A t(n e a r) \text { then } \text { goto(near) end if; goto(far) }
$$

If the robot is initially not near the wall (as in our example), the following sequence is a realization of the program:

$$
\langle\text { goto(near }), \text { goto }(\text { far })\rangle
$$

This high-level trace is much simpler than the trace of the low-level program shown in Equation 1. At the same time, as $\Sigma_{\text {goto }}$ is a sound and complete abstraction of $\Sigma_{\text {move }}$, this sequence may be translated to $\Sigma_{\text {move }}$ by applying the refinement mapping $m$ and the translated program then takes care of noisy sensors and actuators.

## 5 CONCLUSION

In this paper, we have presented a framework for abstraction of probabilistic dynamic domains. More specifically, in a first step, we have defined a transition semantics for Golog programs with noisy actions based on $\mathcal{D S}$, a variant of the situation calculus with probabilistic belief. We have then defined a suitable notion of bisimulation in the logic that allows the abstraction of noisy robot programs in terms of a refinement mapping from a high-level to a low-level basic action theory. This abstraction method allows to obtain a significantly simpler high-level domain, which can be used for reasoning or high-level programming without the need to deal with stochastic actions. Furthermore, the resulting programs and traces are much easier to understand, because they do not contain noisy actions and are often much shorter.

While abstractions need to be manually constructed, future work may explore abstraction generation algorithms based on [6,21]. A further extension might be to provide conditions under which we can modify the low-level program, e.g., with new sensors with different error profiles, without modifying the high-level program.

Interestingly, as the logics $\mathcal{D S}$ and $\mathcal{E S}$ are fully compatible for non-probabilistic formulas not mentioning noisy actions [7] and abstraction allows to get rid of probabilistic formulas and noisy actions, we may construct $\mathcal{E S}$ programs that are sound and complete abstractions of $\mathcal{D S}$ programs. This is a step towards cognitive robotics as envisioned by Reiter [23], where the classical non-probabilistic situation calculus machinery may prove entirely sufficient to define the behavior and termination of real-world robots.

## ACKNOWLEDGMENTS

Till was partly supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) - 2236/1 and the EU ICT-48 2020 project TAILOR (No. 952215). Part of this work was created during a research visit of Till at the University of Edinburgh, which was funded by the German Academic Exchange Service (DAAD). Vaishak was partly supported by a Royal Society University Research Fellowship, UK, and partly supported by a grant from the UKRI Strategic Priorities Fund, UK to the UKRI Research Node on Trustworthy Autonomous Systems Governance and Regulation (EP/V026607/1, 2020-2024).

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[^0]:    Proc. of the 22nd International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2023), A. Ricci, W. Yeoh, N. Agmon, B. An (eds.), May 29 - June 2, 2023, London, United Kingdom. (C) 2023 International Foundation for Autonomous Agents and Multiagent Systems (www.ifaamas.org). All rights reserved.

[^1]:    ${ }^{1}$ Note that although the definitions of formulas (Definition 3) and programs (Definition 4) mutually depend on each other, they are still well-defined: programs only allow static situation formulas and static situation formulas may not refer to programs.
    ${ }^{2}$ The original version of the logic also has an only-knowing modal operator O, which captures the idea that something and only that thing is known. For the sake of simplicity, we ignore this operator in our presentation.

[^2]:    ${ }^{3}$ We use "knowledge" and "belief" interchangeably, but do not require that knowledge be true in the real world (i.e., weak S5).
    ${ }^{4}$ The reason will become apparent later on. Intuitively, if we allow interleaved concurrency, then the low-level program could pause the execution of a high-level action and continue with a different highlevel action, possibly leading to different effects. This significantly complicates the formal treatment relating the probabilities of highlevel worlds to their low-level counterparts.

[^3]:    ${ }^{5}$ As above, although they depend on each other, the semantics is well-defined, as the transition semantics only refers to static formulas which may not contain programs.

[^4]:    ${ }^{6}$ We assume that free variables are universally quantified from the outside, $\square$ has lower syntactic precedence than the logical connectives, and $[\cdot]$ has the highest priority, so that $\square \operatorname{Poss}(a) \equiv \pi$ stands for $\forall a$. $\square(\operatorname{Poss}(a) \equiv \gamma)$ and $\square \operatorname{Poss}(a) \supset\left([a] F(\vec{x}) \equiv \gamma_{F}\right)$ stands for $\forall a, \vec{x} . \square\left(\operatorname{Poss}(a) \supset\left([a] F(\vec{x}) \equiv \gamma_{F}\right)\right)$.
    ${ }^{7}$ Claßen [11] proposes a different solution by allowing an action transition even if the action is impossible and then augmenting the program by guarding each action with a test $\operatorname{Poss}(a)$ ?. We prefer the presented solution because the transition semantics only allows actions that are actually possible without augmenting the program.

[^5]:    ${ }^{8}$ The unary move $(x)$ can be understood as abbreviation move $(x):=$ $\pi y \operatorname{move}(x, y)$, where nature nondeterministically picks the distance $y$ that the robot really moved (similarly for sonar()).

[^6]:    ${ }^{9}$ For the sake of brevity, we do not allow the robot to go to mid.
    ${ }^{10}$ The technical results do not hinge on this, but allowing arbitrary epistemic states would make the main results and proofs more tedious. For the general case, we need to set up for every distribution on the high level a corresponding distribution on the low level and establish a bisimulation for each of those pairs.

[^7]:    ${ }^{11}$ All proofs can be found in the extended technical report [19].
    ${ }^{12}$ While a limited form of concurrency could be permitted by only allowing interleaved execution of high-level actions (i.e., each $m(a)$ must be completely executed before switching to a different branch of execution), we omit this for the sake of simplicity.
    ${ }^{13}$ Notice that we require the real world to have the same physical laws as that believed by the agent, which is fairly standard. We do not require the agent knows everything about the real world, nor that the agent beliefs are also true in the real world.

