A Decentralized Distribution Market Mechanism Considering Renewable Generation Units with Zero Marginal Costs

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Abstract **A key feature of electricity generation in a distribution network is manifested by renewable generation with zero marginal cost. Existing market mechanisms are likely to fail in supporting such decentralized transactions while providing a reasonable price signal to compensate for the investment cost of renewable generators. Given this background, this paper first describes an average pricing market (APM) mechanism for pricing zero marginal cost renewable generation outputs in the distribution network. Then, a decentralized formulation of the APM mechanism is derived using the alternating direction method of multipliers (ADMM). Convergence of the decentralized mechanism can be guaranteed under some mild conditions for parameter setting. Finally, case studies are carried out to demonstrate the presented market mechanism. Simulation results show that the problem of always bidding a zero price by renewable generators in some existing markets can be avoided. The presented method also provides a solution for organizing decentralized electricity transactions in the distribution network and can converge to similar results with those obtained by the centralized one, with a relative error less than 5%.**

***Index Terms*—electricity distribution market, decentralized market mechanism, zero marginal cost, privacy of participants, network constraint.**

I. Introduction

TO enable end-user benefits from distributed renewable generation and manage the behaviors of prosumers, the establishment of electricity distribution markets has gained widespread interests. In particular, some trials and projects on peer to peer (P2P) electricity trading in distribution systems have been carried out in several countries, aiming to increase the engagement of customers in energy transactions. As an important basis that supports trading in the distribution network, the design of market mechanism attracts significant research concerns [1].

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There are already some publications on electricity distribution markets. In [2-6], the distribution market is modelled as an intermediate entity between the wholesale electricity market and distribution network customers. Through coordinating communications between the independent system operator (ISO) and proactive customers, a distribution market operator (DMO) helps customers participate in the wholesale electricity market. The distribution locational marginal price (DLMP) is employed for market settlement in [3, 4, 6], and is similar to the concept of locational marginal price (LMP) in the wholesale electricity market. Due to relatively high power losses, voltage volatilities, and phase imbalances in the distribution network, the determination of DLMP is challenging. Therefore, a three-phase alternating current (AC) optimal power flow (OPF) based approach is developed to define and calculate DLMP in [7]. A framework for designing and simulating electric distribution systems and day-ahead electricity markets in UK is studied in [8], however all generators are assumed to be price-takers and offer at their marginal costs.

Besides, in [9] and [10], decentralized energy trading frameworks are studied for the independent system operator (ISO) and the distribution network operator (DNO) respectively to help organize the transaction between renewable generators with uncertain outputs and price-responsive load aggregators. However, all the market participants can only passively accept the trading prices from the ISO rather than setting up the bidding prices by themselves, which is the focus of this paper. In [11], a day-ahead decentralized coordination method with appliance scheduling and energy sharing among smart homes is proposed, but the pricing of electricity is modelled by a quadratic function of the sold power, while the zero marginal cost of renewable generation is overlooked.

As a special electric energy system, a micro grid (MG) has advantages in accommodating distributed generation resources. Existing energy management algorithms for multi-MG systems are usually based on a hierarchical structure [12-17], where the lower level problem addresses the optimal scheduling of generation resources within a MG and the upper level problem deals with the coordinative trading between the multi-MGs and the wholesale electricity market, respectively.

Various coordination strategies for multi-MGs have been proposed. In [12], under a distributed optimization framework, the bargaining of cooperative MGs with each other to reach a fair and Pareto-optimal solution is modelled using the concept of the Nash bargaining solution (NBS). In [13], a centralized inter-MG transactive market is established in order to coordinate the energy supply and demand among MGs. A three-level hierarchical control framework is proposed in [15] to coordinate power exchanges among MGs in a community MG. A community MG represents a cluster of neighbouring MGs which are linked via interlinking-converters. In [16], a MG central controller (MGCC) is presented to coordinate the operation of multi-MGs through solving a centralized scheduling problem, and the dual theory is adopted to decentralize the original model by relaxing the coupled constraint among multi-MGs. A priority-based approach for energy trading with a time interval of 15min among multi-MGs is proposed in [17], and the presented network management system (DNMS) first collects excess energy from producers and receives demand requests from consumers, and then allocates energy to consumers based on the amount of energy being requested and the priority index (PI) of each consumer. A marginal cost based electricity pricing model is proposed in [18] for coordinating energy trading among multi-MGs, with an objective to minimize the total cost. The special characteristics of trading in the distribution market are neglected by all existing publications, especially the zero marginal cost of renewable energy generation. In existing research [19, 20], the conventional quadratic cost function for thermal generators is still adopted to describe the renewable energy generation cost, and this is not appropriate.

Different from traditional generation technologies, renewable generation is capital-intensive but has zero fuel cost [21]. Meanwhile, one of the objectives of the current marginal cost based electricity market design is to efficiently price the short-term operation cost of a power system [22], but a renewable generator makes decision mainly based on its long-term cost, such as capital and maintenance costs. Therefore, the marginal cost based market mechanism could not reveal the real market value and generation cost of a renewable generator.

The contributions of this paper are summarized as below:

(1) Existing electricity markets are designed based on the marginal cost and marginal revenue theory, while in the scenario with all participating generators being renewable ones, these market mechanisms will fail to price renewable energy generation properly. The presented market mechanism in this paper is aimed to solve this problem, which has not been tackled by existing publications.

(2) Although endeavours have been devoted to developing P2P energy trading systems in some existing publications, an effective market mechanism has not yet been proposed to support such decentralized transactions. This paper proposed a new market mechanism which enables prosumers in an electricity distribution network to trade electricity without a middle man/entity or agent.

(3) Simulation results show that the problem of always bidding a zero price by renewable generators in some existing electricity markets can be avoided. Meanwhile, the proposed decentralized market mechanism can converge to basically the same results with those obtained by the centralized one, with a relative error less than 5%. When transmission congestion happens, these two models can achieve almost the same solution, with the deviation less than 1.5 %.

The rest of the paper is structured as follows. Section II introduces the market mechanism for pricing renewable generations with zero marginal costs. Then, the mathematical formulation of the decentralized distribution market mechanism is proposed in section III. Section IV analyses the safety of transaction under the proposed decentralized market and section V provides the case study results and discussions. Finally, the paper is concluded in section VI.

II. Description of Market Mechanism for Renewable Generation Units with Zero Marginal Costs

A. The Centralized Average Pricing Market Mechanism

In our previous work presented in [23], an APM mechanism for clearing the market bidding of renewable generation with zero marginal costs in the distribution network is proposed. The APM market mechanism is a double-sided bidding one where the i^{th} ($i \in N$) consumer bids a price-quantity pair (r_i^{b}, p_i^{b}) and the j^{th} ($j \in M$) producer offers a price-quantity pair (r_j^{s}, p_j^{s}) to the market. If a consumer wins, he/she will purchase electricity from the distribution market at the market clearing price (MCP), otherwise he/she will still purchase electricity at the incumbent retail price from the grid. Similarly, for a producer, if he/she wins, this producer will sell electricity to the distribution market at the MCP, otherwise he/she will have to sell electricity to the power grid utility company while being paid by the feed-in tariff.

A strategy is a dominant one if it maximizes the agent's expected utility for all possible strategies of other agents [16, 24]. In the APM mechanism, both consumers and producers have dominant strategies. The dominant strategies of participants involve the honest reporting of their self-evaluations and the market clearing mechanism is depicted in Fig.1.

In Fig.1, the weighted average $\bar{r}$ of participants' bid prices is adopted as the MCP, where the weighting factors are their bid quantities. Since it is the average price that acts as the MCP, the presented market clearing mechanism is named as the average pricing market (APM).

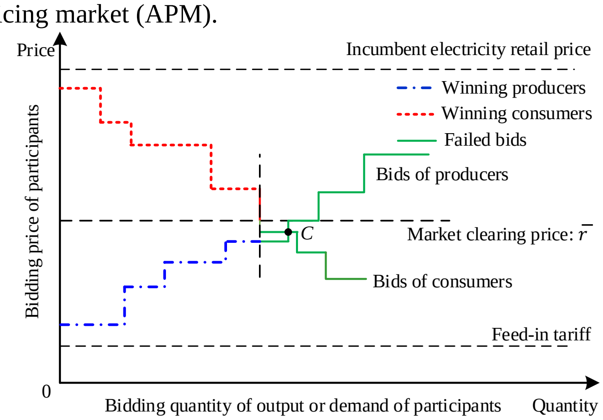


Fig.1. Schematic diagram of the presented market mechanism.

Market rules are elaborated as follows. It is assumed that the i^{th} consumer bids to the market at (r_i^{b}, p_i^{b}) ($i \in N$) and the j^{th} producer offers to the market at (r_j^{s}, p_j^{s}) ($j \in M$). The i^{th} consumer wins the auction only when its bid price r_i^{b} is larger than $\bar{r}$. On the contrary, the j^{th} producer wins the auction only when its offer price r_j^{s} is smaller than $\bar{r}$. Besides, when a participant bids at a price close to $\bar{r}$, there is a risk for this participant to be excluded from trading because the market needs to reach equilibrium between demand and supply. As shown in Fig.1, the last winning producer is the marginal participant under the APM. It can be found that even if there is a producer with a bidding price at $\bar{r}$, this producer still will lose the bidding because the market will finally come to the equilibrium state between supply and demand. However, if the bid quantity of the last winning consumer increases, the producer who bids at $\bar{r}$ could become the marginal unit, namely there is a possibility for the participant to be excluded from trading due to the necessity of equilibrium between demand and supply. Because both the bidding parameters of participants and market clearing outcomes are unknown beforehand, all possible scenarios should be therefore considered when setting the market rules.

The market clearing model of the APM mechanism is given as follows.

$$\max \quad f(p_i^{\text{cb}}, p_j^{\text{cs}}) = \sum_{i\in N} p_i^{\text{cb}} \cdot r_i^{\text{b}} - \sum_{j\in M} p_j^{\text{cs}} \cdot r_j^{\text{s}} \tag{1}$$

$$\text{s.t.} \quad \bar{r} = \Big(\sum_{i\in N} r_i^{\text{b}} \cdot p_i^{\text{b}} + \sum_{j\in M} r_j^{\text{s}} \cdot p_j^{\text{s}}\Big) \Big/ \Big(\sum_{i\in N} p_i^{\text{b}} + \sum_{j\in M} p_j^{\text{s}}\Big) \tag{2}$$

$$\forall i \in N, p_i^{\text{cb}} = 0 \text{ if } r_i^b < \bar{r} \tag{3}$$

$$\forall j \in M, p_j^{\text{cs}} = 0 \text{ if } r_j^{\text{s}} > \bar{r} \tag{4}$$

$$\sum_{i\in N} p_i^{\text{cb}} - \sum_{j\in M} p_j^{\text{cs}} = 0 \tag{5}$$

$$-P_l^{\max} \le \sum_{i\in N} p_i^{\text{cb}} d_{l,i} + \sum_{j\in M} p_j^{\text{cs}} d_{l,j} \le P_l^{\max}, (l = 1,2,\cdots,L) \tag{6}$$

$$0 \le p_i^{\text{cb}} \le p_i^{\text{b}},\ 0 \le p_j^{\text{cs}} \le p_j^{\text{s}} \tag{7}$$

where p_i^{cb} / p_j^{cs} indicates the dispatched load / generation output of the i^{th} consumer / j^{th} producer; $P_l^{\max}$ is the power transfer limit of branch l; L is the set of branches in the distribution network; $d_{l,j}$ denotes the power transfer distribution factor (PTDF) which is used to indicate the relative change of the active power that occurs on a particular branch l due to actual power change at node j; Eqn. (1) is the maximization of social welfare; Eqn. (2) calculates the MCP; Eqn. (3) and Eqn. (4) define the rules of the participants being dispatched; Eqn. (5) means that market clearing ends at an equilibrium state; Eqn. (6) represents distribution network constraints; Eqn. (7) represents the constraints on decision variables.

The application of PTDFs has been proved to be useful and makes it possible to model the impacts of electricity transactions on branch flows in a linear way. PTDFs can be calculated using either an alternating current (AC) or direct current (DC) power flow model [25]. In particular, since the distribution network is characterized by the high R/X ratio and unbalanced operation, the AC approach based PTDFs needs to be adopted in modelling distribution network constraints. Moreover, it is demonstrated in [26] that for a system of an arbitrary topology with losses, PTDFs are relatively insensitive to the operating point of a given power system if the system topology is fixed and there is sufficient reactive power to maintain voltages basically constants at all buses. After taking all these into consideration, the PTDF approach is adopted to model the distribution network constraints, as expressed by Eqn. (6).

B. Proof of the Honesty Dominated Bidding/Offering Strategy

It is defined as honesty if a participant will bid at his/her self-estimated generation cost / electricity utility when re-bidding is not permitted and all participants bid and offer simultaneously. Otherwise, when re-bidding is permitted, it is defined as honesty when the bidding behaviour truly reflects the relationship between a participant's self-estimation of generation cost/electricity utility and the MCP. In other words, being honest, a participant tends to submit a bid / an offer that is not less than/ not larger than the observed MCP if the self-estimated generation cost/electricity utility is indeed not less than/ not larger than the MCP.

Theorem 1: *Honesty is a dominant strategy for participants in the presented market mechanism.*

Proof: It is assumed that when a consumer submits a bid r_i^{b} to the electricity market, he/she is aware of his/her true utility of using electricity, which is represented by b. Without loss of generality, it can also be assumed that the bids of other participants except consumer i can be ordered and plotted as in Fig.1. After the participation of consumer i, the MCP would change from $\bar{r}$ to $\bar{r}_{\text{new}}$. Then, the net utility of consumer i through consuming a unit of electricity can be expressed by $b-\bar{r}_{\text{new}}$. But if the consumer i loses the auction, the attained utility will be 0.

In the APM mechanism, the bidding strategies of consumer i are analysed under different scenarios of utility b, as shown in Table I.

TABLE I Analysis of Bidding Strategies for Consumers

Scenario	Strategy	Value of $\bar{r}_{\text{new}}$	Utility of consumer i
$b>\bar{r}$	if $r_i^{\text{b}} > \bar{r}$	$\bar{r}_{\text{new}} = (1+\theta)\cdot\bar{r}$	$b-(1+\theta)\cdot\bar{r}$
	if $r_i^{\text{b}} = \bar{r}$	$\bar{r}_{\text{new}} = \bar{r}$	0
	if $r_i^{\text{b}} < \bar{r}$	$\bar{r}_{\text{new}} = (1-\theta)\cdot\bar{r}$	0
$b=\bar{r}$	if $r_i^{\text{b}} > \bar{r}$	$\bar{r}_{\text{new}} = (1+\theta)\cdot\bar{r}$	$b-(1+\theta)\cdot\bar{r}<0$
	if $r_i^{\text{b}} = \bar{r}$	$\bar{r}_{\text{new}} = \bar{r}$	0
	if $r_i^{\text{b}} < \bar{r}$	$\bar{r}_{\text{new}} = (1-\theta)\cdot\bar{r}$	0
$b<\bar{r}$	if $r_i^{\text{b}} > \bar{r}$	$\bar{r}_{\text{new}} = (1+\theta)\cdot\bar{r}$	$b-(1+\theta)\cdot\bar{r}<0$
	if $r_i^{\text{b}} = \bar{r}$	$\bar{r}_{\text{new}} = \bar{r}$	0
	if $r_i^{\text{b}} < \bar{r}$	$\bar{r}_{\text{new}} = (1-\theta)\cdot\bar{r}$	0

Note: θ is a parameter and indicates the change of market clearing because of the bids of consumer i.

Thus, when no available market information, to bid at $r_i^{\text{b}}=b$ is the only choice that can be the best strategy for the consumer under all possible conditions.

In the re-bidding process, when $b>\bar{r}$ and consumer i chooses to bid at $r_i^{\text{b}}>\bar{r}$, this consumer needs to ensure $b-(1+\theta)\cdot\bar{r}>0$. Let p_{-i}^{b} denote the bids of other consumers except consumer i, the MCP when consumer i bids at r_i^{b} can be expressed as follows.

$$\bar{r}_{\text{new}} = (1+\theta)\cdot\bar{r} = \left(r_i^{\text{b}} \cdot p_i^{\text{b}} + \sum \bar{r} \cdot p_{-i}^{\text{b}}\right) \Big/ \left(p_i^{\text{b}} + \sum p_{-i}^{\text{b}}\right) \tag{8}$$

$$b-(1+\theta)\cdot\bar{r} > 0 \Rightarrow \bar{r} < r_i^{\text{b}} < b + \left[(b-\bar{r})\cdot\sum p_{-i}^{\text{b}}\right] \Big/ p_i^{\text{b}} \tag{9}$$

Therefore, when $b>\bar{r}$ and $r_i^{\text{b}}>\bar{r}$, the decision space of consumer i can be determined by Eqn. (9).

To summarize, once the current market clearing price and total trading volume is released, the best strategy for consumer i when $b>\bar{r}$ is to bid $r_i^{\text{b}}>\bar{r}$. Meanwhile, the entries in Table I shows that when $b=\bar{r}$ and $b<\bar{r}$, consumer i cannot do better than bidding at $r_i^{\text{b}}=\bar{r}$ and $r_i^{\text{b}}<\bar{r}$, respectively.

Similarly, for the j^{th} producer, under the presented market mechanism, the offering strategies of producer j are analysed under different scenarios of its evaluation s, as shown in Table II. A producer obtains the utility of $\bar{r}_{\text{new}}-s$ by selling a unit of electricity to consumers at the price of $\bar{r}_{\text{new}}$. Besides, if a producer loses the auction, the obtained utility will also be 0.

Thus, when no available market information, to bid at $r_j^{\text{s}}=s$ is the only choice that can be the best strategy for the producer under all possible conditions.

In the re-bidding process, when $s<\bar{r}$ and producer j chooses to offer at $r_j^s<\bar{r}$, the producer needs to ensure $(1-\theta)\cdot\bar{r}-s>0$. Let p_{-j}^s denote the offers of other producers except producer j, the MCP when producer j offers at r_j^s can be expressed as follows.

$$\bar{r}_{\text{new}}=(1-\theta)\cdot\bar{r}=\left(r_j^{\text{s}}\cdot p_j^{\text{s}}+\sum\bar{r}\cdot p_{-j}^{\text{s}}\right)\Big/\left(p_j^{\text{s}}+\sum p_{-j}^{\text{s}}\right) \quad (10)$$

$$(1-\theta)\cdot\bar{r}-s>0\Rightarrow s-\left[(\bar{r}-s)\cdot\sum p_{-j}^{\text{s}}\right]\Big/p_j^{\text{s}}<r_j^{\text{s}}<\bar{r} \quad (11)$$

Therefore, when $s<\bar{r}$ and $r_j^s<\bar{r}$, the decision space of producer j can be determined by Eqn. (11).

Thus, once the current market clearing price and total trading volume is released, the best strategy for producer j when $s<\bar{r}$ is to offer $r_j^s<\bar{r}$. Besides, the entries in Table II shows that when $s=\bar{r}$ and $s>\bar{r}$, producer j cannot do better than offering at $r_j^s=\bar{r}$ and $r_j^s>\bar{r}$.

In other words, when making decision to maximize their own utilities, both a consumer and a producer cannot do better than bidding/offering honestly in the presented market mechanism. Hence, Theorem 1 is proved.

TABLE II ANALYSIS OF OFFERING STRATEGIES FOR PRODUCERS

Scenario	Strategy	Value of $\bar{r}_{\text{new}}$	Utility of producer j
$s>\bar{r}$	if $r_j^s>\bar{r}$	$\bar{r}_{\text{new}}=(1+\theta)\cdot\bar{r}$	0
	if $r_j^s=\bar{r}$	$\bar{r}_{\text{new}}=\bar{r}$	0
	if $r_j^s<\bar{r}$	$\bar{r}_{\text{new}}=(1-\theta)\cdot\bar{r}$	$(1-\theta)\cdot\bar{r}-s<0$
$s=\bar{r}$	if $r_j^s>\bar{r}$	$\bar{r}_{\text{new}}=(1+\theta)\cdot\bar{r}$	0
	if $r_j^s=\bar{r}$	$\bar{r}_{\text{new}}=\bar{r}$	0
	if $r_j^s<\bar{r}$	$\bar{r}_{\text{new}}=(1-\theta)\cdot\bar{r}$	$(1-\theta)\cdot\bar{r}-s<0$
$s<\bar{r}$	if $r_j^s>\bar{r}$	$\bar{r}_{\text{new}}=(1+\theta)\cdot\bar{r}$	0
	if $r_j^s=\bar{r}$	$\bar{r}_{\text{new}}=\bar{r}$	0
	if $r_j^s<\bar{r}$	$\bar{r}_{\text{new}}=(1-\theta)\cdot\bar{r}$	$(1-\theta)\cdot\bar{r}-s$

Note: θ is a parameter and indicates the change of market clearing because of the offers of producer j.

III. Mathematical Formulation of The Decentralized Distribution Market Mechanism

A. Proposed Decentralized Distribution Market Mechanism

Before establishing the decentralized distribution market mechanism, the centralized one has been presented in Section II where bidding behaviours of zero marginal cost renewable generators are also considered.

In the mathematical optimization community, endeavours have been made to seek efficient methods to decompose an intractable problem into several sub-problems. As a method that combines the advantages of dual decomposition and augmented Lagrangian methods for constrained optimization problems, the ADMM is a simple but powerful algorithm for distributed convex optimization problems [27]. Since the ADMM is originally introduced for the special case where there are only two blocks of variables in the optimization problem, in [28] the Gauss-Seidel and Jacobian ADMMs are proposed for cases with three or more blocks of variables. In particular, the Jacobian ADMM is featured by the advantage of enabling a parallelized updating of all variables.

In [29-31], the ADMM algorithm has been studied to develop distributed computational methods for OPF problems. However, there is still no attempt being reported which tries to derive highly efficient distributed electricity market mechanisms using the ADMM algorithm.

In this section, the Proximal Jacobian ADMM algorithm [28] is adopted to develop the distributed market mechanism. Notably, the original Proximal Jacobian ADMM algorithm can only deal with linear equality constraints. However, Eqn. (6) is an inequality constraint. Therefore, the Proximal Jacobian ADMM algorithm is extended in this paper by introducing the slack variable R in order to transform the inequality constraint into equations. Then, Eqn. (6) can be expressed as follows.

$$\sum_{i\in N}p_i^{\text{cb}}d_{l,i}+\sum_{j\in M}p_j^{\text{cs}}d_{l,j}+R_l^{\text{ref}}=P_l^{\max}\quad\forall l\in L \quad (12)$$

$$R_l^{\text{cref}}-(\sum_{i\in N}p_i^{\text{cb}}d_{l,i}+\sum_{j\in M}p_j^{\text{cs}}d_{l,j})=P_l^{\max}\quad\forall l\in L \quad (13)$$

where R_l^{ref} / R_l^{cref} indicates the slack variable of transmission constraints when the power flow is the same as / in contrast to the predefined reference direction of electric power.

Given the centralized optimization problem as expressed by Eqn. (1) - Eqn. (7), its corresponding augmented Lagrangian equation is as follows.

$$\begin{aligned}L_\rho=&(\sum_{i\in N}p_i^{\text{cb}}r_i^{\text{b}}-\sum_{j\in M}p_j^{\text{cs}}r_j^{\text{s}})+\lambda^{\text{T}}(\sum_{i\in N}p_i^{\text{cb}}-\sum_{j\in M}p_j^{\text{cs}})+\\&\frac{\rho}{2}\|\sum_{i\in N}p_i^{\text{cb}}-\sum_{j\in M}p_j^{\text{cs}}\|_2^2+\sum_{l\in L}[\mu_{\text{ref},l}^{\text{T}}(\sum_{i\in N}p_i^{\text{cb}}d_{l,i}+\sum_{j\in M}p_j^{\text{cs}}d_{l,j}+R_l^{\text{ref}}-P_l^{\max})]\\&+\sum_{l\in L}\frac{\rho}{2}\|\sum_{i\in N}p_i^{\text{cb}}d_{l,i}+\sum_{j\in M}p_j^{\text{cs}}d_{l,j}+R_l^{\text{ref}}-P_l^{\max}\|_2^2\\&+\sum_{l\in L}[\mu_{\text{cref},l}^{\text{T}}(R_l^{\text{cref}}-\sum_{i\in N}p_i^{\text{cb}}d_{l,i}-\sum_{j\in M}p_j^{\text{cs}}d_{l,j}-P_l^{\max})]\\&+\sum_{l\in L}\frac{\rho}{2}\|R_l^{\text{cref}}-\sum_{i\in N}p_i^{\text{cb}}d_{l,i}-\sum_{j\in M}p_j^{\text{cs}}d_{l,j}-P_l^{\max}\|_2^2\end{aligned} \quad (14)$$

where λ, $\mu_{\text{ref},l}$, $\mu_{\text{cref},l}$ are dual variables in the augmented Lagrangian equation; ρ is the parameter for the quadratic penalty of the constraint.

Then, the Jacobian ADMM can solve the original centralized optimization problem by solving a decentralized problem in an iterative way. Meanwhile, the calculation of each variable can be carried out in parallel. Through introducing a proximal term in each sub-problem and a damping parameter for the update of dual parameters in the iterative calculation, the Proximal Jacobian ADMM algorithm is developed in [28]. With the added proximal term, the Jacobian ADMM algorithm is usually more stable especially when the sub-problem is not strictly convex.

Consequently, the final decentralized formulation of the APM market clearing mechanism is derived, as detailed below. For the j^{th} producer, the optimization problem to be solved can be formulated as below.

$$\begin{aligned}f_j^{\text{cs}(k+1)}=&\underset{p_j^{\text{cs}},R_{j,l}^{\text{ref}},R_{j,l}^{\text{cref}}}{\operatorname{argmin}}\;p_j^{\text{cs}}r_j^{\text{s}}+\frac{\rho}{2}\|p_j^{\text{cs}}+\sum_{m\in M-j}p_m^{\text{cs}(k)}-\sum_{i\in N}p_i^{\text{cb}(k)}-\frac{\lambda^{(k)}}{\rho}\|_2^2+\\&\sum_{l\in L}[\frac{\rho}{2}\|\sum_{i\in N}p_i^{\text{cb}(k)}d_{l,i}+\sum_{m\in M\text{-}j}p_m^{\text{cs}(k)}d_{l,m}+p_j^{\text{cs}}d_{l,j}+R_{j,l}^{\text{ref}}-P_l^{\max}-\frac{\mu_{\text{ref},l}^{(k)}}{\rho}\|_2^2]+\\&\sum_{l\in L}[\frac{\rho}{2}\|R_{j,l}^{\text{cref}}-\sum_{i\in N}p_i^{\text{cb}(k)}d_{l,i}-\sum_{m\in M-j}p_m^{\text{cs}(k)}d_{l,m}-p_j^{\text{cs}}d_{l,j}-P_l^{\max}-\frac{\mu_{\text{cref},l}^{(k)}}{\rho}\|_2^2]+\\&\frac{1}{2}\|p_j^{\text{cs}}-p_j^{\text{cs}(k)}\|_{H_j}^2+\frac{1}{2}\|R_{j,l}^{\text{ref}}-R_l^{\text{ref,avg}(k)}\|_{H_j}^2+\frac{1}{2}\||R_{j,l}^{\text{cref}}-R_l^{\text{cref,avg}(k)}\|_{H_j}^2\end{aligned} \quad (15)$$

$$\text{s.t.} \quad \forall j \in M,\, p_j^{\text{cs}} = 0 \text{ if } r_j^{\text{s}} > \bar{r} \tag{16}$$

$$0 \le p_j^{\text{cs}} \le p_j^{\text{s}} \tag{17}$$

where $\frac{1}{2}\|p_j^{\text{cs}} - p_j^{\text{cs}(k)}\|_{H_j}^2$ is the proximal term for the p_j^{cs} sub-problem. H_j is a symmetric and positive semi-definite matrix and $\|X_j\|_{H_j}^2 = X_j^{\text{T}} H_j X_j$; $M-j$ represents the set of producers except j; k is the iteration counter.

For the i^{th} consumer, the optimization problem to be solved can be formulated as below.

$$f_i^{\text{cb}(k+1)} = \underset{p_i^{\text{cb}},\, R_{i,l}^{\text{ref}},\, R_{i,l}^{\text{cref}}}{\text{argmin}} (-p_i^{\text{cb}} r_i^{\text{b}}) + \frac{\rho}{2}\|\sum_{j\in M} p_j^{\text{cs}(k)} - \sum_{n\in N-i} p_n^{\text{cb}(k)} - p_i^{\text{cb}} - \frac{\lambda^{(k)}}{\rho}\|_2^2 +$$
$$\sum_{l\in L}[\frac{\rho}{2}\|\sum_{n\in N-i} p_n^{\text{cb}(k)} d_{l,n} + p_i^{\text{cb}} d_{l,i} + \sum_{j\in M} p_j^{\text{cs}(k)} d_{l,j} + R_{i,l}^{\text{ref}} - P_l^{\max} - \frac{\mu_{\text{ref},l}^{(k)}}{\rho}\|_2^2] +$$
$$\sum_{l\in L}[\frac{\rho}{2}\|R_{i,l}^{\text{cref}} - \sum_{n\in N-i} p_n^{\text{cb}(k)} d_{l,n} - p_i^{\text{cs}} d_{l,i} - \sum_{j\in M} p_j^{\text{cs}(k)} d_{l,j} - P_l^{\max} - \frac{\mu_{\text{cref},l}^{(k)}}{\rho}\|_2^2] +$$
$$\frac{1}{2}\|p_i^{\text{cb}} - p_i^{\text{cb}(k)}\|_{H_i}^2 + \frac{1}{2}\|R_{i,l}^{\text{ref}} - R_l^{\text{ref,avg}(k)}\|_{H_j}^2 + \frac{1}{2}\|R_{i,l}^{\text{cref}} - R_l^{\text{cref,avg}(k)}\|_{H_j}^2 \tag{18}$$

$$\text{s.t.} \quad \forall i \in N,\, p_i^{\text{cb}} = 0 \text{ if } r_i^{b} < \bar{r} \tag{19}$$

$$0 \le p_i^{\text{cb}} \le p_i^{\text{b}} \tag{20}$$

where $\frac{1}{2}\|p_i^{\text{cb}} - p_i^{\text{cb}(k)}\|_{H_i}^2$ is the proximal term for the p_i^{cb} sub-problem. H_i is also a symmetric and positive semi-definite matrix; $N-i$ represents the set of consumers except i.

During the iteration, dual parameters also need to be updated.

$$\lambda^{(k+1)} = \lambda^{(k)} - \gamma\rho(\sum_{j\in M} p_j^{\text{cs}(k+1)} - \sum_{i\in N} p_i^{\text{cb}(k+1)}) \tag{21}$$

$$\mu_{\text{ref},l}^{(k+1)} = \mu_{\text{ref},l}^{(k)} - \gamma\rho(\sum_{i\in N} p_i^{\text{cb}(k+1)} d_{l,i} + \sum_{j\in M} p_j^{\text{cs}(k+1)} d_{l,j} + R_l^{\text{ref,avg}(k+1)} - P_l^{\max}) \tag{22}$$

$$\mu_{\text{cref},l}^{(k+1)} = \mu_{\text{cref},l}^{(k)} - \gamma\rho(R_l^{\text{cref,avg}(k+1)} - \sum_{i\in N} p_i^{\text{cb}(k+1)} d_{l,i} - \sum_{j\in M} p_j^{\text{cs}(k+1)} d_{l,j} - P_l^{\max}) \tag{23}$$

where $\gamma>0$ is the damping parameter for the update of dual variables.

Besides, since all the sub-problems share the same transmission constraints of the network, therefore, in the final decentralized results, the slack variables obtained by all sub-problems should be equal. Then, in Eqn. (15) and Eqn. (18) the last two items enable the decisions of sub-problems converge to their average value after the iteration procedure is completed. In this way, all the slack variables calculated by each sub-problem will be equal in the final optimization results. Obviously, the average value of slack variables also needs to be updated during the iteration as follows.

$$R_l^{\text{ref,avg}(k+1)} = (\sum_{i\in N} R_{i,l}^{\text{ref}(k+1)} + \sum_{j\in M} R_{j,l}^{\text{ref}(k+1)}) \Big/ (N+M) \tag{24}$$

$$R_l^{\text{cref,avg}(k+1)} = (\sum_{i\in N} R_{i,l}^{\text{cref}(k+1)} + \sum_{j\in M} R_{j,l}^{\text{cref}(k+1)}) \Big/ (N+M) \tag{25}$$

In [27, 31], it is pointed out that the terminating condition for the ADMM based methods is that the primal and dual residuals must be small enough.

$$\|r^{\text{rd}(k)}\|_2 \le \varepsilon^{\text{pri}} \text{ and } \|s^{\text{rd}(k)}\|_2 \le \varepsilon^{\text{dual}} \tag{26}$$

where $r^{\text{rd}(k)}$ and $s^{\text{rd}(k)}$ are the primal and dual residuals; ε^{pri} and $\varepsilon^{\text{dual}}$ are the feasibility tolerances for the primal and dual residuals, respectively.

In the proposed mechanism, the primal residual is defined as follows for each sub-problem.

For the j^{th} producer and i^{th} consumer,

$$r_j^{\text{rd}(k)} = p_j^{\text{cs}(k)} - p_j^{\text{cs}(k-1)} \tag{27}$$

$$r_i^{\text{rd}(k)} = p_i^{\text{cb}(k)} - p_i^{\text{cb}(k-1)} \tag{28}$$

Besides, the dual residual after each iteration is defined as follows.

$$s^{\text{rd}(k)} = \rho(\sum_{j\in M} p_j^{\text{cs}(k)} - \sum_{i\in N} p_i^{\text{cb}(k)}) \tag{29}$$

In this paper, it is assumed that when both the primal and dual residuals are smaller than the smallest tolerance of all decision makers, then the mechanism reaches its termination criterion of the iterative procedure.

B. Convergence of the Decentralized Market Mechanism

Since the proposed distributed market mechanism is computed in an iterative way, its convergence is hence an essential issue. The proposed decentralized market model is based on a modified algorithm of ADMM, namely the Proximal Jacobian ADMM, all convergence results that hold for the Proximal Jacobian ADMM still hold for the proposed model. In [28], it is proved that if certain conditions for the matrix H_i and the parameter γ are satisfied, the Proximal Jacobian ADMM can achieve global convergence at an $o(1/k)$ convergence rate. Besides, the added proximal term in the objective function also enables the sub-problem to become strictly or strongly convex if it is not originally.

In this paper, since the original centralized problem as given by Eqn. (1) - Eqn. (7) is a linear programming one, then the sub-problem after the decentralized formulation is still a convex optimization one. Thus, the proposed decentralized algorithm can guarantee the convergence of iteration if the specifications for H_i and γ satisfy the conditions as given below. In the Proximal Jacobian ADMM algorithm, there are two commonly adopted specifications for H_i and γ.

(1) $H_i=\tau_i E$ ($\tau_i>0$). This corresponds to the standard proximal method. E is the identity matrix. τ_i is a parameter. The condition for parameter setting is

$$\tau_i > \rho(\frac{N^{\text{sub}}}{2-\gamma} - 1)\|A_i\|^2 \tag{30}$$

(2) $H_i=\tau_i E - \rho A_i^{\text{T}} A_i$ ($\tau_i>0$). This corresponds to the prox-linear method, which not only linearizes the quadratic penalty term of the augmented Lagrangian equation but also adds a proximal term. The condition for parameter setting is

$$\tau_i > [\rho N^{\text{sub}} \big/ (2-\gamma)]\|A_i\|^2 \tag{31}$$

where A_i is the coefficient matrix of the equality constraints in the original optimization problem; N^{sub} is the total number of sub-problems.

IV. Safety of Transaction under the Proposed Decentralized Market

A. Communication of Participants in Decentralized Market

In order to implement the proposed distributed market mechanism, a communication path is demanded to transmit information among all the participants. Regarding the concrete structure of the communication network, different types of network topologies can be considered including the ring topology, the fully connected topology and the bus topology. The design of the communication network topology is beyond the scope of this paper, while the focus is on the requirement of exchanged information for implementing the proposed method. It is also assumed that the message is transmitted in a serial way despite the topology of the actual communication network, as shown in Fig.2.

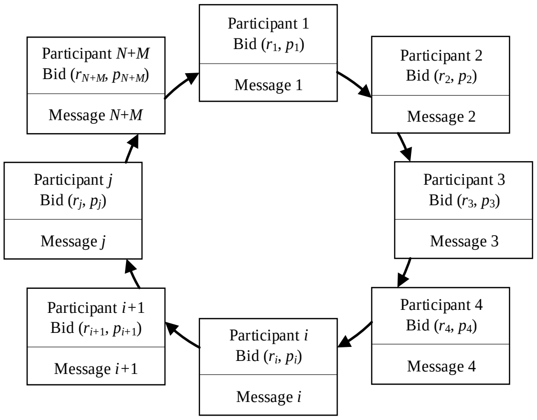


Fig.2. Message transmission among participants in the proposed mechanism.

In the sub-problem of each participant, the proximal terms are introduced in order to increase the stability of the Jacobian ADMM algorithm especially when the sub-problem is not strictly convex. The added proximal terms in the objective function of each participant can help the decentralized model converge to a stable solution. Notably, the whole process of iterative calculation, as shown in Table III, only starts after each participant submitted its own bidding parameters. Under this circumstance, the process of iterative calculation can be designed as an automatic calculation system that prohibits the intervention of participants, and then there will be no involvement of participants in the iterative calculation. Therefore, the market clearing process in the proposed decentralized market mechanism is carried out by the iterative calculation and communication without a centralized market operator.

The pseudo code for the proposed algorithm and details of messages transmitted among participants are given in Table III.

TABLE III Decentralized Market Clearing Algorithm and Details of Messages Transmitted among Participants

Communication before the decentralized calculation	
Round 1	Each participant sets its identity as a producer or consumer, and then adds its own data into the passing message. At the end, the following information is obtained: $\sum_{i\in N} p_i$ (consumers), $\sum_{j\in M} p_j$ (producers); $\sum_{i\in N} p_i r_i$ (consumers), $\sum_{j\in M} p_j r_j$ (producers);
Round 2	$\sum_{i\in N} p_i$, $\sum_{j\in M} p_j$, $\sum_{i\in N} p_i r_i$ and $\sum_{j\in M} p_j r_j$ are transmitted to each participant.
Communication during the decentralized calculation	
1. Initialize the parameters and variables (k=0): $\lambda^{(k)}$, $\mu_{\mathrm{ref},l}^{(k)}$, $\mu_{\mathrm{cref},l}^{(k)}$, ρ, H_i, H_j, γ, $p_j^{\mathrm{cs}(k)}$, $R_{j,l}^{\mathrm{ref}(k)}$, $R_{j,l}^{\mathrm{cref}(k)}$, $p_i^{\mathrm{cb}(k)}$, $R_{i,l}^{\mathrm{ref}(k)}$, $R_{i,l}^{\mathrm{cref}(k)}$.	
2. Start the iteration:	
(1) The j^{th} producer (j=1,…,M) and the i^{th} consumer (i=1,…,N) update ($p_j^{\mathrm{cs}(k+1)}$, $R_{j,l}^{\mathrm{ref}(k+1)}$, $R_{j,l}^{\mathrm{cref}(k+1)}$) and ($p_i^{\mathrm{cb}(k+1)}$, $R_{i,l}^{\mathrm{ref}}$, $R_{i,l}^{\mathrm{cref}}$), respectively.	
Round 1:	Using the optimization results, each participant adds its own data into the passing message. At the end, the following information is obtained: $\sum_{i\in N} p_i^{\mathrm{cb}(k+1)}$, $\sum_{j\in M} p_j^{\mathrm{cs}(k+1)}$, $\sum_{i\in N} R_{i,l}^{\mathrm{ref}(k+1)}$, $\sum_{j\in M} R_{j,l}^{\mathrm{cref}(k+1)}$, $\sum_{i\in N} p_i^{\mathrm{cb}(k+1)} d_{l,i}$, $\sum_{j\in M} p_j^{\mathrm{cs}(k+1)} d_{l,j}$;
Round 2:	Transmit the obtained aggregated values to each participant in the distribution network;
(2) Each participant updates $\lambda^{(k+1)}$, $\mu_{\mathrm{ref},l}^{(k+1)}$, $\mu_{\mathrm{cref},l}^{(k+1)}$, $R_l^{\mathrm{ref}(k+1)}$, and $R_l^{\mathrm{cref}(k+1)}$;	
(3) Continue the iteration until $r_i^{\mathrm{rd}(k+1)}$, $r_j^{\mathrm{rd}(k+1)}$ and $s^{\mathrm{rd}(k+1)}$ satisfy the terminating conditions;	
3. End iteration.	
After the decentralized calculation	
Distribution market participants will be dispatched and settled according to the optimization results obtained through the iterative calculation.	

Note: during the iteration, the optimization calculation of each participant can be carried out in a parallel way.

After the first two rounds of communication, each participant would obtain the MCP $\bar{r}$. Then, the parameters in the optimization model of each participant will be reset according to the market clearing rules, as formulated by Eqn. (16) and Eqn. (19). Next, the decentralized market mechanism starts the iterative calculation until the terminating criterion as expressed by Eqn. (26) is satisfied. In each iteration, there are two rounds of communications for participants to update their parameters, where the aggregated values of solutions are transmitted among participants. Finally, the dispatch and settlement in the distribution network will be implemented based on the optimization results obtained by the iterative calculation.

B. Privacy Analysis of Market Participants

As the bid/offer data from participants are private, the information of each participant should be prevented from leaking to the others. The information leakage may lead to the speculative behaviours of some participants, and even result in the failure of the market mechanism. Compared with the centralized electricity market, the decentralized one does not require a market operator who is in charge of the market operation and manage private information of participants. Therefore, protecting the privacy of participants becomes a more severe problem in a decentralized market mechanism.

In the proposed mechanism, the aggregated value of the bidding/offering data from other participants is used to solve each sub-problem as shown in previous sections. The aggregated values of bids/offers are transmitted in the communication network. Each participant solves its own optimization problem after receiving the transmitted messages from his/her neighbours. Through the aggregating of data, the bid/offer information of participates is being encrypted. These data will not be decrypted during the whole process, thus the distributed market mechanism can be operated in a way without exposing the personal information of each participant to the others. The personal information of each participant will only be used for solving his/her own optimization problem.

V. Case Study and Discussions

A. Data Specifications in the Case Study

In [32], it is reported that the global weighted levelized cost of energy (LOCE) has declined to about 0.05 \$/kWh for onshore wind and 0.06 \$/kWh for solar photovoltaic (PV) based on the latest data in 2017. In this case study, it is assumed that the offer prices of small-scale renewable generators fall within the range between 0.05 and 0.5 \$/kWh. Actual residential solar data in the Australian distribution system are presented in [33], then the bid/offer quantities of participants are assumed to fall within the range between 1 and 5 kW. The standard proximal method is adopted for the Proximal Jacobian ADMM algorithm. The values of other parameters are given in Table IV and initial values of decision variables for each participant are all set as 0.

TABLE IV Parameter Values in Decentralized Market Mechanism

Parameters	$\lambda^{(k)}$	$\mu_{\mathrm{ref},l}^{(k)}$	$\mu_{\mathrm{cref},l}^{(k)}$	ρ	γ	$\varepsilon^{\mathrm{pri}}$	$\varepsilon^{\mathrm{dual}}$
Initial value (k=0)	1	1	1	3×10^{-3}	0.5	0.01	0.01

Note: H_i=$\tau_i E$, H_j=$\tau_j E$, τ=τ_j=τ_i ($j\in M$; $i\in N$). Since Eqn. (30) in Section III.B needs to be satisfied, τ is set according to the total number of participants.

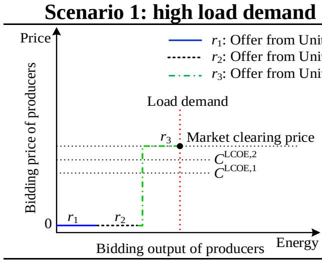
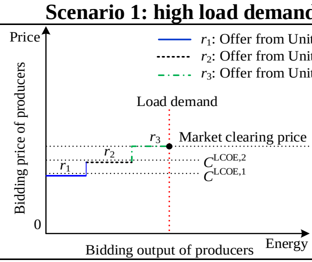
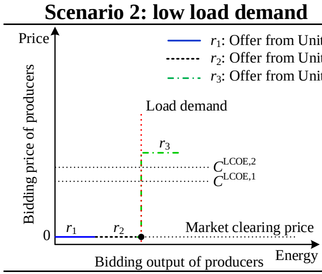
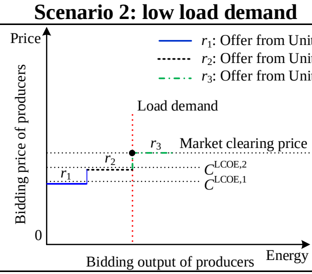
B. Failure of the Marginal Cost Based Market Mechanism

A sample power system with three generation units is taken as an example to elaborate shortcomings of the conventional uniform clearing mechanism in pricing generation from zero marginal cost units, where details about end-users are omitted for simplification. Costs of generation units in this sample power system are given in Table V and the levelized cost of energy (LCOE) is a measurement of long-term generation cost. In Table VI, comparisons between market clearing outcomes of the marginal cost based market and the APM mechanism in this paper are presented.

TABLE V GENERATION COSTS OF EACH UNIT IN THE SAMPLE POWER SYSTEM

Unit #	Long-term Generation Cost	Short-term Generation Cost
	LCOE	Marginal Cost
1	$C^{LCOE,1}$	$C_1^{mgl}=0$
2	$C^{LCOE,2}$	$C_2^{mgl}=0$
3	$C^{LCOE,3}$	$C_3^{mgl}>0$

TABLE VI COMPARISON BETWEEN MARGINAL COST MARKET AND APM

Marginal cost based market	**APM mechanism**
Scenario 1: high load demand	**Scenario 1: high load demand**
	
Analysis of MCP	**Analysis of MCP**
Under the uniform clearing mechanism, the market clearing price will be acceptable for units 1 and 2, even though they would offer zero prices. Because the MCP is determined only by the marginal unit and offers from units 1 and 2 have no impact on the MCP.	Under the APM mechanism, if units 1 and 2 still offer zero prices then the final MCP could be lower than their acceptable values. Therefore, units 1 and 2 would choose non-zero offer prices. If participants are permitted to bid only once, they will bid at the self-estimated generation cost or electricity utility.
Scenario 2: low load demand	**Scenario 2: low load demand**
	
Analysis of MCP	**Analysis of MCP**
Renewable generators are also likely to become marginal units when the load demand in the power system is low. The zero bidding prices from renewable generators result in the zero MCP, and the market is then failed to reveal the genuine non-zero value of renewable generations.	Units 1 and 2 do not bid zero prices to the market under the APM mechanism. Under this circumstance, even if only generation units with zero marginal costs participate in the bidding, the APM market mechanism can still produce a reasonable MCP.
Conclusion	**Conclusion**
Bidding decisions of generators are made based on their short-term generation costs. Generation units with zero marginal costs will still offer zero prices to the market, even though their LCOEs are not zero.	The bidding price from each generator will have impacts on the final MCP. The presented APM mechanism enables generators submit bidding price by considering their long-term LCOEs.

C. Comparisons Study without Network Congestions

Market clearing outcomes obtained by centralized and decentralized models are compared under cases 1, 2 and 3. Details about the calculation results are shown in Tables VII to XIII.

(1) Calculation results for various scenarios in Case 1

In case 1, the bidding data for all participants in each scenario are generated independently. ρ is set as 3×10^{-3} and 1×10^{-3} when the total number of participants is below and over 1000, respectively. τ is given in Table IX. Fig.3 shows the differences between decentralized and centralized market clearing outcomes when the number of participants is 1000.

The statistical results in Table VII show that there are at most 6.6% of participants being affected when the decentralized model is employed. In particular, if the differences with a micro value are neglected, such as those with values much less than 1 kW, the percentage can decrease further. Besides, Table VIII shows that the difference between market volumes attained by the decentralized and centralized models is always within 5%. In terms of the convergence speed, Table VIII manifests the number of iterations needed in each scenario. Meanwhile, with the increase of participants, the difference between market outcomes tends to get larger. This is because with the increase of participants, the scale and complexity of the decentralized optimization problem will also increase, so the accuracy of the algorithm decreases slightly. It is also shown in Table VIII that although the number of participants increased by 10 times, the proposed decentralized model can still converge to the optimal solution quickly.

The comparison of MCP and computation time under those scenarios is presented in Table IX. The results show that the proposed decentralized market model can generate the same MCP as the centralized model. That is because the MCP is determined by the bidding parameters of participants. The decentralization of the market mechanism only changes the way of information transmission but has no impact on the original bidding data. Thus, it does not change the final MCP, which is proved by the case study results.

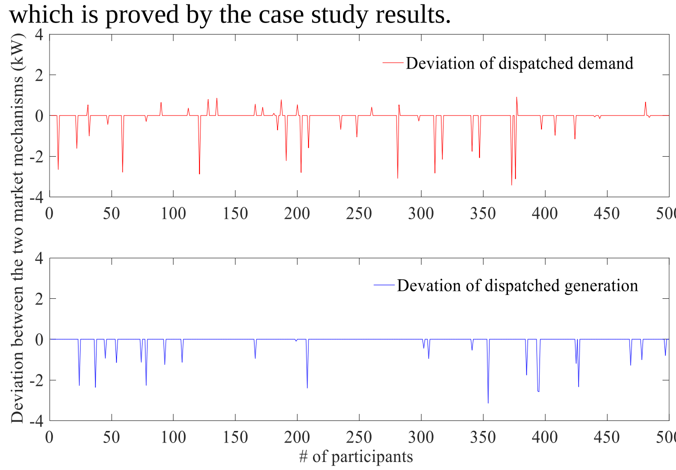


Fig.3. Deviations of market clearing outcomes for 1000 participants in case 1. Note: The deviation in Fig.3 represents the difference between the decentralized market result and the centralized market result.

TABLE VII NUMBER OF PARTICIPANTS HAVING DIFFERENT TRADING OUTCOMES WITHOUT NETWORK CONGESTION

Scenario	Number of participants	NPA	NCA	Percent of (NPA+ NCA)
1	100	0	4	4%
2	200	4	7	5.5%
3	400	5	2	1.75%
4	600	19	13	5.3%
5	800	18	35	6.6%
6	1000	23	41	6.4%
7	1200	23	23	3.83%
8	1400	41	41	5.69%

Note: NPA / NCA represents the number of producers / consumers who obtain different market clearing results under the decentralized and centralized market models; Percent of (NPA+NCA) indicates the proportion of NPA+NCA over the total number of participants.

TABLE VIII COMPARISONS OF MARKET CLEARING OUTCOMES UNDER CASE 1

Number of participants	Market volume (kW)		DBTM (kW)	Percent of DBTM	Number of iterations
	DE-M	CE-M			
100	68	68	0.0	0.0%	228
200	156	158	2.0	1.27%	113
400	290	291	1.0	0.34%	153
600	417	434	17.0	3.92%	105
800	558	586	28.0	4.78%	80
1000	704	738	34.0	4.61%	83
1200	871	888	17	1.91%	148
1400	986	1027	41	3.99%	119

Note: DE-M represents the decentralized market; CE-M represents the centralized market; DBTM represents the difference between the clearing results of the decentralized and centralized markets; Percent of DBTM represents the proportion of DBTM over the market volume attained by the centralized market model.

TABLE IX COMPARISONS OF MCPS AND COMPUTATION TIME UNDER CASE 1

Number of participants	MCP ($/kWh)		τ	ρ	Computation time (min)
	DE-M	CE-M			
100	0.2922	0.2922	1	3.0×10^{-3}	4.22
200	0.2961	0.2961	1	3.0×10^{-3}	4.17
400	0.2811	0.2811	1	3.0×10^{-3}	11.10
600	0.2647	0.2647	1.2	3.0×10^{-3}	11.50
800	0.2752	0.2752	1.7	3.0×10^{-3}	11.82
1000	0.2807	0.2807	2.0	3.0×10^{-3}	17.56
1200	0.2663	0.2663	0.8	1.0×10^{-3}	38.92
1400	0.2710	0.2710	0.94	1.0×10^{-3}	34.30

(2) Calculation results for various scenarios in Case 2

In case 2, the bidding data from all participants in each scenario are generated independently. Differently, ρ is set as 1.5×10^{-3} and 0.5×10^{-3} when the total number of participants is below and over 1000, respectively. τ is given in Table XI. The results in Table X show that the performance of the decentralized market model is improved after the adjustment of parameters. In particular, the difference between results that are attained by the two market models decreases to 0 when the number of participants is below 600.

TABLE X COMPARISONS OF MARKET CLEARING OUTCOMES UNDER CASE 2

Number of participants	Market volume (kW)		DBTM (kW)	Percent of DBTM	Number of iterations
	DE-M	CE-M			
100	68	68	0.0	0.0%	116
200	158	158	0.0	0.0%	42
400	291	291	0.0	0.0%	122
600	434	434	0.0	0.0%	190
800	578	586	8	1.37%	119
1000	716	738	22	2.98%	105
1200	888	888	0	0.0%	198
1400	1009	1027	18	1.75%	80

TABLE XI COMPARISONS OF MCPS AND COMPUTATION TIME UNDER CASE 2

Number of participants	MCP ($/kWh)		τ	ρ	Computation time (min)
	DE-M	CE-M			
100	0.2922	0.2922	0.1	1.5×10^{-3}	2.17
200	0.2961	0.2961	0.2	1.5×10^{-3}	1.52
400	0.2811	0.2811	0.4	1.5×10^{-3}	9.28
600	0.2647	0.2647	0.6	1.5×10^{-3}	20.92
800	0.2752	0.2752	0.8	1.5×10^{-3}	16.67
1000	0.2807	0.2807	1.0	1.5×10^{-3}	19.08
1200	0.2663	0.2663	0.40	0.5×10^{-3}	50.53
1400	0.2710	0.2710	0.48	0.5×10^{-3}	22.83

Table XI also shows that the decentralized and centralized market models generate the same clearing price. Fig. 4 gives the differences between the decentralized and centralized market clearing outcomes when the number of participants is 1000. By comparing the outcomes in Fig. 4 with those in Fig. 3, it can be found after the parameter adjustments in the decentralized model, the performance of the proposed decentralized algorithm is further improved. The same dispatching outcome is attained with more participants by the decentralized and centralized market models.

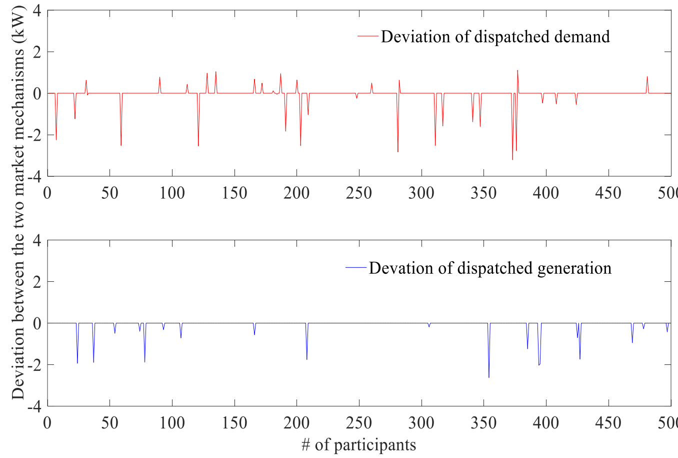


Fig. 4. Deviations of market clearing outcomes for 1000 participants in case 2.

Note: The deviation shown in Fig.4 represents the difference between the decentralized market result and the centralized market result.

(3) Calculation results for various scenarios in Case 3

In case 3, the bidding data from 100 participants are first generated and then additional bidding data added for the increased participants. Therefore, for each scenario in case 3, comparing with its previous adjacent scenario, the incumbent participants have the same bidding data, only the bidding data from newly added participants are randomly generated, which differs from cases 1 and 2. Meanwhile, ρ is still set as 1.5×10^{-3} and 0.5×10^{-3} when the total number of participants is below and over 1000. τ is given in Table XIII.

TABLE XII COMPARISONS OF MARKET CLEARING OUTCOMES UNDER CASE 3

Number of participants	Market volume (kW)		DBTM (kW)	Percent of DBTM	Number of iterations
	DE-M	CE-M			
100	75	75	0.0	0.0%	128
200	137	137	0.0	0.0%	124
400	273	273	0.0	0.0%	157
600	416	416	0.0	0.0%	129
800	562	562	0.0	0.0%	185
1000	716	738	22	2.98%	105
1200	899	904	5	0.55%	100
1400	1048	1054	6	0.57%	138

TABLE XIII COMPARISONS OF MCPS AND COMPUTATION TIME UNDER CASE 3

Number of participants	MCP ($/kWh)		τ	ρ	Computation Time (min)
	DE-M	CE-M			
100	0.2891	0.2891	0.1	1.5×10^{-3}	2.32
200	0.2873	0.2873	0.2	1.5×10^{-3}	4.58
400	0.2799	0.2799	0.4	1.5×10^{-3}	11.45
600	0.2840	0.2840	0.6	1.5×10^{-3}	14.38
800	0.2813	0.2813	0.8	1.5×10^{-3}	27.60
1000	0.2807	0.2807	1.0	1.5×10^{-3}	19.25
1200	0.2793	0.2793	0.40	0.5×10^{-3}	25.20
1400	0.2791	0.2791	0.48	0.5×10^{-3}	42.22

The results of cases 1, 2 and 3 manifest that the convergence of the iterative calculation is impacted by parameter settings, such as the number of participants, the values of ρ and τ, as well as the bidding data of participants. As the quantity of participants increases from 100 to 1400, the number of iterations does not increase accordingly. This is because the bidding data of participants under each scenario is generated independently. Therefore, the bidding data from a large number of participants is possibly to converge more easily than another group randomly generated bidding data for a small group of participants. In order to verify this point, cases 2 and 3 are carried out after the case 1. In case 2, the bidding data for all participants in each scenario is also generated independently. In case 3, the bidding data for 100 participants is firstly generated and then additional bidding data is added onto it for the increased participants. In other words, a proportion of participants in case 3 share the same bidding data, which differs from case 2. However, the calculation results of case 3 show that the increase of the number of participants will not necessarily lead to the increase of iterations. The convergence of the iterative calculation is significantly impacted by the concrete bidding data of participants.

D. Comparisons Study with Network Congestions

The IEEE 33-node distribution system [34] is adopted for testing the proposed model and solving algorithm. It is assumed that end-users 1 to 32 are connected to feeders 2 to 33 sequentially, with end-users 1 to 16 to be consumers and end-users 17 to 32 to be producers. Node 1 is selected as the slack bus. The PTDF matrix is calculated using the data from [34]. Six branches are randomly selected and it is assumed that network congestion would occur on these branches. The number of congested branches is 1/2/3/4/5/6 under scenario 1/2/3/4/5/6, respectively.

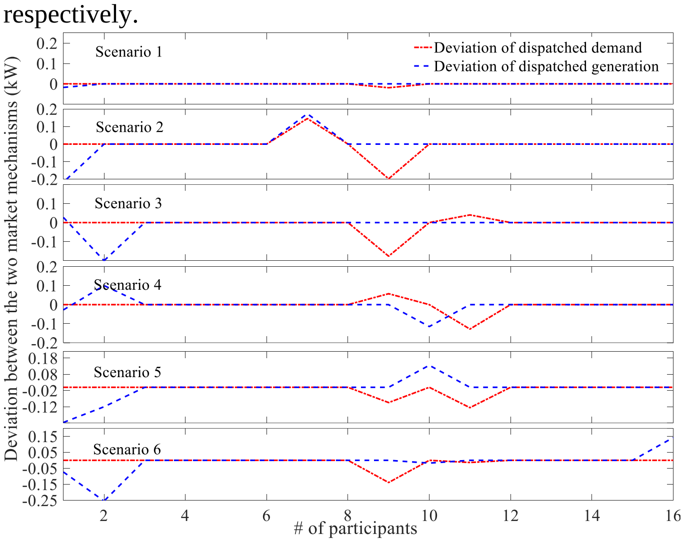


Fig.5. Deviations market outcomes under different scenarios with congestion. Note: The deviation in Fig.5 represents the difference between the decentralized market result and the centralized market result.

TABLE XIV NUMBERS OF PARTICIPANTS HAVING DIFFERENT TRADING RESULTS WITH NETWORK CONGESTION

Scenario	NPA	NCA	Max-deviation (kW)	Min-winning bid/offer (kW)
1	1	1	0.019	1.15
2	2	2	0.22	1.15
3	2	2	0.2	1.15
4	3	2	0.13	1.15
5	3	2	0.22	1.15
6	3	1	0.25	1.15

Note: Max-deviation indicates the maximum value of deviation; Min-winning bid / offer represents the minimum demand / generation output among winning bids / offers in the market clearing results.

Fig.5 presents the differences between market outcomes when congestion happens. Details of the differences are given in Table XIV. Differences between market outcomes are slight in which the maximum deviation for all scenarios is 0.25 kW and is less than 22% of the minimum winning bids/offers. Regarding the market trade volume, Table XV shows that the proposed decentralized algorithm can achieve almost the same solution as the centralized model, where the maximum difference is only 1.35%.

TABLE XV COMPARISONS OF MARKET OUTCOMES UNDER DIFFERENT SCENARIOS WITH NETWORK CONGESTION

Scenario	Market volume (kW)		DBTM (kW)	Percent of DBTM	Number of iterations
	DE-M	CE-M			
1	17.95	17.97	0.02	0.11%	536
2	17.72	17.76	0.04	0.23%	650
3	14.95	15.09	0.14	0.9%	1608
4	14.78	14.82	0.04	0.27%	2057
5	14.59	14.79	0.20	1.35%	2279
6	14.60	14.80	0.20	1.35%	2268

TABLE XVI COMPARISONS OF MCPS AND COMPUTATION TIME UNDER SCENARIOS WITH NETWORK CONGESTION

Scenario	MCP ($/kWh)		τ	ρ	Computation Time (min)
	DE-M	CE-M			
1	0.2842	0.2842	0.065	3×10^{-3}	6.67
2	0.2842	0.2842	0.2	3×10^{-3}	11.00
3	0.2842	0.2842	0.5	3×10^{-3}	28.20
4	0.2842	0.2842	0.6	3×10^{-3}	41.65
5	0.2842	0.2842	0.7	3×10^{-3}	70.52
6	0.2842	0.2842	0.8	3×10^{-3}	72.73

When the number of congested branches increases in the network, the number of iterations also increases. Because when there are more transmission constraints needed to be considered in the sub-problem of each participant, the decision variables will increase accordingly, which leads to more iterations. Once the decentralized one converges, market clearing outcomes, that are close to the centralized market clearing outcomes, can be attained. Thus, the results verify that the proposed algorithm can still clear the market in a decentralized way when transmission constraints in the distribution system are considered.

In Tables IX, XI, XIII and XVI, details of the MCP, computation time, and parameter setting under various scenarios are presented. As mentioned above, the convergence of the iterative algorithm and computation time change with the parameter settings, such as the number of participants, the values of ρ and τ, as well as the bidding data of participants. When there is no network congestion, the decentralized market model can converge in an hour for all scenarios and in most cases within 30 mins for the numerical example. Even if network congestion occurs, the decentralized market model can mostly converge in an hour while in the worst case the required computation time is about 73 mins.

Notably, the serial computation is adopted for the simulations here, and the proposed decentralized market model can converge in a reasonable time. The computation time can be further reduced by employing parallel computation. Because in the proposed decentralized market, the Jacobian ADMM is characterized by the advantage of enabling a parallelized updating of all variables. Compared with the serial computation, parallel computation will significantly reduce the computational time for the proposed mechanism to converge, where the computational time will mainly be spent on communications among participants.

E. Convergence Analysis for the Decentralized Market Model

Because in the proposed decentralized market model, once the values of ρ and γ are determined, the parameter τ can be assigned different values as long as the constraint of Eqn. (30) is respected. Therefore, the convergence speed of the decentralized market model is analysed against different values of τ. Five scenarios with 400 participants in the market are studied for cases with and without network congestions.

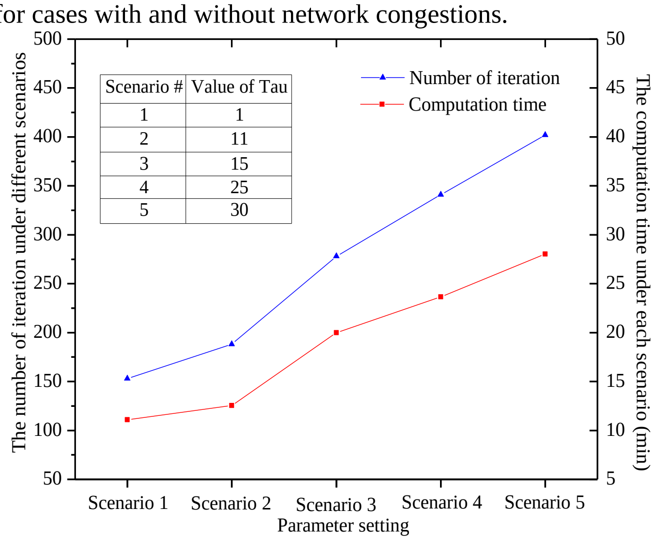


Fig.6. Number of iteration and calculation time without network congestion.

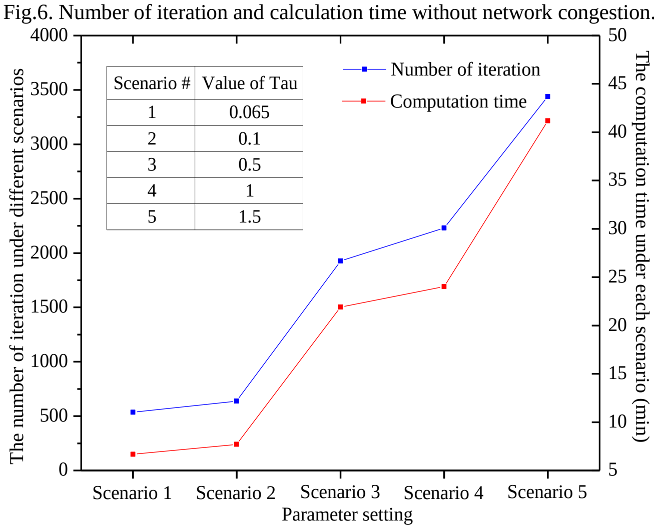


Fig.7. Number of iteration and computation time with network congestion.

TABLE XVII CONVERGENCE ANALYSIS OF DECENTRALIZED MARKET MODEL

Scenario	Results without network congestion			Results with network congestion		
	τ	Iteration number	C-time (min)	τ	Iteration number	C- time (min)
1	1.0	153	11.10	0.065	536	6.67
2	11	188	12.55	0.1	638	7.70
3	15	278	20.00	0.5	1927	21.92
4	25	341	23.65	1.0	2230	24.02
5	30	402	28.03	1.5	3493	41.17

Note: C-time indicates the computation time.

Fig.6 and Fig.7 present the number of iterations and computation time needed by the decentralized market model to converge under scenarios without and with network congestions, respectively. From the results, it can be found that with the increase of τ, the convergence speed of the decentralized model would decrease. Consequently, the number of iterations and computation time increase. Therefore, the value of τ should be controlled to be slightly beyond the limit as calculated by Eqn. (30) in implementing the proposed decentralized market mechanism. Besides, Table XVII shows the details of the convergence analysis results.

VI. CONCLUSIONS

Since the percentage of distributed generation in an actual power system is expected to grow steadily in the coming years, the traditional models of managing the electricity market may not work. The following conclusions are attained:

1) This paper first addresses the distribution market competition problem among renewable generators with zero marginal costs. The presented APM mechanism is able to cope with the situation that all participants in the auction are renewables with zero marginal costs. 2) Then, a decentralized form of the APM mechanism is derived using the ADMM. This provides a solution for organizing the decentralized electricity transactions in a distribution network. 3) The convergence of the decentralized mechanism can be guaranteed under some mild conditions for parameter setting, and information privacy of market participants can also be protected since only the sum of bid/offer data is transmitted during the iteration. Besides, network constraints are also integrated into the proposed model.

In terms of future research, since the convergence speed tends to be slower when network congestion occurs, this issue will be carefully examined in our future research efforts. Another direction of further work will be to investigate the impacts of uncertain trading behaviours on the market operation.

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