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by

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Abstract

We present a theory that systematically and causally links the well-being of native inhabitants with variation in the extent of the assimilation of migrants. Recent empirical findings are yielded as predictions of the theory.

Keywords: Migrants' assimilation; The well-being of native inhabitants

JEL classification: I31, J61

1. Introduction

In a study of the effect of migration into a country on the life satisfaction of the native population, Akay et al. (2014) present an array of findings. These findings will be summarized shortly. Akay et al. search for an explanation for the patterns observed. They dismiss several possible explanations, and suggest ad hoc ones for their reported findings. However, there is no unifying theory on offer, nor an overall model that can yield all the findings they report on assimilation-intensity and well-being. The usefulness of a unifying theory is obvious: it can generate an array of testable predictions and facilitate an orderly and logical interpretation of the findings obtained. A solid theory can also provide a clear guide as to what to look for in harvesting and employing the data. And it can pinpoint where gaps still exist in relating the data to the theory. It is the purpose of this paper to present a theory that systematically and causally links variation in the assimilation of migrants with impact on the well-being of native inhabitants.

The impact of migration on the well-being of the native inhabitants is one of the most intensively studied topics in migration research. A typical approach has been to estimate the elasticity of earnings and / or of the employment rate of the native inhabitants (or of subgroups of the native inhabitants) with respect to migration. A less common approach relates the gain to the native inhabitants to the tax proceeds collected from the migrants. An intermediate variable here is the host country’s specific human capital that the migrants choose to acquire. The idea (cf. Stark, 2010) is that the greater this capital, the greater the migrants’ productivity, the higher their earnings, the higher the income tax collected from them and, consequently, the greater the gain to the native inhabitants. The received literature then assesses the repercussions of migration for the well-being of the native inhabitants via moves occurring in the economic space, which is perfectly reasonable, though this is not the only space that matters. In this paper we build on the idea that the assimilation of migrants impinges on the well-being of the native inhabitants, but we take a different course. In the spirit of Akay et al. (2014), we look at how the well-being of the native inhabitants is impacted by moves made by the migrants in social space. The move places the migrants in the reference group of the natives. The “length” of the move (the intensity of assimilation) determines the migrants’ income.

We characterize a group of individuals (a population) by the multiset of the incomes of the members of the group, *X*. We employ the following operations and definitions. First, by a sum over a multiset we mean a sum over its elements with repetitions accounted for. As an example, for $X = \{1,1,2,2\}$ we have that $\sum_{x \in X} x = 1+1+2+2$. Second, by disjoint groups *X* and *Y* we mean that the sets of the individuals whose incomes constitute the multisets *X* and *Y* are disjoint. For example, having disjoint groups of migrants and of native inhabitants means that there is no individual who is both a migrant and a native inhabitant (although the sets of the incomes of the migrants and of the native inhabitants need not be disjoint). Third, for disjoint groups *X* and *Y* we define the combined group $X \vee Y$ as the multiset sum of *X* and *Y*.[1] For example, for $X = \{1,1,2\}$ and $Y = \{1,2,3\}$ we will have a combined group $X \vee Y = \{1,1,1,2,2,3\}$.[2]

For a reference group characterized by a multiset of incomes of its members, *X*, we define the relative deprivation of an individual whose income is *y* as

$$RD\left(y \mid X\right) \equiv \frac{1}{|X|} \sum_{x \in X} \max\left\{x - y, 0\right\},$$

where $|X|$ is the size of reference group *X* (the number of members who constitute reference group *X*).[3] From this definition it follows that the individual whose relative deprivation we measure need not be a member of the group with respect to which the individual's relative deprivation is calculated. For example, we may compute the relative deprivation of a native inhabitant, henceforth a native, with respect to a reference group of migrants. The aggregate relative deprivation of a group characterized by a multiset of incomes *Y*, with respect to a reference group characterized by a multiset of incomes *X*, is the sum of the levels of relative deprivation of the individuals in *Y*, calculated with respect to the individuals in *X*:

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[1] To simplify notation, we use $\vee$ rather than the often used â as our symbol of a multiset sum.

[2] Because a constellation that combines groups that are not disjoint is not considered in the analysis that follows, we use a notation that does not incorporate such a constellation either.

[3] Because we represent a reference group by the multiset of the incomes of its members, *X*, we refer to the multiset *X* also as a reference group.

$$ARD(Y \mid X) \equiv \sum_{y \in Y} RD\left(y \mid X\right) = \frac{1}{|X|} \sum_{y \in Y} \sum_{x \in X} \max\left\{x - y, 0\right\}.$$

Let the well-being of an individual depend positively on the individual's income, and negatively on the individual's relative deprivation, which arises from comparing his income with the incomes of others in his reference group(s); income is desirable, relative deprivation is undesirable. A brief foray into the subject of relative deprivation and a discussion of the significance of relative comparisons to well-being are in Sorger and Stark (2013). To enable us to draw inference from the aggregate relative deprivation of the native inhabitants to their well-being, we take the utility functions of the native inhabitants to be linear in relative deprivation with the same linear coefficient across all the native inhabitants. That is, the utility function of a native inhabitant who compares his income y with reference group X takes the form $u(y \mid X) = f(y) - aRD(y \mid X)$, where $a > 0$ and $f(\cdot)$ is a strictly increasing function.[4] We measure the well-being of the group of natives, N, whose members compare their incomes with the members of reference group X, as sum of the utility levels of the members of N:

$$W(N \mid X) = \sum_{y \in N} u(y \mid X) = \sum_{y \in N} f(y) - a \sum_{y \in N} RD(y \mid X) = \sum_{y \in N} f(y) - aARD(N \mid X).$$

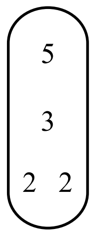
Holding constant the incomes of the natives, we can gauge the change in the well-being of the natives, brought about by assimilation of the migrants (which is tantamount to adding the migrants to the reference group of the natives), by the change in the aggregate relative deprivation of the natives. The magnitude of such a change is determined by the intensity of assimilation of the migrants. Different intensities affect the extent to which the migrants are added to the reference group of the native inhabitants, as well as the different positioning of the assimilating migrants in the native inhabitants' reference group.

With this measure of well-being in hand, we schematically present a series of constellations.

[4] All the results reported below will go through if the function describing the preferences towards absolute income, $f(\cdot)$, were to vary among the natives.

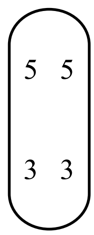
Constellation I: No assimilation

In Constellation I, the migrants, whose incomes are 1 each, constitute their own reference group, as do the natives whose incomes are 3 and 5. There is no assimilation. The migrants do not affect the relative deprivation nor the well-being of the natives.



Constellation II: Intermediate assimilation

In Constellation II, the migrants assimilate, thereby moving, so to speak, into the social space of the natives; the migrants are now included in the reference group of the natives. Because the intensity of assimilation is moderate, the migrants' incomes remain lower than the incomes of the natives. This assimilation lowers the relative deprivation of native "3" and does not affect (leaves at zero) the relative deprivation of native "5." Thus, the aggregate relative deprivation of the natives is lowered and their well-being rises.

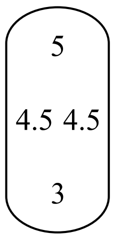


Constellation III: Complete assimilation

In Constellation III, the migrants assimilate perfectly or "completely," replicating the incomes of the natives. This assimilation leaves the aggregate relative deprivation and well-being of the natives intact.

In sum: when the migrants' level of assimilation is intermediate (Constellation II), the natives' well-being is affected positively; when the migrants do not assimilate (Constellation I), the effect on the well-being of the natives is "zero;" and when the migrants assimilate completely (Constellation III), the effect on the natives' well-being is "zero" too. These are exactly the results obtained by Akay et al. In their words: "We find that the positive effect of immigration on natives' life satisfaction is a function of the assimilation of immigrants in the region. Immigration's well-being effect is higher in regions with intermediate assimilation levels and is essentially zero in regions with no or complete assimilation" (p. 72).

Our proposed theory yields an additional prediction, not tested by Akay et al.: when the assimilation level of the migrants is as per constellation IV, the natives' well-being is lowered:



Constellation IV: Intensive assimilation

In Constellation IV, the migrants assimilate more intensively than in Constellation II. An assimilation of such an extent increases the relative deprivation of "3" and leaves unchanged the relative deprivation of "5." The aggregate relative deprivation of the natives increases and their well-being takes a beating.

We next generalize the preceding examples of migrants' assimilation and its impact on the well-being of the natives. To this end, we specify a condition under which assimilation is detrimental or beneficial to the well-being of the natives: for assimilation to favor the well-being of the natives, the relative deprivation experienced by the natives from comparison with the assimilating migrants has to be lower than the relative deprivation experienced by the natives from a comparison with fellow natives.

2. A general framework

To begin with, we state and prove a lemma that will enable us to calculate the aggregate relative deprivation of a group of individuals with respect to a combined reference group.

Lemma 1. For disjoint groups *X* and *Y*, the aggregate relative deprivation of group *Z* calculated with respect to the combined reference group $X \vee Y$ is

$$ARD(Z \mid X \vee Y) = \frac{|X|}{|X \vee Y|} ARD(Z \mid X) + \frac{|Y|}{|X \vee Y|} ARD(Z \mid Y). \tag{1}$$

Proof. For any individual $z \in Z$ we have that

$$\begin{aligned} RD(z \mid X \vee Y) &= \frac{1}{|X \vee Y|} \sum_{x \in X \vee Y} \max\{x - z, 0\} \\ &= \frac{1}{|X \vee Y|} \left\{ \sum_{x \in X} \max\{x - z, 0\} + \sum_{y \in Y} \max\{y - z, 0\} \right\} \\ &= \frac{|X|}{|X \vee Y|} \frac{1}{|X|} \sum_{x \in X} \max\{x - z, 0\} + \frac{|Y|}{|X \vee Y|} \frac{1}{|Y|} \sum_{y \in Y} \max\{y - z, 0\} \\ &= \frac{|X|}{|X \vee Y|} RD(z \mid X) + \frac{|Y|}{|X \vee Y|} RD(z \mid Y). \end{aligned}$$

Summing up over all $z \in Z$ yields (1). □

Example 1. Let $X = Z = \{1,3\}$, $Y = \{5\}$. Then

$$ARD(Z \mid X) = ARD(\{1,3\} \mid \{1,3\}) = \frac{1}{|\{1,3\}|}(3-1) = 1,$$

$$ARD(Z \mid Y) = ARD(\{1,3\} \mid \{5\}) = \frac{1}{|\{5\}|}[(5-1)+(5-3)] = 6,$$

and

$$\begin{aligned}
ARD(Z \mid X \vee Y) &= ARD(\{1,3\} \mid \{1,3,5\}) = \frac{1}{|\{1,3,5\}|}[(5-1)+(5-3)+(3-1)] \\
&= \frac{1}{|\{1,3,5\}|}(3-1) + \frac{1}{|\{1,3,5\}|}[(5-1)+(3-1)] \\
&= \frac{|\{1,3\}|}{|\{1,3,5\}|}\frac{1}{|\{1,3\}|}(3-1) + \frac{|\{5\}|}{|\{1,3,5\}|}\frac{1}{|\{5\}|}[(5-1)+(3-1)] \\
&= \frac{|\{1,3\}|}{|\{1,3,5\}|}ARD(\{1,3\} \mid \{1,3\}) + \frac{|\{5\}|}{|\{1,3,5\}|}ARD(\{1,3\} \mid \{5\}) \\
&= \frac{|X|}{|X \vee Y|}ARD(Z \mid X) + \frac{|Y|}{|X \vee Y|}ARD(Z \mid Y).
\end{aligned}$$

Remark 1. Unlike the *ARD* calculated with respect to a combined reference group (cf. Lemma 1), the *ARD* of a combined population, say combined population $X \vee Y$, where X and Y are disjoint populations, calculated with respect to reference group Z, is a simple sum of the levels of the *ARD* of populations X and Y calculated with respect to Z, namely

$$ARD(X \vee Y \mid Z) = ARD(X \mid Z) + ARD(Y \mid Z).$$

Example 2. Let $X = \{1\}, Y = \{3\}, Z = \{5\}$. Then

$$\begin{aligned}
ARD(X \vee Y \mid Z) &= ARD(\{1,3\} \mid \{5\}) = \frac{1}{|\{5\}|}[(5-1)+(5-3)] = 6 \\
&= \frac{1}{|\{5\}|}(5-1) + \frac{1}{|\{5\}|}(5-3) = ARD(\{1\} \mid \{5\}) + ARD(\{3\} \mid \{5\}) \\
&= ARD(X \mid Z) + ARD(Y \mid Z).
\end{aligned}$$

To track the effect of the assimilation of migrants on the well-being of the natives, we define a group (population) N of natives, and a group (population) M of migrants, where $|N| > 0$ and $|M| > 0$ denote the size of groups N and M, respectively. The fact that N and M are disjoint groups implies that

$$|N \vee M| = |N| + |M|. \tag{2}$$

We measure the change in the aggregate relative deprivation of the population of the natives caused by the assimilation of the migrants; the very act of assimilation implies that the assimilating migrants are included in the reference group of the natives.

Claim 1. The aggregate relative deprivation of the natives increases in the wake of assimilation of the migrants if and only if the aggregate relative deprivation of the natives calculated with respect to the reference group of the migrants is higher than the aggregate relative deprivation of the natives calculated with respect to fellow natives as a reference group, namely, iff

$$ARD(N \mid N \vee M) > ARD(N \mid N) \Leftrightarrow ARD(N \mid M) > ARD(N \mid N).$$

Conversely, the aggregate relative deprivation of the natives decreases in the wake of assimilation of the migrants, namely $ARD(N \mid N \vee M) < ARD(N \mid N)$, iff $ARD(N \mid M) < ARD(N \mid N)$.

Proof. Using Lemma 1 and equation (2), we have that

$$\begin{aligned}
& ARD(N \mid N \vee M) > ARD(N \mid N) \\
& \Leftrightarrow \frac{|N|}{|N \vee M|} ARD(N \mid N) + \frac{|M|}{|N \vee M|} ARD(N \mid M) > ARD(N \mid N) \\
& \Leftrightarrow \frac{|M|}{|N \vee M|} ARD(N \mid M) > \frac{|N \vee M|}{|N \vee M|} ARD(N \mid N) - \frac{|N|}{|N \vee M|} ARD(N \mid N) \\
& \Leftrightarrow \frac{|M|}{|N|+|M|} ARD(N \mid M) > \frac{|N|+|M|}{|N|+|M|} ARD(N \mid N) - \frac{|N|}{|N|+|M|} ARD(N \mid N) \\
& \Leftrightarrow \frac{|M|}{|N|+|M|} ARD(N \mid M) > \frac{|M|}{|N|+|M|} ARD(N \mid N) \\
& \Leftrightarrow ARD(N \mid M) > ARD(N \mid N).
\end{aligned}$$

The proof for the case in which the *ARD* of the natives decreases is analogous. □

Claim 1 nicely aligns with intuition. In the case of intensive assimilation, the well-being of the natives will not be lowered when $ARD(N \mid N) \geq ARD(N \mid M)$, namely, when the aggregate relative deprivation of the natives is relatively high in comparison with the aggregate relative deprivation of the natives calculated with respect to the migrants. Such a configuration can arise when there is a relatively large dispersion of the incomes of the natives. Under intensive assimilation (which, however, income-wise does not place the migrants above all the natives), the relative deprivation experienced by the "wealthiest" natives is not affected. Changes in the well-being of the natives arise from changes in the relative deprivation of the poorer section of the native population. When the incomes of the assimilating migrants are relatively "close" to the incomes of the "poor" natives (namely, when the incomes of the assimilating migrants do not exceed the incomes of the poor natives by much), the "poor" natives can become less relatively deprived as their reference group expands with not-too-wealthy individuals, and the overall dispersion in incomes that they experience will then be reduced. Therefore, the well-being of the natives can increase even if the assimilation of the migrants is intensive and their incomes exceed the incomes of some natives.

We now show how Claim 1 can be applied to assess the repercussions of the four constellations of the assimilation of migrants presented in the Introduction.

2.1 Constellation I: No assimilation

With no assimilation, the migrants do not affect the relative deprivation of the natives; the migrants are outside the reference group of the natives and, thus, the aggregate relative deprivation among the natives, which is

$$ARD(N \mid N) = \frac{1}{|N|} \sum_{z \in N} \sum_{y \in N} \max\{y - z, 0\},$$

does not change in the presence of the migrants. The migrants do not affect the well-being of the natives.

2.2 Constellation II: Intermediate assimilation

We assume that the incomes of the assimilating migrants are lower than or equal to the incomes of the natives: $x \le y$ for any $x \in M$, $y \in N$. Including the migrants in the reference group of the natives, we compute the aggregate relative deprivation of the natives, *N*, with respect to the combined reference group of the migrants and the natives, $N \vee M$. Drawing on the fact that from the assumption $x \le y$ for any $x \in M$, $y \in N$ it follows that $\max\{x - y, 0\} = 0$, we have that $RD(y \mid M) = 0$ for all $y \in N$ and, thus,

$$ARD(N \mid N) \ge ARD(N \mid M) = 0$$

with the inequality being strict if the incomes of the natives are not all the same, in which case $ARD(N \mid N) > 0$. Thus, using Claim 1, we have that

$$ARD(N \mid N \vee M) \le ARD(N \mid N).$$

An alternative way of obtaining this result is to draw on Lemma 1 and equation (2). Indeed,

$$ARD(N \mid N \vee M) = \frac{|N|}{|N \vee M|} ARD(N \mid N) + \frac{|M|}{|N \vee M|} ARD(N \mid M)$$
$$= \frac{|N|}{|N \vee M|} ARD(N \mid N) = \frac{|N|}{|N|+|M|} ARD(N \mid N) \leq ARD(N \mid N).$$

Consequently, with an intermediate assimilation of the migrants, the aggregate relative deprivation of the natives decreases and their well-being rises (or it stays the same if to begin with their *ARD* was zero).

Remark 2. It is also of interest to note that the positive impact of assimilation on the well-being of the natives (arising from a decrease in the natives' *ARD*) is higher, the bigger the difference:

$$\Delta ARD \equiv ARD(N \mid N) - ARD(N \mid N \vee M) = \frac{|M|}{|N|+|M|} ARD(N \mid N).$$

Treating ΔARD as a function of the size of the migrant population $|M|$, it follows that

$$\frac{d\Delta ARD}{d|M|} = \frac{|N|}{\left(|N|+|M|\right)^2} ARD(N \mid N),$$

which, unless $ARD(N \mid N) = 0$, is positive, implying that in this Constellation, the well-being of the natives increases with the number of the assimilating migrants.

2.3 Constellation III: Complete assimilation

We now assume that the extent of the migrants' assimilation is such that they replicate the incomes of the natives; that is, for any $y \in N$ there exists exactly one $x \in M$ such that $y = x$; hence, $M = N$. In such a case, $ARD(N \mid N) = ARD(N \mid M)$ and, thus, from Claim 1 we have that

$$ARD(N \mid N \vee M) = ARD(N \mid N).$$

This result can also be obtained by drawing on Lemma 1 and equation (2). Indeed,

$$ARD(N \mid N \vee M) = \frac{|N|}{|N \vee M|} ARD(N \mid N) + \frac{|M|}{|N \vee M|} ARD(N \mid M)$$
$$= \frac{|N|}{2|N|} 2 \cdot ARD(N \mid N) = ARD(N \mid N).$$

Thus, complete assimilation does not change the aggregate relative deprivation of the natives. Consequently, their well-being remains unchanged.

2.4 Constellation IV: Intensive assimilation

Here we divide the population of the natives into two disjoint groups: $N_- = \{y \in N : y < x, \forall x \in M\}$, and $N_+ = \{y \in N : y \geq x, \forall x \in M\}$, such that $N = N_- \vee N_+$. That is, N_- consists of natives whose incomes are lower than the incomes of all the assimilating migrants, and N_+ consists of natives whose incomes are the same as or higher than the incomes of all the assimilating migrants. In the following lemma we identify the minimal level of the mean income of migrants, x_0, for which in such a configuration of incomes, the aggregate relative deprivation of the natives increases after assimilation.

Lemma 2. For subpopulations N_- and N_+, we have that $ARD(N \mid N \vee M)$ $> ARD(N \mid N)$ if and only if

$$\overline{x} > \overline{y} + \frac{|N_-| ARD(N_- \mid N_-) + |N_+| ARD(N_+ \mid N_+)}{|N||N_-|} \equiv x_0, \tag{3}$$

where $\overline{x} = \frac{1}{|M|}\sum_{x \in M} x$ and $\overline{y} = \frac{1}{|N|}\sum_{y \in N} y$ are, respectively, the average income of the population of the migrants, and the average income of the population of the natives.

Proof. Because $RD(y \mid M) = 0$ for any $y \in N_+$, we have that

$$ARD(N \mid M) = \frac{1}{|M|}\sum_{z \in N_-}\sum_{x \in M} \max\{x - z, 0\} = \frac{1}{|M|}\sum_{z \in N_-}\sum_{x \in M}(x - z)$$
$$= \frac{1}{|M|}\sum_{z \in N_-}\sum_{x \in M} x - \frac{1}{|M|}\sum_{z \in N_-}\sum_{x \in M} z = |N_-|\,\overline{x} - \sum_{z \in N_-} z.$$

In turn, using Lemma 1, Remark 1, and the fact that $ARD(N_+ \mid N_-) = 0$, we have that

$$\begin{aligned}
&ARD(N \mid N) = ARD(N_- \vee N_+ \mid N_- \vee N_+) \\
&= ARD(N_- \mid N_- \vee N_+) + ARD(N_+ \mid N_- \vee N_+) \\
&= \frac{|N_-|}{|N_- \vee N_+|} ARD(N_- \mid N_-) + \frac{|N_+|}{|N_- \vee N_+|} ARD(N_- \mid N_+) \\
&\quad + \frac{|N_-|}{|N_- \vee N_+|} ARD(N_+ \mid N_-) + \frac{|N_+|}{|N_- \vee N_+|} ARD(N_+ \mid N_+) \\
&= \frac{|N_-|}{|N|} ARD(N_- \mid N_-) + \frac{|N_+|}{|N|} ARD(N_- \mid N_+) + \frac{|N_+|}{|N|} ARD(N_+ \mid N_+).
\end{aligned}$$

Expanding the middle term in the last line, we get

$$\begin{aligned}
&\frac{|N_+|}{|N|} ARD(N_- \mid N_+) = \frac{1}{|N|} \sum_{z \in N_-} \sum_{y \in N_+} \max\{y - z, 0\} = \frac{1}{|N|} \sum_{z \in N_-} \sum_{y \in N_+} (y - z) \\
&= \frac{1}{|N|} \sum_{z \in N_-} \left(\sum_{y \in N_+} y - |N_+| z \right) = \frac{1}{|N|} \left(|N_-| \sum_{y \in N_+} y - |N_+| \sum_{z \in N_-} z \right) \\
&= \frac{1}{|N|} \left[|N_-| \sum_{y \in N_+} y - \left(|N| - |N_-| \right) \sum_{z \in N_-} z \right] = \frac{1}{|N|} \left[|N_-| \left(\sum_{y \in N_+} y + \sum_{z \in N_-} z \right) - |N| \sum_{z \in N_-} z \right] \\
&= \frac{|N_-|}{|N|} \sum_{y \in N} y - \sum_{z \in N_-} z = |N_-| \, \overline{y} - \sum_{z \in N_-} z,
\end{aligned}$$

and thus, we have that $ARD(N \mid M) > ARD(N \mid N)$ which, by Claim 1, is equivalent to $ARD(N \mid N \vee M) > ARD(N \mid N)$, if and only if

$$\begin{aligned}
&ARD(N \mid M) - ARD(N \mid N) \\
&= |N_-| \, \overline{x} - \sum_{z \in N_-} z - \frac{|N_-|}{|N|} ARD(N_- \mid N_-) - \frac{|N_+|}{|N|} ARD(N_- \mid N_+) - \frac{|N_+|}{|N|} ARD(N_+ \mid N_+) \\
&= |N_-| \, \overline{x} - \sum_{z \in N_-} z - \frac{|N_-|}{|N|} ARD(N_- \mid N_-) - |N_-| \, \overline{y} + \sum_{z \in N_-} z - \frac{|N_+|}{|N|} ARD(N_+ \mid N_+) \\
&= |N_-| (\overline{x} - \overline{y}) - \frac{1}{|N|} \left[|N_-| \, ARD(N_- \mid N_-) + |N_+| \, ARD(N_+ \mid N_+) \right] > 0.
\end{aligned}$$

The inequality in the last line is equivalent to (3). □

The critical value of the mean income of migrants, x_0, which leads to an increase of the natives' *ARD* is higher (meaning that it is becoming more difficult for migrants to lower the well-being of the natives - in terms of raising the natives' *ARD* - as causing that requires the migrants to attain a higher average income) (I) the higher the average income of the natives, and (II) the higher the magnitudes of the *ARD* of the constituent subpopulations of the natives. Whereas (I) seems to be quite obvious, Lemma 2 reveals an interesting relationship with respect to (II). Clearly, a high dispersion in incomes in any of the two subpopulations of the natives results in a high value of x_0. Thus, the higher the *ARD* experienced by the natives in any of their two subpopulations, the less likely it is that the assimilation of the migrants will lower the well-being of the natives. Moreover, Lemma 2 indicates that the ratio of the sizes of the two subpopulations of the natives determines which *ARD* - that of the richer subpopulation or that of the poorer subpopulation - influences x_0 more strongly. When the number of natives who earn more than the migrants is higher than the number of natives who earn less than the migrants ($|N_+|>|N_-|$), then the *ARD* among the richer natives plays a bigger role in determining x_0; that is, a higher weight is assigned to the *ARD* of the richer natives in comparison with the weight assigned to the poorer natives.

Revisiting the example of the incomes of Constellation IV in the Introduction, we have $N=\{3,5\}$, $N_-=\{3\}$, $N_+=\{5\}$, and $M=\{4.5,4.5\}$. Thus, the aggregate relative deprivation of the natives calculated with respect to their own population as a reference group, is

$$ARD(N\,|\,N)=\sum_{y\in N} RD(y\,|\,N)=\frac{1}{2}\cdot(5-3)=1,$$

and the aggregate relative deprivation of the natives, calculated with respect to the migrants as a reference group, is

$$ARD(N\,|\,M)=\sum_{y\in N} RD(y\,|\,M)=\frac{1}{2}\left[(4.5-3)+(4.5-3)\right]=\frac{3}{2}.$$

Thus,

$$ARD(N\,|\,M)>ARD(N\,|\,N),$$

and, indeed, the aggregate relative deprivation of the natives increases after the inclusion of the migrants in the reference group of the natives, namely,

$$ARD(N \mid N \vee M) = \sum_{y \in N} RD(y \mid N \vee M) = \frac{1}{4}\big[(5-3)+(4.5-3)+(4.5-3)\big]$$
$$= \frac{5}{4} > 1 = ARD(N \mid N).$$

We also verify that (3) is satisfied:

$$\frac{1}{2}\left(4.5+4.5\right) = 4.5 = \overline{x} > x_0 = 4 = \frac{1}{2}\left(3+5\right) + \frac{1\cdot 0 + 1\cdot 0}{2\cdot 1}$$
$$= \overline{y} + \frac{|N_-| \, ARD(N_- \mid N_-) + |N_+| \, ARD(N_+ \mid N_+)}{|N| |N_-|}.$$

3. Conclusion

Inspired by the findings of Akay et al. (2014), we formulated a simple theory that enables us to predict the impact of the assimilation of migrants on the well-being of the native inhabitants. Founded on the concept of relative deprivation, the theory tracks how the inclusion of migrants in the comparison group of the natives affects the well-being of the natives. We find that the crucial determinant in this regard is the relationship between the relative deprivation experienced by the natives from comparisons with other natives, and the relative deprivation experienced by the natives from comparisons with the assimilating migrants.

In this paper, the decisions of the migrants as to how much effort they should put into assimilating, and what influences these decisions are not modeled. Here, as in Akay et al., our interest has been in assessing how different levels of assimilation interact with the natives' sense of well-being. Examples of studies that model the migrants' optimal assimilation effort are Fan and Stark (2007), Stark and Dorn (2013), and Stark and Jakubek (2013).

Our purpose in this paper has been to assess the impact of alternative intensities of assimilation on the well-being of the natives, and to this end we have sought to lay out all feasible intensity categories. By construction, this is a comparative statics approach. We did not inquire how the natives will react when subjected to alternative assimilation intensities. For example, it could be the case that natives who are becoming more relatively deprived will choose to increase their work effort, obtain higher earnings, and thereby contain the increase in their relative deprivation. Similarly, in the spirit of Sorger and Stark (2013), natives who experience reduced relative deprivation as a consequence of assimilation may choose to reduce their work effort and earnings, retaining though their pre-assimilation level of overall well-being. Dynamics of this type could be an intriguing topic of follow-up research.

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