Mathematics Anxiety and Cognition: A Computational Modelling Study

by

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“Mathematics is the language in which God has written the universe.”

- Galileo Galilei

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Statement of Authentication

The work presented in this thesis is, to the best of my knowledge and belief, original except as acknowledged in the text. I hereby declare that I have not submitted this material, either in full or in part, for a degree at this or any other institution.

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Angela Rose

Declaration

I declare that some of my peer reviewed and published journal article that I co-authored during my Master of Research degree, and which was partly based on my initial research proposal prepared during the degree, has been reused in this thesis. Furthermore, the journal article was published under my previous last name, Angela Porter.

> Moustafa, A. A., Porter, A., & Megreya, A. M. (2020). Mathematics anxiety and cognition: An integrated neural network model. *Rev. Neurosci.*, *31*(3), 287–296. https://doi.org/10.1515/revneuro-2019-0068

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List of Abbreviations

ACC……………………………………………………...…….Anterior Cingulate Cortex

AMAS…………………………………..……………Abbreviated Math Anxiety Scale

ANS……………………………………………………..…Approximate Number System

EEG…………………………………………………………….Electroencephalogram

ERP…………………………………………………………...Event-Related Potentials

fMRI……………………………………………Functional Magnetic Resonance Imaging

HMA…………………………………………………………………High Math-Anxious

LMA…………………………………………………………………Low Math-Anxious

MNL…………………………………………………..……….Mental Number Line

OECD………………………Organisation for Economic Co-operation and Development

PISA……………………........................Program for International Student Assessment

sMARS…………………………….……....Abbreviated Mathematics Anxiety Rating Scale

STEM……………………………...Science, Technology, Engineering, and Mathematics

Abstract

Anxiety about performing numerical calculations is becoming an increasingly important issue. Termed *mathematics anxiety*, this condition negatively impacts performance in numerical tasks which can affect education outcomes and future employment prospects. The disruption account proposes this poor performance is from the anxiety and its worrying thoughts disrupting the limited resources of working memory (specifically the attentional and inhibitory functions) leaving less cognitive resources available for the current task. There are many behavioural studies on mathematics anxiety. However, its underlying cognitive and neural mechanisms remain unclear. This thesis examines the relationship between mathematics anxiety and attentional control using neural network modelling, there are no neural network models simulating mathematics anxiety. The numerical Stroop task and the symbolic number comparison task were modelled with a single neural network model architecture examining the effect of modifications to both tasks. Different model modifications were used to simulate high and low math-anxious conditions by modifying attentional processes and learning. The model simulations suggest that mathematics anxiety is associated with reduced attention to numerical stimuli. These results are consistent with attentional control theory where anxiety decreases the influence of the goal-directed attentional system and increases the influence of the stimulus-driven attentional system. Notably, when simulating the numerical Stroop task, the high math-anxious model with reduced attention to numerical stimuli experienced less neural activation in the response layer for the inhibitory condition than the low math-anxious model, suggesting an under activation of working memory resources when experiencing conflict. Furthermore, the model was able to account for several other cognitive conditions, including reduced learning, the physical Stroop task across learning, and the speed-accuracy trade-off.

Chapter 1: Introduction

Numerical skills such as counting and arithmetic calculations are an important part of life and our ability to function effectively in society. However, many people have trouble processing numbers and performing mathematics. Deficiencies in number-related skills can impact education outcomes, career choices, or even affect day-to-day life skills such as calculating the cost of groceries or budgeting to buy a home. Furthermore, anxiety about performing numerical calculations and mathematics is becoming an increasingly important issue. The term *mathematics anxiety* has been defined as “a feeling of tension, apprehension, or even dread that interferes with the ordinary manipulation of numbers and the solving of mathematical problems” (Ashcraft & Faust, 1994, p.98). The Organisation for Economic Co-operation and Development (OECD) Program for International Student Assessment (PISA) that monitors the outcomes of education systems reported an increase in mathematics anxiety from 2003 to 2012 (OECD, 2013). Approximately 30% of fifteen-year-old students across OECD countries in 2012 reported feeling helpless or nervous when solving a mathematics problem, and 59% of students across OECD countries reported they worry about mathematics classes being difficult. Mathematics anxiety negatively impacts performance in numerical and mathematical tasks (Beilock, 2008; Braham & Libertus, 2018; Lyons & Beilock, 2012; Ramirez et al., 2013, 2018). Moreover, mathematics anxiety can lead to avoidance of mathematics subjects at both school and university levels that impact students’ choices in science, technology, engineering, and mathematics (STEM) careers (Ashcraft & Krause, 2007; Daker et al., 2021; Hembree, 1990; Levy et al., 2021). A position paper released by the Australian Government’s Office of the Chief Scientist (2015) discusses the importance of STEM education, suggesting students will enter a very different work force in 2030. Therefore, given the importance of learning numerical and mathematical skills, understanding and treating mathematics anxiety is essential to reducing students' emotional stress around the subject, and improving education and employment outcomes.

1.1 Mathematics Anxiety and Low Achievement

A robust finding in the mathematics anxiety literature is that mathematics anxiety is consistently related to poor math performance (Hembree, 1990; Ma, 1999; Zhang et al., 2019). In Hembree's (1990) influential meta-analysis, mathematics anxiety was negatively correlated with mathematics aptitude measures across Grade 5 to Grade 12, and with mathematics grades in both high school and college. Subsequently, a meta-analysis by Ma (1999) found the relationship between mathematics anxiety and mathematics achievement is consistent across all grades from four to twelve. Confirming these findings, a recent meta-analysis also reported a negative relationship between mathematics anxiety and mathematics performance, which was strongest in senior high school students (Zhang et al., 2019). There are two main frameworks that have been proposed to explain the negative link between mathematics anxiety and mathematics achievement (for reviews see Carey et al., 2016; Ramirez et al., 2018): the disruption account (Ramirez et al., 2018), also referred to as the debilitating anxiety theory (Carey et al., 2016); and the reduced competency account (Ramirez et al., 2018), also referred to as the deficit theory (Carey et al., 2016).

The disruption account proposes that anxiety about performing mathematics results in underperforming in mathematical tasks (Ramirez et al., 2018). Mathematics anxiety can affect learning due to avoidance of mathematics situations (Hembree, 1990). Mathematics anxiety can also affect processing and recall (Carey et al., 2016) whereby anxiety and worrying thoughts can reduce the limited resources of working memory (specifically the attentional and inhibitory functions; see Working Memory and Mathematics Anxiety section for a description) leaving fewer cognitive resources available for the current task (Derakshan & Eysenck, 2009; Eysenck et al., 2007). The reduced competency account proposes that individuals with mathematics anxiety have poorer mathematical skills that leads to compromised learning and performance which results in mathematics anxiety (Ramirez et al., 2018). Within the reduced competency account, it has been suggested that deficits in basic numerical abilities may compromise the learning of more complex mathematical skills (Maloney et al., 2010, 2011). However, Carey et al. (2016) suggests that the findings showing deficits in basic numerical skills for individuals with mathematics anxiety may also be consistent with the disruption account. These studies were conducted on adults and their basic numerical skills may have been impaired because they have avoided mathematics due to their high levels of mathematics anxiety. The direction of the relationship between mathematics anxiety and mathematics performance discriminates the two theories. A third possibility has been proposed by Carey et al. where the relationship between mathematics anxiety and performance influence each other in a bidirectional relationship resulting in a cycle whereby poor performance in some individuals can bring about anxiety which subsequently results in reduced performance that continues in a vicious cycle. This theory of reciprocal influence of anxiety and mathematics ability is referred to as the reciprocal theory.

1.2 The Current Study

The aim of the current research is to model the relationship between mathematics anxiety and cognition in the context of the disruption account and theories of attentional control and anxiety. Specifically, the current research will study the relationship between mathematics anxiety and cognition using neural network modelling. To do this a single neural network model for mathematics anxiety will examine the effects of specific impairments on the outcomes of two different experimental tasks. The effect of impairments of attention and learning due to mathematics anxiety will be modelled and compared to experimental data of these tasks. In addition, novel predictions will be derived from the model to suggest additional ways for assessing the effects of mathematics anxiety on performance and to further isolate the specific source of the impairment.

Mathematics anxiety has primarily been studied in behavioural experiments, and more recently using brain imaging and electrophysiological recording techniques. However, to the best of the author's knowledge, there have been no studies simulating mathematics anxiety with neural network modelling. A neural network model is a computational simulation that is loosely based on a biological neural network (Moustafa et al., 2009, 2017; Moustafa & Gluck, 2011a, 2011b; O'Reilly & Munakata, 2006). It performs mathematical calculations to simulate how information is processed within brain circuits. Neural network modelling is a tool that can be used to test theories and make predictions that suggest directions for future research (Chakravarthy & Moustafa, 2018; Huber et al., 2016; Moustafa, 2017). One of the main strengths of neural network model simulations is that they can identify underlying cognitive mechanisms associated with particular brain impairments (Amos, 2000). Researchers are still in the process of understanding the cognitive factors affecting mathematics anxiety. Therefore, as the cognitive mechanisms underlying mathematics anxiety are still under investigation, neural network modelling will be used in the current study to elucidate a better understanding of those mechanisms.

I have specified a novel and theoretical integrative neural network model to simulate the neural and behavioural studies of mathematics anxiety (Moustafa et al., 2020). This model integrates previous neural network models of numerical cognition to examine the relationship between mathematics anxiety and impairments in inhibition, attention, and working memory. This thesis describes an initial implementation of specific aspects of this model that are related to attention. Accordingly, two mathematical tasks will be simulated on one neural network model architecture to study cognitive factors underlying mathematics anxiety. These tasks are the numerical Stroop task and the symbolic number (single-digit) comparison task. The numerical Stroop task involves deciding which of two numbers has the largest numerical magnitude when they are presented in different physical sizes. It requires attentional control to inhibit the irrelevant physical size dimension during the assessment of numerical size. The symbolic number comparison task involves deciding which of two single-digit numbers has the largest magnitude when the numbers are presented with the same physical sizes. This task can identify deficits in basic numerical skills and does not require inhibiting task irrelevant stimuli. A previous neural network model architecture that simulates multi-symbol number comparison (Huber et al., 2016) will be used as the basis of the neural network model for the current simulations. This architecture will be adapted to simulate the numerical Stroop task and the symbolic number comparison task. Both tasks will be simulated on this one neural network model architecture by making minor adjustments to the architecture to account for different task requirements. High math-anxious (HMA) and low math-anxious (LMA) implementations of the model will be realised for both the numerical Stroop task and the symbolic number comparison task that simulate individuals with and without mathematics anxiety respectively. Adjustments to the parameters of the low math-anxious model to demonstrate the effects of reduced attentional control and/or reduced learning on task performance will simulate these effects to compare with the experimental results from high math-anxious individuals. The pattern of results of the LMA and HMA model simulations will be compared to experimental studies of mathematics anxiety to determine whether the direction of the differences between conditions are analogous. Results of simulations across the difference tasks will be compared to determine whether a single model can reproduce the veridical pattern across both the numerical Stroop task and the symbolic number comparison task. The results will be interpreted in the context of theories of mathematics anxiety and cognition and the deficits that they hypothesise. The models also produce predictions for mathematics performance of high and low mathematics anxiety groups under novel conditions that are discussed in the Discussion section.

1.3 Thesis Structure

This chapter has provided an introduction to mathematics anxiety and described the aims of the current research including an introduction to using neural network modelling as the methodology.

Chapter 2 provides a review of the mathematics anxiety literature. This includes describing working memory, attentional control and theories of anxiety, deficits in basic numerical skills, neural network modelling of numerical cognition effects, and learning. This chapter concludes with a more in-depth explanation of the aims of the current study.

Chapter 3 describes the methodology used for this study. It describes the neural network modelling process that has been employed, prior relevant neural network models that the current model has been based upon, and the model architecture used to simulate the numerical Stroop task and the symbolic number comparison task.

Chapter 4 and Chapter 5 present the neural network model simulations of the numerical Stroop task and the symbolic number comparison task respectively. These chapters describe the procedure and results of the simulations and conclude with a discussion of the results.

Chapter 6 presents a general discussion of the results across both tasks.

Chapter 2: Literature Review

2.1 Inhibitory and Attentional Performance: Relation to Mathematics Anxiety

2.1.1 Working Memory and Mathematics Anxiety

Working memory refers to "a brain system that provides temporary storage and manipulation of the information necessary for such complex cognitive tasks as language comprehension, learning, and reasoning" (Baddeley, 1992, p. 556). Working memory consists of the central executive whose function includes an attentional control system, and a number of subcomponents. Several studies have reported a positive relationship between working memory and performance in mathematical activities (Meyer et al., 2010; see also Alloway et al., 2010; Passolunghi et al., 2016). It has been argued that the impact of mathematics anxiety on mathematical activities is mediated by working memory (Skagerlund et al., 2019). Furthermore, it is well-established that general anxiety impacts working memory performance (Lukasik et al., 2019). Therefore, it is reasonable to conclude that working memory does impact mathematics anxiety, which can in turn impair performance in mathematics activities (Moustafa et al., 2020).

2.1.2 Theories of Attentional Control

Several theories have been proposed to explain the effects of anxiety on cognitive performance. These theories suggest that anxiety leads to an impairment in the cognitive control system, which is the system responsible for the ability to adapt behaviour to the current circumstances. Specifically, to modify behaviour depending on the current goals and reject behaviour that is inappropriate for the circumstances. According to the processing efficiency theory (Eysenck & Calvo, 1992), anxiety and its associated thoughts consume the limited resources of working memory. Specifically, anxiety uses attentional resources of the central executive, leaving fewer resources available for the current task. The attentional control theory (Derakshan & Eysenck, 2009; Eysenck et al., 2007), which is an extension of the processing efficiency theory, describes how anxiety decreases attentional control and impairs the inhibition and shifting functions through the reduced efficiency of working memory. The inhibition function of the central executive component of working memory (as described by Friedman and Miyake (2004) and Miyake et al. (2000)) involves the deployment of attentional control to resist interference or distraction from task-irrelevant stimuli or responses. Eysenck et al. (2007) proposed that anxious individuals have an imbalance between the top-down goal-directed attentional system and the bottom-up stimulus-driven attentional system. Consequently, anxiety is associated with an increased influence of the stimulus-driven attentional system and a decreased influence of the goal-directed attentional system which results in an inability to inhibit distracting or irrelevant information to the task at hand. Furthermore, Eysenck et al. suggested that this inability to inhibit distracting information occurs regardless of whether the distraction is external (such as task irrelevant sensory stimuli), or internal (such as anxious thoughts). Accordingly, it is hypothesized that anxiety impairs processing efficiency to a greater extent than it impairs performance effectiveness. That is, anxious individuals exert increased effort to counter the negative effects of anxiety to attain a comparable quality of task performance (such as response accuracy) compared to less anxious individuals. Moreover, anxiety increases the allocation of attention to threat-related stimuli. Thus, the effects of anxiety on task performance are greater with threat-related stimuli than with neutral stimuli.

Whether anxious individuals continuously monitor conflict is a matter of contention. The conflict-monitoring hypothesis (Botvinick et al., 2001) proposes the existence of a system in the anterior cingulate cortex that monitors for conflict and triggers an adjustment of attention to exert top-down control (see section on Neural Network Modelling: Previous Relevant Models for more information). However, the existence of conflict monitoring is currently under debate within the literature. Other cognitive mechanisms, such as learning and memory biases, have been proposed to explain the various congruency effects either alternatively or in conjunction with conflict monitoring and adaptation (see Schmidt, 2019 for a review). The dual mechanisms of control framework (Braver, 2012; Braver et al., 2009; Hutchison, 2011) further suggests that anxious individuals do not maintain top-down control continuously in a proactive manner, but instead they exert control reactively only as needed when conflict or a task-irrelevant stimulus is detected. To conclude, these theories suggest that anxiety impairs the cognitive control system affecting the individual's ability to inhibit distracting information and adjust top-down control to maintain goal-directed behaviour.

2.1.3 The Stroop Task to Study the Ability to Inhibit Attention

The Stroop task is a standard test of cognitive control assessing the ability to inhibit irrelevant information. In Stroop's (1935) classic article, he sought to investigate the effects of inhibition by comparing word reading (a more automatic process) with colour naming (a less automatic task). By presenting a colour word and an ink colour stimulus simultaneously where the word was incongruent with the ink colour the task produces interference (see MacLeod, 1991 for a review). He defined a measure of interference of the conflicting word stimuli on colour naming as the difference in time between naming the colours and reading the words. Since Stroop's landmark study, variations of the Stroop task have been developed to investigate the role of interference across different types of stimuli. A widely used version is the colour-word Stroop task. Participants are presented with a written coloured word and must name the colour of the ink while ignoring the word's meaning. For example, the word BLUE is presented in a black coloured font, and the participant names the font colour as black while inhibiting the written word BLUE. Stimuli can be incongruent when the colour of the word and word meaning are mismatched (e.g., BLUE written in colour black), or congruent when they are the same (e.g., BLACK written in colour black). The interference effect is manifest by incongruent stimuli having slower response times than congruent stimuli. It is a measure of the extent to which attention is captured by distracting (or conflicting) information.

2.1.4 Mathematics Anxiety Studies on Inhibitory and Attentional Performance

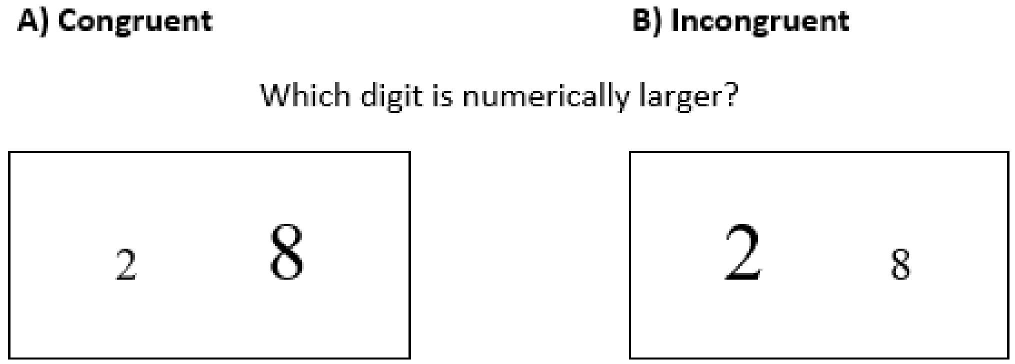
The Stroop task provides an instrument to measure the effects of attentional capture and hence inhibition of this attentional capture, and thus how anxiety may affect inhibition (Hopko et al., 2002; Suárez-Pellicioni et al., 2014). Studies implementing a colour-word version have found that interference is greater under conditions of anxiety and stress (Hochman, 1967, 1969; Kalanthroff et al., 2016; Pallak et al., 1975; Richards et al., 2000). Consistent with these findings in general anxiety, early research on individuals with mathematics anxiety suggests that they may have trouble inhibiting attention to distracting information (Hopko et al., 1998, 2002). Hopko et al. (1998) investigated whether individuals with mathematics anxiety have a deficient inhibition mechanism where they have difficulty inhibiting attention to intrusive thoughts which overburden the working memory system. Participants performed a self-report mathematics anxiety questionnaire (Abbreviated Mathematics Anxiety Rating Scale [sMARS]; Alexander & Martray, 1989) where they answered questions about their feelings towards mathematical situations. A continuous scale of mathematics anxiety was obtained from the self-report questionnaire and participants were divided into low, medium, and high mathematics anxiety groups based on their individual score. Participants then performed a reading task designed to measure their ability to inhibit attention to distracting information. The medium and high math-anxiety groups made more errors and took significantly longer to read paragraphs with distracters than those in the low math-anxiety group. Furthermore, individuals with mathematics anxiety were less able to inhibit attention to distracters even when paragraphs were unrelated to mathematics, indicating a general deficit of the inhibition mechanism.

Subsequently, a math-related Stroop paradigm was employed to investigate the effect of mathematics anxiety on inhibitory deficits (Hopko et al., 2002). The math-related Stroop paradigm was relatively unexplored in the field of mathematics anxiety. Participants who were high and low in mathematics anxiety were tested on a card task in both a numerical and letter condition. In the numerical condition, cards were presented with repeated numerals 1 to 9 (e.g., 444) and participants responded with the quantity of numerals displayed on the card (e.g., three). The task required participants to inhibit reading the numerals and to respond by counting the digits. In the letter condition, cards were presented with repeating letters (e.g., EEEE) and participants responded by counting the number of letters on the card (e.g., four). Additionally, a modified version of the colour-naming Stroop task was performed where mathematical words and neutral words were displayed in different colours. Participants were required to name the colour of the word while ignoring the word meaning. Overall, the researchers found that individuals who were high in mathematics anxiety had longer response times in the letter and numerical counting task than individuals who were low in mathematics anxiety, and this difference between the groups was more pronounced in the numerical than the letter condition. Response times did not differ between the LMA and HMA groups on the modified Stroop colour-word task for either mathematical or neutral words. The results suggested that interference effects of mathematics anxiety may be a function of inhibitory deficits that are compounded when exposed to more salient (i.e., numerical) stimuli. The authors concluded that there needs to be further exploration of the specific deficiencies associated with mathematics anxiety.

To investigate the effect of mathematics anxiety on the execution of attentional control when encountering conflict during processing, Suárez-Pellicioni et al. (2014) used the event-related potentials (ERP) technique to understand whether mathematics anxiety is related to early (i.e., detection) or late stages of the processing of conflict. Their research was first to study interference effects in mathematics anxiety using techniques which assess more automatic responses to numerical stimuli, observed by measuring cortical electrical signalling that may be difficult to observe behaviourally. LMA and HMA groups were formed based on a self-report mathematics anxiety questionnaire (Abbreviated Mathematics Anxiety Rating Scale [sMARS]; Alexander & Martray, 1989). The LMA group comprised individuals who scored within the lowest quartile. The HMA group comprised individuals who scored within the highest quartile. Groups differed in mathematics anxiety but not trait anxiety, state anxiety, or simple math ability. Electroencephalogram (EEG) was recorded while participants were tested on a numerical Stroop paradigm, a standard test for examining cognitive control and the ability to inhibit irrelevant information during a numerical task. In this version a participant is presented with two single-digit numbers each in different physical sizes and must decide which number is numerically larger (see Figure 1). A conflict can occur where the physical size is mismatched with the numerical size and needs to be inhibited. The size congruity effect, or numerical interference effect, is observed where it is easier to decide which number is numerically larger, when this number is also physically larger (the congruent condition), than when this number is physically smaller (the incongruent condition). For example, if presented with 2 and 8 (the numbers as displayed in the left panel of Figure 1), it is easier to decide 8 is numerically larger than if presented with 2 and 8 (as displayed in the right panel of Figure 1). In line with previous studies, the authors calculated a single score index of interference, subtracting congruent from incongruent response times, and incongruent from congruent for hit rates (i.e., accuracy). The greater the value, the greater the interference.

Figure 1

Congruent and Incongruent Stimuli for the Numerical Stroop Task



Note. Panel A shows an example of a congruent trial in the numerical Stroop task, when the numerically larger number is also physically larger, and hence there is no conflict between numerical and physical size. Panel B shows an example of an incongruent trial when the numerically larger number is physically smaller and hence there is conflict between the numerical and physical size.

Suárez-Pellicioni et al. (2014) found that individuals with mathematics anxiety had larger interference effects for response times. This result supports the existence of an impaired inhibition mechanism and the claims of attentional control theory as it relates to mathematics anxiety. Specifically, it suggests that individuals with mathematics anxiety are more easily distracted by task irrelevant, stimulus driven features of the environment. Additionally, through correlation of participants' sMARS test scores with the interference effect for response times, the authors found that those with the greater level of mathematics anxiety, showed the largest difference in responding to incongruent trials compared to congruent trials. Furthermore, they found no differences between the LMA and HMA groups in accuracy when identifying the numerically larger item which supports the hypotheses that anxiety impairs processing efficiency (i.e., response times) to a greater extent than performance effectiveness (i.e., accuracy). Moreover, their results from the ERP technique suggested that mathematics anxiety does not affect the early stages of cognitive control processing, where the system monitors for conflict, but affects the later stages of processing, with an abnormal upregulation of resources to adapt to the conflict that has been encountered. Notably, their results converge with theories of general anxiety. Specifically, the results were interpreted to support the dual mechanisms of control theory where anxious individuals do not maintain top-down attentional control continuously in a proactive manner, but instead exert control reactively only as needed when conflict is encountered. Suárez-Pellicioni et al. therefore concluded that attentional control and susceptibility to distraction are important factors related to mathematics anxiety that deserve further investigation.

Impairments in attentional control and inhibition amongst individuals with mathematics anxiety has been the focus of recent empirical research. Studies have found individuals with mathematics anxiety show deficits in inhibitory abilities (Justicia-Galiano et al., 2016; Mammarella et al., 2018; Passolunghi et al., 2016) and in attentional control (Ashkenazi, 2018; Hartwright et al., 2018; Liu et al., 2019; Pizzie & Kraemer, 2017). In an attentional deployment paradigm using functional magnetic resonance imaging (fMRI), Pizzie and Kraemer (2017) observed that mathematics anxiety is associated with attentional disengagement (threat avoidance) that is specific to numerical stimuli. Ashkenazi (2018) investigated the effect of intentional versus automatic processing in individuals with mathematics anxiety by combining a numerical Stroop paradigm with emotional priming involving mathematically related words. Participants were tested on separate tasks where the physical size was the irrelevant dimension and when the numerical size was the irrelevant dimension. The author found a larger interference effect among individuals with mathematics anxiety when physical size was irrelevant (as in Suárez-Pellicioni et al. (2014)) but not when numerical size was irrelevant. The findings of a greater processing of non-numerical irrelevant information, such as the physical size dimension, aligns with attentional control theory where anxiety reduces attentional control to the intended dimension and increases attention to the irrelevant (and more automatic) dimension.

In summary, early research suggested that individuals with mathematics anxiety may experience deficits in attentional control and inhibition of task irrelevant information during mathematical tasks. Furthermore, results of these studies are in line with research on general anxiety. More recently, neuroimaging and electrophysiological techniques have provided further support for these proposals, attempting to describe the underlying cognitive mechanisms in more detail.

2.2 Deficits in Basic Numerical Skills: Relation to Mathematics Anxiety

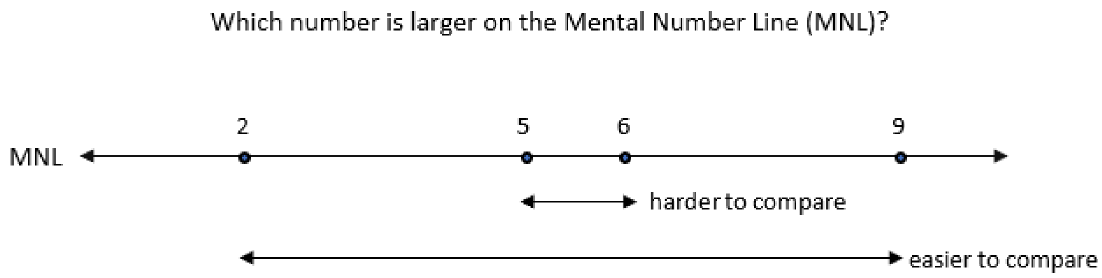
2.2.1 The Magnitude Comparison Task

As outlined previously, deficits in numerical skills can lead to compromised learning and performance that in turn can result in developing mathematics anxiety. Studies have investigated the effects of mathematics anxiety on basic numerical skills by examining performance on magnitude comparison tasks for non-symbolic quantities (e.g., deciding which of two dot arrays presented on a screen has largest magnitude) and for symbolic quantities (e.g., deciding which of two Arabic digits is numerically larger). The non-symbolic magnitude comparison task can be used to assess the acuity of the approximate number system (ANS) (see Dietrich, Huber, & Nuerk, 2015 for a review). The ANS is assumed to represent magnitude in an approximate way and can be used to estimate quantities in comparison tasks without explicitly counting the items (Dehaene, 1992). The symbolic number comparison task includes several cognitive components including digit identification, digit to number-word matching, digit ordering, and general comparison (see Sasanguie et al., 2017 for a review). The task of number comparison is related to the underlying hypothesis that numbers are mentally represented as a number line (Dehaene, 2001). For example, the number 9 is larger than the number 2 because of its spatial location on the number line, as it is located to the right of the number 2 and numbers increase in size as they move to the right. Each number on the mental number line is thought to share representational features with the numbers that are in close proximity to it. Furthermore, the representation of numerical magnitude is thought to vary between individuals due to the degree of overlap between the numbers. A larger degree of overlap indicates a less precise representation of numerical magnitude (Dehaene et al., 1998). Additionally, the representation of numerical magnitude on the mental number line has been described as having tuning curves following a Gaussian distribution (see Feigenson et al., 2004 for a review). Each specific numerical magnitude activates neurons maximally with adjacent magnitudes activated to a lesser extent.

The numerical distance effect and the size effect are robust findings observed in magnitude comparison tasks. The numerical distance effect occurs when it is easier to compare two quantities and decide which is numerically larger when the numbers are further apart than when they are closer together (Moyer & Landauer, 1967). For example, it is easier to compare the numbers 2 and 9 and decide which is larger than it is to compare the numbers 5 and 6 (see Figure 2). Previously, the main theory regarding the origins of the numerical distance effect proposes that it indexes the overlap of numerical representations on the mental number line where magnitudes that are positioned close to each other share more representational overlap than those that are further apart. This overlap results in numbers that are closer together being harder to discriminate during a number comparison task. Another theory regarding the origins of the numerical distance effect in numerical comparisons is that it indexes comparison processes between the numerical stimuli representations and the response (van Opstal et al., 2008; van Opstal & Verguts, 2011; Verguts et al., 2005).

Figure 2

The Numerical Distance Effect Shown on the Mental Number Line



Note. Based on the mental number line, the numerical distance effect is observed in number comparison tasks, where it is easier to decide which number is larger when the numbers are further apart than when they are closer together.

The size effect shows it is easier to compare two numbers that have the same numerical distance between them, when the numbers are small than when they are large (Moyer & Landauer, 1967). For example, it is easier to compare the numbers 1 and 2 and decide which is larger, than it is to compare the numbers 8 and 9. The main theory describing the size effect attributes it to the overlap between ANS representations which increases with numerosity such that larger numbers are represented more vaguely than smaller numbers and are therefore more difficult to discriminate (Feigenson et al., 2004). However, Verguts et al. (2005) provided an alternative explanation for the size effect that it can be explained as (response-related) comparison processes as a result of the frequency of numbers experienced in daily life (Dehaene et al., 1990) where larger numbers occur with a lower frequency.

2.2.2 Mathematics Anxiety Studies of the Number Comparison Task

The number comparison task has been studied within the mathematics anxiety literature with mixed results. Researchers have proposed different theories to explain these mixed findings. Previous research had suggested that the effects of mathematics anxiety affected more complex arithmetic rather than simple arithmetic skills (Ashcraft & Faust, 1994; Faust et al., 1996), so Maloney et al. (2010) investigated whether mathematics anxiety affects basic numerical processing. Participants were first tested on a visual enumeration task, then measures of working memory capacity were administered. In the enumeration task participants were presented with one to nine squares and responded with the number of squares that were displayed. The HMA group performed worse than the LMA group counting squares in the range of five to nine (the counting range). Differences between the groups appeared to occur due to working memory capacity. Maloney et al. (2010) proposed that the effects of mathematics anxiety extend into basic numerical processing where a deficit in basic numerical skills compromises the development of higher-level mathematics. Moreover, the effect of anxiety on working memory further exacerbates these deficits.

The nature of deficits in basic numerical skills in individuals with mathematics anxiety has been further investigated with symbolic number comparison tasks to examine whether mathematics anxiety was associated with deficits in the representation of numerical magnitude (Maloney et al., 2011). Maloney et al. (2011) examined the numerical distance effect in individuals who were high and low in mathematics anxiety. The numerical distance effect can show individual differences in numerical representation and processing. Response times for the numerical distance effect were more pronounced in the HMA group than for the LMA group. The authors interpreted these findings as indicating that individuals high in mathematics anxiety may have a less precise representation of numerical magnitude than individuals who are low in mathematics anxiety. Furthermore, they found no differences overall between the LMA and HMA groups for response times or error rates. In conclusion, the authors proposed that a hybrid theory is likely: a less precise representation of magnitude would lead to a difficulty with higher-level mathematics and this difficulty can lead to mathematics anxiety where reduced working memory further exacerbate the effects of deficits in basic numerical skills.

The extent to which mathematics anxiety is related to a less precise representation of numerical magnitude is contentious. Núñez-Peña and Suárez-Pellicioni (2014) investigated the effects of the symbolic number comparison task on individuals with mathematics anxiety. They presented stimuli as extreme as possible to participants. The numerical distance effect and the size effect were studied as both measures relate to accessing numerical magnitude representations. Overall, the HMA group were slower than the LMA group. No differences were found in error rates. However, there were marginal differences between the groups for both the numerical distance effect and the size effect. Dietrich, Huber, Moeller et al. (2015) and Colomé (2019) claimed that recent research suggests that magnitude representations of non-symbolic numbers and symbolic numbers may be coded differently (Piazza et al., 2006, 2007; Verguts & Fias, 2004). A deficient representation of numerical magnitude refers to a deficient ANS. Therefore, they questioned whether the acuity of the ANS could be tested with a symbolic number comparison task. A more appropriate and standard test to assess the acuity of the ANS is the non-symbolic dot comparison task (see Dietrich, Huber, & Nuerk, 2015 for a review).

To assess ANS representations and the influence of mathematics anxiety on symbolic and non-symbolic magnitude processing, Dietrich, Huber, Moeller, et al. (2015) tested participants on a non-symbolic dot comparison task and a symbolic comparison task. Several indices that measure ANS acuity along with standard measures of error rates, mean response times, and distance and size effects were measured. In the symbolic number comparison task, participants decided which of two single-digit numbers was the largest. All combinations of Arabic digits from 1 to 9 were presented. Dietrich, Huber, Moeller, et al. (2015) found no link between mathematics anxiety and any of the measures of ANS acuity in the non-symbolic number comparison task. Furthermore, in the symbolic number comparison task they replicated results from previous studies (Maloney et al., 2011; Núñez-Peña & Suárez-

Pellicioni, 2014) finding a significant association between mathematics anxiety and the numerical distance effect for response times. Individuals with high mathematics anxiety had more pronounced distance effects than individuals with low mathematics anxiety. They also found no significant association between mathematics anxiety and overall response times or the size effect in the symbolic number comparison task. Moreover, they found no difference in error rates for either of the tasks. The authors concluded that their results suggest that the acuity of the ANS is not impaired in individuals with mathematics anxiety and do not support the conclusion that these individuals may have a less precise representation of numerical magnitude.

Consequently, the authors proposed an alternative theory to explain the findings that mathematics anxiety is associated with larger numerical distance effects in the symbolic number comparison task. Individuals with mathematics anxiety may have impaired comparison processes instead of an impaired representation of numerical magnitude, as the numerical distance effect can reflect comparison processes between the symbolic representation and the response in deciding which number is larger. Impaired comparison processes may be due to less training of the connection between the representation and the "which numeral is larger" response. Individuals with mathematics anxiety may have less trained connections because they may be less motivated to perform or more motivated to avoid numerical calculations. Dietrich, Huber, Moeller, et al. (2015) conclusion fits into the model by Ashcraft et al. (2007) who proposed that deficits in basic numerical skills or low motivation may be risk factors in developing mathematics anxiety.

Other evidence concerning the nature of magnitude representations amongst those high and low in mathematics anxiety comes from the comparison of performance on a non-symbolic number comparison task, a symbolic number comparison task, and a counting Stroop task (Colomé, 2019). The counting Stroop task (Pavese & Umiltà, 1998) involved deciding how many Arabic digits were presented in an array of identical digits. Overall, no differences were found between the LMA and HMA groups on any of the tasks for response times, distance and size effects, or for error rates. The author concluded that the results do not support the hypothesis that individuals with mathematics anxiety have a less precise representation of magnitude for either non-symbolic or symbolic representations, regardless of whether the representations for non-symbolic and symbolic quantities are the same or not. Furthermore, as the counting Stroop task does not require the use of comparison processes, the findings support the claim by Dietrich, Huber, Moeller, et al. (2015) that larger distance effects in individuals with mathematics anxiety may be due to comparison processes (i.e., less trained connections between the representation and the “which numeral is larger” response). However, Colomé questioned this conclusion. Firstly, several studies had contradictory results. Secondly, if individuals with high mathematics anxiety have less trained connections, a size effect (which can reflect comparison processes) would show differences between the LMA and HMA groups. No differences in the size effect, however, were found for any of the above-mentioned studies with the exception that Núñez-Peña and Suárez-Pellicioni (2014) found differences that were marginal. Concluding, Colomé suggested that the differences in findings may be due to either experimental design or that motivation and attitudes towards mathematics were not controlled for in the studies and could explain the variability between them. Low motivation for mathematics could also be the result of a lack of attribution for the importance of mathematics that could be explained in part by the classroom environment (Middleton & Spanias, 1999). Moreover, although there were no differences between the LMA and HMA groups for behavioural measures, Colomé noted that the lack of these findings could also be related to the proposal that individuals high and low in mathematics anxiety could differ in attentional control.

Further to the above-mentioned research, other studies have investigated the link between basic numerical skills and mathematics anxiety. Douglas and Lefevre (2016) did not find a direct link between basic numerical skills and mathematics anxiety when participants were tested on a range of mathematical tasks and memory measures. However, the authors found an indirect link between them that was fully mediated by complex mathematical performance. Furthermore, they didn't find any evidence that basic numerical skills mediated the relationship between mathematics anxiety and complex math performance. Their results support the view that under certain conditions, basic numerical skills can elicit anxiety and impact mathematical skills. Artemenko et al. (2015) found that individuals with high mathematics anxiety showed less efficient neural processing during basic numerical skills. Chang et al. (2017) examined individuals high and low in mathematics anxiety performing simple arithmetic under fMRI. Their findings showed that individuals high and low in mathematics anxiety activated attention-related networks (specifically the front-parietal network) differently even though performance was similar for behavioural measures. Performance in the LMA group improved when the fronto-parietal attentional network was activated less. However, performance in the HMA group was not as dependent on the reduction of this network as the LMA group. The authors suggested several explanations for their findings, that individuals with high mathematics anxiety may recruit resources to reduce the negative thoughts associated with the anxiety induced by numerical stimuli, or that they employed different strategies for problem-solving which are related to avoidance of mathematics across their lifespan, or that they require increased effort and/or decreased efficiency to solve numerical problems. Pletzer et al. (2015) performed a two-digit number comparison task and a number bisection task while participants were imaged using fMRI. Individuals high in mathematics anxiety experienced a reduced deactivation of the default mode network compared to individuals low in mathematics anxiety, possibly due to preoccupation with emotional content. This reduced deactivation was more pronounced when irrelevant stimuli needed to be inhibited. The default mode network is related to emotional processing (see Raichle, 2015 for a review). Deactivating the default mode network supports goal-directed behaviour and processing efficiency (Fales et al., 2008). Ashkenazi (2018) combined a numerical Stroop task with a priming paradigm and found an abnormal numerical distance effect in individuals who were high in mathematics anxiety. It was concluded that impairments in basic numerical processing are context related. Rubinsten et al. (2015) and Batashvili et al. (2020) found that individuals high in mathematics anxiety may have a threat-related response just by the process of observing simple numerical stimuli.

In summary, results from mathematics anxiety studies examining behavioural measures of performance in the symbolic number comparison task are mixed. Some studies have found differences between the LMA and HMA groups for response times, the distance effect, and the size effect, and some have not. There may be several factors influencing the differences in findings. One of the suggestions for these differences between the LMA and HMA groups is that individuals high in mathematics anxiety may have impaired comparison processes from less training of the connections between the symbolic representation and the response (Colomé, 2019; Dietrich, Huber, Moeller, et al., 2015). Differences in training of the connections could be due to several different factors, including low motivation or avoidance. Furthermore, studies examining the effect of mathematics anxiety on basic numerical skills have found that individuals high in mathematics anxiety have less efficient neural processing, a threat-related response observing simple numerical stimuli, and differences in the way they activate attentional control networks.

2.2.3 Neural Network Modelling of the Origins of the Distance Effect and the Size Effect

To examine how numerical magnitude might be represented and to assess the consequences of different representations for behavioural effects, it is possible to model neural network representations and simulate their behavioural results. These methods can be informative in helping to draw conclusions about the underlying factors affecting cognitive performance. Indeed, neural network modelling has been influential in describing the origins of the numerical distance effect and the size effect, and how numerical magnitude may be represented. Verguts and Fias (2004) implemented a neural network model to investigate the representation of numerical information for both non-symbolic and symbolic stimuli. In their first simulation they showed how number-selective neurons developed from unsupervised learning when non-symbolic stimuli (e.g., a collection of dots) were presented to the model. After the model was trained, nodes developed spontaneously that were attuned to a specific numerosity (i.e., number-selective neurons). The model was able to account for both the numerical distance effect and the size effect suggesting that these effects during non-symbolic number comparison emerge from the representation of numerical information. The properties of these number-selective neurons in the model were similar to those discovered in monkeys (Nieder et al., 2002; Nieder & Miller, 2003). Nieder et al. (2002) found number-selective neurons that responded to a specific numerosity maximally, and nearby neurons that responded with decreasing strength as they become further away. This property can account for the numerical distance effect. Numerosities that are closer together have overlapping distributions and are harder to discriminate than numerosities that are further apart. Nieder et al. also found these number-selective neurons had tuning curves that became increasingly broader for larger numerosities. This property can account for the size effect. Larger numerosities have more representational overlap than smaller numerosities. Therefore, larger numerosities are harder to discriminate than smaller numerosities. In line with these findings, the neurons in Verguts and Fias' model showed a filter property where a neuron activates maximally for its preferred numerosity with nearby neurons activating to a lesser extent as a function of distance, and an increasing bandwidth property where larger numbers have larger bandwidths than smaller numbers. Verguts and Fias also fitted Gaussian functions for the tuning curves for each numerosity and found that a logarithmic rather than a linear or power transformation had the best fit for non-symbolic numerosities. They concluded by proposing that the internal mental number line can develop by unsupervised learning.

In Verguts and Fias' (2004) second simulation both non-symbolic and symbolic stimuli were presented to the model simultaneously to simulate learning. These same number-selective neurons that had developed from non-symbolic input in the first simulation learned to represent the meaning of numerical symbols. However, only some of the properties of the number-selective neurons that emerged from presentation of non-symbolic stimuli were transferred due to the presentation of symbolic stimuli. After learning, the filter property was retained where number nodes that preferred a specific non-symbolic quantity also preferred the corresponding symbolic quantity. The bandwidths were smaller for the symbolic stimuli than for the non-symbolic stimuli. Thus, tuning curves became more peaked (i.e., less broad). This resulted in symbolic stimuli being represented more precisely than non-symbolic input and therefore efficiency was increased for symbolic numbers. There was no transfer of the increasing bandwidth property from non-symbolic to symbolic processing thus questioning the origins of the size effect which had occurred when non-symbolic stimuli had been presented to the model. This result suggested that the size effect may have different origins depending on the type of numerical stimuli. The authors proposed an alternative description of the origins of the size effect in their subsequent research. Furthermore, their neural network models have shown that the development of the symbolic representation of numerical magnitude is not just simply linking the symbol to the non-symbolic numerical representation because not all of the properties are transferred across.

Subsequently, Verguts et al. (2005) implemented a neural network model (see section Neural Network Modelling: Prior Relevant Models for more details) to investigate the origins of the distance effect and the size effect for symbolic numbers. The origins of the distance effect and the size effect can be explained by different assumptions about how numerical information is represented and processed. The authors proposed a unified framework that accounted for previous empirical findings for the distance effect and the size effect appropriately across different tasks. Their model proposed that the representation of numerical magnitude for symbolic numbers on the mental number line has place coding, linear scaling, and constant variability properties. In addition to these assumptions, they proposed an alternative to account for the origins of the size effect in the symbolic number comparison task. Numbers were presented to the model during the training phase with the frequencies that they occurred in daily life (Dehaene et al., 1990), where smaller numbers were presented more often than larger numbers. The model simulations suggested that the size effect for the symbolic number comparison task was the result of comparison processes (i.e., nonlinear mappings) between the mental number line and the output fields. These mappings were derived from the frequency that numbers were presented to the model during learning. Importantly, Verguts et al. explained the origins of the numerical distance effect and the size effect in the symbolic number comparison task as developing from the monotonicity (the condition of consistently increasing or decreasing in value) of the connection weights between the stimuli and the response units.

van Opstal et al. (2008) and van Opstal and Verguts (2011) further examined distance effects by neural network modelling proposing that the distance effect has different origins depending on the task context. Specifically, they showed that the "comparison" distance effect that is obtained from a symbolic number comparison task is derived from comparison processes between the stimuli and response and not from the overlap of the numerical representations of the stimuli. Furthermore, the authors claimed the importance of studying the origins of the distance effect, and of choosing the correct experimental task to draw correct conclusions about cognition.

2.2.4 The Symbolic Number Comparison Task Across Learning and/or Development

The symbolic number comparison task is related to children's mathematics achievement (Brankaer et al., 2017; Schneider et al., 2017; Vogel et al., 2015). Mathematical competence across the lifespan is related more strongly with symbolic than non-symbolic numerical magnitude processing and it has been suggested that symbolic magnitude processing may be more suitable for diagnostics and interventions for adults and for children at risk of mathematical difficulties (Schneider et al., 2017). The numerical distance effect has been studied in children across development. Some studies report decreases in the size of numerical distance effect as age increases (Holloway & Ansari, 2008; Moore & Ashcraft, 2015; Sekuler & Mierkiewicz, 1977). Further, de Smedt et al. (2009) found the size of the numerical distance effect predicted individual differences in mathematics achievement where larger distance effects predicted lower mathematics achievement. Other studies have found the size of the numerical distance effect to be stable during development and accompanied with an overall decrease in response times (Landerl, 2013; Reeve et al., 2012). Importantly, the authors of a recent meta-analysis of the comparison distance effect in symbolic and non-symbolic tasks comparing typically developing children with children with mathematical learning difficulties concluded their findings support the view that the distance effect for comparison tasks is not an index of the representation of symbolic magnitude (Schwenk et al., 2017). Their research found that symbolic number comparison was more impaired than non-symbolic number comparison for children with mathematical learning difficulties compared to typically developing children. Furthermore, they found no qualitative differences in the numerical distance effect between typically developing children and children with mathematical learning difficulties for either the symbolic or the non-symbolic number comparison task.

Another task that is studied during development is the physical Stroop task, to investigate the automisation of symbols. This task is similar to the numerical Stroop task. However, the numerical and physical size dimensions are reversed where the physical size is the relevant dimension and the numerical size is the irrelevant dimension. The physical Stroop task entails deciding which of two numbers presented in different sized fonts is physically larger while ignoring their numerical value. The size congruity effect (or interference effect) refers to the difference in response times between incongruent and congruent trials. As the processing of the meaning of numerical symbols is automised they interfere with the physical size dimension in the physical Stroop task thereby increasing the size congruity effect. Consequently, the size congruity effect in the physical Stroop task increases with age as children learn the meaning of symbols (Girelli et al., 2000; Landerl & Kölle, 2009).

2.3 The Current Study

The current study investigates whether mathematics anxiety affects attentional control and inhibition and/or the amount of training in numerical comparisons which negatively impacts performance on numerical tasks. Mathematics anxiety studies have suggested that individuals with mathematics anxiety have impaired attentional control and inhibitory mechanisms (e.g., Hopko et al., 1998, 2002; Suárez-Pellicioni et al., 2014), as proposed by the disruption account that mathematics anxiety is characterised by a reduction in working memory resources which results in underperforming in mathematical tasks. However, the underlying cognitive mechanisms related to these impairments remain unclear. These impairments are proposed to impact processing efficiency (i.e., response times) to a greater extent than performance effectiveness (i.e., accuracy) as proposed by the attentional control theory. Furthermore, individuals with mathematics anxiety may experience a threat-related response while observing numerical stimuli (e.g., Batashvili et al., 2020; Pizzie & Kraemer, 2017; Rubinsten et al., 2015). Moreover, it has been suggested that individuals with mathematics anxiety may have less trained connections (i.e., comparison processes) between the numerical representation and the response in number comparison tasks (Dietrich, Huber, Moeller, et al., 2015).

The current study will use a neural network model which simulates changes to attentional control, inhibition, and learning to examine how changes to these underlying processes impact performance on the numerical Stroop task and the symbolic number comparison task. First, the neural network model will be validated to ensure that it simulates empirical effects of the numerical Stroop task and the symbolic number comparison task. For the numerical Stroop task, the model should be able to simulate the size congruity effect and the numerical distance effect. Furthermore, the amount of energy (i.e., conflict) in the response layer should show the standard empirical effect that incongruent trials experience more conflict than congruent trials due to the interference of the physical size on the judgment of numerical size. The speed-accuracy trade-off and the physical Stroop task will also be simulated as part of the validation process to demonstrate the model's performance in normal functioning individuals across a range of conditions. The speed-accuracy trade-off is a well-known and robust effect in cognitive studies where decisions made more slowly have increased accuracy than those made in a shorter response time (for a review see Heitz, 2014). For the symbolic number comparison task, the model should be able to simulate the standard empirical effects: the distance effect and the size effect. To simulate reduced learning, both models will be validated to ensure that increased learning improves response times and accuracy (i.e., performance outcomes).

Once the models have been validated and are a reasonable low math-anxious model of an individual without mathematics anxiety, the effect of reducing attention and reducing learning will be simulated and compared to the research literature to investigate the effect of these impairments in relation to mathematics anxiety on both tasks. For the numerical Stroop task, a model with mathematics anxiety should have longer response times for the interference effect and for incongruent trials, and no difference in response times for congruent trials or for error rates. Additionally, a high math-anxious model of the numerical Stroop task with reduced attention that simulates mathematics anxiety will be compared to a low math-anxious model without attention impaired to investigate differences in the amount of energy (i.e., conflict) in the response layer during congruent and incongruent trials. For the symbolic number comparison task, response times, accuracy, the numerical distance effect, and the size effect will be compared between the low math-anxious model and the impaired models to investigate the effect impairments have on these empirical effects.

Chapter 3: Methodology: Neural Network Modelling

This chapter describes the neural network architecture used for modelling numerical representations and cognitive processes used in the current research. First it reviews prior relevant neural network models. Next it describes the process for simulating the LMA and HMA models. Finally, it describes the neural network architecture for the simulation of the numerical Stroop task and the symbolic number comparison task.

3.1 Neural Network Modelling: Prior Relevant Models

Although, to date, no neural network models exist simulating mathematics anxiety, neural network modelling has been used extensively to investigate the mechanisms of numerical cognition and cognitive control. In their seminal research article, Verguts et al. (2005) implemented a neural network model that proposed a place-coding system to explain how number-selective neurons, that are attuned to numbers (Nieder et al., 2002), are represented on the mental number line for symbolic numbers. Within the literature there are different assumptions about how numerical magnitude is represented. Summation coding (e.g., Zorzi & Butterworth, 1999) assumes that each number activates the corresponding number of units on the number line. For example, the number 3 activates the units 1, 2, and 3. Larger numbers activate a subset of smaller numbers (e.g., 5 activates a subset of those units activated for the number 3). Compressed scaling (e.g., Dehaene, 1992) assumes that the distance between the representation of numbers decreases as the numbers become larger. Increasing variability (e.g., Gallistel & Gelman, 1992) assumes that units close to the maximally activated unit are activated with increasing variability as the numbers increase with magnitude (thus the increase depends on the distance between the two numbers). These assumptions, however, do not hold across various tasks. Verguts et al. therefore, proposed a unified framework that accounted for findings from previous studies. Their model, called a model of exact small-number representation, proposed that the representation of numerical magnitude for symbolic numbers has place coding, linear scaling, and constant variability properties.

Verguts et al.'s (2005) place-coding model simulated several tasks including a symbolic number comparison task. This task involved deciding which of two numbers (from 1 to 15) has greatest magnitude. The numerical magnitude representation reflected place-coding properties as each input number presented to the model activates the same number of units on the number line. For example, if the number 2 or the number 5 is presented to the model only the second or fifth nodes respectively are maximally activated and not all preceding nodes as well. Each number presented to the model demonstrated constant variability by maximally activating its corresponding number line node with adjacent nodes being activated with decreasing strength the further they are from the number. For example, if number 4 is presented to the model the fourth node is activated maximally, adjacent nodes 3 and 5 are activated to a lesser degree, nodes 2 and 6 are activated to a lesser degree than nodes 3 and 5, and so on. The model exhibited the properties of linear scaling where the distance between each of the number line nodes is fixed and has the same distance. When two numerical stimuli, presented on the left and right, were input to the model for comparison as to which was largest, the model activated either the left or right response unit corresponding to the left or right input stimuli, depending on which number was the largest. The model was able to simulate the numerical distance effect and the size effect. These effects originated from the monotonicity of the connection weights between the mental number line and the response units after training (see section The Connection Weights Between the Stimuli and the Response for a description). Verguts et al.'s place-coding model has been seminal in the development of computational models of numerical cognition. It has been the basis of subsequent neural network models simulating number magnitude comparison tasks (e.g., Chen & Verguts, 2010; van Opstal et al., 2008).

Santens and Verguts (2011) adapted the Verguts et al. (2005) model to simulate the numerical Stroop task, which involved deciding which of two single-digit numbers had greater magnitude when they were presented in different physical sizes. The numerical size and physical size dimensions can interact at different levels of processing, such as the input, representation, decision, or output level (Verguts & Fias, 2008). Thus, the size congruity effect which is central to the numerical Stroop task can originate at different levels of processing. The level of processing that the size congruity effect originates from is still under investigation. According to the shared representation account, numerical size and physical size interact at the representation level (Schwarz & Heinze, 1998). According to the shared decisions account, numerical size and physical size are initially processed separately then interact at the decision level of the task (Schwarz & Heinze, 1998). Santens and Verguts' neural network model implemented a dual route architecture to simulate the shared decisions account. This model was able to account for effects of the numerical Stroop task and several other tasks to explain findings that the shared representation account could not.

Moeller et al. (2011) extended the Verguts et al. (2005) model to simulate a two-digit number comparison task, that is, deciding which of two two-digit numbers had the largest magnitude. The authors investigated how two-digit numbers are represented by creating three neural network models: a holistic model (e.g., Dehaene et al., 1990) where the entire two-digit number is holistically represented on just one mental number line; a strictly decomposed model (e.g., Verguts & de Moor, 2005) that proposed a separate mental number line that is recycled for each place-value (for example both the tens and units of a two-digit number are decomposed to their own mental number line); and a hybrid model (e.g., Nuerk et al., 2001) containing both holistic and decomposed representations. The authors concluded that the strictly decomposed model simulated the empirical effects of two-digit number comparison in the best and most parsimonious way. The strictly decomposed model consisted of a single-digit number comparison network for each of the tens and units of a two-digit number. These networks represented the tens and units having their own mental number line respectively.

An extension of the strictly decomposed model to include the modelling of cognitive control required to resolve conflict which may arise between tens and units in a two-digit number comparison was developed by Huber, Moeller, Nuerk, Macizo, et al. (2013). It was based on a neural network model by Verguts and Notebaert (2008) who proposed a conflict-modulated Hebbian learning rule to show how the cognitive control system knows where to intervene when it detects conflict. The Verguts and Notebaert model consisted of a conflict monitoring unit that monitored the amount of conflict in the system during a cognitive task and signalled to strengthen connections between active representations (thereby strengthening task-relevant associations) when conflict was encountered. By integrating their two-digit number comparison model with a cognitive control network, Huber, Moeller, Nuerk, Macizo, et al. (2013) were able to model the conflict that occurs during a two-digit number comparison task, where the tens are initially compared while ignoring the units.

Conflict processing has been modelled previously by Botvinick et al. (2001) who implemented a series of neural network models to propose how the cognitive control system detects the need to intervene during information processing. Botvinick et al.'s conflict monitoring hypothesis proposed that a conflict monitoring system evaluates the amount of conflict in the system and consequently regulates the amount of top-down cognitive control based on current task demands. The authors hypothesised that the detection of conflict may be a function of the anterior cingulate cortex (ACC) based on data from empirical studies . In a series of simulations, they extended existing neural network models to include a conflict monitoring unit that calculated the amount of response conflict at each step of processing. Conflict in the models was defined as the simultaneous activation of incompatible (i.e., alternative) responses of mutually inhibiting units. Quantitatively, conflict was measured as the amount of energy (Hopfield, 1982) across the units in the response layer of each model. A model simulating the colour-word version of the Stroop task showed an increase in activation of the conflict monitoring unit for the incongruent condition compared to the congruent condition, which was in line with previous neuroimaging studies of ACC activation. In subsequent simulations, Botvinick et al. extended their models to create a feedback loop from the conflict monitoring unit, to use the amount of conflict detected on previous trials as a signal to adjust top-down control. When a large amount of conflict was detected, cognitive control was strengthened. Conversely, if a low amount of conflict was detected, cognitive control was weakened.

Subsequently to their previous cognitive control model, Huber, Moeller, Nuerk, Macizo, et al.'s (2013) neural network model was extended to include number comparison of three-digit numbers (Huber, Moeller, Nuerk, & Willmes, 2013) and decimals (Huber et al., 2014). Eventually, all the models were integrated into one general framework for multi-symbol number comparison which also included negative numbers (Huber et al., 2016). The resulting model was validated by simulating most of the standard empirical effects for the number comparison task (e.g., the distance effect for two-digit and three-digit numbers). It is this general framework by Huber et al. (2016) which will be used as a basis for the neural network model utilised in the current project. This model will be adapted to integrate aspects of Santens and Verguts' (2011) numerical Stroop model.

3.1.1 The Connection Weights Between the Stimuli and the Response

As described previously, Verguts et al. (2005) explained the origins of the numerical distance effect and the size effect in the symbolic number comparison task as developing from the monotonicity of the connection weights between the stimuli and the response nodes. In the symbolic number comparison task, a set of input number nodes (representing the mental number line) for the left and right stimuli are connected to the left and right response nodes via a set of connection weights. When the model is trained to decide which input number is larger, the connection weights between the stimuli and the response are modified by the learning algorithm to improve accuracy. Subsequently, the left and right response nodes are activated in proportion to the strength of the connection weights between the input nodes and the response nodes (in addition to the strength of the numerical representation and other parameters within the model). The response node with the strongest activation becomes the winning node and the model decides that the input stimulus corresponding to that node (i.e., the left or the right) has the largest numerical value.

After the model has been trained to perform the number comparison task these connection weights have a monotonic pattern. Small numbers from the left input stimulus and large numbers from the right input stimulus have weak connections to the left response node and strong connections to the right response node as they need to activate the left response node weakly and the right response node strongly to decide that the input number on the right is larger. Conversely, large numbers from the left input stimulus and small numbers from the right input stimulus have strong connections to the left response node and weak connections to the right response node, as they need to activate the left response node strongly and the right response node weakly to decide that the input number on the left is larger. The connection weights, therefore, are monotonically increasing or decreasing. Verguts et al. (2005) demonstrated that the comparison distance effect could emerge as a result of monotonic connection weights. When comparing two numbers that are close together, the similar connection weights (due to the monotonicity) activate both response nodes to a comparable degree thereby causing competition between the response nodes which increases the response time. When comparing two numbers that are further apart, the connection weights result in one of the response nodes being activated much more strongly than the other, reducing the amount of competition between the response nodes which decreases the response time. Additionally, by training the model with numbers that mirror the frequency with which those numbers occur in daily life, the pattern of connection weights between the stimuli and response was compressive. This compressive pattern of monotonic connection weights resulted in the size effect. The difference in weights between smaller numbers was larger than the difference in weights between larger numbers. Thus, larger numbers activated the response nodes with more competition than smaller numbers thereby increasing response times.

3.2 Procedure for the Model Simulations

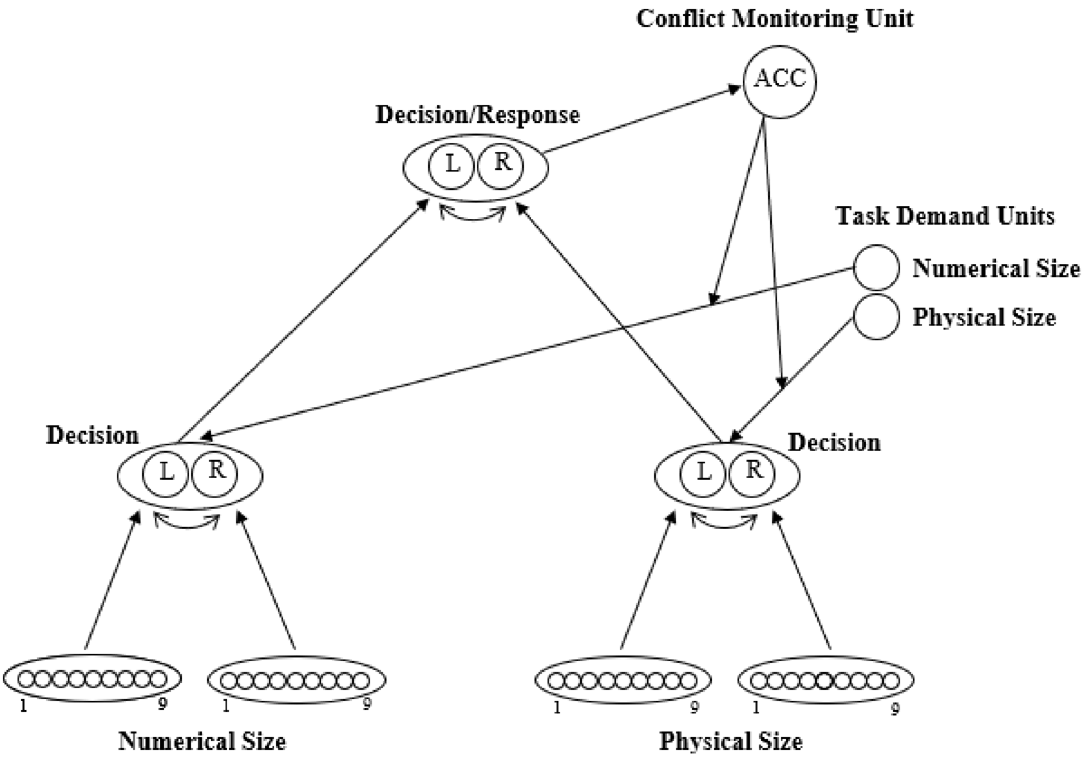
As mentioned in the introduction, this study creates a single neural network model for mathematics anxiety examining the effects of modifications to the model across two different experimental tasks to investigate how particular impairments influence task performance, which can then be compared to experimental findings and to make predictions for novel tasks or conditions. A previous neural network model architecture that simulates mulit-symbol number comparison (Huber et al., 2016) will be used as the framework for the current research. This architecture will be adapted to simulate the numerical Stroop task and the symbolic number comparison task. Both tasks will be simulated on this neural network model architecture, making minor changes to the architecture to account for different task requirements. The model architecture will subsequently be validated to ensure it is a suitable model of these tasks. This involves comparing the model's results to experimental findings. After validation this model will become the LMA model for each task simulating an individual who does not have mathematics anxiety. To create the HMA model of an individual with mathematics anxiety, different parts of the LMA model will be impaired to simulate different cognitive conditions (e.g., for impairing a neural network model see Amos, 2000; Yeung & Cohen, 2006). The models' results will be compared to experimental findings of studies of participants with mathematics anxiety. The model which is able to accurately simulate the pattern of experimental findings amongst individuals with mathematics anxiety on the numerical Stroop task and the symbolic number comparison task will be offered as an initial model of mathematics anxiety. This model can then be tested against future experimental results and used to generate novel predictions against experimental findings that can be compared.

3.3 Model Architecture for the Current Study

The integrated framework for the comparison of multi-symbol numbers developed by Huber et al. (2016) was adapted to simulate the numerical Stroop task (see Figure 3 for schematic illustration). Huber et al.'s (2016) model simulates Stroop-like effects of two-digit number comparison where the comparison of the tens digits have more relevance than the comparison of the units digits. Consequently, numerical size and physical size from the numerical Stroop model of Santens and Verguts (2011) were each mapped onto the tens and units networks of the Huber et al. (2016) model, respectively. This mapping facilitates the Stroop effect in the numerical Stroop task where numerical size has more relevance than physical size. The model uses a dual route architecture to reflect the shared decisions account (as in the study of Santens and Verguts) where numerical size and physical size only interact at the decision level. The model consists of four layers: an input layer, comparison layer, task demand layer, and a response layer; and includes a cognitive control network as implemented by Verguts and Notebaert (2008) and adapted by Huber, Moeller, Nuerk, Macizo, et al. (2013) and Huber et al. (2016). Additionally, features of the numerical Stroop model for magnitude judgment implemented in Experiment 1 of the study by Santens and Verguts were incorporated into the adapted architecture. The programming code written in Matlab by Huber and colleagues was downloaded from the supplementary materials of Huber et al. (2016) which is based on their previous neural network models of number comparison (Huber et al., 2014; Huber, Moeller, Nuerk, & Willmes, 2013; Huber, Moeller, Nuerk, Macizo, et al., 2013; Moeller et al., 2011) and on previous neural network models of Verguts and colleagues (Verguts et al., 2005; Verguts & Notebaert, 2008).

Figure 3

Schematic Illustration of the Neural Network Model Architecture



Note. Schematic illustration of the neural network model architecture for the simulation of the numerical Stroop task and the symbolic number comparison task. The model consists of two single-digit comparison networks, one for the numerical size and one for the physical size. The task demand units serve as an attentional bias to specify the relevant and irrelevant dimension of the task. Information is propagated to the decision nodes where a response is made. The conflict monitoring unit calculates the amount of conflict during the task and adjusts attention accordingly. ACC refers to the anterior cingulate cortex.

3.3.1 Single-digit Comparison Networks

The input layer consists of a single-digit comparison network each for numerical size and physical size. The numerical size network contains two number line fields that code the representation of numerical magnitude for the left and right Arabic digits to be compared that are presented to the model. Each number line field is implemented as in Santens and Verguts (2011) and is a vector of nine nodes. Each of the nine nodes represent one Arabic digit to create an ordered sequence of natural numbers, allowing for the comparison of sizes 1 to 9. A number line field represents numerical magnitude using a place-coding system with linear scaling and constant variability as in the model of exact small number representation by Verguts et al. (2005) (see also Huber et al., 2016; Santens & Verguts, 2011). The single-digit number comparison network for physical size is represented identically to that of numerical size. Nine physical sizes (a - i) are mapped onto each of the nine number line nodes respectively as in the model of Santens and Verguts. The equation for the activation of node j when input number i is presented to the model is as per Huber et al. (2016) (and is analogous to Verguts et al. and Santens and Verguts who used an exponent of -1 instead of -10) as follows:

$$f(i, j) = exp(-10 * |i - j|) \text{ where } 1 \leq i \leq 9;\ 1 \leq j \leq 9. \qquad (1)$$

The numerical magnitude representation reflects place-coding properties as each input number presented to the model activates the same number of units on the number line. Each number presented to the model demonstrates constant variability by maximally activating its corresponding number line node with adjacent nodes being activated with decreasing strength as they become further away. The model exhibits the properties of linear scaling as the exponent -10 * |$i - j$| relies on the distance between the number nodes and not on the actual value of the corresponding numbers i and j.

3.3.2 Propagation of Input Layer to Comparison Layer

The propagation of the input layer to the comparison layer is identical to the study of Huber et al. (2016). All nodes in the number line fields for a single-digit comparison network are propagated via feed forward connections to all nodes in the hidden comparison layer for that network. The comparison layer for each single-digit comparison network consists of a left and right node coding for “left larger” or “right larger”. Activity is propagated similar to Equation (1) of Moeller et al. (2011):

$$\overline{net_i}(t) = \tau\, net_i(t) + (1 - \tau)\overline{net_i}(t - 1). \tag{2}$$

where $\overline{net_i}(t)$ is a weighted sum of inputs across time t for node i, τ is a constant of value 0.01 reflecting the rate of activation, and $net_i(t)$ represents the activation of place-coding nodes multiplied by the connection weights between the input and comparison nodes. The net input activation is then transferred by a sigmoid function with a gain of value 2. Lateral inhibitory connections between the left and right comparison nodes with w^{inh} = -2 create competition between the nodes thereby strengthening the node with the largest amount of activation and weakening the node with the smallest activation. The activation $f_i(t)$ of comparison layer node i is calculated as follows:

$$f_i(t) = \frac{1}{1 + e^{-2(\overline{net_i}(t) + w^{inh} \sum_{j \neq i} \overline{net_j}(t))}}. \tag{3}$$

3.3.3 Training of Connection Weights Between Input and Comparison Layer

The connection weights between the input layer and comparison layer of the single-digit comparison networks were trained identically to the general model framework of Huber et al. (2016) with the exception that the numerical size and physical size comparison networks were trained independently of each other. Huber et al. (2016) only trained one single-digit number comparison network and reused the weights for the other single-digit comparison network to implement their decomposed model of number magnitude comparison that recycles the number line for the tens and units of a two-digit number (Verguts & de Moor, 2005). As the numerical size and physical size dimensions are independent of each other in the numerical Stroop task, training was done similar to Santens and Verguts (2011) where the numerical size and physical size comparison networks were trained separately and independently of each other.

The model was trained prior to running the simulations. Initial weights were random numbers generated from a uniform distribution in the interval U(-1,1). Training was performed using the delta rule (Widrow & Hoff, 1960) with a learning rate of 0.01. Each single-digit number comparison network was trained to compare all combinations of single-digit numbers from 1 to 9 with the exception of the numbers being equal. The frequency of each number presented to the model during training was taken from a Google survey which observes the frequency of numbers observed in daily life and allows simulation of the problem size effect (see also Verguts et al., 2005; Verguts & Fias, 2006). The model was trained for 100,000 trials to ensure all combinations were compared correctly. Huber et al. (2016) arbitrarily chose the number of training trials as 100,000. Tuning the learning parameters to increase performance was outside the scope of the authors' study whose objective was to create an abstract model to capture multi-symbol number comparison instead of creating a biologically plausible neural network model. Similarly, performance tuning of the model is outside the scope of the present study as the aim was to create a model that simulates cognitive mechanisms related to mathematics anxiety and not a biological plausible model.

3.3.4 Cognitive Control Network

The activation x_i^{in} of node *i* in the comparison layer at time *t* is calculated by equation (1) of Huber et al. (2016) with the exception that no noise was added to the calculation (as in Santens and Verguts (2011)) as follows:

$$x_i^{in}(t + 1) = (1 - \tau)\, x_i^{in}(t) + \tau\, (f_i(t) + \beta_{in}). \tag{4}$$

Huber et al. (2016) adapted the equation from equation (A1) of Verguts and Notebaert (2008) whereby the output of the single-digit comparison networks serves as input to the cognitive control network. The values of the constants τ = 0.25 and β_{in} = 0.2 are the same as in Huber et al. (2016) and $f_i(t)$ is the activation of the comparison nodes from the single-digit comparison network.

3.3.4.1 Task Demand Layer. The numerical size and physical size single-digit comparison networks have feed forward connections to the decision layer. Activation from the comparison layer to the decision layer is modulated by the task demand layer which comprises of two nodes, one node for numerical size and one node for physical size. The task demand layer serves as an attentional bias (Botvinick et al., 2001; Cohen et al., 1990) to specify the relevance of the numerical size dimension and irrelevance of the physical size dimension in the numerical Stroop task. The stronger the activation of the task demand nodes, the more relevant the dimension and greater the influence on the comparison process. The activation of the numerical size task demand node is set at 1.0 and the activation of the physical size task demand node is set at 0.15 allowing for more attention directed to the relevant numerical size dimension. These values are identical to the amount of attentional bias of the relevant and irrelevant dimensions respectively of the numerical Stroop model of Santens and Verguts (2011). However, Santens and Verguts' model did not include a task demand layer and instead multiplied the representation layer of the irrelevant dimension by a parameter theta of 0.15 to act as a proxy in which the size congruity effect is modulated (Schwarz & Ischebeck, 2003).

3.3.4.2 Decision Layer. The decision layer is implemented as in Huber et al. (2016). A left and a right node code for the decision “left larger” or “right larger” respectively. The nodes have lateral inhibitory connections between them with w^{inh} = -0.5 that cause response competition and reduce the amount of time taken for the model to make a decision. When the activation of one of the decision nodes reaches the prespecified threshold parameter theta, the model records the number of time steps *t* to reach that decision as the simulated response time. If the left node reaches the threshold value first, then the model has decided that the left input stimulus number has the largest numerical size while ignoring its physical size. If the right node reaches threshold first, then the right input stimulus number has the largest numerical size while ignoring its physical size. The value of the threshold parameter θ is 0.75. A maximum number of time steps *t* is set at 200 in case the activation threshold is not reached. Similar to the comparison layer, the decision layer calculates a weighted sum of activation over time with a constant value of τ = 0.25 as the rate of activation that impacts the amount of time it takes to reach the decision unit threshold.

The activation x_j^{res} of decision layer node *j* at time *t* is the same as equation (2) of Huber et al. (2016) with the exception that no noise is added to the formula. The equation is identical to equation (A2) of Verguts and Notebaert (2008) as follows:

$$x_j^{res}(t+1) = (1-\tau)\, x_j^{res}(t) + \tau \left\{ \sum_i w_i^{ir}\, x_i^{in}(t) \left[C + \sum_{k=1}^{n_{task}} w_{ki}^{ti} x_k^{td}(n_{trial}) \right] + w^{inh} \sum_{k \neq j} x_k^{res}(t) \right\}. \quad (5)$$

where w_i^{ir} are the bottom-up connection weights between the comparison and decision layers and x_i^{in} is the activation in the comparison layer for node *i*. The top-down attentional weighting of the task demand layer to the comparison layer is indicated by the term $\left[C + \sum_{k=1}^{n_{task}} w_{ki}^{ti} x_k^{td}(n_{trial}) \right]$, where w_{ki}^{ti} are the connection weights between the task demand layer for node *k* and the comparison layer for node *i*, $x_k^{td}(n_{trial})$ is the activation of task demand nodes for trial n_{trial}, n_{task}= 2 for the two nodes in the task demand layer one for each of the tasks of comparing numerical size and physical size, and *C* is a constant with value 0.7 that ensures irrelevant digits always contribute to the activation in the decision layer regardless of the attentional bias in the task demand layer (Huber et al., 2016; Verguts & Notebaert, 2008). The term $w^{inh} \sum_{k \neq j} x_k^{res}$ (*t*) represents lateral inhibition between the decision nodes.

3.3.4.3 Connection Weights between Comparison Layer and Decision Layer. As in Huber et al. (2016) the connection weights between the comparison layer and decision layer in the present model were fixed. The values of these weights reflect how automatic the processing route is where the larger the connection weight the more automatic and faster the task is. In the classical Stroop task where the font colour of the word is named while ignoring the meaning of the word, word processing is a more automatic and faster task than naming the font colour (Cohen et al., 1990). In the numerical Stroop task studies have shown that judging the physical size of the digit is a more automatic task than judging the numerical size of the digit and is therefore processed faster (Henik & Tzelgov, 1982; Szũcs et al., 2007; Szũcs & Soltész, 2007). Szũcs et al. (2007) investigated the speed of magnitude processing on numerical size comparison versus physical size comparison. Participants responded faster on a physical task than a numerical task. The ratio of response time from the numerical task to the physical task in the study was equal to 0.94 and this ratio was applied to the connection weights for the numerical and physical size dimensions to the decision layer in the present model. Additionally, the values of these weights also affect the size of the size congruity effect and the amount of errors in the model. As in Huber et al. (2016), the values of these weights were arbitrarily chosen to ensure the size congruity effect and error rate were similar to empirical studies. The connection weights w^{ir} between the comparison layer and decision layer for the numerical size dimension are 0.85 and for the physical size dimension are 0.9.

3.3.4.4 Conflict Monitoring Unit. As in the models of Huber, Moeller, Nuerk, Macizo, et al. (2013) and Verguts and Notebaert (2008) a conflict monitoring unit calculates the amount of conflict during a trial as the energy in the decision layer which is calculated by the product of the activation of the decision nodes (Botvinick et al., 2001). At the end of each trial, if the level of conflict on the current trial is high compared to previous trials, the conflict monitoring unit can adapt the weights between the task demand layer and the comparison layer via the conflict-modulated Hebbian learning rule as described in equations (A3) and (A4) of Verguts and Notebaert. The learning rule has the effect of strengthening attention to the relevant numerical size dimension and weakening attention to the irrelevant physical size dimension as needed. The current research does not study conflict adaptation effects, therefore the conflict monitoring unit does not adapt the weights in the simulations. However, subsequent research could investigate conflict adaptation effects as they relate to mathematics anxiety. The initial weights between task demand nodes and comparison layer nodes were set as 0.5 (Huber, Moeller, Nuerk, Macizo, et al., 2013). All other parameters for the conflict monitoring unit equations remained the same as in Huber, Moeller, Nuerk, Macizo, et al. (2013).

Chapter 4: Neural Network Model Simulations of the Numerical Stroop Task

This chapter describes the neural network model simulations of the numerical Stroop task. This task involves deciding which number is numerically larger when the stimuli are presented in different physical sizes. First, the LMA model is validated to ensure it simulates experimental results. Next, the HMA model is simulated by making parameter changes to the LMA model. The HMA model is compared to results of experimental studies to evaluate whether it is a suitable model of mathematics anxiety.

4.1 Validation of the Numerical Stroop Model

Before impairing the LMA model to create the HMA model, the LMA model was validated to ensure it can simulate various experimental effects. Below is a description of the simulations of the numerical Stroop task assessing the behaviour of the model on empirical effects for this task. In addition to simulating standard effects of numerical Stoop and reduced learning, the speed-accuracy trade-off and the physical Stroop task were simulated to demonstrate the model's behaviour in other cognitive conditions.

4.1.1 Simulation of the Numerical Stroop Task

4.1.1.1 Procedure. The numerical Stroop neural network model created by Santens and Verguts (2011) simulated experimental effects which are reproduced using simulations of the current LMA model. These include the size congruity effect (or interference effect) where it is faster to compare stimulus pairs that are congruent than when they are incongruent, and the numerical distance effect where it is faster to compare stimulus pairs when the distance between the numbers is further apart than when the distance between the numbers is closer together. In these simulations the relevant dimension was the numerical size and the irrelevant dimension was the physical size. The current data set was constructed in a similar way to Experiment 1 of Santens and Verguts (2011) where all combinations of four sizes of Arabic digits for both the numerical size and physical size create 12 x 12 = 144 stimulus pairs, excluding instances where the numerical size or the physical size are equal (e.g., numerical size 1 cannot be compared with numerical size 1 as trials are either congruent or incongruent and cannot be neutral). Congruent stimuli and incongruent stimuli were presented to the model in equal proportions. The four numerical sizes presented to the input layer of the current model were the Arabic digits 1, 2, 8, and 9 as used in the mathematics anxiety study of Suárez-Pellicioni et al. (2014) for which the current study's HMA models' results are compared. There were four physical sizes presented to the model a, b, h, and i. These were mapped onto the Arabic digits 1, 2, 8, and 9 for the purposes of the simulation (see Santens and Verguts for a similar approach which used Arabic digits 1, 2, 7, and 8 mapped onto physical sizes a, b, g, and h). Suárez-Pellicioni et al. used a reduced set of numerical sizes and physical sizes in their mathematics anxiety study and the data set for the current study was reduced to their data set for the HMA model simulations once the LMA model was validated. The trial-to-trial adaptation of the conflict monitoring unit was turned off. The activation of neurons was reset at the beginning of each trial. Therefore, all 144 trials presented to the model were independent. The model simulated 30 participants who were low math-anxious. Distance effects between the numerical sizes and between the physical sizes were modelled. The distance was classified as small (also termed *close*) when the distance between the numerical sizes or between the physical sizes was 1 or 6. The distance was classified as large (also termed *far*) when the distance between the numerical sizes or between the physical sizes was 7 or 8. This allowed an equal amount of observations at each level (see Santens and Verguts (2011) for a similar approach who classified small when the distance between numerical or physical sizes was 1 or 5, and large when the distance between them was 6 or 7).

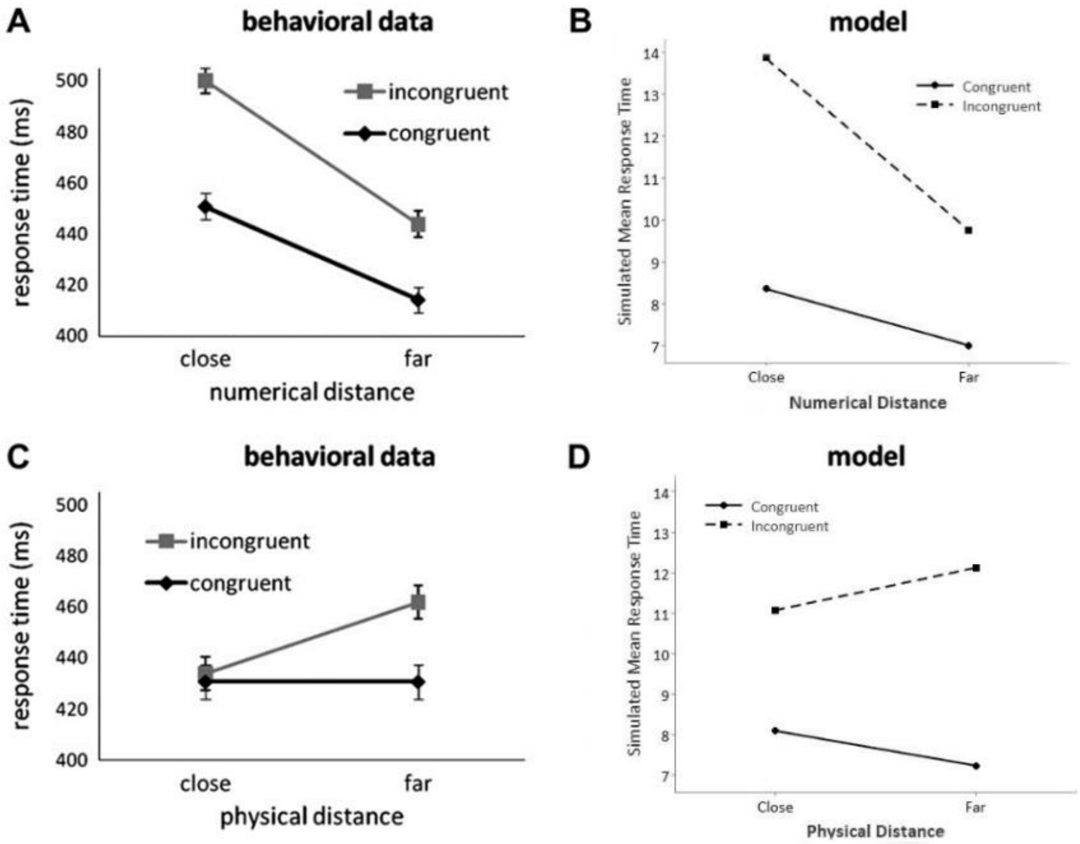
4.1.1.2 Results. The results for the LMA model were based on replicating the results from Santens and Verguts (2011) results in Experiment 1. The response time for each trial is the number of time steps the model takes to reach a decision. The mean response times for successful trials was calculated in each condition. The size congruity effect was calculated as the mean response time for incongruent trials minus congruent trials. The results of the LMA model simulations are described below and were compared to the response patterns of Santens and Verguts as shown in Figure 4.

1. The model was able to simulate the size congruity effect for response times where congruent trials were faster than incongruent trials (see Figure 4E and 4F).
2. The model was able to simulate the numerical distance effect for response times with faster decision times when the numbers were far apart than when they were close (see Figure 4E).
3. Importantly, the model simulated an interaction between the congruity effect and the numerical distance. The congruity effect was larger for a small numerical distance than for a large numerical distance (see Figure 4E).
4. The current LMA model did **not** produce a difference in mean response times between the physical distance being small or large (see Figure 4F). Whereas, response times were faster when the distance between the physical sizes was small than when it was large in the behavioural data from Santens and Verguts (see Figure 4B).
5. The model simulated an interaction between the congruity effect and the physical distance. The congruity effect was larger for a large physical distance than for a small physical distance (see Figure 4F).
6. Participants did not respond in time or made an error on 1.8% of trials in Santens and Verguts' behavioural study. In their simulations the model produced 1.5% errors. In both instances the errors were all made in the slower conditions (i.e., incongruent, small numerical distance, large physical distance). The current LMA model produced

5.3% errors that all occurred in the slower conditions where trials were incongruent and the distance between the numerical sizes was small.

Figure 4

Validation of the Numerical Stroop Model



Note. Results for the numerical Stroop task with numerical size as the relevant dimension and physical size as the irrelevant dimension. Panels A and C are mean response times of behavioural data from Santens and Verguts' (2011). Panels B and D are the current study's simulated LMA model mean response times. Panels A and B depict the numerical distance and panels C and D depict the physical distance. Error bars for behavioural data represent 95% confidence intervals. Panels A and C: From "The Size Congruity Effect: Is Bigger Always More?," by S. Santens and T. Verguts, 2011, *Cognition*, 118(1), p. 98 (https://doi.org/10.1016/j.cognition.2010.10.014).

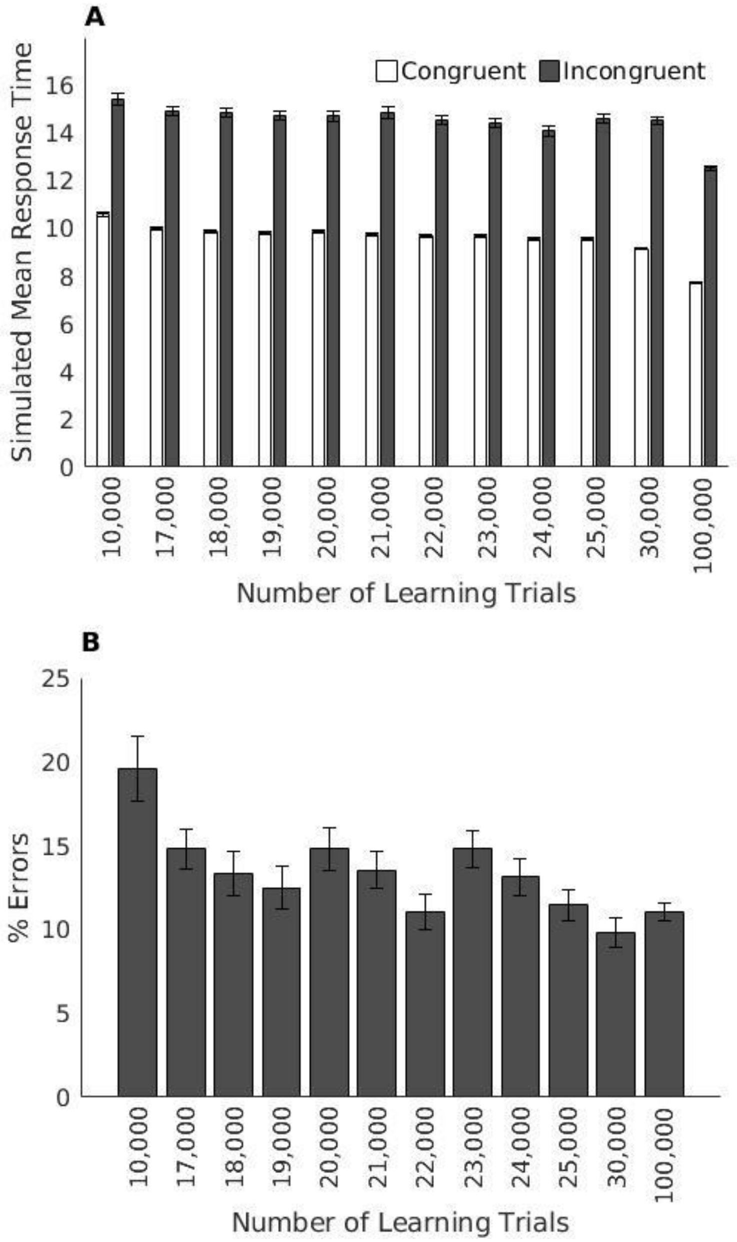
4.1.2 Simulation of Changes in the Amount of Learning

4.1.2.1 Procedure. This simulation demonstrates the effect of training the connections less between the numerical representation and the response on the numerical Stroop task. The numerical size and physical size single-digit comparison networks were trained with different values for the number of learning trials (see section Model Architecture for the Current Study in Chapter 3 for a description of the training). This created a different set of weights for each training run. Training was performed such that the connection weights were identical for equal values of the number of learning trials. For example, on each trial, numerical stimuli were presented to the model such that when the training of the model reached 10,000 learning trials, during the training of a total of 17,000 trials, the connection weights were identical to the end weights of the previous training when the model was trained for a total of 10,000 learning trials. After training, model simulations were then run for these different amounts of learning trials. Larger numbers of learning trials represent increased learning. The mean response times for successful trials was calculated in each condition.

4.1.2.2 Results. In previous simulations the number of learning trials was 100,000 as in the general model framework of multi-symbol number comparison of Huber et al. (2016) which resulted in 100% accuracy for the single-digit comparison networks. In the current simulations, the model predicts that as the amount of learning increases, response times will decrease and errors will decrease (see Figure 5). This result is consistent with empirical research showing that learning of basic numerical processing skills across a variety of different tasks improves response times and accuracy (e.g., Landerl, 2013).

Figure 5

The Effect of Changing the Amount of Learning in the Numerical Stroop Task



Note. Simulated models' results for different numbers of learning trials on performance in the numerical Stroop task. Panel A shows mean simulated response times. Panel B shows the percentage of errors. Error bars depict the standard error of the mean.

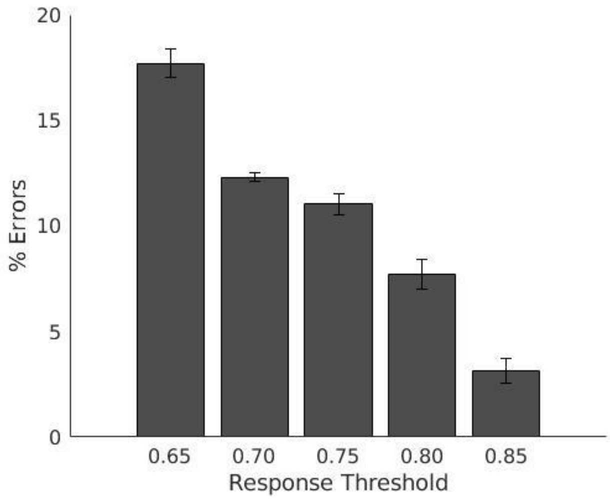
4.1.3 Simulation of the Speed-Accuracy Trade-Off

4.1.3.1 Procedure. This simulation demonstrates the speed-accuracy trade-off during performance of the numerical Stroop task. When one of the left or right response nodes in the numerical Stroop model reaches an activation threshold of 0.75 the model records the number of time steps as the simulated response time. To simulate the speed-accuracy trade-off, the value of the activation threshold parameter in the model was adjusted. Arbitrary values of 0.65, 0.70, 0.75, 0.80, and 0.85 were simulated. Reducing the activation threshold parameter has the effect of reducing the simulated response time, and conversely increasing the activation threshold parameter has the effect of increasing the simulated response time. The number of learning trials was 100,000.

4.1.3.2 Results. For each of the model simulations the percentage of errors was calculated. Figure 6 shows the speed-accuracy trade-off where accuracy increases as the simulated response threshold increases. Some studies have noted speed-accuracy trade-offs while solving calculation problems in individuals with mathematics anxiety, which may have occurred due to avoidance of the numerical stimuli (Ashcraft & Faust, 1994; Faust et al., 1996). The current study does not investigate the effect of mathematics anxiety on the speed-accuracy trade-off. However, future work could involve simulating this condition.

Figure 6

Model Simulations of the Speed-Accuracy Trade-Off During the Numerical Stroop Task



Note. Simulated models' results showing the percentage of errors (on the y-axis) for different values of the response activation threshold (on the x-axis). Error bars depict the standard error of the mean.

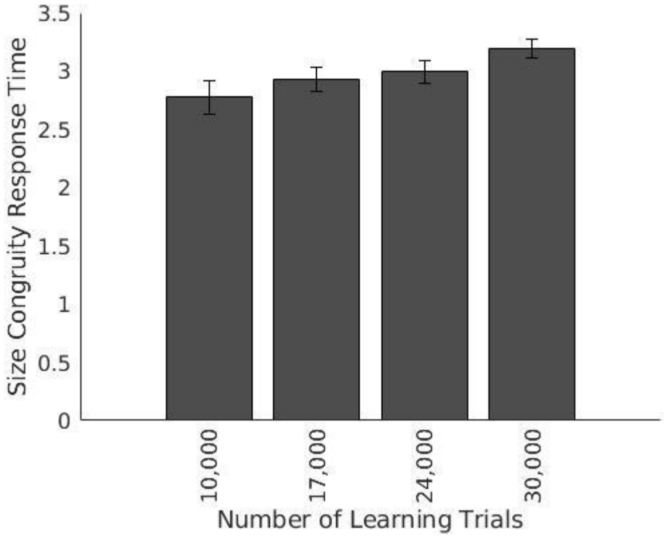
4.1.4 Simulation of the Physical Stroop Task

4.1.4.1 Procedure. The physical Stroop task involves deciding which number has the largest physical size while ignoring the numerical value. The relevant and irrelevant dimensions are reversed from the numerical Stroop task. The physical size is the relevant dimension in this task. The numerical size is the irrelevant dimension. The conditions for the simulations of the physical Stroop task were the same as used to validate the LMA model of the numerical Stroop task but the relevant dimension in the model was physical size and the irrelevant dimension was numerical size. Several simulations were performed to demonstrate the change in the size congruity effect (i.e., the difference between mean response times for incongruent minus congruent trials) for different amounts of training (which were chosen arbitrarily). The size congruity effect was calculated for response times of successful trials in each of the simulations for the different amounts of learning. The congruity by numerical distance was examined for one of the simulations where the number of learning trials was 17,000 (chosen arbitrarily). For this simulation, the simulated mean response time was calculated in the congruent and incongruent conditions by the irrelevant numerical distance.

4.1.4.2 Results. The results for the simulation of the size congruency effect on response times in the physical Stroop task are shown in Figure 7. The results of the model suggest that the size congruity effect increases as the training trials increase. These results are consistent with empirical results where the size congruity effect increases as children learn the meaning of symbols (e.g., Landerl & Kölle, 2009). The results for the simulation as a function of the differences in numerical size are shown in Figure 8. In the congruent condition it is faster to decide which number has the largest physical size when the distance between the numerical sizes is large than when the distance between them is small. In the incongruent condition it is faster to decide which number has the largest physical size when the distance between the numerical sizes is small than when the distance between them is large. Furthermore, the size congruity effect is larger when the distance between the numerical sizes is large than when the distance between them is small. These results are consistent with empirical studies examining the speed of processing in the physical Stroop task (e.g., Henik & Tzelgov, 1982; Landerl & Kölle, 2009).

Figure 7

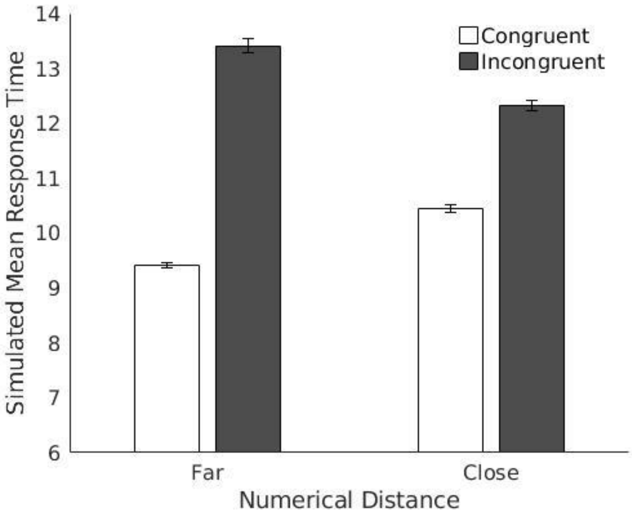
Model Simulations of the Physical Stroop Task Across Learning



Note. The size congruity effect for different numbers of learning trials in the physical Stroop task. The y-axis depicts the size congruity effect which is the difference in the simulated mean response times for incongruent trials minus congruent trials. Error bars depict the standard error of the mean.

Figure 8

Congruity by Numerical Distance for the Physical Stroop Task



Note. Simulation of the physical Stroop task. The physical size is the relevant dimension. The numerical size is the irrelevant dimension. The mean simulated response time is shown for the congruent and incongruent conditions when the distance between the numerical sizes is far and close. Error bars depict the standard error of the mean.

4.2 Mathematics Anxiety Experimental Results for the Numerical Stroop Task

The results from the model simulations for the current study aim to reproduce the pattern of results from the mathematics anxiety study by Suárez-Pellicioni et al. (2014). These authors tested LMA and HMA participants performing the numerical Stroop task. Their data set consisted of all combinations of the Arabic digit pairs 1-2, 1-8, 2-9, and 8-9 as numerical stimuli. Their physical stimuli consisted of two sizes, font size 40 as small and font size 80 as large. This produced 16 unique stimulus combinations. Congruent and incongruent trials were presented to participants in equal proportions. They calculated median response times for successful trials and the percentage of hits for both congruent and incongruent trials for LMA and HMA participants. Subsequently, they calculated an overall mean of medians. An interference score was calculated for response times, incongruent minus congruent response times (i.e., the size congruity effect), and for accuracy, congruent minus incongruent percentage of hits. For both measures the larger the score the larger the degree of interference. Post-error trials were removed from the analysis of reaction times as they can have slower response times. Suárez-Pellicioni et al. did not report results for congruent and incongruent trials separately. However, results were acquired by contacting them. Their results are compared with the current study's model simulation results to determine whether the simulation captures the important differences between LMA and HMA groups. A summary of the central characteristics of the behavioural data are as follows:

1. There were no significant differences in response times between the LMA and HMA groups for congruent trials.
2. The HMA group had significantly longer response times than the LMA group for incongruent trials.
3. The interference effect for responses times was significantly larger for the HMA group than the LMA group.
4. There were no significant differences in the interference score for accuracy between the LMA and HMA groups. Furthermore, the percentage of hits for the LMA group was 21.62%, and for the HMA group was 20%.

4.3 Procedure for the Simulation of HMA Models

For the HMA model simulations the data set was reduced to the stimulus pairs used by Suárez-Pellicioni et al. (2014). All combinations of the Arabic digit pairs 1-2, 1-8, 2-9, and 8-9 as numerical sizes with two physical sizes (small and large) were presented to the model producing 16 unique stimulus combinations. Physical sizes consisted of small font size 40 presented to the model as number 2, and large font size 80 presented to the model as number 8. The distance between the physical sizes was the same for all stimuli as there were only two sizes. The physical sizes presented to the model were chosen arbitrarily. The number of congruent and incongruent trials were presented in equal proportions. For these simulations the trial-to-trial adaptation of the conflict monitoring unit was turned off. The activation of neurons was reset at the beginning of each trial. Therefore, all 16 trials presented to the model were independent. Stimulus pairs were randomly presented to the model, however for these simulations randomising the data had no effect on the results as trials were independent and there is no noise in the model. The response time for each simulated participant is the number of time steps taken for the model to reach the specified threshold and decide which input stimuli has the largest numerical size.

The single-digit comparison networks for numerical size and physical size were trained separately for 30 simulated participants, so that each simulated participant had a different set of weights. Those weights were subsequently used for each participant for both the LMA and HMA models. For example, the weights for simulated LMA participant 1 were the same as for simulated HMA participant 1. The weights for simulated LMA participant 2 were the same for simulated HMA participant 2, and so on. Keeping the connection weights the same for the LMA and HMA models facilitated a comparison of results. The mean of response times for successful trials and percentage of hits were calculated for the congruent and incongruent conditions for the LMA and the HMA models. Similar to the study of Suárez-Pellicioni et al. (2014), the single score index of interference was then calculated from these means by taking incongruent response times minus congruent response times and congruent percentage of hits minus incongruent percentage of hits. The model does not account for post-error trials, they have not been excluded in the analysis for any of the study’s simulations.

4.4 Simulation of the HMA Model with Impaired Attention

4.4.1 Procedure

These simulations test the effect of reduced attentional control during performance of the numerical Stroop task. Model simulations of reduced attention to the numerical size dimension, reduced attention to the physical size dimension, and reduced attention to both the numerical and physical size dimensions were performed. To simulate reduced attentional control, the attention module of the model was impaired by reducing the activation of the neurons in the task demand units for the numerical size or physical size as appropriate.

4.4.2 Results

Figure 9 shows the experimental and simulated findings on the numerical Stroop task. Results from the model simulations of reduced attentional control were compared to the experimental findings of Suárez-Pellicioni et al. (2014).

4.4.2.1 Reduced Attention to Numerical Size. The HMA model was simulated with attention reduced to the numerical size dimension and not reduced to the physical size dimension. The results for reducing attention to 95% on the numerical size dimension for the HMA model are reported as this level of impairment was generally consistent with the experimental findings. Percentages close to 95% were also consistent with the above-mentioned experimental findings. The more attention was reduced, the longer the response time. The HMA model produced longer response times than the LMA model in the incongruent condition, which had the effect of increasing the interference effect for response times compared to the LMA model (Figure 9B). The HMA model did not differ from the LMA model on response times for congruent trials or on the size of the interference effect for accuracy. The interference effect for the percentage of hits was similar to the experimental data with the LMA model having a value of 22.08% (experimental was 21.62%) and the HMA model was 20.83% (experimental was 20%). Further analysis of the comparison of the

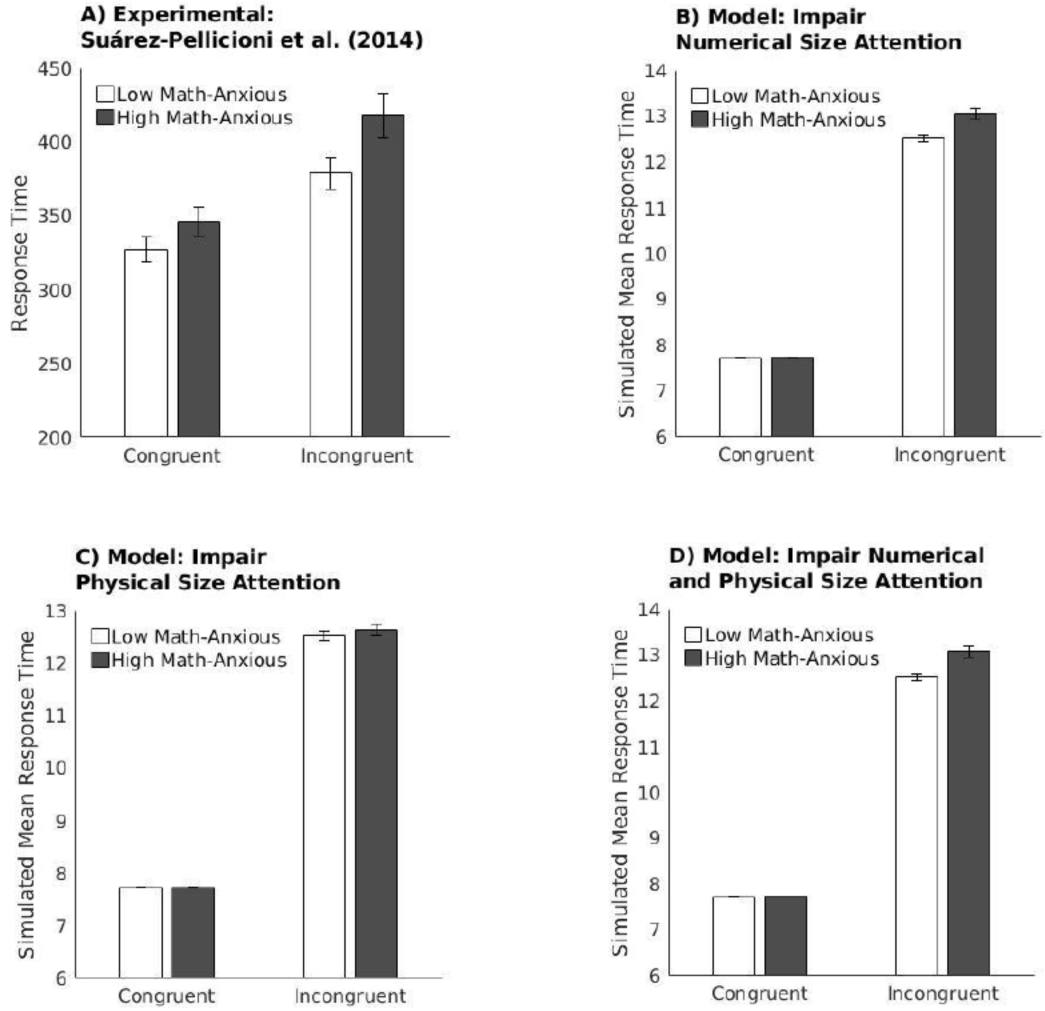
LMA model and the HMA model for response times found that all congruent trials (with the exception of one trial where the distance between the numerical sizes was small) were equal. Furthermore, all incongruent trials that differed between the LMA model and the HMA model were by a simulated response time of 1. The model with reduced attentional control to the numerical size dimension produced results qualitatively consistent with the above-mentioned experimental findings on mathematics anxiety.

4.4.2.2 Reduced Attention to Physical Size. The HMA model was simulated with attention reduced to the physical size dimension and not reduced to the numerical size dimension. The results for reducing attention to 95% on the physical size dimension for the HMA model have been reported. Results were similar for other values of reduced attention. For response times, the HMA model did not produce a difference to the LMA model in the congruent or incongruent conditions, or for the interference effect (Figure 9C). The model with reduced attention to the physical size dimension did not produce results qualitatively consistent with the above-mentioned experimental findings on mathematics anxiety.

4.4.2.3 Reduced Attention to Numerical Size and Physical Size. The HMA model was simulated with attention reduced to both the numerical and physical size dimensions. The results for reducing attention to 95% on both numerical and physical size dimensions for the HMA model are reported. Percentages close to 95% were also consistent with the above-mentioned experimental findings. The more attention was reduced, the larger the response time. The HMA model produced longer response times than the LMA model in the incongruent condition and for the interference effect (Figure 9D). The HMA model did not differ from the LMA model on response times for congruent trials or on the size of the interference effect for accuracy. The model with reduced attention to the numerical and physical size dimensions produced results qualitatively consistent with the above-mentioned experimental findings on mathematics anxiety.

Figure 9

Model Simulations of the Numerical Stroop Task With Attention Impaired

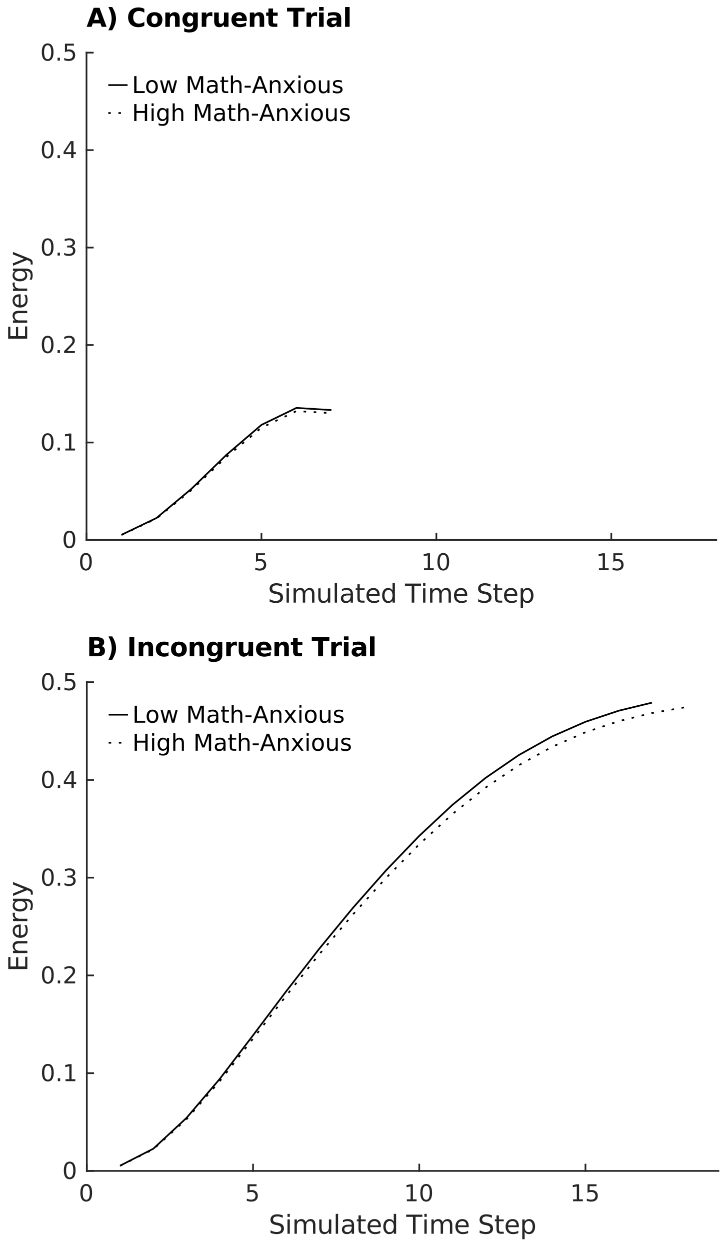


Note. Mean response times for the numerical Stroop task in the congruent and incongruent conditions for high and low mathematics anxiety are shown for experimental and modelling results. Panel A: Experimental results from Suárez-Pellicioni et al. (2014). The y-axis shows the mean of the median of response times. Panels B, C and D: show the results of the model simulations where the high math-anxious model has attention impaired. Panel B: Attention is impaired to the numerical size. Panel C: Attention is impaired to the physical size. Panel D: Attention is impaired to both the numerical and physical sizes. The y-axis shows the simulated mean response time. Error bars depict the standard error of the mean.

4.4.2.4 The Amount of Energy (Conflict) in the Response Layer. To examine the amount of energy in the response layer when attention is impaired, the model with reduced attention to the numerical size dimension that was consistent with studies of mathematics anxiety was examined further. One simulated LMA and corresponding HMA participant was examined on a congruent and an incongruent trial. A congruent trial with stimulus small 1 and large 8 where the LMA and HMA models produced the same simulated response time was chosen and graphed (see Figure 10A). An incongruent trial with stimulus large 1 and small 2 where the HMA model produced a larger simulated response time (by one time step) than the LMA model was chosen and graphed (see Figure 10B). The amount of conflict, which is defined as the amount of energy in the response layer, is graphed at each time step. The amount of energy in the response layer was calculated as the product of the activation of the decision nodes (see Botvinick et al., 2001 for a similar approach). The modelling shows that conflict is higher on incongruent trials than on congruent trials (as in previous studies). The HMA model with reduced attention to the numerical size dimension experienced less conflict than the LMA model.

Figure 10

The Amount of Energy in the Response Layer During the Numerical Stroop Task



Note. An example of the simulated conflict, which is defined as energy in the response layer (shown on the y-axis), at each simulated time step (shown on the x-axis) across the course of a trial for one simulated participant. Each panel shows the results for the low math-anxious model and the high math-anxious model with attention impaired to the (relevant) numerical size dimension. Panel A shows a congruent trial with stimulus small 1 and large 8. Panel B shows an incongruent trial with stimulus large 1 and small 2.

4.5 Simulation of the HMA Model with Reduced Learning

4.5.1 Procedure

Dietrich, Huber, Moeller, et al. (2015) examined the effects of mathematics anxiety on the symbolic number comparison task and suggested that individuals with mathematics anxiety may have less trained connections between the numerical representations and the response. These simulations test the effect of reduced learning with and without reduced attentional control to the numerical size dimension during performance of the numerical Stroop task. To examine the effect of reduced learning, an LMA model and an HMA model were chosen initially with a specific amount of learning trials such that the size of the interference effect for the percentage of hits for these models was close to the results of experimental studies and was consistent with experimental findings where there were no differences between them. A Mann-Whitney equivalent test was performed on the interference effect for the percentage of hits in the LMA and HMA models to confirm there were no significant differences as the data was not normally distributed. The LMA model was chosen with 20,000 learning trials and an interference effect for the percentage of hits of 23.75% (experimental was 21.62%). The HMA model was chosen with 18,000 learning trials and an interference effect for the percentage of hits of 21.67% (experimental was 20%). These values were chosen so that there were no differences in the percentage of hits between the LMA and HMA models, yet the number of learning trials were far enough apart for there to be a difference between the models in response times. Subsequently, the chosen LMA and HMA models were simulated with and without reduced attention. To simulate reduced attention, the numerical size dimension of the HMA model was reduced to 95%. This impairment was chosen because it produced a model of mathematics anxiety in previous simulations.

4.5.2 Results

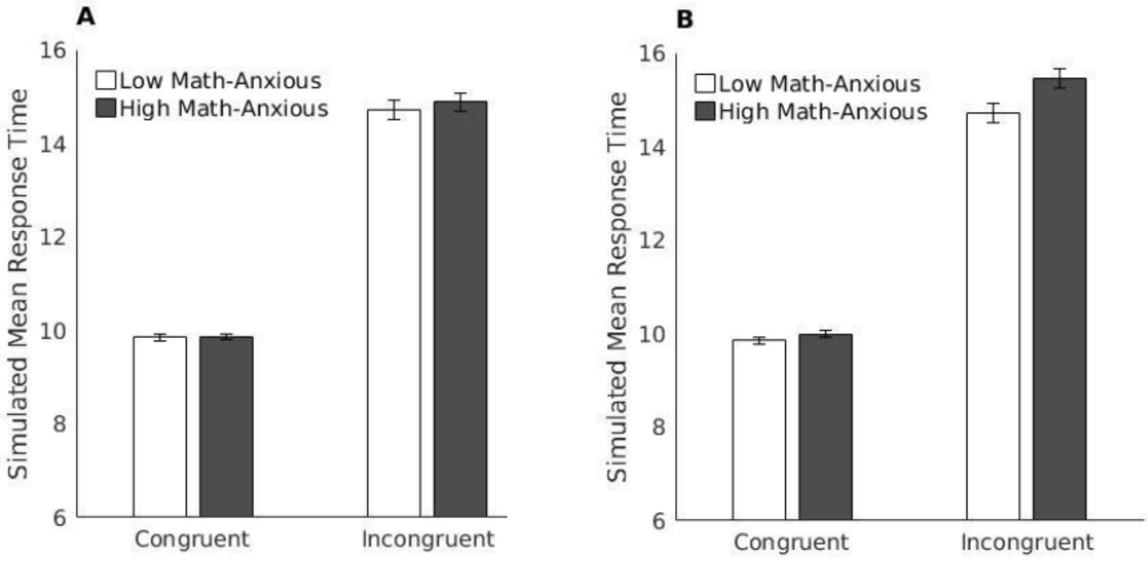
Figure 11 shows the simulated findings of the numerical Stroop task. Results from the model simulations of reduced learning with and without reduced attentional control to the numerical size were compared to the experimental findings of Suárez-Pellicioni et al. (2014) (see Figure 9A for their experimental results).

4.5.2.1 Reduced Learning Without Attention Impaired. The results of the model simulation of reduced training trials on the response times for congruent and incongruent trials when there was no impairment in attention are presented. From Figure 11B you can see that there are no differences in the response times to congruent and incongruent trials as a result of the reduction in the number of training trials in the HMA model results. The model with reduced learning did not produce results qualitatively consistent with the above-mentioned experimental findings on mathematics anxiety.

4.5.2.2 Reduced Learning with Reduced Attention to Numerical Size. The HMA model was simulated with reduced learning trials and attention reduced to the numerical size dimension. The HMA model produced longer response times than the LMA model in the incongruent condition and for the interference effect (Figure 11C). The HMA model did not differ from the LMA model on response times for congruent trials or on the size of the interference effect for accuracy. The model with reduced learning and reduced attention produced results qualitatively consistent with the above-mentioned experimental findings on mathematics anxiety.

Figure 11

Model Simulations of the Numerical Stroop Task With Reduced Learning



Note. Mean simulated response times in the congruent and incongruent conditions for the low and high math-anxious models. Panel A: The low math-anxious model has 20,000 learning trials. The high math-anxious model has 18,000 learning trials. Panel B: The low math-anxious model has 20,000 learning trials. The high math-anxious model has 18,000 learning trials and attention reduced to the numerical size dimension. The y-axis shows the simulated mean response time. Error bars depict the standard error of the mean.

4.6 Discussion

4.6.1 The Simulation Process

As described previously, the aim of the numerical Stroop simulations was to investigate the effect of reductions in attentional control and inhibition on task performance to compare against experimental data from groups with mathematics anxiety. First, a neural network model of the numerical Stroop task was created to simulate the LMA condition. It was based on previous neural network models of the numerical Stroop task (Santens &

Verguts, 2011) and multi-symbol number comparison (Huber et al., 2016). The LMA model was validated to ensure it replicated various empirical effects. Importantly, the model simulated effects of the numerical Stroop task including reduced learning. It also simulated the speed accuracy trade-off and the physical Stroop task demonstrating the model's behaviour across a variety of tasks. The results of the LMA model simulation of the numerical Stroop task were compared to the behavioural results of Santens and Verguts (2011). This included simulating the size congruity effect where incongruent trials had longer response times than congruent trials and the numerical distance effect where comparison of numbers that are further apart was faster than comparison of numbers that are closer together. The model also simulated changes in learning where increased learning improves response times and accuracy.

Next, the HMA model simulations were performed. Several parameter modifications were made to the LMA model to simulate different cognitive conditions and impairments. The results of these HMA models were qualitatively compared to the experimental results of the mathematics anxiety study by Suárez-Pellicioni et al. (2014) to decide whether they were a suitable model of mathematics anxiety. Suárez-Pellicioni et al. found that individuals with mathematics anxiety had longer response times for the interference effect (incongruent minus congruent) than individuals without mathematics anxiety. Furthermore, they found that high math-anxious individuals had longer response times in the incongruent condition than low math-anxious individuals, and there was no difference in response times between the groups for congruent trials or for error rates.

4.6.2 Reduced Attentional Control

In the first series of simulations the effect of impairments of attention were investigated. Results of the simulations suggested that when attention was impaired (i.e., reduced) to the numerical stimuli (either by impairing attention to the numerical size dimension only, or by impairing attention to both the numerical size and physical size dimensions together), that the models' results qualitatively match those of experimental studies on mathematics anxiety. However, when attention was impaired to the non-numerical and irrelevant dimension (by impairing attention to the physical size dimension in the model only), the results did not match those of experimental studies on mathematics anxiety. These results are consistent with previous studies that suggest that mathematics anxiety is associated with reduced attentional control and an attentional disengagement to numerical stimuli (e.g., Ashkenazi, 2018; Hartwright et al., 2018; Liu et al., 2019; Pizzie & Kraemer, 2017). This response is specific to numerical stimuli, as it may be perceived as threatening information. Consequently, this response to numerical stimuli decreases attention to the relevant numerical dimension which increases attention to the irrelevant physical dimension. Therefore, the model predicts that mathematics anxiety reduces attention and triggers an attentional disengagement that is specific to numerical stimuli. The HMA model consistent with these findings (with attention reduced to the numerical size dimension only) simulated longer response times overall, in the incongruent condition, and for the interference effect than simulated by the LMA model. However, there were no differences in the error rates between these LMA and HMA models. These results are consistent with previous studies where individuals high in mathematics anxiety experience more interference than individuals low in mathematics anxiety (Hopko et al., 1998, 2002; Suárez-Pellicioni et al., 2014). Furthermore, they are consistent with the attentional control theory (Derakshan & Eysenck, 2009; Eysenck et al., 2007) suggesting anxiety reduces the attentional resources of working memory. This results in an increased influence of the bottom-up stimulus-driven attentional system and a decreased influence of the top-down goal-directed attentional system. Consequently, mathematics anxiety is associated with a reduced ability to inhibit distracting or irrelevant information (such as to the physical size dimension) during the numerical Stroop task.

Moreover, the HMA model supports the view by attentional control theory that anxiety impairs processing efficiency (i.e., response times) to a greater extent than it impairs performance effectiveness (i.e., accuracy). The HMA model exerted more effort as shown by longer response times than the LMA model to achieve a similar quality of response accuracy where there were no differences between the error rates. Therefore, the model predicts that reduced attention due to the presence of mathematics anxiety results in longer response times in the incongruent condition and for the interference effect. When attention was reduced further to the numerical size dimension in the HMA model, response times for the interference effect increased. Therefore, the model further predicts that the more attention is reduced due to an increased level of mathematics anxiety, the larger the interference for response times. Concluding, these results support the disruption account that mathematics anxiety disrupts attentional control resulting in poor performance on numerical tasks.

4.6.3 Conflict Processing

The HMA model which involved reducing attention to the numerical size dimension (and not the physical size dimension) was examined further in relation to conflict processing. The amount of conflict experienced during a trial was compared for the LMA and HMA models. Conflict was defined as the amount of energy in the response layer. Consistent with previous studies (Botvinick et al., 2001), for both models incongruent trials experienced more conflict than congruent trials. This is because there is minimal competition during a congruent trial as both the numerical and physical size comparisons activate the same response nodes. However, during an incongruent trial the numerical and physical size comparisons activate competing response nodes, thereby resulting in conflict (that needs to be overcome). Comparison of the LMA and the HMA models showed that in a congruent trial both models activated a similar amount of conflict. However, during an incongruent trial where the HMA model had a longer response time than the LMA model, the HMA model experienced less conflict. Reducing attention to the numerical stimuli for the HMA model produced a weaker activation of the response units which produced less of an opportunity for the existence of conflict. These results suggest that the ability to recognise conflict may be beneficial. Recognising conflict would allow a potential adapting of cognitive control to improve performance. However, as the current neural network model architecture did not have the conflict adaptation module turned on, the simulations did not investigate the effects of conflict adaptation. Therefore, it is unclear as to the potential benefit of recognising the existence of conflict by the LMA model. Future work can examine conflict adaptation effects due to reduced attentional control from mathematics anxiety. In conclusion, the models predict that reduced attention to the numerical stimulus dimension due to mathematics anxiety reduces the amount of conflict (i.e., energy in the response layer) which may affect the ability to be able to adapt to the presence of conflict during processing. Bishop (2009) found that individuals with high trait anxiety showed less prefrontal cortex activation and slower responses when processing competition than individuals with low trait anxiety in a response-conflict task that required attentional resources. The findings suggested a dysregulation of the recruitment of prefrontal mechanisms required to adjust attentional control when conflict is experienced. Klados et al. (2015) used ERP to investigate neural activity in individuals with mathematics anxiety during working memory and arithmetic tasks. They found that individuals with higher levels of self-reported mathematics anxiety showed lower cortical activation at frontocentral and centroparietal locations during the early stages of cognitive processing during simple arithmetic tasks. The results were independent of state and trait anxiety levels.

4.6.4 Reduced Learning

The effect of reduced learning on the numerical Stroop task was investigated by changing the parameter for the number of learning trials in the model. These simulations were motivated by the suggestion of previous authors (Dietrich, Huber, Moeller, et al., 2015) that individuals with mathematics anxiety may have less trained connections between the numerical representations and the response. An LMA model and an HMA model with reduced learning were chosen with error rates similar to those of experimental studies. Subsequently, the LMA model and the HMA model with and without reduced attention were simulated and compared. Only the HMA model with reduced learning and reduced attention were consistent with empirical findings of mathematics anxiety. This model produced longer response times than the LMA model for the interference effect and in the incongruent condition. Therefore, this model supports the disruption account where if individuals with mathematics anxiety have less trained connections, mathematics anxiety is characterised by a disruption to attentional control that leads to poor performance on numerical tasks. Carey et al. (2016) describes that mathematics anxiety can impact learning due to avoidance and subsequently impact processing and recall because the anxiety disrupts working memory resources (the disruption account). The model simulations did not investigate the effect of anxiety during learning. This was outside the scope of the current study. Future work could involve impairing attention to the models during training.

Chapter 5: Neural Network Model Simulations of the Symbolic Number Comparison Task

This chapter describes the neural network model simulations of the symbolic number comparison task. This task involves deciding which number is numerically larger when the stimuli are presented in the same physical sizes. First, the LMA model is validated to ensure it simulates experimental results. Next, the HMA model is simulated by making parameter changes to the LMA model to assess the effects of these changes and to compare with the results from a group of participants with mathematics anxiety.

5.1 Validation of the Symbolic Number Comparison Model

Before impairing the LMA model to create the HMA model, the LMA model was validated to ensure it can simulate various experimental effects. Below is a description of the simulations of the symbolic number comparison task assessing the behaviour of the model on the distance effect, the size effect, and reduced learning for this task.

5.1.1 Simulation of the Symbolic Number Comparison Task

5.1.1.1 Procedure. To simulate the symbolic number comparison task, the neural network model that simulated the numerical Stroop task was adapted such that the single-digit comparison network for physical size (used for the irrelevant dimension of the numerical Stroop task) was turned off and the response is generated by comparing numerical sizes. The model simulations use the data set from Dietrich, Huber, Moeller, et al. (2015) which consists of all combinations of single-digit numbers from 1 to 9 resulting in 72 pairs. The model simulated 30 participants who were low math-anxious. The trial-to-trial adaptation of the conflict monitoring unit was turned off. The activation of neurons was reset at the beginning of each trial. Stimulus pairs were randomly presented to the model, however for these simulations randomising the data has no effect on the results as trials are independent and there is no noise in the model. Means for response times on correct trials and error rates were calculated for each simulated participant.

5.1.1.2 Results. The simulations that validated the numerical Stroop model in Chapter 4 included a validation of the symbolic number comparison task, a symbolic number comparison is an inherent requirement of the numerical Stroop task. Consequently, no additional validation of the comparison process of symbolic numbers was required. The model simulations successfully simulated the numerical distance effect and the size effect.

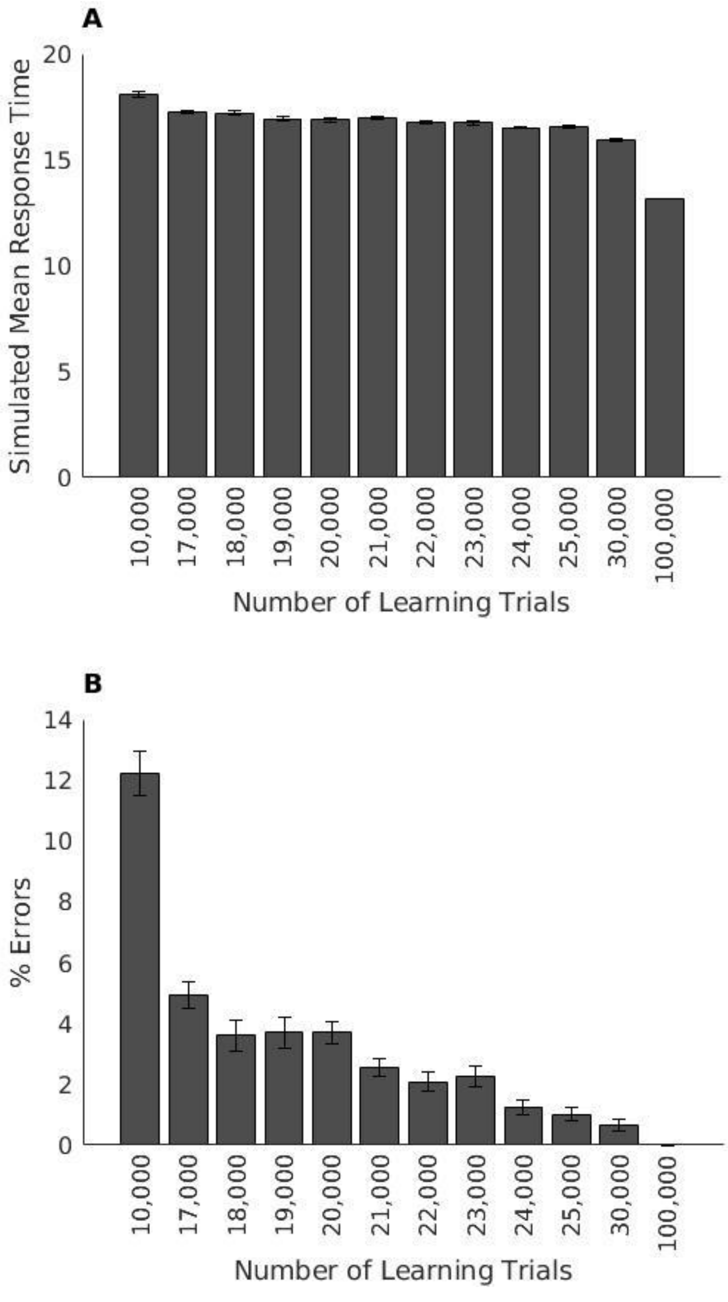
5.1.2 Simulation of Changes in the Amount of Learning

5.1.2.1 Procedure. The following simulation demonstrates the effect that a reduction in the training of the connections between the numerical representation and the response has on response times, accuracy, and the distance effect. The previous chapter investigated the effect of a reduction in the training of connection weights between the numerical representations and the response on simulations of the numerical Stroop task. In those simulations the single-digit comparison networks were trained with different values for the number of learning trials, which generated a different set of weights for each training run. The following simulations use these same weights. Model simulations were run for different amounts of training and no other impairments were made to the model (i.e., all models retained 100% attention). The procedure for training the single-digit comparison networks has been described previously in Chapter 3. Mean response times on correct trials and error rates were calculated for each condition.

5.1.2.2 Results. The results of the simulations on response times, accuracy, and the numerical distance effect were graphed for arbitrary values of the amount of training. In previous simulations when the number of learning trials was 100,000, the models produced 100% accuracy. In the current simulation, the model predicts that as the amount of learning increases, response times will decrease and errors will decrease (see Figure 12). This result is consistent with studies that show learning of basic numerical processing skills across a variety of different tasks improves response times and accuracy (e.g., Landerl, 2013). Figure 13 shows the results for the numerical distance effect where overall response times decrease for each numerical distance as learning increases. This result is consistent with studies showing changes in the numerical distance effect for symbolic number comparison during development (e.g., Landerl & Kölle, 2009).

Figure 12

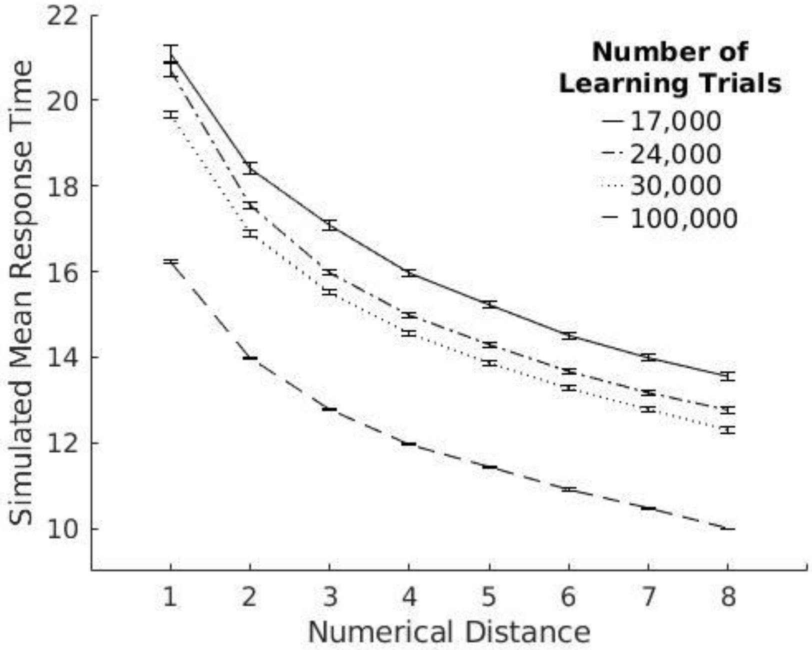
The Effect of Changing the Amount of Learning in the Symbolic Number Comparison Task



Note. Simulated models' results for different numbers of training trials on performance in the symbolic number comparison task. Panel A shows mean simulated response times. Panel B shows the percentage of errors. Error bars depict the standard error of the mean.

Figure 13

The Numerical Distance Effect Across Learning in the Symbolic Number Comparison Task



Note. The numerical distance effect is shown for different values of learning trials. The x-axis depicts the distance between the numerical stimuli.

5.2 Mathematics Anxiety Experimental Results for the Symbolic Number Comparison Task

The symbolic number comparison task has been studied within the mathematics anxiety literature with mixed results. Some studies have found differences between individuals high and low in mathematics anxiety in overall response times, for distance and size effects, and some have not. Most studies have found that there are no differences in error rates between individuals high and low in mathematics anxiety.

5.3 Procedure for the Simulation of HMA Models

The model simulations use the same data set and conditions that were used to validate the symbolic number comparison model in the previous section. There were 30 simulated participants for both the LMA and the HMA models. As in the numerical Stroop model, the connection weights between the input layer and the comparison layer of the single-digit comparison network for numerical size were different for each simulated participant within each LMA or HMA model, and were the same for each matched LMA and HMA participant. Keeping the connection weights the same for the LMA and HMA models facilitated a comparison of the results of the LMA and HMA models. Several model simulations were performed investigating the difference between the LMA and HMA models for changes in parameter values that simulated the impairments. Means for response times on correct trials and error rates were calculated for each simulated participant in each condition.

5.4 Simulation of the HMA Model with Impaired Attention

5.4.1 Procedure

This simulation investigates the effect of reduced attention during performance of the symbolic number comparison task. The numerical size single-digit comparison networks were trained to 100% accuracy in the initial simulations of the numerical Stroop task. The errors in those simulations resulted from the conflict between the relevant and irrelevant dimensions. The first simulation here retains these same connection weights between the input layer and the comparison layer resulting in 100% accuracy for the single-digit comparison task. The results from the numerical Stroop model simulations proposed that impairing attention to the numerical size dimension qualitatively replicated results from experimental studies on mathematics anxiety. Therefore, for this simulation attention was impaired to the HMA model on the numerical size dimension. This was achieved by reducing the activation of the neurons in the task demand units for the numerical size to 95% as in the numerical Stroop model simulations.

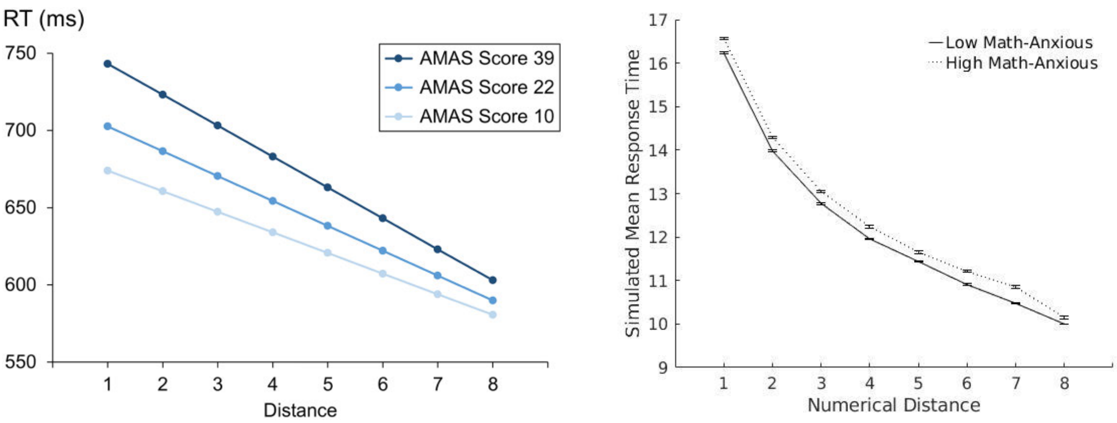
5.4.2 Results

Figure 14 shows the results for the mathematics anxiety experimental data of Dietrich, Huber, Moeller, et al. (2015) and the model simulations. These authors did not find overall differences in response times between the LMA and HMA groups. However, they found a more pronounced distance effect in response times for the HMA group than the LMA group. The simulation results show that the HMA model produced longer response times than the LMA model. The more attention was reduced, the longer the response time. The LMA model and the HMA model produced reliable numerical distance effects where it is faster to compare two numbers and decide which is the largest when the distance between the numbers is large than when the distance between the numbers is small. The size of the distance effect for the HMA model was similar to the distance effect in the LMA model. As the single-digit comparison network was trained to 100% accuracy, there were no differences in error rates between the LMA and the HMA model.

Figure 14

Model Simulations of the Symbolic Number Comparison Task With Attention Impaired

A **B**



Note. Response times for the distance effect in symbolic number comparison, as a function of mathematics anxiety, are shown for experimental and modelling results. Panel A: Experimental results of the estimated distance effects for participants with low mathematics anxiety (Abbreviated Math Anxiety Scale (AMAS) score = 10), middle mathematics anxiety (AMAS score = 22), and high mathematics anxiety (AMAS score = 39). Reprinted from "The Influence of Math Anxiety on Symbolic and Non-Symbolic Magnitude Processing" by J. F. Dietrich, S. Huber, K. Moeller, and E. Klein, 2015, Frontiers in Psychology, 6(1621), p. 6 (https://doi.org/10.3389/fpsyg.2015.01621). CC BY 4.0. Panel B: Results of the high math-anxious model simulation with attention to the numerical sizes impaired. The y-axis shows the simulated mean response time. The x-axis shows the distance between the numerical stimuli. Error bars depict the standard error of the mean.

5.5 Simulation of the HMA Model with Reduced Learning

5.5.1 Procedure

These simulations test the effect of reduced learning with and without reduced attentional control to the numerical size dimension during performance of the symbolic number comparison task. To examine the effect of reduced learning, an LMA model and an HMA model were chosen initially with a specific amount of learning trials such that the error rates for these models were close to the results of experimental studies and were consistent with experimental findings where there were no differences between them. A Mann-Whitney equivalent test was performed on the percentage of errors in the LMA and HMA models to confirm there were no significant differences as the data was not normally distributed. Dietrich, Huber, Moeller, et al. (2015) reported an overall error rate for symbolic number comparison of 3.82%. The LMA model was chosen with 20,000 learning trials and 3.7% errors. The HMA model was chosen with 18,000 learning trials and 3.61% errors. These values were chosen so that there were no differences in the percentage of errors between the LMA and HMA models, yet the number of learning trials were far enough apart for there to be a difference between the models in response times. Subsequently, the chosen LMA and HMA models were simulated with and without reduced attention. To simulate reduced attention, the numerical size dimension of the HMA model was reduced to 95%. This impairment was chosen because it produced a model of mathematics anxiety in previous simulations. Incidentally, these values are the same as the LMA model and the HMA model with reduced learning that simulated the numerical Stroop task. To simulate reduced attention, the numerical size dimension of the HMA model was reduced to 95%. This impairment was chosen because it produced a model of mathematics anxiety in previous simulations.

5.5.2 Results

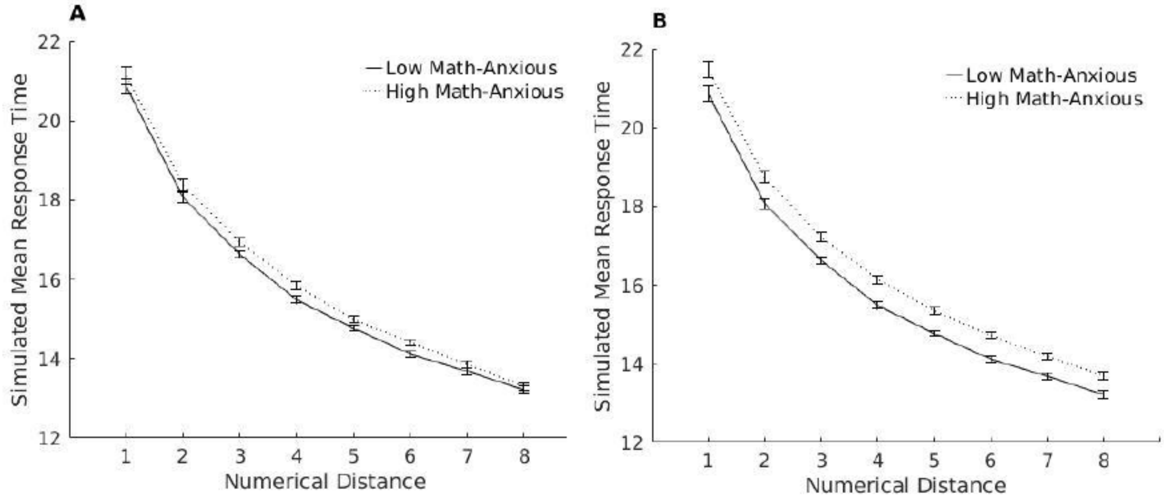
Figure 15 shows the results of the simulations with reduced learning, with and without reduced attentional control.

5.5.2.1 Reduced Learning Without Attention Impaired. The results of the model simulations of reduced training trials on response times when there is no impairment in attention are presented. From Figure 15A you can see that there are no differences in the response times as a result of the reduction in the number of training trials in the HMA model results. The LMA model and the HMA model produced reliable numerical distance effects. The size of the distance effect for the HMA model was similar to the distance effect in the LMA model.

5.5.2.2 Reduced Learning with Reduced Attention. The HMA model was simulated with reduced training trials and attention reduced to the numerical size dimension. The HMA model produced longer overall response times than the LMA model (Figure 15B). The HMA model did not differ from the LMA model on the percentage of errors. Both models produced a reliable numerical distance effect. The size of the distance effect for the HMA model was similar to the distance effect in the LMA model. The size effect was examined for the combination of stimulus pairs “1 2” (which includes “2 1’) and “8 9” (which includes “9 8”) as they were as extreme as possible to compare, as in the study by Núñez-Peña and Suárez-Pellicioni (2014) who found marginal differences between the LMA and HMA groups for the size effect. As in these authors’ study, an interference effect was calculated for the size effect where the response times for small numbers was subtracted from large numbers. Both models produced a reliable size effect. The interference effect for the size effect of the HMA model was similar to the interference effect in the LMA model.

Figure 15

Model Simulations of the Symbolic Number Comparison Task With Reduced Learning



Note. Mean simulated response times for the numerical distance effect for the low and high math-anxious models. Panel A: The low math-anxious model has 20,000 learning trials, the high math-anxious model has 18,000 learning trials. Panel B: The low math-anxious model has 20,000 learning trials, the high math-anxious model has 18,000 learning trials and attention reduced to the numerical stimuli. The y-axis shows the simulated mean response time. The x-axis shows the distance between the numbers. Error bars depict the standard error of the mean.

5.6 Discussion

5.6.1 The Simulation Process

The aim of the symbolic number comparison simulations was to investigate the effect of reductions in attention to numerical stimuli and reductions in learning of simple numerical discrimination on basic numerical skills to see whether they approximate results of individuals with mathematics anxiety. The neural network model architecture that simulated the numerical Stroop task was modified so that the single-digit comparison network for the physical size dimension was turned off. The model was validated to ensure it simulates various empirical effects of the symbolic number comparison task. The network's parameters were then modified to simulate reductions in attention to numerical stimuli and a reduction in training. The results were compared to experimental results on mathematics anxiety. Previous research has generally found that there are no differences in error rates between individuals high and low in mathematics anxiety on basic numerical skills. However, research findings have been mixed between the two groups for overall response times, the distance effect, and the size effect.

5.6.2 Reduced Attentional Control

The model firstly simulated the effect of reducing attentional control to numerical stimuli. This resulted in longer response times for the HMA model than for the LMA model. Furthermore, the more attention was reduced, the longer the response times. Studies on mathematics anxiety have found that performance is affected more on tasks that require more working memory resources, as these resources are specifically disrupted by anxiety (Ashcraft & Faust, 1994; Faust et al., 1996). Recent research involving neuroimaging and ERP during numerical tasks have shown that cognitive processing differs between individuals with and without mathematics anxiety, even though they may achieve similar performance outcomes (see Artemenko et al., 2015). Furthermore, Rubinsten et al. (2015) and Batashvili et al. (2020) found that individuals with mathematics anxiety experienced a threat-related response just by observing simple numerical stimuli. Therefore, the model of reduced attention to numerical stimuli is consistent with the findings that mathematics anxiety may reduce attention to numerical stimuli, even on basic numerical tasks if there is sufficient anxiety.

However, there are limitations of the simulations of reduced attention to numerical stimuli as a model of mathematics anxiety. The LMA and HMA models did not produce any errors as the single-digit numerical size comparison network had been trained to successfully compare the numbers and decide which was numerically largest. Even though error rates are extremely low for the symbolic comparison task and that generally there are no differences between the LMA and HMA groups in the literature for accuracy, this model simulation did not account for error rates. Further, in some studies individuals with and without mathematics anxiety show similar performance outcomes for response times and accuracy, yet they have shown differences in the processing of numerical stimuli as demonstrated by ERP measures (e.g., Pletzer et al., 2015). The current simulations did not account for the condition where the LMA and HMA models produced similar performance outcomes for response times. However, the model was not designed to model all aspects of working memory and it is not a biologically plausible model. Instead, it’s aim was to identify the underlying cognitive factors associated with mathematics anxiety, and the modelling supports the view that mathematics anxiety affects attentional processes.

5.6.3 Reduced Learning

The effect of reduced learning on the symbolic number comparison task was investigated by changing the parameter for the number of learning trials in the models. These simulations reduced the accuracy of trials. As previously described, these simulations were motivated by the suggestion of previous authors (Dietrich, Huber, Moeller, et al., 2015) that individuals with mathematics anxiety may have less trained connections between the numerical representations and the response. The result of these simulations was similar to those in the numerical Stroop simulations. The model simulations showed the standard empirical effect where increased learning improves response times and accuracy. An LMA model and an HMA model with reduced learning were chosen with error rates similar to those of experimental studies. Subsequently, the LMA model and the HMA model with and without reduced attention were simulated and compared. Only the HMA model with reduced learning and reduced attention produced longer response times than the LMA model. As in the numerical Stroop simulations, the model supports the disruption account where if individuals with mathematics anxiety have less trained connections, mathematics anxiety is characterised by a disruption to attentional control that leads to poor performance on numerical tasks. Furthermore, as in the numerical Stroop simulations, the modelling did not investigate the effect of anxiety during learning as it was outside the scope of the current study. Future work could involve investigating this.

5.6.4 The Numerical Distance Effect and the Size Effect

The distance effect and the size effect were simulated with reduced learning and reduced attention to the numerical stimuli. The size effect was examined for number pairs that were as extreme as possible, as in the study by Núñez-Peña and Suárez-Pellicioni (2014). These authors found marginal differences between the LMA and HMA groups for the distance effect and the size effect. Some subsequent findings in the research literature show no differences between groups suggesting that the marginal differences between LMA and HMA groups may be unreliable. In the current modelling, both the LMA and HMA models produced reliable distance and size effects. The HMA model did not produce more pronounced distance or size effects than the LMA model. Dietrich, Huber, Moeller, et al.'s (2015) suggestion that individuals with mathematics anxiety may have less trained connections was motivated by the fact that in some experimental studies individuals high in mathematics anxiety had more pronounced distance effects than individuals low in mathematics anxiety, and the distance effect in symbolic number comparison indexes comparison processes between the numerical representation and the response. Furthermore, Colomé (2019) suggested that the differences in the behavioural studies may also be due to either experimental design or that motivation and attitudes towards mathematics were not controlled for in the studies and could explain the variability between them. The results of the current modelling do not predict that reduced attention due to mathematics anxiety affects the distance effect and the size effect. These findings are consistent with the results of behavioural studies where more pronounced distance and size effects in individuals with mathematics anxiety is not a robust finding. Furthermore, the current modelling suggests that mathematics anxiety may be more effectively modelled by changes in attention than modelling distance or size effects. Alternatively, another interpretation of the results of the current modelling is that there may be other factors related to working memory or attentional control that the model does not account for that could produce more pronounced effects.

The model's performance of the distance effect across learning was also simulated as part of the model validation prior to the HMA model simulations. The results suggest that response times decrease with training across all numerical distances. Furthermore, it is worth mentioning that the model presented here is not a biologically plausible model. Accordingly, the neural network modelling process for learning involves setting the initial connection weights to randomly generated numbers between the numerical representation and the response, presenting numbers to the model with specific frequencies which were taken from a Google survey of those experienced in daily life, and performing a learning algorithm to train the model to decide which number is the largest that updates the connection weights between the numerical representation and the response. These learning processes, such as the starting position of the connection weights between the numerical representation and the response, the type of numerical stimuli exposed to while learning, and the type of learning, may influence response outcomes of the numerical distance effect. It is outside the scope of the current study to examine the effect of differences in the type of numerical experiences and the types of number comparisons that are presented during training on task performance, as the current research focus is on attentional control. Moreover, research suggests that number skills attained before starting school improve education outcomes (Butterworth, 2019), and exposure to numerical activities in everyday situations may support mathematical learning (see Hannula-Sormunen et al., 2019 for a review).

Chapter 6: General Discussion

The aim of the current study was to simulate using neural network modelling the effects of various types of impairments of numerical processing that are hypothesised to be important in mathematics anxiety. Specifically, to simulate the consequences for accuracy and response times on numerical tasks which have been empirically investigated amongst individuals with mathematics anxiety. The disruption account suggests that individuals with mathematics anxiety experience a disruption of working memory resources (in particular the attentional control and inhibitory mechanisms) that leads to poor performance during numerical tasks. Research has shown that mathematics anxiety impacts attentional control. However, the underlying mechanisms remain unclear and deserve further research. The methodology used for this study was neural network modelling. To the author's knowledge, this is the first study simulating mathematics anxiety by neural network modelling. By using this methodology, underlying cognitive factors related to mathematics anxiety were able to be examined in such a way that they could be compared to the outcomes of behavioural experimental conditions. Consequently, neural network modelling has provided a means to test theories on mathematics anxiety and attentional control. It has allowed investigation of specific impairments in addition to performing exploratory work. Two numerical tasks were modelled on one neural network model architecture by making similar modifications to the network that resulted in similar conclusions for both tasks. The first task modelled was the numerical Stroop task as it specifically requires attentional control to inhibit irrelevant information during a numerical task. There are limited mathematics anxiety studies of this task. The second task modelled was the symbolic number comparison task. This task is a basic numerical task and the role of attention during a basic numerical task was investigated. Previously, it has been proposed that tasks that involve more working memory resources are affected more by disruptions to attentional control. However, recent studies have suggested that cognitive function operates differently between individuals with and without mathematics anxiety, regardless of whether performance outcomes of these individuals differ. Furthermore, mathematics anxiety studies of the symbolic number comparison task have mixed results. One suggestion for the differences in these studies could be due to less training of the connection between the numerical representation and the response, possibly due to avoidance of mathematics because of the high levels of anxiety. This idea was proposed based on differences in the distance effect. The distance effect in the symbolic number comparison task has been proposed to be an index of comparison processes between the numerical representation and the response (van Opstal et al., 2008; van Opstal & Verguts, 2011; Verguts et al., 2005). Therefore, the current network architecture simulated both tasks with reduced learning and additionally with and without impaired attentional control.

Both tasks were firstly simulated with impaired attention to the numerical stimuli by reducing the activation of the task demand units representing attention. The models both showed that reduced attention to numerical stimuli resulted in longer response times and no changes in accuracy. Further, the more attention was reduced, the longer the response times. The numerical Stroop task additionally involves inhibiting attention to the irrelevant physical size dimension. When the modelling involved simulating reducing attention to the numerical size dimension it produced results qualitatively similar to those of experimental studies on mathematics anxiety. However, when attention was only reduced to the physical size dimension it did not produce results that matched those of experimental studies on mathematics anxiety. These findings are in support of the attentional control theory which claims that anxiety increases the influence of the stimulus-driven attentional system and decreases the influence of the goal-directed attentional system which results in an inability to inhibit distracting or irrelevant information for the current task. Moreover, they support recent research that numerical information may be perceived as a threat for individuals with mathematics anxiety and can be associated with impaired attentional control, even on basic numerical tasks.

Next, both tasks were simulated with reduced learning with and without reduced attention to the numerical stimuli. For both tasks results were again in agreement. The models with reduced learning and no impairments of attention did not produce a good qualitative fit to empirical data. However, the models with reduced learning and reduced attention to the numerical stimuli provided the best qualitative fit to previous studies in mathematics anxiety and resulted in impaired performance. These results support the view that if individuals have less trained connections possibly due to avoidance of numerical tasks, that mathematics anxiety is further characterised by a disruption of attention to numerical stimuli during processing and recall. Overall, the results provide further support of the disruption account that mathematics anxiety disrupts working memory resources that results in underperformance in mathematical tasks (Carey et al., 2016; Ramirez et al., 2018). Interestingly, the HMA model with attention reduced to the numerical stimuli showed less conflict then the LMA model over the time of an incongruent trial during the numerical Stroop task. Conflict in the model was defined as the amount of energy in the response layer. These results predict an impairment of prefrontal mechanisms due to mathematics anxiety.

The model is not a biologically plausible model and does not provide a simulation of all of the features of the empirical findings. The model simulates reduced attention but it does not account for other factors related to working memory and attention that may be impacting performance due to mathematics anxiety. The modelling provides a first neural network architecture for simulating mathematics anxiety and provides evidence that reduced attention impacts mathematics anxiety. It is the authors hope that it will open a new field of enquiry on the topic using this method as an adjunct to behavioural experiments. Further, the model could be used to investigate other conditions related to numerical processing. For example, studies show that individuals with mathematics learning disability show mathematics anxiety (Carey et al., 2016). Mathematics learning disability is a condition related to difficulty in understanding numbers (Soares et al., 2018).

Mathematics anxiety is highly prevalent and developing interventions to help treat the underlying cognitive factors is important for improved education outcomes and STEM career prospects for students. Current interventions that focus on the cognitive processes underlying mathematics anxiety include expressive writing about emotions before completing mathematical tasks to reduce the intrusive thoughts associated with anxiety and release working memory resources (Park et al., 2014), reappraisal therapy to assist in emotion regulation (Pizzie et al., 2020), and focused relaxation training (Brunyé et al., 2013). Brunyé et al. (2013) examined a behavioural mindfulness intervention that included a focused breathing exercise to train attentional control. The exercise was designed to reduce the feelings of worry and anxiety by assisting students to effortfully control their attention. Individuals with mathematics anxiety experienced an increase in calmness and enhanced performance on an arithmetic test that was performed immediately afterwards. Besides Brunyé et al.'s study, mindfulness meditation has shown potential to reduce anxiety and increase attentional control (see Tang et al., 2015 for a review on the neuroscience of mindfulness meditation). Moreover, several authors have suggested the importance of specifically tailoring interventions to the individual to reduce their mathematics anxiety and increase performance (Moustafa et al., 2021; Ramirez et al., 2018; Skagerlund et al., 2019). A novel individualised cognitive behavioural therapy (i-CBT) has been suggested by Moustafa et al. (2021) where students are initially assessed to determine the factors that are underlying their mathematics anxiety. Subsequently, the students will have targeted sessions based on their assessment. These could include the above-mentioned interventions.

6.1 Conclusion

This study created neural network models to simulate the effect of impaired attentional control on mathematics anxiety. The models were consistent with the disruption account of mathematics anxiety as it relates to the attentional control theory, finding that mathematics anxiety is characterised by impaired attentional control on mathematical tasks that affects performance. Further investigations on the underlying factors related to impaired attention due to mathematics anxiety are recommended.

References

Alexander, L., & Martray, C. (1989). The development of an abbreviated version of the mathematics anxiety rating scale. *Measurement and Evaluation in Counseling and Development*, *22*(3), 143–150. https://doi.org/10.1080/07481756.1989.12022923

Alloway, T. P., Banner, G. E., & Smith, P. (2010). Working memory and cognitive styles in adolescents' attainment. *British Journal of Educational Psychology*, *80*(4), 567–581. https://doi.org/10.1348/000709910X494566

Amos, A. (2000). A computational model of information processing in the frontal cortex and basal ganglia. *Journal of Cognitive Neuroscience*, *12*(3), 505–519. https://doi.org/10.1162/089892900562174

Artemenko, C., Daroczy, G., & Nuerk, H.-C. (2015). Neural correlates of math anxiety – an overview and implications. *Frontiers in Psychology*, *6*, Article 1333. https://doi.org/10.3389/fpsyg.2015.01333

Ashcraft, M. H., & Faust, M. W. (1994). Mathematics anxiety and mental arithmetic performance: An exploratory investigation. *Cognition & Emotion*, *8*(2), 97–125. https://doi.org/10.1080/02699939408408931

Ashcraft, M. H., & Krause, J. A. (2007). Working memory, math performance, and math anxiety. *Psychonomic Bulletin & Review*, *14*(2), 243–248. https://doi.org/10.3758/BF03194059

Ashcraft, M. H., Krause, J. A., & Hopko, D. R. (2007). Is math anxiety a mathematical learning disability? In D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 329–348). Paul H. Brookes Publishing Co.

Ashkenazi, S. (2018). Intentional and automatic processing of numerical information in mathematical anxiety: Testing the influence of emotional priming. *Cognition and Emotion*, *32*(8), 1700–1707. https://doi.org/10.1080/02699931.2018.1435504

Batashvili, M., Staples, P., Baker, I. S., & Sheffield, D. (2020). The neurophysiological relationship between number anxiety and the EEG gamma-band. *Journal of Cognitive Psychology*, *32*(5–6), 580–585. https://doi.org/10.1080/20445911.2020.1778006

Beilock, S. L. (2008). Math performance in stressful situations. *Current Directions in Psychological Science*, *17*(5), 339–343. https://doi.org/10.1111/j.1467-8721.2008.00602.x

Bishop, S. J. (2009). Trait anxiety and impoverished prefrontal control of attention. *Nature Neuroscience*. *12*(1), 92–99. https://doi.org/10.1038/nn.2242

Botvinick, M. M., Braver, T. S., Barch, D. M., Carter, C. S., & Cohen, J. D. (2001). Conflict monitoring and cognitive control. *Psychological Review*, *108*(3), 624–652. https://doi.org/10.1037/0033-295x.108.3.624

Braham, E. J., & Libertus, M. E. (2018). When approximate number acuity predicts math performance: The moderating role of math anxiety. *PLoS ONE*, *13*(5), Article e0195696. https://doi.org/10.1371/journal.pone.0195696

Brankaer, C., Ghesquière, P., & de Smedt, B. (2017). Symbolic magnitude processing in elementary school children: A group administered paper-and-pencil measure (SYMP Test). *Behav Res*, *49*, 1361–1373. https://doi.org/10.3758/s13428-016-0792-3

Braver, T. S. (2012). The variable nature of cognitive control: A dual mechanisms framework. *Trends in Cognitive Sciences*, *16*(2), 106–113. https://doi.org/10.1016/j.tics.2011.12.010

Braver, T. S., Paxton, J. L., Locke, H. S., Barch, D. M., & Smith, E. E. (2009). Flexible neural mechanisms of cognitive control within human prefrontal cortex. *Proceedings of* *the National Academy of Sciences*, *106*(18), 7351–7356. https://doi.org/10.1073/pnas.0808187106

Brunyé, T. T., Mahoney, C. R., Giles, G. E., Rapp, D. N., Taylor, H. A., & Kanarek, R. B. (2013). Learning to relax : Evaluating four brief interventions for overcoming the negative emotions accompanying math anxiety. *Learning and Individual Differences*, *27*, 1–7. https://doi.org/10.1016/j.lindif.2013.06.008

Butterworth, B. (2019). Forward. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), *International handbook of mathematical learning difficulties: From the laboratory to the classroom* (pp. vii–ix). Springer. https://doi.org/10.1007/978-3-319-97148-3

Carey, E., Hill, F., Devine, A., & Szűcs, D. (2016). The chicken or the egg? The direction of the relationship between mathematics anxiety and mathematics performance. *Frontiers in Psychology*, *6*, Article 1987. https://doi.org/10.3389/fpsyg.2015.01987

Chakravarthy, V. S., & Moustafa, A. A. (2018). *Computational Neuroscience Models of the Basal Ganglia*. Springer Singapore. https://doi.org/10.1007/978-981-10-8494-2

Chang, H., Sprute, L., Maloney, E. A., Beilock, S. L., & Berman, M. G. (2017). Simple arithmetic: Not so simple for highly math anxious individuals. *Social Cognitive and Affective Neuroscience*, *12*(12), 1940–1949. https://doi.org/10.1093/scan/nsx121

Chen, Q., & Verguts, T. (2010). Beyond the mental number line: A neural network model of number-space interactions. *Cognitive Psychology*, *60*(3), 218–240. https://doi.org/10.1016/j.cogpsych.2010.01.001

Cohen, J. D., Dunbar, K., & McClelland, J. L. (1990). On the control of automatic processes: A parallel distributed processing account of the Stroop effect. *Psychological Review*, *97*(3), 332–361. https://doi.org/10.1037/0033-295X.97.3.332

Colomé, À. (2019). Representation of numerical magnitude in math-anxious individuals. *Quarterly Journal of Experimental Psychology*, *72*(3), 424–435. https://doi.org/10.1177/1747021817752094

Daker, R. J., Gattas, S. U., Sokolowski, H. M., Green, A. E., & Lyons, I. M. (2021). First-year students' math anxiety predicts STEM avoidance and underperformance throughout university, independently of math ability. *Npj Science of Learning*, *6*, Article 17. https://doi.org/10.1038/s41539-021-00095-7

de Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, *103*(4), 469–479. https://doi.org/10.1016/j.jecp.2009.01.010

Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, *44*(1–2), 1–42. https://doi.org/10.1016/0010-0277(92)90049-N

Dehaene, S. (2001). Précis of the number sense. *Mind & Language*, *16*(1), 16–36. https://doi.org/10.1111/1468-0017.00154

Dehaene, S., Dehaene-Lambertz, G., & Cohen, L. (1998). Abstract representations of numbers in the animal and human brain. *Trends in Neurosciences*, *21*(8), 355–361. https://doi.org/10.1016/S0166-2236(98)01263-6

Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digit number comparison. *Journal of Experimental Psychology: Human Perception and Performance*, *16*(3), 626–641. https://doi.org/10.1037/0096-1523.16.3.626

Derakshan, N., & Eysenck, M. W. (2009). Anxiety, processing efficiency, and cognitive performance: New developments from attentional control theory. *European Psychologist*, *14*(2), 168–176. https://doi.org/10.1027/1016-9040.14.2.168

Dietrich, J. F., Huber, S., Moeller, K., & Klein, E. (2015). The influence of math anxiety on symbolic and non-symbolic magnitude processing. *Frontiers in Psychology, 6*, Article 1621. https://doi.org/10.3389/fpsyg.2015.01621

Dietrich, J. F., Huber, S., & Nuerk, H.-C. (2015). Methodological aspects to be considered when measuring the approximate number system (ANS) - a research review. *Frontiers in Psychology, 6,* Article 295. https://doi.org/10.3389/fpsyg.2015.00295

Douglas, H. P., & LeFevre, J.-A. (2017). Exploring the influence of basic cognitive skills on the relation between math performance and math anxiety. *Journal of Numerical Cognition*, *3*(3), 642–666. https://doi.org/10.5964/jnc.v3i3.113

Eysenck, M. W., & Calvo, M. G. (1992). Anxiety and performance: The processing efficiency theory. *Cognition and Emotion*, *6*(6), 409–434. https://doi.org/10.1080/02699939208409696

Eysenck, M. W., Derakshan, N., Santos, R., & Calvo, M. G. (2007). Anxiety and cognitive performance: Attentional control theory. *Emotion*, *7*(2), 336–353. https://doi.org/10.1037/1528-3542.7.2.336

Fales, C. L., Barch, D. M., Rundle, M. M., Mintun, M. A., Snyder, A. Z., Cohen, J. D., Mathews, J., & Sheline, Y. I. (2008). Altered emotional interference processing in affective and cognitive-control brain circuitry in major depression. *Biological Psychiatry*, *63*(4), 377–384. https://doi.org/10.1016/j.biopsych.2007.06.012

Faust, M. W., Ashcraft, M. H., & Fleck, D. E. (1996). Mathematics anxiety effects in simple and complex addition. *Mathematical Cognition*, *2*(1), 25–62. https://doi.org/10.1080/135467996387534

Feigenson, L., Dehaene, S., & Spelke, E. (2004). Core systems of number. *Trends in Cognitive Sciences*, *8*(7), 307–314. https://doi.org/10.1016/j.tics.2004.05.002

Friedman, N. P., & Miyake, A. (2004). The relations among inhibition and interference control functions: A latent-variable analysis. *Journal of Experimental Psychology: General*, *133*(1), 101–135. https://doi.org/10.1037/0096-3445.133.1.101

Gallistel, C. R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, *44*(1–2), 43–74. https://doi.org/10.1016/0010-0277(92)90050-r

Girelli, L., Lucangeli, D., & Butterworth, B. (2000). The development of automaticity in accessing number magnitude. *Journal of Experimental Child Psychology*, *76*(2), 104–122. https://doi.org/10.1006/jecp.2000.2564

Hannula-Sormunen, M. M., McMullen, J., & Lehtinen, E. (2019). Everyday context and mathematical learning: On the role of spontaneous mathematical focusing tendencies in the development of numeracy. In A. Fritz, V. G. Haase, & P. Räsänen (Eds.), *International handbook of mathematical learning difficulties: From the laboratory to the classroom* (pp. 25–42). Springer. https://doi.org/10.1007/978-3-319-97148-3_3

Hartwright, C. E., Looi, C. Y., Sella, F., Inuggi, A., Santos, F. H., González-Salinas, C., Santos, J. M. G., Kadosh, R. C., & Fuentes, L. J. (2018). The neurocognitive architecture of individual differences in math anxiety in typical children. *Scientific Reports*, *8*(1), Article 8500. https://doi.org/10.1038/s41598-018-26912-5

Heitz, R. P. (2014). The speed-accuracy tradeoff: History, physiology, methodology, and behavior. *Frontiers in Neuroscience*, *8*, Article 150. https://doi.org/10.3389/fnins.2014.00150

Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, *21*(1), 33–46. https://doi.org/https://doi.org/10.5951/jresematheduc.21.1.0033

Henik, A., & Tzelgov, J. (1982). Is three greater than five: The relation between physical and semantic size in comparison tasks. *Memory & Cognition*, *10*(4), 389–395. https://doi.org/10.3758/BF03202431

Hochman, S. H. (1967). The effects of stress on Stroop color-word performance. *Psychonomic Science*, *9*, 475–476. https://doi.org/10.3758/BF03330904

Hochman, S. H. (1969). Stress and response competition in children's color-word performance. *Perceptual and Motor Skills, 28*(1), 115–118. https://doi.org/https://doi.org/10.2466/pms.1969.28.1.115

Holloway, I. D., & Ansari, D. (2008). Domain-specific and domain-general changes in children's development of number comparison. *Developmental Science, 11*(5), 644–649. https://doi.org/10.1111/j.1467-7687.2008.00712.x

Hopfield, J. J. (1982). Neural networks and physical systems with emergent collective computational abilities. *Proc. Natl. Acad. Sci. USA*, *79*(8), 2554–2558. https://doi.org/10.1073/pnas.79.8.2554

Hopko, D. R., Ashcraft, M. H., Gute, J., Ruggiero, K. J., & Lewis, C. (1998). Mathematics anxiety and working memory: Support for the existence of a deficient inhibition mechanism. *Journal of Anxiety Disorders*, *12*(4), 343–355. https://doi.org/10.1016/S0887-6185(98)00019-X

Hopko, D. R., McNeil, D. W., Gleason, P. J., & Rabalais, A. E. (2002). The emotional Stroop paradigm: Performance as a function of stimulus properties and self-reported mathematics anxiety. *Cognitive Therapy and Research*, *26*(2), 157–166. https://doi.org/10.1023/A:1014578218041

Huber, S., Klein, E., Willmes, K., Nuerk, H.-C., & Moeller, K. (2014). Decimal fraction representations are not distinct from natural number representations - evidence from a combined eye-tracking and computational modeling approach. *Frontiers in Human Neuroscience*, *8*, Article 172. https://doi.org/10.3389/fnhum.2014.00172

Huber, S., Moeller, K., Nuerk, H.-C., Macizo, P., Herrera, A., & Willmes, K. (2013). Cognitive control in number processing-a computational model. In R. West & T. Stewart (Eds.), *Proceedings of the 12th International Conference on Cognitive Modeling* (pp. 185–190). Ottawa, Canada: Carleton University. https://doi.org/10.13140/2.1.4137.4729

Huber, S., Moeller, K., Nuerk, H.-C., & Willmes, K. (2013). A computational modeling approach on three-digit number processing. *Topics in Cognitive Science*, *5*(2), 317–334. https://doi.org/10.1111/tops.12016

Huber, S., Nuerk, H.-C., Willmes, K., & Moeller, K. (2016). A General Model Framework for Multisymbol Number Comparison. *Psychological Review*, *123*(6), 667–695. https://doi.org/10.1037/rev0000040

Hutchison, K. A. (2011). The interactive effects of listwide control, item-based control, and working memory capacity on Stroop performance. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, *37*(4), 851–860. https://doi.org/10.1037/a0023437

Justicia-Galiano, M.-J., Pelegrina, S., Lechuga, M., Gutiérrez-Palma, N., Martín-Puga, E.-M., & Lendínez, C. (2016). Math anxiety and its relationship to inhibitory abilities and perceived emotional intelligence. *Anales de Psicología*, *32*(1), 125–131. https://doi.org/10.6018/analesps.32.1.194891

Kalanthroff, E., Henik, A., Derakshan, N., & Usher, M. (2016). Anxiety, emotional distraction, and attentional control in the Stroop task. *Emotion*, *16*(3), 293–300. https://doi.org/10.1037/emo0000129

Klados, M. A., Simos, P., Micheloyannis, S., Margulies, D., & Bamidis, P. D. (2015). ERP measures of math anxiety: How math anxiety affects working memory and mental calculation tasks? *Frontiers in Behavioral Neuroscience*, *9*, Article 282. https://doi.org/10.3389/fnbeh.2015.00282

Landerl, K. (2013). Development of numerical processing in children with typical and dyscalculic arithmetic skills-a longitudinal study. *Frontiers in Psychology*, *4*, Article 459. https://doi.org/10.3389/fpsyg.2013.00459

Landerl, K., & Kölle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. *Journal of Experimental Child Psychology*, *103*(4), 546–565. https://doi.org/10.1016/j.jecp.2008.12.006

Levy, H. E., Fares, L., & Rubinsten, O. (2021). Math anxiety affects females' vocational interests. *Journal of Experimental Child Psychology*, *210*, Article 105214. https://doi.org/10.1016/j.jecp.2021.105214

Liu, J., Li, J., Peng, W., Feng, M., & Luo, Y. (2019). EEG correlates of math anxiety during arithmetic problem solving: Implication for attention deficits. *Neuroscience Letters*, *703*, 191–197. https://doi.org/10.1016/j.neulet.2019.03.047

Lukasik, K. M., Waris, O., Soveri, A., Lehtonen, M., & Laine, M. (2019). The relationship of anxiety and stress with working memory performance in a large non-depressed sample. *Frontiers in Psychology*, *10*, Article 4. https://doi.org/10.3389/fpsyg.2019.00004

Lyons, I. M., & Beilock, S. L. (2012). Mathematics anxiety: Separating the math from the anxiety. *Cerebral Cortex*, *22*(9), 2102–2110. https://doi.org/10.1093/cercor/bhr289

Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education*, *30*(5), 520–540. https://doi.org/10.2307/749772

MacLeod, C. M. (1991). Half a century of research on the Stroop effect: An integrative review. *Psychological Bulletin*, *109*(2), 163–203. https://doi.org/10.1037/0033-2909.109.2.163

Maloney, E. A., Ansari, D., & Fugelsang, J. A. (2011). The effect of mathematics anxiety on the processing of numerical magnitude. *Quarterly Journal of Experimental Psychology*, *64*(1), 10–16. https://doi.org/10.1080/17470218.2010.533278

Maloney, E. A., Risko, E. F., Ansari, D., & Fugelsang, J. (2010). Mathematics anxiety affects counting but not subitizing during visual enumeration. *Cognition*, *114*(2), 293–297. https://doi.org/10.1016/j.cognition.2009.09.013

Mammarella, I. C., Caviola, S., Giofrè, D., & Borella, E. (2018). Separating math from anxiety: The role of inhibitory mechanisms. *Applied Neuropsychology: Child*, *7*(4), 342–353. https://doi.org/10.1080/21622965.2017.1341836

Meyer, M. L., Salimpoor, V. N., Wu, S. S., Geary, D. C., & Menon, V. (2010). Differential contribution of specific working memory components to mathematics achievement in 2nd and 3rd graders. *Learning and Individual Differences*, *20*(2), 101–109. https://doi.org/10.1016/j.lindif.2009.08.004

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, *30*(1), 65–88. https://doi.org/https://doi.org/10.2307/749630

Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: A latent variable analysis. *Cognitive Psychology*, *41*(1), 49–100. https://doi.org/10.1006/cogp.1999.0734

Moeller, K., Huber, S., Nuerk, H.-C., & Willmes, K. (2011). Two-digit number processing: Holistic, decomposed or hybrid? A computational modelling approach. *Psychological Research*, *75*(4), 290–306. https://doi.org/10.1007/s00426-010-0307-2

Moore, A. M., & Ashcraft, M. H. (2015). Children's mathematical performance: Five cognitive tasks across five grades. *Journal of Experimental Child Psychology*, *135*, 1–24. https://doi.org/10.1016/j.jecp.2015.02.003

Moustafa, A. A. (Ed.). (2017). *Computational models of brain and behavior*. Wiley Blackwell. https://doi.org/10.1002/9781119159193

Moustafa, A. A., Al-Emadi, A. A., & Megreya, A. M. (2021). The need to develop an individualized intervention for mathematics anxiety. *Frontiers in Psychology*, *12*, Article 723289. https://doi.org/10.3389/fpsyg.2021.723289

Moustafa, A. A., & Gluck, M. A. (2011a). A neurocomputational model of dopamine and prefrontal-striatal interactions during multicue category learning by Parkinson patients. *Journal of Cognitive Neuroscience*, *23*(1), 151–167. https://doi.org/10.1162/jocn.2010.21420

Moustafa, A. A., & Gluck, M. A. (2011b). Computational cognitive models of prefrontal-striatal-hippocampal interactions in Parkinson's disease and schizophrenia. *Neural Networks*, *24*(6), 575–591. https://doi.org/10.1016/j.neunet.2011.02.006

Moustafa, A. A., Misiak, B., & Frydecka, D. (2017). Neurocomputational models of schizophrenia. In A. A. Moustafa (Ed.), *Computational models of brain and behavior* (pp. 73–84). Wiley Blackwell. https://doi.org/10.1002/9781119159193.ch6

Moustafa, A. A., Myers, C. E., & Gluck, M. A. (2009). A neurocomputational model of classical conditioning phenomena: A putative role for the hippocampal region in associative learning. *Brain Research*, *1276*, 180–195. https://doi.org/10.1016/j.brainres.2009.04.020

Moustafa, A. A., Porter, A., & Megreya, A. M. (2020). Mathematics anxiety and cognition: An integrated neural network model. *Rev. Neurosci.*, *31*(3), 287–296. https://doi.org/10.1515/revneuro-2019-0068

Moyer, R. S., & Landauer, T. K. (1967). Time required for judgements of numerical inequality. *Nature*, *215*, 1519–1520. https://doi.org/10.1038/2151519a0

Nieder, A., Freedman, D. J., & Miller, E. K. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science*, *297*(5587), 1708–1711. https://doi.org/10.1126/science.1072493

Nieder, A., & Miller, E. K. (2003). Coding of cognitive magnitude: Compressed scaling of numerical information in the primate prefrontal cortex. *Neuron*, *37*(1), 149–157. https://doi.org/10.1016/s0896-6273(02)01144-3

Nuerk, H.-C., Weger, U., & Willmes, K. (2001). Decade breaks in the mental number line? Putting the tens and units back in different bins. *Cognition*, *82*(1), B25–B33. https://doi.org/10.1016/S0010-0277(01)00142-1

Núñez-Peña, M. I., & Suárez-Pellicioni, M. (2014). Less precise representation of numerical magnitude in high math-anxious individuals: An ERP study of the size and distance effects. *Biological Psychology*, *103*, 176–183. https://doi.org/10.1016/j.biopsycho.2014.09.004

Organisation for Economic Co-operation and Development. (2013). *PISA 2012 results: Ready to learn: Students' engagement, drive and self-beliefs (Volume III): Vol. III*. OECD Publishing. https://doi.org/10.1787/9789264201170-en

Office of the Chief Scientist. (2015). *Transforming STEM teaching in Australian primary schools: Everybody's business*. https://www.chiefscientist.gov.au/sites/default/files/Transforming-STEM-teaching_FINAL.pdf

O'Reilly, R. C., & Munakata, Y. (2006). Computational neuroscience: From biology to cognition. In A. Sell, E. H. Hagen, L. Cosmides, & J. Tooby (Eds.), *Encyclopedia of Cognitive Science* (pp. 1–10). Wiley. https://doi.org/10.1002/0470018860.s00051

Pallak, M. S., Pittman, T. S., Heller, J. F., & Munson, P. (1975). The effect of arousal on Stroop color-word task performance. *Bulletin of the Psychonomic Society*, *6*(3), 248–250. https://doi.org/10.3758/BF03336652

Park, D., Ramirez, G., & Beilock, S. L. (2014). The role of expressive writing in math anxiety. *Journal of Experimental Psychology: Applied*, *20*(2), 103–111. https://doi.org/10.1037/xap0000013

Passolunghi, M. C., Caviola, S., de Agostini, R., Perin, C., & Mammarella, I. C. (2016). Mathematics anxiety, working memory, and mathematics performance in secondary-school children. *Frontiers in Psychology*, *7*, Article 42. https://doi.org/10.3389/fpsyg.2016.00042

Pavese, A., & Umiltà, C. (1998). Symbolic distance between numerosity and identity modulates Stroop interference. *Journal of Experimental Psychology: Human Perception and Performance*, *24*(5), 1535–1545. https://doi.org/10.1037/0096-1523.24.5.1535

Piazza, M., Mechelli, A., Price, C. J., & Butterworth, B. (2006). Exact and approximate judgements of visual and auditory numerosity: An fMRI study. *Brain Research*, *1106*(1), 177–188. https://doi.org/10.1016/j.brainres.2006.05.104

Piazza, M., Pinel, P., le Bihan, D., & Dehaene, S. (2007). A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron*, *53*(2), 293–305. https://doi.org/10.1016/j.neuron.2006.11.022

Pizzie, R. G., & Kraemer, D. J. M. (2017). Avoiding math on a rapid timescale: Emotional responsivity and anxious attention in math anxiety. *Brain and Cognition*, *118*, 100–107. https://doi.org/10.1016/j.bandc.2017.08.004

Pizzie, R. G., McDermott, C. L., Salem, T. G., & Kraemer, D. J. M. (2020). Neural evidence for cognitive reappraisal as a strategy to alleviate the effects of math anxiety. *Social*

Cognitive and Affective Neuroscience, *15*(12), 1271–1287. https://doi.org/10.1093/scan/nsaa161

Pletzer, B., Kronbichler, M., Nuerk, H.-C., & Kerschbaum, H. H. (2015). Mathematics anxiety reduces default mode network deactivation in response to numerical tasks. *Frontiers in Human Neuroscience*, *9*, Article 202. https://doi.org/10.3389/fnhum.2015.00202

Raichle, M. E. (2015). The brain's default mode network. *Annual Review of Neuroscience*, *38*, 433–447. https://doi.org/10.1146/annurev-neuro-071013-014030

Ramirez, G., Gunderson, E. A., Levine, S. C., & Beilock, S. L. (2013). Math anxiety, working memory, and math achievement in early elementary school. *Journal of Cognition and Development*, *14*(2), 187–202. https://doi.org/10.1080/15248372.2012.664593

Ramirez, G., Shaw, S. T., & Maloney, E. A. (2018). Math anxiety: Past research, promising interventions, and a new interpretation framework. *Educational Psychologist*, *53*(3), 145–164. https://doi.org/10.1080/00461520.2018.1447384

Reeve, R., Reynolds, F., Humberstone, J., & Butterworth, B. (2012). Stability and change in markers of core numerical competencies. *Journal of Experimental Psychology: General*, *141*(4), 649–666. https://doi.org/10.1037/a0027520

Richards, A., Richards, L. C., & McGeeney, A. (2000). Anxiety-related Stroop interference in adolescents. *The Journal of General Psychology*, *127*(3), 327–333. https://doi.org/10.1080/00221300009598587

Rubinsten, O., Eidlin, H., Wohl, H., & Akibli, O. (2015). Attentional bias in math anxiety. *Frontiers in Psychology*, *6*, Article 1539. https://doi.org/10.3389/fpsyg.2015.01539

Santens, S., & Verguts, T. (2011). The size congruity effect: Is bigger always more? *Cognition*, *118*(1), 94–110. https://doi.org/10.1016/j.cognition.2010.10.014

Sasanguie, D., Lyons, I. M., de Smedt, B., & Reynvoet, B. (2017). Unpacking symbolic number comparison and its relation with arithmetic in adults. *Cognition*, *165*, 26–38. https://doi.org/10.1016/j.cognition.2017.04.007

Schmidt, J. R. (2019). Evidence against conflict monitoring and adaptation: An updated review. *Psychonomic Bulletin & Review*, *26*(3), 753–771. https://doi.org/10.3758/s13423-018-1520-z

Schneider, M., Beeres, K., Coban, L., Merz, S., Schmidt, S. S., Stricker, J., & de Smedt, B. (2017). Associations of non-symbolic and symbolic numerical magnitude processing with mathematical competence: A meta-analysis. *Developmental Science, 20*(3), 1–16. https://doi.org/10.1111/desc.12372

Schwarz, W., & Heinze, H.-J. (1998). On the interaction of numerical and size information in digit comparison: A behavioral and event-related potential study. *Neuropsychologia*, *36*(11), 1167–1179. https://doi.org/10.1016/S0028-3932(98)00001-3

Schwarz, W., & Ischebeck, A. (2003). On the relative speed account of number-size interference in comparative judgments of numerals. *Journal of Experimental Psychology: Human Perception and Performance*, *29*(3), 507–522. https://doi.org/10.1037/0096-1523.29.3.507

Schwenk, C., Sasanguie, D., Kuhn, J. T., Kempe, S., Doebler, P., & Holling, H. (2017). (Non-)symbolic magnitude processing in children with mathematical difficulties: A meta-analysis. *Research in Developmental Disabilities*, *64*, 152–167. https://doi.org/10.1016/j.ridd.2017.03.003

Sekuler, R., & Mierkiewicz, D. (1977). Children's judgments of numerical inequality. *Child Development*, *48*(2), 630–633. https://doi.org/https://doi.org/10.2307/1128664

Skagerlund, K., Östergren, R., Västfjäll, D., & Träff, U. (2019). How does mathematics anxiety impair mathematical abilities? Investigating the link between math anxiety, working memory, and number processing. *PLoS ONE*, *14*(1), Article e0211283. https://doi.org/10.1371/journal.pone.0211283

Soares, N., Evans, T., & Patel, D. R. (2018). Specific learning disability in mathematics: A comprehensive review. *Transl Pediatr*, *7*(1), 48–62. https://doi.org/10.21037/tp.2017.08.03

Stroop, J. R. (1935). Studies of interference in serial verbal reactions. *Journal of Experimental Psychology*, *18*(6), 643–662. https://doi.org/10.1037/h0054651

Suárez-Pellicioni, M., Núñez-Peña, M. I., & Colomé, À. (2014). Reactive recruitment of attentional control in math anxiety: An ERP study of numeric conflict monitoring and adaptation. *PLoS ONE*, *9*(6), Article e99579. https://doi.org/10.1371/journal.pone.0099579

Szũcs, D., & Soltész, F. (2007). Event-related potentials dissociate facilitation and interference effects in the numerical Stroop paradigm. *Neuropsychologia*, *45*(14), 3190–3202. https://doi.org/10.1016/j.neuropsychologia.2007.06.013

Szũcs, D., Soltész, F., Jármi, É., & Csépe, V. (2007). The speed of magnitude processing and executive functions in controlled and automatic number comparison in children: An electro-encephalography study. *Behavioral and Brain Functions*, *3*, Article 23. https://doi.org/10.1186/1744-9081-3-23

Tang, Y., Hölzel, B. K., & Posner, M. I. (2015). The neuroscience of mindfulness meditation. *Nature Reviews Neuroscience*, *16*, 213–225. https://doi.org/10.1038/nrn3916

van Opstal, F., Gevers, W., de Moor, W., & Verguts, T. (2008). Dissecting the symbolic distance effect: Comparison and priming effects in numerical and nonnumerical orders. *Psychonomic Bulletin & Review*, *15*(2), 419–425. https://doi.org/10.3758/PBR.15.2.419

van Opstal, F., & Verguts, T. (2011). The origins of the numerical distance effect: The same–different task. *Journal of Cognitive Psychology*, *23*(1), 112–120. https://doi.org/10.1080/20445911.2011.466796

Verguts, T., & de Moor, W. (2005). Two-digit comparison: Decomposed, holistic, or hybrid? *Experimental Psychology*, *52*(3), 195–200. https://doi.org/10.1027/1618-3169.52.3.195

Verguts, T., & Fias, W. (2004). Representation of number in animals and humans: A neural model. *Journal of Cognitive Neuroscience*, *16*(9), 1493–1504. https://doi.org/10.1162/0898929042568497

Verguts, T., & Fias, W. (2006). Lexical and syntactic structures in a connectionist model of reading multi-digit numbers. *Connection Science*, *18*(3), 265–283. https://doi.org/10.1080/09540090600639396

Verguts, T., & Fias, W. (2008). Symbolic and nonsymbolic pathways of number processing. *Philosophical Psychology*, *21*(4), 539–554. https://doi.org/10.1080/09515080802285545

Verguts, T., Fias, W., & Stevens, M. (2005). A model of exact small-number representation. *Psychonomic Bulletin and Review*, *12*(1), 66–80. https://doi.org/10.3758/BF03196349

Verguts, T., & Notebaert, W. (2008). Hebbian learning of cognitive control: Dealing with specific and nonspecific adaptation. *Psychological Review*, *115*(2), 518–525. https://doi.org/10.1037/0033-295X.115.2.518

Vogel, S. E., Remark, A., & Ansari, D. (2015). Differential processing of symbolic numerical magnitude and order in first-grade children. *Journal of Experimental Child Psychology*, *129*, 26–39. https://doi.org/10.1016/j.jecp.2014.07.010

Widrow, B., & Hoff, M. E. (1960). Adaptive switching circuits. *Institute of Radio Engineers, Western Electronic Show and Convention Record, Part 4*, 96–104.

Yeung, N., & Cohen, J. D. (2006). The impact of cognitive deficits on conflict monitoring: Predictable dissociations between the error-related negativity and N2. *Psychological Science*, *17*(2), 164–171. https://doi.org/10.1111/j.1467-9280.2006.01680.x

Zhang, J., Zhao, N., & Kong, Q. P. (2019). The relationship between math anxiety and math performance: A meta-analytic investigation. *Frontiers in Psychology*, *10*, Article 1613. https://doi.org/10.3389/fpsyg.2019.01613

Zorzi, M., & Butterworth, B. (1999). A computational model of number comparison. In M. Hahn & S. C. Stoness (Eds.), *Proceedings of the Twenty First Annual Conference of the Cognitive Science Society* (pp. 778–783). Lawrence Erlbaum. https://doi.org/10.4324/9781410603494