Charging Infrastructure Planning and Resource Allocation for Electric Vehicles



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Thesis submitted for the degree of Master of Philosophy Nov 2022

Statement of Authentication

The work presented in this thesis is, to the best of my knowledge and belief, original except as acknowledged in the text. I hereby declare that I have not submitted this material, either in full or in part, for a degree at this or any other institution.

...... (Signature)

Acknowledgements

I would like to express my most sincere gratitude to my principal supervisor Associate Professor Dongmo Zhang. His academic attitude and principles have deeply influenced me, and I am very grateful to A/Prof. Zhang for creating a good academic environment for me. A/Prof. Zhang not only helped me to solve my academic problems but also taught me the principles of living and working. Since I enrolled in the university, he has spent at least 10 hours per week discussing my research with me. When I retreated in the face of difficulties, he constantly encouraged me and told me that I was the best. I would like to thank my co-supervisor, Dr. Bo Du. He has guided me to develop the right attitude toward research since I was a student at UNSW. Instead of thinking that I was not determined enough because I changed my research topic so often, he encouraged me to keep trying different topics until I found one that I liked. I would like to thank Prof. Yan Zhang and Felicity for providing me with a good office environment, a monitor and computer stand, and a luxurious separate office.

Moreover, I would like to express my gratitude to my parents for their unconditional support for my tuition and living expenses. I am very grateful to them for helping me to take care of my three dogs so that I can study without worries. I would also like to thank my friends Jingyi Zhou, Yijun Zhou, Mingxue Ma, Elisa, Qinjun Liu, Lei Liu, Wenzhai Bi, and Cheng Jiang for making me enjoyable on the campus, where there are more kangaroos than people. Also, I would like to thank the only two cafes on campus for the free breakfast and good coffee. From the bottom of my heart, I would like to thank Piccolo Me for providing me with free chocolate sauce and fun stickers.

Finally, I want to thank myself for always believing that I am the best, the loveliest. I thank myself for going choose an unknown and interesting path, on which I have encountered storms and seen rainbows. As the saying, cease to struggle and you cease to live.

Abstract

With the increasing uptake of electric vehicles (EVs) and relative lag in the development of charging facilities, how to plan charging infrastructure and effectively use existing charging resources have become the top priority for governments, related industry and research communities. This study aims to address two key issues related to EV charging - charging station planning and charging resource allocation. The major contributions of the study are: (1) Introduced a model for charging infrastructure planning based on origin-destination data of EV traffic flows. I first showed how to use the gravity model to calculate point-to-point traffic flows from traffic data at each intersection and further induce the origin-to-destination flow data. Then, I introduced an optimization model for charging allocation based on origin-destination traffic flow data and extended it into a formal model for charging station planning by minimizing the total waiting time of EVs. (2) Applied the charging infrastructure planning model to Sydney Metropolitan charging station planning. I selected a set of representative areas from Sydney metropolitan and collected traffic data for these areas. I then used the gravity model to calculate the EV flow for each route based on possible portions of EVs among all traffic. The optimisation constraints under consideration include charging

station locations, total budget and feasibility of charging allocations. Optimisation for chargers at each intersection for different scenarios is solved using the least squares method. (3) Designed an algorithm for charging facility allocation to balance the load of charging stations. By considering the maximum driving range, the number of chargers at charging stations, and waiting time and queue length at each charging station, a queue balancing algorithm is proposed. Numerical experiments were conducted to validate the algorithm based on a linear road scenario. I believe that the outcomes of this research have a great potential to be used for government/industry planning of charging stations and improvement of utilization of charging stations resources.

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Chapter 1

Introduction

1.1 Background

Over the past decade, electric vehicles (EVs) are becoming more and more popular due to their advantages in terms of reducing carbon dioxide emissions and improving energy efficiency [1]. However, EVs are accounted for only 0.6% of new vehicle sales in Australia in 2020 [2]. One of the main obstacles for EV uptake is insufficient charging facilities [3; 4]. Although many cities are actively promoting the development of charging infrastructure, it still cannot meet the fast-growing charging demand from EV users [5]. In real life, with limited charging resources and uneven distribution of charging demand, the high-demand charging activities may cause long queue at some charging stations while other charging stations are underutilised. Therefore, both planning and utilising of charging infrastructure are crucial for local/federal governments, related industries and EV users not only for now but also for the future. Although great amount of work in both topics has been done in the literature, most approaches are only applicable to the data sources they designed for. This thesis investigates the approaches of charging infrastructure planning and charging resource allocation to origin-destination EV flow data. We found that most publicly available data for traffic flows only record traffic volumes on road segments or crossing intersections. A vehicle would have been counted several times in different road segments and/or intersections even though it only needs to be charged once or twice for its whole trip. To solve the problem, we introduced an approach to induce origin-destination traffic flows from traffic volume data and developed an specific model for charging station planning and charging facility allocation. Based on the model, we conducted a case study of charging station planning for Sydney metropolitan road network and designed an algorithm of queue balancing for charging facility allocation for highway scenario with linear road network. We believe that our approach can be applied to a wide range of application domains in EV charging.

1.2 Major contributions

The research aims to plan charging infrastructure and provide charging guidance for EV users to make the best use of available charging resources. The main contributions of this thesis can be summarized as follows:

• Introduced the optimization model for charging station planning based on origin-destination traffic flow data. I first showed how to use the gravity model to calculate the point-to-point traffic volume data of each intersections, which can induce the origin-destination traffic data. Then, I proposed a optimization model for charging station assignment based on origin-destination traffic flow data. Finally, I extended the model to a formal model for charging station planning by minimizing the total waiting time for electric vehicles. Based on this model, we can calculate the number of chargers that should be installed at each charging station and the optimal allocation of vehicles at each charging station.

- Conducted a case study for Sydney metropolitan charging station planning. Firstly, I selected several representative areas in Sydney metropolitan and collected traffic volume data from the New South Wales Department of Transport in Australia (Transport for NSW). Then, I used the gravity model to calculate EV flows for each origin-destination route. Finally, I applied the optimization model to calculate the number of charging posts required for each intersection under different scenarios by the least squares method.
- Designed a queue-balancing algorithm to minimise EV charging time and test it with a scenario of linear road network. Given the traffic volume of EV flow in a linear road network and distribution of allowed driving range, we first estimate the length of queues at each charging station based on M/M/n queuing model. We then developed an algorithm that can recursively reallocation of EVs to charging stations based on their remaining driving range. The algorithm is tested with a scenario simulating EV charging on a highway.

1.3 Literature review

This research covers three main research categories, including charging infrastructure planning, queuing theory and the resource allocation for electric vehicles. I will provide a brief review on each of the topic in the following subsections.

1.3.1 EV charging demand

The charging demand of electric vehicles has significant implications for the planning of charging stations. Early studies have used questionnaires and historical data to understand the charging demand and behavior of electric vehicle users [6; 7; 8]. With the further research, there are two widely applied models for electric vehicle charging demands. One is a spatial-temporal probabilistic model [9; 10; 11] and the other is a rough probabilistic model [12; 13; 14; 15]. Zhang et al. proposed a method to predict the spatial and temporal distribution of EV charging load based on EV parking behavior. The spatial and temporal distributions of EV charging demand are developed by quantifying the EV parking behavior and parking generation rate thereby [16]. A model of electric vehicle user driving and charging behavior based on random trip chains and Markov decision processes is developed by Tang and Wang. The model is used to evaluate the charging demand resulting from the temporal and spatial distribution of electric vehicles [17]. Mu et al. have incorporated a Monte Carlo simulation approach in the spatial and temporal models to obtain the charging load of electric vehicles. That model to evaluate the impact of large-scale deployment of electric vehicles on the urban distribution grid and compare different types of electric vehicle charging strategies to verify the validity of the model [18]. Shun et al. developed a trip-chain stochastic simulation-based approach to model the driving patterns of electric vehicle users. In this way, the spatial and temporal distribution characteristics of electric vehicle charging demand are reflected [19].

In [20], the authors identify a probabilistic distribution model for electric vehicle charging demand based on the U.S. National Household Travel Survey dataset and propose a queuing theory based probabilistic approach to calculate electric vehicle charging demand. Zhou et al. considered the charging characteristics of various types of electric vehicles and improved the sampling method of the initial charging state of electric vehicles, thereby establishing a probabilistic model of the charging load of electric vehicles [21]. Lojowska et al. obtained stochastic properties of vehicle behavior based on Dutch traffic dataset, and proposed a Monte Carlo simulation method to model the charging demand of electric vehicles from simulated electric vehicle user commuting and electric vehicle charging scenarios [22]. In [23], an improved model of Monte Carlo simulation is presented to predict the charging demand of electric vehicles. The modified model takes into consideration the number of electric vehicles, the charging duration and the driving distance of electric vehicles. Yang et al. developed a probabilistic electric vehicle charging load model through a set of equations that introduces charging traffic flow to describe a discrete sequence of electric vehicle charging events, which contains both spatial and temporal properties of the electric vehicles charging load [24]. An auto-regressive integrated moving average method is proposed by Amini et al. to predict the charging demand for conventional electrical loads and electric vehicle parking lots. This model is used as input based on driving patterns and distances, and the accuracy of the model prediction is improved by optimizing the integrated and auto-regressive order parameters [25].

1.3.2 Waiting time at charging station

In order to specify waiting time of charging, queuing theory is widely used in both charging infrastructure planning and charging resource allocation [26; 27; 28]. Seyedhoesini et al. [29] proposes the application of two M/M/C queues being used to describe the operation of indoor and outdoor trucks in each terminal, and proposes a mixed integer model that incorporates queuing theory to optimize the location and operational success of the terminal. Selinka et al. [30] developed a fixed backlog-handling method based on a queuing model to evaluate the performance of truck loading and unloading operations at air cargo terminals. A three-level supply network system consisting of supplier, cross-docks, and producer-distributors is designed using M/M/M queues by Maghsoudlou et al. [31]. And the authors proposed a bi-objective optimization model. The first objective is to minimize supplier, producer, and distributor costs. The second objective is to minimize the vehicle travel time and waiting time. Shahram fard and vahdani 32 proposed a bi-objective optimization model for the truck assignment problem at terminals. The model provides an M/M/1 queuing system to minimize the waiting time of trucks. The second objective function is to minimize the energy consumption of trucks at the terminal, and the authors use a multi-objective heuristic to perform the optimization. Chen et al. [33] developed a data-driven planning model for plug-in electric vehicle charging stations. The model uses real data on electric taxicab trips and uses queuing theory to model and analyze the charging congestion of service sharing electric taxicab in central Beijing. In [34], a charging model is proposed to reduce the cost of electricity as well as the waiting time of electric vehicle users. The authors show that queues of multiple charging stations in a certain area are consolidated into one queue in advance, i.e., all electric vehicles in multiple charging stations are centrally assigned to available charging stations. The results obtained from model simulations show that concentrating charging at one charging station not only reduces the cost of battery degradation by 80% but also reduces the queuing time by more than 60%. Jung et al. [35] proposed a two-level simulation optimization framework that incorporates an M/M/K queuing system to determine the optimal location of charging stations and the allocation of chargers.

1.3.3 Charging infrastructure planning

With the increase in the number of EVs, the demand for charging stations from EV users is also gradually increasing. How to plan electric vehicle charging service areas and charging stations has also attracted the interest of more and more scholars [36; 37; 38; 39; 40]. Zhu et al.[41] proposed a genetic algorithm based approach to determine the location of charging stations and the number of chargers that should be built at each charging station. Liu et al. [42] developed a method that combines the sitting of charging stations and the reduction of charging station costs. In [43], they presented a method to determine the location and optimize the size of charging stations for EVs. The model introduces the concept of candidate charging stations and develops a mathematical model that minimizes the construction of charging stations as well as the consumption of electric vehicle users. Micari et al. [44] proposed a methodology to incorporate charging stations in a road network by determining 1) the total number required of charging stations and 2) their corresponding locations. Zhu et al. [28] presented

a model for planning charging stations. The model takes the minimum cost of EV users and charging station construction as the optimization objective, the EV charging queuing time and the safety of distribution network operation as the constraints, and finally the charging stations division by a weighted Voronoi diagram. In [45], the authors proposed a smart charging strategy that supports multiple charging options. The model considers multiple charging options as a multi-server queuing system to estimate the waiting time at the charging station for each charging option. The model is a multi-objective optimization problem with the goal of minimizing travel time, waiting time, and charging cost. Ge et al. [46] aims to minimize the travel cost of EV users to the charging station, and on the basis of considering the traffic flow and the electric capacity constraint of the chargers, the charging stations is partitioned by the grid division method and finally adjusted iteratively to obtain the location of the charging station. To overcome the limited driving range of EVs and satisfy the increasing charging demand, researchers have made great efforts on locating EV charging stations and allocating charging resources for both private cars [47; 48] and public transport such as electric buses [49; 50]. Nie & Ghamami [51] investigated the problem of battery sizing, charging facilities locating with the objective to minimise the total social cost considering a given level of service. Zheng et al. 52 proposed a bi-level model to optimise the deployment of charging stations with traffic equilibrium. The upper level aimed to optimise the locations of charging stations, and the lower level determined the users' path selection considering traffic equilibrium based on the charging stations obtained in the upper level. Micari et al. [44] proposed an approach to determine the number of required charging stations in the road network of a highway and corresponding locations with simple

assumption of the same remaining battery of each EV. By applying the queuing theory, Lu & Hua [53] developed a flow-refuelling location model to investigate the location-sizing problem for EV charging stations considering customers' tolerance. With a focus on users' benefits, Jung et al. [54] proposed a stochastic dynamic itinerary-interception refuelling location model to optimise the locations and sizing of charging stations to minimise the average delay, which referred to the travel time to the facility and waiting time at the facility. Yang et al. [55] formulated an integer linear programming model to optimise the locations and size of charging stations with the objective to minimise the total investment cost. The M/M/x/s queuing model was adopted to estimate the probability of EV taxis being charged, and a data-driven approach was employed to analyse taxis' dwell patterns. Xiao et al. ^[5] proposed a charging station planning model with consideration of finite queue length and charging queuing behaviour. Konara et al. [56] propose a charging operation strategy that coordinates and reallocates charging resources by giving priority to fast charging EVs. This strategy can improve the utilization of charging stations and maximize the profitability of charging stations.

1.3.4 Charging resource allocation

The majority of current research on electric vehicle resource allocation is directed towards electric vehicle route selection. There are many studies on EV routing selection. [57; 58; 59; 60; 61]. A general Electric Vehicle Routing Problem (EVRP) was proposed by Lin et al. [62]. The model finds the optimal routing strategy that results in the shortest travel time and lowest energy cost for EVs. Goeke and Schneider [63] present the Electric Vehicle Routing Problem with Time Window and Hybrid Fleet (E-VRPTWMF) to optimize the routing of two hybrid fleets. They developed an adaptive large domain search algorithm, which, according to experimental results, was shown to find better results for EVRP in less time. Zhang et al. [64] proposed a metaheuristic approach based on ant colony (AC) algorithm to reduce the energy consumption of electric vehicles and developed a corresponding mathematical model to find a routing plan for EVs. Yang et al. [65] proposes an EV route selection and charging navigation optimization model that reduces travel costs as well as charging costs for EV users. Olle and Carl [66] demonstrate a new method for planning electric vehicle charging with grid constraints, including voltage and power. The method creates an individual charging plan for each EV, avoiding congestion on the distribution grid while meeting the requirements of individual vehicle owners.

1.4 Summary

Although many studies have investigated the location and sizing problem of charging stations for EVs, the majority focused on seeking the optimal locations of charging stations and the number of chargers at corresponding locations. This study not only addresses the charging station planning problem but also proposes a corresponding solution to the problem of uneven distribution of charging resources. For charging station planning, the main contribution of this study is the introduction of a charging infrastructure planning model based on the origin and destination data of EV flow, which is extended to a formal model for charging station planning by minimizing the total waiting time for EVs. For the charging resources allocation, the investigation on balancing the charging load between charging stations to maximize the utilization of existing charging resources and minimize the total queuing time of EV users is still missing. To fill the gap, this research formulates an optimization model to allocate the charging demand to a proper charging station to minimize the total queuing time while satisfying each EV user's charging demand.

The remainder of this thesis is organized as follows. In Chapter 2, we will introduce the optimization model for charging station planning based on origindestination traffic flow data. In Chapter 3, we will conduct a case study for Sydney metropolitan charging station planning. In Chapter 4, we will design a queue-balancing algorithm to minimize EV charging time and test it with a scenario of a linear road network.

Chapter 2

A formal Model for Charging Infrastructure Planning

In this chapter, I will introduce an origin-destination based model for charging station planning. Section 2.1 will briefly describe the idea of the origin-destination approach. Section 2.2 shows how to use the gravity model to induce origin-destination flows based on traffic flows of intersections. Section 2.3 will present an optimization model for charging station planning over a road network based on origin-destination traffic flow data. The target of charging station planning is to minimise waiting time for EV charging by optimise the number and location of charging stations and chargers under certain road configuration and budget. Section 2.4 will give a brief summary of the chapter.

2.1 Introduction

Charging infrastructure planning is to determine where and how many charging stations (chargers) to be built in an area/region based on EV charging demand. However, the data for EV charging demand is not always available, especially for the areas where EVs are relatively low. Most of existing studies on charging infrastructure planning are based on traffic data for generic vehicles with a projection of generic traffic flow onto EV traffic flow in terms of the percentage of EV over general traffic [36; 37; 38; 39; 40; 41; 42; 45]. Although there are many sources to acquire data for general traffic flows, the data mostly record the traffic volumes on a road segment or an intersection. A plan of charging stations based on such data can be very inaccurate because a vehicle may travel allow different segments of a road across different intersections along his trip but would just need to charge once for the whole trip. In this research, I will introduce an approach to charging station planning based on origin-destination (O-D) flows. To describe my idea, let us consider a simple scenario for charging station planning. Figure 2.1 shows two origin-destination paths, o_1 - d_1 and o_2 - d_2 , in a road network. If each vehicle from its origin to destination only needs to charge once and can be charged at any charging station along its path 1 . For any given EV traffic flow on each path, the cost of building a charging station at each location and a total budget, I can optimise the selection of locations to build charging stations and number of chargers so that the overall waiting time for charging is minimal.

¹In Chapter 4, I will consider more complicated scenarios in which choice of charging stations of a vehicle is subject to its remaining battery level.

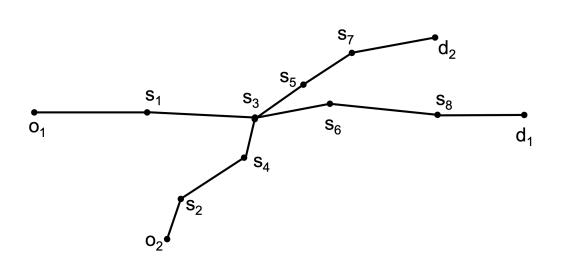


Figure 2.1: A simple scenario for origin-destination based charging station planning

2.2 Estimation of point-to-point traffic flow

As mentioned above, most available data for traffic flow record only traffic volumes on road segments or crossing intersections while our approach requires origin-destination flow data. There have been quite a number of approaches to estimate point-to-point traffic flows from traffic volumes [67; 68; 69; 70]. In this research, I use a gravity model to estimate the point-to-point matrix based on the data of intersection flows. This approach has been widely used for trip distribution, which is inspirited by Newton's fundamental law of attraction [71; 72].

Consider a road network G = (V, E) where the set V of vertices represents a set of intersections and the set E of edges represents the set of road segments that link the intersections. For each vertex (intersection) $i \in V$ in a road network, let O_i denote the number of EVs that enter the road network at i and D_i denote the number of EVs that exit from the network at vertex i. For each i and j, let c_{ij} represent the distance between i and j. Furthermore, we let $f(\cdot)$ be the determined function for the gravity model, which takes into account the fact that the accessibility of an opportunity decreases as the distance increases between two road nodes [71]. Finally let Q_{ij} be the number of vehicles that travel from ito j. $(Q_{ij})_{i,j\in V}$ is called the point-to-point matrix.

Based on the gravity model [71], the point-to-point matrix is recursively defined by the following equation:

$$Q_{ij} = A_i B_j O_i D_j f(c_{ij}), \text{ for each } i, j \in V$$
(2.1)

where A_i and B_j are the balancing factors defined as follows:

$$A_i = \frac{1}{\sum_{j \in V} (B_j D_j f(c_{ij}))} \tag{2.2}$$

$$B_{j} = \frac{1}{\sum_{i \in V} (A_{i}O_{i}f(c_{ij}))}$$
(2.3)

while O_i and D_j are determined by the following constraints on total outmovement and in-movement:

$$O_i = \sum_{j \neq i} Q_{ij} \tag{2.4}$$

$$D_j = \sum_{i \neq j} Q_{ij} \tag{2.5}$$

According to our data setting shown in Chapter 3, I will use the following negative exponential function as the deterrence function:

$$f(c_{ij}) = e^{-\beta c_{ij}} \tag{2.6}$$

Intuitively β reflects the average travel cost of vehicles - the lower the β value, the higher the average travel cost of vehicles. Obviously, the value of β affects the iteration of point-to-point matrix (see Equation 2.1). In Chapter 3, I will use Hyman's method [73] to determine the value of β .

With the gravity model we can get point-to-point traffic flows in a road network. In Chapter 3, I will use this approach to induce traffic flows for origindestination based on the traffic data in Sydney Metropolitan and use the information to plan charging stations for Sydney Metropolitan. To facilitate the data analysis, let me present a generic formal model for charging station planning. The model is a simplification of Roughgarden's selfish routing game [74].

2.3 The model for charging allocation

As mentioned above, I use a directed graph G = (V, E) to represent a road network, where the set V of vertices represents intersections and the set E of edges represents the roads that link the intersections. I assume that there are a set of k paths, $P = \{p_1, \dots, p_k\}$, in the road network that link a set of origins and a set of destinations¹. We assume that all EVs that need to be charged are charged only once on the road network. We also assume that charging stations are all

¹Different from Roughgarden's model [74], our purpose of formalisation of road networks is to present an optimisation model rather than a game theoretical model. We do not specify which intersections in the network are origins and which are destinations but simply assume that the collection of the starting points of the paths are origins and the collection of the end points of the paths are destinations.

located in a vertex of the road network rather than on an edge (linked road)¹.

For each $p \in P$, $v \in p$ means that v is a vertex (intersection) in path p. For each $p \in P$, r_p denotes the number of EVs on path p that need to be charged². Since each vehicle only needs to be charged at most once, $r = \sum_{p \in P} r_p$ represents the total demand of charging in the whole road network (per time unit under consideration).

A charging allocation $f : P \times V \to \mathbb{Z}$ is a function that maps a path and vertex into an integer. Intuitively, for each path $p \in P$ and $v \in V$, f(p, v) represents the number of EVs travel on path p choose to charge at v. Obviously, we require for each $p \in P$

$$f(p,v) = 0 \text{ if } v \notin p \tag{2.7}$$

which means that if a path p does not go through intersection v, no vehicle on the flow of path p can choose to charge at v.

Furthermore, we require any vehicle which needs to be charged has to be charged at a vertex on its way:

Definition 2.1. A charging allocation f is feasible if it satisfies Equation (2.7) and the following condition

$$\sum_{v \in p} f(p, v) = r_p \tag{2.8}$$

Giving a charging allocation function f, for each $v \in V$, let

$$f_v = \sum_{p \in P \text{ s.t } v \in p} f(p, v)$$
(2.9)

¹If a charging station is on the way between two intersections, we consider the charging station as an intersection, or more accurately, a vertex in the road network.

²it also represents the total vehicles travel on the path from the orign (the start point of the path) to the destenation (the end point of the path) in a period of time under consideration.

It is easy to see that f_v represents the number of vehicles that charge at vertex v. Put all the assumption together, we have

Lemma 2.1. Given a road network and a charging allocation f for the network, if f is feasible,

$$\sum_{v \in V} f_v = r \tag{2.10}$$

Proof. The proof is straightforward based on the assumptions and definitions above. In fact, we know $r = \sum_{p \in P} r_p$. By Equation (2.7 and 2.8) we further have

$$\begin{aligned} r &= \sum_{p \in P} \sum_{v \in V} f(p, v) \\ &= \sum_{v \in V} \sum_{p \in P} f(p, v) \\ &= \sum_{v \in V} (\sum_{p \in P \text{ s.t. } v \in p} f(p, v) + \sum_{p \in p \text{ s.t. } v \notin p} f(p, v)) \\ &= \sum_{v \in V} \sum_{p \in P \text{ s.t. } v \in p} f(p, v) \\ &= \sum_{v \in V} f_v \end{aligned}$$

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Next, we consider the cost of the EV users. For simplicity, we assume that all the chargers installed in the road network are with the same type and each vehicle is charged for the same time (more complicated cases will be considered in Chapter 4). Since all vehicles are charged only on the path they travel, the basic time cost of each vehicle except for waiting for chargers is the same when we plan for charging stations. Therefore the objective of charging station plan is to minimise the total waiting time for charging for all the vehicles. Formally, let $\ell_v(h)$ be waiting time of each vehicle for charging at vertex vwhen there are h vehicles choose to charge there during the time period under consideration¹. Assume that for each $v \in V$ the waiting function $\ell_v(\cdot)$ is nonnegative and non-decreasing.

For any feasible charging allocation f, we let C(f) be the total cost of waiting time for EV charging in the road network, that is,

$$C(f) = \sum_{v \in V} \ell_v(f_v) f_v \tag{2.11}$$

If we let $\ell(f)$ represent the sum of the waiting time of all the vehicles travel on path p:

$$\ell_p(f) = \sum_{v \in p} \ell_v(f_v) f(p, v)$$
(2.12)

we will have the following

Lemma 2.2. If charging allocation f is feasible,

$$C(f) = \sum_{p \in P} \ell_p(f) \tag{2.13}$$

Proof. Similar to the proof of Lemma 2.1 and the condition of feasibility of a

¹The vehicles are not necessarily queue at v at the same time but they have chosen to charge there during that period of time.

charging allocation, we have

$$C(f) = \sum_{v \in V} \ell_v(f_v) f_v$$

= $\sum_{v \in V} \ell_v(f_v) \sum_{p \in P \text{ s.t } v \in p} f(p, v)$
= $\sum_{v \in V} \ell_v(f_v) \sum_{p \in P} f(p, v)$
= $\sum_{v \in V} \sum_{p \in P} \ell_v(f_v) f(p, v)$
= $\sum_{p \in P} \sum_{v \in V} \ell_v(f_v) f(p, v)$
= $\sum_{p \in P} \ell_p(f)$

as desired.

I would like to emphasize that the total cost of vehicles on a path $\ell_p(f)$ is a crucial concept in a game theoretical model of charging infrastructure planning because it determines the utility of the agent representing all the vehicles from the same origin to the same destination if it chooses the path to travel. Unfortunately I have no time to go into the detail of the game-theoretical model. I leave this investigation for my PhD research.

Now I am ready to present the model of charging allocation. To decide which spots are needed to install chargers and how many chargers to be installed at each of the spots, we first find the charging demand of each path in a road network and then find the best charging allocation that minimises the total waiting time of the EVs. In other words, we need to find a solution to the following optimisation

problem:

$$\min_{f} \sum_{p \in P} \sum_{v \in p} \ell_v(f_v) f(p, v)$$
(2.14)

subject to

$$f(p,v) = 0 \text{ if } v \notin p, \forall p \in P \& v \in V$$

$$(2.15)$$

$$\sum_{v \in p} f(p, v) \ge r_p, \forall p \in P$$
(2.16)

$$\sum_{p \in P: v \in p} f(p, v) = f_v, \forall v \in V$$
(2.17)

$$f(p,v) \ge 0, \forall p \in P \& v \in V$$

$$(2.18)$$

Here, Equation 2.15 and 2.16 require an optimal charging allocation must be feasible (note that f is non-decreasing). Equation 2.17 defines the variable f_v . Equation 2.17 sets the boundary conditions for the charging allocation function.

By Lemma 2.2, Equation (2.19) can be simplified as

$$\min_{f} \sum_{p \in P} \ell_p(f) \tag{2.19}$$

2.3.1 The model for charging station planning

The major concerns for an EV user to decide when and where to charge include the remaining battery level of their vehicles, the distance to the charging stations they choose to charge, and the waiting time at each charging station. For charging infrastructure builders, the major concerns are where to build a charging station and how many charger to install. Note that since land prices vary from place to place. The costs of maintenance also vary from region to region, a detailed cost analysis can be very complicated and goes beyond the scope of the research. For simplicity, I will ignore the cost of building the infrastructure of a charging station, including the cost of land, parking space and roadside facilities, but only the cost of chargers at each charging station. We also assume the chargers installed at each charging station are in the same type with the same costs.

For planning purpose, we assume that the total budget for building chargers is W (for simplicity, we assume that no charging facility exists) while the cost of each charger is w.

Let $n: V \to \mathbb{Z}$ be a function such that for each vertex v, n(v) represents the number of chargers to build at vertex v. Then we have the following budget limit:

$$\sum_{v \in V} wn(v) \le W \tag{2.20}$$

With the optimal model of charging allocation shown in the previous section, we can calculate the demand of charging at each vertex in a road network. Based on the demand, we can decide how many chargers to install at each charging station based on the available budget. However, to use the optimal model, we need to estimate the waiting time at each charging station. Ideally we can calculate the average waiting time at a charging station based on queuing theory (see Chapter 4). This can be very hard at the stage of charging infrastructure planning because there is no running data support the estimation. In this research, we use a simple method to estimate the waiting time at each charging station [75].

Assume that f_v is the number of EVs that choose to charge at station v as defined in Equation (2.9). These m_v EVs can be roughly divided into the queues shown in the following equation:

$$\hat{m_v} = \lceil \frac{f_v}{n(v)} \rceil \tag{2.21}$$

where $\hat{m_v}$ is the number of EVs waiting for each charger at charging station v. Next, we assume that the length of charging time per EV is t. Based on [75], the average waiting time of each EV can be calculated as:

$$\ell_v(f_v) = \frac{t}{\hat{m}_v} \cdot \sum_{i=1}^{\hat{m}_v} i \tag{2.22}$$

where $\hat{m_v}$ is determined by Equation (2.21). Note that I do not consider the actual time each electric vehicle arrives the charging station but assume they will be charged at the charging station in the given time period thus the estimated time could be longer than actually needed.

Finally we can present our model for charging station planning as an extension of the optimisation model for charging allocation:

$$\min_{n,f} \sum_{v \in V} \ell_v(f_v) f_v \tag{2.23}$$

subject to

$$f(p,v) = 0 \text{ if } v \notin p, \forall p \in P \& v \in V$$

$$(2.24)$$

$$\sum_{v \in p} f(p, v) \ge r_p, \forall p \in P$$
(2.25)

$$\sum_{p \in P: v \in p} f(p, v) = f_v, \forall v \in V$$
(2.26)

$$\sum_{v \in V} wn(v) \le W \tag{2.27}$$

$$f(p,v) \ge 0, \forall p \in P \& v \in V$$

$$(2.28)$$

$$n(v) \ge 0, \forall v \in V \tag{2.29}$$

Since the cost function $\ell_v(\cdot)$ is defined by Equations (2.21 and 2.22), these two equations can also be viewed as constraints for the cost minimisation.

Note that the optimisation problem is non-linear with two optimisation variables, n and f, both of which are functions thus the computation of optimisation for real-world applications can be quite challenging. I will leave the computational issues for charging infrastructure planning for PhD research.

2.4 Summary

In this Chapter, I introduced an optimisation model for charging station planning. Firstly, I showed how to use the gravity model to calculate point-to-point traffic flows from traffic data at each intersection, which can induce the originto-destination flow data. Then, I presented an optimization model for charging allocation based on origin-destination traffic flow data. Finally, I develop an optimization model for charging station planning by minimizing the total waiting time of EVs. Based on the model, we can calculate the number of chargers should be installed at each charging station and the optimal allocation of vehicles to each station. The real life examples of charging station planning is analyzed in detail in the Chapter 2.

Chapter 3

Charging Station Planning for Sydney Road Network

In this Chapter, I will apply the model presented in Chapter 2 to calculate optimal locations and number of chargers of each charging station for Sydney road network. Section 3.1 will briefly describe the chosen study area, as well as the data source. Section 3.2 will present the EV flows for each route, which obtained by the gravity model. And the number of chargers required at each intersection for different scenarios solved by the least squares method. Section 3.3 will give a brief summary of the chapter.

3.1 Introduction

In the last few years, the number of EVs in Australia has increased significantly, as shown in Figure 3.1. From this figure, it is clear that the increase in the number of charging stations has not fluctuated significantly. I analyzed a 10-year public dataset from the New South Wales Department of Transport in Australia (Transport for NSW), which data contains the number of EVs and the number of charging stations in Australia from 2011 to 2021. Based on Figure 3.2, it is clear that although New South Wales has the highest number of charging stations, the number of the public charging station is far from meeting the future demand for EVs relative to the annual growth of EVs. Therefore, I use the model introduced in the previous Chapter to plan the location and the number of chargers for part of the road network in Sydney Metropolitan, using the city of Sydney as an example.

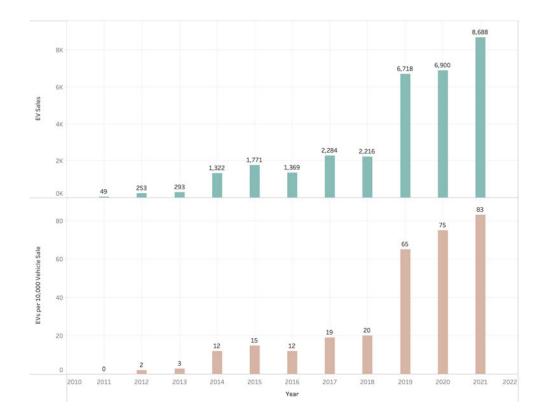


Figure 3.1: The number of electric vehicles in Australia

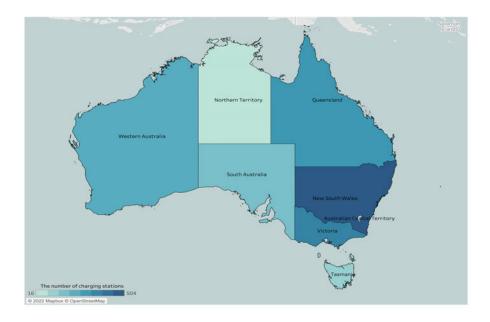


Figure 3.2: The number of public charging stations in Australia

3.2 Data preparation

In the case study, an application has been made on a real case study considering the city of Sydney. Based on the percentage of EVs in 2021 in Figure 3.1, the number of EVs at each intersection in Figure 3.3 can be roughly estimated. Note that the number of EVs in Figure 3.3 is the traffic volume for all day.

I obtained the intersection traffic volumes from the NSW open access data and based on the proportion of EVs to total vehicles in Australia in 2021, I can roughly estimate the number of EVs in one hour at each of the 17 intersections in this experiment. It is clear from Figure 3.3 that there are 5 routes with 17 intersections. Parramatta and Ryde, North Wahroonga, and Sydney Center have more EVs, and it also can be said that the North and East Districts have more EVs and high concentrations of EVs compared to the South and West Districts.

Since I am with a real dataset, but only intersection traffic is available in

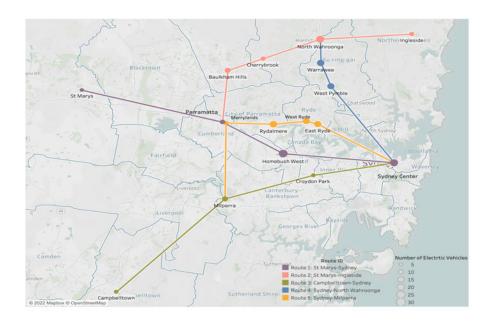


Figure 3.3: Number of electric vehicles at intersections on the Sydney road network

that dataset. Therefore, I use a gravity model to estimate the O-D matrix by the intersection traffic [71]. Based on the gravity model and the number of EVs at the intersection, I obtain the EV flow for each route in this experiment after iterations of processing, as shown in Table 3.1. Further, I can obtain the hourly intersection EV flow after redistribution through the gravity model, as shown in Figure 3.4.

| 10010 0 | | anatea engin | acconnacton | 1110001111 | m sjanej eng | |
|-----------------|---------|---------------|--------------|------------|-----------------|----------|
| | St Mary | Sydney Center | Campbelltown | Ingleside | North Wahroonga | Mipkerra |
| St Mary | 0.02 | 0.20 | 0.20 | 3.14 | 0.29 | 0.15 |
| Sydney Center | 0.09 | 0.00 | 0.28 | 0.00 | 6.81 | 3.82 |
| Campbelltown | 0.11 | 0.36 | 0.94 | 0.50 | 1.39 | 0.71 |
| Ingleside | 2.37 | 0.00 | 0.70 | 0.37 | 1.04 | 0.53 |
| North Wahroonga | 0.13 | 7.40 | 1.18 | 0.63 | 1.76 | 0.89 |
| Mipkerra | 0.07 | 4.48 | 0.65 | 0.34 | 0.96 | 0.49 |

Table 3.1: Calculated origin-destination matrix in Sydney city

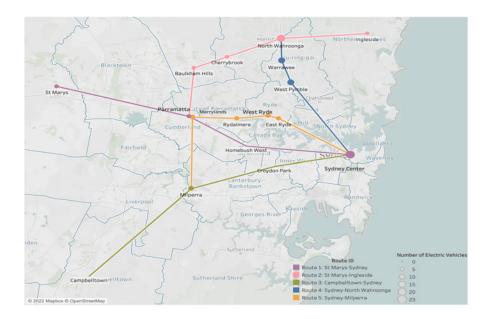


Figure 3.4: Number of electric vehicles at intersections on the Sydney road network (After gravity modeling)

3.2.1 Numerical results

In this case study, I assume that EVs at each intersection need to be charged if the EV flow at each intersection is greater than 1 in one hour. Assuming the cost of per charger is $25,000 \ AUD$, and the total cost of charger investment is $2,000,000 \ AUD$. Based on the model and the above assumptions, I can obtain the location and number of chargers to be allocated as shown in Figure 3.5.

According to Table 3.1 I can know the EV flow of these five routes after the gravity model redistribution. Let us take route 1 as an example. In Table 3.2, baseline indicates the hourly EV flow at each intersection after gravity redistribution. Then, baseline+20% indicates that the EV flow in route 1 is increased by 20%, and the flow of other routes remains unchanged. By analogy, I can get baseline+40% etc. In this case study, I have five routes. Taking route 1 as an

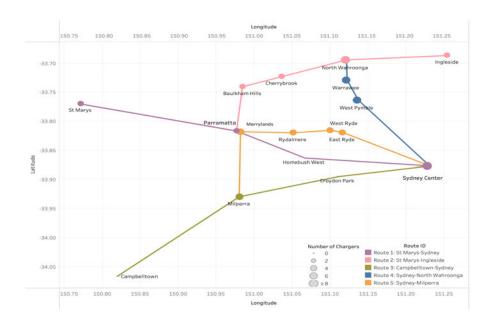


Figure 3.5: Number of chargers at intersections on the Sydney road network (After gravity modeling)

example, I analogously obtain the number of EVs per hour at the intersections for route 2, route 3, route 4, and route 5 in the road network under different situations. They are shown in Tables 3.3, Table 3.4, Table 3.5 and Table 3.6, respectively.

3.2.2 Optimal results for different scenarios

When the EV flow in the road network changes, the allocation of the number of chargers in the whole road network will change accordingly. Therefore, after I obtain the number of EVs per intersection for each route separately in different situations, I can calculate the number of chargers allocated to each intersection by minimizing the mathematical model of waiting time.

Although the EV flow increases proportionally for each route, the number of

| Intersection | | Number | of electric | vehicles (p | er hour) | |
|-----------------|----------|----------|-------------|-------------|----------|----------|
| (Suburb) | baseline | baseline | baseline | baseline | baseline | baseline |
| (Suburb) | | +20% | +40% | +60% | +80% | +100% |
| St Marys | 5.79 | 5.85 | 5.91 | 5.97 | 6.03 | 6.08 |
| Parramatta | 5.79 | 5.85 | 5.91 | 5.97 | 6.03 | 6.08 |
| Homebush West | 0.29 | 0.35 | 0.41 | 0.46 | 0.52 | 0.58 |
| Sydney Center | 23.11 | 23.17 | 23.23 | 23.29 | 23.34 | 23.40 |
| Rydalmere | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| Merrylands | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 |
| Baulkham Hills | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Cherrybrook | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| North Wahroonga | 19.72 | 19.72 | 19.72 | 19.72 | 19.72 | 19.72 |
| Ingleside | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Campbelltown | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Milperra | 8.61 | 8.61 | 8.61 | 8.61 | 8.61 | 8.61 |
| Croydon Park | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| West Pymble | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| Warrawee | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| West Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| East Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |

Table 3.2: The number of electric vehicles at intersection for route 1

Table 3.3: The number of electric vehicles at intersection for route 2

| Intersection | | Number | of electric | vehicles (p | er hour) | |
|-----------------|----------|----------|-------------|-------------|----------|----------|
| | baseline | baseline | baseline | baseline | baseline | baseline |
| (Suburb) | | +20% | +40% | +60% | +80% | +100% |
| St Marys | 5.79 | 6.90 | 8.00 | 9.10 | 10.20 | 11.30 |
| Parramatta | 5.79 | 6.90 | 8.00 | 9.10 | 10.20 | 11.30 |
| Homebush West | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 |
| Sydney Center | 23.11 | 23.11 | 23.11 | 23.11 | 23.11 | 23.11 |
| Rydalmere | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| Merrylands | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 |
| Baulkham Hills | 5.51 | 6.90 | 8.00 | 9.00 | 10.10 | 11.20 |
| Cherrybrook | 5.51 | 6.90 | 8.00 | 9.00 | 10.10 | 11.20 |
| North Wahroonga | 19.72 | 20.82 | 21.92 | 23.02 | 24.12 | 25.22 |
| Ingleside | 5.51 | 6.61 | 7.71 | 8.81 | 9.91 | 11.01 |
| Campbelltown | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Milperra | 8.61 | 8.61 | 8.61 | 8.61 | 8.61 | 8.61 |
| Croydon Park | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| West Pymble | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| Warrawee | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| West Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| East Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |

| Intersection | Number of electric vehicles (per hour) | | | | | |
|-----------------|--|----------|----------|----------|----------|----------|
| | baseline | baseline | baseline | baseline | baseline | baseline |
| (Suburb) | | +20% | +40% | +60% | +80% | +100% |
| St Marys | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 |
| Parramatta | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 |
| Homebush West | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 |
| Sydney Center | 23.11 | 23.17 | 23.23 | 23.29 | 23.35 | 23.41 |
| Rydalmere | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| Merrylands | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 |
| Baulkham Hills | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Cherrybrook | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| North Wahroonga | 19.72 | 19.72 | 19.72 | 19.72 | 19.72 | 19.72 |
| Ingleside | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Campbelltown | 0.30 | 0.36 | 0.42 | 0.48 | 0.54 | 0.60 |
| Milperra | 8.61 | 8.67 | 8.73 | 8.79 | 8.85 | 8.91 |
| Croydon Park | 0.30 | 0.36 | 0.42 | 0.48 | 0.54 | 0.60 |
| West Pymble | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| Warrawee | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| West Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| East Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |

Table 3.4: The number of electric vehicles at intersection for route 3

Table 3.5: The number of electric vehicles at intersection for route 4

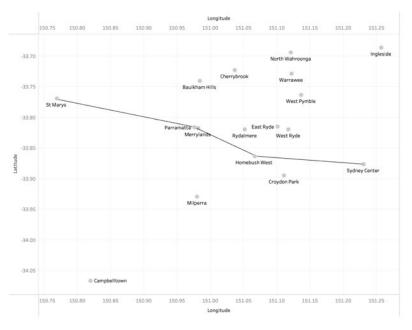
| Intersection | | Number | of electric | vehicles (p | er hour) | |
|-----------------|----------|----------|-------------|-------------|----------|----------|
| | baseline | baseline | baseline | baseline | baseline | baseline |
| (Suburb) | | +20% | +40% | +60% | +80% | +100% |
| St Marys | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 |
| Parramatta | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 |
| Homebush West | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 |
| Sydney Center | 23.11 | 25.95 | 28.79 | 31.63 | 34.47 | 37.31 |
| Rydalmere | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| Merrylands | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 | 8.59 |
| Baulkham Hills | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Cherrybrook | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| North Wahroonga | 19.72 | 22.56 | 25.40 | 28.24 | 31.08 | 33.92 |
| Ingleside | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Campbelltown | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Milperra | 8.61 | 8.61 | 8.61 | 8.61 | 8.61 | 8.61 |
| Croydon Park | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| West Pymble | 14.22 | 17.06 | 19.90 | 22.74 | 25.58 | 28.42 |
| Warrawee | 14.22 | 17.06 | 19.90 | 22.74 | 25.58 | 28.42 |
| West Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |
| East Ryde | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 | 8.30 |

| Intersection | | Number | of electric | vehicles (p | er hour) | |
|-----------------|----------|----------|-------------|-------------|----------|----------|
| (Suburb) | baseline | baseline | baseline | baseline | baseline | baseline |
| (auburb) | | +20% | +40% | +60% | +80% | +100% |
| St Marys | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 |
| Parramatta | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 | 5.79 |
| Homebush West | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 |
| Sydney Center | 23.11 | 24.77 | 26.43 | 28.09 | 29.75 | 31.41 |
| Rydalmere | 8.30 | 9.96 | 11.62 | 13.28 | 14.94 | 16.60 |
| Merrylands | 8.59 | 10.25 | 11.91 | 13.57 | 15.23 | 16.89 |
| Baulkham Hills | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Cherrybrook | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| North Wahroonga | 19.72 | 19.72 | 19.72 | 19.72 | 19.72 | 19.72 |
| Ingleside | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 | 5.51 |
| Campbelltown | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| Milperra | 8.61 | 10.27 | 11.93 | 13.59 | 15.25 | 16.91 |
| Croydon Park | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 |
| West Pymble | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| Warrawee | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 | 14.22 |
| West Ryde | 8.30 | 9.96 | 11.62 | 13.28 | 14.94 | 16.60 |
| East Ryde | 8.30 | 9.96 | 11.62 | 13.28 | 14.94 | 16.60 |

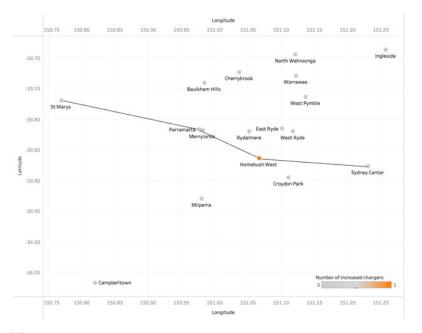
Table 3.6: The number of electric vehicles at intersection for route 5

chargers increases differently for each route because of the different numbers of EVs on each route. For example, in route 1, although the intersection EVs flow of route 1 increases by 20%, 40%, etc. in sequence, however, the flow of EVs in route 1 is much smaller than the flow of the other routes. Therefore the change in the number of chargers for route 1 is small. In Figure 3.6, the change in the number of chargers because of changes in the flow of EVs on route 1 is shown. Only two graphs are shown in this figure for comparison because the number of chargers required is the same for the EV flow scenarios from baseline in sequence increment to baseline+60%. The number of chargers at the Homebush West intersection increases by one for baseline+80% and baseline+100% EV flows.

Figure 3.7 and Figure 3.8 show that the increase in EV flow at the intersection in route 2 leads to a change in the number of chargers across the whole network. The increase in the number of chargers at intersections on route 2 also leads to

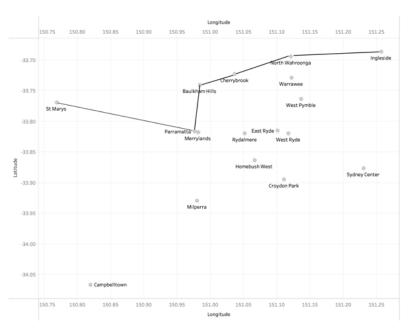


(a) Various changes in charger demands as EV flows increase from Baseline to Baseline+60\%

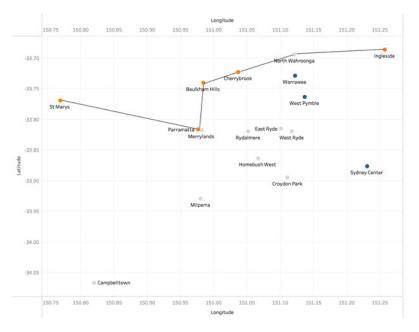


(b) Various changes in charger demands as EV flows increase from Baseline+80% to Baseline+100%

Figure 3.6: Number of chargers increased at each intersection of route 1 for different EV flows scenarios

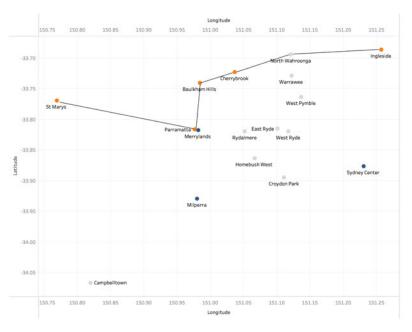


(a) Various changes in charger demands as EV flows increase from Baseline to Baseline+20\%

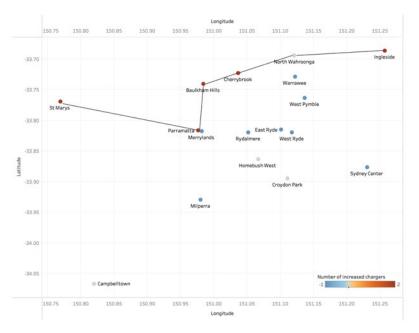


(b) Various changes in charger demands as EV flows increase from Baseline+40% to Baseline+60%

Figure 3.7: Number of chargers increased at each intersection of route 2-1 for different EV flows scenarios



(a) Various changes in charger demands when EV flows is Baseline+80\%



(b) Various changes in charger demands when EV flows is Baseline+100%

Figure 3.8: Number of chargers increased at each intersection of route 2-2 for different EV flows scenarios

a decrease in the number of chargers allocated to other road intersections when the total investment of chargers is kept constant. It is easy to find that the intersection that reduce the number of chargers are different when the EV flow is baseline+40% and baseline+80%. This is because our objective function is to reduce the overall queuing waiting time.

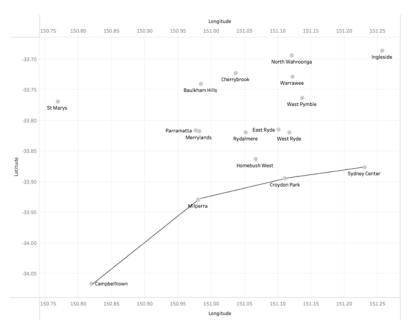
Figure 3.9 has only two comparison graphs because route 3 has less EV flow than the other routes. Therefore, when the EV flow increases, it still does not have a large impact on the increase of the number of chargers.

In Figure 3.10 and Figure 3.11, which is worth noting that when the EV flow is Baseline+40%, both intersections Warrawee and West Pymble in route 4 increase by one charger. In contrast, North Wahroonga decreases by one charger. This is because the North Wahroonga is also on route 2, and in addition to that the number of chargers allocated to this intersection is much higher than other intersections, and the optimization algorithm is aimed at minimizing the waiting time. Reducing the number of chargers at North Wahroonga intersection has the least impact on the overall waiting time.

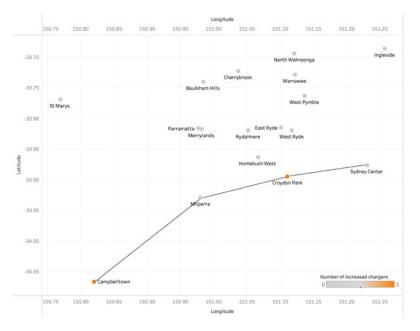
In Figure 3.12, Figure 3.13 the number of chargers added to the Rydalmere intersection on route 5 is two, while Merrtlands and Ryde have only one additional charger.

After I have introduced the effect of the change in EV flow on each route on the number of charger on the whole road network. Next, I move on to discuss the relationship between EV flow and the amount of investment at all intersections in the whole road network.

In Figure 3.14, I show the relationship between these six different EV flow scenarios and the budget. Where scenario 1 represents the hourly flow of EVs at

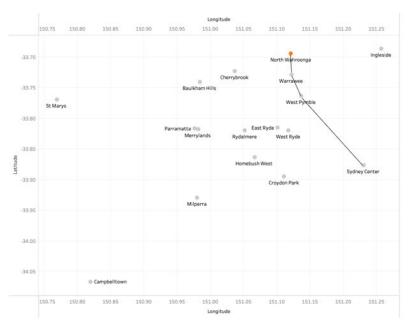


(a) Various changes in charger demands as EV flows increase from Baseline to Baseline+60\%

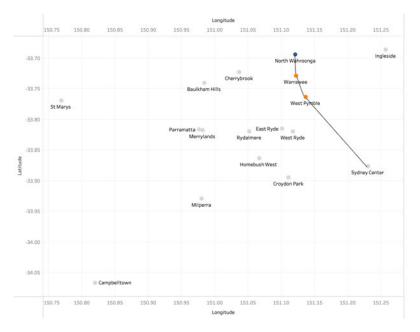


(b) Various changes in charger demands as EV flows increase from Baseline+80% to Baseline+100%

Figure 3.9: Number of chargers increased at each intersection of route 3 for different EV flows scenarios

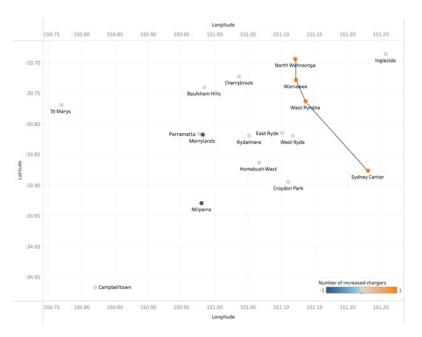


(a) Various changes in charger demands as EV flows increase from Baseline to Baseline+20\%



(b) Various changes in charger demands when EV flows is Baseline+40\%

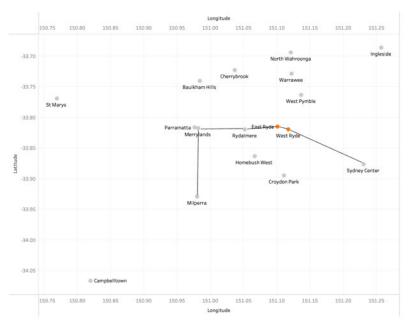
Figure 3.10: Number of chargers increased at each intersection of route 4-1 for different EV flows scenarios



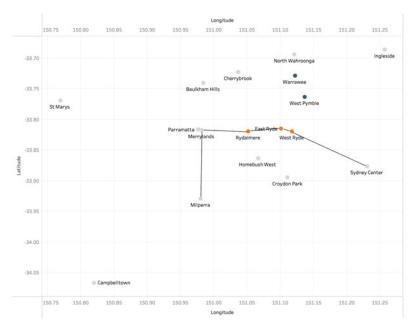
(a) Various changes in charger demands as EV flows increase from Baseline+60% to Baseline+100%

Figure 3.11: Number of chargers increased at each intersection of route 4-2 for different EV flows scenarios

each intersection after redistribution by the gravity model, scenario 2 represents a 20% increase in the flow of EVs based on scenario 1, and by analogy I get scenarios 3, 4, 5 and 6. By observing Figure 3.14, it can be found that the functions of waiting time and budget for scenario 1 and scenario 2 can be almost approximated as the same curve. That is, when the EV flow increases by 20%, the relationship between the waiting time and the budget in the road network is the same as the relationship function between the two in the initial flow scenario 1. And with this figure, I can see that although the waiting time decreases when the budget becomes larger, the waiting time for EVs will infinitely converge to a constant value when the budget increases infinitely. That is, when the budget of chargers increases from AU\$1,000,000 to AU\$4,000,000, the sum of the average

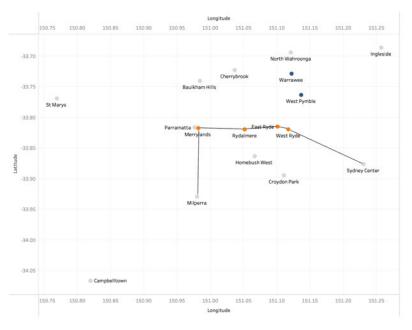


(a) Various changes in charger demands as EV flows increase from Baseline to Baseline+20\%

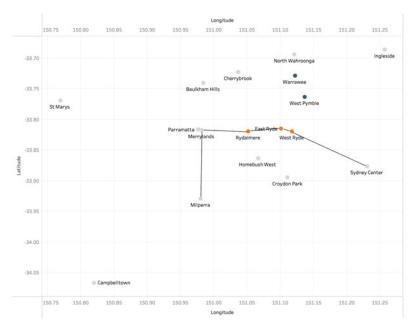


(b) Various changes in charger demands when EV flows is Baseline+40\%

Figure 3.12: Number of chargers increased at each intersection of route 5-1 for different EV flows scenarios



(a) Various changes in charger demands as EV flows increase from Baseline+60% to Baseline+80%



(b) Various changes in charger demands when EV flows is Baseline+100%

Figure 3.13: Number of chargers increased at each intersection of route 5-2 for different EV flows scenarios

waiting time at all intersections in the whole road network decreases from 60 - 120 minutes to 30 - 40 minutes.

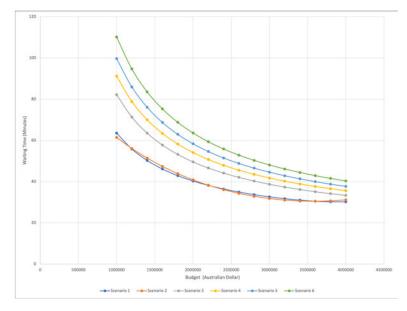


Figure 3.14: The relationship between budget and waiting time in different scenarios

3.3 Summary

In this chapter, a portion of the Sydney city road network is selected as a case study based on the model in the previous chapter. The optimal location and number of chargers to deploy EV charging infrastructure within the constraints of various charging station locations, budgets and number of chargers. First, I selected several representative areas in the Sydney metropolitan area. Then, I used a gravity model to calculate the traffic flow on each route. Finally, the intersection flows processed by the gravity model are brought into the optimization model in the previous chapter to solve for the number of chargers required at each intersection under different scenarios using the least squares method.

Chapter 4

A Queue Balancing Approach for Electric Vehicle Charging Allocation

Based on Chapter 2 and Chapter 3, I have determined the location and number of charging stations in the road network. Next, this Chapter is about how to efficiently utilize and manage the available charging resources. This Chapter presents a proposed queue balancing algorithm to balance the uneven charging demand at different charging stations and to minimize the total average waiting time. Section 4.1 will present the background and significance of this study. Section 4.2 will provide a detailed description of the definition of linear road network. Section 4.3 will introduce Remaining battery modelling. Section 4.4 will present a multi-server queuing model with finite waiting time to describe the queuing process of electric vehicles. Section 4.5 will show a charging resource allocation model to minimize the total waiting time in the charging network. Section 4.6 will present a queue balancing algorithm. Section 4.7 will present a case study to demonstrate the effectiveness of the optimization model and algorithm. A brief summary of the chapter will be presented in Section 4.8.

4.1 Introduction

With increasing number of electric vehicles but relatively insufficient charging facilities, how to utilise and manage the existing charging resource in an efficient and effective way becomes an important challenge to address. In this Chapter, a charging resource allocation model is proposed to balance the uneven charging demand at different charging stations and minimise the total of average waiting time. A queue balancing algorithm is proposed to solve the problem. Case study is conducted on a linear travel corridor. The results show that the proposed approach can balance the charging load among different charging stations and maximise the utilisation of charging resources. Moreover, the proposed model can help reduce the total of average waiting time at charging stations.

4.2 Basic definition

Given multiple charging stations distributed in a road network and each charging station with a limited number of chargers, EV drivers are allowed to book their charging activities in advance. A centralised management system will collect all the charging request and make decision to allocate charging stations to drivers to minimise the total of average waiting time at charging stations while satisfying charging demand. To simplify the presentation, a major travel corridor is adopted as a linear road network in this study [76]. As shown in Figure 4.1, it is assumed that there are a set of EVs leaving from the origin to the destination along a travel corridor. There are *n* charging stations $V = \{1, 2, \dots, v\}$ along the corridor, and each charging station consists of at least one charger. Charging station 1 is located in the origin area and charging station *v* is in the destination area. The distance from the origin to charging station *v* is denoted by d_v . Note that $d_1 = 0$.

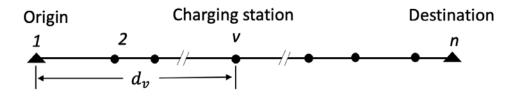


Figure 4.1: EV charging stations along a travel corridor

It is assumed that the number of EVs departing from the origin and the driving range that an EV can travel with the remaining battery level conform to a certain probability distribution. Based on the assumptions, I first calculate the current queuing length at each charging station and the average waiting time of each EV based on its remaining battery level, and then allocate those available charging stations to corresponding EV drivers with the objective to minimise the total of average waiting time.

Relevant parameters and variables used in the model formulation are introduced in Table 4.1, followed by the remaining battery capacity model and queuing theory model presented in the subsequent sections.

| | Table 4.1: Parameters and variables |
|---------------------|--|
| Notation | Description |
| Parameters | |
| V | The sets of charging station, $v \in V$ |
| d_v | The distance from the origin to the charging station v |
| n_v | The number of chargers at charging station v |
| m_v | The number of EVs at charging station v |
| $ ho_v$ | The charging service intensity at charging station v |
| λ_v | The arrival rate of EVs at charging station v |
| μ_v | The service rate of EVs at charging station v |
| $P_v(m_v)$ | The probability of m_v EVs waiting at charging station v |
| Variables | |
| L_v | The average queuing length at charging station v |
| $\overline{\ell_v}$ | The average waiting time at charging station v |
| T_v | The maximum waiting time at charging station v |

4.3 Remaining battery modelling

The related literature shows that the state-of-charge (SOC) of EV battery has a stochastic characteristic [77; 78; 79; 80]. When the number of EVs becomes larger, the remaining battery level of EVs shows a certain regularity, and as the number of EVs increases to a certain extent, the probability distribution of the allowed driving range conforms to a certain probability density function.

As aforementioned, I assume that the distance that an EV can travel with its remaining battery level from the origin is a random variable X, which follows a probability density function, f(x) (see Figure 4.2). x denotes the number of kilometres that can be traveled by the remaining power of an EV. The shaded part indicates the probability that an EV can travel with the remaining battery capacity between the distance $[\alpha, \beta]$, i.e., $P(\alpha \leq X \leq \beta)$.

Assume the total number of EVs that leave the origin in a unit of time is M, $M \in [0, \infty)$, Equation 4.1 shows the total number of EVs, $N(\alpha, \beta)$, which can

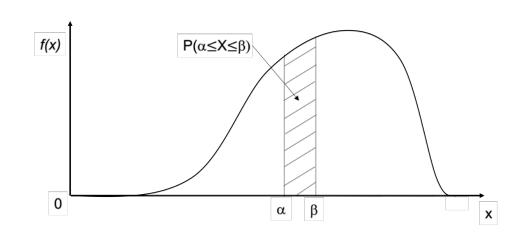


Figure 4.2: Probability distribution of driving range

travel between the range of $[\alpha, \beta]$ with the remaining battery level.

$$N(\alpha,\beta) = M \cdot P(\alpha \le X \le \beta) = M \cdot \int_{\alpha}^{\beta} f(x) dx$$
(4.1)

4.4 Charging queues

To better understand how to allocate a charging station to an EV along the travel corridor, a simple scenario is used for illustration. Assume that an EV is running along the corridor from the origin to the destination, and its remaining battery level can only afford the vehicle to reach the second charging station as furthest. In this case, this EV driver can only choose to charge at the origin, the first or the second charging station. Therefore, each vehicle's candidate charging stations are determined by its remaining battery capacity. Accordingly, I can calculate the number of EVs queuing for charging at each charging station.

I propose to use queuing theory to calculate the queuing time of each charging station. [81; 82; 83; 84], which will be used to describe our queuing charging model

for EV users. That means at each charging station, the arrival of EVs forms a Poisson flow. We assume that EVs arrive as Poisson flow and the charging service at each charging station is based on first-come, first-served (FCFS) [27; 85; 86; 87; 88]. To calculate the average waiting time at each charging station, I use the M/M/n queuing model.

It should be noted that a necessary condition for a stable queuing system is that there is a finite queue in the steady state, which is $\rho_v < 1$. In there, λ_v is denote the arrival rate of electric vehicles at vertex v per time unit, and μ_v is represent the service rate of electric vehicles at vertex v per time unit. We can use η_v to represent λ_v/μ_v . According to queuing theory, I need to consider two cases. For each charging station $v \in V$, assume there are m_v electric vehicles go to a simultaneously for charging, and n_v is the number of chargers at charging station v, then I have:

Case 1: The number of EVs is less than the number of chargers at charging station v, *i.e.*, $m_v < n_v$. Then the average charging service intensity is expressed as follows:

$$\rho_v = \eta_v / m_v \tag{4.2}$$

Case 2: The number of EVs is no less than the number of chargers at charging station $v, i.e., m_v \ge n_v$. The average charging service intensity is:

$$\rho_v = \eta_v / n_v \tag{4.3}$$

Based on the principle of Markov chains [89; 90], there are always EVs coming in for charging and EVs leaving after charging, hence the length of the queuing system is always changing. However, eventually the queuing system will reach an equilibrium status that the number of EVs coming in for charging and the number of EVs completing charging are considered to be equal, as described below [91; 92]:

$$\begin{cases} \lambda_{v}P_{v}(0) = \mu_{v}P_{v}(1) \\ \lambda_{v}P_{v}(k-1) + (k+1)\mu_{v}P_{v}(k+1) = (\lambda_{v} + k\mu_{v})P_{v}(k), \\ 1 \le k < n_{v} \\ \lambda_{v}P_{v}(k-1) + n_{v}\mu_{i}P_{v}(k+1) = (\lambda_{v} + n_{v}\mu_{v})P_{v}(k), \\ k \ge n_{v} \end{cases}$$
(4.4)

Based on the above derivation, I can obtain the expression for $P_v(0)$, which stands for the probability that there is no EV at charging station v as follows:

$$P_{v}(0) = \left[\sum_{m_{v}=0}^{n_{v}} \frac{\eta_{v}^{m_{v}}}{m_{v}!} + \frac{\eta_{v}^{n_{v}}}{n_{v}!(1-\rho_{v})}\right]^{-1}$$
(4.5)

According to the function of state equilibrium described above. When there are m_v EVs at charging station v. There exist two cases of $P_v(m_v)$:

$$P_{v}(m_{v}) = \begin{cases} \frac{1}{m_{v}!} (\eta_{v})^{m_{v}} P_{v}(0), \ m_{v} < n_{v} \\ \frac{1}{n_{v}! n_{v}^{m_{v} - n_{v}}} (\eta_{v})^{m_{v}} P_{v}(0), \ m_{v} \ge n_{v} \end{cases}$$
(4.6)

Then, I can calculate the average waiting length at charging station v as follows:

$$L_v = \frac{(n_v \rho_v)^{n_v} \rho_v}{(1 - \rho_v)^2 n_v!} P_v(0)$$
(4.7)

Furthermore, according to the Little law [93], the average waiting time can be expressed as

$$\overline{\ell_v} = \frac{L_v}{\lambda_v} \tag{4.8}$$

By integrating Equation 4.6, Equation 4.7 and Equation 4.8, the average waiting time at charging station v can be calculated as follows:

$$\overline{\ell_v} = \begin{cases} \frac{(n_v \rho_v)^{n_v} \rho_v}{\lambda_v (1-\rho_v)^2 n_v!} P_v(0), & m_v < n_v \\ \frac{(\eta_v)^{n_v} \rho_v}{\lambda_v (1-\rho_v)^2 n_v!} P_v(0), & m_v \ge n_v \end{cases}$$
(4.9)

4.5 Optimisation of charging resource allocation

Combining the finite average waiting time constraint and the associated equations in the previous section, a charging resource allocation model is proposed in this section to re-balance the charging demand between charging stations to minimise the total of average waiting time along the travel corridor.

With the remaining battery level, if the maximal driving range of an EV covers charging station v, then charging station v is a feasible charging location for this vehicle. However, when the total number of EVs choosing to charge at charging station v is larger than the amount of chargers available at charging station v, queue will form at charging station v, and it will cause uneven distribution of charging demand. Therefore, a balance coefficient $\sigma_v \in [0, 1]$ is introduced to reallocate the vehicles at different changing station. Moreover, based on the arrival rate of EVs at each charging station, the waiting time for EVs at each charging station can be calculated by combining with the queuing theory model.

Assume $\sigma_v \in [0, 1]$ for each $v \in V$ and $\sum_{v \in V} \sigma_v = 1$. I will use these coefficients

to rebalance queues at the charging stations. The idea is that if the total number of vehicles from the origin is M, the number of vehicles allocated to charging station v will be $\sigma_v M$. Due to the limitation of battery capacity, a vehicle can only be reallocated to an earlier charging station rather than a later charging station. This means

$$\sum_{i=0}^{v} \sigma_i M \ge M \int_0^{d_{v+1}} f(x) dx, \text{ for all } v \in V$$

$$(4.10)$$

Let's calculate the waiting time at each charging station after rebalance. Firstly, we calculate the arrival rate of EVs at each charging station after balance. Assume that the arrival rate of EVs at station v before balance is λ_v . The total arrival rate of EVs at all stations will be

$$\Lambda = \sum_{v \in V} \lambda_v \tag{4.11}$$

The arrival rate of EVs, λ_v^* , at station v after the balance is:

$$\lambda_v^* = \sigma_v \Lambda \tag{4.12}$$

Secondly, we calculate the average waiting time of EVs at charging station v. Combing with Equation 4.9 and Equation 4.12, we can get the function as follows:

Case 1: The number of EVs is less than the number of chargers at charging station v, $\sigma_v M < n_v$.

$$\overline{\ell_v}^* = \frac{\left(\frac{n_v \sigma_v \Lambda}{m_v \mu_v}\right)^{n_v} \frac{\sigma_v \Lambda}{m_v \mu_v}}{\sigma_v \Lambda (1 - \frac{\sigma_v \Lambda}{m_v \mu_v})^2 n_v!} P_v(0)$$
(4.13)

Case 2: The number of EVs is more than the number of chargers at charging station $v, \sigma_v M \ge n_v$.

$$\overline{\ell_v}^* = \frac{\left(\frac{\sigma_v \Lambda}{\mu_v}\right)^{n_v} \frac{\sigma_v \Lambda}{n_v \mu_v}}{\sigma_v \Lambda (1 - \frac{\sigma_v \Lambda}{n_v \mu_v})^2 n_v!} P_v(0)$$
(4.14)

Finally I optimise the total of average waiting time by adjusting the balance coefficients.

$$\min_{\forall v \in V, \ \sigma_v \in [0,1]} \sum_{v \in V} \overline{\ell_v}^* \tag{4.15}$$

s.t.

$$\sum_{v=1}^{n} \sigma_v = 1 \tag{4.16}$$

$$\sum_{i=0}^{v} \sigma_i \ge \int_0^{d_{v+1}} f(x) dx, \text{ for all } v \in V$$

$$(4.17)$$

$$\overline{\ell_v}^* \in [0, \varphi], \ v \in V \tag{4.18}$$

$$\frac{m_v \overline{\ell_v}^*}{n_v} \le T_v, \ v \in V \tag{4.19}$$

Equation 2.17 denotes that the sum of balance coefficients equals to 1. Equation 2.18 describes that the probability of EVs charged at charging station vobtained in the optimal solution should be greater than or equal to the EVs that may be charged at charging station v. To restrain the charging waiting time to a limited range, I use φ to represent the maximum waiting time, then $\overline{\ell_v}^*$ is restricted as Equation 2.27. Considering the user satisfaction, Equation 2.21 express the total of average waiting time of EV users at each charger should not exceed its accepted maximum value T_v .

4.6 Solution algorithms

To solve the charging resource allocation model, queuing time algorithm and queue balancing algorithm are developed as follows.

4.6.1 Queuing time algorithm

Here I use queuing theory to calculate the average queuing time at a charging station, which requires knowing the following information: (a) How many electric vehicles are at charging station v. (b) How many chargers are at charging station v. (c) The service rate of charging station v. (d) The relationship between the number of electric vehicles and the number of charging stations in terms of numbers. After obtaining those information, I can use the Algorithmic 1 to estimate the queuing time of electric vehicles arriving at charging station v at a moment.

4.6.2 Queue balancing algorithm

Initialization: Input parameters: N, m_v, n_v, μ_v, P .

Step 1: Based on the input, I can get μ_v for each charging station, and the sum of μ_v on the path p.

Step 2: Comparing the service intensity of the second charging station with the first charging station. Scenario (1) - if the service intensity of EVs at the first charging station is greater than the intensity at the second charging station, then

Algorithm 1 Queuing time algorithm

Input: n_v : number of chargers;

 m_v : number of electric vehicles to charging station v;

 η_v : Ratio of arrival rate to service rate at charging station v

Output: $\overline{\ell_v}$: average waiting time of each electric vehicle at charging station v

```
1: if m_v \leq n_v then

\rho_v := \frac{\eta_v}{m_v}

 2:
 3: else
      \rho_v := \frac{\eta_v}{n_v}
 4:
 5: end if
 6: FACTORIAL(i)
 7: if i := 1 then
 8:
      return 1
 9: else
      return i * FACTORIAL(i)
10:
11: end if
12: EXPONENTIAL(i)
13: return \rho_v * *i
14: SUM(j)
15: sum := 0
16: for i := 0 to j do
      sum := EXPONENTIAL(i) / FACTORIAL(i)
17:
18: end for
19: return \ell_v
```

the service intensity of EVs at the charging station does not need to be adjusted. Scenario (2) - if the service intensity of EVs at the first charging station is smaller than the intensity at the second charging station, then the service intensity of EVs allocated to these two charging stations will be averaged.

Step 3: In Scenario (2) of Step 2, if the service intensity of EVs allocated to the second charging station is smaller than the intensity at the third charging station, then the service intensity of EVs allocated to the first three charging stations will be averaged. If the service intensity of EVs allocated to the second charging station is greater than the intensity at the third charging station, then the service intensity at these three charging station remain the same. Next, comparing the service intensity of EVs allocated to the first three charging stations with the intensity of EVs at the fourth charging station. Following this procedure, comparison and adjustment will continue until the last charging station.

Step 4: According to Step 3, the service intensity of EVs at each charging station and the number of EVs allocated to each charging station can be calculated to reach a balanced allocation result.

Step 5: When the allocation of path p is completed, the loop is entered again and the same calculation is used to obtain the allocation of other paths.

Algorithm 2 Queue balancing algorithm

Input: N: number of charging stations on path p; m_v : number of EVs allocated to charging station v; n_v : number of chargers at charging station v; μ_v : service rate of EVs at charging station v; P: number of paths in the whole network

Output: m_v : number of EVs allocated to charging station v; Array: number of EVs allocated to each charging station in the whole network

```
1: for i := 1 to P do
         Array := [NULL]
 2:
 3:
         total_{\mu} := 0
 4:
         for i := 0 to N do
 5:
            total_{\mu} := total_{\mu} + \mu_i
         end for
 6:
         for i := 0 to N do
 7:
            \mu_i := \frac{\mu_i}{total_\mu}
 8:
         end for
 9:
         for i := 1 to N do
10:
            j := i - 1
11:
            \begin{array}{l} hold \mathrel{\mathop:}= \frac{m_i}{\mu_i * n_i} \\ sum \mathrel{\mathop:}= hold \end{array}
12:
13:
            while j \ge 0 and hold > \frac{m_j}{\mu_i * n_i} do
14:
               hold := \frac{(sum + \frac{m_j}{\mu_i * n_i})}{(i-j+1)}sum := sum + \frac{m_i}{\mu_i * n_i}
15:
16:
                j := j - 1
17:
18:
            end while
            total := 0
19:
            chargers := 0
20:
            for k := 0 to (i - j) do
21:
                total := total + m_{i-k}
22:
                chargers := chargers + n_{i-k} * \mu_{i-k}
23:
            end for
24:
            total := \frac{total}{chargers}
25:
            for k := 0 to (i - j) do
26:
27:
                m_{i-k} := n_{i-k} * \mu_{i-k} * total
            end for
28:
         end for
29:
         Array := extend([m_v])
30:
31: end for
```

4.7 Case study

In this section, a case study is conducted with results discussed as follows. The variation of the arrival flow of EVs obeys a Poisson distribution. The relevant parameter values are given as follows.

- Length of the travel corridor: 500km.
- Total number of EVs: 1000.
- Total number of charging stations: 10.
- Number of chargers at each charging station: [25, 30, 20, 15, 30, 40, 30, 30, 20, 30].
- Service rate of EVs: [1/12, 2/15, 7/60, 3/20, 1/12, 1/20, 1/10, 7/60, 2/15, 1/12].
- Arrival rate of EVs: [1.12, 2.92, 1.38, 0.70, 0.57, 0.98, 2.22, 3.30, 2.48, 1.00].

The probability density function of the allowable driving range of an EV used in this experiment is as follows:

$$f(x) = \frac{500}{\sqrt{2\pi}(x(500-x))} e^{-\frac{(\ln(\frac{x}{500-x}) - 0.65)^2}{2}}$$
(4.20)

According to the logistic probability density distribution, the majority of EVs have remaining battery level between 50% and 80%. Usually EV users will not charge the battery as long as it is within a safe threshold. They are likely to recharge the battery at charging stations close to their destination if the remaining battery level is enough. In this case, if all EV users choose to charge when they almost run out the battery, then the distribution of charging load at each charging station will be unbalanced. As a result, long queue and significant waiting time will be caused at charging stations close to the destination.

To test the proposed algorithm, I define the following two situations.

Unbalanced: The average waiting time is calculated based on the logistic probability density function without using the proposed algorithm for optimised results.

Balanced: The number of EVs at each charging station is redistributed using the proposed algorithm, and the average waiting time is calculated again.

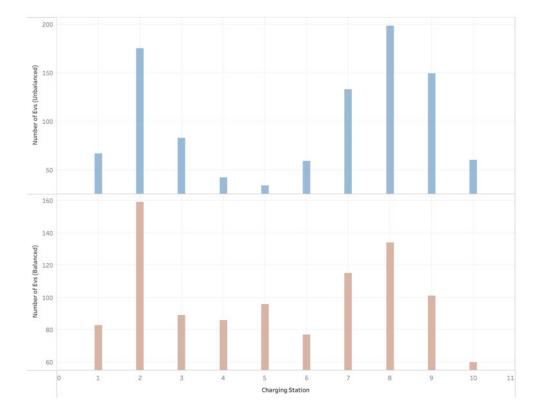


Figure 4.3: Comparison of EV amounts at each charging station under different situations

Figure 4.3 illustrates the difference of the amounts of EVs at each charging station before and after optimising the charging resource allocation. It can be observed that, in an unbalanced situation, the number of EVs is much higher at charging stations 2 and 8, followed by charging stations 7 and 9, which shows uneven charging distribution. In a balanced situation, the difference at each charging station is significantly reduced based on the optimised charging resource allocation.

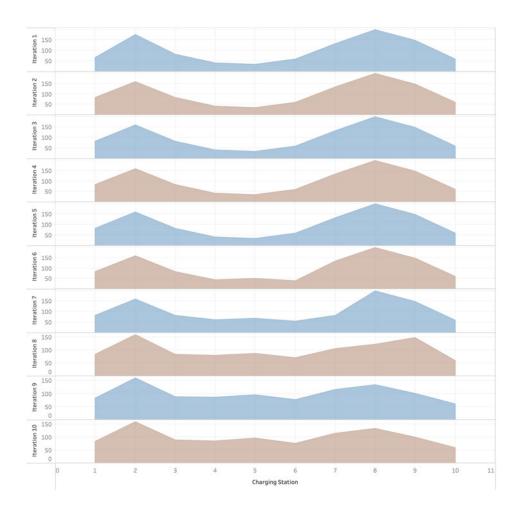


Figure 4.4: Number of vehicles for each charging station in queuing balancing progress

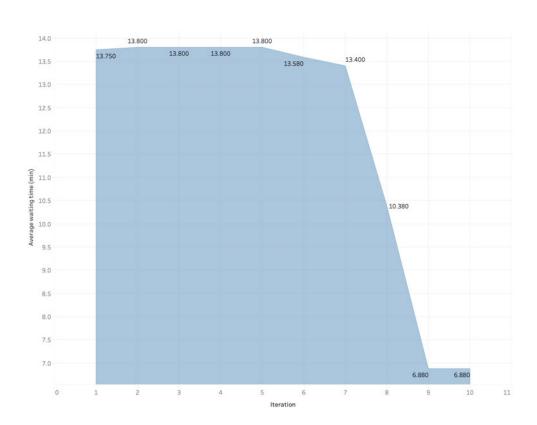


Figure 4.5: Average waiting time for each charging station in queuing balancing progress

By analysing the number of vehicles for each charging station in Figure 4.3 and total average waiting time in Figure 4.4, it can be observed that the number of electric vehicles per charging station is more even, and the total average waiting time is significantly reduced by using the queue balancing algorithm. It indicates that the proposed model and algorithm can better help to utilise the charging resources for EVs according to their driveable mileage and the service intensity at each charging station. Thus, the overall charging efficiency and the charging resource utilisation are improved.

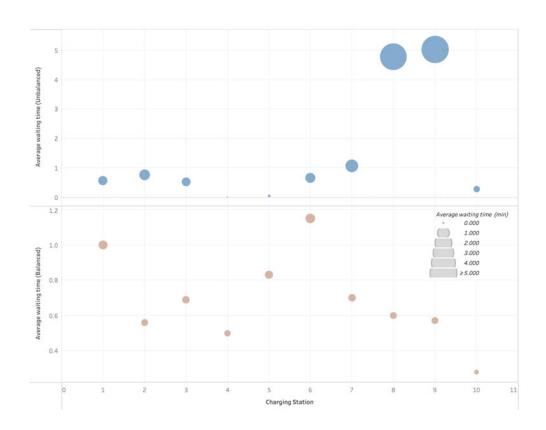


Figure 4.6: Average waiting time under different situations

Figure 4.5 shows the average waiting time for EVs at each charging station under the unbalanced and balanced situations. It can be found that in the unbalanced situation, as shown in the blue circles in Figure 4.5, the average waiting time for EVs at each charging station is unevenly distributed. The larger the blue circles in the figure, which shows significantly longer waiting time at charging stations 8 and 9. At the initial stage, when EV drivers depart from the origin of the corridor, most EVs have the remaining battery level equal to a driving range between 300km and 400km, so the EV users tend to charge at a further charging station such as 8 and 9 when the power almost runs out, which causes the average waiting time at these two charging stations increase significantly. In the balanced situation, after optimisation, the average waiting time at each charging station is quite similar, as shown in the brown circles in Figure 4.5 Compared to the total average waiting time at all charging stations in the unbalanced situation, the total average waiting time in the balanced situation is significantly lower, which further validates the proposed model and algorithm.

To sum up, the numerical results show that the charging resource allocation model and the proposed queue balancing algorithm can, on the one hand, balance the charging load between each charging station and maximise the utilisation of charging resources; on the other hand, reduce the total average waiting time at charging stations for EV users.

4.8 Summary

In this chapter, to maximise the utilisation of existing charging resources and reduce EV users' average waiting time at charging stations, a charging resource allocation model and the corresponding queue balancing algorithm were proposed. The model took into account various practical situations, such as the remaining battery level and corresponding maximal driving range, different number of chargers at different charging stations, and heterogeneous departure rates of EVs at different charging stations. Numerical experiments were conducted in consideration of both unbalanced and balanced situations to validate the proposed model and algorithm with promising results.

Chapter 5

Conclusion and Future Work

This research addresses two research questions related to charging station planning and charging resource allocation. Three specified research tasks will be conducted from the perspectives of charging station construction, electric vehicle queuing, and electric vehicle allocation. The research will be significantly different from the existing research in the following aspects:

• In Chapter 2, I introduced the optimization model for charging station planning. And show how to use the gravity model to calculate the point-topoint traffic flow from the traffic data of each intersection, which can induce the origin-destination traffic data. Then, I propose an optimization model for toll assignment based on origin-destination traffic flow data. Finally, I build an optimization model to plan charging stations by minimizing the total waiting time for electric vehicles. Based on this model, we can calculate the number of chargers that should be installed at each charging station, and the optimal allocation of vehicles at each charging station.

- In Chapter 3, a portion of the Sydney city road network is selected as a case study based on the model in the previous chapter. First, I selected several representative areas in the Sydney metropolitan area. Then, I used a gravity model to calculate the traffic flow on each route. Finally, the intersection flows processed by the gravity model are brought into the optimization model in the previous chapter to solve for the number of chargers required at each intersection under different scenarios using the least squares method.
- The Chapter 4 is about how to efficiently utilize and manage the available charging resources and presents a proposed queue balancing algorithm to balance the uneven charging demand at different charging stations and minimize the total average waiting time. I conducted a case study on a linear travel corridor. The results show that the proposed approach can balance the charging load among different charging stations and maximize the utilization of charging resources. In addition, the proposed model can help reduce the average waiting time at charging stations.

All the research tasks and research training during the course of Master of Philosophy will provide a solid foundation for me to further study and work in the research direction. I plan to further work on game-theoretic models for charging infrastructure planning and resource allocation, comparing the outcome of equilibria and optimization to gain the price of anarchy. I believe that further theoretical and experimental investigations will make a significant contribution to the area of research.

List of Publications

Papers that have been published:

 A Queue Balancing Approach for Electric Vehicle Charging Allocation Authors: Qi Wang, Dongmo Zhang, and Bo Du Conference: The 25th IEEE International Conference on Intelligent Transportation Systems

Date: October 8-12, 2022, Location: Macau, China

During my master study, based on my personal interest, I also worked on data analysis, resulting in the following papers.

- Observational study on multi-type conflicts between passengers and cyclists at the bus stop – A case study in Nanjing Authors: Cheng Zhang, Bo Du, Qi Wang, Jun Shen Journal: (Elsevier) Travel Behaviour and Society
- Observational Study on Traffic Conflicts between Passengers and Cyclists at Bus Stop

Authors: Cheng Zhang, Bo Du, **Qi Wang**, Jun Shen

Conference: Pedestrian and Evacuation Dynamics Conference 2021

 Analysis of COVID-19 Impact on Public Transport Usage based on Smart Card Data – A Regional Case Study in Australia

Authors: Tianyang Qu, Bo Du, Cheng Zhang, **Qi Wang**, Hao Hu, and Pascal Perez

Conference: The 25th IEEE International Conference on Intelligent Transportation Systems

Date: October 8-12, 2022, Location: Macau, China

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